Dynamical modelling and analysis of COVID-19 in India

R. Gopal[1], V. K. Chandrasekar[1] and M. Lakshmanan[2]
[1] *Centre for Nonlinear Science & Engineering, School of Electrical & Electronics Engineering, SASTRA Deemed University, Thanjavur -613 401, Tamilnadu, India.*
[2] *Department of Nonlinear Dynamics, School of Physics, Bharathidasan University, Tiruchirappalli -620 014, Tamil Nadu, India.*

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We consider the pandemic spreading of COVID-19 in India after the outbreak of the coronavirus in Wuhan city, China. We estimate the transmission rate of the initial infecting individuals of COVID-19 in India by using the officially reported data at the early stage of the epidemic with the help of Susceptible (S), Exposed (E), Infected (I), and Removed (R) population model, the so called SEIR dynamical model. Numerical analysis and model verification are performed to calibrate the system parameters with official public information about the number of people infected, and then to evaluate several COVID -19 scenarios potentially applicable to India. Our findings provide an estimation of disease occurrence in the near future, and also demonstrate the importance of governmental and individual efforts to control the effects and time of the pandemic-related critical situations. We also give special emphasis to individual reactions in the containment process.

PACS numbers: epidemic mathematical model,COVID-19, Governmental action

I. INTRODUCTION

COVID-19, a disease caused by the severe acute respiratory syndrome coronavirus 2 (SARS-COV 2), was first identified in December 2019 in Wuhan, the capital of Hubei, China, and has since spread globally [1]. The World Health Organization (WHO) announced COVID-19 as an international public health emergency on 30^{th} January, 2020, and subsequently a pandemic on 11^{th} March, 2020 [2]. The number of patients is growing exponentially and thousands of people are losing their lives in many a countries, almost every day due to COVID-19 [3–5]. The complexity of the situation can be realized from the fact that as on 15^{th} May, 2020 the number of coronavirus cases worldwide has reached a staggering 46,39,427, with currently infected patients being 25,64,442 and more than 3,08,810 confirmed deaths have been reported due to this disease [6]. Moreover, the outbreak has also spread to more than two hundred countries [6].

Much work indicates that COVID-19 could spread from animal to human (zoonotic) [7]. In addition, a rapid increase of COVID-19 infections show the main finding that secondary transmission can occur through human-to-human contacts or through droplets transmitted by coughing or sneezing from an infected person [4, 7] or even when an infected person speaks to a non-infected one. With the above trend, this spread of human-to-human disease is growing significantly almost everywhere in the world and the infection is rapidly increasing in many countries through local transmission [4].

In India, the infection due to COVID-19 was first identified on 02^{nd} March, 2020. The Indian Government announced a 21-day country-wide lock-down as a preventive measure for the COVID-19 outbreak on 24^{th} March, 2020. The aim of the lockdown was to slow down the spread of the novel coronavirus, to allow the Government to follow a multi-pronged strategy to add more beds in its hospital network, to increase the development of the COVID-19 test kits and of personal protective equipments (PPEs) for health workers, etc. The government frequently uses different platforms to keep the public aware of COVID -19. In India, conditions including very high population density in urban areas, unavailability of vaccines and inadequate data about the disease's transmitting process also make it a herculean task to adequately fight the disease.

Mathematical simulations were often used to forecast the effects of various epidemics and also to test the efficacy of the different prevention approaches in reducing the burden of the epidemics [8]. Recently, concerning the current COVID-19 pandemic, considerable research has been carried out using actual occurrence from the impacted countries, analyzing various aspects of the epidemic, as well as assessing the impact of preventive approaches adopted in order to limit the epidemic in the countries concerned [9]. In particular, various kinds of dynamical models have been employed, essentially considering nonlinear governing equations. For instance, the nature of human coronavirus infection and defining contact between human cells and the virus have been described in Ref [10]. The statistical model for estimating virus transmission, taking into account a condensed version of the bats-hosts-reservoir-people transmission model, known as a reservoir-people model was also reported in Ref [11].

Susceptible-exposed-infectious-recovered (SEIR) model is an important tool for tackling coronavirus transmission statistical simulations. Lin and his co-workers have proposed a method to develop analytical models for the COVID-19 outbreak in Wuhan taking into account human behavioral responses and government decisions, such as holiday extension, travel restriction, hospitalization, and quarantine [3]. Some recent studies based on these works use numerical simulations and attempt to provide a reliable real-time forecast of COVID-19 cases in various countries with the help of this mathematical model. For instance, Savi *et al* have obtained the general transmission of the novel coronavirus to test various scenarios of coronavirus propagation in different countries, taking into account the model testing of the evolution of infected populations in China, Italy, Iran and Brazil based on government and individual reactions [12].

In this work, we consider the situation in India starting from the initial outbreak period and fitted SEIR model to the daily infected cases reported between 2^{nd} March, 2020 to 15^{th} May, 2020. We estimate the basic transmission rate of COVID-19 in the initial stage of the epidemic between 2^{nd} March, 2020 and 24^{th} March, 2020. Further, the general propagation of the novel coronavirus is also studied in our investigation to evaluate different scenarios of the propagation of coronavirus. In addition, the model verification takes into consideration the evolution of the infected population and simulates different scenarios based on the rate of transmission and the governmental and, especially individual reactions. Finally it proposes potential future evolution of the spread and possible mitigations, specifically emphasizing the significance of individual reactions at the societal level.

The structure of the paper is as follows. In section II, we briefly describe the SEIR dynamical model. The estimation of transmission rate and detailed numerical analysis of the mathematical model with actual data is described in section III. Finally, we discuss the findings obtained from our study in Section IV.

TABLE I. Summary table of the parameters discussed in Eqs. (1) and (2)

Parameter	Description	value/remarks/reference
N_0	Initial number of population	India populations [15]
S_0	Initial number of susceptible population	$0.9N_0$ (constant)
E_0	Exposed persons for each infected person	$20I_0$ (assumed)
I_0	Initial state of infected persons	3 [15]
α	Government action strength	varied in each lock-down period
k	intensity of individual reaction	1117.3 [3, 12]
σ^{-1}	Mean latent period	3 (days)
γ^{-1}	Mean infectious period	5 (days)
d	Proportion of severe cases	0.2
λ^{-1}	Mean duration of public reaction	11.2 (days)

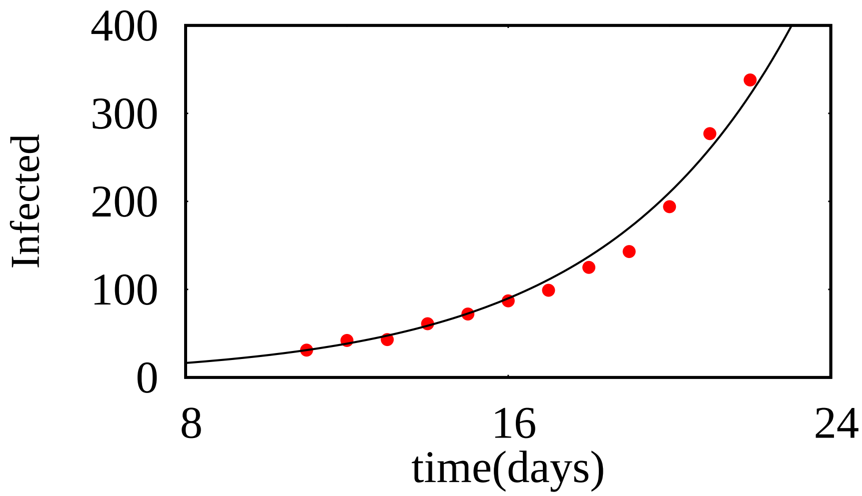

FIG. 1. The number of infected individuals (excluding number of initial quarantined people) after the first few days of outbreak on 2^{nd} March, 2020. Data were fitted by the nonlinear least-square fit for India. The solid black line is fitting data and red circles denote real data of the number of infected individuals.

II. THE DYNAMICAL MODEL

Lin and his co-workers have proposed a susceptible-exposed-infectious-removed (SEIR) model to explain coronavirus disease (COVID-19) [3]. This model was inspired by He *et al's* original influenza model [13, 14]. The SEIR model also has two supplementary terms: D is a public perception of risk with respect to serious cases and deaths, and C is the number of recorded and unreported incidents. In addition, S is the susceptible population, E is the population exposed, I is the currently infectious population (excluding the recovered and death cases) and R is the population removed which includes both the cases of recovered and deaths. The simplified version of the governing equations takes account of the interaction between all these populations, and is represented by the following set of coupled nonlinear differential equations [3],

$$\dot{S} = -\beta(t)\frac{SI}{N}, \tag{1a}$$

$$\dot{E} = \beta(t)\frac{SI}{N} - \sigma\ E, \tag{1b}$$

$$\dot{I} = \sigma E - \gamma I, \tag{1c}$$

$$\dot{R} = \gamma_R I, \tag{1d}$$

$$\dot{D} = d\gamma I - \lambda D, \tag{1e}$$

$$\dot{C} = \sigma E, \tag{1f}$$

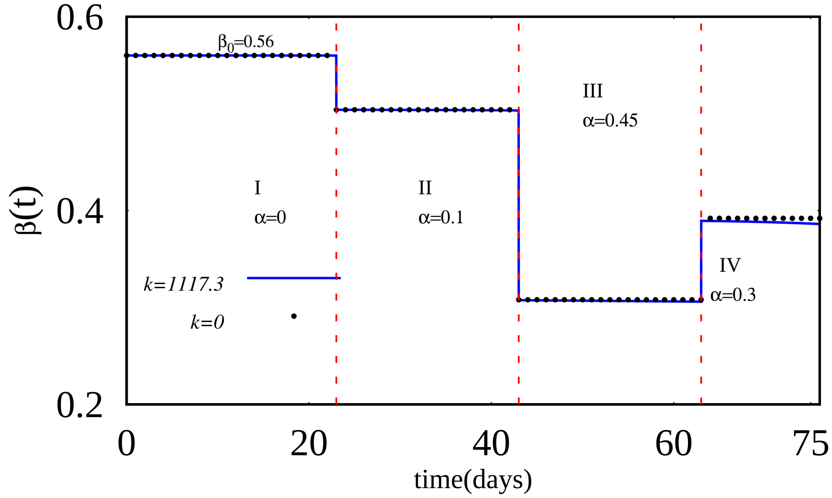

FIG. 2. Variation of transmission rate considered in our model (1), through time with the impact of α in (2) with respect to initial transmission value $\beta_0 = 0.56$. The continuous curve is for intensity of individual reaction $k = 1117.3$ and the dotted curve is for $k = 0$.

where γ is the mean infectious period, γ_R is the delayed removed period, which denotes the relation between removed population and the infected one, σ is the mean latent period, d denotes the proportion of severe cases and λ is the mean duration of public reaction [3, 4, 12].

In Eq.(1), $\beta(t)$ denotes the transmission rate function which incorporates the impact of governmental action $(1-\alpha)$, and the individual action, which is denoted by the function $\left(1 - \frac{D}{N}\right)^k$ [3, 12]. Here, the parameter k defines the intensity of individual reaction, which is measured on a scale of 0 to 10^5 with a normal value of 1117.3 obtained from previous and recent epidemic and pandemic studies [3, 13]. We also assume that the effect of governmental action is different during different lock-down periods. Therefore, the transmission rate $\beta(t)$ is defined as

$$\beta(t) = \beta_0(1-\alpha)\left(1 - \frac{D}{N}\right)^k . \tag{2}$$

The value of β_0 is derived by assuming that the basic reproduction number is $R_0 = \frac{\beta_0}{\gamma}$, which measures the average number of new infections generated by each infected person. The values of the system parameters are mentioned in Table I, based on the information deduced from the references in [3, 4, 12, 13, 17].

The above parameters must be modified for each state/country which is important for the analysis of COVID-19. In general, the physical meanings of the parameters are based on a variety of facts, identifying which constitute a difficult task [3]. It should be also pointed out in this regard that the real data has spatial aspects that are not covered by the above set of governing equations. Consequently, this kind of study is a sort of average activity that needs careful adjustment to suit real data as followed by earlier studies [3, 12].

In our present study, we use step-like functions to define certain parameters that allow for a proper representation of different scenarios, especially the rate of transmission. It is also important to remember that all governmental or individual decisions have a delayed impact on the dynamics of the system. Further, virus mutation is another important factor related to the definition of COVID-19 dynamics which can affect the reaction of the system significantly but is not discussed in our present study [12]. The next sections treat the COVID-19 dynamics considering two different objectives. To start with, we identify the initial value of transmission rate with the data from initially infected people, and then it examines various scenarios for the COVID -19 situation in India, using different transmission rates and government and individual action strengths.

III. DATA ANALYSIS WITH NUMERICAL MODEL

As a first phase of the established study, model verification is performed using information available on India covid-19 tracker and worldometers.info [6]. We follow the SEIR implementation methodology described for various

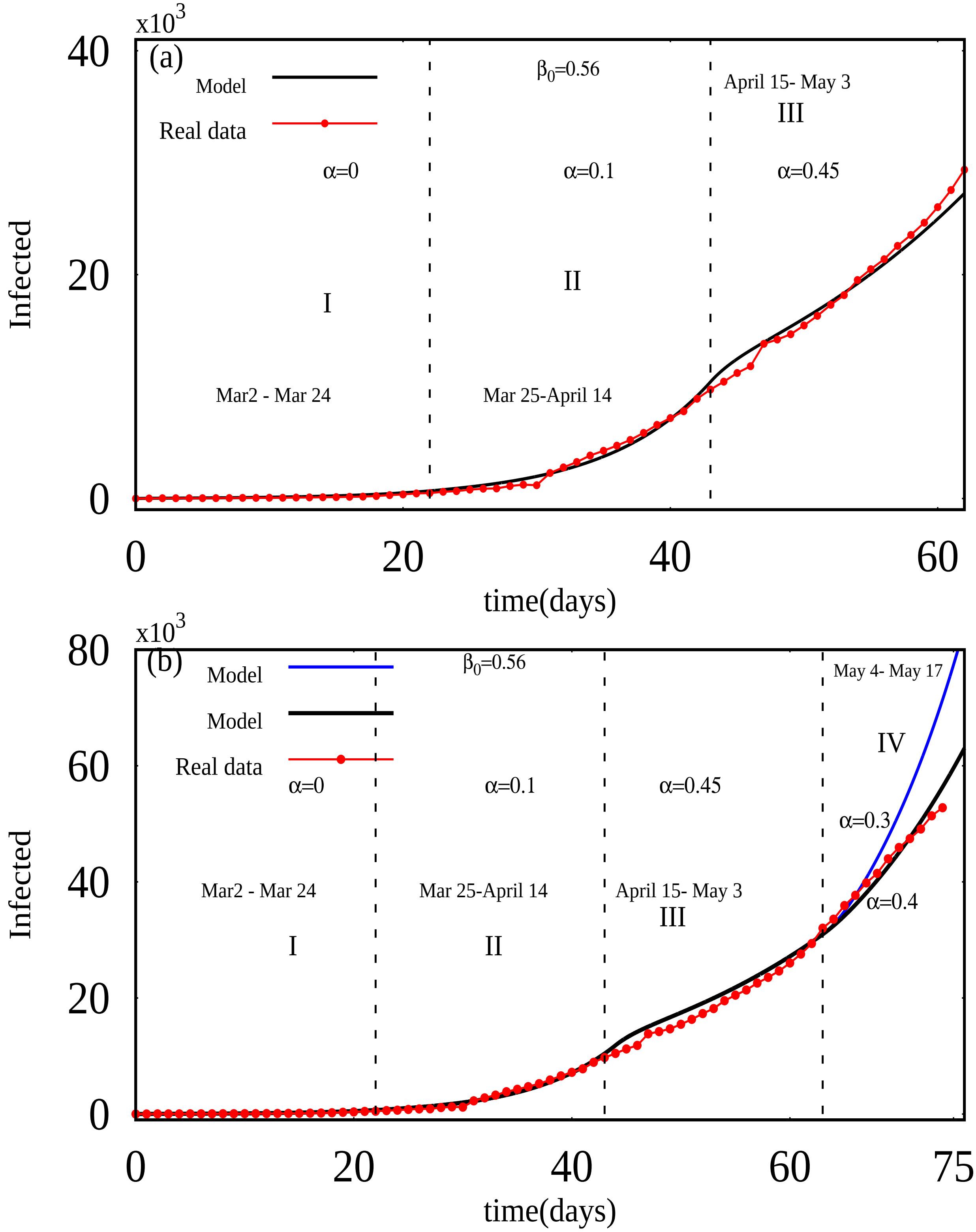

FIG. 3. Numerical simulation of the number of infected individual people (excluding both recovered and deaths) with the value of initial transmission rate taken as $\beta_0 = 0.56$ and for different governmental action strengths before and after the lock-down period. The prediction of the mathematical model (continuous blue and black curve) compared with actual data (red dotted curve) for the daily infected people in India upto (a) 3^{rd} May, 2020 and, (b) 17^{th} May, 2020 is depicted. The strength of individual reaction is considered as $k = 1117.3$ and the remaining parameters are taken from Table I.

countries in [12]. The model employed for simulations is with a total population of India at $N \approx 139 \times 10^7$ [15], and 3 infected COVID-19 confirmed as on 02-Mar-2020. The initial state is taken to be with $I_0 = 3$ and a susceptible initial population is assumed to be $S_0 = 0.9N$. Another information needed for the model is the number of individuals exposed to each infected person. Each infected individual is believed to have the potential to infect a further 20 individuals, $E_0 = 20I_0$ [3, 12]. Initially, we started with the estimation of the initial transmission rate.

A. Estimation of initial transmission rate

To start with, we note that, intensified precautionary measures to curb the spread of COVID -19 was carried out in India. In particular, when patients were found at a location, the Committee of Physicians and Experts checked those people who had contacts with the patients. After that, both the patients and the contaminated people were quarantined in a hospital or in other isolated areas. In India's data published, in the initial few days, the active cases were automatically moved to quarantined cases [19]. Therefore, the active cases correlate after a few days with the omitted quarantined cases. From the recorded data we can match real infected individuals (excluding quarantined people) as a function of time at the early stage of the disease spreading bar the initial quarantined people [20].

In order to solve the model (1), we consider the susceptible population to be close to the overall population in the early stages of transmission of the disease [20], and we can rewrite the dynamic equation of the epidemic with $\frac{S}{N} \approx 1$.

In this case, from Eqs. (1c) and (1b), the equation for the presently infectious population (which excludes the recovered and death cases) becomes the second order linear ordinary differential equation of damped linear type,

$$\ddot{I} + (\gamma + \sigma)\dot{I} - \sigma(\beta - \gamma)I = 0. \tag{3}$$

By integrating the above equation, we obtain the solution as [16]

$$I(t) = I_{01} e^{\frac{-1}{2}(\gamma+\sigma-\sqrt{(\gamma-\sigma)^2+4\sigma\beta})t} + I_{02} e^{\frac{-1}{2}(\gamma+\sigma+\sqrt{(\gamma-\sigma)^2+4\sigma\beta})t}, \tag{4}$$

where $I(0) = I_{01} + I_{02}$ is the initial number of individual infected people. Now the curve is fitted with the initial real infectious population data available in [6]. Further we identify an exact fit with the curve $3e^{0.473t}$. From this we obtain the parameter values as $I_{01} = 3$, $I_{02} = 0$ and the initial value of β is 0.56. This β value can be considered as the initial transmission rate β_0 in (2). It gives the reproduction number $R_0 = \frac{\beta_0}{\gamma} = 2.8$. One may note that a few recent studies suggest a reproduction rate of R_0 about 2.52 for the first 22 days before lockdown, and then identify the value of R_0 to vary between 1.9 to 3.0 in various places in India, due to the variation of transmission rates [17, 18, 20], but in our analysis the estimation of R_0 from the model is confirmed with real data of daily infected individuals (excluding recovered and death cases). In Fig. 1 we represent the curve fit of the infected people as a function of time at the early stage of the disease spreading in India.

B. Verification through Simulations

In this section, the first scenario for model verification based on the results of India is presented. It should be noted that our analysis considers all infected cases in entire India, and it is not restricted to any specific state or place. The parameters which are used in the model are specified in Table I, and these parameters should be treated as average since they are appropriate for the country as a whole. Initially, we started with the value of transmission rate starting with $\beta_0 = 0.56$ and the value attributable to governmental action strength is also considered as different during the periods before and after lock-down. Fig. 2 charts the transmission rate $\beta(t)$, before and after the lock-down period with respect to various governmental action strengths. One may note that one important constraint on the parameter $\beta(t)$ is that this variable should be a step-like function with time due to the impact of government action strength. Therefore, using the daily COVID-19 incidence data, numerical simulation is carried out for the model (2) with the value of $\beta_0 = 0.56$. Fig. 3 shows the evolution of the number of infected individuals (excluding both recovered and death population), indicating a strong agreement between simulation and actual data. The infected people regions are marked as I, II,III and IV and they correspond to the periods of Mar 02 -Mar 24 (before lock-down), Mar 25-April 14 (first lock-down), April 15 - May 3 (second lock-down) and May 4 - May 17 (third lock-down), respectively.

It is apparent that our estimation of the number of infected individuals in the model (1), with basic transmission rate, system parameters and different governmental action strengths upto the end of the second lock-down period starting from initial outbreak (Mar 2 -May 3) shows good agreement with the actual data of infected individuals (See Fig.3(a)). This study helps to predict newly infected individuals in the near future. For instance, we choose low values of governmental action strengths, $\alpha = 0.3$ and 0.4, in the third lock-down period (due to the announcement of relaxation in the lock-down by the Government of India) compared to $\alpha = 0.45$ during the second lock-down period,

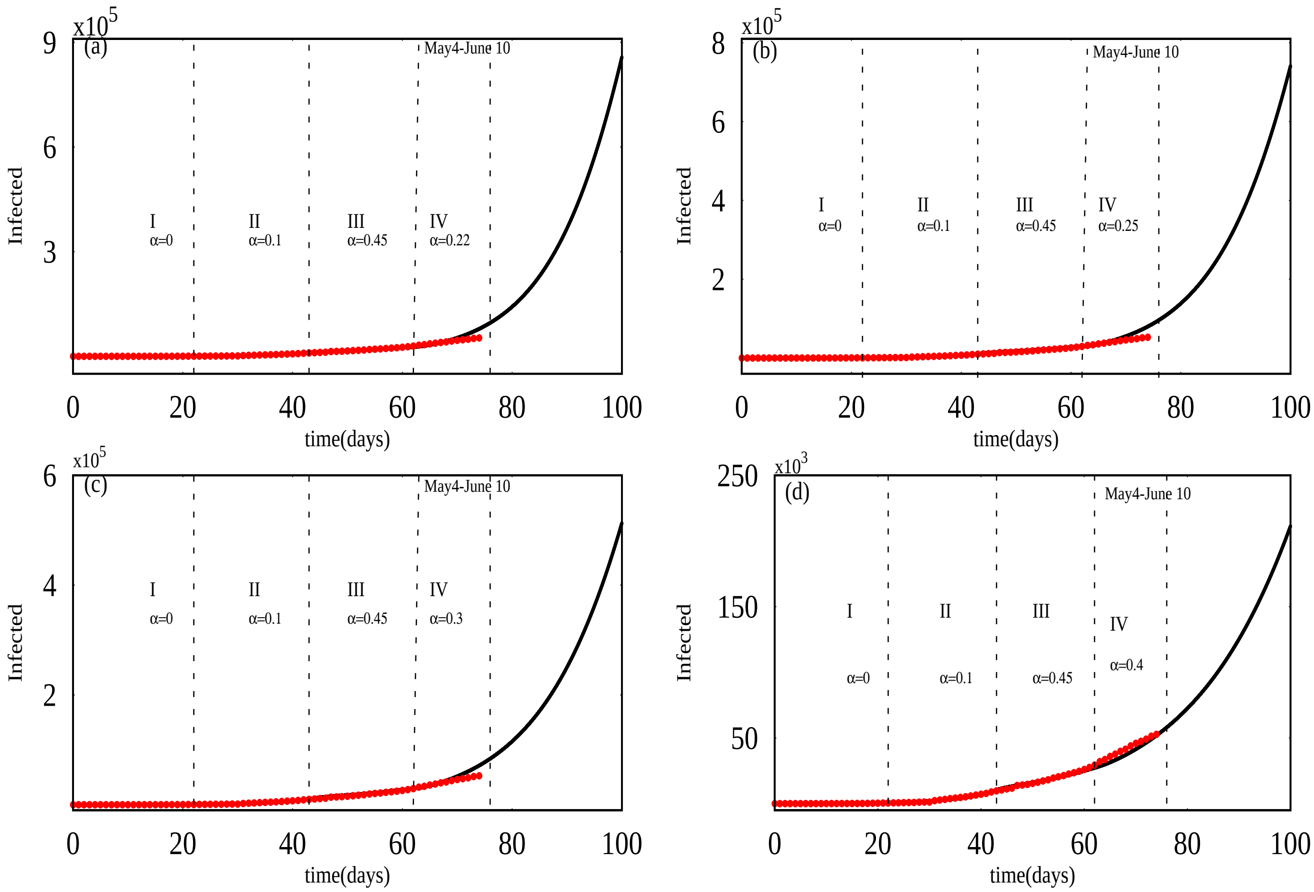

FIG. 4. Numerical simulation of the evolution of number of infected individuals with the value of initial transmission rate $\beta_0 = 0.56$. The red curve denotes actual data for the daily infected people (excluding recovered and death cases) in India up-to 15^{th} May, 2020. Note that this simulation (See column IV) shows the rate of newly infected population after May 3 with respect to the values of government action strength α in (2) with (a) $\alpha = 0.22$, (b) $\alpha = 0.25$, (c) $\alpha = 0.3$, and (d) $\alpha = 0.4$. The strength of individual reaction considered is a rather low value of k = 1117.3 and the remaining parameters are taken from Table I.

and our model study predicts the number of infected individuals reasonably well with the real data for $\alpha = 0.4$ in the region IV (See black curve in Fig.3(b)). Note that in Figs.3, we have also included the strength of individual reaction at a rather low value of $k = 1117.3$, see Eq. (2) above, but the contribution due to this term is only minimal. A sample summary of the actual infected people in India [6], and prediction of infected individuals from the model (1) are given in Table.II.

C. Role of governmental action and individual reaction

In addition, we also analyze the possibility of impact of prevention approaches in reducing new cases infected with COVID-19 via the above mathematical model, after 3^{rd} May, 2020 (For instance 4^{th} May, 2020 - 10^{th} June, 2020, and beyond, marked as region IV in Figs. 4(a)-(d)). Strategies for prevention include preventive mechanisms such as lock-down, information campaign by newspapers and television, adequate hand sanitation, social distancing etc. which results in slowing down the COVID-19 transmission process. These strategies of prevention as modeled in terms of the parameter α and k in Eq. (2), which imply that there will be a reduction in the transmission rate $\beta(t)$. Now, we start with the efficiency of prevention by varying government action strength alone, keeping k at fixed low strength $k = 1117.3$ [3]. For such low values of strengths, that is for $\alpha = 0.22$ and $\alpha = 0.25$, it is observed that the number of infected individuals will peak around 8×10^5 and 7×10^5 respectively by June 10 (see Fig.4(a)-(b)). If the strength is increased (for $\alpha = 0.30$), the peak of the infected/active people cases may decrease and the occurrence of the peak is shifted down to 5×10^5 on June 10 (see Fig.4(c)). Now, we consider a further increased value of government action strength (for $\alpha = 0.4$), and the newly infected cases can decrease and reach around 2.5×10^5 by June 10 (see Fig.4(d)).

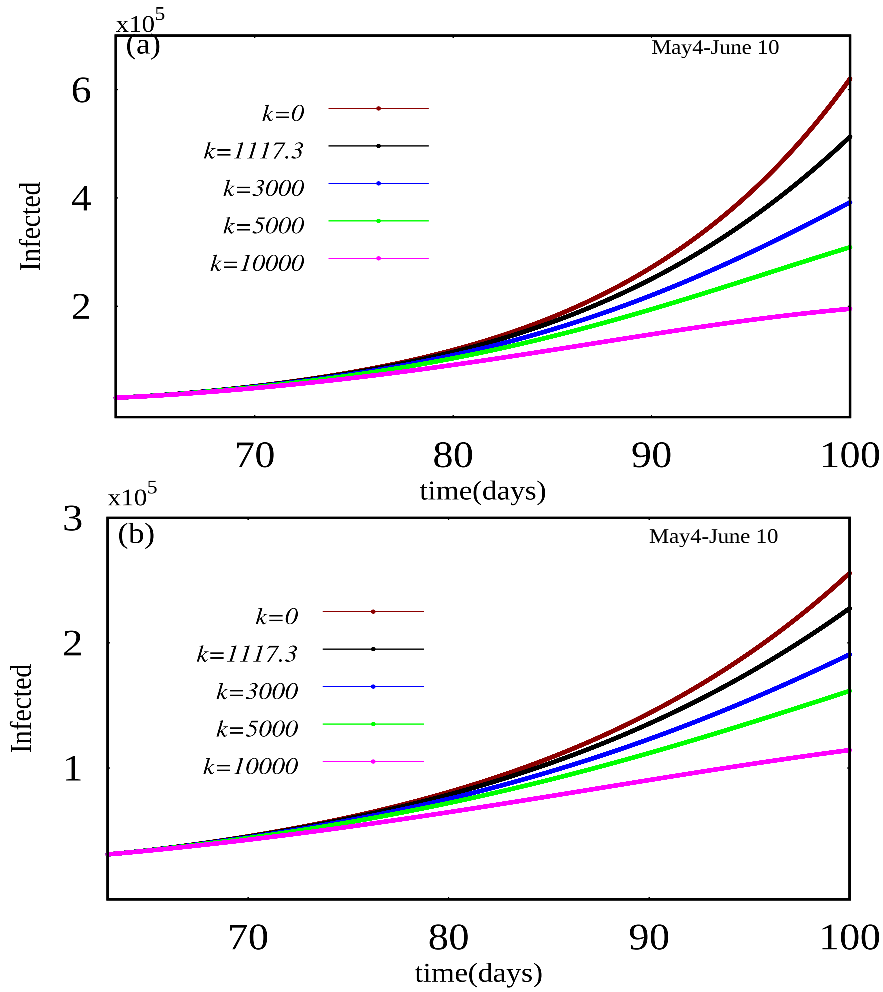

FIG. 5. Numerical simulation of the evolution of the number of infected individuals with the specific value of initial transmission rate $\beta_0 = 0.56$, after May 3 (expanded view of column IV in Figs. 4(c)-(d)) with respect to different values of intensity of individual response k in Eq.(2) for different values of government action strength in (a) $\alpha = 0.3$, and (b) $\alpha = 0.4$. The remaining parameters are taken from Table I.

TABLE II. Sample summary of actual data of COVID-19 pandemic in India [6] and predicted infected people (I(t)) from SEIR model given by Eqs. (1) and (2)

Date	Number of Infected People [6](worldometers)	SEIR Model (I(t))
04/03/2020	26	29
10/03/2020	58	84
24/03/2020	486	679
2/04/2020	2280	2249
11/04/2020	7189	7103
14/04/2020	9735	10417
15/04/2020	10440	11564
19/04/2020	14202	14660
23/04/2020	17306	17575
24/04/2020	18171	18370
25/04/2020	19519	19198
03/05/2020	29339	27261
10/05/2020	43980	43910
13/05/2020	49104	55958 ($\alpha = 0.3$); 46368 ($\alpha = 0.4$)
14/05/2020	51379	60668 ($\alpha = 0.3$); 49073 ($\alpha = 0.4$)
15/05/2020	52773	65767 ($\alpha = 0.3$); 51904 ($\alpha = 0.4$)

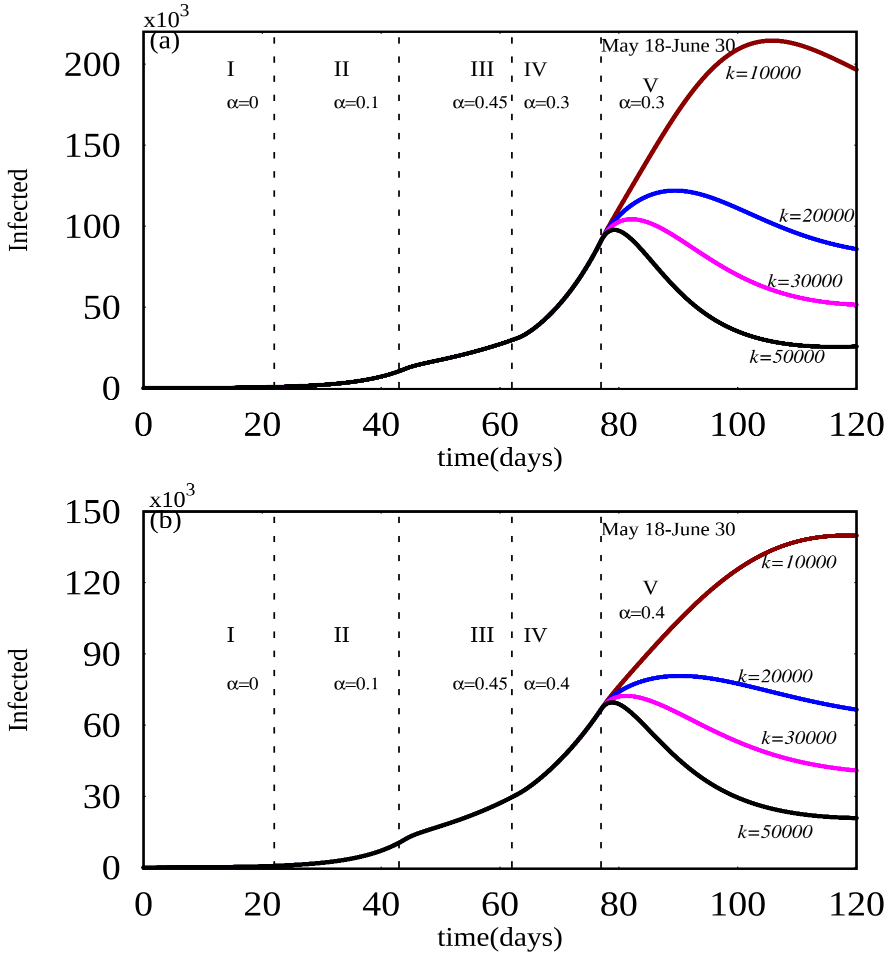

FIG. 6. Numerical simulation of the evolution of the number of infected individuals for the value of initial transmission rate $\beta_0 = 0.56$ and $k = 1117.3$ (I - IV). Note that this simulation shows rate of infected people after May 17 (see column V) with respect to the different values individual reaction in Eq. (2) with (a) $\alpha = 0.3$ and (b) $\alpha = 0.4$. The remaining parameters are taken from Table I.

Analyzing the results of the above, the present dynamical model clearly shows that when the value of α is reduced, new infected cases continue to quickly increase, while a larger value of α decreases the infected cases to a considerable extent. However, the later will not also help in reducing the infection to approach zero unless perhaps α approaches a value close to unity. Moreover, it is not practical to keep increasing the governmental action to a higher and higher level even in the fourth lock-down period and further due to the necessity of opening up the economic front for survival of the nation. On the other hand, with relaxation of governmental action over time, particularly in the fourth lock-down period and subsequent period, the above type of prevention method alone will not end up in the eradication of the disease.

In the above, in Figs. 4, we have shown our simulations, based on the various governmental action strengths with fixed low value of intensity of individual reaction ($k = 1117.3$) in Eq.(2). Individual reactions or behavior can include social distancing, personal hygiene, health habits and avoiding crowded places and so on [2]. It will also include alerting fellow citizens to wear masks, to follow personal hygiene and social distancing, political and social organizations urging fellow citizens to follow social norms and so on. Now, we also address the importance of individual reactions with low value of governmental action strength $\alpha = 0.3$ and $\alpha = 0.4$ in the region V (which corresponds to the period beyond May 18) in Figs. 5. We observe from Figs. 5 that the number of infected people increases for no action of individual response (for $k = 0$), while the number of infected people decreases when the value of individual response increases to k=3000, 5000 and 10000. From these figures we also learn that individual behavioral responses are also very important along with governmental action.

In the above scenario, we further considered different low values of government action strength α in the region IV in Figs.4(a)-(d) (i-e May 3- June 10) and various values of intensity of individual reaction in the same region in Figs. 5(a)-(b). The corresponding figures show that the disease continues to infect more and more people due to low value of governmental action strength, but it can become controllable with respect to appropriate individual reactions. In order to break the chain of infection spread and to get more controllable handle on infected individuals one may choose immediate action of the individual reaction response after third lock-down period, that is range IV, and consider appropriate values of α and k in the time window V in Figs. 6. For $\alpha = 0.3$ or 0.4 and $k = 10000$ or $k = 20000$ the newly infected cases tend to decrease within a few weeks, after May 18 (see Figs. 6(a)-(b)). If the individual reaction is increased further ($k = 30000$ and $k = 50000$) we see that the disease can be effectively eradicated within 1 to 2 months from 18^{th} May, 2020 (see Figs. 6(a)-(b)).

Based on our analysis in Figs. 4-6, we find that if we introduce appropriate values of individual reaction strength k, then even for low values of governmental action, the reduction can become substantially impressive and one can approach a regime of complete controlling of the disease in a reasonably short period in the absence of appropriate vaccination and so on.

IV. CONCLUSION

On 3^{rd} May, 2020 the total number of active infected cases registered for COVID-19 and deceased cases in India were 29,339 and 1391, respectively and on May 15^{th} they stood at 52,773 and 2753, receptively [19]. This amount of rise in the infected cases has happened after some minimal relaxation in the government lock-down, and several hundred new cases are reported every day from different locations across India. This is a worrying situation, because India is going to encounter a larger number of infected people amidst a huge population within a few days or weeks due to various local situations and other factors. Therefore, predictive mathematical/dynamical models can also provide useful insights to strengthen our understanding of COVID-19 transmission and control.

In our study, we considered a dynamical model of Susceptible-Exposed-Infectious-Removed (SEIR) spreading epidemic, and estimated the initial rate of COVID-19 transmission by considering the initially infected people in India. In addition, a verification procedure is also performed with respect to different transmission rate values based on the data available from India. Our findings also indicate that the government and in particular individual efforts are important in reducing infected populations and also in reducing the overall epidemic period. In addition, we would like to note that these kind of epidemic mathematical models and their predictive simulations are also valuable resources which can be helpful for public health planning and in governmental as well as individual acts.

Our model and current data seem to indicate that the confirmed infected individuals continue to grow in India every day, in spite of rapid response by the government to the pandemic through various quarantine measures, nationwide lock-down and risk-based zoning and so on when the individual reaction rate (k) is taken as low. But we find that for appropriate increased values of individual reaction, even with low governmental action strength, there can be dramatic reduction in the total number of infected people. Depending upon the increased individual contribution the disease can be effectively controlled in a rather short period. It is then imperative that society as a whole contributes its might by simple social measures, besides appropriate governmental action. These combined efforts can contribute towards a total control of the disease in a short period in India and perhaps elsewhere as well.

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