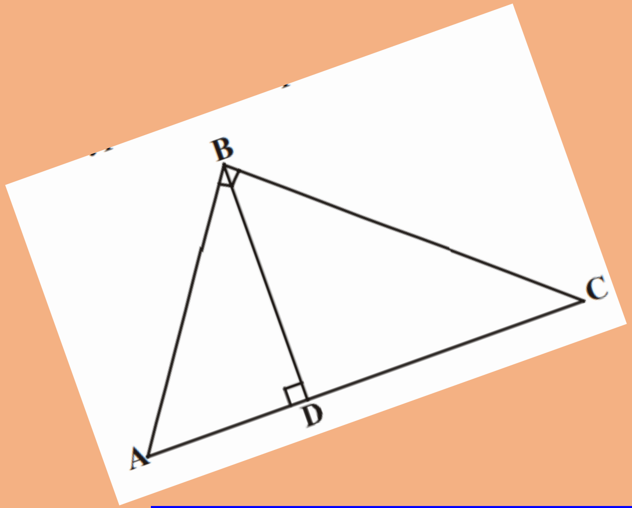
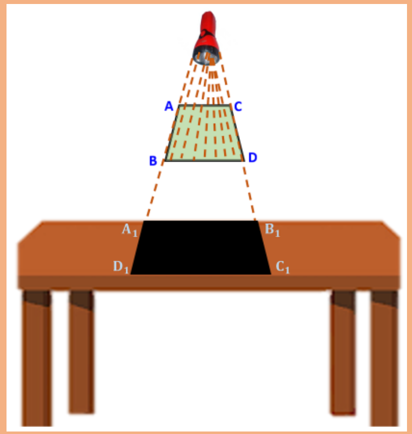
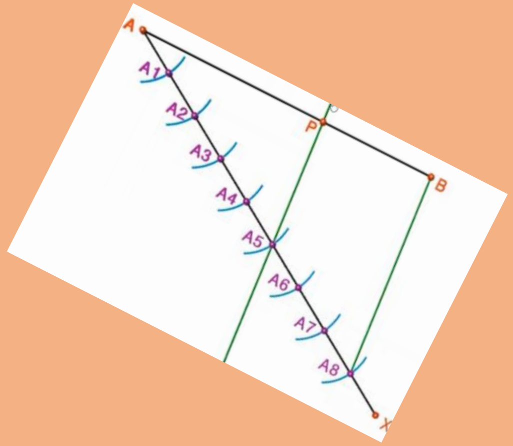
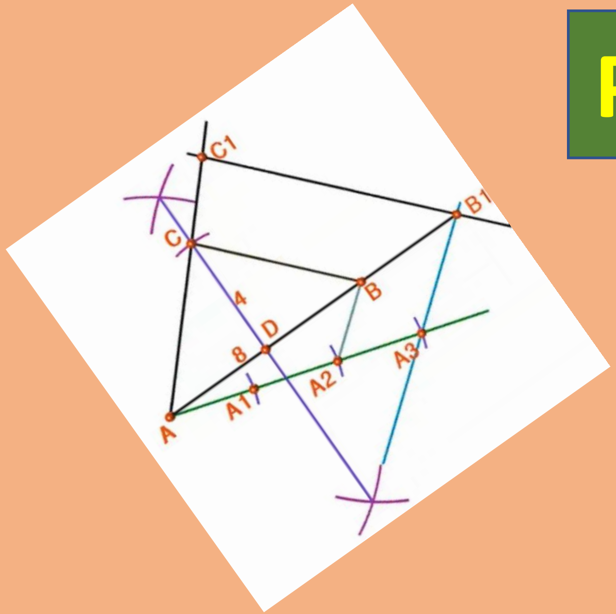


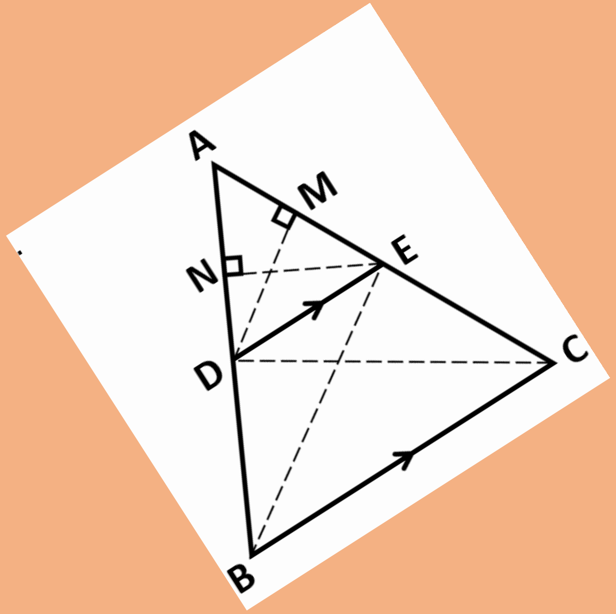




SIMILAR TRIANGLES

Pdf version with video links





Click on this blue logo for 1st part video lesson

Click on this blue logo for 2nd part video lesson

Click on this blue logo for 3rd part video lesson

Click on this blue logo for 4th part video lesson

Click on this blue logo for 5th part video lesson

Click on this blue logo for 6th part video lesson

Click on this blue logo for 7th part video lesson

Click on this blue logo for 8th part video lesson

For construction problems touch on the below images.
Construction video will be played.

Construct an isosceles triangle whose base is 8cm and altitude is 4 cm. Then, draw another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

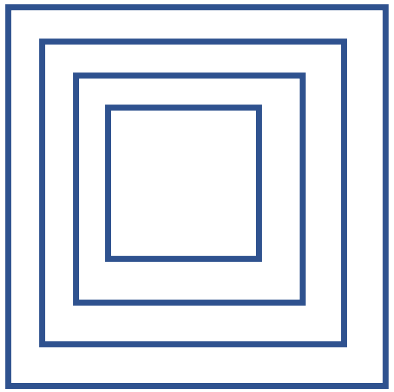
Construct a triangle similar to the given ΔABC, with its sides equal to the $\frac{5}{3}$ of the corresponding sides of the Δ ABC

12. Construct a triangle of sides 4cm, 5 cm and 6 cm. Then, construct a triangle similar to it, whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Draw a line segment of length 7.2 cm and divide it in the ratio 5 : 3. Measure the two parts.

Two polygons of the same number of sides are similar if their corresponding angles are equal and their corresponding sides are in the same rartio or proportion.

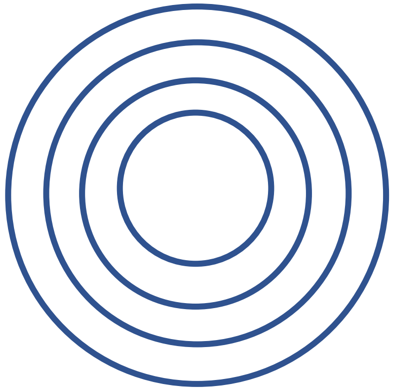
All regular polygons having the same number of sides are always similar.



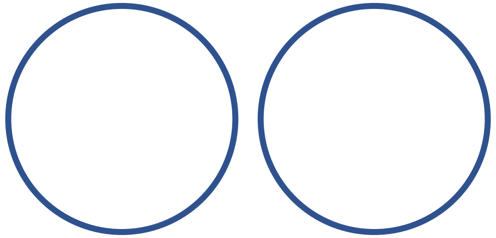
Similar squares

Similar equilateral triangles

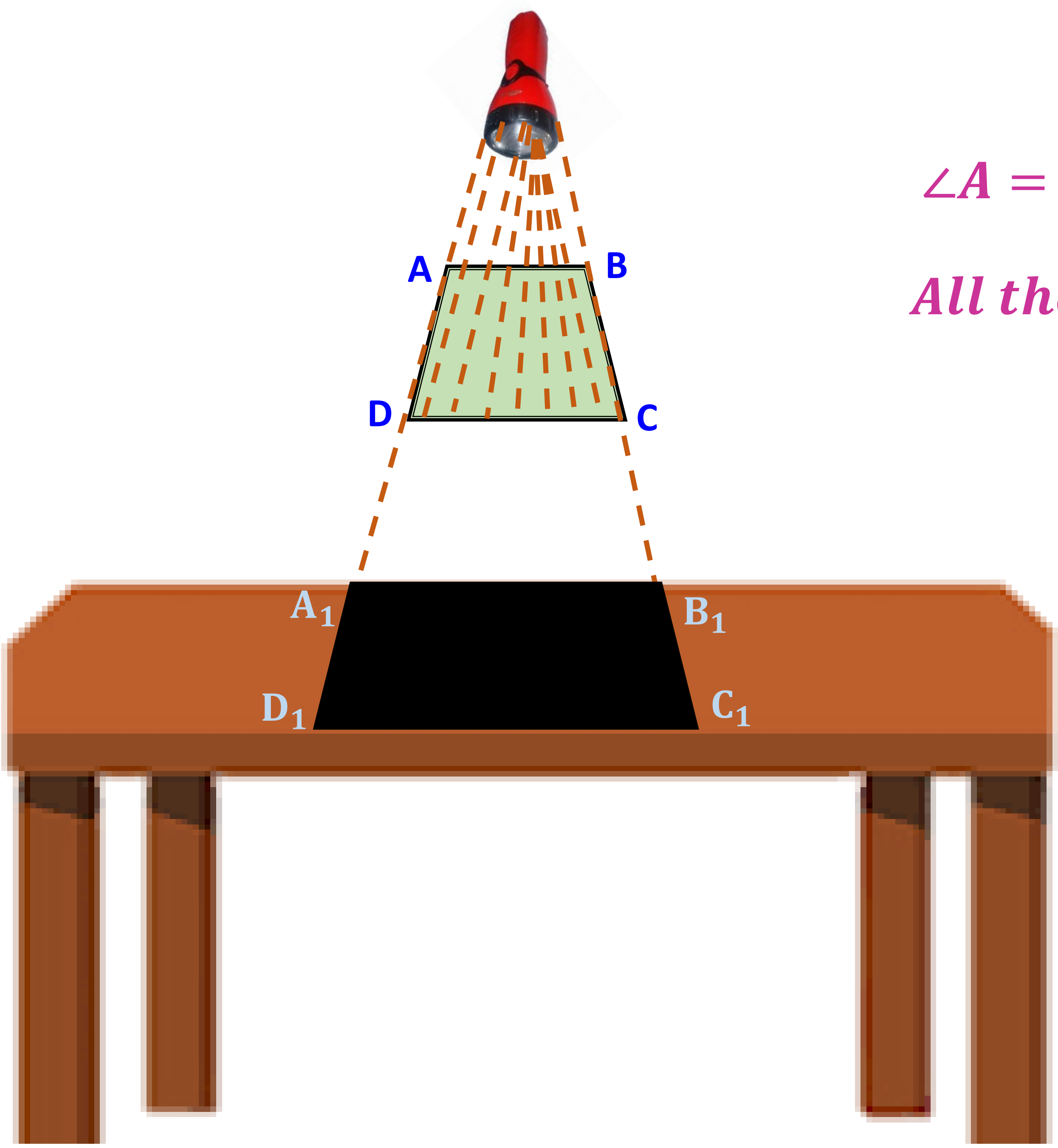
All circles are similar and the circles with same radius are congruent.



Similar circles



congruent circles



$\angle A = \angle A_1, \angle B = \angle B_1, \angle C = \angle C_1, \angle D = \angle D_1,$

All the corresponding angles are equal

$$\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CD}{C_1D_1} = \frac{DA}{D_1A_1}$$

All the corresponding sides are in same ratio

DO THIS

1. Fill in the blanks with similar / not similar.

(i) All squares are ...similar.....................

(ii) All equilateral triangles are similar........................

(iii) All isosceles triangles are ...not similar.........................

(iv) Two polygons with same number of sides are ....similar........... if their corresponding angles are equal and corresponding sides are equal.

(v) Reduced and Enlarged photographs of an object are ..similar...........

(vi) Rhombus and squares are ....not similar........ to each other.

2. Write True / False for the following statements.

(i) Any two similar figures are congruent. False

(ii) Any two congruent figures are similar. True

(iii) Two polygons are similar if their corresponding angles are equal. False

3. Give two different examples of pair of (i) Similar figures (ii) Non similar figures

Similar figures : (i) any two circles (ii) any two squares

Non Similar figures : (i) Rectangle and parallelogram (ii) right angle triangle and equilateral triangle

Similarity of Triangles :

Two triangles are similar if

(i) Corresponding angles are equal and

(ii) Corresponding sides are in the same ratio (in proportion)

In $\Delta\mathrm{ABC}$ and $\Delta\mathrm{DEF}$,

$\angle\mathrm{A} = \angle\mathrm{D}, \angle\mathrm{B} = \angle\mathrm{E}, \angle\mathrm{C} = \angle\mathrm{F}$ and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF} = K$

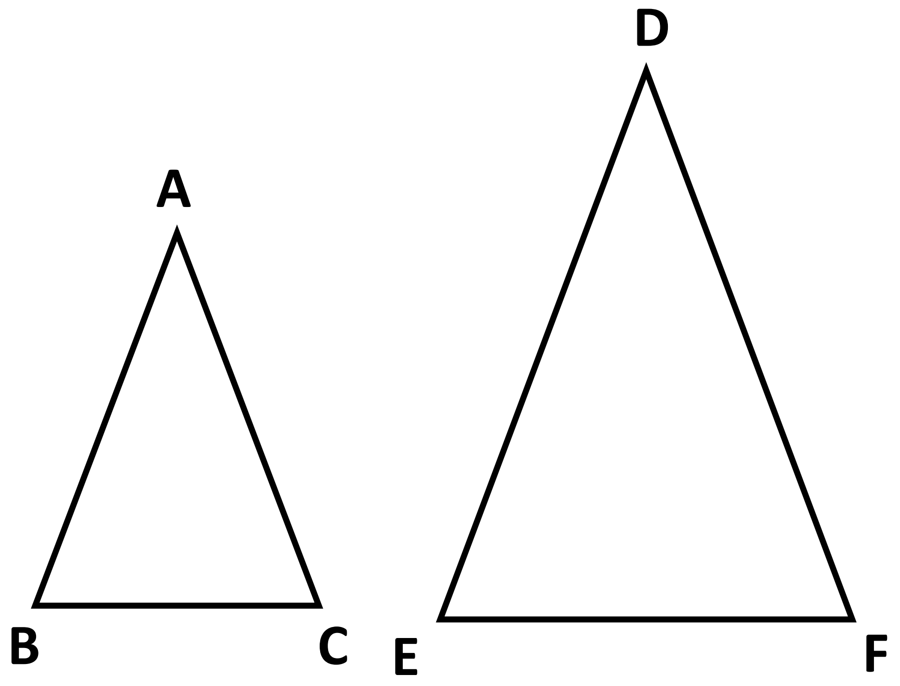
then $\Delta\mathrm{ABC}$ is similar to $\Delta\mathrm{DEF}$

Symbolically we write it as $\Delta\mathrm{ABC} \sim \Delta\mathrm{DEF}$

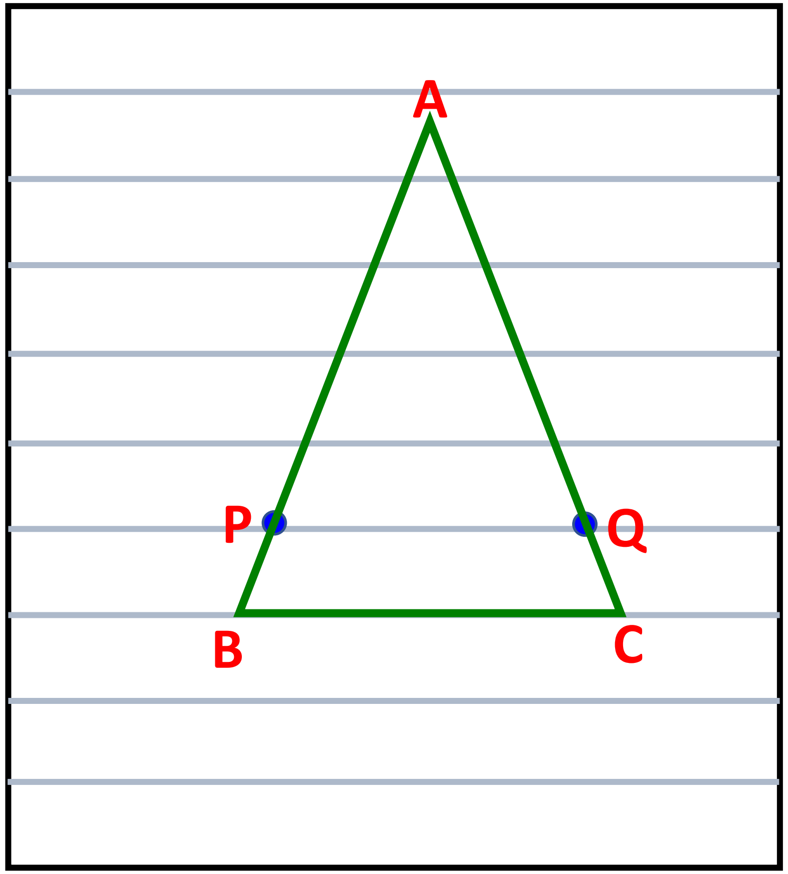
If $K < 1$,then we get reduced figure.

If $K = 1$,then we get congruent figure.

If $K > 1$,then we get enalrged figure.



Activity



$$\frac{AP}{PB} \text{ and } \frac{AQ}{QC}$$

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

$$In\ \Delta ABC, if\ PQ \parallel BC\ then\ \frac{AP}{PB} = \frac{AQ}{QC}$$

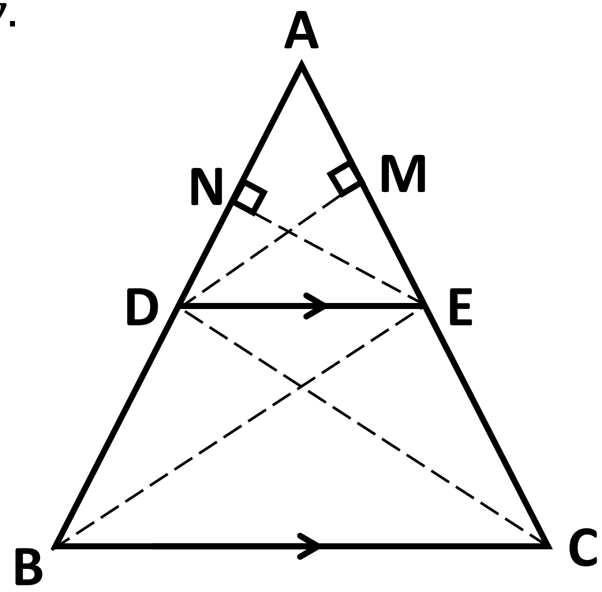
Basic Proportionality Theorem
(Thales Theorem)

Basic Proportionality Theorem (Thales Theorem)

Theorem : *If a line is drawn parallel to one side of a triangle to intersect the other two sides in district points, then the other two sides are divided in the same ratio.*

Given : In ΔABC, $DE \parallel BC$ which intersects sides AB and AC at D and E respectively.

RTP : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$



Construction : Join B, E and C, D and then draw $DM \perp AC$ and $EN \perp AB$.

Proof : Area of $\Delta ADE = \dfrac{1}{2} \times AD \times EN$ and Area of $\Delta BDE = \dfrac{1}{2} \times DB \times EN$

$$\text{Now, } \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\cancel{\frac{1}{2}} \times AD \times \cancel{EN}}{\cancel{\frac{1}{2}} \times DB \times \cancel{EN}} = \frac{AD}{DB} \longrightarrow (1)$$

Also, Area of $\Delta ADE = \dfrac{1}{2} \times AE \times DM$

and Area of $\Delta CDE = \dfrac{1}{2} \times EC \times DM$

$$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\cancel{\frac{1}{2}} \times AE \times \cancel{DM}}{\cancel{\frac{1}{2}} \times EC \times \cancel{DM}} = \frac{AE}{EC} \longrightarrow (2)$$

But we can observe that ΔBDE and ΔCDE are on same base BC and between the parallel lines DE and BC

so that area of $\Delta BDE =$ area of $\Delta CDE \longrightarrow (3)$

From equations $(1), (2)$ and (3) we can write

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved.

Another form of Basic Proportionality Theorem

$$\frac{AD}{DB} = \frac{AE}{EC}$$

by adding 1 *on both sides,*

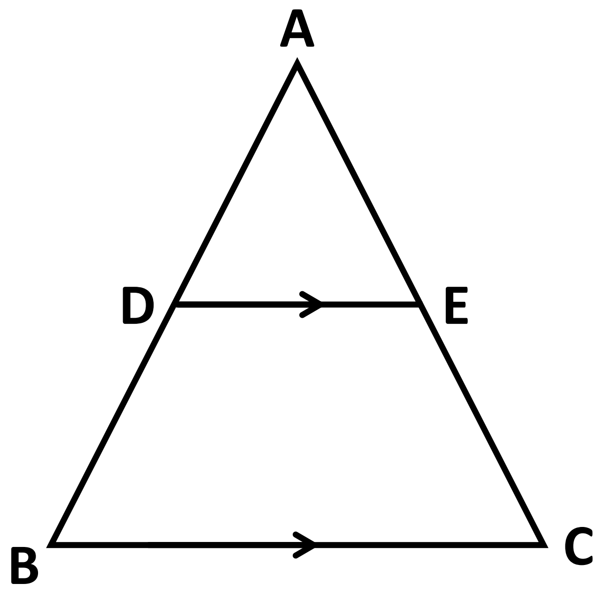
$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\frac{AD + DB}{DB} = \frac{AE + EC}{EC}$$

$$\boxed{\frac{AB}{DB} = \frac{AC}{EC}}$$

we can also write this as

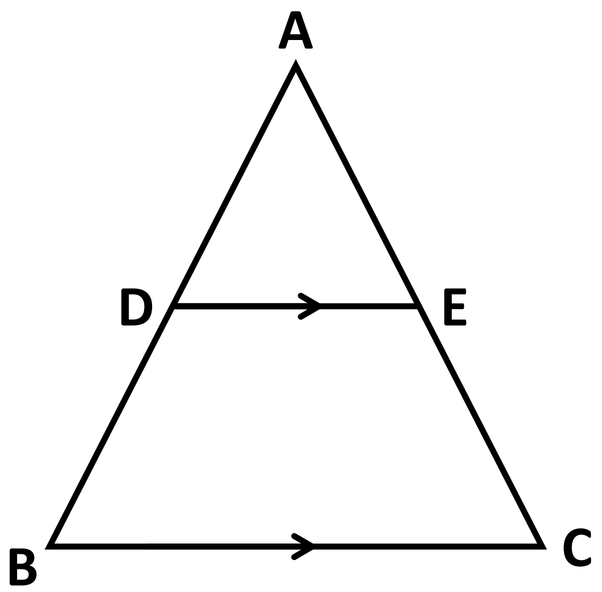
$$\boxed{\frac{AD}{AB} = \frac{AE}{AC}}$$



Converse of Basic Proportionality Theorem

Theorem : *If a line diides two sides of a triangle in the same ratio. then the line is parallel to the third side.*

$$if\ \frac{AD}{DB} = \frac{AE}{EC}\ then\ DE \parallel BC$$



TRY THIS :

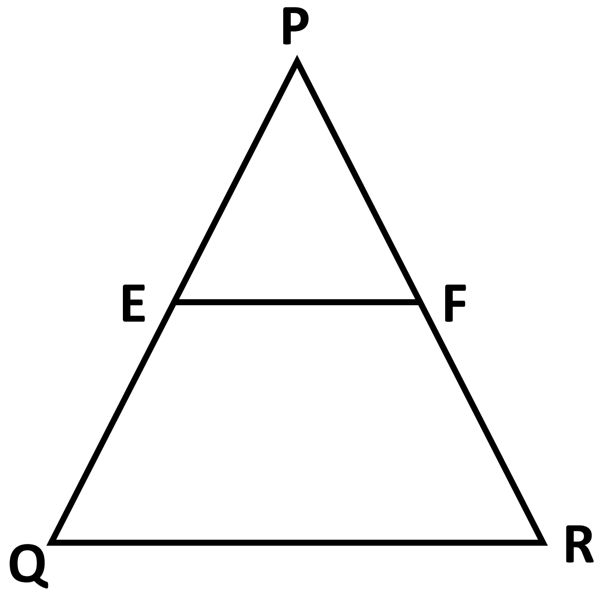
1. In triangle DPQR, E and F are points on the sides PQ and PR respectively. For each of the following, state whether EF ||QR or not?

(i) PE = 3.9 cm EQ = 3 cm PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.

(iii) PQ = 1.28 cm PR = 2.56 cm PE = 1.8 cm and PF = 3.6 cm

(i) $Given\ PE = 3.9\ cm, EQ = 3\ cm, PF = 3.6\ cm\ and\ FR = 2.4\ cm$



$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3 \quad and \quad \frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2}$$

$$\frac{PE}{EQ} \neq \frac{PF}{FR}$$

$$\therefore In\ this\ case\ EF \nparallel QR$$

TRY THIS :

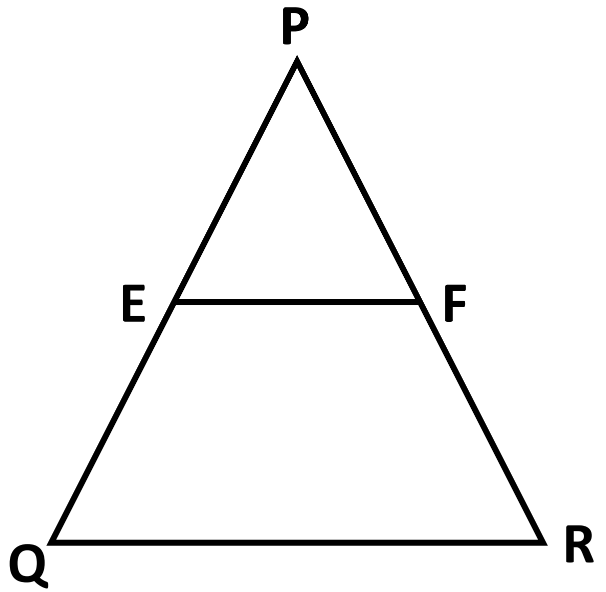
1. In triangle DPQR, E and F are points on the sides PQ and PR respectively. For each of the following, state whether EF ||QR or not?

(i) PE = 3.9 cm EQ = 3 cm PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.

(iii) PQ = 1.28 cm PR = 2.56 cm PE = 1.8 cm and PF = 3.6 cm

(ii) $Given\ PE = 4\ cm, EQ = 4.5\ cm, PF = 8\ cm\ and\ FR = 9\ cm$



$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9} \quad and \quad \frac{PF}{FR} = \frac{8}{9}$$

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$$\therefore In\ this\ case\ EF \parallel QR \quad (\because converse\ of\ B.P.T.)$$

TRY THIS :

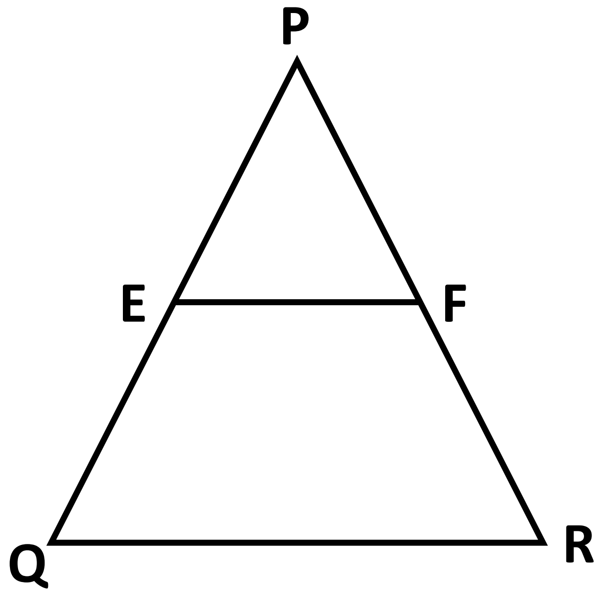
1. In triangle DPQR, E and F are points on the sides PQ and PR respectively. For each of the following, state whether EF | |QR or not?

(i) PE = 3.9 cm EQ = 3 cm PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.

(iii) PQ = 1.28 cm PR = 2.56 cm PE = 1.8 cm and PF = 3.6 cm

(iii) $Given\ PQ = 1.28\ cm, PR = 2.56\ cm, PE = 1.8\ cm\ and\ PF = 3.6\ cm$



$$\frac{PQ}{PE} = \frac{1.28}{1.8} = \frac{128}{180} = \frac{32}{45} \quad and \quad \frac{PR}{PF} = \frac{2.56}{3.6} = \frac{256}{360} = \frac{32}{45}$$

$$\frac{PQ}{PE} = \frac{PR}{PF}$$

$\therefore$ *In this case* $EF \parallel QR$ $(\because$ *converse of B.P.T.*$)$

TRY THIS :

1. In the following figures, DE ||BC.

(ii) Find AD.

$$Given\ AE = 1.8\ cm, EC = 5.4\ cm, DB = 7.2\ cm$$

$$also, DE \parallel BC \quad so\ that\ \frac{AD}{DB} = \frac{AE}{EC} \ (\because\ B.P.T.)$$

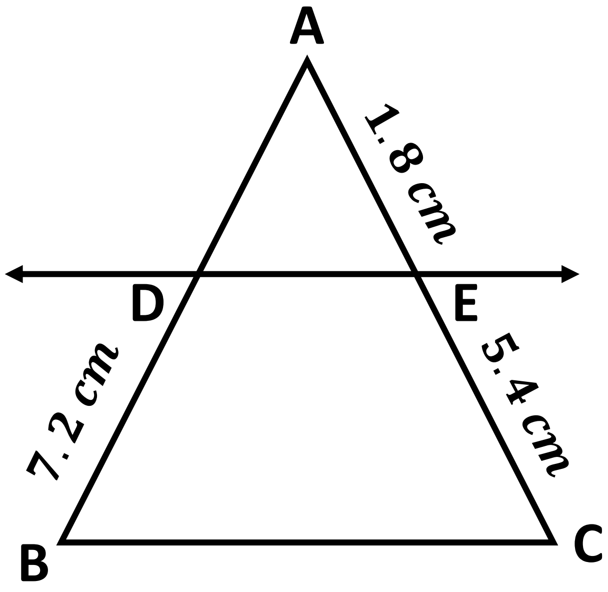
$$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\Rightarrow \frac{AD}{7.2} = \frac{1}{3}$$

$$\Rightarrow AD = \frac{7.2}{3}$$

$$\Rightarrow AD = 2.4$$

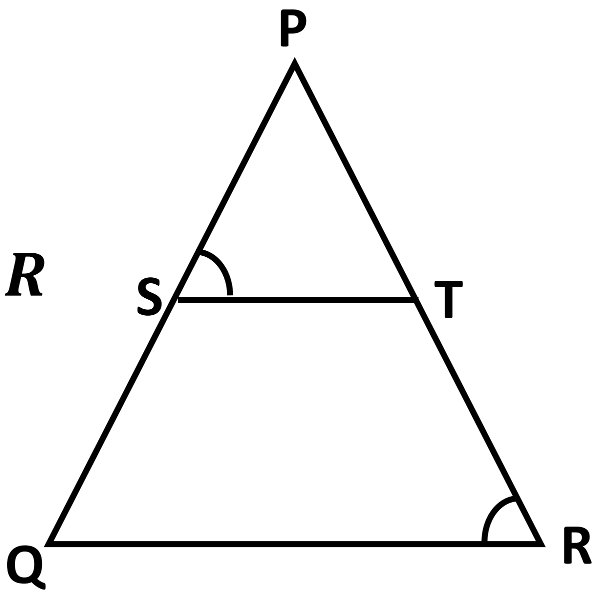
$$\therefore AD = 2.4\ cm$$



EXERCISE – 8.1

1. $In\ \Delta PQR, ST\ is\ a\ line\ such\ that\ \frac{PS}{SQ} = \frac{PT}{TR}\ and\ also\ \angle PST = \angle PRQ$

$Prove\ that\ \Delta PQR\ is\ an\ isosceles\ triangle.$



$Sol:$ $Given\ that\ in\ \Delta PQR, \frac{PS}{SQ} = \frac{PT}{TR}$

$By\ converse\ of\ Basic\ Proportionality\ theorem, ST \parallel QR$

$Now, ST \parallel QR\ and\ PQ\ is\ a\ transversal.$

$then\ \angle PST = \angle PQR (\because corresponding\ angles)$ ⟶ (1)

$also\ given\ \angle PST = \angle PRQ$ ⟶ (2)

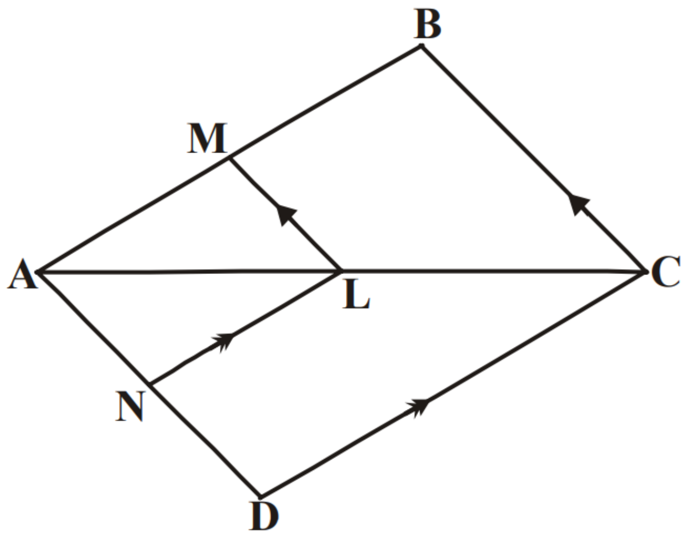
$From\ equations\ (1)\ and\ (2), \angle PQR = \angle PRQ$

$\therefore \Delta PQR\ is\ an\ isosceles\ triangle.$

EXERCISE – 8.1

2. $In\ the\ given\ figure, LM \parallel CB\ and\ LN \parallel CD. Prove\ that\ \frac{AM}{AB} = \frac{AN}{AD}.$

Sol : $In\ \Delta ABC, LM \parallel CB$



$By\ Basic\ Proportionality\ theorem,$

$$\frac{AM}{AB} = \frac{AL}{AC} \longrightarrow (1)$$

$In\ \Delta ADC, LN \parallel CD$

$By\ Basic\ Proportionality\ theorem,$

$$\frac{AL}{AC} = \frac{AN}{AD} \longrightarrow (2)$$

$From\ equations\ (1)\ and\ (2), \frac{AM}{AB} = \frac{AN}{AD}$

$Hence\ proved.$

EXERCISE – 8.1

3. *In the given figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.*

Sol : $In\ \Delta ABE, DF \parallel AE$

By Basic Proportionality theorem,

$$\frac{BD}{DA} = \frac{BF}{FE} \longrightarrow (1)$$

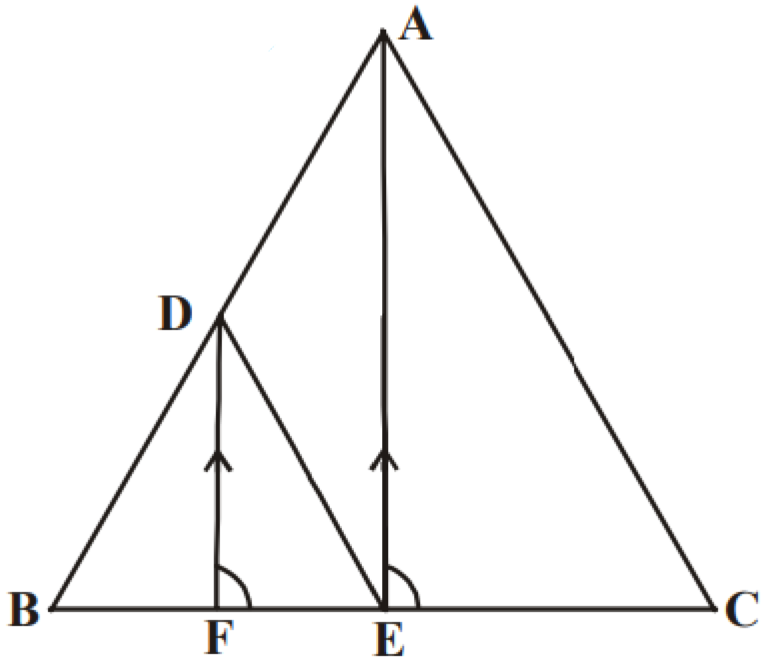
$In\ \Delta ABC, DE \parallel AC$

By Basic Proportionality theorem,

$$\frac{BD}{DA} = \frac{BE}{EC} \longrightarrow (2)$$

From equations (1) *and* (2), $\frac{BF}{FE} = \frac{BE}{EC}$

Hence proved.



EXERCISE $- 8.1$

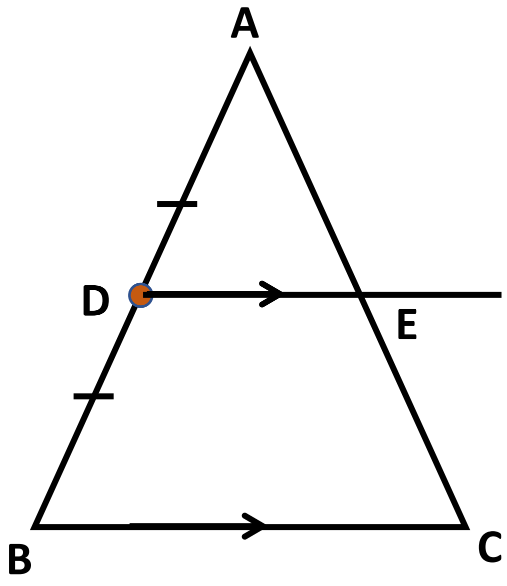
4. *Prove that a line drawn through the mid − point of one side of a triangle parallel to another side bisecets the third side.*

Sol : $In\ \Delta ABC, let\ D\ be\ the\ mid\ point\ of\ AB.$

$then\ AD = DB$

$$\Rightarrow \frac{AD}{DB} = 1 \longrightarrow (1)$$

$Draw\ a\ line\ through\ D, parallel\ to\ BC\ meets\ AC\ at\ E$



$By\ Basic\ Proportionality\ theorem,$

$$\frac{AD}{DB} = \frac{AE}{EC} \longrightarrow (2)$$

$$From\ equations\ (1)\ and\ (2), \frac{AE}{EC} = 1$$

$$\Rightarrow AE = EC$$

$$i.e. DE\ bisects\ AC$$

$\therefore A\ line\ drawn\ through\ mid\ point\ of\ one\ side\ of\ a\ tringle\ parallel\ to\ another\ side\ bisects\ the\ third\ side.$

EXERCISE – 8.1

5. ***Prove that a line joining the mid points of any two sides of a triangle is parallel to the third side.***

Sol : *In ΔABC, let D be the mid point of AB.*

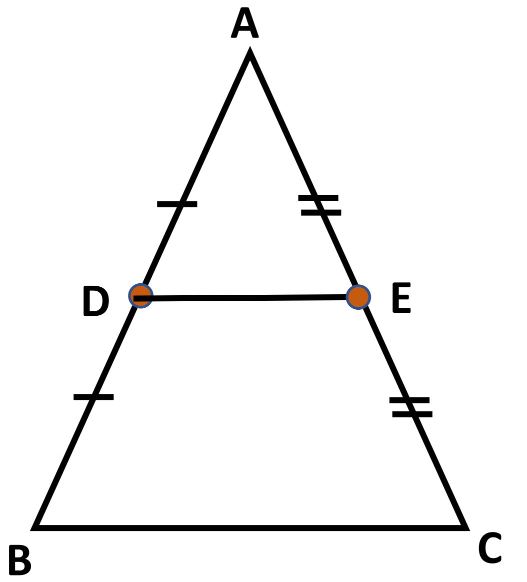
then $AD = DB$

$$\Rightarrow \frac{AD}{DB} = 1 \longrightarrow (1)$$

In ΔABC, let E be the mid point of AC.

then $AE = EC$

$$\Rightarrow \frac{AE}{EC} = 1 \longrightarrow (2)$$



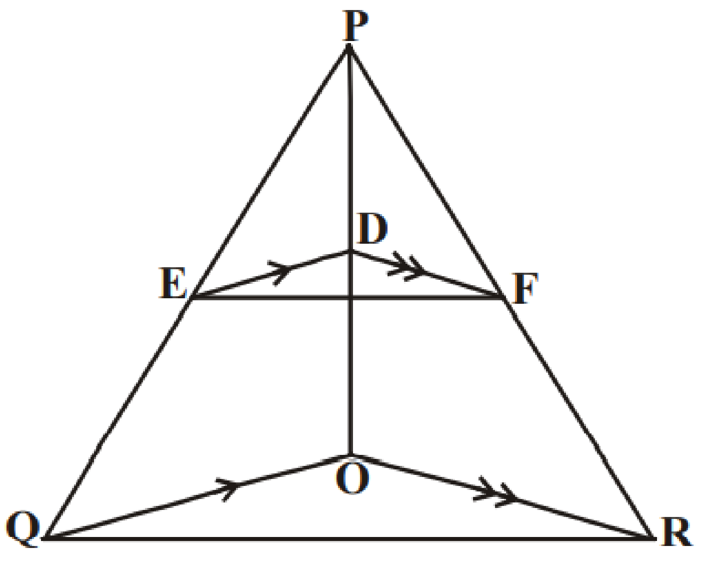
From equations (1) *and* (2), $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

i.e. DE divides AB and AC in the same ratio

By the converse of B.P.T, $DE \parallel BC$

$\therefore$ *A line joining the mid points of any two sides of a traingle is parallel to the third side.*

6. In the given figure, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.



Sol : In ΔPOQ, $DE \parallel OQ$

By Basic Proportionality theorem,

$$\frac{PE}{EQ} = \frac{PD}{DO} \longrightarrow (1)$$

In ΔPOR, $DF \parallel OR$

By Basic Proportionality theorem,

$$\frac{PD}{DO} = \frac{PF}{FR} \longrightarrow (2)$$

From equations (1) and (2), $\dfrac{PE}{EQ} = \dfrac{PF}{FR}$

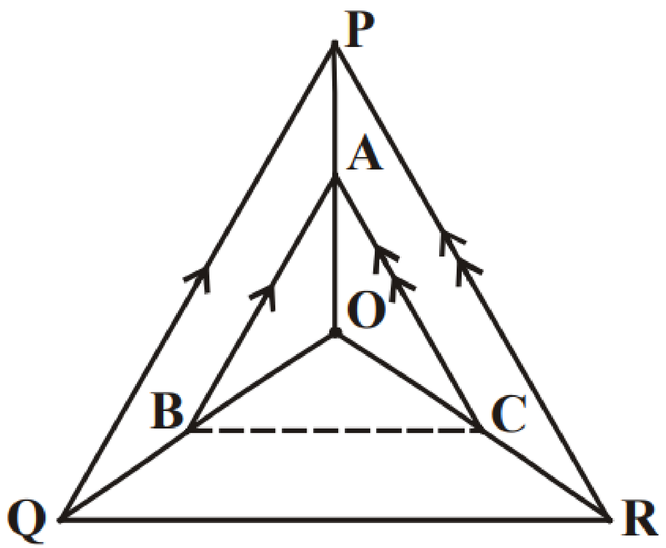
i.e. In ΔPQR, EF divides PQ and QR in the same ratio.

By converse of Basic Proportionality theorem,

$$EF \parallel QR$$

Hence proved.

7. In the adjacent figure, A, B and C are points on OP, OQ and OR respectively such that AB ∥ PQ and AC ∥ PR. Show that BC ∥ QR.



Sol : In ΔOPQ, $AB \parallel PQ$

By Basic Proportionality theorem,

$$\frac{OA}{AP} = \frac{OB}{BQ} \longrightarrow (1)$$

In ΔOPR, $AC \parallel PR$

By Basic Proportionality theorem,

$$\frac{OA}{AP} = \frac{OC}{CR} \longrightarrow (2)$$

From equations (1) and (2), $\dfrac{OB}{BQ} = \dfrac{OC}{CR}$

i.e. In ΔOPR, BC divides OQ and OR in the same ratio.

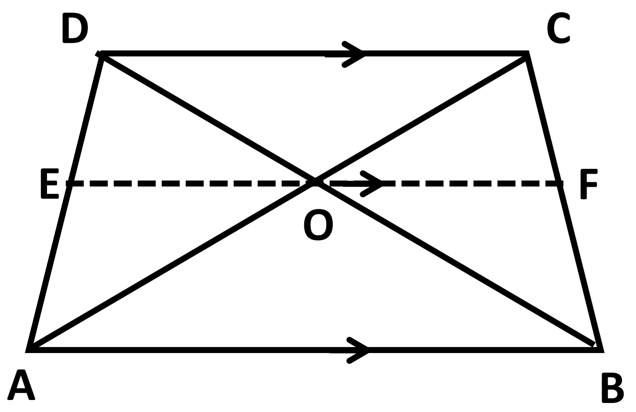
By converse of Basic Proportionality theorem,

$BC \parallel QR$

Hence proved.

EXERCISE $-$ 8.1

8. $ABCD$ *is a trapezium in which* $AB \parallel DC$ *and its diagonals intersect each other at point* $'O'$. *Show that* $\dfrac{AO}{BO} = \dfrac{CO}{DO}$



Sol : *Draw a line EF through* $'O'$ *and parallel to AB and DC*

$\text{In } \Delta ACD, EO \parallel DC$

By Basic Proportionality theorem,

$$\frac{AO}{CO} = \frac{AE}{DE} \longrightarrow (1)$$

$\text{In } \Delta ABD, EO \parallel AB$

By Basic Proportionality theorem,

$$\frac{AE}{DE} = \frac{BO}{DO} \longrightarrow (2)$$

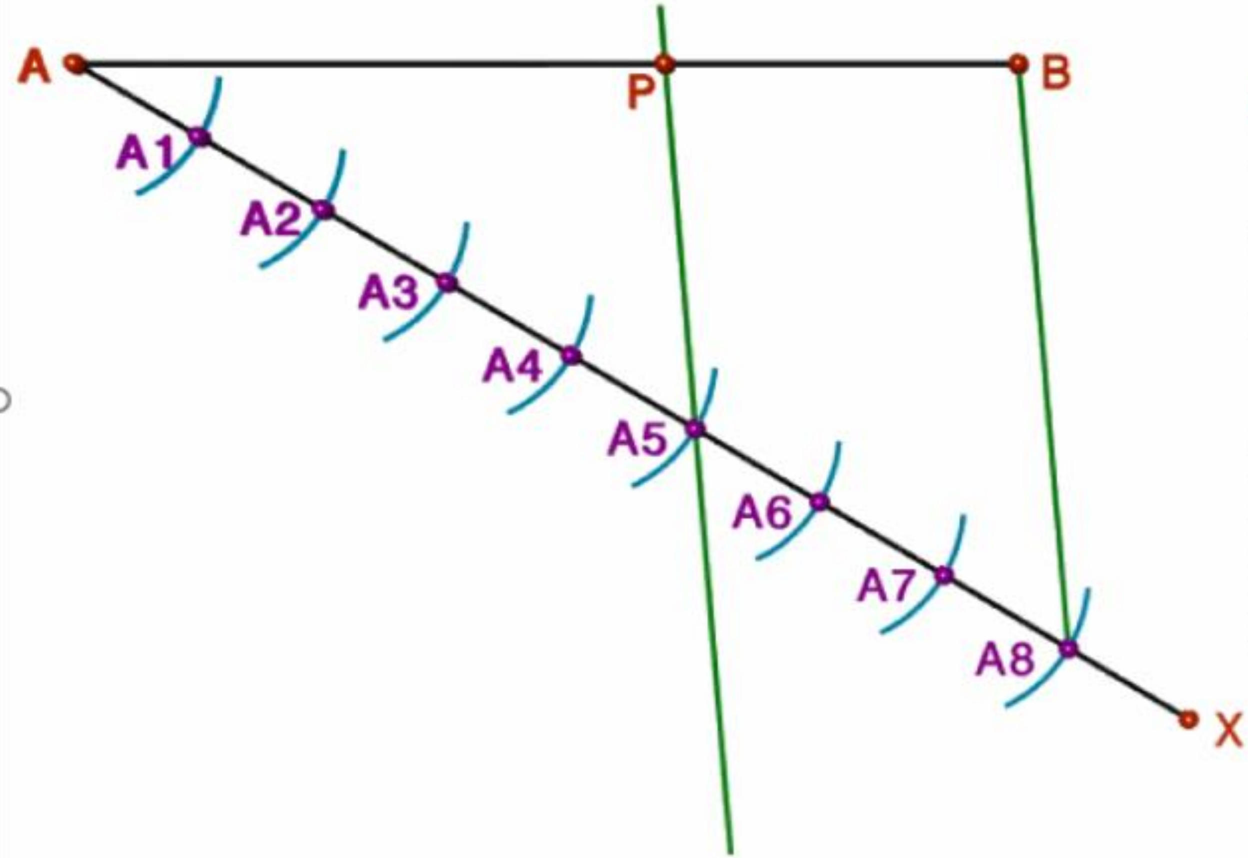
From equations (1) *and* (2), $\dfrac{AO}{CO} = \dfrac{BO}{DO}$

$$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

Hence proved.

9. Draw a line segment of length 7.2 cm and divide it in the ratio 5 : 3. Measure the two parts.

Sol:



Steps of construction :

1. Draw a line segment AB of length 7.2 cm .
2. Draw a ray AX which making an acute angle with AB .
3. Mark off 5+3 = 8 equl parts (A 1,A2 , ...A8) on AX with same radius .
4. Join A8 and B .
5. Draw a line parallel to A8B at A5 meeting AB at P .
6. P is the required point which divides AB in the ratio 5 : 3 .
7. By measuring with scale we can oberve that AP = 4.5 cm and PB = 2.7 cm .

AAA CRITERION FOR SIMILARITY OF TRAINGLES

Theorem : *In two triangles, if the angles are equal, then the sides opposite to the equal angles are in the same ratio and hence the two triangles are similar*

Given : In triangles ABC and DEF, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

RTP : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Construction : Locate points P and Q on DE and DF respectively, such that $AB = DP$ and $AC = DQ$. Join PQ

Proof : In ΔABC and ΔDPQ,

$AB = DP$ ($\because$ construction)

$\angle A = \angle D$ ($\because$ given)

$AC = DQ$ ($\because$ construction)

then, $\Delta ABC \cong \Delta DPQ$ ($\because$ SAS congruency)

$\Rightarrow \angle B = \angle P$ ($\because$ CPCT)

but $\angle B = \angle E$ ($\because$ given)

so that $\angle P = \angle E$

$\Rightarrow PQ \parallel EF$ ($\because$ corresponding angles are equal)

then, $\frac{DP}{DE} = \frac{DQ}{DF}$ ($\because$ B.P.T)

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF} \quad (\because DP = AB,\ DQ = AC) \longrightarrow (1)$$

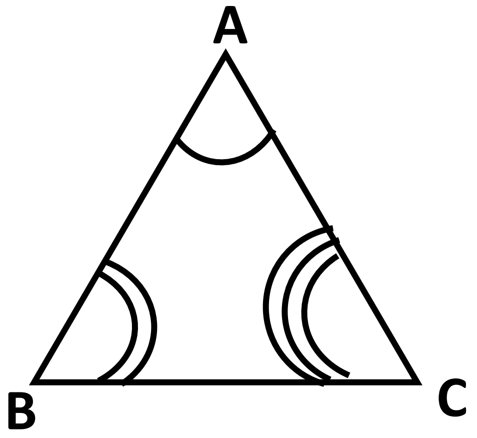
similary we can show that

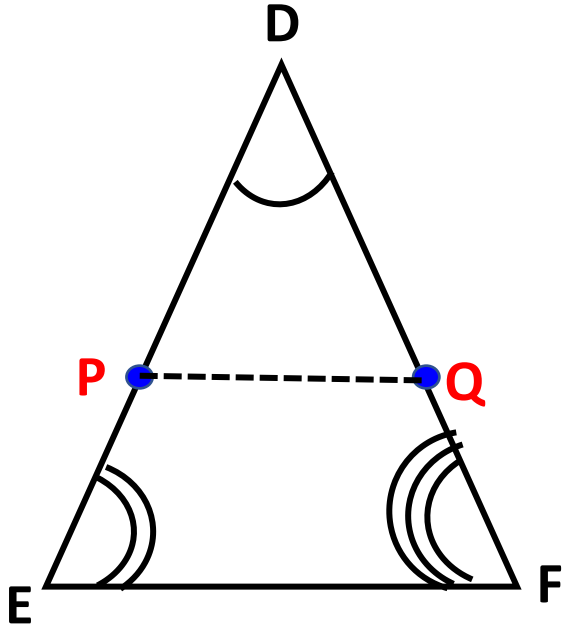
$$\frac{AB}{DE} = \frac{BC}{EF} \longrightarrow (2)$$

from eq. (1) and (2)

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Hence proved.





AAA CRITERION FOR SIMILARITY OF TRAINGLES

Theorem : *In two triangles, if the angles are equal, then the sides opposite to the equalangles are in the same ratio and hence the two triangles are similar*

If two angles of a triangle are respectively equal to the two angles of another triangle, then by the angle sum property, third angles will also be equal.

So, AA similarity criterion stated as if two angles of one triangle are respectvely equal to the two angles of another triangle, then the two triangles are similar.

Converse : *In two triangles, if the ratio of corresponding sides are equal, then corresponding angles are equal*

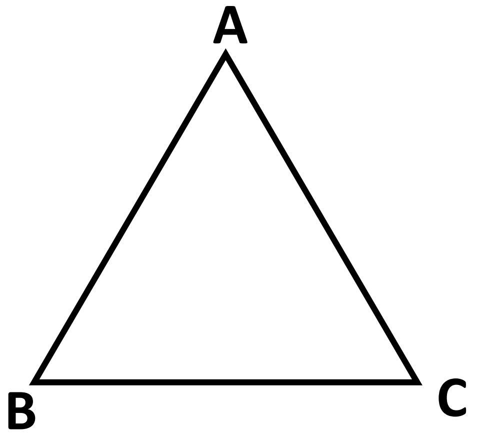
SSS CRITERION FOR SIMILARITY OF TRAINGLES

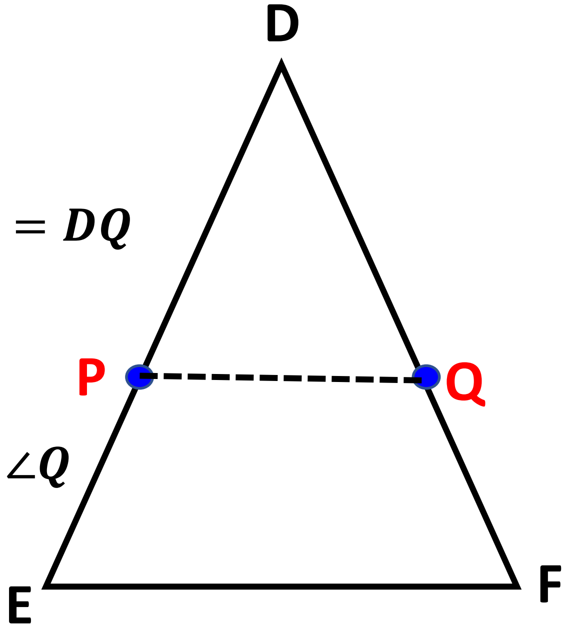
Theorem : In two triangles, if the sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal hence the triangles are similar

Given : In triangles ABC and DEF, $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

RTP : $\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$

Construction : Locate points P and Q on DE and DF respectively, such that $AB = DP$ and $AC = DQ$. Join PQ





Proof : In ΔABC and ΔDPQ,

$$\frac{AB}{DE} = \frac{AC}{DF} \quad (\because \text{given})$$

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} \quad (\because DP = AB,\ DQ = AC)$$

$$\Rightarrow PQ \parallel EF \quad (\because \text{converse of B.P.T.})$$

so, $\angle P = \angle E$ and $\angle Q = \angle F$ $(\because$ corresponding anles$)$

then, $\Delta DPQ \sim \Delta DEF$ $(\because$ AAA similarity$)$

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF} \longrightarrow (1)$$

also we have, $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$ $(\because$ given$)$

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} = \frac{BC}{EF} \longrightarrow (2)$$

$$(\because DP = AB,\ DQ = AC)$$

from eq. (1) and (2)

$$\frac{PQ}{EF} = \frac{BC}{EF} \quad \Rightarrow BC = PQ$$

also we have, $AB = DP$ and $AC = DQ$

by SSS congruency,

$$\Delta ABC \cong \Delta DPQ,$$

$\Rightarrow \angle A = \angle D, \angle B = \angle P$ and $\angle C = \angle Q$

$(\because CPCT)$

but, $\angle P = \angle E$, $\angle Q = \angle F$

$\therefore \angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$

Hence proved.

SAS CRITERION FOR SIMILARITY OF TRAINGLES

Theorem : *If one angle of a triangle is to one angle of the other triangle and the including sides of these angles are proportional, then the two triangles are similar.*

Given : In triangles ABC and DEF, $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ and $\angle A = \angle D$

RTP :, $\Delta\, ABC \sim \Delta DEF$

Construction : Locate points P and Q on DE and DF respectively , such that $AB = DP$ and $AC = DQ$. Join PQ

Proof : In $\Delta\, ABC$ and ΔDPQ,

$$\frac{AB}{DE} = \frac{AC}{DF} \quad (\because given)$$

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} \quad (\because DP = AB,\ DQ = AC)$$

$$\Rightarrow PQ \parallel EF \quad (\because \text{converse of } B.P.T.)$$

so, $\angle P = \angle E$ and $\angle Q = \angle F$ $(\because$ corresponding anles$)$

then, $\Delta\, DPQ \sim \Delta DEF$ $(\because$ AAA similarity$)$

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF} \longrightarrow (1)$$

also we have, $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$ $(\because$ given$)$

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} = \frac{BC}{EF} \longrightarrow (2)$$

$$(\because DP = AB,\ DQ = AC)$$

from eq. (1) and (2)

$$\frac{PQ}{EF} = \frac{BC}{EF} \quad \Rightarrow BC = PQ$$

also we have, $AB = DP$ and $AC = DQ$

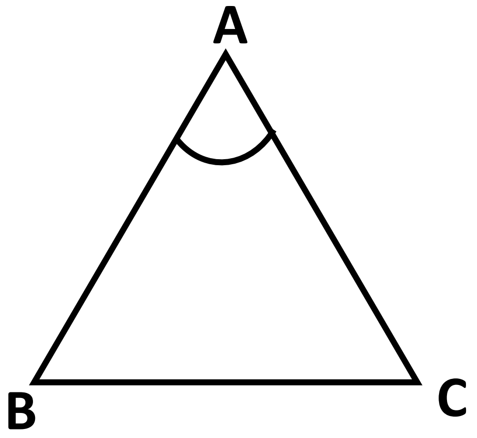
by SSS congruency,

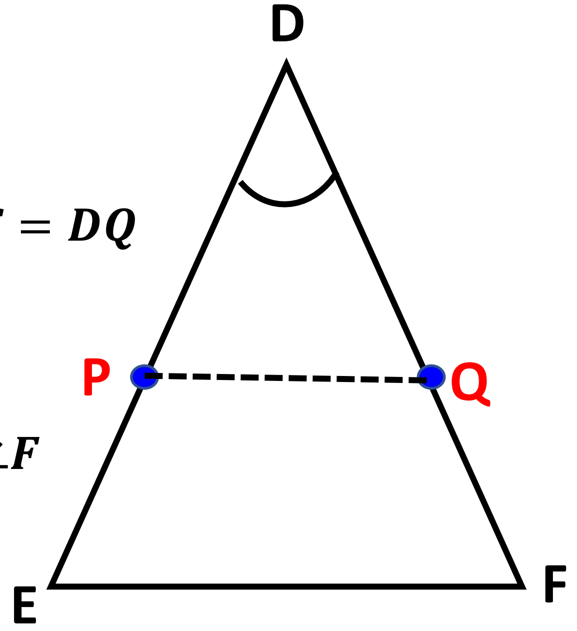
$$\Delta\, ABC \cong \Delta DPQ,$$

$\therefore \angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$

$$(\because CPCT)$$

Hence proved.

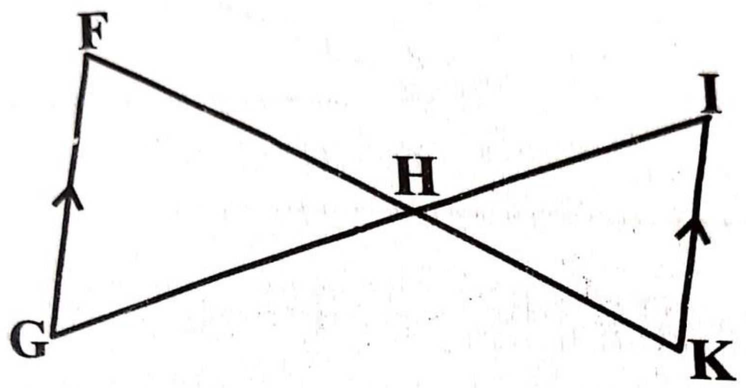




TRY THIS :

1. Are triangles formed in each figure similar? If so, name the criterion of similarity. Write the similarity relation in symbolic form.

(i) $In\ \Delta FGH\ and\ \Delta IKH,$



$\angle FHG = \angle IHK \quad (\because vertically\ opposite\ angles)$

$as\ FG \parallel IK,$

$\angle F = \angle K \quad (\because alternate\ interior\ angles)$

$also\ we\ have$

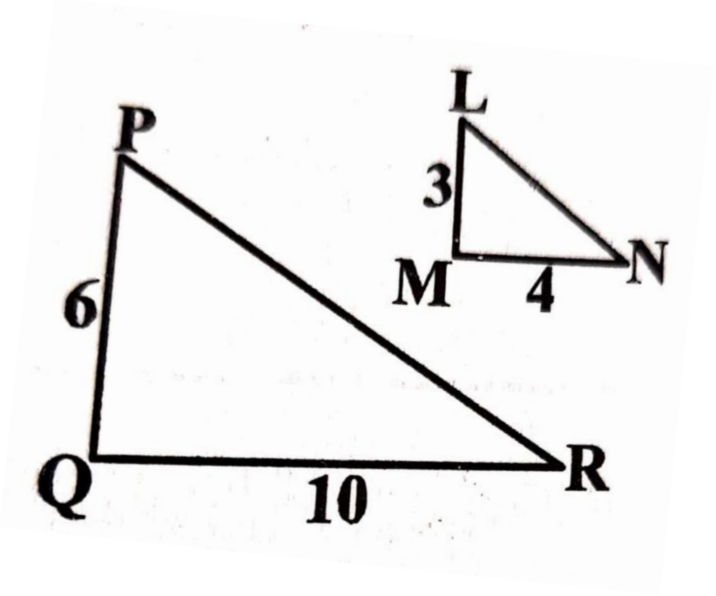
$\angle G = \angle I \quad (\because alternate\ interior\ angles)$

$\therefore By\ AAA\ similarity,$

$\Delta FHG \sim \Delta KHI$

TRY THIS :

1. Are triangles formed in each figure similar? If so, name the criterion of similarity. Write the similarity relation in symbolic form.



(ii) $In\ \Delta PQR\ and\ \Delta LMN,$

$$\frac{PQ}{LM} = \frac{6}{3} = 2$$

$$\frac{QR}{MN} = \frac{10}{4} = \frac{5}{2}$$

$$\frac{PQ}{LM} \neq \frac{QR}{MN}$$

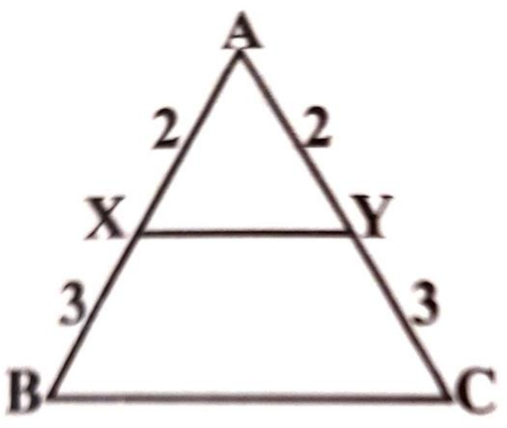
$i.e.\ corresponding\ sides\ are\ not\ proportional$

$\Delta PQR\ and\ \Delta LMN\ are\ not\ similar.$

TRY THIS :

1. Are triangles formed in each figure similar? If so, name the criterion of similarity. Write the similarity relation in symbolic form.

(iii) $In\ \Delta AXY\ and\ \Delta ABC,$



$$\frac{AX}{AB} = \frac{AX}{AX + XB} = \frac{2}{2+3} = \frac{2}{5}$$

$$\frac{AY}{AC} = \frac{AY}{AY + YC} = \frac{2}{2+3} = \frac{2}{5}$$

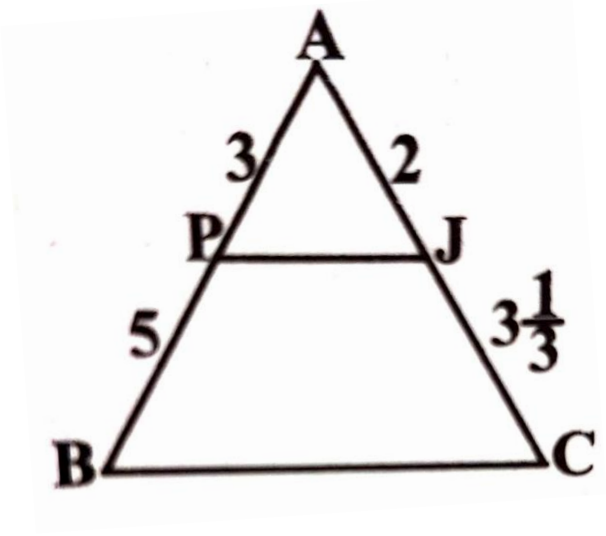
$$\frac{AX}{AB} = \frac{AY}{AC} \qquad also\ \angle A = \angle A\ (\because common\ angles)$$

$i.e.\ in\ the\ two\ triangles,\ one\ angle\ is\ equal\ and\ included\ sides\ are\ in\ proportion$

$\therefore By\ SAS\ similarity,\ \Delta AXY \sim \Delta ABC$

TRY THIS :

1. Are triangles formed in each figure similar? If so, name the criterion of similarity. Write the similarity relation in symbolic form.



(iv) $In\ \Delta APJ\ and\ \Delta ABC,$

$$\frac{AP}{AB} = \frac{AP}{AP + PB} = \frac{3}{3+5} = \frac{3}{8}$$

$$\frac{AJ}{AC} = \frac{AJ}{AJ + JC} = \frac{2}{2 + 3\frac{1}{3}} = \frac{2}{\left(\frac{16}{3}\right)} = 2 \times \frac{3}{16} = \frac{3}{8}$$

$$\frac{AP}{AB} = \frac{AJ}{AC} \quad also\ \angle A = \angle A\ (\because common\ angles)$$

$i.e.\ in\ the\ two\ triangles, one\ angle\ is\ equal\ and\ included\ sides\ are\ in\ proportion$

$$\therefore By\ SAS\ similarity,\ \ \Delta APJ \sim \Delta ABC$$

TRY THIS :

1. Are triangles formed in each figure similar? If so, name the criterion of similarity. Write the similarity relation in symbolic form.

(v) $In\ \Delta OAQ\ and\ \Delta OBP,$

$\angle A = \angle B \quad (= 90°)$

$\angle AOQ = \angle BOP \ (\because vertically\ opposite\ angles)$

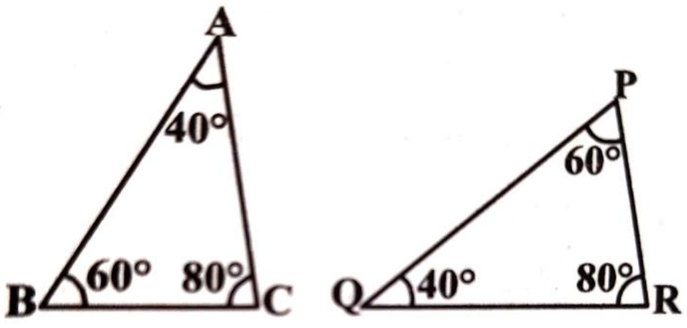
$i.e.\ in\ the\ two\ triangles, two\ corresponding\ angles\ are\ equal.$

$\therefore By\ AA\ similarity,\ \Delta OAQ \sim \Delta OBP$

TRY THIS :

1. Are triangles formed in each figure similar? If so, name the criterion of similarity. Write the similarity relation in symbolic form.

(vi) $In\ \Delta ABC\ and\ \Delta PQR,$



$\angle A = \angle Q \quad (= 40°)$

$\angle B = \angle P \quad (= 60°)$

$\angle C = \angle R \quad (= 80°)$

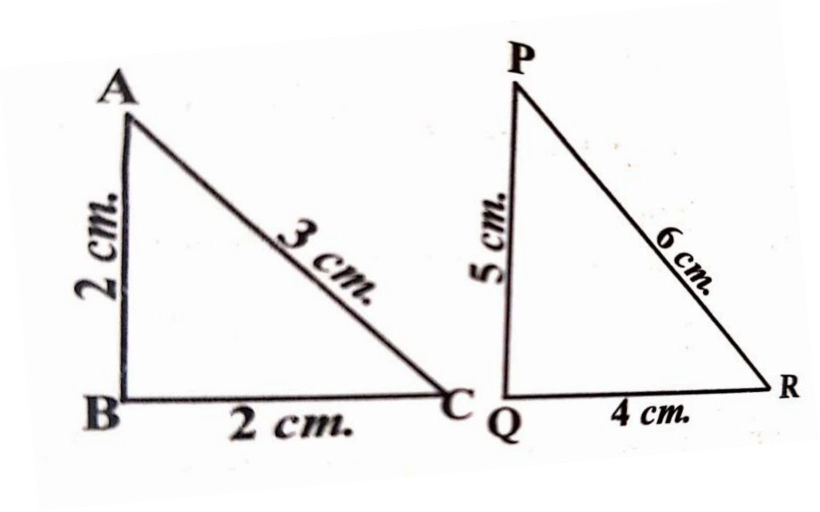
$i.e.\ in\ \Delta ABC\ and\ \Delta QPR,\ all\ corresponding\ angles\ are\ equal.$

$\therefore\ By\ AAA\ similarity, \Delta ABC \sim \Delta QPR$

TRY THIS :

1. Are triangles formed in each figure similar? If so, name the criterion of similarity. Write the similarity relation in symbolic form.

(vii) $In\ \Delta ABC\ and\ \Delta PQR,$



$$\frac{AB}{PQ} = \frac{2}{5}$$

$$\frac{BC}{QR} = \frac{1}{4} = \frac{1}{2}$$

$$\frac{AB}{PQ} \neq \frac{BC}{QR}$$

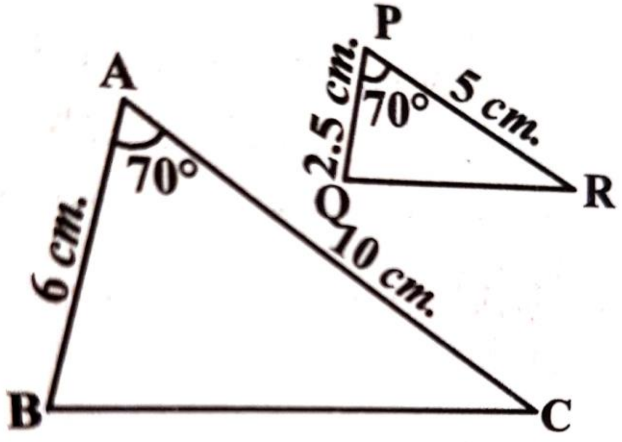
$i.e.\ corresponding\ sides\ are\ not\ proportional$

$\Delta ABC\ and\ \Delta PQR\ are\ not\ similar.$

TRY THIS :

1. Are triangles formed in each figure similar? If so, name the criterion of similarity. Write the similarity relation in symbolic form.

(viii) $In\ \Delta ABC\ and\ \Delta PQR,$



$$\frac{AB}{PQ} = \frac{6}{2.5} = \frac{60}{25} = \frac{12}{5}$$

$$\frac{AC}{PR} = \frac{10}{5} = 2$$

$$\frac{AB}{PQ} \neq \frac{BC}{QR}$$

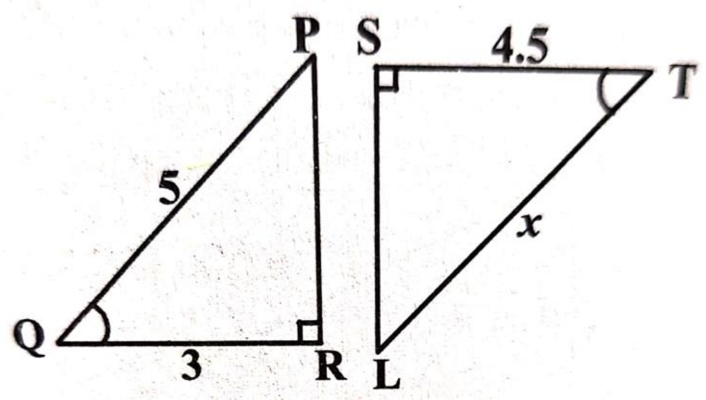
$i.e. in\ the\ two\ triangles, included\ sides\ of\ equal\ angles\ are\ not\ in\ proportion$

$\Delta ABC\ and\ \Delta PQR\ are\ not\ similar.$

TRY THIS :

2. If pairs of the triangles are similar and then find the value of x.

(i) $Given\ \Delta PRQ \sim \Delta LST$



$$then\ \frac{QR}{TS} = \frac{PQ}{LT}$$

$$\Rightarrow \frac{3}{4.5} = \frac{5}{x}$$

$$\Rightarrow x = \frac{5 \times 4.5}{3}$$

$$\Rightarrow x = \frac{5 \times 1.5}{1}$$

$$\Rightarrow x = 7.5$$

TRY THIS :

2. If pairs of the triangles are similar and then find the value of x.

(ii) $Given\ \Delta ABC \sim \Delta PQC$

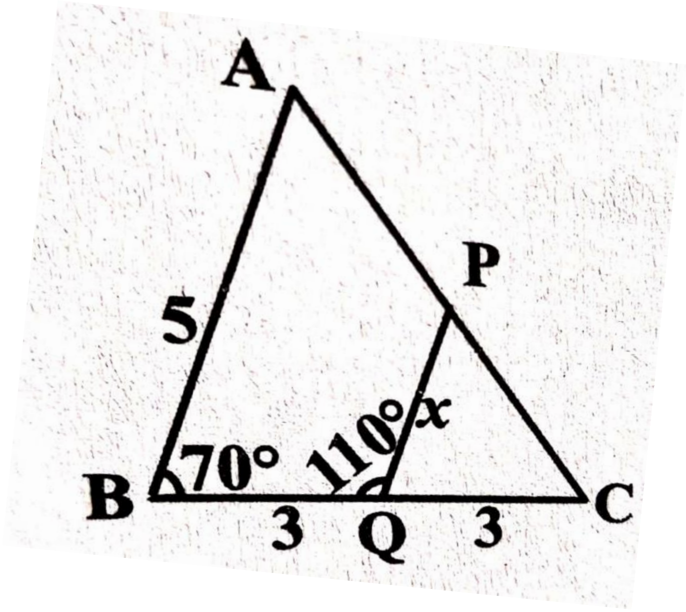
$$then\ \frac{AB}{PQ} = \frac{BC}{QC}$$

$$\Rightarrow \frac{5}{x} = \frac{6}{3}$$

$$\Rightarrow x = \frac{5 \times 3}{6}$$

$$\Rightarrow x = \frac{5}{2}$$

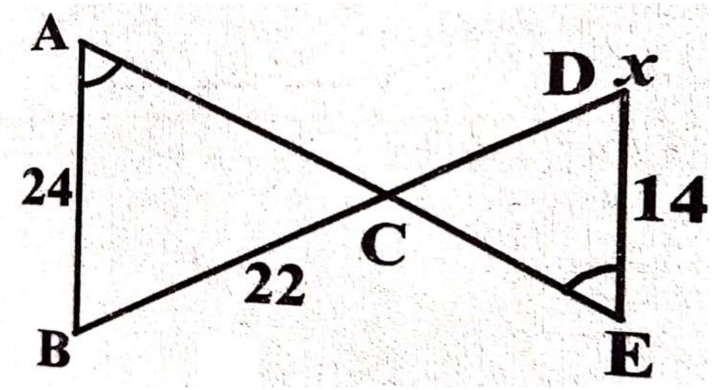
$$\Rightarrow x = 2.5$$



TRY THIS :

2. If pairs of the triangles are similar and then find the value of x.

(iii) $Given\ \Delta ABC \sim \Delta EDC$



$$then\ \frac{AB}{ED} = \frac{BC}{DC}$$

$$\Rightarrow \frac{24}{14} = \frac{22}{x}$$

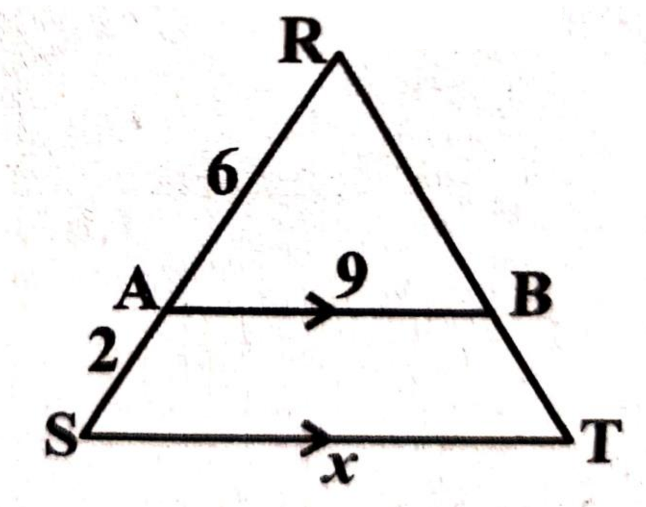
$$\Rightarrow x = \frac{22 \times 14}{24}$$

$$\Rightarrow x = 12.8$$

TRY THIS :

2. If pairs of the triangles are similar and then find the value of x.

(iv) $Given\ \Delta RAB \sim \Delta RST$



$$then\ \frac{RA}{RS} = \frac{AB}{ST}$$

$$\Rightarrow \frac{6}{8} = \frac{9}{x}$$

$$\Rightarrow x = \frac{8 \times 9}{6}$$

$$\Rightarrow x = 12$$

TRY THIS :

2. If pairs of the triangles are similar and then find the value of x.

(v) $Given\ \Delta PMN \sim \Delta PQR$

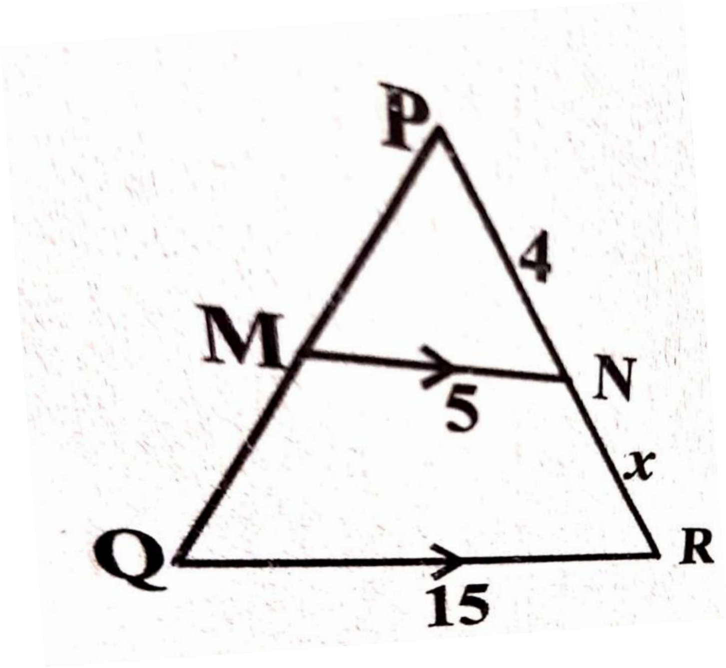
$$then\ \frac{MN}{QR} = \frac{PN}{PR}$$

$$\Rightarrow \frac{5}{15} = \frac{4}{4+x}$$

$$\Rightarrow 4 + x = \frac{15 \times 4}{5}$$

$$\Rightarrow 4 + x = 12$$

$$\Rightarrow x = 12 - 4 = 8$$



TRY THIS :

2. If pairs of the triangles are similar and then find the value of x.

(vi) $Given\ \Delta XAB \sim \Delta XZY$

$$then\ \frac{XA}{XZ} = \frac{AB}{ZY}$$

$$\Rightarrow \frac{x}{x+7.5} = \frac{12}{18}$$

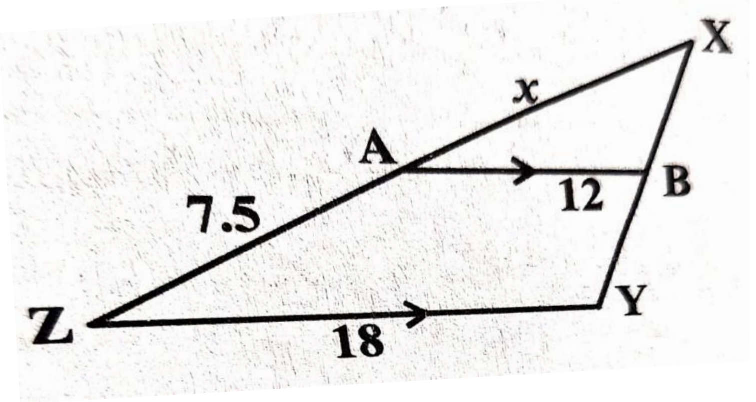
$$\Rightarrow \frac{x}{x+7.5} = \frac{2}{3}$$

$$\Rightarrow 3x = 2(x+7.5)$$

$$\Rightarrow 3x = 2x + 15$$

$$\Rightarrow 3x - 2x = 15$$

$$\Rightarrow x = 15$$



TRY THIS :

2. If pairs of the triangles are similar and then find the value of x.

(vii) $Given\ \Delta ABC \sim \Delta EDC$

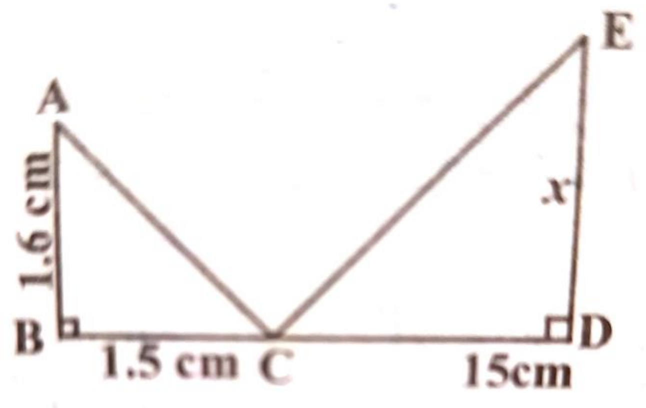
$$then\ \frac{AB}{ED} = \frac{BC}{DC}$$

$$\Rightarrow \frac{1.6}{x} = \frac{1.5}{15}$$

$$\Rightarrow x = \frac{1.6 \times 15}{1.5}$$

$$\Rightarrow x = 1.6 \times 10$$

$$\Rightarrow x = 16$$



AAA CRITERION FOR SIMILARITY OF TRAINGLES

Theorem : *In two triangles, if the angles are equal, then the sides opposite to the equal angles are in the same ratio and hence the two triangles are similar*

SSS CRITERION FOR SIMILARITY OF TRAINGLES

Theorem : *In two triangles, if the sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal hence the triangles are similar*

SAS CRITERION FOR SIMILARITY OF TRAINGLES

Theorem : *If one angle of a triangle is to one angle of the other triangle and the including sides of these angles are proportional, then the two triangles are similar.*

$$Let\ \Delta\ ABC\ \sim\ \Delta DEF$$

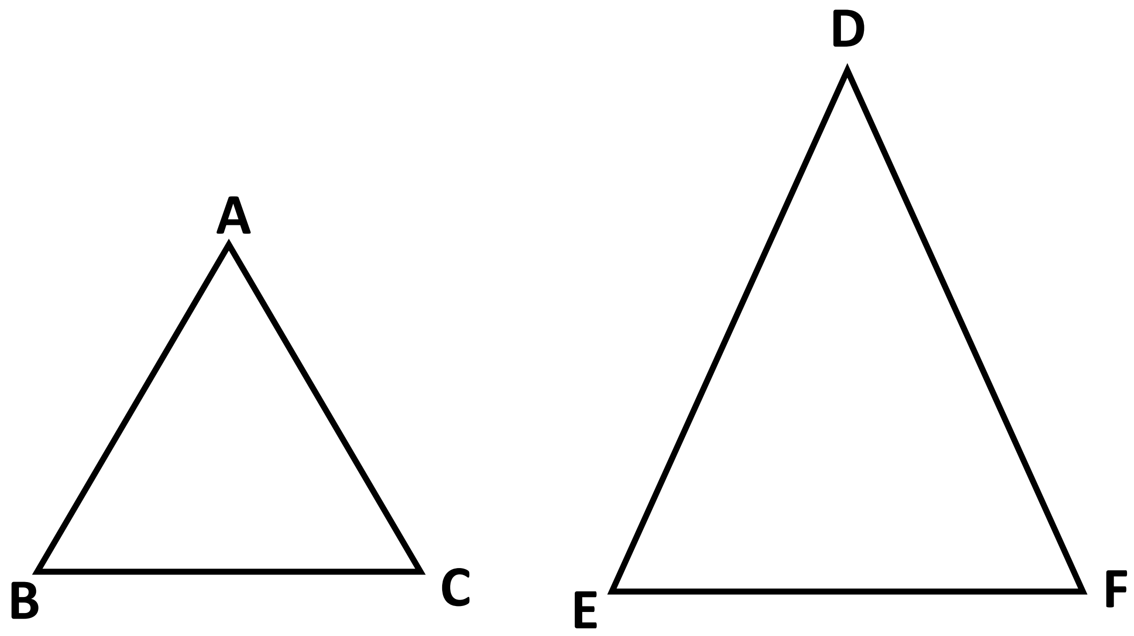
$$then\ we\ have,$$

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$$

$$\boxed{\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}}$$

$$\Longrightarrow \frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}=\frac{AB+BC+CA}{DE+EF+FA}$$

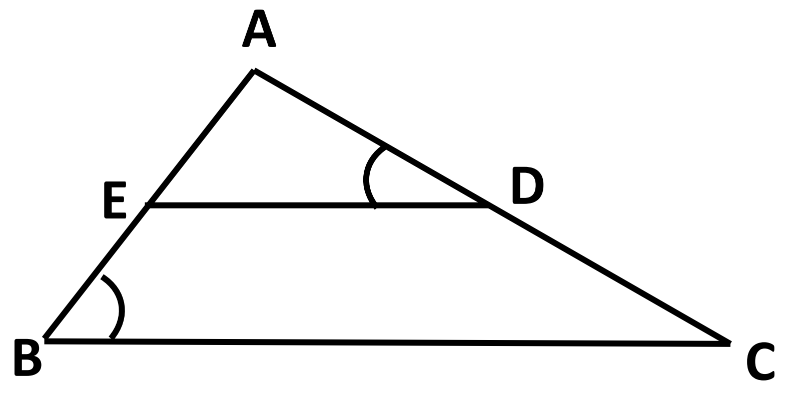
$$\Longrightarrow \frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}=\frac{perimeter\ of\ \Delta ABC}{perimeter\ of\ \Delta DEF}$$



$$i.e, ratio\ of\ the\ perimeters\ of\ similar\ triangles\ is\ equal\ to\ ratio\ of\ corresponding\ sides$$

EXERCISE – 8.2

1. In the given figure, ∠ ADE= ∠B (i) Show that Δ ABC ~ Δ ADE (ii) If AD = 3.8 cm, AE = 3.6cm, BE = 2.1 cm and BC = 4.2 cm, find DE



Sol : (i) $In\ \Delta\ ABC\ and\ \Delta ADE,$

$\angle A = \angle A \quad (\because common\ angle)$

$\angle ADE = \angle B \ (\because given)$

$\therefore \Delta\ ABC \sim \Delta ADE \ (\because AA\ similarity)$

(ii) $Given\ AD = 3.8\ cm, AE = 3.6\ cm$

$BE = 2.1\ cm, BC = 4.2\ cm$

$To\ find\ DE.$

$we\ have\ \Delta\ ABC \sim \Delta ADE$

$$\Rightarrow \frac{AB}{AD} = \frac{BC}{DE} \quad (\because ratios\ of\ correspondng\ angles\ are\ equal)$$

$$\Rightarrow \frac{AE + BE}{AD} = \frac{BC}{DE}$$

$$\Rightarrow \frac{3.6 + 2.1}{3.8} = \frac{4.2}{DE}$$

$$\Rightarrow \frac{5.7}{3.8} = \frac{4.2}{DE}$$

$$\Rightarrow DE = \frac{4.2 \times 3.8}{5.7}$$

$$\Rightarrow DE = \frac{4.2 \times 2}{3}$$

$$\Rightarrow DE = 1.4 \times 2$$

$$\Rightarrow DE = 2.8$$

$$\therefore DE = 2.8\ cm$$

$\underline{EXERCISE\ \ -8.2}$

2. The perimeters of two similar triangles are 30 cm and 20 cm respectively. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

Sol : $Let\ \Delta\ ABC \sim \Delta DEF$

$Perimeter\ of\ \Delta ABC = 30\ cm$

$Perimeter\ of\ \Delta DEF = 20\ cm$

$one\ side\ of\ \Delta\ ABC\ is\ AB = 12\ cm$

$let\ the\ corrresponding\ side\ of\ AB\ in\ \Delta\ DEF\ is\ DE =' x'\ cm$

$$then\ \frac{AB}{DE} = \frac{perimeter\ of\ \Delta ABC}{perimeter\ of\ \Delta DEF}$$

$$\Rightarrow \frac{12}{x} = \frac{30}{20}$$

$$\Rightarrow x = \frac{12 \times 20}{30}$$

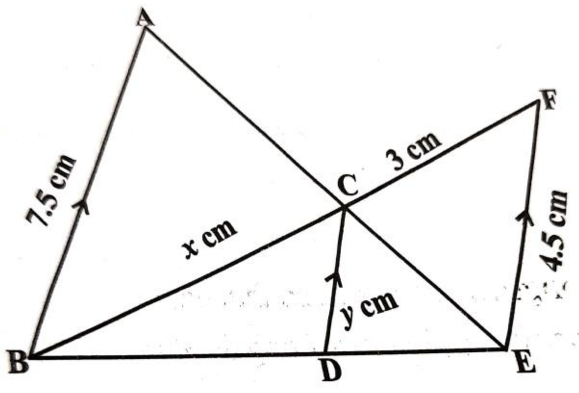
$$\Rightarrow x = \frac{12 \times 2}{3}$$

$$\Rightarrow x = 4 \times 2 = 8$$

$\therefore\ length\ of\ the\ corresponding\ side\ of\ length\ 12\ cm\ is\ 8\ cm$

EXERCISE – 8.2

3. In the given figure, AB || CD || EF. given that AB=7.5 cm, DC= y cm EF = 4.5 cm and BC = x cm, find the values of x and y.



Sol : $Given\ AB \parallel CD \parallel EF$

$$AB = 7.5\ cm, DC = 'y'cm, EF = 4.5\ cm,$$
$$BC = 'x'cm, CF = 3\ cm$$

$In\ \Delta ABC\ and\ \Delta EFC,$

$\angle ACB = \angle ECF \quad (\because vertically\ opposite\ anles)$

$\angle BAC = \angle FEC \quad (\because alternate\ interior\ angles)$

$\angle ABC = \angle EFC \quad (\because alternate\ interior\ angles)$

$then, \Delta\ ABC \sim\ \Delta EFC \quad (\because AAA\ similarity)$

$$\Rightarrow \frac{AB}{EF} = \frac{BC}{FC}$$

$$\Rightarrow \frac{7.5}{4.5} = \frac{x}{3}$$

$$\Rightarrow x = \frac{7.5 \times 3}{4.5}$$

$$\Rightarrow x = \frac{7.5}{1.5} = 5$$

$In\ \Delta BDC\ and\ \Delta BEF,$

$\angle B = \angle B \quad (\because common\ angle)$

$\angle D = \angle E \quad (\because corresponding\ angles)$

$\angle C = \angle F \quad (\because corresponding\ angles)$

$by\ AAA\ similarity,$

$\Delta\ BDC \sim\ \Delta BEF$

$$\Rightarrow \frac{BC}{BF} = \frac{DC}{EF}$$

$$\Rightarrow \frac{x}{x+3} = \frac{y}{4.5}$$

$$\Rightarrow \frac{5}{5+3} = \frac{y}{4.5}$$

$$\Rightarrow \frac{5}{8} = \frac{y}{4.5}$$

$$\Rightarrow y = \frac{4.5 \times 5}{8}$$

$$\Rightarrow y = \frac{22.5}{8}$$

$$\Rightarrow y = 2.8$$

$\therefore x = 5\ cm\ and\ y = 2.8\ cm$

EXERCISE – 8.2

4. A girl of height 90 cm is walking away from the base of a lamp post at a speed of 1.2 m/sec. If the lamp post is 3.6m above the ground, find the length of her shadow after 4 seconds

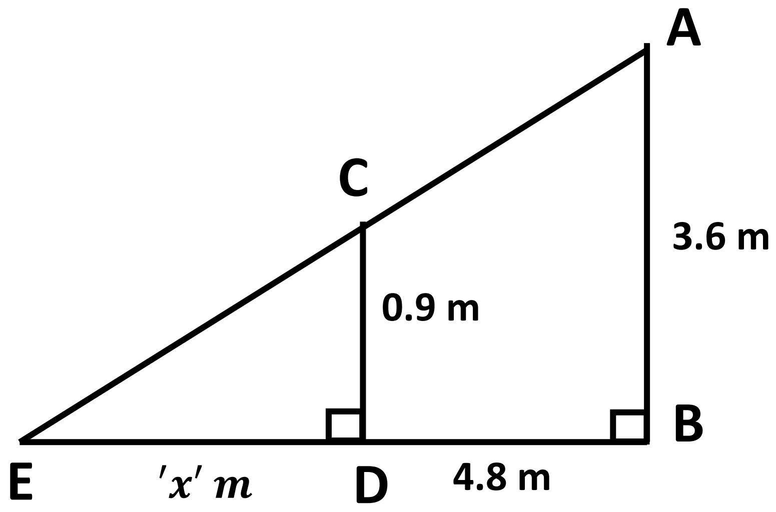
Sol : $Let\ the\ height\ of\ the\ lamp\ post\ be\ AB = 3.6\ m$

$height\ of\ the\ girl\ be\ CD = 90\ cm = 0.9m$

$speed\ of\ the\ girl\ is\ 1.2\ m/sec$

$distance\ walked\ by\ girl\ from\ the\ foot\ of\ the\ lamp\ post\ in\ 4\ seconds\ is\ BD = 4 \times 1.2 = 4.8\ m$

$Let\ the\ length\ of\ the\ shadow\ of\ girl\ be\ DE = 'x'\ m$



$In\ \Delta ABE\ and\ \Delta CDE,$

$\angle E = \angle E\ (\because common\ angle)$

$\angle ABE = \angle CDE\ \ (= 90°)$

$then, \Delta\ ABE \sim \Delta CDE\ \ (\because AA\ similarity)$

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DE}$$

$$\Rightarrow \frac{3.6}{0.9} = \frac{x + 4.8}{x}$$

$$\Rightarrow 4 = \frac{x + 4.8}{x}$$

$$\Rightarrow 4x = x + 4.8$$

$$\Rightarrow 4x - x = 4.8$$

$$\Rightarrow 3x = 4.8$$

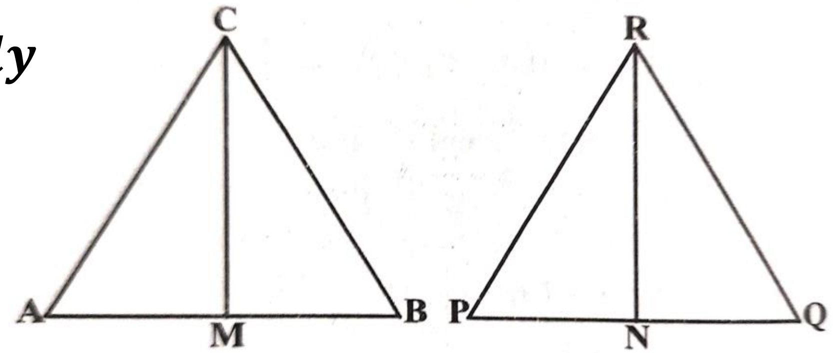
$$\Rightarrow x = \frac{4.8}{3}$$

$$\Rightarrow x = 1.6$$

$\therefore length\ of\ \ shadow\ of\ the\ girl\ is\ 1.6\ m$

EXERCISE – 8.2

5. Given that $\Delta ABC \sim \Delta PQR$. CM and RN are respectively the medians of ΔABC and ΔPQR. Prove that (i) $\Delta AMC \sim \Delta PNR$ (ii) $\frac{CM}{RN} = \frac{AB}{PQ}$ (iii) $\Delta CMB \sim \Delta RNQ$



Sol : *CM and RN are the medians of ΔABC and ΔPQR respectively*

so that $AM = BM = \frac{1}{2}AB$ *and*

$$PN = QN = \frac{1}{2}PQ$$

(i) *In ΔAMC and ΔPNR,*

$\angle A = \angle P$ $(\because \Delta ABC \sim \Delta PQR)$

$\frac{AC}{PR} = \frac{AB}{PQ}$ $(\because \Delta ABC \sim \Delta PQR)$

$$= \frac{\frac{1}{2}AB}{\frac{1}{2}PQ}$$

$$= \frac{AM}{PN}$$

$\frac{AC}{PR} = \frac{AM}{PN}, \angle A = \angle P$

by SAS similarity,

$\Delta AMC \sim \Delta PNR$

(ii) *we have* $\Delta AMC \sim \Delta PNR$

$$\frac{CM}{RN} = \frac{AM}{PN}$$

$$= \frac{\frac{1}{2}AB}{\frac{1}{2}PQ}$$

$$= \frac{AB}{PQ}$$

$$\therefore \frac{CM}{RN} = \frac{AB}{PQ}$$

(iii) $\angle B = \angle Q$ $(\because \Delta ABC \sim \Delta PQR)$

$\frac{BC}{QR} = \frac{AB}{PQ}$ $(\because \Delta ABC \sim \Delta PQR)$

$$= \frac{\frac{1}{2}AB}{\frac{1}{2}PQ} = \frac{BM}{QN}$$

In ΔCMB and ΔRNQ,

$\frac{BC}{QR} = \frac{BM}{QN}, \angle B = \angle Q$

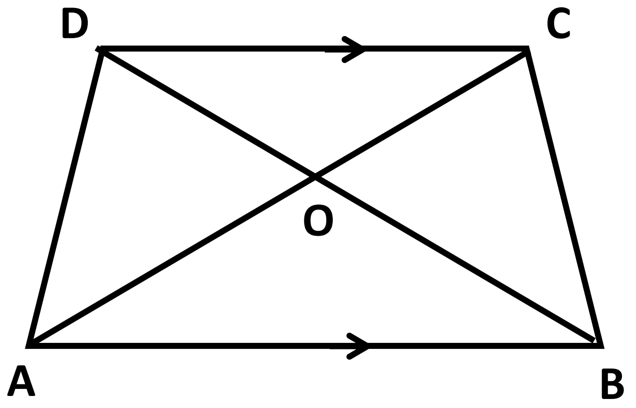
by SAS similarity,

$\Delta CMB \sim \Delta RNQ$

$EXERCISE\ -8.2$

6. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point 'O'. Using the criterion of similarity for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Sol : $In\ trapezium\ ABCD, AB \parallel DC\ and\ intersection\ point\ of\ diagonals\ AC\ and\ BD\ is\ 'O'$



$In\ \Delta OAB\ and\ \Delta OCD,$

$\angle AOB = \angle COD$ ($\because$ vertically opposite angles)

$\angle OAB = \angle OCD$ ($\because$ alternate interior angles)

$\angle OBA = \angle ODC$ ($\because$ alternate interior angles)

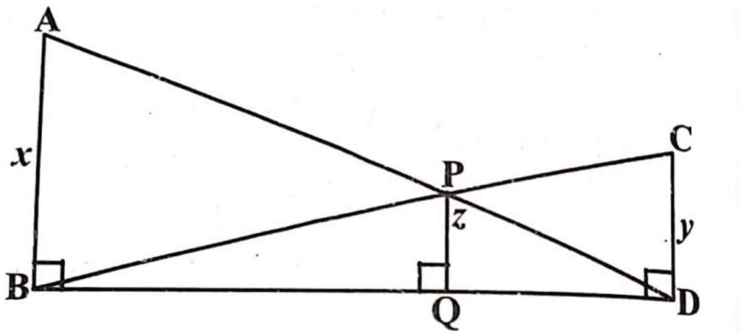
by AAA similarity, $\Delta OAB \sim \Delta OCD$

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD} \quad (\because \text{ratio of corresponding sides})$$

Hence proved.

EXERCISE − 8.2

7. AB, CD, PQ are perpendiculars to BD. $AB = x. CD = y$ and $PQ = z$ prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$



Sol : Given $\angle B = \angle Q = \angle D = 90°$

so that, $AB \parallel PQ \parallel CD$

In ΔBQP and ΔBDC,

$\angle B = \angle B$ ($\because$ common angle)

$\angle Q = \angle D$ ($= 90°$)

$\angle P = \angle C$ ($\because$ corresponding angles)

by AAA similarity, $\Delta BQP \sim \Delta BDC$

$$\Rightarrow \frac{BQ}{BD} = \frac{PQ}{CD} \longrightarrow (1)$$

In ΔDQP and ΔDBA,

$\angle D = \angle D$ ($\because$ common angle)

$\angle Q = \angle B$ ($= 90°$)

$\angle P = \angle A$ ($\because$ corresponding angles)

by AAA similarity, $\Delta DQP \sim \Delta DBA$

$$\Rightarrow \frac{QD}{BD} = \frac{PQ}{AB} \longrightarrow (1)$$

by adding eq. (1) and (2),

$$\frac{BQ}{BD} + \frac{QD}{BD} = \frac{PQ}{CD} + \frac{PQ}{AB}$$

$$\Rightarrow \frac{BQ+QD}{BD} = PQ\left(\frac{1}{CD}+\frac{1}{AB}\right)$$

$$\Rightarrow \frac{BD}{BD} = z\left(\frac{1}{y}+\frac{1}{x}\right)$$

$$\Rightarrow 1 = z\left(\frac{1}{y}+\frac{1}{x}\right)$$

$$\Rightarrow \frac{1}{z} = \left(\frac{1}{y}+\frac{1}{x}\right)$$

$$\therefore \frac{1}{x}+\frac{1}{y}=\frac{1}{z}$$

EXERCISE – 8.2

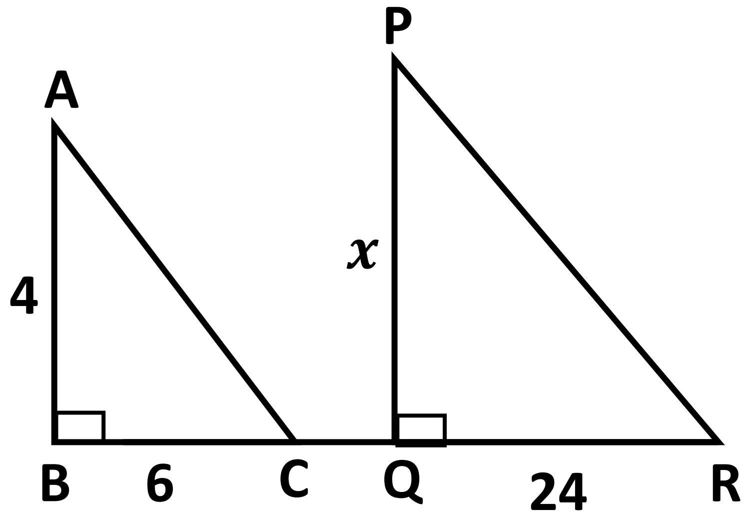
8. A flag pole 4m tall casts a 6 m shadow. At the same time, a nearby building casts a shadow of 24m. How tall is the building ?

Sol : $Height\ of\ the\ flag\ pole\ is\ AB = 4m$

$length\ of\ the\ shadow\ of\ flagpole\ is\ BC = 6m$

$height\ of\ the\ building\ is\ PQ = 'x'm$

$length\ of\ the\ shadow\ of\ building\ is\ QR = 24m$



$In\ \Delta\ ABC\ and\ \Delta PQR,$

$\angle A = \angle P \quad (\because\ inclination\ of\ the\ sun)$

$\angle B = \angle Q \quad (= 90°)$

$\therefore \Delta\ ABC\ \sim\ \Delta PQR\ (\because\ AA\ similarity)$

$$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}\ (\because\ ratios\ of\ correspondng\ sides\ are\ equal)$$

$$\Rightarrow \frac{4}{6} = \frac{x}{24}$$

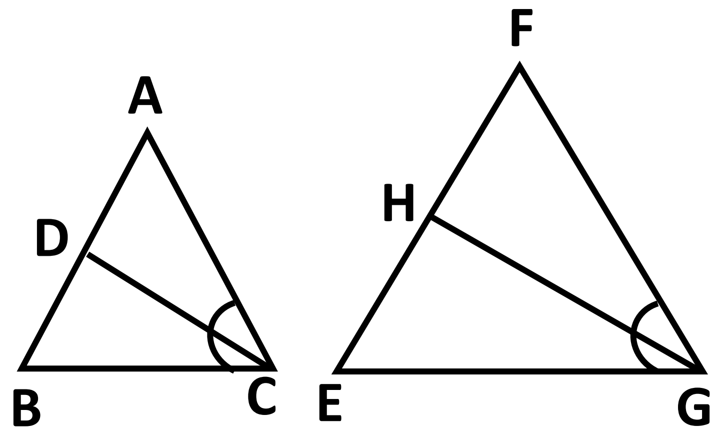
$$\Rightarrow x = \frac{4 \times 24}{6}$$

$$\Rightarrow x = 4 \times 4 = 16$$

$\therefore\ height\ of\ the\ building\ is\ 16m$

9. *CD and GH are respectively the bisectors of* $\angle ACB$ *and* $\angle EGF$ *such that* D *and* H *lie on sides* AB *and* FE *of* ΔABC *and* ΔEFG *respectively.*

If $\Delta ABC \sim \Delta FGE$ *then show that* (i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ (ii) $\Delta DCB \sim \Delta HGE$ (iii) $\Delta DCA \sim \Delta HGF$



Sol : Given $\Delta ABC \sim \Delta FGE$

so that $\angle ACB = \angle FGE \Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\Rightarrow \angle ACD = \angle FGH \longrightarrow (1)$

similarly $\angle DCB = \angle HGE \longrightarrow (2)$

(i) *In* ΔACD *and* ΔFGH,

$\angle A = \angle F \quad (\because \Delta ABC \sim FEG)$

$\angle ACD = \angle FGH \ (\because \text{eq.}\,(1))$

by AA similarity,

$\Delta ACD \sim \Delta FGH$

$\Rightarrow \dfrac{CD}{GH} = \dfrac{AC}{FG}$

(ii) *In* ΔDCB *and* ΔHGE,

$\angle B = \angle E \quad (\because \Delta ABC \sim FEG)$

$\angle DCB = \angle HGE \ (\because \text{eq.}\,(2))$

by AA similarity,

$\Delta DCB \sim \Delta HGE$

(iii) *In* ΔDCA *and* ΔHGF,

$\angle A = \angle F \quad (\because \Delta ABC \sim FEG)$

$\angle ACD = \angle FGH \ (\because \text{eq.}\,(1))$

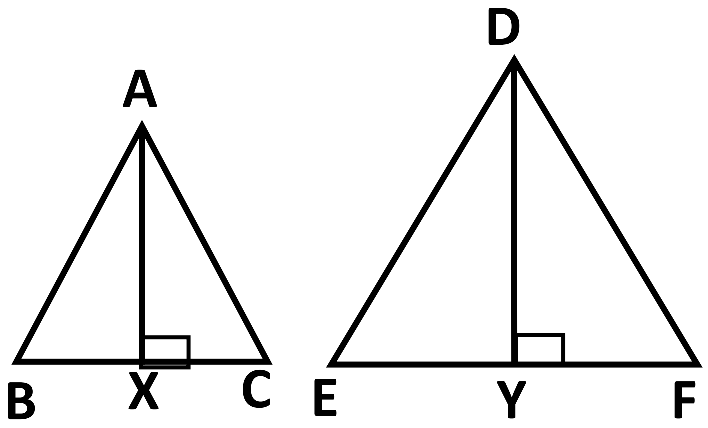
by AA similarity,

$\Delta DCA \sim \Delta HGF$

EXERCISE – 8.2

10. AX and DY are altitudes of two similar triangles ΔABC and ΔDEF .

Prove that AX : DY = AB : DE



Sol : $\textit{Given } \Delta\, ABC \sim \Delta DEF$

$AX \perp BC \text{ and } DY \perp EF$

$\text{In } \Delta ABX \text{ and } \Delta DEY,$

$\angle B = \angle E\ \ (\because \Delta \mathrm{ABC} \sim DEF)$

$\angle X = \angle Y\ \ (= 90°)$

by AA similarity,

$\Delta\mathrm{ABX} \sim \Delta\mathrm{DEY}$

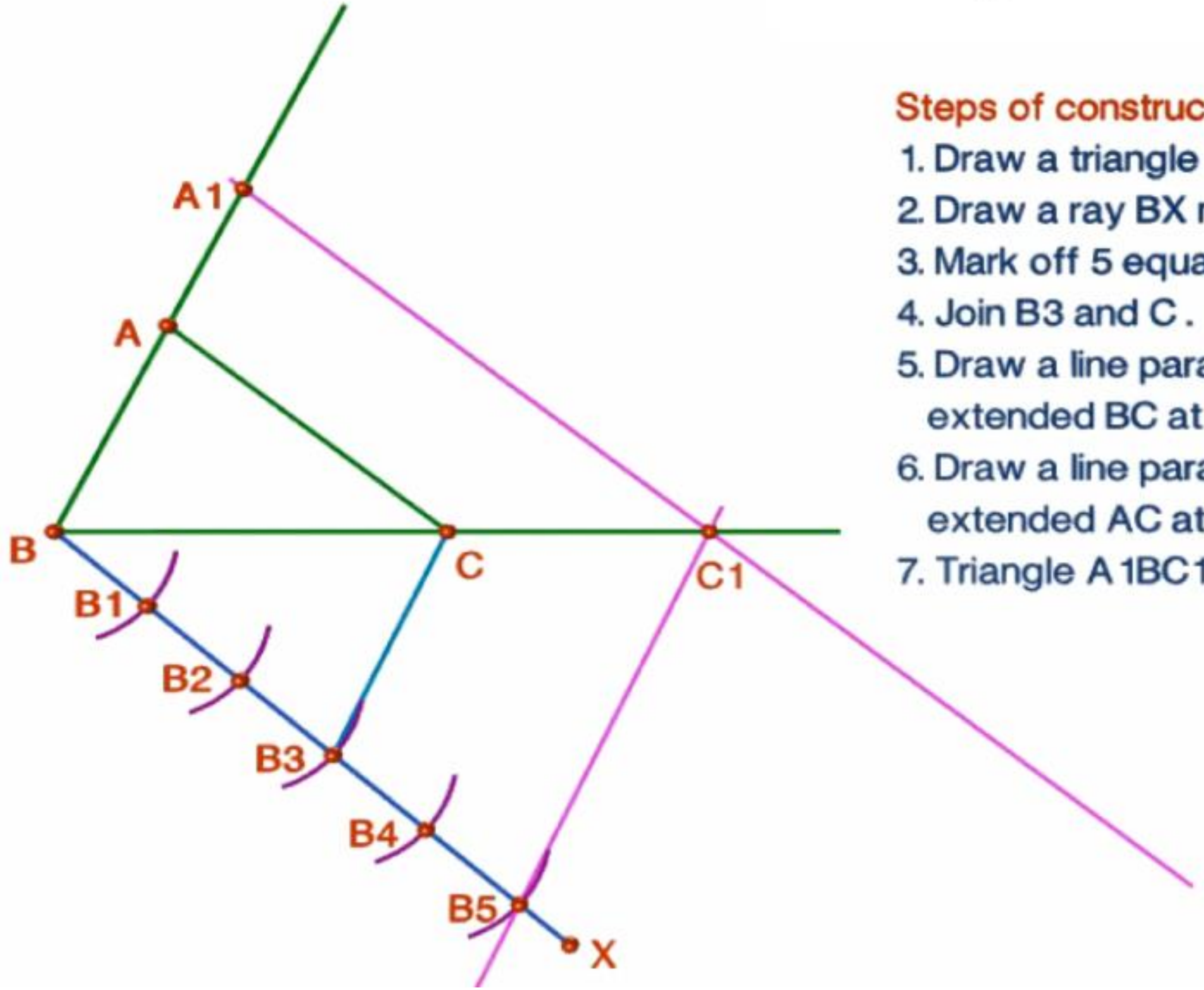
$$\Rightarrow \frac{AX}{DY} = \frac{AB}{DE}\ \ (\because \textit{ratios of correspondng sides are equal})$$

$$\Rightarrow \mathrm{AX : DY = AB : DE}$$

Hence proved.

11. Construct a triangle similar to the given ΔABC, with its sides equal to the $\frac{5}{3}$ of the corresponding sides of the triangle ABC

Sol:

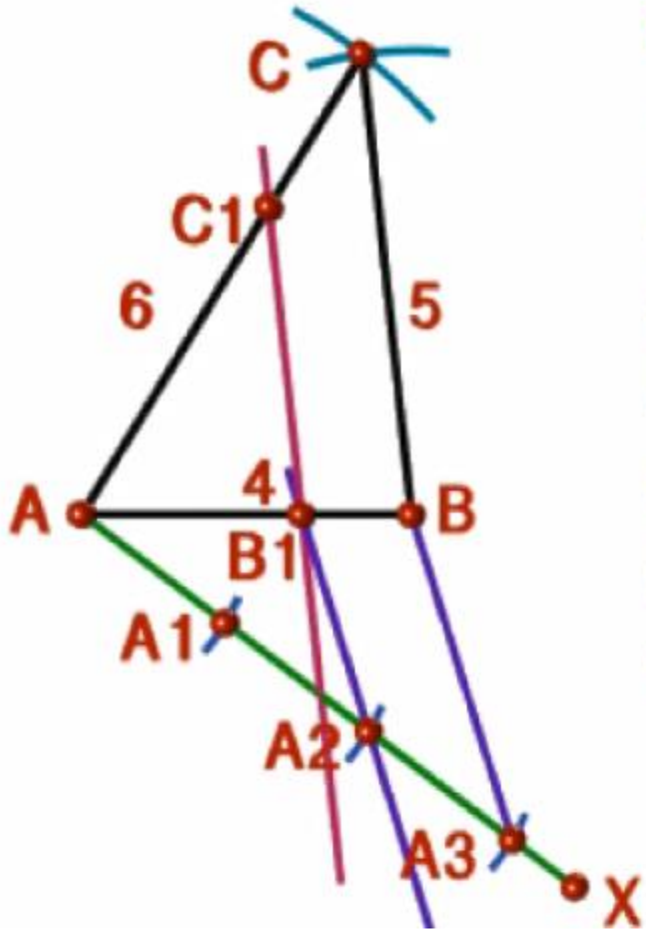


Steps of construction :

1. Draw a triangle ABC with certain measurements .
2. Draw a ray BX making an acute angle with the side BC .
3. Mark off 5 equal parts (B1, B2, . . B5) on BX with same radius
4. Join B3 and C .
5. Draw a line parallel B3C, through B5 which intersects extended BC at C1 .
6. Draw a line parallel to AC, through C1which intersects extended AC at A1 .
7. Triangle A1BC1 is the required triangle .

12. Construct a triangle of sides 4cm, 5 cm and 6 cm. Then, construct a triangle similar to it, whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Sol:

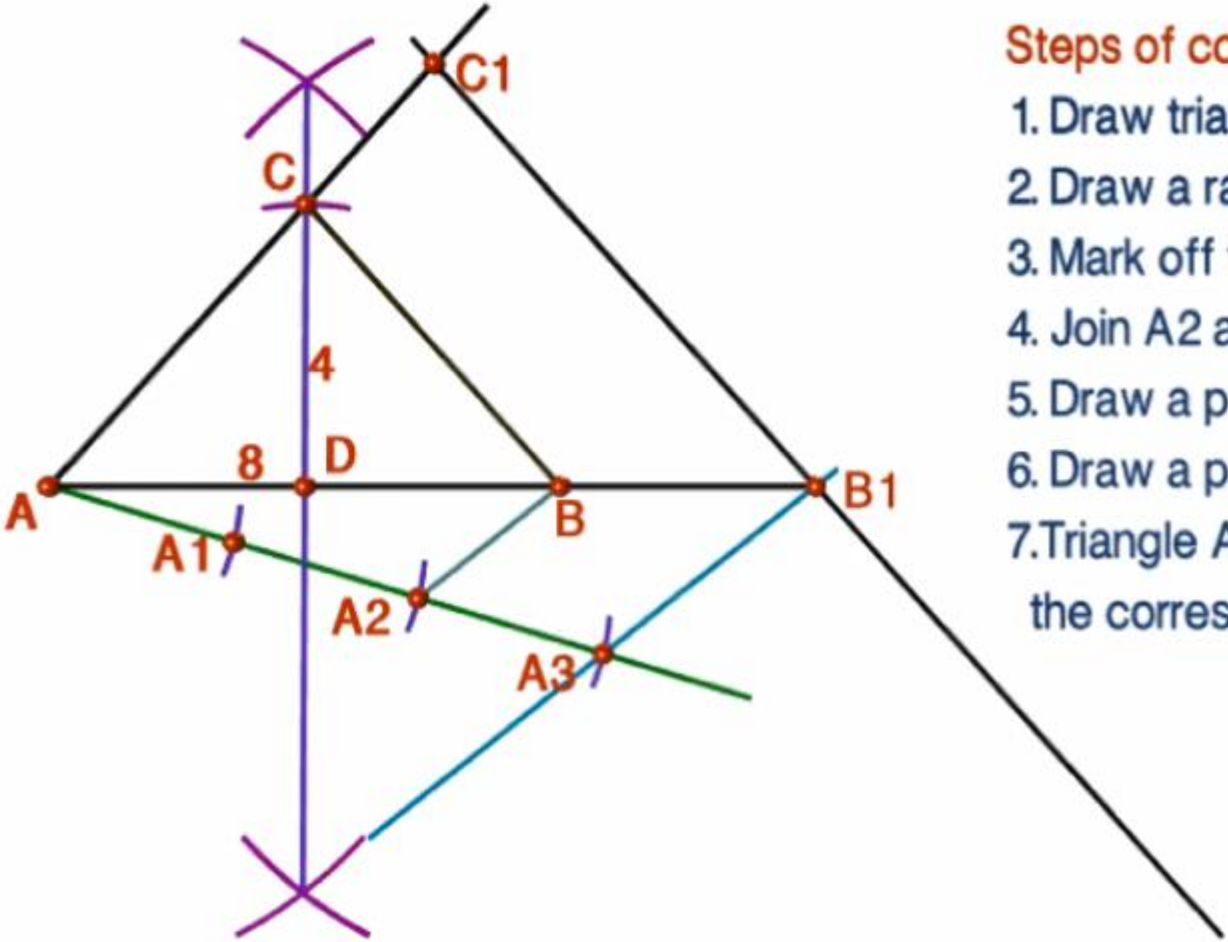


Steps of construction :

1. Draw a triangle ABC with AB = 4 cm, BC = 5 cm, and CA = 6 cm
2. Draw a ray AX making an acute angle with AB .
3. Mark off three equal parts (A1, A2, A3)on AX with same radius .
4. Join A3 and B .
5. Draw a line parallel to A3B through A2 meeting AB at B1 .
6. Draw a line parallel to BC through B1 meeting AC at C1 .
7. Triangle AB1C1 is the required triangle .

13. Construct an isosceles triangle whose base is 8cm and altitude is 4 cm. Then, draw another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Sol: $1\frac{1}{2}\ times = \frac{3}{2}\ times$



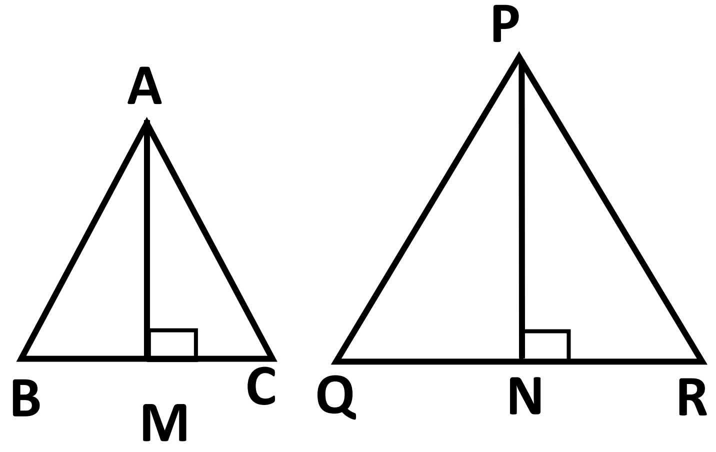
Steps of construction :

1. Draw triangle ABC with base AB = 8 cm and altitude CD = 4 cm .
2. Draw a ray AX making an acute angle with AB .
3. Mark off three equal parts (A1, A2, A3) on AX with equal radius .
4. Join A2 and B .
5. Draw a parallel to A2B through A3, meeting extended AB at B1 .
6. Draw a parallel to BC through B1, meeting extended AC at C1 .
7. Triangle AB1C1 is the required triangle, whose sides are 3/2 times the corresponding sides of triangle ABC .

Theorem : The ratio of the areas of two similar triangles is equal to the squares of ratio of their corresponding sides

Given : $\Delta ABC \sim \Delta PQR$

RTP : $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{AC}{PR}\right)^2$



Construction : *Draw* $AM \perp BC$ *and* $PN \perp QR$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN} \longrightarrow (1)$$

In ΔABM *and* ΔPQN,

$\angle B = \angle Q \quad (\because \Delta ABC \sim \Delta PQR)$

$\angle M = \angle N \quad (= 90°)$

$\therefore \Delta ABM \sim \Delta PQN$ $(\because$ *by AA similarity*$)$

$$\Rightarrow \frac{AM}{PN} = \frac{AB}{PQ} \longrightarrow (2)$$

also $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \longrightarrow (3)$$

from eq. (1), (2) *and* (3)

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB \times AB}{PQ \times PQ} = \left(\frac{AB}{PQ}\right)^2$$

from eq. (3), *we can write*

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

EXERCISE – 8.3

1. D, E, F are mid points of sides BC, CA, AB of ΔABC. Find the ratio of areas of ΔDEF and ΔABC.

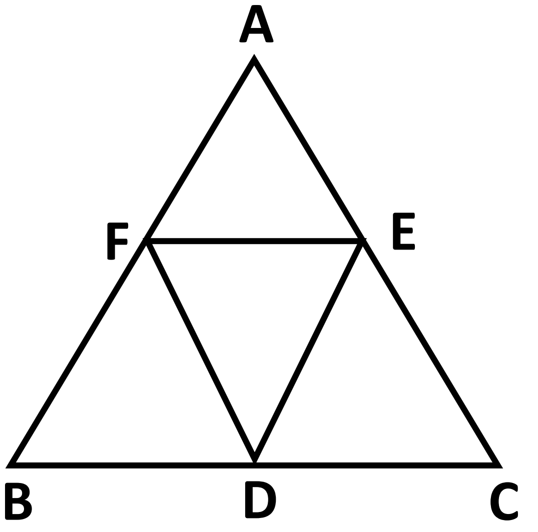
Sol : In $\Delta\, ABC$, mid points of BC, CA, AB are D, E, F

$\Rightarrow AF = FB$

$\Rightarrow \dfrac{AF}{FB} = 1 \longrightarrow (1)$

$\Rightarrow AE = EC$

$\Rightarrow \dfrac{AE}{EC} = 1 \longrightarrow (2)$



from eq. (1) and (2), $\dfrac{AF}{FB} = \dfrac{AE}{EC}$

$\Rightarrow FE \parallel BC$ ($\because$ converse of B.P.T.)

similarly we can show that $ED \parallel AB$

$\therefore BDEF$ is a parallelogram.

$\Rightarrow FE = BD$

$\Rightarrow FE = \dfrac{1}{2}BC$

$\Rightarrow \dfrac{FE}{BC} = \dfrac{1}{2} \longrightarrow (3)$

similarly $\dfrac{DE}{AB} = \dfrac{1}{2} \longrightarrow (4)$

$\dfrac{DF}{AC} = \dfrac{1}{2} \longrightarrow (5)$

In ΔDEF and ΔABC,

$\dfrac{FE}{BC} = \dfrac{DE}{AB} = \dfrac{DF}{AC}$

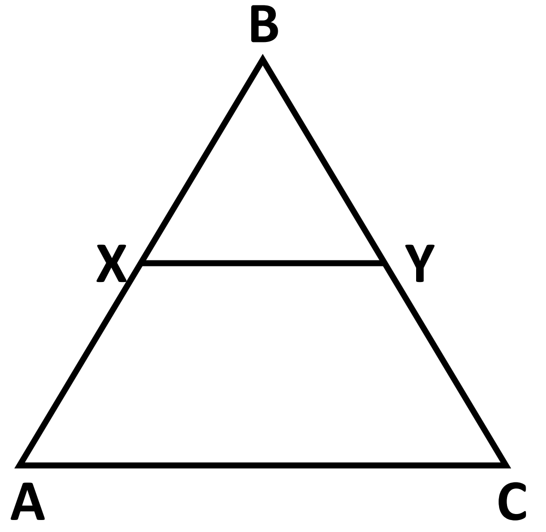
$\therefore \Delta DEF \sim \Delta ABC$ ($\because$ by SSS similarity)

$\Rightarrow \dfrac{ar(\Delta DEF)}{ar(\Delta ABC)} = \left(\dfrac{FE}{BC}\right)^2 = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

$\therefore$ Ratio of areas ΔDEF and ΔABC is $1:4$

EXERCISE – 8.3

2. In $\Delta ABC, XY \parallel AC$ and XY divides the triangle into two parts of equal area. Find the ratio of $\frac{AX}{XB}$.



Sol : $In\ \Delta\ ABC, XY \parallel AC$

$$given\ ar(\Delta BXY) = ar(ACXY) = \frac{1}{2} ar(\Delta ABC)$$

$In\ \Delta XBY\ and\ \Delta ABC,$

$\angle BXY = \angle BAC \quad (\because XY \parallel AC)$

$\angle BYX = \angle BCA \quad (\because XY \parallel AC)$

$\angle XBY = \angle ABC \quad (\because common\ angle)$

$\therefore \Delta XBY \sim \Delta ABC \quad (\because by\ AAA\ similarity)$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta XBY)} = \left(\frac{AB}{XB}\right)^2$$

$$\Rightarrow \frac{2}{1} = \left(\frac{AB}{XB}\right)^2$$

$$\Rightarrow \frac{AB}{XB} = \sqrt{2}$$

$$\Rightarrow \frac{AX + XB}{XB} = \sqrt{2}$$

$$\Rightarrow \frac{AX}{XB} + \frac{XB}{XB} = \sqrt{2}$$

$$\Rightarrow \frac{AX}{XB} + 1 = \sqrt{2}$$

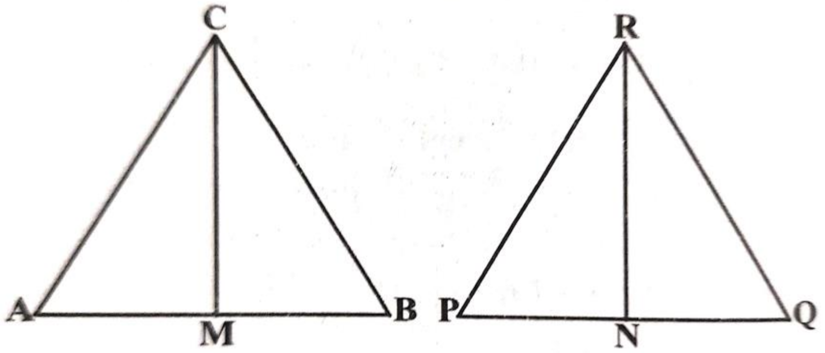
$$\Rightarrow \frac{AX}{XB} = \sqrt{2} - 1$$

$$\therefore \frac{AX}{XB} = \sqrt{2} - 1$$

3. Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding medians

Sol : Let $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AC}{PR}\right)^2 \longrightarrow (1)$$



In ΔABC, CM is median drawn from C on AB

$$AM = BM = \frac{1}{2}AB$$

In ΔPQR, RN is median drawn from R on PQ

$$PN = QN = \frac{1}{2}PQ$$

In ΔAMC and ΔPNR,

$\angle A = \angle P$ $(\because \Delta ABC \sim \Delta PQR)$

$$\frac{AC}{PR} = \frac{AB}{PQ} \quad (\because \Delta ABC \sim \Delta PQR)$$

$$= \frac{\frac{1}{2}AB}{\frac{1}{2}PQ}$$

$$= \frac{AM}{PN}$$

$$\Rightarrow \frac{AC}{PR} = \frac{AM}{PN}$$

by SAS similarity,

$\Delta AMC \sim \Delta PNR$

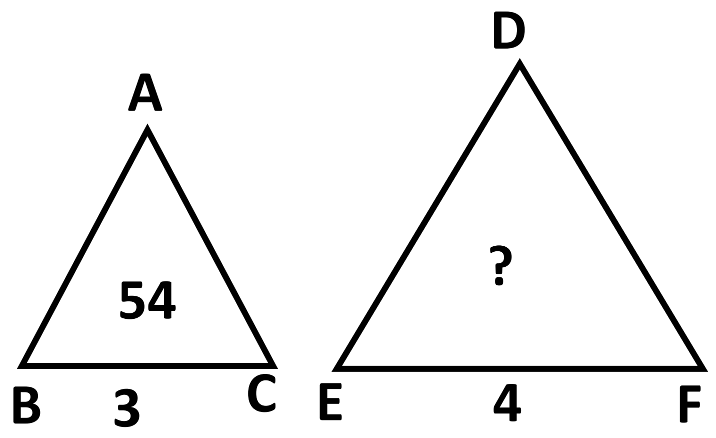
$$\Rightarrow \frac{AC}{PR} = \frac{CM}{RN} \longrightarrow (2)$$

from eq. (1) and (2)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{CM}{RN}\right)^2$$

$\therefore$ Ratio of areas of similar triangles is equal to the square of the ratio of corresponding medians.

4. $\Delta ABC \sim \Delta DEF, BC = 3cm, EF = 4cm$ and area of $\Delta ABC = 54$ sq. cm. Determine the area of ΔDEF.



Sol : Given $\Delta ABC \sim \Delta DEF$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \frac{54}{ar(\Delta DEF)} = \left(\frac{3}{4}\right)^2$$

$$\Rightarrow \frac{54}{ar(\Delta DEF)} = \frac{9}{16}$$

$$\Rightarrow ar(\Delta DEF) = \frac{54 \times 16}{9}$$

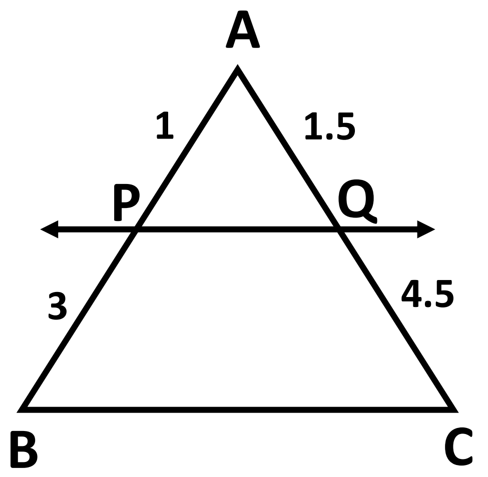
$$\Rightarrow ar(\Delta DEF) = 6 \times 16$$

$$\Rightarrow ar(\Delta DEF) = 96$$

$\therefore$ Area of ΔDEF is $96cm^2$

$EXERCISE - 8.3$

5. ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If $AP = 1cm, BP = 3cm, AQ = 1.5cm, CQ = 4.5cm,$ prove that $(area\ of\ \Delta APQ) = \frac{1}{16}(area\ of\ \Delta ABC)$



Sol : In $\Delta ABC, \frac{AP}{PB} = \frac{1}{3}, \frac{AQ}{QC} = \frac{1.5}{4.5} = \frac{1}{3}$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

$$\Rightarrow PQ \parallel BC\ (\because converse\ of\ B.P.T)$$

In ΔAPQ and ΔABC,

$\angle A = \angle A$ (common angle)

$\angle P = \because \angle B$ ($\because$ corresponding angles)

$\angle Q = \angle C$ ($\because$ corresponding angles)

$\therefore \Delta APQ \sim \Delta ABC$ ($\because$ by AAA similarity)

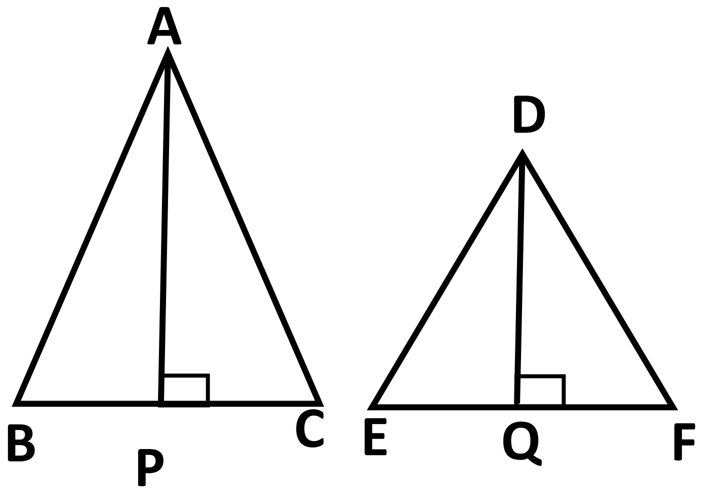
$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{1}{1+3}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{16}$$

$$\therefore area(\Delta APQ) = \frac{1}{16} area(\Delta ABC)$$

EXERCISE – 8.3

6. *The areas of two similar triangles are $81cm^2$ and $49cm^2$ respectively. If the altitudes of the bigger triangle is 4.5 cm, find the corresponding altitude of the smaller triangle.*



Sol : *Let the areas of the similar triangles ΔABC and ΔDEF be $81cm^2$ and $49cm^2$ respectively*

In ΔABC, length of altitude drawn from A is $AP = 4.5\ cm$

Let the length of corresponding altitude in ΔDEF is $DQ = x\ cm$

In ΔABP and ΔDEQ,

$\angle B = \angle E \quad (\because \Delta ABC \sim \Delta DEF)$

$\angle P = \angle Q \quad (= 90°)$

by AA similarity,

$\Delta ABP \sim \Delta DEQ$

$$\Rightarrow \frac{AB}{DE} = \frac{AP}{DQ} \longrightarrow (1)$$

$$\Rightarrow \frac{ar(\Delta ABP)}{ar(\Delta DEQ)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AP}{DQ}\right)^2 \quad (\because eq. 1)$$

$$\Rightarrow \frac{81}{49} = \left(\frac{4.5}{x}\right)^2$$

$$\Rightarrow \frac{4.5}{x} = \sqrt{\frac{81}{49}}$$

$$\Rightarrow \frac{4.5}{x} = \frac{9}{7}$$

$$\Rightarrow x = \frac{4.5 \times 7}{9}$$

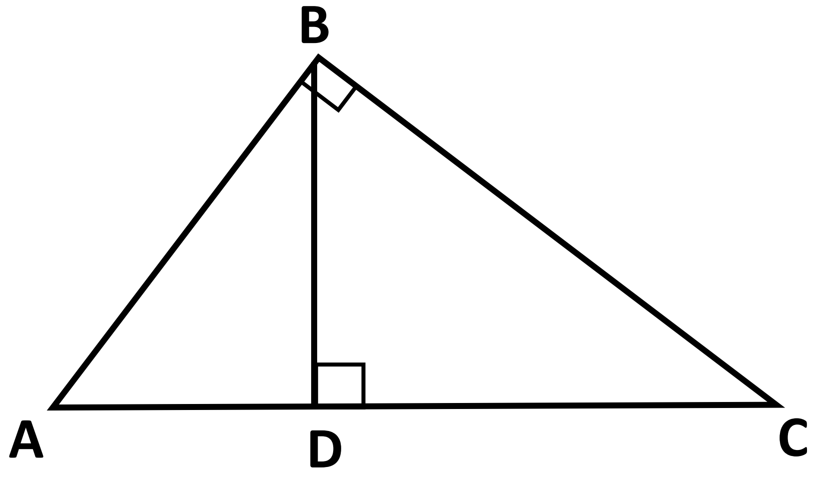
$$\Rightarrow x = 0.5 \times 7 = 3.5$$

$\therefore$ *corresponding altitude of smaller triangle is 3.5 cm*

Theorem : **If a perpendicular is drawn from the vertex of the rigt angle triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and to each other.**

$\boldsymbol{Proof}$: $\Delta\, ABC$ *is a right triangle , in which* $\angle B = 90°$

Let BD *be the perpendicular to hypotenuse* AC.



In ΔADB *and* ΔABC,

$\angle A = \angle A$ $\quad (\because$ *common angle*$)$

$\angle ADB = \angle ABC \; (= 90°)$

$\therefore \Delta ADB \sim \Delta ABC \quad (\because$ *by AA similarity*$) \longrightarrow$ **(1)**

similarly we can show $\quad \Delta BDC \sim \Delta ABC \longrightarrow$ **(2)**

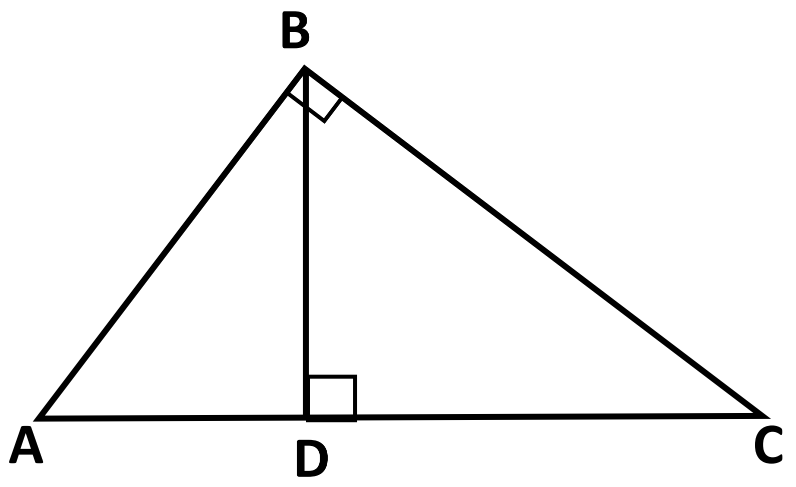
from eq. (1) *and* (2)

$\Delta ADB \sim \Delta BD\, C$

$\therefore$ *In a right angle triangle if a perpendicular drawn from the vertex on hypotenuse then the triangles on both sides the perpendicular are similar to the whole triangle and also they are similar to each other.*

PYTHAGORAS THEOREM (BAUDHAYAN THEOREM)

Theorem : In a right angle triangle, the square of length of the hypotenuse is equal to the sum of the squares of lengths of the other two sides.



Given : $\Delta\, ABC$ *is a right triangle, in which* $\angle B = 90°$

RTP : $AC^2 = AB^2 + BC^2$

Construction : *Draw* $BD \perp AC$

Proof : $\Delta ADB \sim \Delta ABC$

$$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$$

$$\Rightarrow AD.AC = AB^2 \longrightarrow (1)$$

also, $\Delta BDC \sim \Delta ABC$

$$\Rightarrow \frac{CD}{BC.} = \frac{BC}{AC}$$

$$\Rightarrow CD.AC = BC^2 \longrightarrow (1)$$

on adding eq. (1) *and* (2)

$$AD.AC + CD.AC = AB^2 + BC^2$$

$$\Rightarrow (AD + CD)AC = AB^2 + BC^2$$

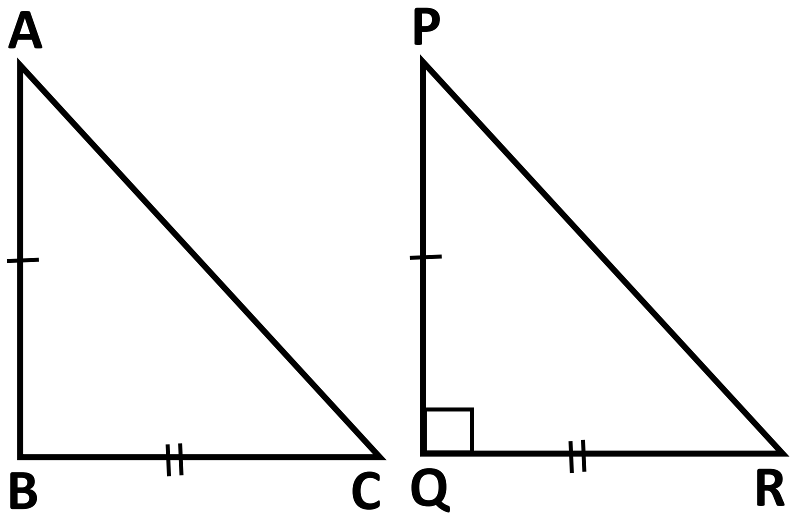
$$\Rightarrow AC.AC = AB^2 + BC^2$$

$$\Rightarrow AC^2 = AB^2 + BC^2$$

Hence proved.

CONVERSE OF PYTHAGORAS THEOREM

Theorem : In a right angle triangle, if the square of the length of one side is equal to the sum of th lengths of the other two sides, then the angle opposite to the first side is a right angle and the triangle is a right angled triangle.



Given : $In\ \Delta\ ABC, AC^2 = AB^2 + BC^2$

RTP : $\angle B = 90°$

Construction : $Construct\ a\ right\ angled\ triangle\ \Delta PQR, right\ angled\ at\ Q\ such\ that\ PQ = AB\ and\ QR = BC$

Proof : $In\ \Delta PQR, PR^2 = PQ^2 + QR^2 (\because Pythagoras\ theorem)$

$\Longrightarrow PR^2 = AB^2 + BC^2 \quad (\because construction) \longrightarrow (1)$

$but, AC^2 = AB^2 + BC^2 \quad (\because given) \longrightarrow (2)$

$from\ eq. (1) and\ (2), AC = PR$

$Now\ in\ \Delta ABC\ and\ \Delta PQR,$

$AB = PQ \quad (\because by\ construction)$

$BC = QR \quad (\because by\ construction)$

$AC = PR \quad (\because proved)$

$\therefore \Delta ABC \cong \Delta PQR\ (\because by\ SSS\ cogruency)$

$\Longrightarrow \angle B = \angle Q \quad (\because CPCT)$

$but\ \angle Q = 90°$

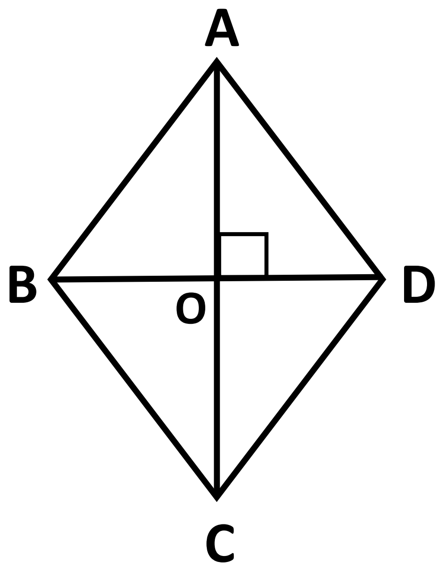
$\therefore\ \angle B = 90°$

$Hence\ proved.$

EXERCISE – 8.4

1. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals

Sol : *Let the diagonals of the rhombus ABCD are intersecting at 'O'*



$AB = BC = CD = AD$

$\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$

$In\ \Delta AOB, AB^2 = OA^2 + OB^2$

$In\ \Delta BOC, BC^2 = OB^2 + OC^2$

$In\ \Delta COD, CD^2 = OC^2 + OD^2$

$In\ \Delta AOD, AD^2 = OA^2 + OD^2$

Now, sum of the squares of the sides is

$AB^2 + BC^2 + CD^2 + AD^2$

$= OA^2 + OB^2 + OB^2 + OC^2 + OC^2 + OD^2 + OA^2 + OD^2$

$= 2(OA^2 + OB^2 + OC^2 + OD^2)$

$$= 2\left[\left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 + \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2\right]$$

$$= 2 \times \frac{1}{4}\left[AC^2 + BD^2 + AC^2 + BD^2\right]$$

$$= \frac{1}{2}\left[2AC^2 + 2BD^2\right]$$

$$= \frac{2}{2}\left[AC^2 + BD^2\right]$$

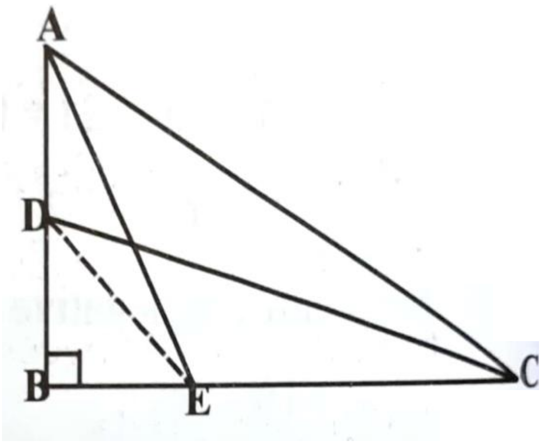
$$= AC^2 + BD^2$$

$\therefore$ *The sum of the squares of the sides is equal to the sum of the squares of the diagonals.*

$EXERCISE - 8.4$

2. ABC is a right triangle right angled at B. Let D and E be any points on AB and BC respectively. Prove that $AE^2 + CD^2 = AC^2 + DE^2$

Sol : $Given, \angle B = 90°$



$In\ \Delta ABC, AC^2 = AB^2 + BC^2$

$In\ \Delta ABE, AE^2 = AB^2 + BE^2$

$In\ \Delta DBC, CD^2 = BD^2 + BC^2$

$In\ \Delta DBE, DE^2 = BD^2 + BE^2$

$Now, L.H.S = AE^2 + CD^2$

$= AB^2 + BE^2 + BD^2 + BC^2$

$= (AB^2 + BC^2) + (BD^2 + BE^2)$

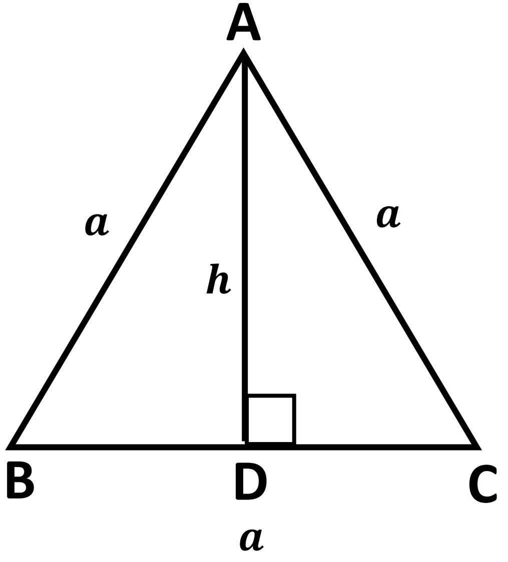
$= AC^2 + DE^2$

$= R.H.S.$

3. Prove that three times the square of any side of an equilateral triangle is equal to four times the square of the altitude.

Sol : $In\ an\ equilateral\ traingle\ ABC\ AB = BC = AC = a$

$Let\ the\ length\ of\ altitude\ drawn\ from\ A\ on\ BC\ is\ AD = h$



$In\ \Delta ADB\ and\ \Delta ADC,$

$\angle ADB = \angle ADC\ (= 90°)$

$AD = AD\ \ (\because\ common\ side)$

$AB = AC\ \ (\because\ \Delta ABC\ is\ equilateral\ triangle)$

$\therefore\ \Delta ADB \cong \Delta ADC\ \ (\because\ R.H.S.congruency)$

$\Longrightarrow BD = CD\ \ (\because\ CPCT)$

$\therefore\ BD = CD = \frac{a}{2}$

$Now, AB^2 = AD^2 + BD^2$

$$\Longrightarrow a^2 = h^2 + \left(\frac{a}{2}\right)^2$$

$$\Longrightarrow a^2 = h^2 + \frac{a^2}{4}$$

$$\Longrightarrow a^2 - \frac{a^2}{4} = h^2$$

$$\Longrightarrow \frac{4a^2 - a^2}{4} = h^2$$

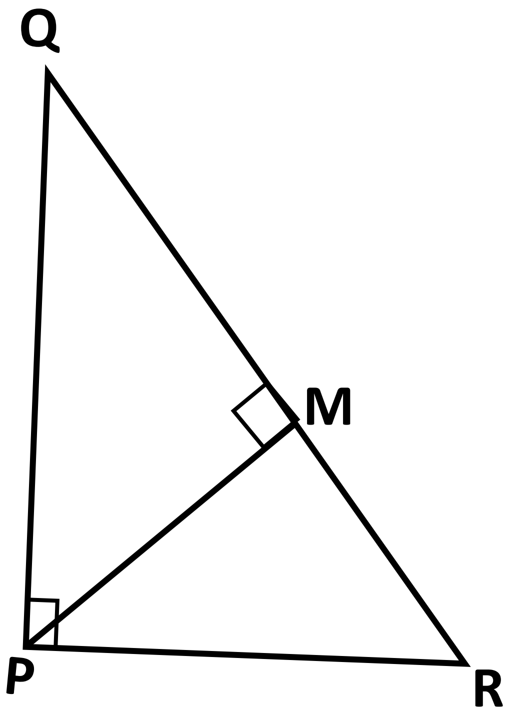
$$\Longrightarrow \frac{3a^2}{4} = h^2$$

$$\Longrightarrow 3a^2 = 4h^2$$

$i.e.three\ times\ the\ square\ of\ a\ side\ is\ equal\ to\ four\ times\ the\ altitude$

$Hence\ proved.$

4. PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR.
Show that $PM^2 = QM.MR$



$Sol:$ $In\ \Delta PQR, \angle P = 90°, and\ PM \perp QR$

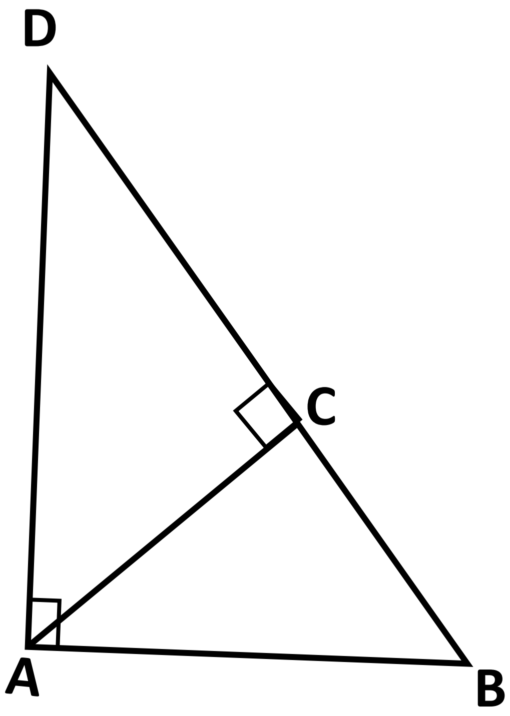
$$\Rightarrow \Delta RMP \sim \Delta PMQ$$

$$\Rightarrow \frac{PM}{QM} = \frac{MR}{PM}$$

$$\Rightarrow PM^2 = QM.MR$$

$\underline{EXERCISE\ -8.4}$

5. ABD is a triangle right angled at A and AC ⊥ BD . Show that (i) $AB^2 = BC.BD$ (ii) $AC^2 = BC.DC$ (iii) $AD^2 = BD.CD$



Sol : $In\ \Delta ABD, \angle A = 90°, and\ AC \perp BD$

$$\Longrightarrow \Delta BCA \sim \Delta BAD \sim \Delta ACD$$

(i) $\Delta BCA \sim \Delta BAD$

$$\Longrightarrow \frac{AB}{BD} = \frac{BC}{AB}$$

$$\Longrightarrow AB^2 = BC.BD$$

(ii) $\Delta BCA \sim \Delta ACD$

$$\Longrightarrow \frac{AC}{DC} = \frac{BC}{AC}$$

$$\Longrightarrow AC^2 = BC.DC$$

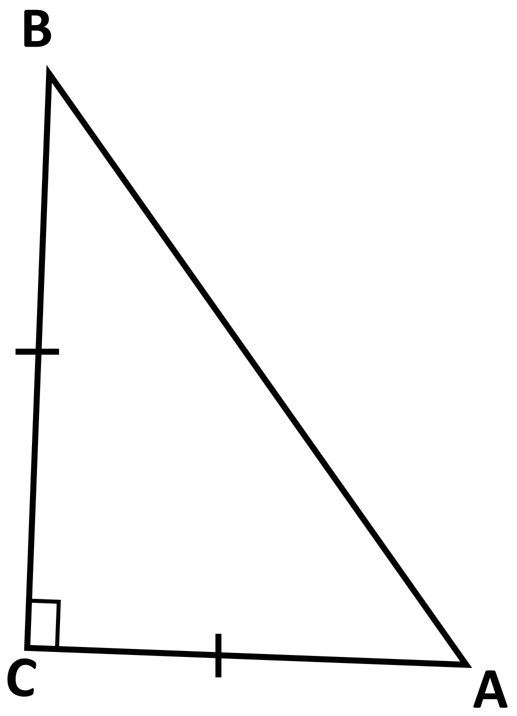
(iii) $\Delta BAD \sim \Delta ACD$

$$\Longrightarrow \frac{AD}{CD} = \frac{BD}{AD}$$

$$\Longrightarrow AD^2 = BD.CD$$

6. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$

Sol : $In\ ABC, \angle C = 90°, and\ AC = BC$



$\Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

$\Rightarrow AB^2 = 2AC^2$

$Hence\ proved$

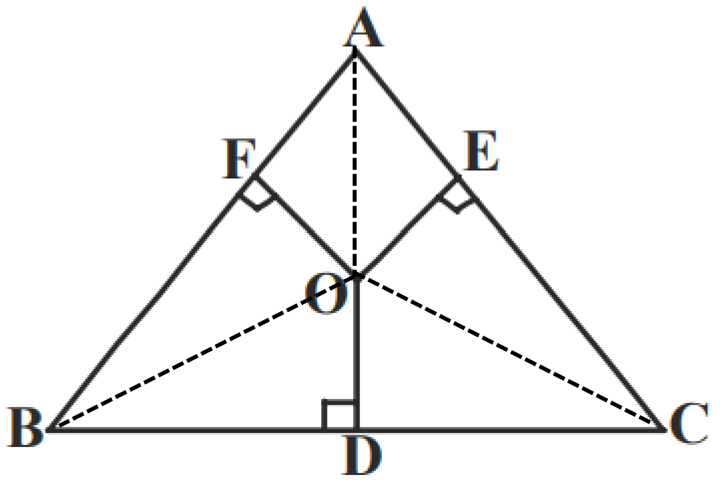
EXERCISE – 8.4

7. 'O' is any point in the interior of a triangle ABC. If OD⊥BC, OE ⊥ AC and OF⊥AB, show that

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Sol : Join 'O' with A, B and C



In ΔOAF, $OA^2 = OF^2 + AF^2$

$\Rightarrow AF^2 = OA^2 - OF^2 \longrightarrow (1)$

In ΔOBD, $OB^2 = OD^2 + BD^2$

$\Rightarrow BD^2 = OB^2 - OD^2 \longrightarrow (2)$

In ΔOCE, $OC^2 = OE^2 + CE^2$

$\Rightarrow CE^2 = OC^2 - OE^2 \longrightarrow (3)$

In ΔOAE, $OA^2 = OE^2 + AE^2$

$\Rightarrow AE^2 = OA^2 - OE^2 \longrightarrow (4)$

In ΔOBF, $OB^2 = OF^2 + BF^2$

$\Rightarrow BF^2 = OB^2 - OF^2 \longrightarrow (5)$

In ΔOCD, $OC^2 = OD^2 + CD^2$

$\Rightarrow CD^2 = OC^2 - OD^2 \longrightarrow (6)$

(i) by adding eq. (1), (2) and (3)

$$AF^2 + BD^2 + CE^2 = OA^2 - OF^2 + OB^2 - OD^2 + OC^2 - OE^2$$

$$\Rightarrow AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$$

(ii) from above result, we have

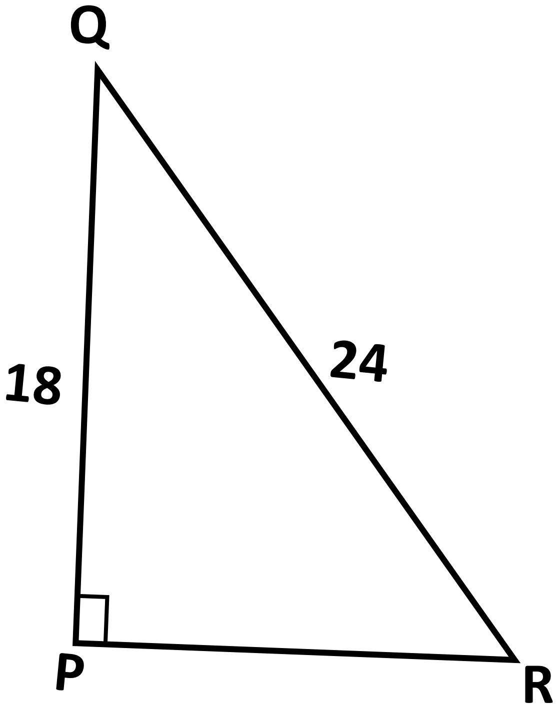
$$AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$$

$$\Rightarrow AF^2 + BD^2 + CE^2 = (OA^2 - OE^2) + (OB^2 - OF^2) + (OC^2 - OD^2)$$

$$\Rightarrow AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2 \quad (\because \text{from eq}(4), (5) \text{ and } (6))$$

$EXERCISE\ -8.4$

8. A wire attached to a vertical pole of height 18m is 24m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?



$Sol:$ $Height\ of\ the\ pole\ is\ PQ = 18m$

$length\ of\ wire\ is\ QR = 24m$

$distance\ between\ the\ foot\ of\ the\ pole\ and\ stake\ is\ PR$

$In\ \Delta PQR, QR^2 = PQ^2 + PR^2$

$\Longrightarrow 24^2 = 18^2 + PR^2$

$\Longrightarrow 576 = 324 + PR^2$

$\Longrightarrow PR^2 = 576 - 324$

$\Longrightarrow PR^2 = 252$

$\Longrightarrow PR = \sqrt{252}$

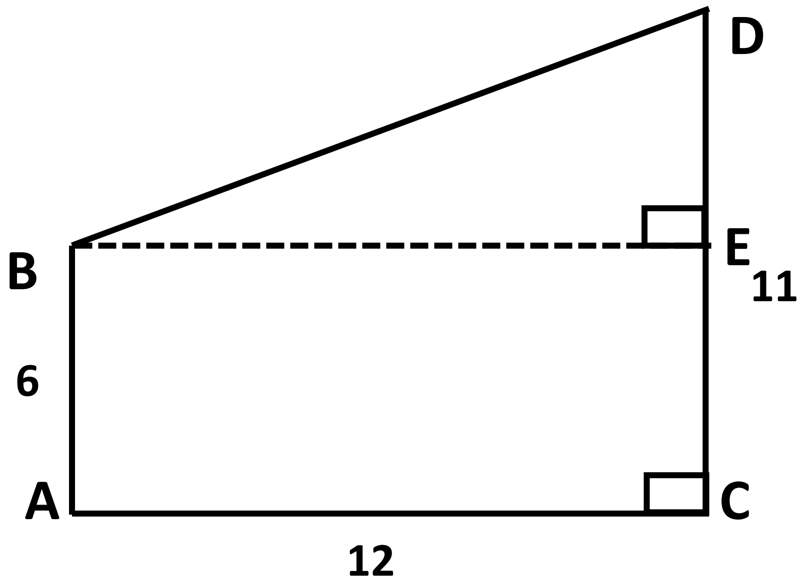
$\Longrightarrow PR = \sqrt{36 \times 7}$

$\Longrightarrow PR = 6\sqrt{7}$

$\therefore The\ stake\ should\ be\ driven\ at\ a\ distance\ of\ 6\sqrt{7}\ m\ from\ the\ foot\ of\ the\ pole.$

$\underline{EXERCISE\ -8.4}$

9. Two poles of heights 6m and 11m stand on a plane ground. If the distance between the feet of the poles is 12m find the distance between their tops.



$Sol:$ $Height\ of\ the\ pole\ AB = 6\ m$

$Height\ of\ the\ pole\ CD = 11\ m$

$distance\ between\ the\ feet\ of\ the\ poles\ is\ AC = 12\ m$

$draw\ a\ line\ BE\ parallel\ to\ AC$

$In\ the\ rectangle\ ABEC, AC = BE = 12\ m$

$$AB = CE = 6\ m$$

$In\ the\ triangle\ BED, DE = DC - EC = 11 - 6 = 5m$

$In\ \Delta BED, BD^2 = BE^2 + DE^2$

$$\Rightarrow BD^2 = 12^2 + 5^2$$

$$\Rightarrow BD^2 = 144 + 25$$

$$\Rightarrow BD^2 = 169$$

$$\Rightarrow BD = \sqrt{169}$$

$$\Rightarrow BD = 13$$

$\therefore The\ distance\ between\ the\ tops\ of\ the\ poles\ is\ 13\ m.$

EXERCISE – 8.4

10. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$

Sol : $Given\ AB = BC = AC\ and\ BD = \frac{1}{3}BC$

$Join\ A\ and\ D$

$Draw\ \ AE\ perpendicular\ to\ BC$

$then, \Delta ABE \cong \Delta ACE$

$\Longrightarrow BE = CE = \frac{1}{2}BC$

$In\ \Delta ABE, AB^2 = AE^2 + BE^2$

$also, AD^2 = AE^2 + DE^2$

$$Now, AB^2 - AD^2 = AE^2 + BE^2 - \left(AE^2 + DE^2\right)$$
$$= AE^2 + BE^2 - AE^2 - DE^2$$
$$= BE^2 - DE^2$$
$$= BE^2 - (BE - BD)^2$$
$$= \left(\frac{1}{2}BC\right)^2 - \left[\frac{1}{2}BC - \frac{1}{3}BC\right]^2$$
$$= \frac{1}{4}BC^2 - \left[\frac{1}{6}BC\right]^2$$

$$AB^2 - AD^2 = \frac{1}{4}BC^2 - \frac{1}{36}BC^2$$
$$= \left(\frac{1}{4} - \frac{1}{36}\right)BC^2$$
$$= \left(\frac{9-1}{36}\right)BC^2$$
$$= \frac{8}{36}BC^2$$
$$= \frac{2}{9}BC^2$$

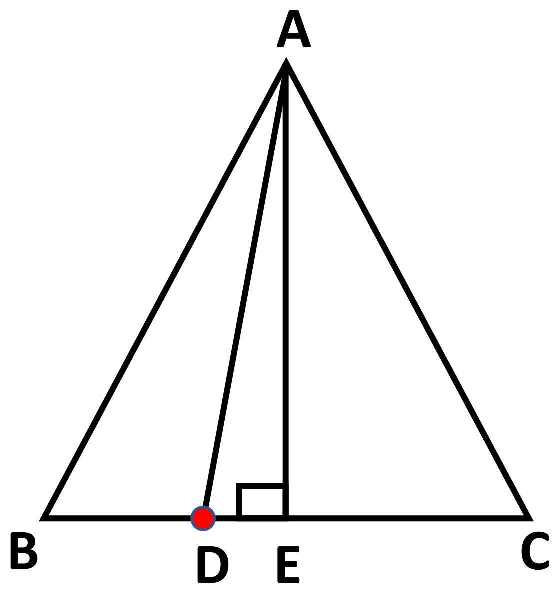
$$AB^2 - AD^2 = \frac{2}{9}BC^2$$

$$\Longrightarrow AB^2 - AD^2 = \frac{2}{9}AB^2$$

$$\Longrightarrow 9\left(AB^2 - AD^2\right) = 2AB^2$$

$$\Longrightarrow 9AB^2 - 9AD^2 = 2AB^2$$

$$\Longrightarrow 9AB^2 - 2AB^2 = 9AD^2$$

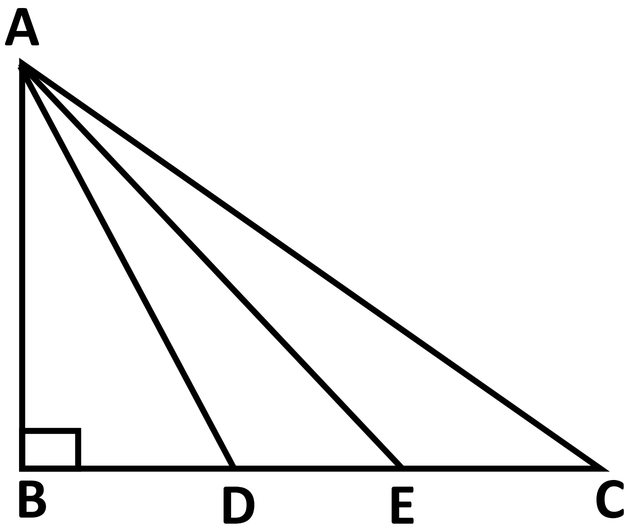


$$\Longrightarrow 7AB^2 = 9AD^2$$

$$\Longrightarrow 9AD^2 = 7AB^2$$

Hence proved

11. In the given figure, ABC is a triangle right angled at B. D and E are ponts on BC trisect it. Prove that $8AE^2 = 3AC^2 + 5AD^2$



Sol : $Given\ \angle B = 90°$ and BD = DE = EC $= \frac{1}{3}BC$

$In\ \Delta ABC, AC^2 = AB^2 + BC^2 \longrightarrow (1)$

$In\ \Delta ABE, AE^2 = AB^2 + BE^2 \longrightarrow (2)$

$In\ \Delta ABD, AD^2 = AB^2 + BD^2 \longrightarrow (3)$

$from\ eq.(1)$

$3AC^2 = 3AB^2 + 3BC^2 \longrightarrow (4)$

$from\ eq.(3)$

$5AD^2 = 5AB^2 + 5BD^2 \longrightarrow (5)$

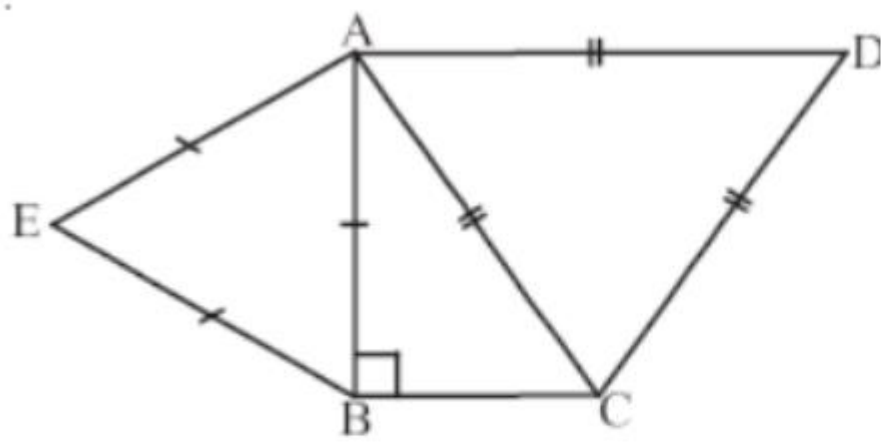
$by\ addding\ eq.(4)\ and\ (5)$

$$3AC^2 + 5AD^2 = 3AB^2 + 3BC^2 + 5AB^2 + 5BD^2$$
$$= 8AB^2 + 3BC^2 + 5BD^2$$
$$= 8AB^2 + 3\left(\frac{3}{2}BE\right)^2 + 5\left(\frac{1}{2}BE\right)^2$$
$$= 8AB^2 + \frac{27}{4}BE^2 + \frac{5}{4}BE^2$$

$$3AC^2 + 5AD^2 = 8AB^2 + \frac{32}{4}BE^2$$
$$= 8AB^2 + 8BE^2$$
$$= 8(AB^2 + BE^2)$$
$$= 8AE^2$$

$$\therefore 3AC^2 + 5AD^2 = 8AE^2$$

12. ABC is an isosceles triangle right angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and Δ ACD.



Sol : $In\ \Delta ABC, \angle B = 90°, and\ AB = BC$

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = AB^2 + AB^2$$

$$\Rightarrow AC^2 = 2AB^2$$

$$\Rightarrow \frac{AB^2}{AC^2} = \frac{1}{2}$$

$$\Rightarrow \left(\frac{AB}{AC}\right)^2 = \frac{1}{2} \longrightarrow (1)$$

$we\ have\ \Delta ABE \sim \Delta ACD$

$$\Rightarrow \frac{area(\Delta ABE)}{area(\Delta ACD)} = \left(\frac{AB}{AC}\right)^2$$

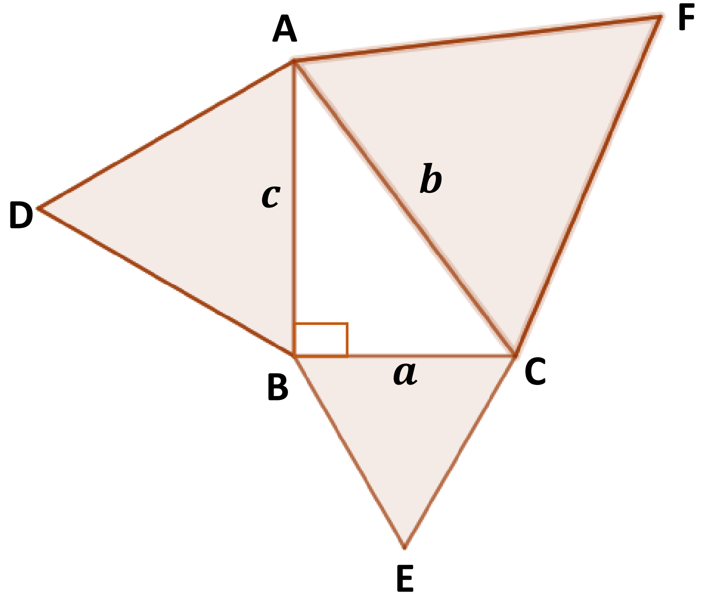
$$\Rightarrow \frac{area(\Delta ABE)}{area(\Delta ACD)} = \frac{1}{2} \ (\because from\ eq. (1))$$

$\therefore The\ ratio\ of\ the\ areas\ of\ \Delta ABE\ and\ \Delta ACD\ is\ 1:2$

13. Equilateral triangles are drawn on the three sides of a right angled triangle. Show that the area of the triangle on the hypotenuse is equal to the sum of the areas of triangles on the other two sides.

Sol : $In\ \Delta ABC, \angle B = 90°, and\ BC = a, AC = b, AB = c$

$$\Rightarrow b^2 = c^2 + a^2 \longrightarrow (1)$$



$$area\ of\ the\ equilateral\ triangle\ BCE = \frac{\sqrt{3}}{4}a^2$$

$$area\ of\ the\ equilateral\ triangle\ ACF = \frac{\sqrt{3}}{4}b^2$$

$$area\ of\ the\ equilateral\ triangle\ ABD = \frac{\sqrt{3}}{4}c^2$$

$sum\ of\ the\ areas\ of\ \Delta ABD\ and\ \Delta BCE$

$$= \frac{\sqrt{3}}{4}c^2 + \frac{\sqrt{3}}{4}a^2$$

$$= \frac{\sqrt{3}}{4}(c^2 + a^2)$$

$$= \frac{\sqrt{3}}{4}b^2 \quad (\because from\ eq(1))$$

$$= area\ of\ \Delta ACF$$

$\therefore$ *In a right angle triangle, area of the equilateral triangle on the hypotenuse is equal to the sum of the areas of the equilateral triangles on the remaining two sides*

14. Prove that the area of the equilateral triangle described on the side of a square is half of the area of equilateral triangle described on its diagonal.

Sol : $ABCD$ *is a square with side* $'a'$

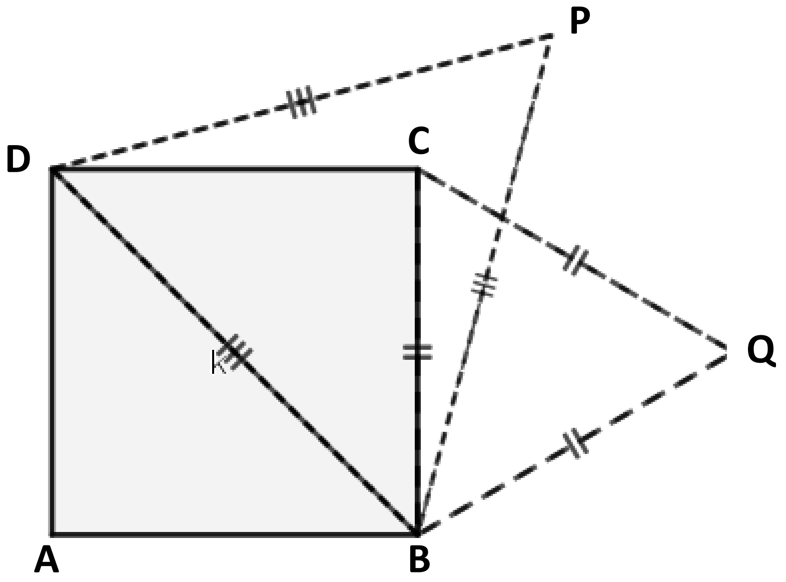
length of its diagonal $BD = \sqrt{2}.a$

area of the equilateral triangle on the diagonal BD

$$= \frac{\sqrt{3}}{4}\left(\sqrt{2}\,a\right)^2$$

$$= \frac{\sqrt{3}}{4} \times 2a^2$$

$$= \frac{\sqrt{3}}{2} a^2$$



area of the equilateral triangle on the side BC

$$= \frac{\sqrt{3}}{4} a^2$$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} a^2$$

$$= \frac{1}{2} \times \text{area of the equilateral triangle on the diagonal } BD$$

$\therefore$ *The area of the equilateral triange on the side of a square is equal to half of the area of equilateral triangle on its diagonal*

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