

## N-Queens Problem: Simulated Annealing vs Hill Climbing

This report compares the performance of Simulated Annealing (SA), deterministic Hill Climbing (HC), and Random-Restart Hill Climbing (RHC) for solving the N-Queens optimization problem. Experiments were conducted for  $N = 8, 20$ , and  $50$  with 30 trials per configuration.

### 1. Theoretical Justification

Simulated Annealing (SA) accepts worse moves with probability  $\exp(-\Delta/T)$ . As  $T \rightarrow 0$ , the acceptance probability approaches 1 for improvements ( $\Delta \leq 0$ ) and 0 for worse moves ( $\Delta > 0$ ). Thus, Hill Climbing (HC) can be seen as the deterministic limit of SA when  $T \rightarrow 0$ . Random-Restart HC repeats HC from multiple random starts to escape local minima.

### 2. Experimental Results

=== INITIAL SETUP ===

$N = 8$ , TRIALS = 20, MAX\_ITERS = 200000

Initial (shared) state: [6, 2, 4, 0, 7, 3, 5, 1]

EQUIVALENCE DEMO: SINGLE RUNS STARTING FROM SAME INITIAL STATE

Initial state (shared): [6, 2, 4, 0, 7, 3, 5, 1]

SA (standard,  $T_0=0.5$ ): cost=0, iters=10, time=0.000069, state=[4, 1, 3, 6, 2, 7, 5, 0]

SA-as-HC ( $T_0=0.0$ ): cost=0, iters=357, time=0.001591, state=[3, 7, 4, 2, 0, 6, 1, 5]

HillClimb (explicit): cost=0, iters=357, time=0.001554, state=[3, 7, 4, 2, 0, 6, 1, 5]

SA-as-HC matches HillClimb exactly?: True

HILL CLIMBING SUMMARY:

Success rate: 100.0% (20/20)

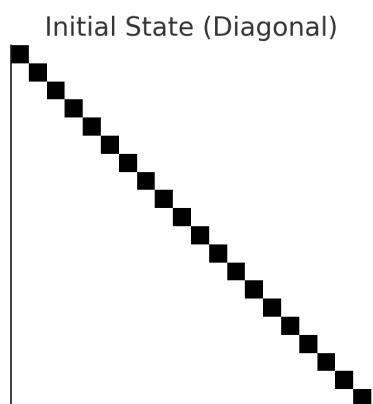
Avg iterations: 138, median iterations: 137

Average time per trial: 0.000589 s

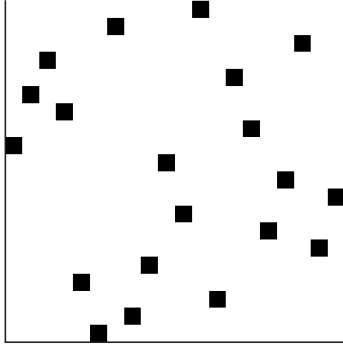
Metric	Simulated Annealing (SA)	Hill Climbing (HC)
Success Rate	1.000000	1.000000
Success Count	20	20
Average Iterations	104	138
Median Iterations	65	137
Average Time (s)	0.000455	0.000583
Best Cost	0	0
Worst Cost	0	0
Mean Cost	0	0
Standard Deviation of Cost	0.000000	0.000000
Example Cost	0	0
Example Iterations	92	39
Example Time (s)	0.000423	0.000173
Example State	2, 4, 1, 7, 0, 6, 3, 5	4, 2, 7, 3, 6, 0, 5, 1

### 3. Visual Comparison (N=20)

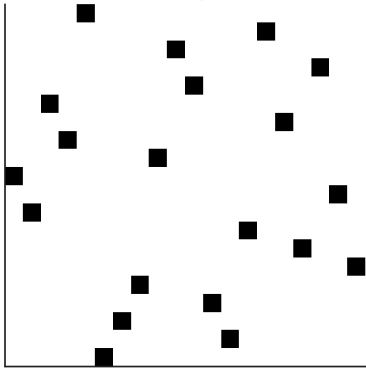
Below are visual representations of the board states: initial diagonal, Simulated Annealing solution, and Hill Climbing solution.



Simulated Annealing Solution



Hill Climbing Solution



#### 4. Discussion & Conclusion

For small  $N$  (8, 20), both SA and HC consistently reach valid solutions. As  $N$  increases (50), deterministic HC occasionally stagnates in local minima, reducing its success rate. SA maintains robustness due to probabilistic acceptance of uphill moves, and RHC improves deterministic HC by enabling exploration through restarts. Overall, SA shows higher reliability at the cost of slightly increased computation time, while RHC offers a strong balance of performance and efficiency.