Optimization for Data Science

By

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Introduction:

Optimization is an act, process, or methodology of obtaining the best result under given circumstances.

An Example

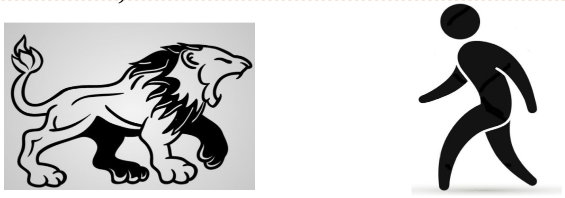
❖Task: Optimal Strategy for survival

Case-1, Situation

An Example

❖Task: Optimal Strategy for survival

Case-1, Situation



Optimal Strategy

An Example

❖ Task: Optimal Strategy for survival

Case-1, Situation

Optimal Strategy

Case-2, Situation

An Example

❖Task: Optimal Strategy for survival

Case-1, Situation

Optimal Strategy

Case-2, Situation

OPTIMISATION IN ENGINEERING

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Process: Water Sprinkling

Objective: Cover Maximum area from one location

Variables:
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Constraint

- Tensile strength of water pipe

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Optimization is a mathematical tool to find the optimal values of the variables, under given circumstances, for best performance.

Mathematical Modeling of Optimization

❖ Objective Function

$$\boldsymbol{f(x)}_{\mathrm{max/min}} : A \rightarrow \Re$$

❖ Variables

$$X \rightarrow \left[x_1, \ x_2, \ x_3, \ \dots \ x_n \right]$$

❖ Constraints

$$\boldsymbol{g(x_1, \ x_2, \dots x_n)} \ \mathrm{f} \ \boldsymbol{Condition}$$

❖ Solutions

- Feasible Solution : Satisfies all Constraints
- Optimal Solution : Feasible and Best Objective Function
- Near-Optimal Solution : Feasible but not necessarily the best

Classification of Optimisation Algorithm

Exact Algorithms :

- It is guaranteed that it will find the optimal solution in a finite amount of time.
- It is based on finding the global solutions of an optimization problem, using <u>classical mathematics</u>, <u>mathematical modeling</u> and <u>predefined logic</u>.

Heuristic Algorithms :

- The optimal solution in a finite amount of time is not guaranteed.
- It is based on finding the global solutions of an optimization problem, using <u>random variables</u> and <u>searching logics</u>.

Meta-Heuristics Optimisation Algorithm

(Greek (ενρισκειν) : Beyond to - find)

- Meta-heuristics are strategies that “guide” the search process.
- The goal is to explore the search space in order to find (near) optimal solutions.
- Meta-heuristics are not problem-specific.
- The basic concepts of meta-heuristics permit an abstract level description.
- It is based on random variables and searching logic.

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Is there any problem which is not possible to solve using “Classical Mathematics”.

Example - I

$$\boldsymbol{f} = \max(\mathrm{x}^2 + \mathrm{y}^2)$$

Where,

$$\begin{cases} 2x + 3y \le 3 \\ 6x + y \ge -1 \\ x \ge -2 \\ y \ge -1 \end{cases}$$

Solution,

Example - I

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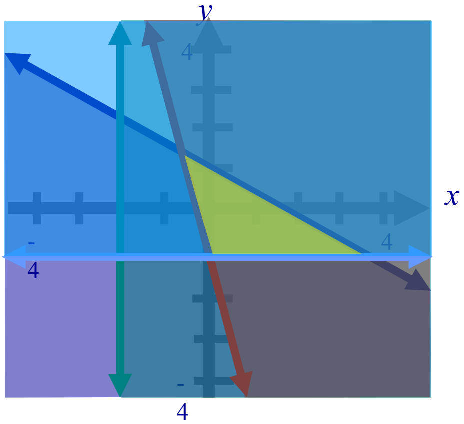


Solution,

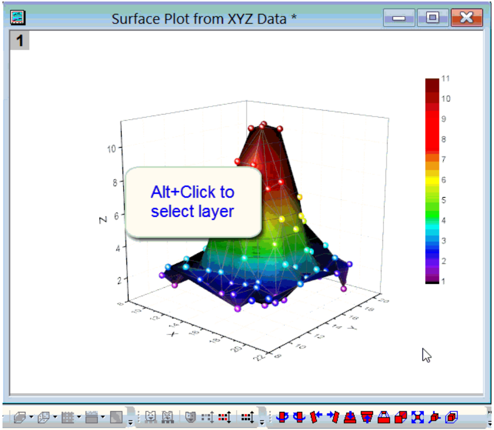
All corner co-ordinates
(0,-1) , (3, -1) & (-3/8, 10/8)

f = 10
For, x = 3 & y = -1

Example - II

$$f = \max(x^2 + y^2 + z^2)$$

Where,
$$\begin{cases} 3x + 5y - 9z < 5 \\ -2x + 10y + z \geq -7 \\ x \times \left(\sin^{-1} z\right) + y \leq 0.7 \\ x > -3 \\ y \leq 13 \\ z < 1 \end{cases}$$



Solution…

Example - III

$$f = \max(w^2 + x^2 + y^2 + z^2)$$

Where,

$$\begin{cases} 3x + 5y - 9z + w < 5 \\ e^x \times \left(\cos^{-1} z\right) + y^3 + \sqrt{w} \le 33 \\ -2x + 10y + z - w^2 \ge -7 \\ x \times \left(\sin^{-1} z\right) + y + \sqrt{2}w \le 0.7 \\ x + y > -3 \\ y + z^2 \le 13 \\ z < 1 \\ w^z \ge 17 \end{cases}$$

Solution…

Not possible using Classical Mathematics. So, Requires Heuristic and Meta-heuristic Algorithms.

Introduction

Optimization : The procedure or procedures used to make a system or design as effective or functional as possible.

Why Optimization?

- Helps improve the quality of decision-making
- Applications in Engineering, Business, Economics, Science, Military Planning etc.

Mathematical Program

Mathematical Program : A mathematical formulation of an optimization problem:

$$\text{Minimize } f(x) \text{ subject to } x \in S$$

Essential Components of a Mathematical program:

x: *variables or parameters*

f: *objective function*

S: *feasible region*

What is a solution of this Mathematical Program?

$$x^* \in S \text{ such that } f(x^*) \leq f(x) \ \forall x \in S$$

x^*: solution, $f(x^*)$: optimal objective function value

x^* may not be unique and may not even exist.

Maximize $f(x) \equiv$ Minimize $-f(x)$

Mathematical Optimization

The problem,

$$\text{Minimize } f(x) \text{ subject to } x \in S$$

can be written as

$$\begin{aligned} \min_{x} \quad & f(x) \\ \text{s.t.} \quad & x \in S \end{aligned} \qquad (1)$$

Some Optimization Problems

- Find the *shortest* path between the two points A and B in a horizontal plane