Probing primordial power spectrum with CMB anisotropies

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Introduction

- Current observations indicate that CMB anisotropies :
  1. are isotropic (statistically)
  2. are Gaussian
  3. have no contribution from tensor perturbations
- In this case the scalar/curvature power spectrum $P_{\mathcal{R}}(k)$ contains all the information we have about the primordial fluctuations which seed galaxy formation in the Universe.

$$\langle \mathcal{R}_k \mathcal{R}_{k'} \rangle = \delta^3(k - k') \frac{2\pi^2}{k^3} P_{\mathcal{R}}(k), \tag{1}$$

- CMB anisotropies as we observe today are related to the primordial power spectrum in the following way:

$$C_l^{TT} \propto \int k^2 dk \Delta_{Tl}^2(k) P_{\mathcal{R}}(k) \quad \text{or} \quad C_l = \sum_{k=k_{\min}}^{k_{\max}} G_{lk} f_k \tag{2}$$

where $\Delta_{Tl}^2(k)$ is the transfer function and

$$\langle a_{lm}^* a_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l^{TT}, \tag{3}$$

with

$$a_{lm} = \int d\hat{n} Y_{lm}^*(\hat{n}) T(\hat{n}). \tag{4}$$

$Y_{lm}(\hat{n})$ are the standard spherical harmonics on a two-sphere and $T(\hat{n})$ are CMB anisotropies.

Probing primordial power spectrum

- Forward Problem:

$$P_{\mathcal{R}}(k) \xrightarrow{\Delta^2_{Tl}(k)} C_l^{TT} \tag{5}$$

This is easy and there are standard numerical codes available i.e., CMBFAST,CAMB,CMBANS...

- Inverse Problem:

$$P_{\mathcal{R}}(k) \xleftarrow{\Delta^2_{Tl}(k)} C_l^{TT} \tag{6}$$

This is hard and there is no unique solution as is common to inverse problems due to incomplete and noisy data.

Common approaches

1. Assume a model of PPS motivated from early universe (inflation) physics and try to constrain the parameters of that from the current cosmology (CMB) data.
2. Do not assume any model of $P_{\mathcal{R}}(k)$and try to deconvolve (reconstruct) it from C_l^{TT}:
   - Richardson-Lucy deconvolution [Shafieloo & Souradeep (2004)].
   - SVD deconvolution [Nicholson et al. (2010)].
   - Maximum Entropy deconvolution [Goswami & Prasad (2013)].
   - Tikhonov regularization [Hunt & Sarkar (2014)]
3. Consider different parameterized models of $P_{\mathcal{R}}(k)$and find out which one best-fits (?) the data.

My Work

1. **Prasad Jayanti** and Souradeep Tarun, Phys. Rev. D (2012) **85**, 123008, [astro-ph.CO/1108.5600], *Cosmological parameter estimation using Particle Swarm Optimization (PSO)*.
2. Gaurav Goswami and **Jayanti Prasad**, Phys. Rev. D (2013) **88**, 023522, [arXiv:1303.4747], *Maximum Entropy deconvolution of Primordial Power Spectrum*.
3. Suratna Das, Gaurav Goswami, **Jayanti Prasad**, Raghavan Rangarajan (2014) [minor comments from JCAP], [arXiv:1412.7093 [astro-ph.CO]], *Revisiting a pre-inflationary radiation era and its effect on the CMB power spectrum.*
4. Asif Iqbal, **Jayanti Prasad**, Tarun Souradeep, Manzoor A. Malik (2015),[minor comments from JCAP], [arXiv:1501.02647 [astro-ph.CO]], *Joint Planck and WMAP Assessment of Low CMB Multipoles.*

Regularization

- Reconstructing $P_{\mathcal{R}}(k)$from C_l^{TT} is an ill posed problem - there are many models of $P_{\mathcal{R}}(k)$which will give the same C_l^{TT}.
- One of the common ways to solve ill-posed problems is regularization : models of $P_{\mathcal{R}}(k)$which do not have required (constraints) features are penalized.
- In place of minimizing χ^2, which may over-fit the data, we minimize:

$$Q(f) = C(f) + \lambda S(f) \tag{7}$$

- In Maximum Entropy approach we identity $S(f)$ with some "entropy" and try to find a f which minimizes $C(f)$ and maximizes $S(f)$ simultaneously for a given λ.

Regularization

- In [Goswami & Prasad (2013)] we demonstrated that the MEM algorithm of Skilling & Bryan [Skilling & Bryan (1984)] can be used to reconstruct $P_{\mathcal{R}}(k)$from C_l^{TT} for WMAP seven year data.
- In Skilling & Bryan algorithm $Q(f)$ is minimized making quadratic approximations of $C(f)$ and $S(f)$ in a subspace constructed by ∇C and ∇S on which "metric" is defined by the Hessian $\nabla\nabla C$ and $\nabla\nabla S$ of chi-square and entropy respectively.

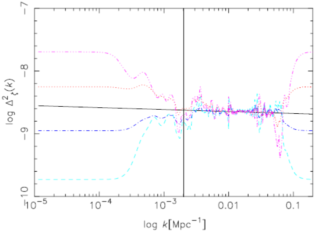


FIG. 8 (color online). The result of applying the algorithm to binned WMAP 7-year TT data. The solid verticle straight line corresponds to the pivot scale of $k_0 = 0.002$ (Mpc)$^{-1}$. The solid line corresponds to the maximum likelihood sPPS that one gets if one assumes the sPPS to be a power law. The curves correspond to the following values of priors on A_S (in units of 2×10^{-9}): dashed (0.09), dash-dot (0.56), dotted (2.8), and dash-dot-dot-dot (10).

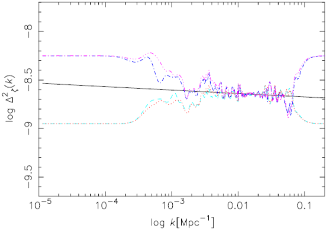


FIG. 10 (color online). The recovery of PPS using MEM with WMAP 9-year data (the difference between the two datasets is that the corresponding error bars are slightly smaller for WMAP 9-year data) for two values of A. The dotted curve with $A = 3000$ (corresponding prior on $A_S = 5.61$ in units of 2×10^{-9}) is obtained using WMAP 9-year data while the dashed curve is for WMAP 7. For the other value of A (corresponding prior on $A_S = 2.8$ in units of 2×10^{-9}), dash-dot curve is for WMAP 9-year data while dash-dot-dot-dot curve is for WMAP 7-year data. Notice that on very small and very large scales, the improved data does not change the recovery.

[Goswami & Prasad (2013)]

Large scales CMB anomalies

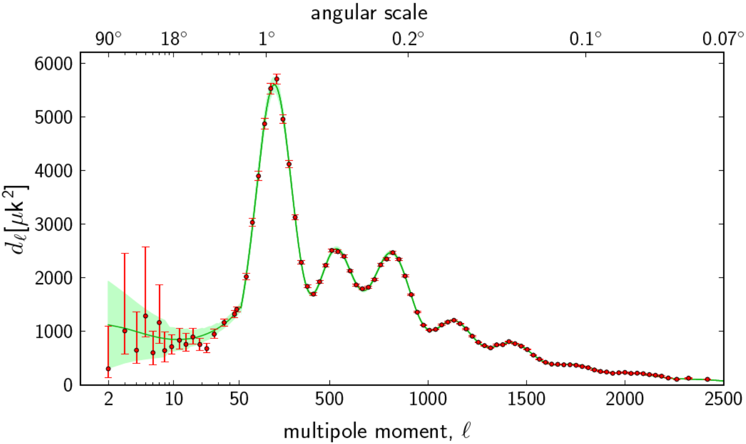


Fig. 20. Temperature angular power spectrum of the primary CMB from *Planck*, showing a precise measurement of seven acoustic peaks that are well-fitted by a six-parameter ΛCDM model (the model plotted is the one labelled [Planck+WP+highL] in Planck Collaboration XVI 2014). The shaded area around the best-fit curve represents cosmic/sample variance, including the sky cut used. The error bars on individual points also include cosmic variance. The horizontal axis is logarithmic up to $\ell = 50$, and linear beyond. The vertical scale is $\ell(\ell+1)C_l/2\pi$. The measured spectrum shown here is exactly the same as the one shown in Fig. 1 of Planck Collaboration XVI (2014), but it has been rebinned to show better the low-ℓ region.

Large scales CMB anomalies

- It should be noted that the best-fit cosmological model is determined almost exclusively by CMB anisotropies at small scales.
- WMAP & Planck both show that the quadruple amplitude is too low i.e., WMAP 9 quadruple still within cosmic variance $(< 2\sigma)$ [Bennett et al. (2013)].
- Using "Hausman test" Planck reported the statistical significance of low-l power deficiency $2 - 2.5\sigma$ [Planck Collaboration et al. (2014)].
- There have been proposed many theoretical models, represented by a form of $P_{\mathcal{R}}(k)$, to resolve the problem.
- One of the scenarios to can explain low-l power deficiency is pre-inflation.

Pre-Inflation

- If inflation lasts just long enough to solve the horizon and flatness problems, it is quite possible that pre-inflationary cosmology will leave imprints on the presentday Universe.
- In particular, preinflationary physics can lead to modulations in the primordial power spectrum, which may be observed in the largescale temperature anisotropies of the cosmic microwave background (CMB).

[Powell & Kinney (2007)]

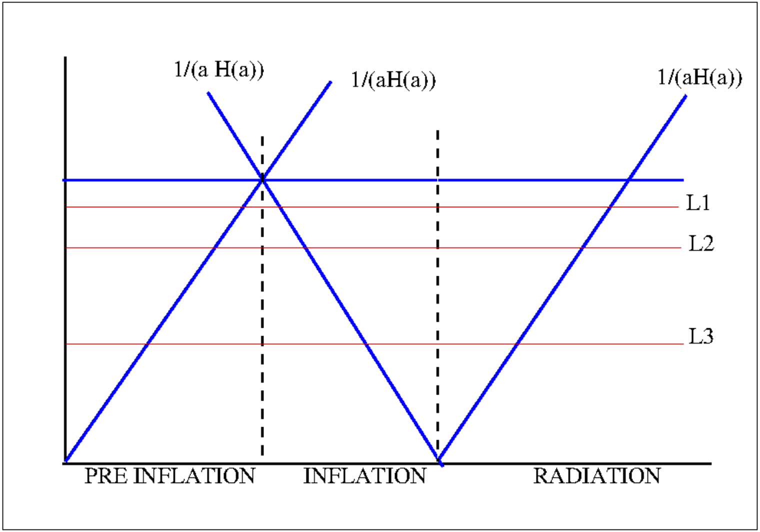
The usual quantization condition between the fields and their canonical momenta yields $[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = \delta^3(\mathbf{k} - \mathbf{k}')$ and the vacuum satisfies $a_{\mathbf{k}}|0\rangle = 0$. If the inflaton field had zero occupation prior to inflation then $\langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}}\rangle = 0$ and we would obtain a correlation function $\sim |f_k(\tau)|^2$. However, if the inflaton field was in thermal equilibrium at some earlier epoch [12] it will retain its thermal distribution even after decoupling from the other radiation fields and its occupation number will be given by:

$$\langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}'}\rangle = \frac{1}{e^{E_k/\mathcal{T}_f} - 1}\delta^3(\mathbf{k} - \mathbf{k}'), \qquad (5)$$

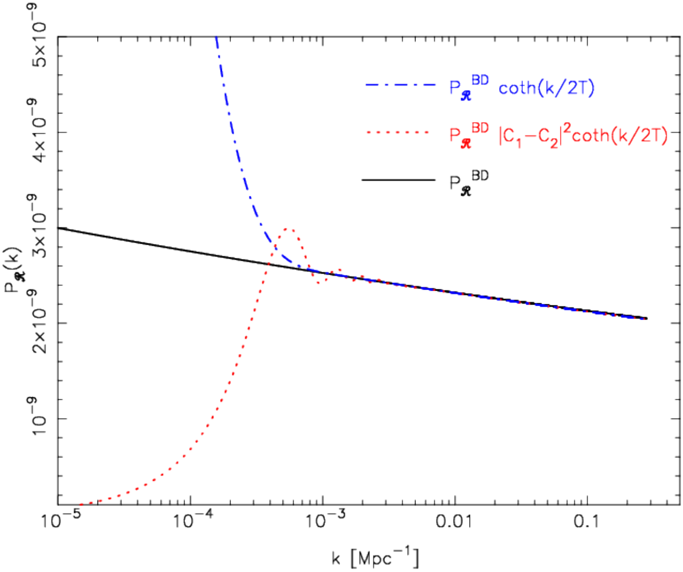
where E_k is the energy corresponding to the k mode at the inflaton decoupling temperature $\mathcal{T}_f$.

[Bhattacharya et al. (2006)]

Pre-Inflation

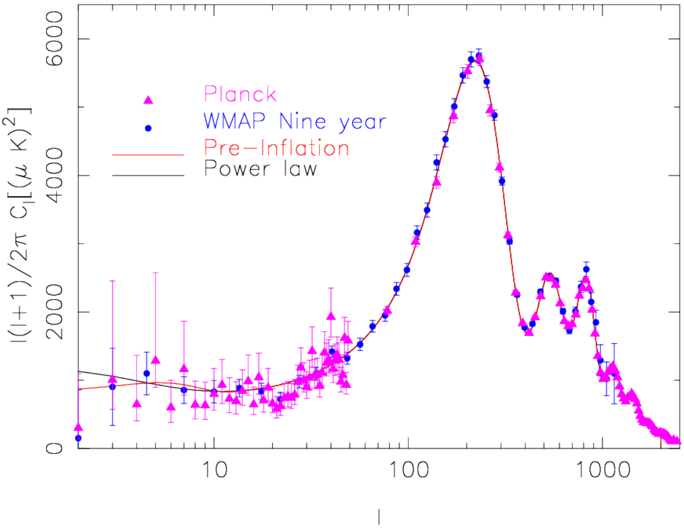


Pre-Inflation



[SD, GG, JP & RR (2014), arXiv:1412.7093]

Pre-Inflation



[SD, GG, JP & RR (2014), arXiv:1412.7093]

Cut-off models

- We have carried out a "mini-survey" of theoretical models (of PPS) which have power deficiency and large scales ($k < k_c$) and perfectly match the standard power law model at small scales.
- This survey can be considered an extension of [Sinha & Souradeep (2006)] with a couple of extra models.
- We use WMAP nine year and Planck (temperature only) data to constrain the parameters of the theoretical models using Markov-Chain Monte Carlo analysis.
- We use Akaike information criteria (AIC) and Bayesian Information criteria (BIC) to "rank" the models.
- We find that cut-off models always give better χ^2 but when we incorporate penalty for the extra parameters their significance (statically) drops.

Cut-off models

- Model 1 : Power law (PL)

$$\ln P_{\mathcal{R}}(k) = \ln A_s + (n_s - 1)\ln\left(\frac{k}{k_0}\right). \tag{8}$$

All the models we consider in the present work can be written as modulation over the power law model:

$$P_{\mathcal{R}}(k) = \mathcal{P}_0(k) \times \mathcal{F}(k, \boldsymbol{\Theta}), \tag{9}$$

where $\mathcal{F}(k, \boldsymbol{\Theta})$ is the "modulation" part and $\boldsymbol{\Theta}$ is a vector which characterizes the extra parameters.

- Model 2: Running Spectral Index Model (RN)

$$\ln P_{\mathcal{R}}(k) \;=\; \ln A_s + (n_s - 1)\ln\left(\frac{k}{k_0}\right) + \frac{\alpha_s}{2}\left[\ln\left(\frac{k}{k_0}\right)\right]^2. \tag{10}$$

Cut-off models

- Model 3 : Sharp cut off (SC)

$$P_{\mathcal{R}}(k) = \begin{cases} A_s \left(\frac{k}{k_c}\right)^{n_s - 1}, & \text{for} \quad k > k_c \\ 0, & \text{otherwise.} \end{cases}$$

- Model 4 : Pre-inflationary radiation domination (PIR)

$$P_{\mathcal{R}}(k) = A_s \, k^{1-n_s} \, \frac{1}{4y^4} \left| e^{-2iy}(1 + 2iy) - 1 - 2y^2 \right|^2, \quad (11)$$

where $y = k/k_c$. The cut off scale k_c is set by the Hubble parameter and is proportional to k^2.
[Vilenkin & Ford (1982); Powell & Kinney (2007); Wang & Ng (2008)]

Cut-off models

- Model 5 : Pre-inflationary kinetic domination (PI)
  - In this case it is assumed that the usual inflationary (de-Sitter) phase is preceded by a kinetic dominated phase ($\dot{\phi}$ not small).
  - Evolution of inflationary mode function is given by Mukhanov-Sasaki equation:

$$v_k'' + \left[k^2 - \frac{z''}{z}\right] v_k = 0, \tag{12}$$

with

$$v_k = a\left(\delta\phi_k + \frac{\phi'}{h}\Phi_k\right), \quad z = \frac{a\phi'}{h}, \quad h = \frac{a'}{a} \tag{13}$$

  - In Kinetic dominated regime we get :

$$a \approx \sqrt{1 + 2h\eta} \tag{14}$$

In de-Sitter we get :

$$a \approx \frac{1}{1 - h\eta} \tag{15}$$

Cut-off models

- After solving the Mukhanov-Sasaki equation in different regimes and matching boundary conditions we get:

$$P_{\mathcal{R}}(k) = A'_s \left(\frac{k}{k_0}\right)^{n_s-1} \frac{H_{inf}^2}{2\pi^2} k \mid A - B \mid^2, \qquad (16)$$

with

$$A_s = A'_s \frac{H_{inf}^2}{2\pi^2} k_0 \mid A(k_0) - B(k_0) \mid^2. \qquad (17)$$

H_{inf} denotes the Hubble parameter in physical units during inflation.
[Contaldi et al. (2003)]

Cut-off models

- Model 6 : Exponential cut off (EC)

$$P_{\mathcal{R}}(k) \;=\; \mathcal{P}_o(k)\left[1 - e^{-(k/k_c)^{\alpha}}\right], \tag{18}$$

where α is a measure of the steepness of the cut off.

- Model 7 : Starobinsky (SB)

$$P_{\mathcal{R}}(k) \;=\; \mathcal{P}_o(k)\,\mathcal{D}^2(y,\Delta), \tag{19}$$

where

$$\begin{aligned}\mathcal{D}^2(y,\Delta) \quad &= \quad \left[1 + \frac{9\Delta^2}{2}\left(\frac{1}{y}+\frac{1}{y^3}\right)^2 + \frac{3\Delta}{2}\left(4+3\Delta-\frac{3\Delta}{y^4}\right)\frac{1}{y^2}\cos(2y)\right. \\ &\left. +3\Delta\left(1-(1+3\Delta)\frac{1}{y^2}-\frac{3\Delta}{y^4}\right)\frac{1}{y}\sin(2y)\right] \end{aligned} \tag{20}$$

where $y = k/k_c$ and Δ are fitting parameters.
[Starobinski (1992)]

Cut-off models

- Model 8 : Starobinsky cut off (SBC)

$$P_{\mathcal{R}}(k) = \mathcal{P}_o(k) \left[1 - e^{-(\varepsilon\, k/k_c)^{\alpha}}\right] \mathcal{D}^2(y, \Delta), \qquad (21)$$

where $\mathcal{D}^2(y, \Delta)$ is the transfer function of the Starobinsky feature described in the previous section and ε sets the ratio of the two cutoff scales involved.

Parameter Estimation

Parameter Name	Symbol	Prior Ranges
Baryon Density	$\Omega_b h^2$	0.005-0.1
Cold Dark Matter Density	$\Omega_c h^2$	0.001-0.99
Angular size of Acoustic Horizon	θ	0.5-10.0
Optical Depth	τ	0.01-0.8
Scalar Spectral Index	n_s	0.5-1.5
Scalar Amplitude	$\log 10^{10} A_s$	2.7-4.0
Hubble Parameter at Inflation	H_{inf} (Mpc^{-1})	10^{-2}-10^{-7}
Running Index	α_s	-1-1
Cut off Parameter	k_c (Mpc^{-1})	0.0-0.01
Cut off Steepness Parameter	α	1.0-15.0
Starobinsky Parameter	Δ	0.0-1

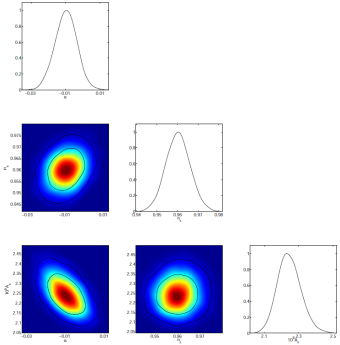
Table : Uniform prior used in parameter estimation.

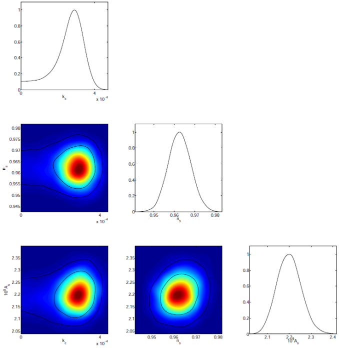
Parameter Estimation

		WMAP 9			WMAP 9+Planck		
Model	Parameter	Best Fit	68% Limit	$\chi^2 = -2\log\mathcal{L}$	Best Fit	68 % Limit	$\chi^2 = -2\log\mathcal{L}$
1 (PL)				7558.0160			15382.9400
2 (RN)	α	-0.012	-0.012± 0.022	7557.7340	-0.009	-0.009 ± 0.006	15380.6580
3 (SC)	$10^4 k_c$	2.9149	2.3597± 0.9809	7555.6080	3.0449	2.5653±0.8250	15378.2840
4 (PR)	$10^4 k_c$	0.3910	0.5909±0.5324	7557.9100	0.3941	<0.4296	15382.1560
5 (PI)	$10^4 H_{inf}$	2.0846	2.07836 ± 1.0052	7556.1900	2.1485	2.0934± 0.8973	15380.0950
6 (EC)	$10^4 k_c$	2.9244	2.4876±1.1406	7555.6700	2.9780	2.7752± 0.9237	15378.6420
	α	7.6167	8.1354[NL]		9.22328	8.2764[NL]	
7 (SB)	$10^4 k_c$	1.4724	< 12.83404	7556.1760	14.641	< 14.5739	15375.7760
	Δ	0.3893	< 0.2583		0.0558	0.0696±0.0667	
8 (SBC)	$10^4 k_c$	3.1313	< 0.1839	7555.7640	2.9149	< 0.23354	15378.3460
	α	12.502	8.022[NL]		12.6627	8.1238[NL]	
	Δ	0.0037	<0.4509		0.055492	<0.2896	

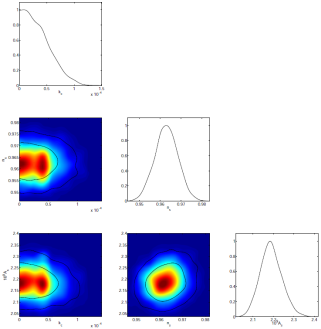
Table 2: The best fit and mean values of the extra parameters of PPS models we consider for the WMAP 9 year and join WMAP 9 year and Planck data. We find good constrains on the cut off scales k_c, however, our constrains on other parameters are poor. We were able to put upper limit on the Starobinsky parameter Δ but no limit (NL) was found for the exponential cut off parameter α.

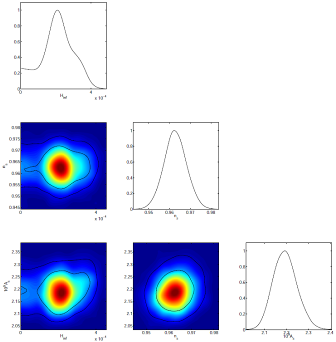
Model 2 (RN) & Model 3 (SC)



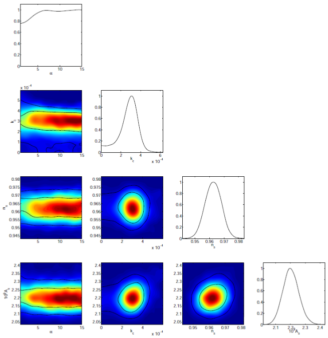


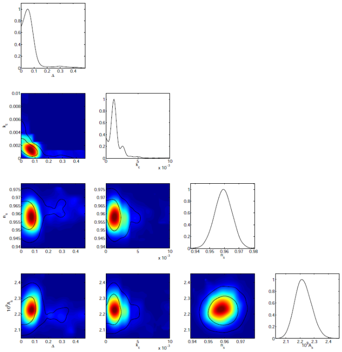
Model 4 (PIR) & Model 5 (PIK)



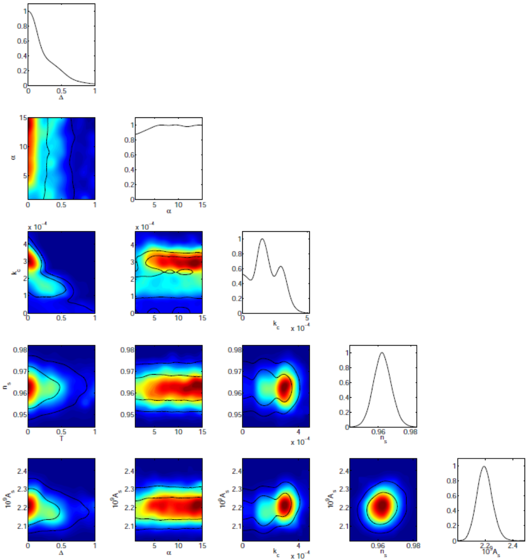


Model 6 (EC) & Model 7 (SB)





Model 8 (SBC)



Best fit PPS)

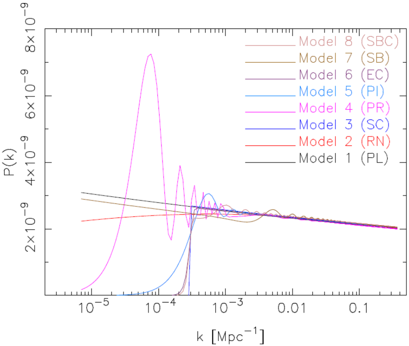


Figure 8: The Best-fit primordial spectra for the models consider for our analysis using WMAP 9 + Planck data. Note that all the models we considered in this work have cut off at large scale $k < k_c$ and matched with the standard power law model. All the models we consider give better likelihood than the pure power law model.

Best fit C_l)

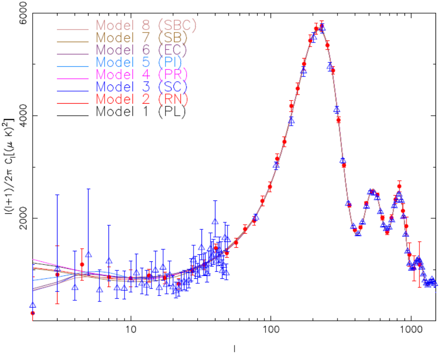


Figure 9: The best fit angular power spectra C_l^{TT} for the models of PPS we consider using WMAP 9 + Planck data. The observed data points for WMAP 9 + Planck data are also shown by red dots and blue triangles respectively with error bars.

Best fit C_l)

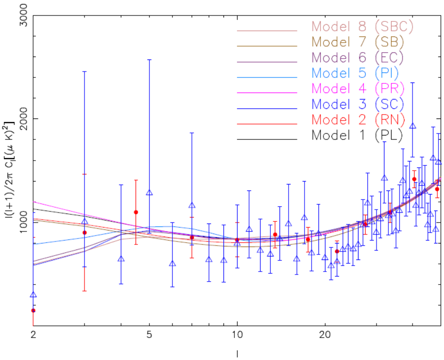


Figure 10: Same as in Fig. (9) at low-ℓ. This figure shows that the model 7 (SB) gives power suppression up to a large values of l so fit the Planck data better than other models.

With four parameters I can fit an elephant, and with five I can make him wiggle his trunk. - Von Neumann

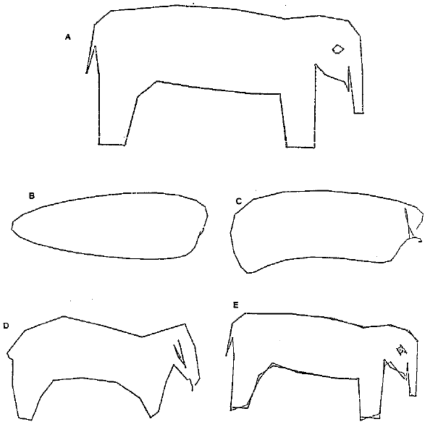


FIGURE 1.2. "How many parameters does does it take to fit an elephant?" was answered by Wel (1975). He started with an idealized drawing (A) defined by 36 points and used least squares Fourier sine series fits of the form $x(t) = \alpha_0 + \sum \alpha_i \sin(it\pi/36)$ and $y(t) = \beta_0 + \sum \beta_i \sin(it\pi/36)$ for $i = 1, \ldots, N$. He examined fits for $K = 5, 10, 20$, and 30 (shown in B–E) and stopped with the fit of a 30 term model. He concluded that the 30-term model "may not satisfy the third-grade art teacher, but would carry most chemical engineers into preliminary design."

Model comparison

- There are many ways to compare theoretical models which fit a given data set and it is not clear which one is the best.
- In the present work we use Akaike information criterion (AIC) and Bayesian information criteria (BIC) for model comparison.
- AIC is based on Kullback-Leibler divergence measure : how much information is lost when probability distribution $q(x)$ [true] is approximated by $p(x)$.

$$D_{\mathrm{KL}}(p||q) = \int p(x) \ln \left(\frac{p(x)}{q(x)} \right) dx \tag{22}$$

Model comparison

- One of easiest ways to compare the models with different number of fitting parameters is not to compare just $\chi^2(-2\ln L)$ but some other measure which have penalty for models with more number of parameters.
- AIC and BIC exactly do that:

$$AIC = -2\ln \mathcal{L}(\mathbf{d}|\theta) + 2k, \tag{23}$$

where $\mathcal{L}(\mathbf{d}|\theta)$ is likelihood, $\mathbf{d}$ data vector, $\theta \in \mathbb{R}^k$ is the parameter vector and k is the number of parameters. The best model is one which have minimum value of AIC. BIC is also defined in the similar way:

$$BIC = -2\ln \mathcal{L}(\mathbf{d}|\theta) + k\ln N, \tag{24}$$

where N is the number of data points.

Model comparison

	WMAP 9		WMAP 9+Planck	
Model	ΔAIC	ΔBIC	ΔAIC	ΔBIC
1 (PL)	0.408	-5.632	3.164	-10.036
2 (RN)	2.126	2.126	2.882	-3.718
3 (SC)	0.000	0.000	0.508	-6.092
4 (PIR)	2.302	2.302	4.380	-2.22
5 (PIK)	0.582	0.582	4.319	-4.281
6 (EC)	2.062	8.102	2.866	2.866
7 (SB)	2.56	8.608	0.000	0.000
8 (SBC)	4.156	16.236	4.570	11.17

Table : For WMAP 9 data we find that the sharp cut (SC) model (model 3) gives lowest AIC, however, for WMAP 9 year + Planck, Starobinsky model (SB) model (model 7) gives the lowest AIC. The reason behind model 7 being preferred by WMAP 9 year + Planck data is that it suppresses power at higher angular scales $\ell \leq 30$.

Discussion, conclusions & future direction

- We discussed different approaches to probe primordial power spectra from CMB data.
- It is discussed that maximum entropy regularization may be the best way to avoid models which fit features are not there in the data.
- A scenario of pre-inflationary model was also discussed.
- We presented a survey of a set of models of PPS having cut of at large scales for the WMAP 9 and Planck data.
- Using AIC and BIC we provided a "ranking" of models.
- Reconstruction of PPS from C_ls with regularization and constraining parameters of different models of PPS both are important exercises and we plan to keep working in this direction.

Work in Progress

- Jayanti Prasad (2014), [arXiv:1412.3298 [astro-ph.CO]], *Revisiting Cosmological parameter estimation*.
- Suratna Das, Gaurav Goswami, **Jayanti Prasad**, Raghavan Rangarajan (2015) [manuscript in preparation], *Constraints on just enough in inflation preceded by a thermal era.*

Thank You !

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