Cosmolgical Parameter Estimation

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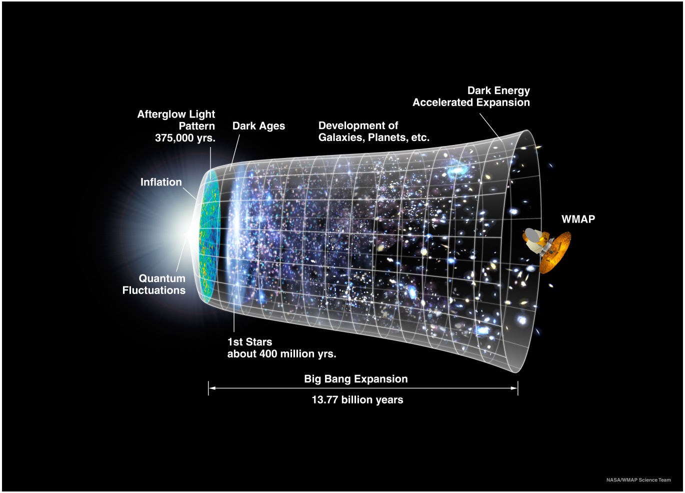
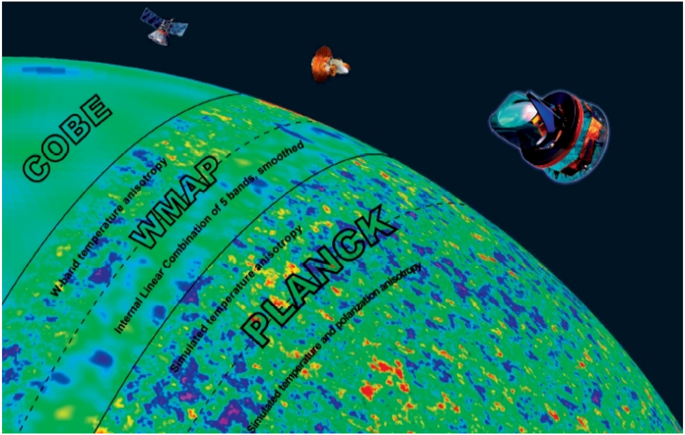
October 18, 2014

A representation of the evolution of the universe over 13.77 billion years



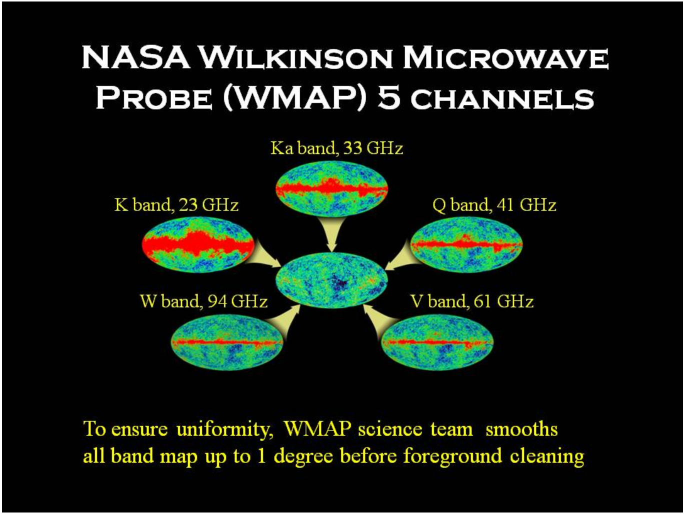
[Reference: http://map.gsfc.nasa.gov/]

Part of the CMB sky, as measured by the COBE, WMAP, and Planck satellite experiments.



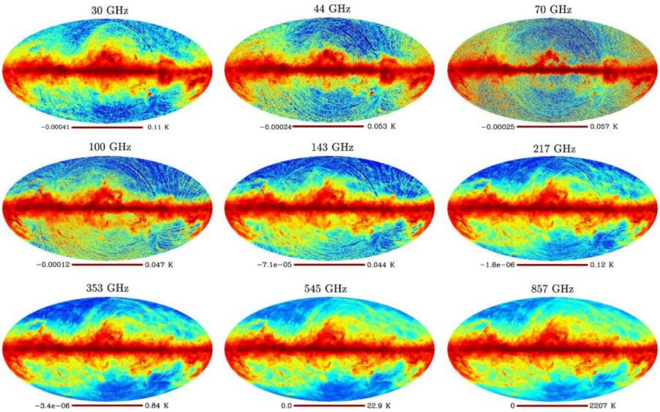
[Reference: http://scidacreview.org]

CMB and foreground Maps from WMAP



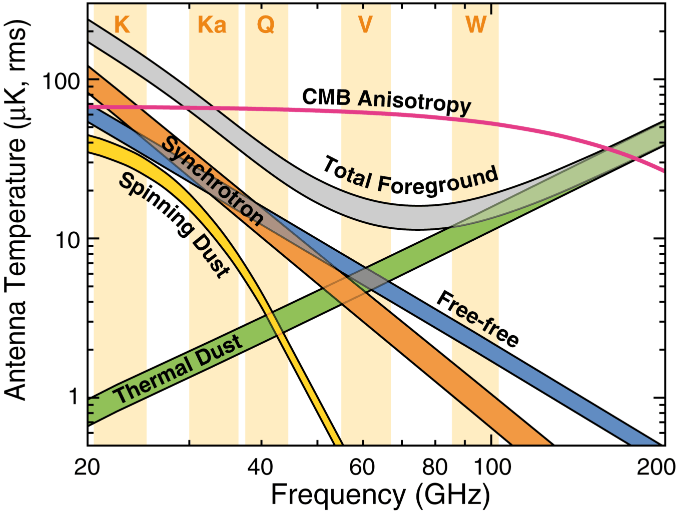
[Reference: http://map.gsfc.nasa.gov/]

Multi-frequnecy maps from Planck



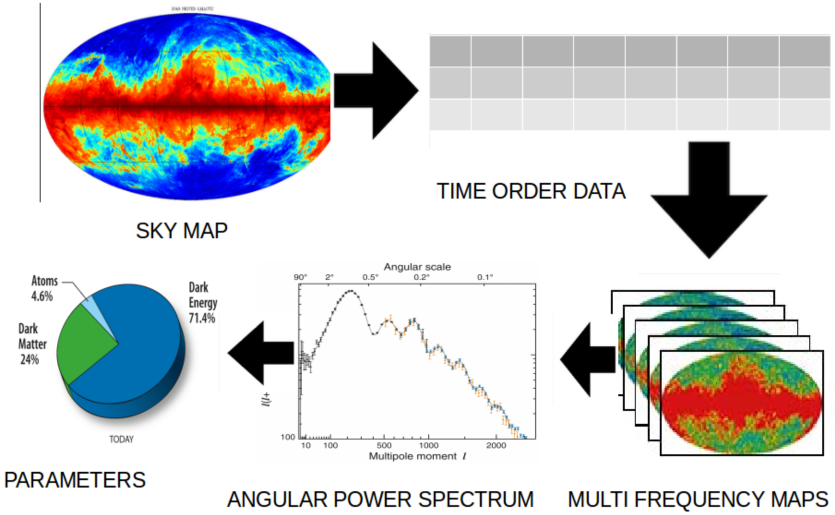
[Reference:
http://www.esa.int/Our_Activities/Space_Science/Planck]

CMB foregrounds



[Reference: http://lambda.gsfc.nasa.gov/]

CMB Data Analysis Pipeline



CMB anisotropies

- In general CMB anisotropies are represented in spherical harmonic basis:

$$\frac{\delta T(\theta,\phi)}{T_0} = \sum_{l=0}^{l=l_{\max}} \sum_{m=-l}^{m=l} a_{lm} Y_{lm}(\theta,\phi) \tag{1}$$

- CMB anisotropies are stochastic in nature and if they are Gaussian and isotropic (statistically) also (as seems to be the case) then they can be completely characterized by their two point correlation function.

$$\langle a_{lm} a^*_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l \tag{2}$$

- Since we only have one Universe, so we have limited m-modes for every l-mode i.e., (2l+1) so the power spectrum (angular) can be written as:

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{m=l} \langle |a_{lm}|^2 \rangle \tag{3}$$

CMB anisotropies

- CMB anisotropies which are represented by C_l depend on :
  - Primordial (curvature) fluctuations (scalars, tensors) generate during inflation, an epoch of accelerated expansion of the universe which preceded the usual big bang expansion.

$$P(k) = A_S \left(\frac{k}{k_0}\right)^{n_s - 1} \tag{4}$$

  - Background cosmological model represented by spatial curvature Ω_k, energy densities of different species $\Omega_b, \Omega_c, \Omega_\Lambda$ and the expansion rate of the universe H_0.
  - Events along the way which affect CMB photons i.e., reionization, interaction with hot electrons in clusters etc.,
- In order to get CMB anisotropies from the primordial fluctuations we must integrate CMB photons "along the line of sight" for which excellent codes (CMBFAST, CAMB) are available.

Cosmological Parameter Estimation

- Cosmological parameters from CMB are estimated in Bayesian framework in which the joint (posterior) distribution $P(\Theta|\mathbf{d})$ of a set of parameter Θ given data $\mathbf{d}$ is given by:

$$P(\Theta|\mathbf{d}) \propto P(\mathbf{d}|\Theta)P(\Theta), \tag{5}$$

  where $P(\mathbf{d}|\Theta)$ is called the "likelihood" and $P(\Theta)$ the priors.
- In most cases we take "flat" priors and in that situation posterior and likelihood are proportional.
- Primary aim of cosmological parameter estimation is to find the theoretical parameters $\Theta_{\rm max}$ which maximize the likelihood function which is an optimization problem.
- In general, we are not only interested in the best fit values $\Theta_{\rm max}$ we are interested in the error bars $\Delta\Theta$ also.

- At present methods based on Markov-Chain Monte (MCMC) Carlo methods are most common [Lewis & Bridle (2002)] and parameter estimation from the WMAP and Planck data have been done with MCMC methods.
- In a recent study [Prasad & Souradeep (2012)] we have shown that Particle Swarm Optimization which was given by Kennedy & and Eberhart [Kennedy & Eberhart (1995)] in 1995 can also be used for cosmological parameter estimation which has many advantages over usual MCMC based methods.
- PSO is more useful for problems with very large number of dimensions and/or multiple local maxima in the multi-dimensional parameter space.

Particle Swarm Optimization

- In a typical PSO algorithm [Prasad & Souradeep (2012)] a team of particles (computational agents) is launched in a multi-dimensional parameter space. Apart from moving along their original directions, particles are "accelerated" towards:
  - The location of the past point in the parameter space where it has found the maximum value of the cost function called it personal best or `Pbest`.
  - The location of the particle at present which have the highest value of `Pbest`, called the global best or `Gbest`.
- PSO is very effective in parameter estimation when parameters are large and/or the likelihood surface has multiple local peaks.

Particle Swarm Optimization

- According to the PSO algorithm if the the position and velocities of the i^{th} particle at step ("time") t are represented by $X^i(t)$ and $V^i(t)$ respectively the position at time $(t+1)$ is given by:
$$X^i(t+1) = X^i(t) + V^i(t+1), \tag{6}$$
- The velocity $V^i(t+1)$ is given by:

$$V^i(t+1) = wV^i(t) + c_1\xi_1[P^i(t) - X^i(t)] + c_2\xi_2[G - X^i(t)] \tag{7}$$

where P and G are locations of `Pbest` and `Gbest` respectively and ξ_1 and ξ^2 are two uniform random numbers in the range $[0, 1]$.

PHYSICAL REVIEW D **85,** 123008 (2012)

Cosmological parameter estimation using particle swarm optimization

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(Received 4 September 2011; revised manuscript received 16 May 2012; published 19 June 2012)

Constraining theoretical models, which are represented by a set of parameters, using observational data is an important exercise in cosmology. In Bayesian framework this is done by finding the probability distribution of parameters which best fits to the observational data using sampling based methods like Markov chain Monte Carlo (MCMC). It has been argued that MCMC may not be the best option in certain problems in which the target function (likelihood) poses local maxima or have very high dimensionality. Apart from this, there may be examples in which we are mainly interested to find the point in the parameter space at which the probability distribution has the largest value. In this situation the problem of parameter estimation becomes an optimization problem. In the present work we show that particle swarm optimization (PSO), which is an artificial intelligence inspired population based search procedure, can also be used for cosmological parameter estimation. Using PSO we were able to recover the best-fit Λ cold dark matter (LCDM) model parameters from the WMAP seven year data without using any prior guess value or any other property of the probability distribution of parameters like standard deviation, as is common in MCMC. We also report the results of an exercise in which we consider a binned primordial power spectrum (to increase the dimensionality of problem) and find that a power spectrum with features gives lower chi square than the standard power law. Since PSO does not sample the likelihood surface in a fair way, we follow a fitting procedure to find the spread of likelihood function around the best-fit point.

DOI: 10.1103/PhysRevD.85.123008 PACS numbers: 98.70.Vc

TABLE II. The first column in the above table shows the PSO fitting parameters and the second, third, fourth and fifth columns show the search range, the location of Gbest, the average position of PSO particles and the error or standard deviation (which is computed by fitting the sampled function) respectively. In the sixth and seventh columns we give the best fit (ML) and the average values of the cosmological parameters derived from WMAP seven years likelihood estimation respectively. In the last column we give the difference between our best-fit parameters (PSO parameters) and WMAP team's best-fit parameters (difference between ML and Gbest values). From this table it is clear that roughly there is good agreement between the PSO best-fit parameters and WMAP team's best-fit parameters from the seven year data.

		Cosmological parameters from PSO					
		PSO best fit			WMAP best fit [9]		
Variable	Range	Gbest ($\chi^2_{\text{eff}} = 7469.73$)	Mean	Standard Deviation	ML ($\chi^2_{\text{eff}} = 7486.57$)	Mean	Difference (Gbest-ML)
$\Omega_b h^2$	(0.01,0.04)	0.022036	0.022030	0.000456	0.02227	$0.02249^{0.00056}_{-0.00057}$	$-0.000234(-1.05\%)$
$\Omega_c h^2$	(0.01,0.20)	0.112313	0.112435	0.005276	0.1116	0.1120 ± 0.0056	0.000713 (0.63%)
Ω_Λ	(0.50,0.75)	0.721896	0.720353	0.029047	0.729	$0.727^{+0.030}_{-0.029}$	$-0.007104(-0.97\%)$
n_s	(0.50,1.50)	0.963512	0.963278	0.011730	0.966	0.967 ± 0.014	$-0.002488(-0.25\%)$
$A_s/10^{-9}$	(1.0,4.0)	2.448498	2.454202	0.106615	2.42	2.43 ± 0.11	0.028498(1.17%)
τ	(0.01,0.11)	0.08009	0.083930	0.012113	0.0865	0.088 ± 0.015	$-0.00641(-7.41\%)$

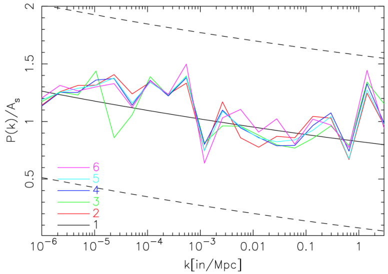


FIG. 8 (color online). This figure shows how the binned primordial power spectrum (20 logarithmic bins over the k range) changes as PSO progresses (line 1 is for the initial PPS and 6 is for the final PPS). The lower and upper values of the power in bins are represented by the dashed line. Starting with a power law PPS we found that a power spectrum which has low power in some bins and high in others fits better than a power law model.

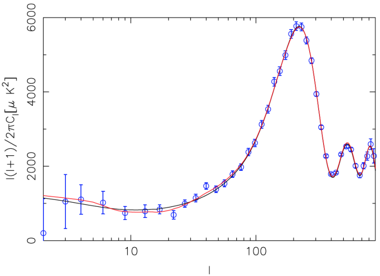
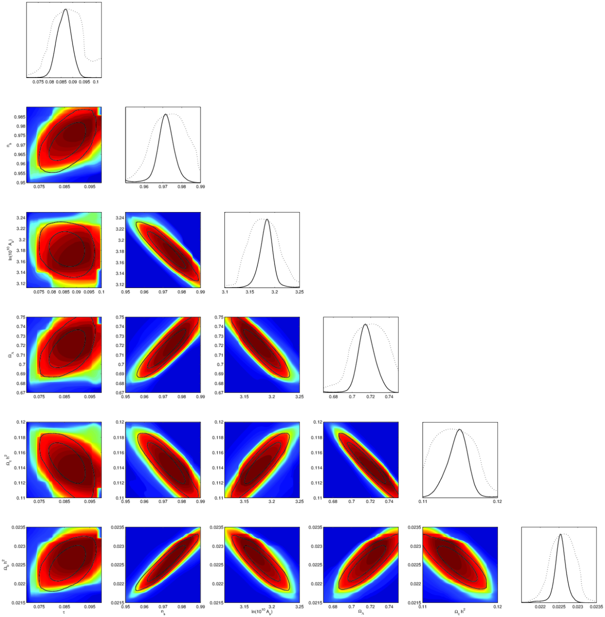


FIG. 9 (color online). The red, black and blue lines in the above figure represent the best-fit angular power spectrum recovered from PSO, standard LCDM power spectrum and the binned power spectrum of WMAP seven year data, respectively. Note that at low l the angular power spectrum with binned PPS fits better as compared to the standard power law PPS to the observed data (the improvement in $\Delta\chi^2_{\rm eff}$ is around 7).



τ
n_s
ln(10^10 A_s)
Ω_Λ
Ω_c h^2
Ω_b h^2

Discussion and Conclusions

- Methods based on MCMC (COSMOMC) need (1) covariance matrix (2) good starting point (3) rough estimate of the error estimates on the parameters which we want to estimate.
- We showed that PSO which is a stochastic optimization which needs only the search range can be used for cosmological parameter estimation.
- Recently we have found that PSO not only can find the best fit point but does sample the parameter space quite fairly. The result of our this finding will be reported very soon.

Thank You !

References

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