Basics of Radio Astronomy

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1 Black body radiation

It is useful to discuss electromagnetic radiation considering black body radiation as a *template*, which is emitted by systems in thermal equilibrium, and characterized by a single parameter i.e., the temperature. If we let a surface perpendicular to black body radiation then the energy received by unit area in unit frequency range is called the brightness B_ν which is given by the Planck radiation formula

$$B_\nu = \left(\frac{2h\nu^3}{c^2}\right)\frac{1}{e^{h\nu}k_BT-1} = k_BT\left(\frac{2h\nu^2}{c^2}\right)\frac{\frac{h\nu}{k_BT}}{e^{h\nu}k_BT-1} \tag{1}$$

Note that here the factor of two is due to two polarizations. From the above equation we can get the following important results:

- The total power radiated by a black body radiator:-

$$P = \int_0^\infty B_\nu d\nu = \frac{2k_B^4T^4}{c^2h^3}\int_0^\infty \frac{x^3}{e^x-1}dx = \frac{2k_B^4T^4}{c^2h^3}\times\frac{\pi^4}{15} = \frac{2\pi^4k_B^4}{15c^2h^3}T^4 = \sigma T^4 \tag{2}$$

  where

$$\sigma = \frac{2\pi^4k_B^4}{15c^2h^3} = 1.8047\times10^{-5}\text{erg cm}^{-2}\text{ sec}^{-1}\text{ Kelvin}^{-4} \tag{3}$$

- Rayleigh-Jeans limit:- If we consider $h\nu < k_BT$ (which is generally true in radio astronomy) we get

$$B_\nu = \frac{2\nu^2}{c^2}k_BT \tag{4}$$

  from the above expression we can find a "temperature equivalent" of brightness, called the brightness temperature T_B as

$$T_B = \frac{B_\nu c^2}{2\nu^2k_B} \tag{5}$$

There is an equivalent of brightness called "specific intensity" I_ν, which is defined by the power P_ν received by a unit surface area perpendicular to the direction of the receiving, in unit frequency range, from a unit angle. If we have a surface of area $d\sigma$ and receive radiation at frequency ν in a window of $d\nu$ around that, from a solid angle $d\Omega$, from a direction which makes angle θ from the normal on the surface then

$$P_\nu = I_\nu d\nu d\Omega d\sigma \cos\theta \tag{6}$$

The total flux can be computed from I_ν by integrating over the solid angle

$$S_\nu = \int I_\nu(\theta,\phi)\cos\theta d\Omega \tag{7}$$

The quantity S_ν is measured in the unit of Watt mt^{-2} Hz^{-1} or in units of Jensky (J) where

$$1J = 10^{-26} \text{ Watt mt}^{-2} \text{ Hz}^{-1} \tag{8}$$

One interesting property of brightness or specific intensity I_ν is that it does not depend on the distance of the source. If we consider a bundle of rays crossing two surfaces S_A and S_B, separated by distance R, of area $d\sigma_A$ and $d\sigma_B$ respectively, then the power received by S_A and S_B are given by

$$dP_A = I_\nu^A \Delta\nu d\sigma_A \frac{d\sigma_B}{R^2} \tag{9}$$

and

$$dP_B = I_\nu^B \Delta\nu d\sigma_B \frac{d\sigma_A}{R^2} \tag{10}$$

if $dP_A = dP_B$ we get $I_\nu^A = I_\nu^B$.

1.1 Antenna temperature

Antennas receive power and we can find a temperature corresponding to the power received in unit frequency range, by using the fact that in an electric conductor temperature increment set ups random motions in that. Due to random motions, the total current in the conductor is zero, however, the power is non-zero, since it depends on the square of the current. If an antenna receives power P_ν then its temperature, called the antenna temperature T_A is defined as

$$T_A = \frac{P_\nu}{k_B} \tag{11}$$

and the power

$$dP = k_B \Delta\nu T_A \tag{12}$$

1.2 System temperature

The output power of a radio receiver depends on the signal sent by antenna and the noise added by the electronics. Corresponding to receiver output we can defined a temperature called the system temperature T_{sys}

$$T_{sys} = \frac{P_\nu}{k_B} \tag{13}$$

In general, T_{sys} has the following compoeenents

$$T_{sys} = T_{sky} + T_{spill} + T_{loss} + T_{rec} \tag{14}$$

1.3 Sensitivity and SNR

The RMS noise of a receiver output taken over bandwidth $\Delta\nu$ and averaged over time τ is given by

$$\mathrm{RMS} = \frac{T_{sys}}{\sqrt{\tau\Delta\nu}} \tag{15}$$

if the gain of the receiver if G and the amplitude of the signal is S then the signal to noise ration (SNR) is defined as

$$\mathrm{SNR} = \frac{GS}{T_{sys}/\sqrt{\Delta\nu\tau}} = G\sqrt{\Delta\nu\tau}\frac{S}{T_{sys}} \tag{16}$$

It turns out that for a system of two antennas A and B with identical gains G the above equation becomes

$$\mathrm{SNR} = G\sqrt{2\Delta\nu\tau}\frac{S}{\sqrt{T^A_{sys}T^B_{sys}}} \tag{17}$$

This SNR is $\sqrt{2}$ times leass than what one would get from an antenna of the area of the sum of the areas of two anteanns and equivalent system temperature $\sqrt{T^A_{sys}T^B_{sys}}$. In that case

$$\mathrm{SNR} = 2G\sqrt{\Delta\nu\tau}\frac{S}{\sqrt{T^A_{sys}T^B_{sys}}} \tag{18}$$

From the above equations we can find the limiting sensitivity of an anteann

$$S = \mathrm{SNR}\frac{T_{sys}}{G\sqrt{\Delta\nu\tau}} \tag{19}$$

since the gain G is proportional to the collecting area A of the antenna therefore

$$S = \mathrm{SNR}\frac{T_{sys}}{A\sqrt{\Delta\nu\tau}} \tag{20}$$

In proper physiacal units the sensity Φ of an antenn in Watt mt^{-2} is given by

$$\Phi = \mathrm{SNR}\frac{k_B T_{sys}}{A\sqrt{\Delta\nu\tau}} \tag{21}$$

or

$$\Phi \approx \mathrm{SNR}\left(\frac{T_{sys}}{100\ \ \mathrm{Kelvin}}\right)\left(\frac{A}{\pi 45^2 \mathrm{mt}^2}\right)^{-1}\left(\frac{\Delta\nu}{16\ \mathrm{Mhz}}\times\frac{\tau}{1\ \mathrm{sec}}\right)^{-1/2} 5.425\times10^{-29}\ \ \mathrm{Watt}\ \ \mathrm{mt}^{-2} \tag{22}$$

2 Imaging

The electric field intensity E_ν at a position $\mathbf{r}$ due a distribution of sources, specified by their position vectors $\mathbf{R}$, over a surface S on sky is given by

$$E_\nu(\mathbf{r}) = \int \mathcal{E}_\nu \frac{e^{2\pi i\nu(|\mathbf{R}-\mathbf{r}|)/c}}{|\mathbf{R}-\mathbf{r}|} dS \tag{23}$$

In the above expression $(|\mathbf{R}-\mathbf{r}|)/c$ is the time taken by the signal to reach at the position $\mathbf{r}$ from the position $\mathbf{R}$ which causes a change in the phase of the signal by amount $2\pi\nu(|\mathbf{R}-\mathbf{r}|)/c$. The correlation function between the electric field intensities at two positions $\mathbf{r_1}$ and $\mathbf{r_2}$ is defined as follows:

$$V(\mathbf{r_1}, \mathbf{r_2}) = V_{12} = \langle E_\nu(\mathbf{r}_1) E^*_\nu(\mathbf{r_2})\rangle \tag{24}$$

from the above two equations:

$$V(\mathbf{r_1}, \mathbf{r_2}) = \int \langle \mathcal{E}_\nu(\mathbf{r}_1)\mathcal{E}^*_\nu(\mathbf{r_2})\rangle |\mathbf{R}|^2 \frac{e^{2\pi i\nu(|\mathbf{R}-\mathbf{r}_1|)/c}}{|\mathbf{R}-\mathbf{r}_1|}\frac{e^{-2\pi i\nu(|\mathbf{R}-\mathbf{r}_2|)/c}}{|\mathbf{R}-\mathbf{r}_2|} dS \tag{25}$$

defining the following variables:

$$\mathbf{s} = \frac{\mathbf{R}}{|\mathbf{R}|} \quad \text{and} \quad I_\nu(s) = \langle \mathcal{E}_\nu(\mathbf{r}_1)\mathcal{E}^*_\nu(\mathbf{r_2})\rangle |\mathbf{R}|^2 \tag{26}$$

the correlation function can be written as:

$$V_\nu(\mathbf{r_1}, \mathbf{r_2}) = \int I_\nu(\mathbf{s}) e^{-2\pi i\, \mathbf{s}.(\mathbf{r_1}-\mathbf{r_2})/c} d\Omega \tag{27}$$

Let is define a coordinate system (u,v,w) such that the baseline $\mathbf{r_1}-\mathbf{r_2}$ is in the plane $u-v$. Now if the components of the vector $\mathbf{s}$ along which the source is observed, along the direction u,v and w are l,m and $\sqrt{1-l^2-m^2}$ respectively then the cross product

$$\mathbf{s}.(\mathbf{r_1}-\mathbf{r_2}) = ul + vm + w\sqrt{1-l^2-m^2} = ul + vm \tag{28}$$

since the baseline does not have any components along w direction, by definition. From this we can write an expression for the visibility $V_\nu(u,v,w=0)$ as

$$V(u,v,w=0)_\nu = \int I_\nu(l,m) e^{-2\pi i(ul+vm)} \frac{dl dm}{\sqrt{1-l^2-m^2}} \tag{29}$$

Where we have substituted

$$d\Omega = \frac{dl dm}{dS_\perp} = \frac{dl dm}{dS\sqrt{1-l^2-m^2}} \tag{30}$$

note that dS is the unit area perpendicular to the direction along which the source is observed i.e., $\mathbf{s}$, and $dS_\perp$ is the unit area perpendicular to the direction w. It has been found useful to split the unit vector $\mathbf{s}$, one component along the direction of the beam (called phase center) $\mathbf{s_0}$, and another one perpendicular to that in the sky plane σ

$$\mathbf{s} = \mathbf{s_0} + \sigma \tag{31}$$

Now if we define a coordinate system in which $\mathbf{s_0} = (0,0,1)$ then in that coordinate system baseline will be given by $\mathbf{r_1}-\mathbf{r_2} = (u,v,w)$ and

$$\mathbf{s} = \mathbf{s_0} + \sigma = (0,0,1) + (l,m,0) \tag{32}$$

therefore

$$\mathbf{s}.(\mathbf{r_1}-\mathbf{r_2}) = ul + vm + w \tag{33}$$

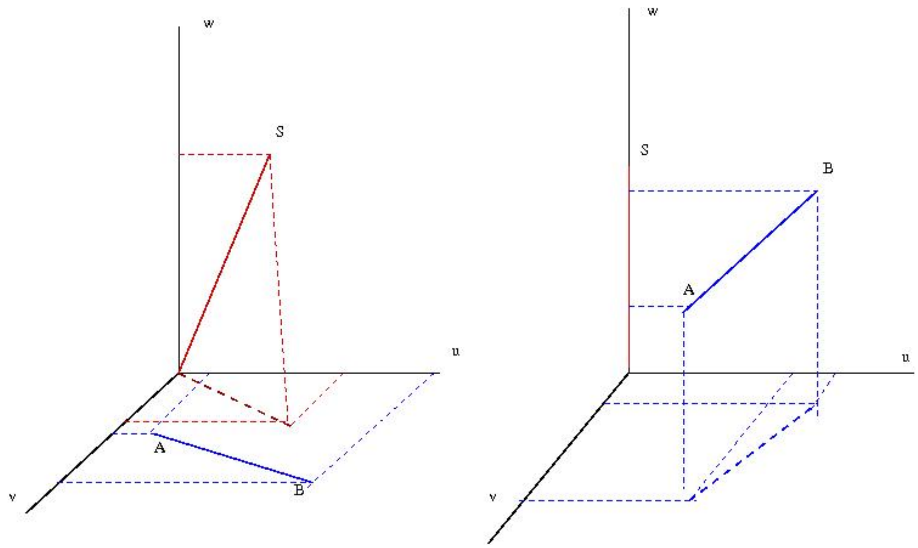

Figure 1: The left figure shows a coordinate system in which the baseline (shown by blue line) is a two dimensional vector (is in u-v plane) and the direction (shown by red line) along which the source is located is a three dimensional vector. The right figure shows a coordinate system in which the source direction is along the third axis and the baseline is a three dimensional vector.

and

$$V'_\nu(u,v,w) = e^{-2\pi i w} \int I_\nu(l,m) e^{-2\pi i (ul+vm)} dl dm \tag{34}$$

or

$$V'_\nu(u,v,w) e^{2\pi i w} = \int I_\nu(l,m) e^{-2\pi i (ul+vm)} dl dm = V_\nu(u,v) \tag{35}$$

here $V_\nu(u,v)$ is the coherence function with respect to the phase tracking center $\mathbf{s_0}$. In summary

$$\boxed{\begin{aligned} V(u,v)_\nu &= \int\int I_\nu(l,m) e^{-2\pi i (ul+vm)} dl dm \\ I_\nu(l,m) &= \int\int V_\nu(u,v) e^{2\pi i (ul+vm)} du dv \end{aligned}}$$

This set of equation is identical to Fourier transforms and by measuring one quantity V_ν or I_ν another can be measured. So far I have considered ideal situation, in what follows I will discuss how various practical considerations the above relationship gets modified.

2.1 Discrete sampling of uv-plane

We do not measure visibility at all the points in uv-plane. In fact it is not just the discrete sampling, generally we have visibilities at arbitrary locations in the uv-plane. If we define a sampling function $S(u,v)$ which is non-zero only at the points at which we have visibilities then the expression for image can be written as

$$I^D_\nu(l,m) = \int\int S(u,v) V_\nu(u,v) e^{2\pi i (ul+vm)} du dv \tag{36}$$

In the above expression I_D is called the *dirty image* which is different from the actual image (when we have visibilities at all points in the uv-plane). In fact the dirty image is the convolution of the actual image and the *synthesized beam* or *dirt beam*.

$$I_\nu(l,m) = I_\nu^D(l,m) * B(l,m) \tag{37}$$

where

$$B(l,m) = \int\int S(u,v) e^{2\pi i(ul+vm)} dudv \tag{38}$$

and

$$I_\nu(l,m) = \int\int V_\nu(u,v) e^{2\pi i(ul+vm)} dudv \tag{39}$$

2.2 Directional sensitivity

In general the beam is not equally sensitive for all direction and there is an extra term $A_\nu(l,m)$ which characterize this property

$$V_\nu(u,v) = \int\int I_\nu(l,m) A_\nu(lm) e^{-2\pi i(ul+vm)} dldm \tag{40}$$

3 Two element interferometer

Let us consider the voltage received by two antennas

$$V_1(t) = v_1 \cos 2\pi\nu(t-\tau_g) \tag{41}$$

and

$$V_2(t) = v_2 \cos 2\pi\nu t \tag{42}$$

where $\tau_g = \mathbf{b}.\,\mathbf{s}/c$ is the geometric time delay. The output of a correlator for the above voltages is given by

$$r(\tau_g) = \langle V_1(t)V_2(t)\rangle = v_1 v_2 \cos 2\pi\tau_g \tag{43}$$

Note that when the Earth rotates the angle between the baseline $\mathbf{b}$ and the direction of the synthesized beam $\mathbf{s}$ changes, therefore the output of the correlator change with time in a periodic manner (fringes). Now if we carryout observations with a finite bandwidth $d\nu$ then the correlator output

$$r = d\nu \int A(s)I(s) \cos \frac{2\pi\nu(\,\mathbf{b}.\,\mathbf{s})}{c} d\Omega \tag{44}$$

where $A(s)I(s)\Delta\nu d\Omega$ is the power received in bandwidth $\Delta\nu$ from direction $d\Omega$.

3.1 Effect of finite bandwidth

The complex visibility is defined as

$$V = |V|e^{i\phi} = \Delta\nu \int A(s)I(s) \cos \frac{2\pi\nu(\,\mathbf{b}.\,\mathbf{s})}{c} d\Omega \tag{45}$$

When we have a finite bandwidth we get the following expression for the interferometer response

$$r = A_0|V|\Delta\nu \left(\frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g}\right)\cos(2\pi\nu_0\tau_g - \phi_V) \tag{46}$$

From the above expression we can find that in order to keep the output 1 % of the maximum we should have

$$|\pi\Delta\nu\tau_g| << 1 \tag{47}$$

$$\boxed{\begin{gathered}\frac{\nu\ \mathbf{b}.\ \mathbf{s}}{c} = ul + vm + wn \\ \frac{\nu\ \mathbf{b}.\ \mathbf{s_0}}{c} = w \\ d\Omega = \frac{dldm}{n} = \frac{dldm}{\sqrt{1-l^2-m^2}} \\ V(u,v,w) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} A(l,m)I(l,m)\exp\left[2\pi i\left(ul+vm+w\sqrt{1-l^2-m^2}\right)\right]\frac{dldm}{\sqrt{1-l^2-m^2}}\end{gathered}}$$

where integrand in the last equation is taken zero for $l^2+m^2 \geq 1$.

For a east west baseline $w = 0$:

$$V(u,v,w) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} A(l,m)I(l,m)\exp\left[2\pi i\left(ul+vm\right)\right]\frac{dldm}{\sqrt{1-l^2-m^2}} \tag{48}$$

and

$$\frac{A(l,m)I(l,m)}{\sqrt{1-l^2-m^2}} = \int_{\infty}^{\infty}\int_{\infty}^{\infty} V(u,v,w)\exp\left[-2\pi i\left(ul+vm\right)\right] \tag{49}$$

3.2 Frequency conversion

Before processing the signal received by an antenna it is converted to a different frequency by mixing the radio frequency ν_{RF} signal with a signal from from a local oscillator with frequency:

$$\nu_{RF} = \nu_{LO} \pm \nu_{IF} \tag{50}$$

where ν_{IF} is the resulting intermediate frequency. Due to frequency conversion there are phase changes in the signal from antennas. For upper side band (USB) the phases for antenna 1 and 2 are

$$\phi_1 = 2\pi\nu_{RF}\tau_g = 2\pi(\nu_{IF}+\nu_{LO})\tau_g \tag{51}$$

and

$$\phi_2 = 2\pi\nu_{IF}\tau_i + \phi_{LO} \tag{52}$$

where τ_l is the instrument delay and ϕ_{LO} is the difference in phase of local oscillators at the two mixers. For the upper side band correlator output

$$r_u = A_0|V|\Delta\nu\left(\frac{\sin\pi\Delta\nu\Delta\tau}{\pi\Delta\nu\Delta\tau}\right)\cos\left[2\pi(\nu_{LO}\tau_g + \nu_{IF_0}\Delta\tau) - \phi_V - \phi_{LO}\right] \tag{53}$$

note that in the above expression the signal is from $[\nu_{IF_0} - \Delta\nu/2, \nu_{IF_0} + \Delta\nu/2]$ and $\Delta\tau = \tau_g - \tau_i$ is the tracking error of the tracking delay τ_i. In the above expression if we adjust ϕ_{LO} such that $2\pi\nu_{LO}\tau_g - \phi_{LO}$ remains constant, the correlator output will vary only as a result of change in V and slow drift in the instrumental parameters. This procedure in which ϕ_{LO} is usually controlled by the same computer that regulates, the delay tracking is called *fringe rotation* or *fringe stopping*.