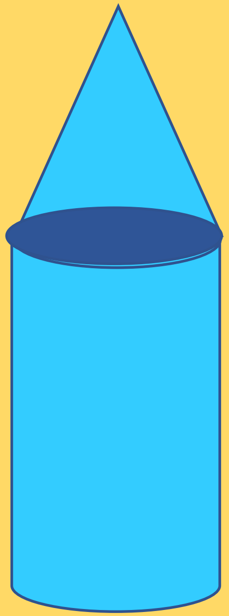
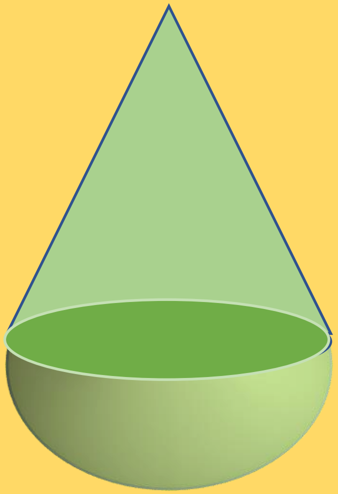
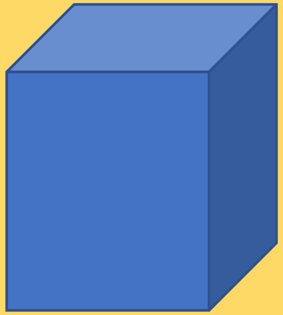
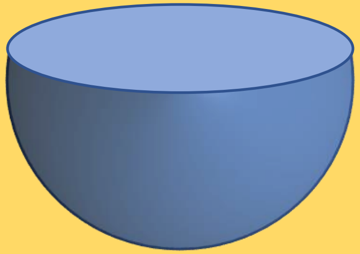
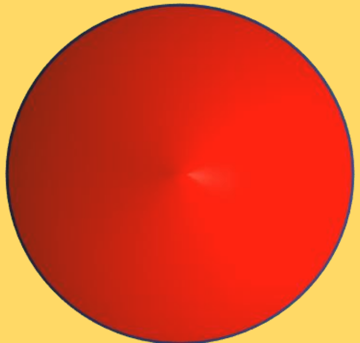
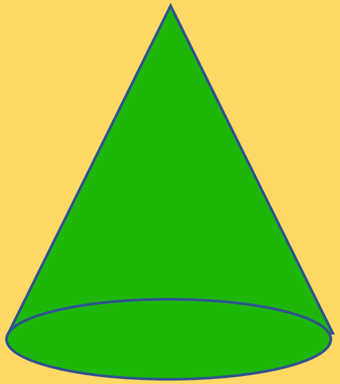
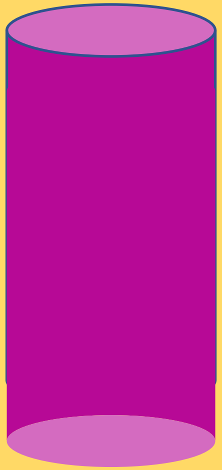




MENSURATION

Pdf version with video links

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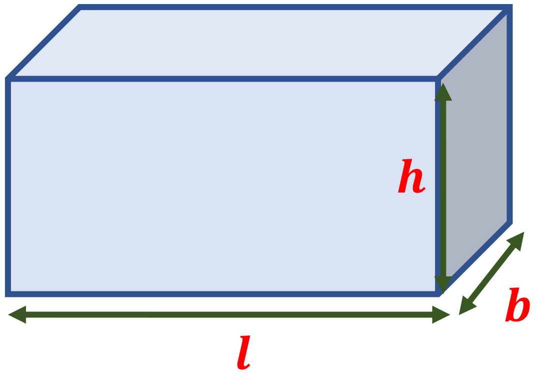
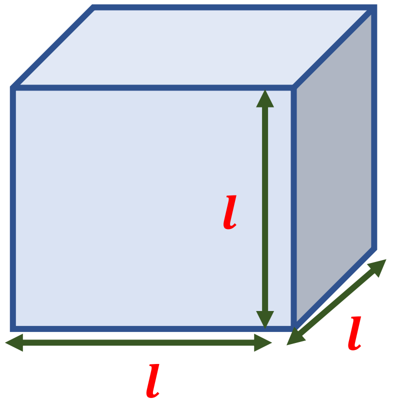


Cuboid

Lateral surface area $= 2h(l + b)$

Total surface area $= 2(lh + bh + lb)$

Volume $= lbh$

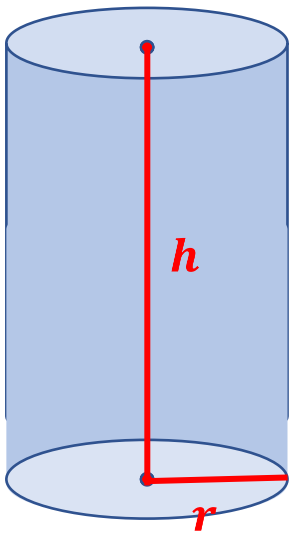


Cube

Lateral surface area $= 4l^2$

Total surface area $= 6l^2$

Volume $= l^3$

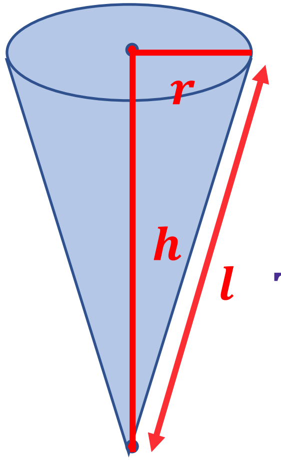


Cylinder

Lateral surface area $= 2\pi rh$

Total surface area $= 2\pi r(h + r)$

Volume $= \pi r^2 h$

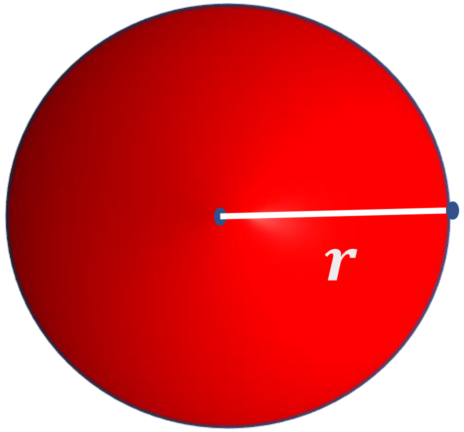


Cone

Lateral surface area $= \pi rl$

Total surface area $= \pi r(l + r)$

Volume $= \dfrac{1}{3}\pi r^2 h$



Sphere

surface area $= 4\pi r^2$

Volume $= \dfrac{4}{3}\pi r^3$

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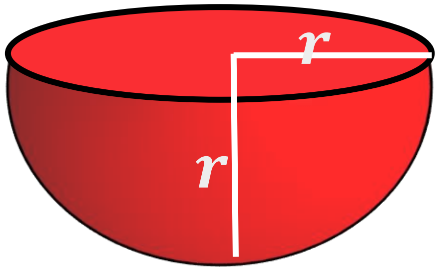
Hemi sphere



Lateral surface area $= 2\pi r^2$

Total surface area $= 3\pi r^2$

Volume $= \dfrac{2}{3}\pi r^3$

EXERCISE – 10.1

1. A Joker's cap is in the form of right circular cone whose base radius is 7 cm and height is 24 cm. Find the area of the sheet required to make 10 such caps.

Sol : $Radius\ of\ the\ base\ of\ cap\ is\ r = 7\ cm$

$Height\ of\ the\ cap\ is\ h = 24\ cm$

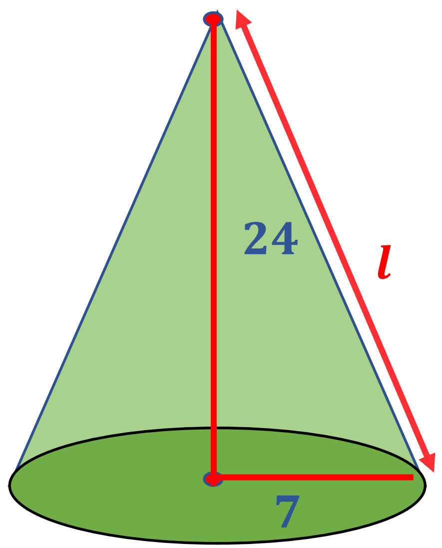
$slant\ height\ of\ the\ cap\ is\ l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24^2}$

$= \sqrt{49 + 576}$

$= \sqrt{625} = 25$

$\therefore l = 25\ cm$



$lateral\ surface\ area\ of\ the\ cap\ = \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 22 \times 25$

$= 550$

$\therefore lateral\ surface\ area\ of\ the\ cap = 550\ cm^2$

$i.e.\ area\ of\ the\ sheet\ required\ for\ 1\ cap\ is\ 550\ cm^2$

$Now,\ area\ of\ the\ sheet\ required\ for\ 10\ caps$

$= 10 \times 550$

$= 5500$

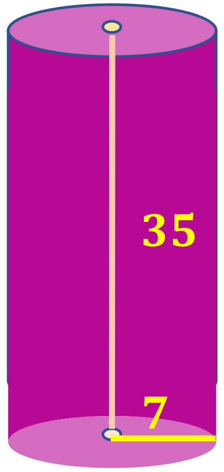
$\therefore Area\ of\ the\ sheet\ required\ to\ make\ 10\ caps\ is\ 5500\ cm^2$

EXERCISE – 10.1

2. A sports company was ordered to prepare 100 paper cylinders for packing shuttle cocks. The required dimensions of the cylinder are 35 cm length/height and its radius is 7 cm. Find the required area of thick paper sheet needed to make 100 cylinders.

Sol : $Radius\ of\ required\ cylinder\ is\ r = 7\ cm$

$Height\ of\ the\ cylinder\ is\ h = 35\ cm$



$$Total\ surface\ area\ of\ cylinder = 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 7 \times (35 + 7)$$

$$= 2 \times 22 \times 42$$

$$= 1848$$

$\therefore Total\ surface\ area\ of\ the\ cylinder = 1848\ cm^2$

$i.e.\ area\ of\ thick\ paper\ sheet\ required\ for\ 1\ cylinder\ is\ 1848\ cm^2$

$Now, area\ of\ the\ paper\ sheet\ required\ for\ 100\ cylinders$

$$= 100 \times 1848$$

$$= 184800$$

$\therefore Area\ of\ the\ paper\ sheet\ required\ make\ 100\ cyliders\ \ is\ 184800\ cm^2$

EXERCISE – 10.1

3. Find the volume of right circular cone with radius 6 cm. and height 7 cm.

Sol : $Given\ radius\ of\ cone\ is\ r = 6\ cm$

$height\ of\ the\ cone\ is\ h = 7\ cm$

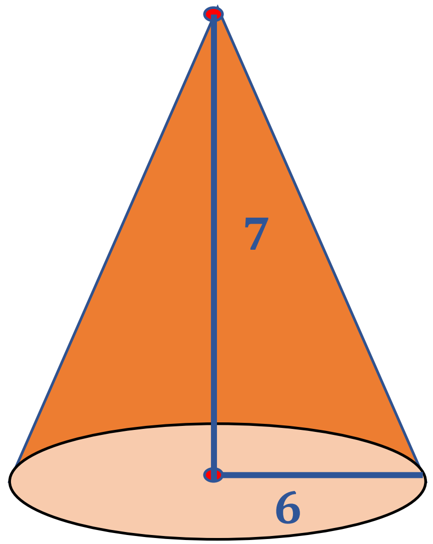
$$volume\ of\ the\ cone = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7$$

$$= 22 \times 2 \times 6$$

$$= 264$$

$$\therefore Volume\ of\ the\ cone = 264\ cm^3$$



EXERCISE – 10.1

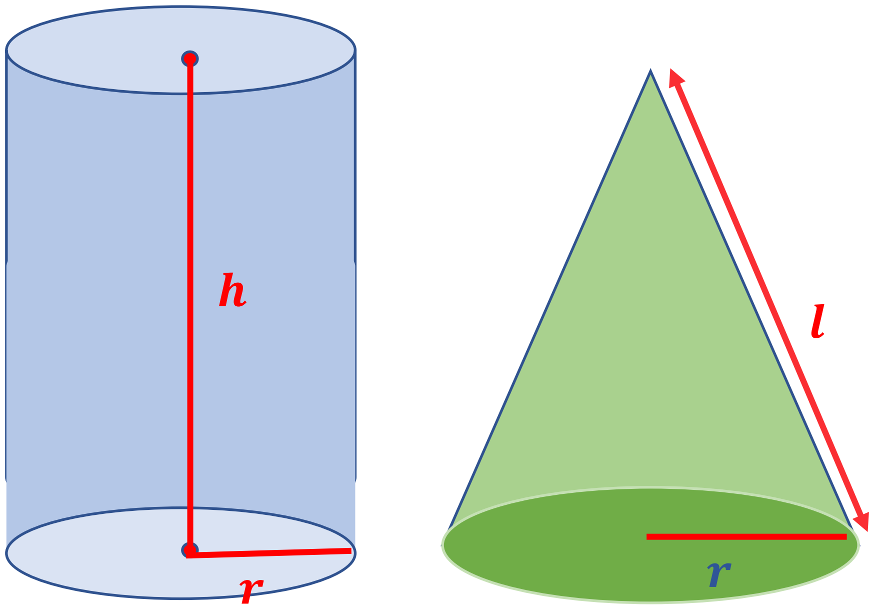
4. The lateral surface area of a cylinder is equal to the curved surface area of cone. If their bases be the same, find the ratio of the height of the cylinder to the slant height of the cone.

Sol : $\boldsymbol{Given\ radii\ of\ cylinder\ and\ cone\ are\ equal.}$

$\boldsymbol{Let\ the\ radius\ of\ cylinder = radius\ of\ cone = r}$

$\boldsymbol{height\ of\ the\ cylinder = h}$

$\boldsymbol{slant\ height\ of\ the\ cone = l}$



$\boldsymbol{Given,\ lateral\ surface\ area\ of\ cylinder}$

$$\boldsymbol{= curved\ surface\ area\ of\ cone}$$

$$\Rightarrow 2\pi rh = \pi rl$$

$$\Rightarrow \frac{h}{l} = \frac{\pi r}{2\pi r}$$

$$\Rightarrow \frac{h}{l} = \frac{1}{2}$$

$\therefore$ $\boldsymbol{The\ ratio\ of\ \ the\ height\ of\ the\ cylider\ to\ the\ slant\ height\ of\ the\ cone\ is\ 1:2}$

EXERCISE – 10.1

5. A self help group wants to manufacture joker's caps of 3 cm. radius and 4 cm. height. If the available paper sheet is 1000 cm^2, then how many caps can be manufactured from that paper sheet.

Sol : $Given, radius\ of\ the\ cap\ is\ r = 3\ cm$

$height\ of\ the\ cap\ is\ h = 4$ cm

$$slant\ height\ of\ the\ cap\ is\ l = \sqrt{r^2 + h^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25} = 5$$

$$\therefore l = 5\ cm$$

$$lateral\ surface\ area\ of\ the\ cap = \pi r l$$

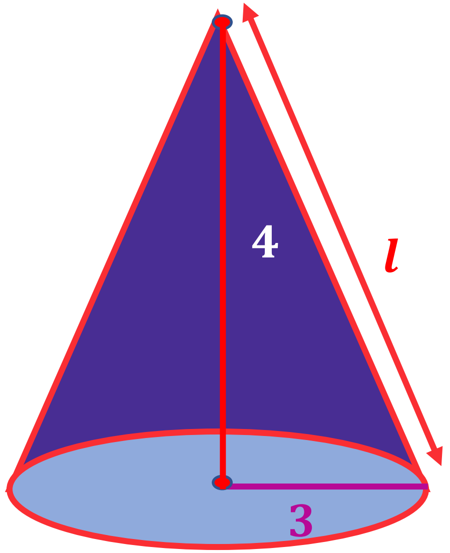
$$= \frac{22}{7} \times 3 \times 5$$

$$= \frac{330}{7}$$

$$\therefore lateral\ surface\ area\ of\ the\ cap = \frac{330}{7} cm^2$$

$i.e.\ area\ of\ the\ sheet\ required\ for\ 1\ cap\ is \frac{330}{7} cm^2$

$but\ the\ area\ of\ available\ paper\ sheet\ is\ 1000\ cm^2$



$number\ of\ caps\ that\ can\ be\ made\ from\ sheet\ of\ 1000\ cm^2$

$$= \frac{1000}{\left(\frac{330}{7}\right)} = 1000 \times \frac{7}{330} = \frac{7000}{330} = 21.21$$

$Number\ of\ caps\ that\ can\ be\ manufactured\ from\ sheet\ of\ 1000\ cm^2\ is\ 21$

EXERCISE – 10.1

6. A cylinder and cone have bases of equal radii and are of equal heights. Show that their volumes are in the ratio of 3:1.

Sol : $\text{Let the radius of cylinder} = \text{radius of cone} = r$

$$\text{height of cylinder} = \text{height of cone} = h$$

$$\text{then, volume of cylinder} = \pi r^2 h$$

$$\text{volume of cone} = \frac{1}{3}\pi r^2 h$$

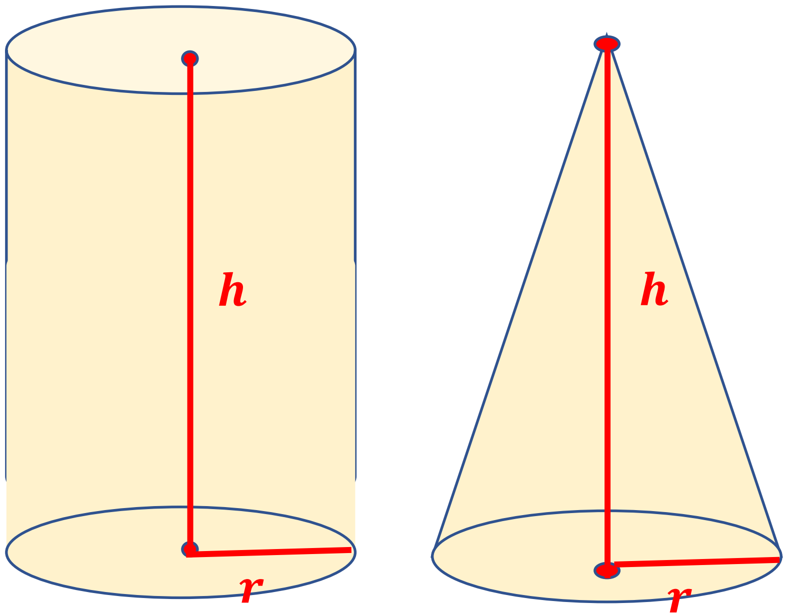
$\text{Now, the ratio of the volumes of cylinder and cone}$

$$= \cancel{\pi r^2} h : \frac{1}{3}\cancel{\pi r^2} h$$

$$= 1 : \frac{1}{3}$$

$$= 1 \times 3 : \frac{1}{3} \times 3$$

$$= 3 : 1$$



$\therefore$ $\text{ratio of volumes of cylinder and cone having same radii and same height is } 3 : 1$

7. The shape of solid iron rod is a cylindrical. Its height is 11 cm. and base diameter is 7 cm. Then find the total volume of 50 such rods.

Sol : $Height\ of\ cylindrical\ rod\ is\ h = 11\ cm.$

$diameter\ of\ the\ rod\ is\ d = 7\ cm.$

$then, radius\ of\ the\ rod\ is\ r = \frac{7}{2}\ cm$

$volume\ of\ cylindrical\ rod = \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 11$$

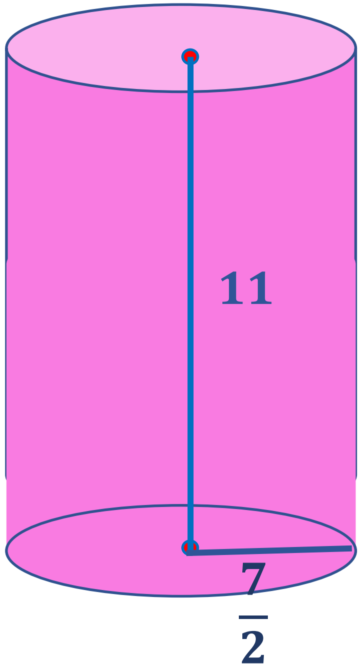
$$= 423.5$$

$volume\ of\ each\ cylindrical\ rod = 423.5\ cm^3$

$volume\ of\ such\ 50\ rods = 50 \times 423.5$

$$= 21175$$

$\therefore total\ volume\ of\ 50\ cylindrical\ rods\ is\ 21175\ cm^3$



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EXERCISE $-$ 10.1

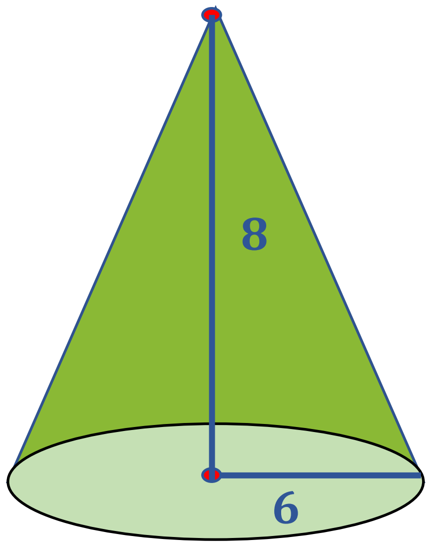
8. A heap of rice is in the form of a cone of diameter 12 m. and height is 8 m. Find its volume. How much canvas cloth is required to cover the heap? (Use $\pi = 3.14$)

Sol :

$$\textit{Height of cone is } h = 8\ m.$$

$$\textit{diameter of cone is } d = 12\ m.$$

$$\textit{then, radius of the cone is } r = \frac{12}{2} = 6\ m.$$



$$\textit{volume of conical heap} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$$

$$= 301.7$$

$$\textit{volume of conical heap of rice} = 301.7 m^3$$

$$\textit{slant height of cone} = \sqrt{r^2 + h^2}$$

$$= \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100} = 10$$

$$\therefore l = 10\ m.$$

$$\textit{lateral surface area of cone} = \pi r l$$

$$= \frac{22}{7} \times 6 \times 10$$

$$= 188.5$$

$\therefore$ *Area of canvas cloth required to cover the conical heap of rice is* $188.5\ m^2$

EXERCISE – 10.2

1. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm. and 4 cm. respectively. Determine the surface area of the toy

Sol : $Diameter\ of\ the\ base\ of\ the\ cone\ is\ d = 6\ cm.$

$then, radius\ of\ the\ cone\ is\ r = \frac{6}{2} = 3\ cm$

$height\ of\ the\ cone\ is\ h = 4\ cm$

$slant\ height\ of\ cone\ is\ l = \sqrt{r^2 + h^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25} = 5$

$\therefore l = 5\ cm.$

$now, curved\ surface\ area\ of\ cone = \pi r l$

$= 3.14 \times 3 \times 5$

$= 47.1$

$\therefore curved\ surface\ area\ of\ cone = 47.1\ cm^2$

$radius\ of\ the\ hemisphere\ is\ r = \frac{6}{2} = 3\ cm$

$curved\ surface\ area\ of\ hemisphere$

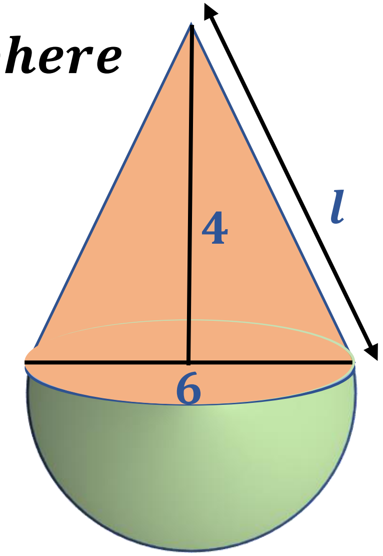
$= 2\pi r^2$

$= 2 \times 3.14 \times 3 \times 3$

$= 56.52$

$\therefore CSA\ of\ hemisphere$

$= 56.52\ cm^2$



$Now, surface\ area\ of\ toy$

$= CSA\ of\ cone + CSA\ of\ hemisphere$

$= 47.1 + 56.52$

$= 103.62$

$\therefore surface\ area\ of\ the\ toy = 103.62 cm^2$

EXERCISE – 10.2

2. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. The radius of the common base is 8 cm. and the heights of the cylindrical and conical portions are 10 cm. and 6 cm. respectively. Find the total surface area of the solid.

Sol : ***Cone :***

$radius\ is\ r = 8\ cm$

$height\ is\ h = 6\ cm$

$slant\ height\ is\ l = \sqrt{r^2 + h^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100} = 10$

$\therefore l = 10\ cm.$

$curved\ surface\ area = \pi rl$

$= 3.14 \times 8 \times 10$

$= 251.2$

$CSA\ of\ cone\ is\ 251.2\ cm^2$

Cylinder:

$radius\ is\ r = 8\ cm$

$height\ is\ h = 10\ cm$

$curved\ surface\ area = 2\pi rh$

$= 2 \times 3.14 \times 8 \times 10$

$= 502.4$

$CSA\ of\ cylinder\ is\ 502.4 cm^2$

Hemisphere :

$radius\ is\ r = 8\ cm$

$curved\ surface\ area = 2\pi r^2$

$= 2 \times 3.14 \times 8 \times 8$

$= 401.92$

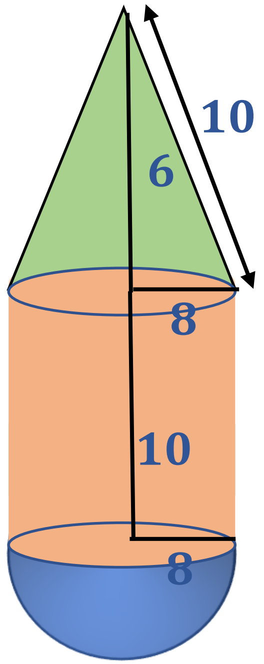
$CSA\ of\ hemisphere\ is\ 401.92\ cm^2$

$Now, total\ surface\ area\ of\ given\ solid$

$= CSA\ of\ cone + CSA\ of\ cylinder + CSA\ of\ hemisphere$

$= 251.2 + 502.4 + 401.92 = 1155.52$

$\therefore Total\ surface\ area\ of\ given\ solid\ is\ 1155.52\ cm^2$



3. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the capsule is 14 mm. and the width is 5 mm. Find its surface area.

Sol : *Given that length of the capsule is 14 mm and the width is 5 mm.*

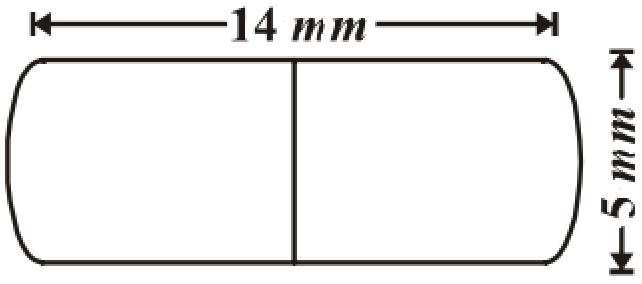
$\Rightarrow radius\ of\ cylinder\ is\ r = \frac{5}{2} = 2.5\ mm\ \ (\because width = d = 2r)$

$\Rightarrow height\ of\ cylinder\ is\ h = length\ of\ capsule - 2 \times radius\ of\ hemisphere$

$\Rightarrow h = 14 - 2 \times 2.5$

$\Rightarrow h = 14 - 5 = 9$

$\therefore height\ of\ cylinder\ is\ h = 9\ mm$



$Now, surface\ area\ of\ capsule$

$= CSA\ of\ cylinder + 2 \times CSA of\ hemisphere$

$= 2\pi rh + 2 \times 2\pi r^2$

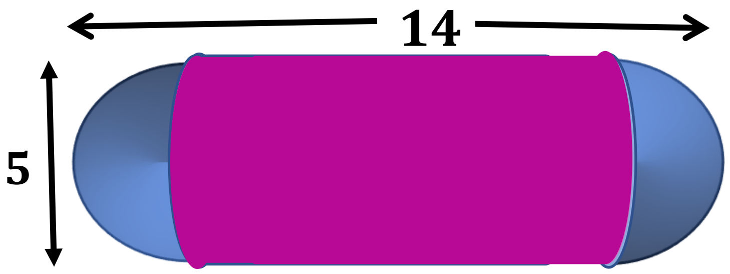
$= 2\pi r(h + 2r)$

$= 2 \times 3.14 \times 2.5 \times (9 + 5)$

$= 5 \times 3.14 \times 14$

$= 219.8$

$\therefore Surface\ area\ of\ the\ capsule = 219.8\ mm^2$



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EXERCISE – 10.2

4. Two cubes each of volume 64 cm^3 area joined end to end together. Find the surface area of the resulting cuboid.

Sol : $Given\ that\ volume\ of\ each\ cube\ is\ 64\ cm^3.$

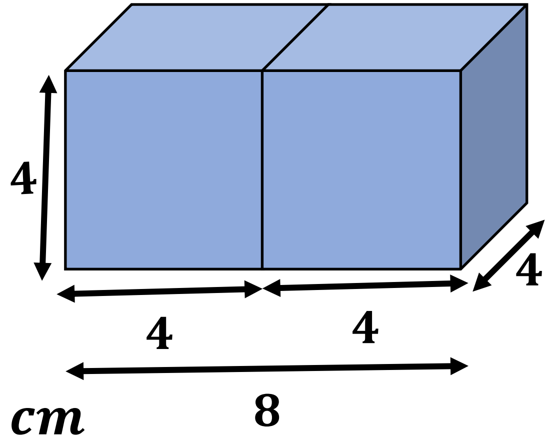
$Let\ the\ side\ of\ each\ cube\ be\ 'a'\ cm.$

$then, volume\ of\ the\ cube\ is\ a^3 = 64$

$$\Rightarrow a = (64)^{\frac{1}{3}}$$

$$\Rightarrow a = 4$$

$\therefore\ side\ of\ each\ cube\ is\ a = 4\ cm.$



$Now, the\ length\ of\ the\ cuboid\ formed\ by\ joining\ the\ cubes\ is\ l = a + a = 8\ cm$

$It's\ breadth\ is\ b = 4\ cm$

$height\ is\ h = 4\ cm$

$$surface\ area\ of\ the\ resulting\ cuboid = 2(lh + bh + lb)$$

$$= 2(8 \times 4 + 4 \times 4 + 8 \times 4)$$

$$= 2(32 + 16 + 32)$$

$$= 2(80)$$

$$= 160$$

$\therefore\ surface\ area\ of\ the\ resulting\ cuboid = 160\ cm^2$

EXERCISE – 10.2

5. A storage tank consists of a circular cylinder with a hemisphere struck on either end. If the external diameter of the cylinder be 1.4 m. and its length be 8 m. Find the cost of painting it on the outside at rate of Rs.20 per m^2.

Sol : $\boldsymbol{Given\ that\ \ diameter\ of\ the\ cylinder\ is\ 1.4\ m.\ and\ length\ of\ cylinder\ is\ 8\ m.}$

$$\Rightarrow radius\ of\ cylinder\ is\ r = \frac{1.4}{2} = 0.7\ m\ (\because width = d = 2r)$$

$length\ of\ \ cylinder\ is\ h = 8\ m.$

$Now, surface\ area\ of\ tank$

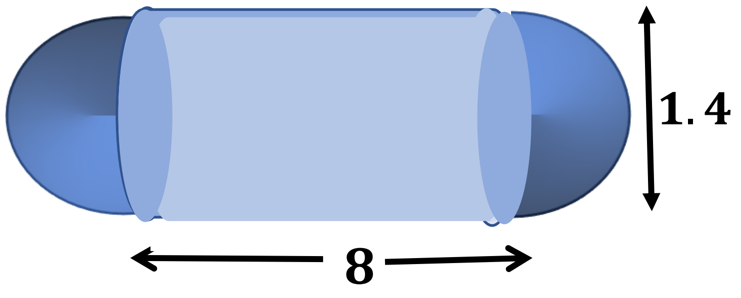
$= CSA\ of\ cylinder + 2 \times CSA\ of\ hemisphere$

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r(h + 2r)$

$$= 2 \times \frac{22}{7} \times 0.7 \times (8 + 1.4)$$

$= 2 \times 22 \times 0.1 \times 9.4 \quad = 41.36$



$\therefore Surface\ area\ of\ the\ tank = 41.36\ m^2$

$rate\ of\ painting\ per\ 1m^2. = \text{Rs}.20$

$cost\ of\ painting\ per\ 41.36\ m^2. = 41.36 \times 20$

$= 827.20$

$\therefore Cost\ of\ painting\ the\ taink\ is\ Rs.827.20$

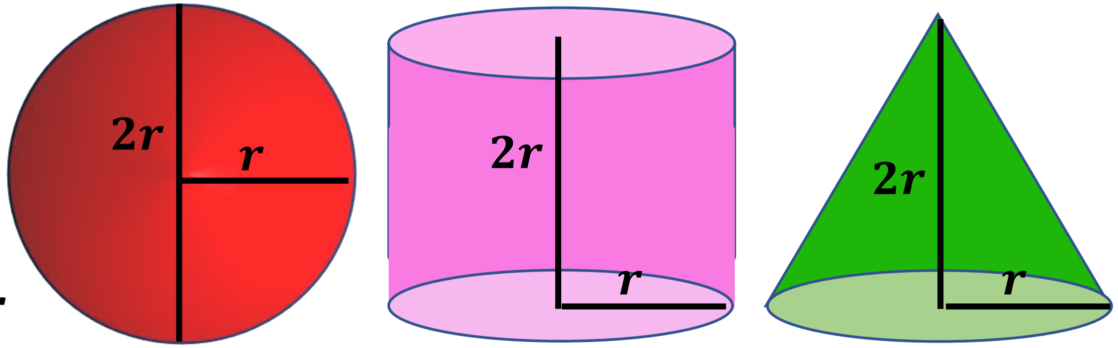
EXERCISE – 10.2

6. A sphere, a cylinder and a cone have the same radius and same height. Find the ratio of their volumes.

Sol : ***Sphere :***

Let the radius of sphere be 'r'

$$\text{volume of sphere} = \frac{4}{3}\pi r^3$$



cylinder :

$$\text{radius of cylinder} = \text{radius of sphere} = r$$

$$\text{height of cylinder} = \text{diameter of sphere} = 2r$$

$$\text{volume of cylinder} = \pi r^2 h$$

$$= \pi r^2 (2r) = 2\pi r^3$$

cone :

$$\text{radius of cone} = \text{radius of sphere} = r$$

$$\text{height of cone} = \text{diameter of sphere} = 2r$$

$$\text{volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2 (2r)$$

$$= \frac{2}{3}\pi r^3$$

Now, the ratio of volumes of sphere, cylinder and cone

$$= \frac{4}{3}\pi r^3 : 2\pi r^3 : \frac{2}{3}\pi r^3$$

$$= \frac{4}{3} : 2 : \frac{2}{3}$$

$$= \frac{2}{3} : 1 : \frac{1}{3} = 2 : 3 : 1$$

$\therefore$ *ratio of volumes of sphere, cone and cylinder is* $2 : 3 : 1$

7. A hemisphere is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the length of the cube. Determine the surface area of the remaining solid.

Sol : *Let the length of the edge of the cube be 'a'*

$\Longrightarrow$ *total surface area of cube* $= 6a^2$

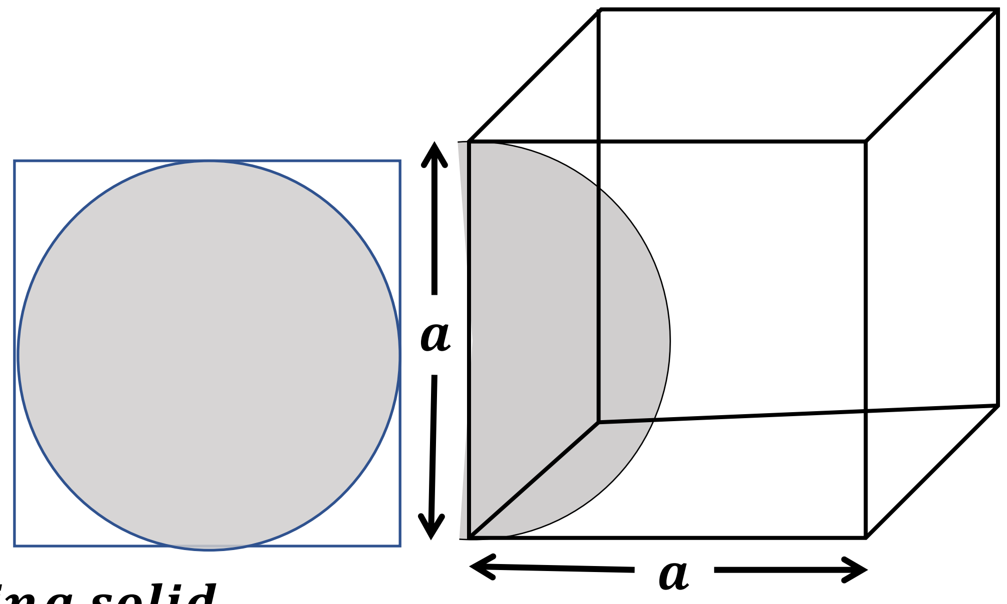
diameter of hemisphere $= a$

$\Longrightarrow$ *radius of hemisphere is* $r = \dfrac{a}{2}$

$\Longrightarrow$ *curved surface area of hemisphere* $= 2\pi r^2$

$$= 2\pi\left(\frac{a}{2}\right)^2$$

$$= 2\pi\frac{a^2}{4} = \frac{\pi a^2}{2}$$



after cutting the hemisphere, surface area of remaining solid

$= TSA\ of\ cube - area\ of\ circle\ having\ radius\ a/2 + CSA\ of\ hemisphere$

$$= 6a^2 - \pi\left(\frac{a}{2}\right)^2 + \frac{\pi a^2}{2}$$

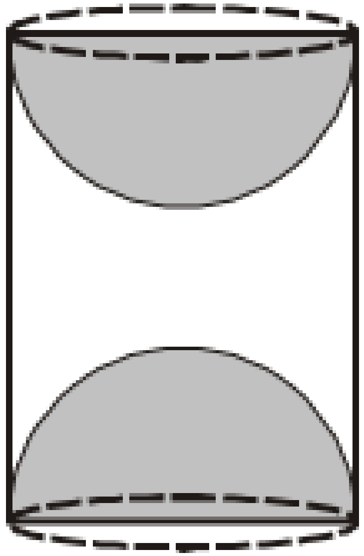
$$= 6a^2 - \frac{\pi a^2}{4} + \frac{\pi a^2}{2} = 6a^2 + \frac{\pi a^2}{4} = a^2\left(6 + +\frac{\pi}{4}\right)$$

$\therefore$ *surface area of remaining solid is* $a^2\left(6 + +\dfrac{\pi}{4}\right)$

EXERCISE – 10.2

8. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the length of the cylinder is 10 cm. and its radius of the base is of 3.5 cm., find the total surface area of the article.

Sol : *Given that radius of cylinder is* $r = 3.5\ cm.$ *and length of cylinder is* $h = 10\ cm.$



curved surface area of cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 3.5 \times 10$$

$$= 2 \times 22 \times 0.5 \times 10$$

$$= 220$$

$\therefore$ *curved surface area of cylinder* $= 220\ cm^2$

radius of the hemisphere is $r = 3.5\ cm$

curved surface area of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 2 \times 22 \times 0.5 \times 3.5$$

$$= 77$$

$\therefore$ *curved surface area of hemisphere* $= 77\ cm^2$

TSA of the article

$= CSA\ of\ cylinder + 2 \times CSA\ of\ hemisphere$

$= 220 + 2 \times 77$

$= 220 + 154 \ = 374$

$\therefore$ *TSA of the article* $= 374\ cm^2$

EXERCISE – 10.3

1. An iron pillar consists of a cylindrical portion of 2.8m. height and 20 cm. in diameter and a cone of 42 cm. height surmounting it. Find the weight of the pillar if 1 cm^3 of iron weighs 7.5 g.

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Sol : ***Cylinder*:**

$$diameter\ is\ d = 20\ cm$$

$$radius\ is\ r = \frac{d}{2} = \frac{20}{2} = 10\ cm$$

$$height\ is\ h = 2.8\ m = 280\ cm$$

$$volume = \pi r^2 h$$

$$= \frac{22}{7} \times 10 \times 10 \times 280$$

$$= 22 \times 10 \times 10 \times 40$$

$$= 88000$$

$$\therefore volume\ of\ cylinder = 88000\ cm^3$$

***Cone* :**

$$radius\ is\ r = 10\ cm$$

$$height\ is\ h = 42\ cm$$

$$volume = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 42$$

$$= 22 \times 10 \times 10 \times 2$$

$$= 4400$$

$$\therefore volume\ of\ cone = 4400\ cm^3$$

$$Now, volume\ of\ iron\ pillar$$

$$= volume\ of\ cylinder + volume\ of\ cone$$

$$= 88000 + 4400 = 92400\ cm^3$$

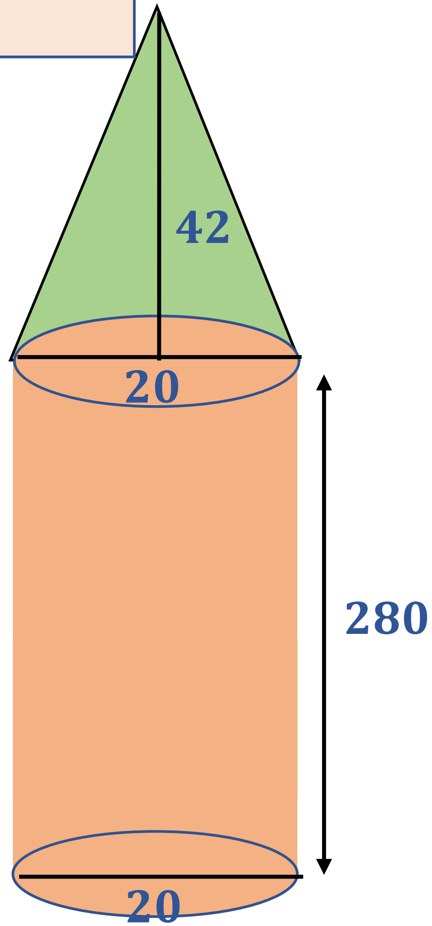
$$weight\ of\ 1\ cm^3\ iron = 7.5\ g$$

$$weight\ of\ iron\ pillar = 7.5 \times 92400$$

$$= 693000\ g.$$

$$= 693\ kg.$$

$$\therefore weight\ of\ iron\ pillar = 693\ kg.$$



EXERCISE – 10.3

2. A toy is made in the form of hemisphere surmounted by a right cone whose circular base is joined with the plane surface of the hemisphere. The radius of the base of the cone is 7 cm. and its volume is 3/2 of the hemisphere. Calculate the height of the cone and the surface area of the toy correct to 2 places of decimal.

Sol : *Given the radius of both hemisphere and cone is* $r = 7\ cm.$

Let 'h' be the height of cone.

$$volume\ of\ cone = \frac{3}{2} \times volume\ of\ hemisphere$$

$$\frac{1}{3}\pi r^2 h = \frac{3}{2} \times \frac{2}{3}\pi r^3$$

$$\Rightarrow h = \frac{3\pi r^3}{\pi r^2}$$

$$\Rightarrow h = 3r$$

$$\Rightarrow h = 3 \times 7 = 21$$

$\therefore$ *height of the cone is 21 cm.*

slant height of the cone is $l = \sqrt{r^2 + h^2}$

$$= \sqrt{7^2 + 21^2}$$

$$= \sqrt{49 + 441} = \sqrt{490} = 22.135$$

Now, the surface area of toy.

$= CSA\ of\ cone$
$+ CSA\ of\ hemisphere.$

$$= \pi r l + 2\pi r^2.$$

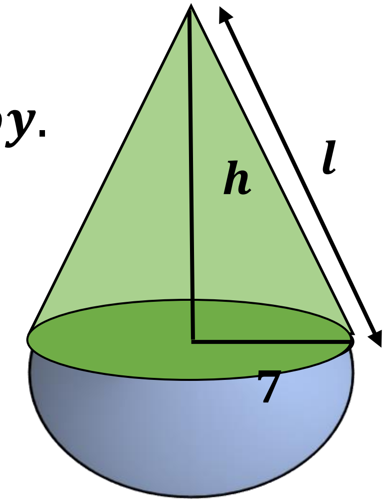
$$= \pi r(l + 2r).$$

$$= \frac{22}{7} \times 7 \times (22.135 + 2 \times 7)$$

$$= 22 \times 36.135$$

$$= 794.17$$

$\therefore$ *surface are of the toy is* $794.17\ cm^2$



3. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 7 cm.

Sol : $\boldsymbol{Given\ the\ edge\ of\ cube\ is\ a = 7\ cm.}$

$\boldsymbol{diameter\ of\ the\ largest\ cone\ that\ can\ be\ cut\ out\ from\ cube\ is\ d}$

$$= edge\ of\ cube = 7cm$$

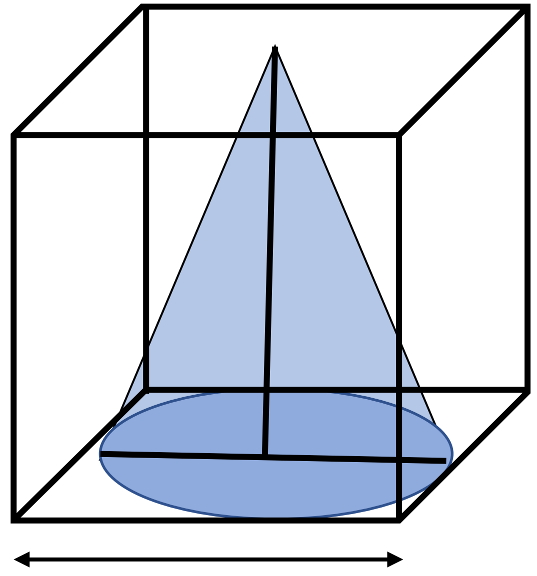
$\boldsymbol{radius\ of\ the\ cone\ is\ r = \frac{d}{2} = \frac{7}{2}\ cm}$

$\boldsymbol{height\ of\ the\ cone\ is\ h = edge\ of\ cube = 7\ cm}$

$$volume\ of\ cone\ = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7$$

$$= 89.83$$



$\boldsymbol{\therefore\ volume\ of\ cone\ that\ can\ be\ cut\ out\ from\ given\ cube\ is\ 89.83\ cm^2.}$

EXERCISE – 10.3

4. A cylindrical tub of radius 5 cm and length 9.8 cm is full of water. A solid in the form of right circular cone mounted on a hemisphere is immersed into the tub. The radius of the hemisphere is 3.5 cm and height of cone outside the hemisphere is 5 cm. Find the volume of water left in the tub.

Sol : *Given that radius of cylinder is* $R = 5\ cm.$

height of cylinder is $H = 9.8\ cm.$

$$volume\ of\ cylinder\ = \pi R^2 H$$

$$= \frac{22}{7} \times 5 \times 5 \times 9.8 = 770\ cm^3$$

radius of hemisphere is $r = 3.5\ cm.$

height of cone is $h = 5\ cm.$

volume of object immersed in waater

$$= volume\ of\ cone + volume\ of\ hemisphere$$

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times (5 + 7) = 154$$

volume of object immersed in waater $= 154 cm^3$

After immersion of solid into tub,

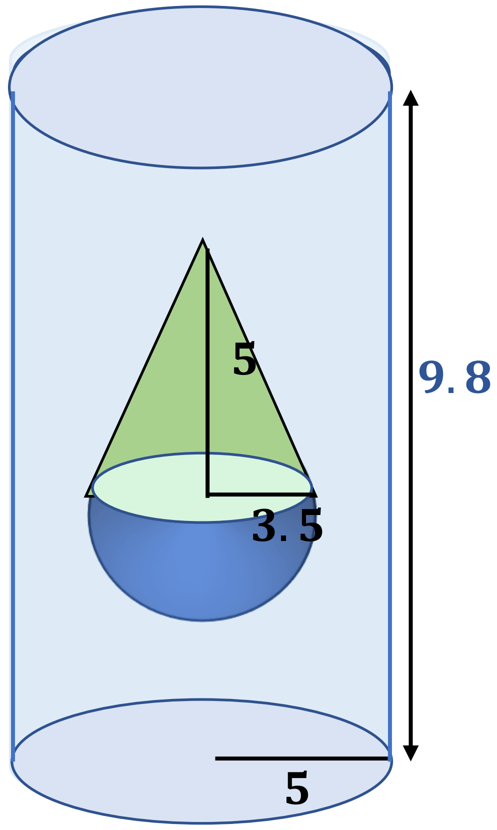
volume of water left in tub

$$= vol.of\ tub - vol.of\ solid$$

$$= 770 - 154$$

$$= 616$$

$\therefore$ *volume of water left in tub* $= 616\ cm^3$



5. In the adjacent figure, the height of a solid cylinder is 10 cm and diameter is 7 cm. Two equal conical holes of radius 3 cm and height 4 cm are cut off as shown in the figure. Find the volume of remaining solid.

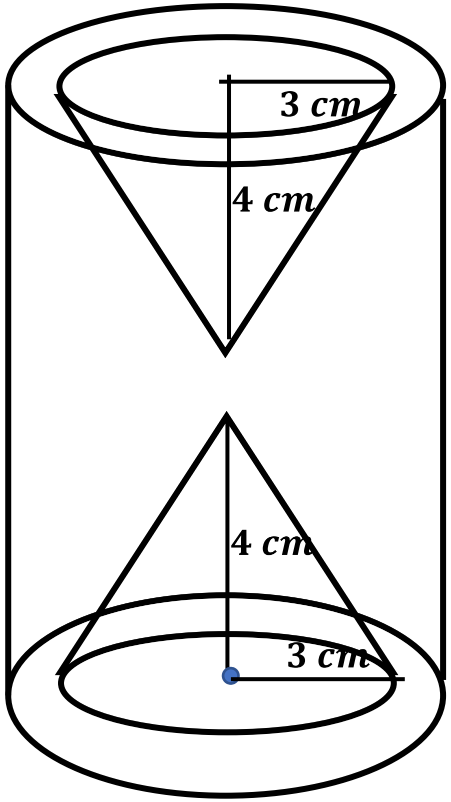
Sol : *Given, the height of cylinder is* $H = 10\ cm$.

diameter of the cylinder is $d = 7\ cm$.

$\Rightarrow$ *radius of the cylinder is* $R = \frac{7}{2} = 3.5\ cm$

radius of each conical hole is $r = 3\ cm$.

height of each conical hole is $h = 4cm$.



Now, the volume of remaining solid after two conical holes cut off from cylinder

$= volume\ of\ cylinder\ hole - 2 \times volume\ of\ conical\ hole$

$= \pi R^2 H - 2 \times \frac{1}{3} \pi r^2 h$

$= \frac{22}{7} \times 3.5 \times 3.5 \times 10 - -2 \times \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 4$

$= \frac{2695}{7} - \frac{528}{7} = \frac{2167}{7} = 309.57$

$\therefore$ *volume of remianing solid is* $309.57\ cm^2$

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EXERCISE – 10.3

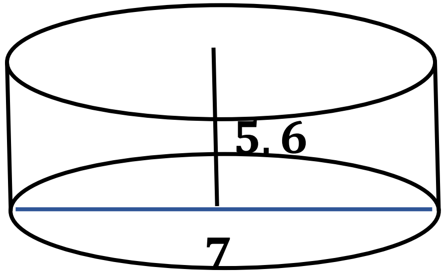
6. Spherical marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm., which contains some water. Find the number of marbles that should be dropped into the beaker, so that water level rises by 5.6 cm.

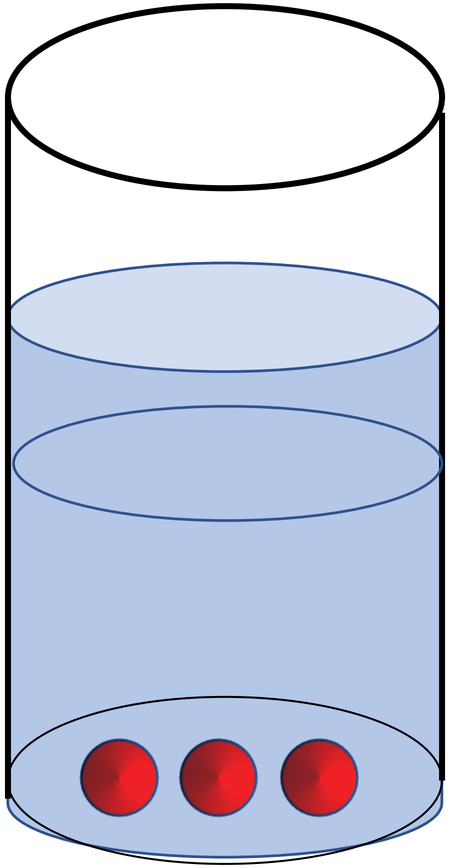
Sol : $\textit{Given the diameter of cylindrical beaker is } d = 7\ cm.$

$\textit{radius of cylindrical beaker is } R = \frac{7}{2} = 3.5\ cm.$

$\textit{rise of the water level is in shape of cylinder of radius } 3.5\ cm$

$\textit{height of the water rise is } h = 5.6\ cm$





$\textit{volume of the water rise} = \pi R^2 h$

$= \frac{22}{7} \times 3.5 \times 3.5 \times 5.6 \quad = 215.6$

$\textit{radius of spherical ball is } = \frac{1.4}{2} = 0.7 cm.$

$\textit{volume of spherical ball } = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 0.7$

$= 1.4373$

$\textit{Let the number of marbles be 'n'}$

$\textit{Now, the volume of water rise}$

$= \textit{volume of 'n' marbles}$

$\Rightarrow 215.6 = n \times 1.4373$

$\Rightarrow n = \frac{215.6}{1.4373}$

$\Rightarrow n = 150$

$\therefore \textit{number of marbles to be dropped in beaker is } 150$

EXERCISE – 10.3

10. A pen stand is made of wood in the shape of cuboid with three conical depreesion to hold the pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of the each of the depression is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.

Sol : *Given the length of the cuboid is* $L = 15\ cm.$

$breadth\ is\ B = 10\ cm.$

$height\ is\ H = 3.5\ cm.$

$volume\ of\ wooden\ cuboid = L \times B \times H$

$= 15 \times 10 \times 3.5$

$= 525$

$\therefore volume\ of\ wooden\ cuboid = 525\ cm^3$

$radius\ of\ each\ conical\ depression\ is\ r = 0.5\ cm$

$and\ it's\ depth\ is\ h = 1.4cm$

$$volume\ of\ each\ conical\ depression = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$$

$$= \frac{1.1}{3}$$

$Now, the\ volume\ of\ wood\ in\ the\ stand\ after\ making\ three\ conical\ depressions$

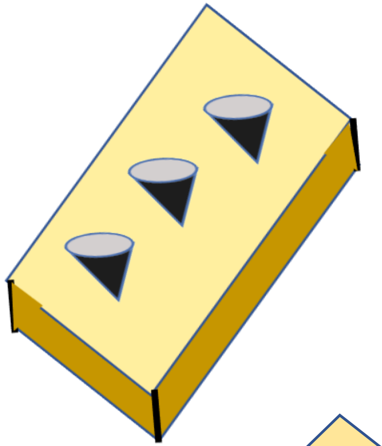
$= vol. of\ wooden\ cuboid - 3 \times vol. of\ conical\ depression$

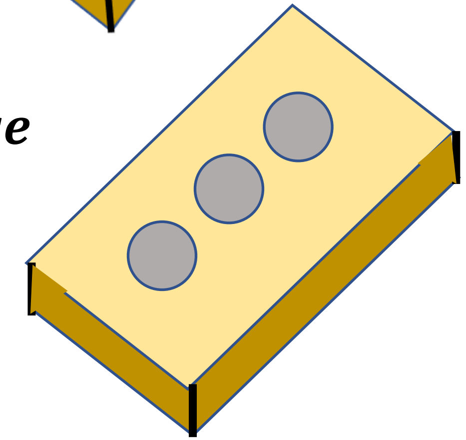
$$= 525 - 3 \times \frac{1.1}{3}$$

$$= 525 - 1.1$$

$$= 523.9$$

$\therefore volume\ of\ wood\ in\ entire\ stand = 523.9\ cm^3$





EXERCISE – 10.4

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder

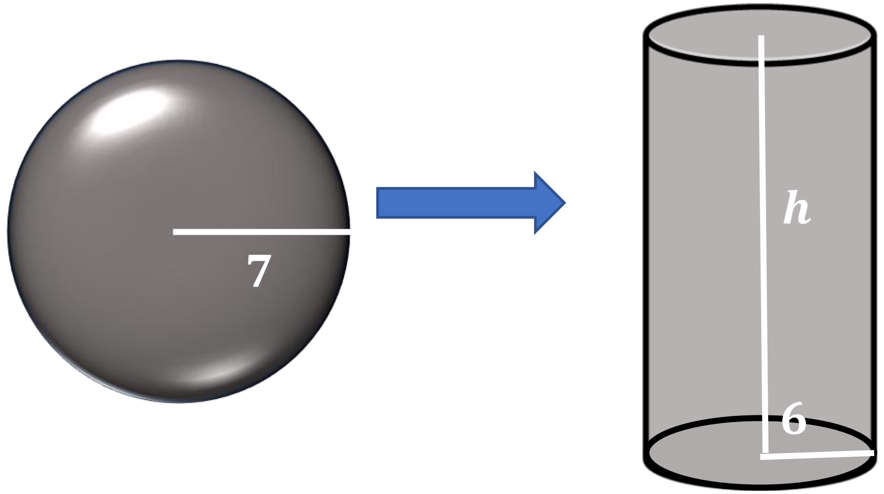
Sol : $Given, the\ radius\ of\ sphere\ is\ R = 4.2\ cm.$

$volume\ of\ the\ sphere = \frac{4}{3}\pi R^3$

$radius\ of\ cylinder\ is\ r = 6\ cm.$

$let\ the\ height\ of\ cylinder\ be\ 'h'\ cm.$

$volme\ of\ cylinder = \pi r^2 h$



$but, volume\ of\ sphere = volume\ of\ cylinder$

$$\Rightarrow \frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\Rightarrow \frac{4\pi R^3}{3\pi r^2} = h$$

$$\Rightarrow h = \frac{4R^3}{3r^2}$$

$$\Rightarrow h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$$

$$\Rightarrow h = 4 \times 1.4 \times 0.7 \times 0.7$$

$$\Rightarrow h = 2.744$$

$\therefore height\ of\ the\ cylinder = 2.744\ cm$

EXERCISE – 10.4

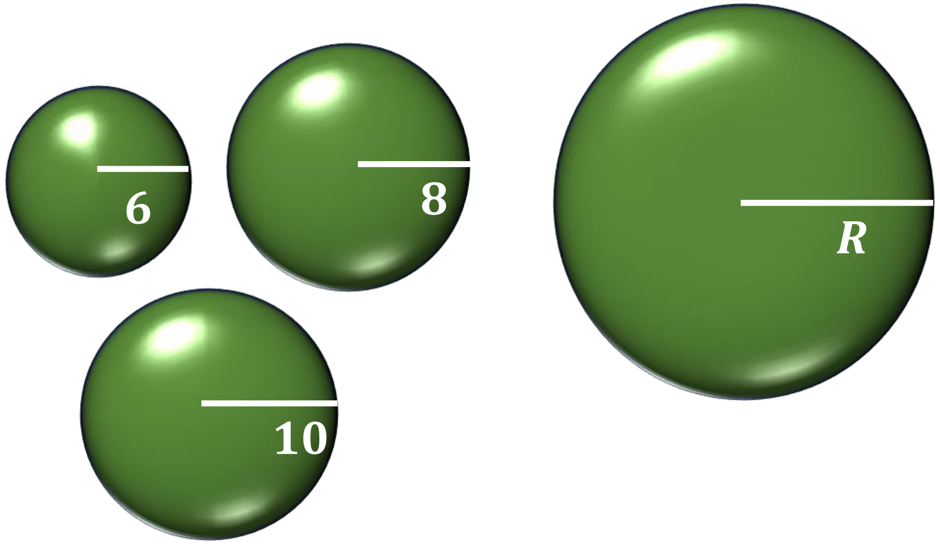
2. Three metallic spheres of radii 6 cm., 8 cm. and 10 cm. respectively are melted together to form a single solid sphere. Find the radius of the resulting sphere.

Sol : *Given, the radii of small spheres are* $r_1 = 6\ cm, r_2 = 8\ cm, r_3 = 10\ cm,$

Let the radius of big sphere be 'R' cm.

Now, volume of big sphere

= sum of the volumes of small spheres



$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\Rightarrow \cancel{\frac{4}{3}\pi} R^3 = \cancel{\frac{4}{3}\pi}(r_1^3 + r_2^3 + r_3^3)$$

$$\Rightarrow R^3 = r_1^3 + r_2^3 + r_3^3$$

$$\Rightarrow R^3 = 6^3 + 8^3 + 10^3$$

$$\Rightarrow R^3 = 216 + 512 + 1000$$

$$\Rightarrow R^3 = 1728$$

$$\Rightarrow R = \sqrt[3]{1728}$$

$$\Rightarrow R = 12$$

$\therefore$ *Radius of the resulting big sphere is 12 cm.*

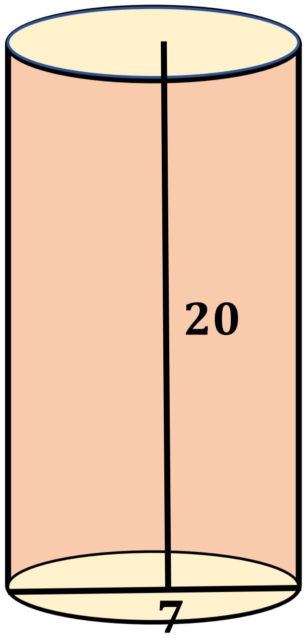
EXERCISE – 10.4

3. A 20 m. deep well of diameter 7 m. is dug and the earth got by digging is evenly spread out to form a rectangular platform of base 22 m × 14 m. Find the height of the platform.

Sol : *Given the diameter of well is* $d = 7\ m$.

so that, the radius of well is $r = \frac{7}{2} = 3.5\ m$.

depth of well is $H = 20\ m$.



and for the rectangular platform formed by earth dugout from well,

length is $l = 22\ m$ *and breadth is* $b = 14\ m$

let the height of platform be 'h'

Now, the volume of earth dugout from well

$= $ *volume of platform which is in the shape of cuboid*

$$\Rightarrow \pi r^2 H = lbh$$

$$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22 \times 14 \times h$$

$$\Rightarrow h = \frac{22 \times 7 \times 7 \times 20}{7 \times 2 \times 2 \times 22 \times 14}$$

$$\Rightarrow h = 2.5$$

$\therefore$ *The height of the platform is* $2.5\ m$

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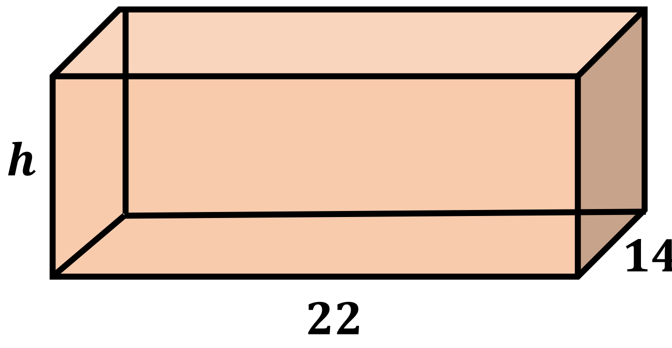


EXERCISE – 10.4

4. A well of diameter 14 m. is dug 15 m. deep. The earth taken out of it has been spread evenly to form circular embankment of width 7 m. Find the height of the embankment.

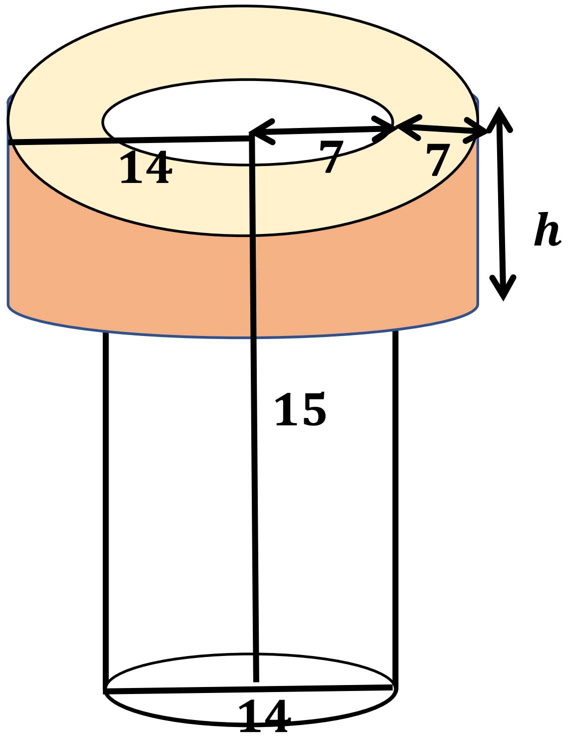
Sol :

Given the height of well is $H = 15\ m.$, *diameter is* $d = 14\ m.$ *and width of embankment is* $w = 7\ m.$

radius of well is $r = \frac{d}{2} = \frac{14}{2} = 7\ m.$

For the embankment, inner radius is $r_1 = 7\ m.$

outer radius is $r_2 = r_1 + w = 7 + 7 = 14\ m.$



let the height of embankment be 'h'

Now, volume of well = volume of embankment

Now, volume of well = volume of embankment

$$\Rightarrow \pi r^2 H = \pi r_2^2 h - \pi r_1^2 h$$

$$\Rightarrow \cancel{\pi} r^2 H = \cancel{\pi} h(r_2^2 - r_1^2)$$

$$\Rightarrow h = \frac{r^2 H}{r_2^2 - r_1^2}$$

$$\Rightarrow h = \frac{7 \times 7 \times 15}{14 \times 14 - 7 \times 7}$$

$$\Rightarrow h = \frac{735}{147}$$

$$\Rightarrow h = 5$$

$\therefore$ *height of the embankment is 5 m*

EXERCISE – 10.4

5. A container shaped like a right circular cylinder having diameter 12 cm. and height 15 cm. is full of ice cream. The icecream is to be filled into cones of height 12 cm. and diameter 6 cm. having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Sol : **For cylinder :**

$diameter\ is\ d = 12\ cm.$

$then, radius\ is\ R = \frac{d}{2} = 6\ cm.$

$height\ is\ H = 15\ cm.$

$volume = \pi R^2 H$

$= \pi \times 6 \times 6 \times 15$

$= 540\pi$

For cone :

$diameter = 6\ cm.$

$then, radius\ is\ r = \frac{6}{2} = 3\ cm.$

$height\ is\ h = 12\ cm.$

$volume = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3 \times 3 \times 12 \ = 36\pi$

For hemisphere :

$radius\ = radius\ of\ cone\ is\ r = 3\ cm.$

$volume = \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 3 \times 3 \times 3$

$= 18\pi$

$Now, the\ volume\ of\ icecream\ cone$

$= vol. of\ cone + vol. of\ hemisphere$

$= 36\pi + 18\pi$

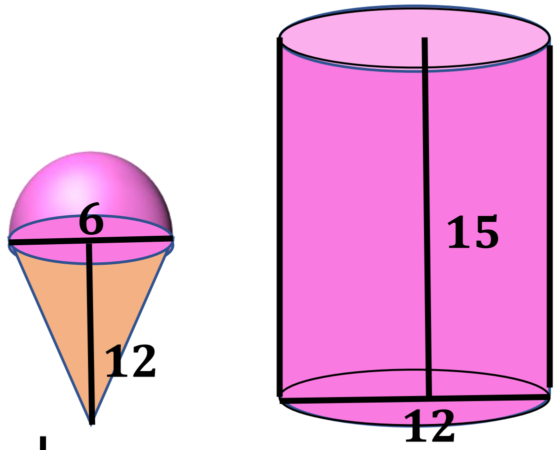
$= 54\pi$

$Let\ the\ number\ of\ icecream\ cones\ be\ 'n'$

$volume\ of\ icecream\ cylinder$

$= n \times vol. of\ icecream\ cone$

$\Rightarrow 540\pi = n \times 54\pi$



$\Rightarrow n = \frac{540\pi}{54\pi} = 10$

$\therefore The\ number\ of\ cones\ that\ can\ be\ filled\ with\ icecream\ is\ 10$

6. How many silver coins, 1.75 cm. in diameter and thickness 2 mm., need to be melted to form a cuboid of dimensions 5.5 cm.×10 cm.×3.5 cm. ?

Sol : For cylindrical silver coin :

$diameter\ is\ d = 1.75\ cm.$

$then, radius\ is\ r = \frac{1.75}{2} cm.$

$height\ is\ h = 2\ mm. = \frac{2}{10} cm$

$volume = \pi r^2 h$

$= \frac{22}{7} \times \frac{1.75}{2} \times \frac{1.75}{2} \times \frac{2}{10}$

For cuboid :

$length\ is\ L = 5.5\ cm.$

$breadth\ is\ B = 10\ cm.$

$height\ is\ H = 3.5\ cm.$

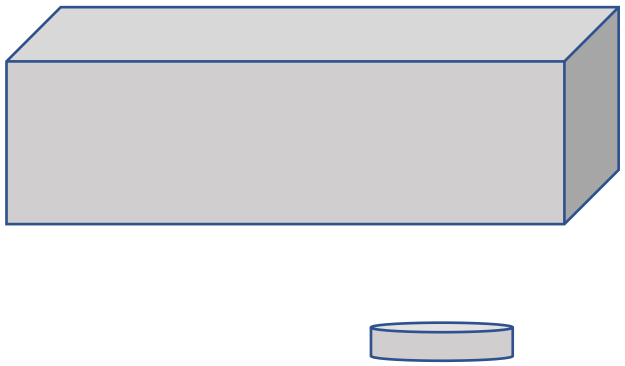
$volume = L \times B \times H$

$= 5.5 \times 10 \times 3.5$

$let\ the\ number\ of\ coins\ needed\ to\ form\ a\ cuboid\ be\ 'n'$

$then, volume\ of\ cuboid$

$= n \times volume\ of\ each\ silver\ coin$



$\Longrightarrow 5.5 \times 10 \times 3.5$

$= n \times \frac{22}{7} \times \frac{1.75}{2} \times \frac{1.75}{2} \times \frac{2}{10}$

$$\Longrightarrow n = \frac{5.5 \times 10 \times 3.5 \times 7 \times 2 \times 2 \times 10}{22 \times 1.75 \times 1.75 \times 2}$$

$\Longrightarrow n = 400$

$\therefore The\ number\ of\ silver\ coins\ needed\ to\ form\ a\ cuboid\ is\ 400$

$\underline{\text{EXERCISE} - 10.4}$

7. A vessel in the form an inverted cone. Its height is 8 cm. and the radius of its top is 5 cm. It is filled with water upto the rim. When lead shots, each of which is a sphere of raidius 0.5 cm.are dropped into the vessel, $\frac{1}{4}$ of the water flows out. Find the number of lead shots dropped into the vessel.

Sol :

EXERCISE – 10.4

7. A vessel in the form an inverted cone. Its height is 8 cm. and the radius of its top is 5 cm. It is filled with water upto the rim. When lead shots, each of which is a sphere of raidius 0.5 cm.are dropped into the vessel, $\frac{1}{4}$ of the water flows out. Find the number of lead shots dropped into the vessel.

Sol : For conical vessel

$radius\ is\ R = 5\ cm.$

$height\ is\ h = 8\ cm.$

$$volume = \frac{1}{3}\pi R^2 h$$

$$= \frac{1}{3} \times \pi \times 5 \times 5 \times 8$$

$$= \frac{200\pi}{3}$$

For spherical lead shots :

$$radius\ is\ r = 0.5 = \frac{5}{10} = \frac{1}{2}\ cm.$$

$$volume = \frac{4}{3}\pi r^3$$

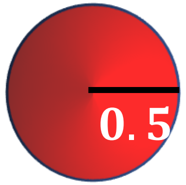
$$= \frac{4}{3} \times \pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

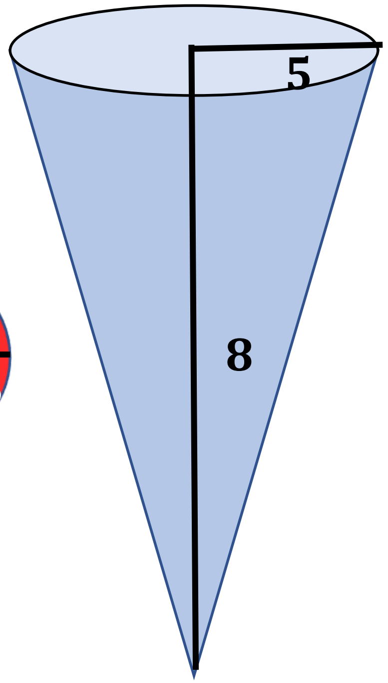
$$= \frac{\pi}{6}$$

$for, \frac{1}{4} th\ of\ water\ flows\ out\ from\ conical\ vessel,$

$let\ the\ number\ of\ lead\ shots\ to\ bedropped$

$into\ vessel\ be\ 'n'$





$now, volume\ of\ water\ flows\ out$

$$= n \times volume\ of\ lead\ shots$$

$$\Longrightarrow \frac{1}{4} \times vol.of\ vessel = n \times vol.of\ sphere$$

$$\Longrightarrow \frac{1}{4} \times \frac{200\pi}{3} = n \times \frac{\pi}{6}$$

$$\Longrightarrow \frac{50\pi}{3} = n \times \frac{\pi}{6}$$

$$\Longrightarrow n = \frac{50\pi \times 6}{3 \times \pi}$$

$$\Longrightarrow n = 100$$

$\therefore Number\ of\ lead\ shots\ dropped\ into\ the\ vessel\ is\ 100$

8. A solid metallic sphere of diameter 28 cm. is melted and recast into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm. and height 3 cm. Find the number of cones so formed.

Sol : **For metallic sphere :**

$$diameter\ is\ d = 28\ cm$$

$$\Rightarrow radius\ is\ R = \frac{d}{2} = 14\ cm$$

$$volume = \frac{4}{3}\pi R^3$$

$$= \frac{4}{3} \times \pi \times 14 \times 14 \times 14$$

$$= \frac{10976\pi}{3}$$

For smaller cone :

$$diameter\ is\ d = 4\frac{2}{3} = \frac{14}{3}\ cm$$

$$\Rightarrow radius\ is\ r = \frac{d}{2} = \frac{7}{3}\ cm$$

$$\Rightarrow height\ is\ h = 3\ cm$$

$$volume = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times \frac{7}{3} \times \frac{7}{3} \times 3 = \frac{49\pi}{9}$$

$$let\ the\ number\ of\ cones\ so\ formed\ is\ 'n'$$

$$now\ vol.\ of\ sphere = n \times vol.\ of\ cone$$

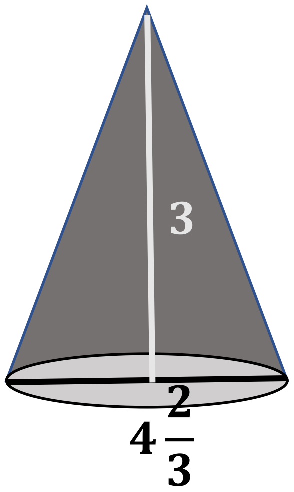
$$\Rightarrow \frac{10976\pi}{3} = n \times \frac{49\pi}{9}$$

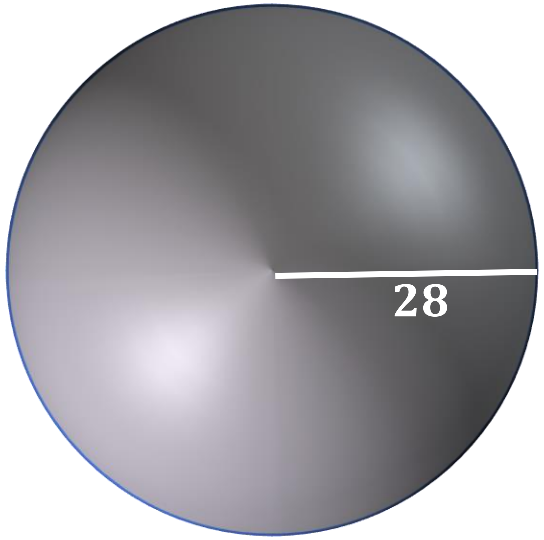
$$\Rightarrow n = \frac{10976\pi \times 9}{49\pi \times 3}$$

$$\Rightarrow n = \frac{10976\pi \times 9}{49\pi \times 3}$$

$$\Rightarrow n = 224 \times 3$$

$$\Rightarrow n = 672$$





$$\therefore The\ number\ of\ smaller\ cones\ so\ formed\ is\ 672$$

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