Cosmological Parameter Estimation using Particle Swarm Optimization (PSO)

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Abstract

Finding a theoretical model which is "best fit" to an observational data set is an important exercise in cosmology, which involves finding the global maximum of the likelihood function. In general, the likelihood function is sampled at discrete points using some stochastic methods like Markov Chain Monte Carlo (MCMC) from which the best fit values of the parameters as well as error bars are computed, after obtaining the one and two dimensional probability distributions. In the present work we demonstrate that another stochastic method named Particle Swarm Optimization (PSO) can also be used to compute the best fit parameters. We apply PSO on the WMAP seven year data and get consistent results.

Parameter Estimation

The probability $P(\theta|d)$ (posterior) of a model, represented by a parameter θ, being true for a given data set d is related to the probability $P(d|\theta)$ (Likelihood) of obtaining the data d, given the model θ by Bayes' theorem:

$$P(\theta|d) = \frac{P(d|\theta)P(\theta)}{P(d)} \tag{1}$$

In the above equation $P(\theta)$ is called the prior. In the case of "flat" priors, the posterior and likelihood are proportional to each other.
The main task of cosmological parameter estimation is to find the parameter θ_0 for which the likelihood is maximum. Apart from that, one and two dimensional probability distributions for parameters are also obtained from which various statistics can be computed.
In general, in place of using cosmological parameters like Hubble parameter H, energy densities $\Omega_m, \Omega_b, \Omega_c$ etc., directly, we firstly compute the angular power spectrum C_l using some code (like `CAMB`) and from that and observational data, compute the likelihood function. We use the likelihood code provided by the WMAP team for computing the likelihood function from the angular power spectrum.

Particle Swarm Optimization

Birds in a bird flock can reach the place where the food is maximum and fish in a fish school can stay away from predators, by using their own learning experiences of the past and that of the other members of the group, in an effective way.

Particle Swarm Optimization (PSO) algorithm is motivated from the dynamics of the bird flock or fish school in which every member of the group remembers at which place in the past it has found the maximum of something (optimizing function) which is called its "personal best" or `Pbest` (given by the location $X^i_{\texttt{Pbest}}$) and every member also knows which of the members has the maximum `Pbest`, called the `Gbest` (and is given by $X_{\texttt{Gbest}}$) of the group.

In PSO the position $X^i(t)$ and velocity $V^i(t)$ of a the particle labelled by "i" are updated in the following way:

$$X^i(t+1) = X^i(t) + V^i(t) \tag{2}$$

and

$$V^i(t+1) = wV^i(t) + c_1\xi_1[X^i(t) - X^i_{\texttt{Pbest}}] + c_2\xi_2[X^i(t) - X_{\texttt{Gbest}}] \tag{3}$$

where ξ_1 and ξ_2 are two uniform random numbers in the range $[0,1]$ and w, c_1 and c_2 are PSO design parameters called the inertia weight, cognition and social learning coefficients respectively.

In any PSO implementation the following considerations have to be taken account:

1. Values of the design parameters.
2. Prescription for setting the initial positions and velocities of the particles.
3. Maximum velocity along every dimension.
4. Boundary conditions.
5. Stopping criteria.

In our PSO implementation we consider a group of 30 particles with $w = 0.72$, $c_1 = c_2 = 1.193$ and set the initial positions and velocities of the particles randomly. We use reflecting wall boundary conditions and set the maximum velocity such that the maximum distance travelled by a particle in a single jump is not more than half of the search range.

We consider a six dimensional cosmological model $(\Omega_b h^2, \Omega_c h^2, \Omega_\Lambda, n_s, A_s, \tau)$ and compute the best fit values of the parameters from the WMAP seven year data (see the table below).

Results

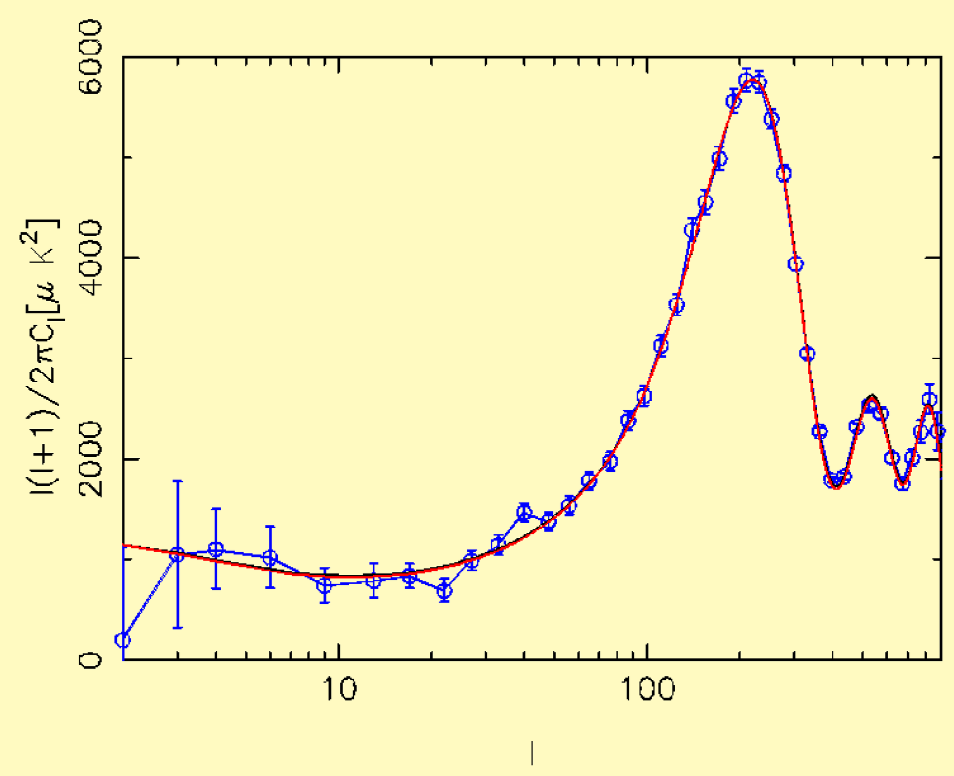
Cosmological parameters from PSO

Parameters		PSO best fit		WMAP 7 year best fit		Difference
	Range	`Gbest`	std dev.	ML	Mean	(`Gbest` -ML)
$\Omega_b h^2$	(0.01,0.04)	0.02219	0.00049	0.02227	$0.02249^{0.00056}_{-0.00057}$	-0.0001 (0.36 %)
$\Omega_c h^2$	(0.01,0.20)	0.1109	0.0058	0.1116	0.1120 ± 0.0056	- 0.0007(0.63%)
Ω_Λ	(0.50,0.75)	0.731	0.034	0.729	$0.727^{+0.030}_{-0.029}$	+0.0020 (0.27%)
n_s	(0.50,1.50)	0.962	0.012	0.966	0.967 ± 0.014	-0.0040(0.41%)
$A_s/10^{-9}$	(1.0,4.0)	2.43	0.13	2.42	2.43 ± 0.11	0.01(0.41%)
τ	(0.01,0.11)	0.084	0.012	0.0865	0.088 ± 0.015	-0.0021(2.43%)

The first and second columns in the above table give the cosmological parameters and the range used for searching the best fit values of the parameters. The third, fifth and sixth columns give the best fit values obtained from PSO, maximum likelihood value and the mean value of parameters computed from MCMC. In the fourth column we give 1σ errors in PSO, which are computed using a fitting procedure.

It has been claimed that PSO can be more useful for the problems which have very high dimensionality and/or the optimization function has a large number of local maxima.

Results



In this figure red and black lines represent the WMAP seven year best fit parameters angular power spectrum computed using MCMC and PSO respectively. The blue line with symbols and error bars shows the WMAP seven year binned power spectrum provided by the WMAP team.

References

[1] J. Prasad, T. Souradeep (2011) ArXiv:1108.5600 *Cosmological parameter estimation using Particle Swarm Optimization*