Cosmological parameter estimation using Particle Swarm Optimization

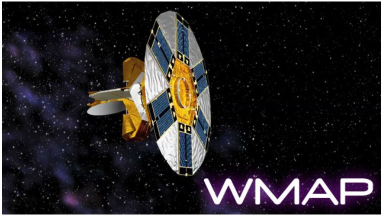
Jayanti Prasad
in collaboration with Tarun Souradeep

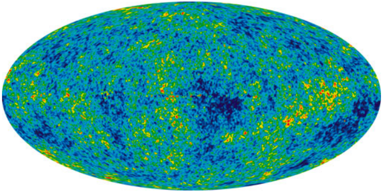
Phys. Rev. D 85, 123008 (2012)

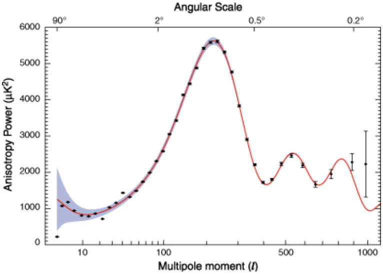
Inter-University Centre for Astronomy & Astrophysics (IUCAA)
Pune, India (411007)

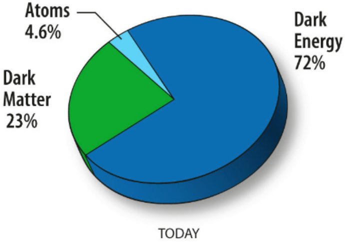
February 27, 2013

Cosmic Microwave Background Data Analysis









Cosmological Parameter Estimation

- Temperature Map

$$T(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{m=l} a_{lm} Y_{lm}(\hat{n}) \tag{1}$$

and

$$a_{lm} = \int d^2\hat{n} Y_{lm}(\hat{n}) T(\hat{n}) \tag{2}$$

- Angular Power Spectrum

$$\langle a_{lm} a^*_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l \tag{3}$$

- Cosmological Parameters

$$C_l = \int dlnk G_l(k) P(k) \tag{4}$$

Where $G_l(k)$ is the transfer function and $P(k)$ is Primordial power spectrum (PPS)

Bayesian Analysis

- Maximum Likelihood

$$P(\theta|d) = \frac{P(d|\theta)P(\theta)}{P(d)} \tag{5}$$

Where $P(\theta|d)$ is posterior i.e., probability distribution for the parameter θ given data d, $P(d|\theta)$ is Likelihood $\mathcal{L}$ and $P(\theta)$ is prior.

- In the case of Gaussian noise

$$\mathcal{L} \propto e^{-\chi^2/2} \quad \text{where} \quad \chi^2 = \sum_i \frac{(d_i - d(\theta_i))^2}{\sigma_i^2} \tag{6}$$

- Note that d and θ are n and m dimensional vectors respectively and our goal is to find a vector $\hat{\theta}$ which can maximize the likelihood $\mathcal{L}$ or minimize the χ^2.

Markov Chain Monte Carlo (MCMC)

- Markov Chain Monte Carlo (MCMC) is the most common method which is used for cosmological parameter estimation at present [Lewis & Bridle (2002)].
- In MCMC we discretely sample the Likelihood surface using some prescription (Metropolis-Hastings algorithm) and after marginalization find one and two dimensional probability distributions.
- The best fit values of cosmological parameters and error bars are computed from the marginalized probability distribution.
- In the present work we have developed a new method for cosmological parameter estimation from CMBR data which is based on Particle Swarm Optimization [Prasad & Souradeep (2012)].

Particle Swarm Optimization (PSO)

- PSO is a non-linear optimization method which is motivated by the movement of organisms in a bird flock or fish school [Kennedy & Eberhart (1995, 2001)].
- In PSO, a "team of particles" (computational agents) search for the global maximum/minimum of a non-linear function (fitness/coast function) exhibiting random motions.
- Random motion of particles is PSO are governed by their "personal experience" (`Pbest`) and "social learning" (`Gbest`).

Mathematical Framework

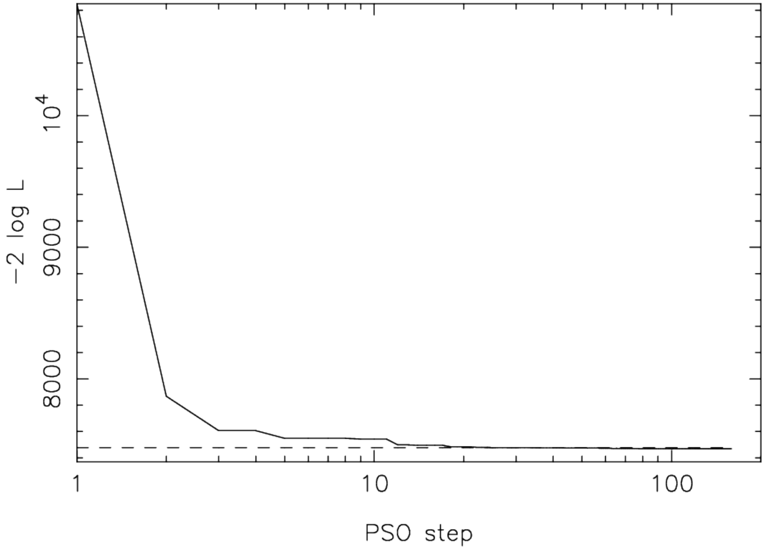
- Equation of Motion :

$$X_{t+1}^{i} = X_{t}^{i} + V_{t+1}^{i} \tag{7}$$

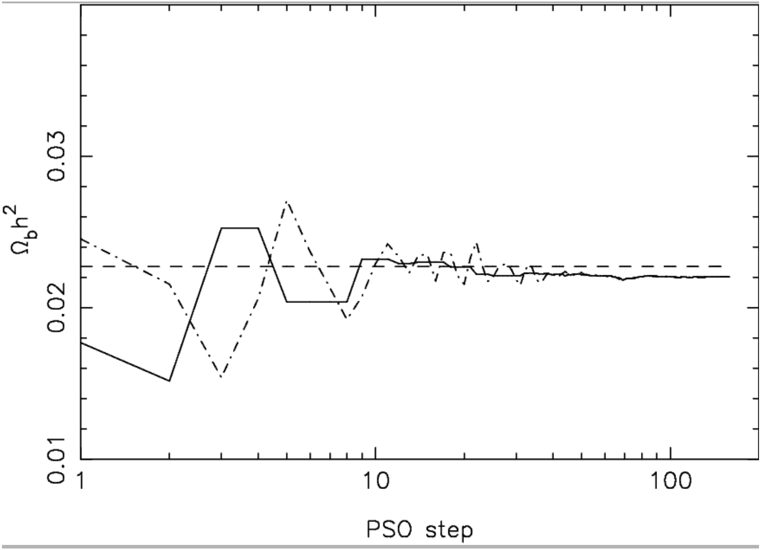
$$V_{t+1}^{i} = wV_{t}^{i} + c_1\xi_1(X_{t}^{i} - X_{\texttt{Pbest}}^{i}\ ) + c^2\xi_2(X_{t}^{i} - X_{\texttt{Gbest}}\ ) \tag{8}$$

-
  - Initial conditions - Random
  - Boundary conditions- Reflecting Boundary
  - Stopping criteria - Gelman -Rubin

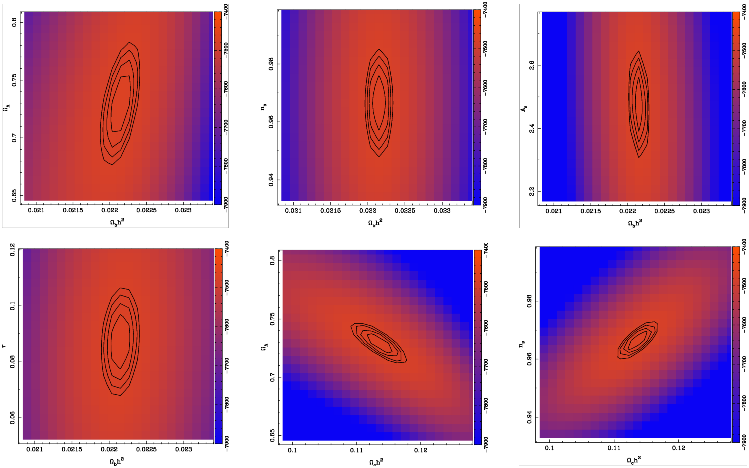
Gbest



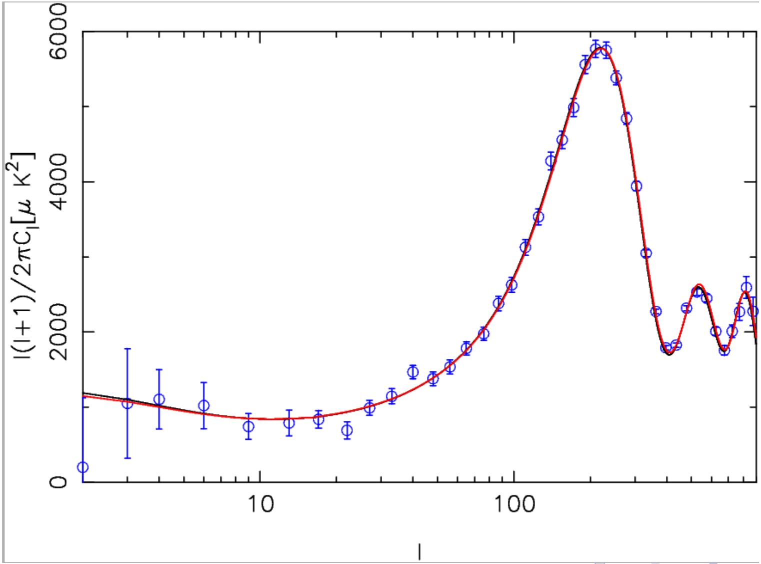
Best fit cosmological parameters



Contour Plots



Best fit Angular Power Spectrum



Best fit Cosmological Parameters

TABLE II. The first column in the above table shows the PSO fitting parameters and the second, third, fourth and fifth columns show the search range, the location of Gbest, the average position of PSO particles and the error or standard deviation (which is computed by fitting the sampled function) respectively. In the sixth and seventh columns we give the best fit (ML) and the average values of the cosmological parameters derived from WMAP seven years likelihood estimation respectively. In the last column we give the difference between our best-fit parameters (PSO parameters) and WMAP team's best-fit parameters (difference between ML and Gbest values). From this table it is clear that roughly there is good agreement between the PSO best-fit parameters and WMAP team's best-fit parameters from the seven year data.

Cosmological parameters from PSO							
		PSO best fit			WMAP best fit [9]		
Variable	Range	Gbest ($\chi^2_{\rm eff} = 7469.73$)	Mean	Standard Deviation	ML ($\chi^2_{\rm eff} = 7486.57$)	Mean	Difference (Gbest-ML)
$\Omega_b h^2$	(0.01,0.04)	0.022036	0.022030	0.000456	0.02227	$0.02249^{0.00056}_{-0.00057}$	$-0.000234(-1.05\%)$
$\Omega_c h^2$	(0.01,0.20)	0.112313	0.112435	0.005276	0.1116	0.1120 ± 0.0056	0.000713 (0.63%)
Ω_Λ	(0.50,0.75)	0.721896	0.720353	0.029047	0.729	$0.727^{+0.030}_{-0.029}$	$-0.007104(-0.97\%)$
n_s	(0.50,1.50)	0.963512	0.963278	0.011730	0.966	0.967 ± 0.014	$-0.002488(-0.25\%)$
$A_s/10^{-9}$	(1.0,4.0)	2.448498	2.454202	0.106615	2.42	2.43 ± 0.11	0.028498(1.17%)
τ	(0.01,0.11)	0.08009	0.083930	0.012113	0.0865	0.088 ± 0.015	$-0.00641(-7.41\%)$

Discussion and conclusions

- In the present work we have demonstrated the application of Particles Swarm Optimization or PSO for cosmological parameter estimation from CMB data.
- Based on a very simple algorithm, PSO has many interesting features some are as follows:
  1. PSO has very few design parameters , the values of which can be easily fixed. By tuning the values of the design parameters, PSO can be made more efficient for global or a local search although it is more useful for a global search.
  2. PSO is very efficient in searching for the global maximum when dimensionality of the search space is very high or there are a large number of local maxima present.
  3. In PSO we need to give only the search range as an input and no other information (as is needed in MCMC) about parameters, like covariance matrix, width of the final 1-d probability distribution or starting point is needed.

Thank You !

References

Kennedy, J., & Eberhart, R. 1995, IEEE, 1942

—. 2001, Swarm Intelligence (Morgan Kaufmann Publishers)

Lewis, A., & Bridle, S. 2002, Phys. Rev. D , 66, 103511

Prasad, J., & Souradeep, T. 2012, Phys. Rev. D , 85, 123008

References:

Jayanti Prasad, Tarun Souradeep, Phys. Rev. D 85, 123008 (2012) [arXiv:1108.5600v2]