DSAT (August 2025)
Assignment 1

Instructions:

1. *In questions asking for algorithm you should write pseudocode or give detailed explanation in words. In case you are writing algorithms in words, make sure you are not missing necessary details.*
2. *You are allowed to take help but do not copy-paste the answers.*
3. *Doing the problems on your own will help you in exams as extensions of some of these problems may come in exams.*
4. *All questions are of* 5 *marks, but they are not of equal difficulty level.*

1. Let $A[1:n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an inversion of A. Give an $O(n \log n)$-time algorithm that determines the number of inversions in A.

2. Describe an $O(n \log n)$ time and $O(n)$ space algorithm that, given a set S of n integers and another integer x, determines whether S contains two elements that sum exactly x. (Hint: Read about binary search)

3. Let T be a BST. Describe an $O(n)$ time algorithm that on input $T.root$ can find the minimum absolute difference of any two keys of T. For instance, if keys of T are 3,8,1,12,7,15, then answer will be $8 - 7 = 1$.

4. An array A is called k-unique if it does not contain a pair of duplicate elements within k positions of each other, that is, there is no i and j such that $A[i] = A[j]$ and $|j - i| \leq k$. Design an $O(n \log k)$ time algorithm to test if A is k-unique.

5. A node x is inserted into a red-black tree and then is immediately deleted using the procedures discussed in the class. Is the resulting red-black tree always the same as the initial red-black tree? Justify your answer.

6. Given an element x in an n-node augmented tree (the one which can find the rank of an element or find the element of the given rank) and a natural number i, show how to determine the ith successor of x in $O(\log n)$ time.