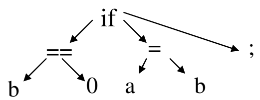
Syntax Analysis

- Check syntax and construct abstract syntax tree

if	(	b	==	0	)	a	=	b	;



- Error reporting and recovery
- Model using context free grammars
- Recognize using Push down automata/Table Driven Parsers

What syntax analysis cannot do!

- To check whether variables are of types on which operations are allowed
- To check whether a variable has been declared before use
- To check whether a variable has been initialized
- These issues will be handled in semantic analysis

Limitations of regular languages

- How to describe language syntax precisely and conveniently. Can regular expressions be used?
- Many languages are not regular, for example, string of balanced parentheses
  - ((((…))))
  - $\{ (^i)^i \mid i \geq 0 \}$
  - There is no regular expression for this language
- A finite automata may repeat states, however, it cannot remember the number of times it has been to a particular state
- A more powerful language is needed to describe a valid string of tokens

Syntax definition

- Context free grammars
  - a set of tokens (terminal symbols)
  - a set of non terminal symbols
  - a set of productions of the form nonterminal →String of terminals & non terminals
  - a start symbol

  <T, N, P, S>
- A grammar derives strings by beginning with a start symbol and repeatedly replacing a non terminal by the right hand side of a production for that non terminal.
- The strings that can be derived from the start symbol of a grammar G form the language L(G) defined by the grammar.

Examples

- String of balanced parentheses
  S → (S) S | ϵ

- Grammar
  list → list + digit
  | list – digit
  | digit
  digit → 0 | 1 | … | 9

  Consists of the language which is a list of digit separated by + or -.

Derivation

list → <u>list</u> + digit
→ <u>list</u> – digit + digit
→ <u>digit</u> – digit + digit
→ 9 – <u>digit</u> + digit
→ 9 – 5 + <u>digit</u>
→ 9 – 5 + 2

Therefore, the string 9-5+2 belongs to the language specified by the grammar

The name context free comes from the fact that use of a production X → … does not depend on the context of X

Examples …

- Grammar for Pascal block

  block → begin statements end
  statements → stmt-list | ϵ
  stmt–list → stmt-list ; stmt
  | stmt

Syntax analyzers

- Testing for membership whether w belongs to L(G) is just a "yes" or "no" answer

- However the syntax analyzer
  - Must generate the parse tree
  - Handle errors gracefully if string is not in the language

- Form of the grammar is important
  - Many grammars generate the same language
  - Tools are sensitive to the grammar

Derivation

- If there is a production $A \rightarrow \alpha$ then we say that A derives α and is denoted by $A \Rightarrow \alpha$
- $\alpha A \beta \Rightarrow \alpha \gamma \beta$ if $A \rightarrow \gamma$ is a production
- If $\alpha_1 \Rightarrow \alpha_2 \Rightarrow \dots \Rightarrow \alpha_n$ then $\alpha_1 \overset{+}{\Rightarrow} \alpha_n$
- Given a grammar G and a string w of terminals in $L(G)$ we can write $S \overset{+}{\Rightarrow} w$
- If $S \overset{*}{\Rightarrow} \alpha$ where α is a string of terminals and non terminals of G then we say that α is a sentential form of G

Derivation …

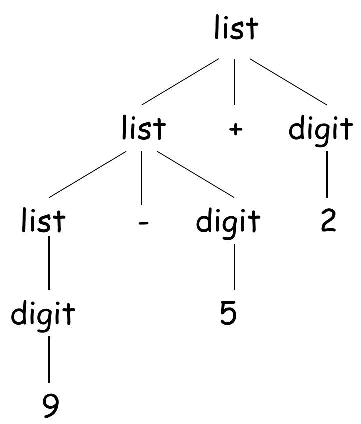
- If in a sentential form only the leftmost non terminal is replaced then it becomes leftmost derivation
- Every leftmost step can be written as
  $wA\gamma \Rightarrow^{lm*} w\delta\gamma$
  where w is a string of terminals and $A \rightarrow \delta$ is a production
- Similarly, right most derivation can be defined
- An ambiguous grammar is one that produces more than one leftmost/rightmost derivation of a sentence

Parse tree

- It shows how the start symbol of a grammar derives a string in the language
- root is labeled by the start symbol
- leaf nodes are labeled by tokens
- Each internal node is labeled by a non terminal
- if A is a non-terminal labeling an internal node and $x_1, x_2, \dots x_n$ are labels of the children of that node then $A \rightarrow x_1\ x_2 \dots x_n$ is a production

Example

Parse tree for 9-5+2

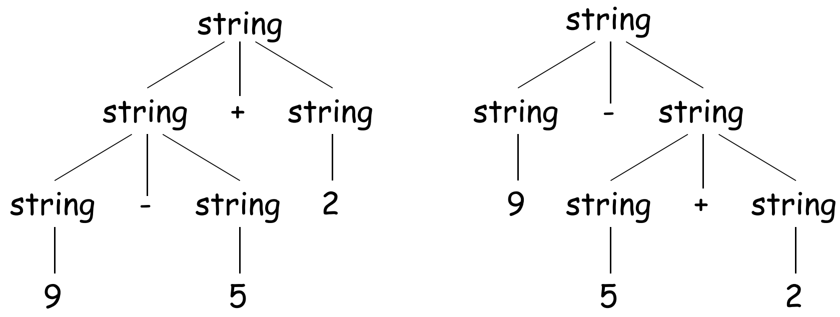


Ambiguity

- A Grammar can have more than one parse tree for a string

- Consider grammar
  string → string + string
  | string – string
  | 0 | 1 | … | 9

- String 9-5+2 has two parse trees



Ambiguity …

- Ambiguity is problematic because meaning of the programs can be incorrect

- Ambiguity can be handled in several ways
  - Enforce associativity and precedence
  - Rewrite the grammar (cleanest way)

- There are no general techniques for handling ambiguity

- It is impossible to convert automatically an ambiguous grammar to an unambiguous one

Associativity

- If an operand has operator on both the sides, the side on which operator takes this operand is the associativity of that operator

- In a+b+c b is taken by left +

- +, -, *, / are left associative

- ^, = are right associative

- Grammar to generate strings with right associative operators
  right → letter = right | letter
  letter → a| b |…| z

Precedence

- String a+5*2 has two possible interpretations because of two different parse trees corresponding to
  (a+5)*2 and a+(5*2)
- Precedence determines the correct interpretation.

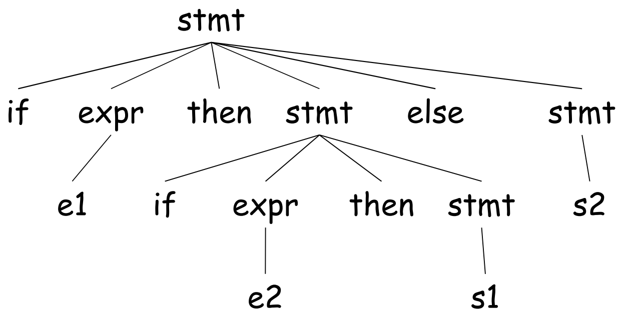
Ambiguity

- Dangling else problem

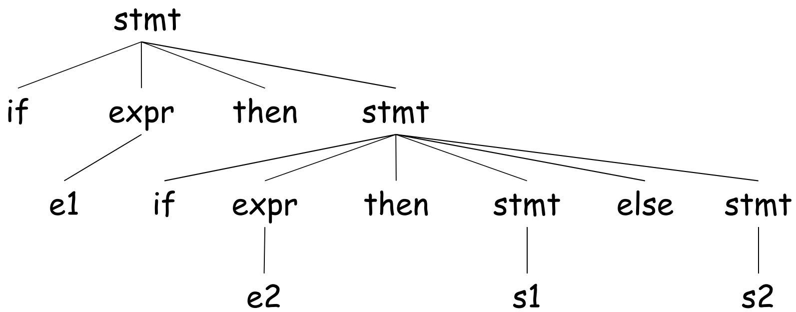
  Stmt $\rightarrow$ if expr then stmt
  | if expr then stmt else stmt

- according to this grammar, string
  if el then if e2 then S1 else S2
  has two parse trees


```
if e1
  then if e2
         then s1
  else s2
```




```
if e1
  then if e2
         then s1
         else s2
```



Resolving dangling else problem

- General rule: match each **else** with the closest previous **then**. The grammar can be rewritten as

  stmt $\rightarrow$ matched-stmt
  | unmatched-stmt

  matched-stmt $\rightarrow$ if expr then matched-stmt
  else matched-stmt
  | others

  unmatched-stmt $\rightarrow$ if expr then stmt
  | if expr then matched-stmt
  else unmatched-stmt

Parsing

- Process of determination whether a string can be generated by a grammar
- Parsing falls in two categories:
  - Top-down parsing:
    Construction of the parse tree starts at the root (from the start symbol) and proceeds towards leaves (token or terminals)
  - Bottom-up parsing:
    Construction of the parse tree starts from the leaf nodes (tokens or terminals of the grammar) and proceeds towards root (start symbol)

Example: Top down Parsing

- Following grammar generates types of Pascal

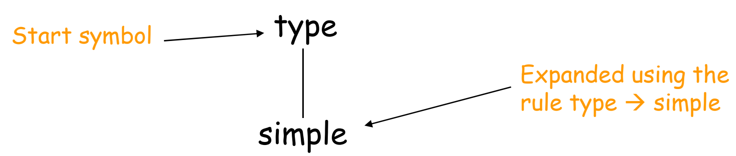
type → simple
       | ↑ id
       | array [simple] of type

simple → integer
         | char
         | num dotdot num

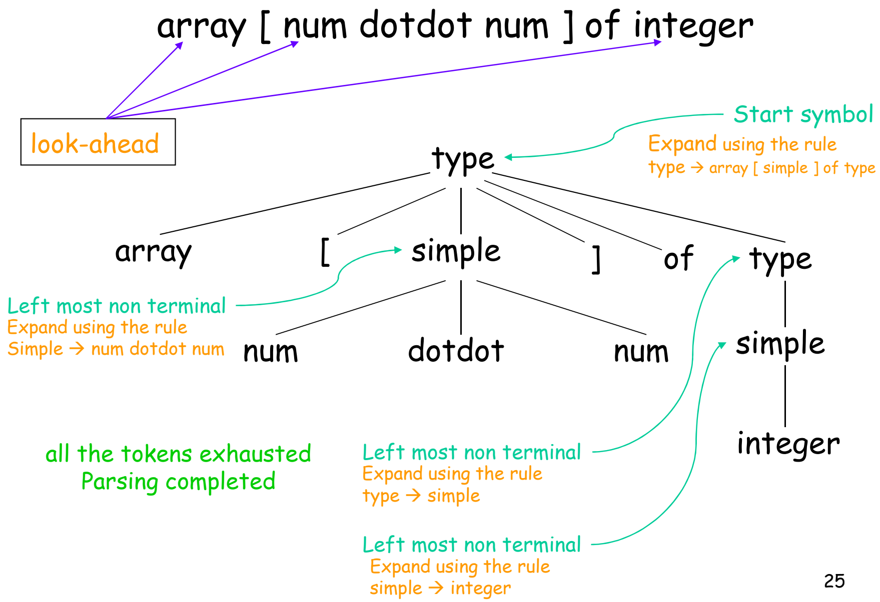
Example …

- Construction of a parse tree is done by starting the root labeled by a start symbol
- repeat following two steps
  - at a node labeled with non terminal A select one of the productions of A and construct children nodes (Which production?)
  - find the next node at which subtree is Constructed (Which node?)

- Parse
  array [num dotdot num] of integer



- Cannot proceed as non terminal "simple" never generates a string beginning with token "array". Therefore, requires back-tracking.
- Back-tracking is not desirable, therefore, take help of a "look-ahead" token. The current token is treated as look-ahead token. (restricts the class of grammars)



Recursive descent parsing

First set:

Let there be a production

$A \to \alpha$

then First(α) is the set of tokens that appear as the first token in the strings generated from α

For example :
First(simple) = {integer, char, num}
First(num dotdot num) = {num}

Define a procedure for each non terminal

```
procedure type;
  if lookahead in {integer, char, num}
    then simple
    else if lookahead = ↑
            then begin match( ↑ );
                       match(id)
                 end
            else if lookahead = array
                   then begin match(array);
                              match([);
                              simple;
                              match(]);
                              match(of);
                              type
                        end
                   else error;
```



```
procedure simple;
  if lookahead = integer
    then match(integer)
    else if lookahead = char
           then match(char)
           else if lookahead = num
                  then begin match(num);
                             match(dotdot);
                             match(num)
                       end
                  else
                       error;

procedure match(t:token);
    if lookahead = t
       then lookahead = next token
       else error;
```

Left recursion

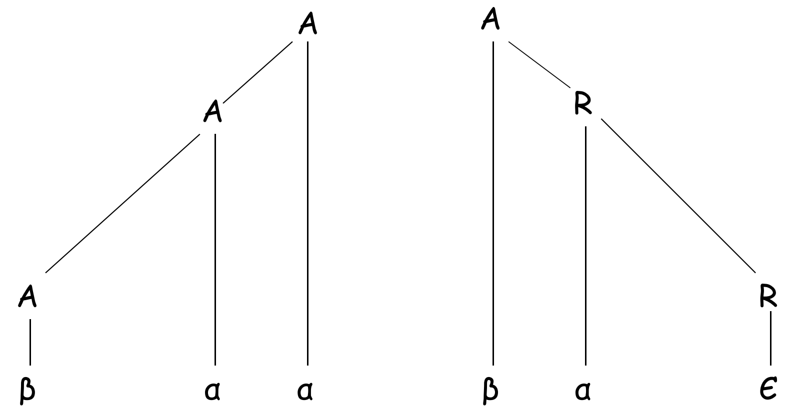
- A top down parser with production $A \rightarrow A\ \alpha$ may loop forever

- From the grammar $A \rightarrow A\ \alpha \mid \beta$ left recursion may be eliminated by transforming the grammar to

  $A \rightarrow \beta\ R$
  $R \rightarrow \alpha\ R \mid \varepsilon$

Parse tree corresponding to a left recursive grammar

Parse tree corresponding to the modified grammar



Both the trees generate string $\beta\alpha^*$

Example

- Consider grammar for arithmetic expressions

  $E \rightarrow E + T \mid T$
  $T \rightarrow T * F \mid F$
  $F \rightarrow (E) \mid id$

- After removal of left recursion the grammar becomes

  $E \rightarrow T E'$
  $E' \rightarrow + T E' \mid \epsilon$
  $T \rightarrow F T'$
  $T' \rightarrow * F T' \mid \epsilon$
  $F \rightarrow (E) \mid id$

Removal of left recursion

In general

$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \ldots\ldots \mid A\alpha_m$
$\quad \mid \beta_1 \mid \beta_2 \mid \ldots\ldots \mid \beta_n$

transforms to

$A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \ldots\ldots \mid \beta_n A'$
$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \ldots\ldots \mid \alpha_m A' \mid \epsilon$

Left recursion hidden due to many productions

- Left recursion may also be introduced by two or more grammar rules. For example:

  $S \rightarrow Aa \mid b$
  $A \rightarrow Ac \mid Sd \mid \epsilon$

  there is a left recursion because

  $S \rightarrow Aa \rightarrow Sda$

- In such cases, left recursion is removed systematically
  - Starting from the first rule and replacing all the occurrences of the first non terminal symbol
  - Removing left recursion from the modified grammar

Removal of left recursion due to many productions …

- After the first step (substitute S by its rhs in the rules) the grammar becomes

  $S \rightarrow Aa \mid b$
  $A \rightarrow Ac \mid Aad \mid bd \mid \epsilon$

- After the second step (removal of left recursion) the grammar becomes

  $S \rightarrow Aa \mid b$
  $A \rightarrow bdA' \mid A'$
  $A' \rightarrow cA' \mid adA' \mid \epsilon$

Left factoring

- In top-down parsing when it is not clear which production to choose for expansion of a symbol
  defer the decision till we have seen enough input.

  In general if $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$

  defer decision by expanding A to $\alpha A'$

  we can then expand A' to β_1 or β_2

- Therefore $A \rightarrow \alpha\, \beta_1 \mid \alpha\, \beta_2$

  transforms to

  $A \rightarrow \alpha A'$
  $A' \rightarrow \beta_1 \mid \beta_2$

Dangling else problem again

Dangling else problem can be handled by left factoring

stmt → if expr then stmt else stmt
| if expr then stmt

can be transformed to

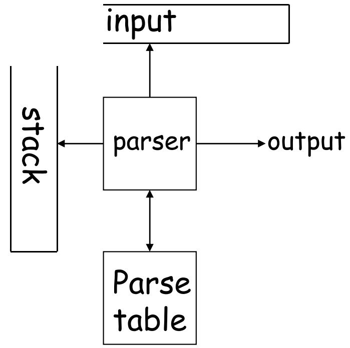
stmt → if expr then stmt S'
S' → else stmt | ϵ

Predictive parsers

- A non recursive top down parsing method
- Parser "predicts" which production to use
- It removes backtracking by fixing one production for every non-terminal and input token(s)
- Predictive parsers accept LL(k) languages
  - First L stands for left to right scan of input
  - Second L stands for leftmost derivation
  - k stands for number of lookahead token
- In practice LL(1) is used

Predictive parsing

- Predictive parser can be implemented by maintaining an external stack



Parse table is a two dimensional array M[X,a] where "X" is a non terminal and "a" is a terminal of the grammar

Example

- Consider the grammar

$E \rightarrow T E'$
$E' \rightarrow +T E' \mid \epsilon$
$T \rightarrow F T'$
$T' \rightarrow * F T' \mid \epsilon$
$F \rightarrow (E) \mid id$

Parse table for the grammar

	id	+	*	(	)	$
E	E→TE'			E→TE'		
E'		E'→+TE'			E'→ε	E'→ε
T	T→FT'			T→FT'		
T'		T'→ε	T'→*FT'		T'→ε	T'→ε
F	F→id			F→(E)		

Blank entries are error states. For example E cannot derive a string starting with '+'

Parsing algorithm

- The parser considers 'X' the symbol on top of stack, and 'a' the current input symbol
- These two symbols determine the action to be taken by the parser
- Assume that '$' is a special token that is at the bottom of the stack and terminates the input string

```
if X = a = $ then halt

if X = a ≠ $ then pop(x) and ip++

if X is a non terminal
      then if M[X,a] = {X → UVW}
              then begin pop(X); push(W,V,U)
                   end
              else error
```


Example

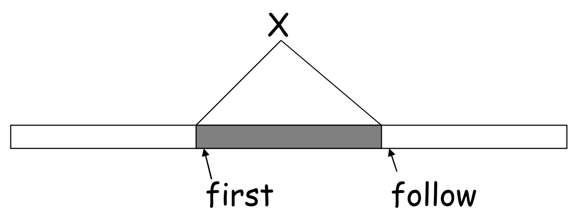
Stack	input	action
$E	id + id * id $	expand by E→TE'
$E'T	id + id * id $	expand by T→FT'
$E'T'F	id + id * id $	expand by F→id
$E'T'id	id + id * id $	pop id and ip++
$E'T'	+ id * id $	expand by T'→є
$E'	+ id * id $	expand by E'→+TE'
$E'T+	+ id * id $	pop + and ip++
$E'T	id * id $	expand by T→FT'

Example …

Stack	input	action
$E'T'F	id * id $	expand by F→id
$E'T'id	id * id $	pop id and ip++
$E'T'	* id $	expand by T'→*FT'
$E'T'F*	* id $	pop * and ip++
$E'T'F	id $	expand by F→id
$E'T'id	id $	pop id and ip++
$E'T'	$	expand by T'→є
$E'	$	expand by E'→є
$	$	halt

Constructing parse table

- Table can be constructed if for every non terminal, every lookahead symbol can be handled by at most one production
- First(α) for a string of terminals and non terminals α is
  - Set of symbols that might begin the fully expanded (made of only tokens) version of α
- Follow(X) for a non terminal X is
  - set of symbols that might follow the derivation of X in the input stream



Compute first sets

- If X is a terminal symbol then First(X) = {X}
- If X → ε is a production then ε is in First(X)
- If X is a non terminal
  and X → $Y_1 Y_2 \ldots Y_k$ is a production
  then
  if for some i, a is in First(Y_i)
  and ε is in all of First(Y_j) (such that j<i)
  then a is in First(X)
- If ε is in First (Y_1) … First(Y_k) then ε is in First(X)

Example

- For the expression grammar
  E → T E'
  E' → +T E' | ε
  T → F T'
  T' → * F T' | ε
  F → (E) | id

  First(E) = First(T) = First(F) = { (, id }
  First(E') = {+, ε}
  First(T') = { *, ε}

Compute follow sets

1. Place $ in follow(S)
2. If there is a production $A \rightarrow \alpha B \beta$
   then everything in first(β) (except ε) is in follow(B)
3. If there is a production $A \rightarrow \alpha B$
   then everything in follow(A) is in follow(B)
4. If there is a production $A \rightarrow \alpha B \beta$
   and First(β) contains ε
   then everything in follow(A) is in follow(B)

Since follow sets are defined in terms of follow sets last two steps have to be repeated until follow sets converge

Example

- For the expression grammar
  E → T E'
  E' → + T E' | ε
  T → F T'
  T' → * F T' | ε
  F → (E) | id

  follow(E) = follow(E') = { $,) }
  follow(T) = follow(T') = { $,), + }
  follow(F) = { $,), +, *}

Construction of parse table

- for each production A → α do
  - for each terminal 'a' in first(α)

    M[A,a] = A → α
  - If Є is in First(α)

    M[A,b] = A → α

    for each terminal b in follow(A)
  - If ε is in First(α) and $ is in follow(A)

    M[A,$] = A → α
- A grammar whose parse table has no multiple entries is called LL(1)

LL Parser Generators

- ANTLR
- LLGen
- LLnextGen
- Many more like Tiny Parser Generator, Wei parser generator, SLK parser generator, Yapps ….

Error handling

- Stop at the first error and print a message
  - Compiler writer friendly
  - But not user friendly
- Every reasonable compiler must recover from errors and identify as many errors as possible
- However, multiple error messages due to a single fault must be avoided
- Error recovery methods
  - Panic mode
  - Phrase level recovery
  - Error productions
  - Global correction

Panic mode

- Simplest and the most popular method
- Most tools provide for specifying panic mode recovery in the grammar
- When an error is detected
  - Discard tokens one at a time until a set of tokens is found whose role is clear
  - Skip to the next token that can be placed reliably in the parse tree

Panic mode …

- Consider following code

```
begin
        a = b + c;
        x = p r ;
        h = x < 0;
end;
```

- The second expression has syntax error
- Panic mode recovery for begin-end block
  skip ahead to next ';' and try to parse the next expression
- It discards one expression and tries to continue parsing
- May fail if no further ';' is found

Phrase level recovery

- Make local correction to the input
- Works only in limited situations
  - A common programming error which is easily detected
  - For example insert a ";" after closing "}" of a class definition
- Does not work very well!

Error productions

- Add erroneous constructs as productions in the grammar
- Works only for most common mistakes which can be easily identified
- Essentially makes common errors as part of the grammar
- Complicates the grammar and does not work very well

Global corrections

- Considering the program as a whole find a correct "nearby" program
- Nearness may be measured using certain metric
- PL/C compiler implemented this scheme: anything could be compiled!
- It is complicated and not a very good idea!

Error Recovery in LL(1) parser

- Error occurs when a parse table entry M[A,a] is empty
- Skip symbols in the input until a token in a selected set (synch) appears
- Place symbols in follow(A) in synch set. Skip tokens until an element in follow(A) is seen.
  Pop(A) and continue parsing
- Add symbol in first(A) in synch set. Then it may be possible to resume parsing according to A if a symbol in first(A) appears in input.

Bottom up parsing

- Construct a parse tree for an input string beginning at leaves and going towards root

  OR
- Reduce a string w of input to start symbol of grammar

Consider a grammar

S → aABe

A → Abc | b

B → d

And reduction of a string

a <u>b</u> b c d e

a <u>A b c</u> d e

a A <u>d</u> e

<u>a A B e</u>

S

Right most derivation

S → a A <u>B</u> e

→ a <u>A</u> d e

→ a <u>A</u> b c d e

→ a b b c d e

Shift reduce parsing

- Split string being parsed into two parts
  - Two parts are separated by a special character "."
  - Left part is a string of terminals and non terminals
  - Right part is a string of terminals
- Initially the input is .w

Shift reduce parsing …

- Bottom up parsing has two actions
- Shift: move terminal symbol from right string to left string

  if string before shift is α.pqr

  then string after shift is αp.qr
- Reduce: immediately on the left of "." identify a string same as RHS of a production and replace it by LHS

  if string before reduce action is αβ.pqr

  and A→β is a production

  then string after reduction is αA.pqr

Example

Assume grammar is $E \rightarrow E+E \mid E*E \mid id$
Parse id*id+id

String	action
.id*id+id	shift
id.*id+id	reduce E→id
E.*id+id	shift
E*.id+id	shift
E*id.+id	reduce E→id
E*E.+id	reduce E→E*E
E.+id	shift
E+.id	shift
E+id.	Reduce E→id
E+E.	Reduce E→E+E
E.	ACCEPT

Shift reduce parsing …

- Symbols on the left of "." are kept on a stack
  - Top of the stack is at "."
  - Shift pushes a terminal on the stack
  - Reduce pops symbols (rhs of production) and pushes a non terminal (lhs of production) onto the stack
- The most important issue: when to shift and when to reduce
- Reduce action should be taken only if the result can be reduced to the start symbol

Bottom up parsing …

- A more powerful parsing technique
- LR grammars – more expensive than LL
- Can handle left recursive grammars
- Can handle virtually all the programming languages
- Natural expression of programming language syntax
- Automatic generation of parsers (Yacc, Bison etc.)
- Detects errors as soon as possible
- Allows better error recovery

Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce
  - Which production to use for reduction?
- Sometimes parser can reduce but it should not:
  $X \rightarrow \epsilon$ can always be reduced!
- Sometimes parser can reduce in different ways!
- Given stack δ and input symbol a, should the parser
  - Shift a onto stack (making it δa)
  - Reduce by some production $A \rightarrow \beta$ assuming that stack has form $\alpha\beta$ (making it αA)
  - Stack can have many combinations of $\alpha\beta$
  - How to keep track of length of β?

Handle

- A string that matches right hand side of a production and whose replacement gives a step in the reverse right most derivation
- If $S \rightarrow^{rm*} \alpha Aw \rightarrow^{rm} \alpha\beta w$ then β (corresponding to production $A \rightarrow \beta$) in the position following α is a handle of $\alpha\beta w$. The string w consists of only terminal symbols
- We only want to reduce handle and not any rhs
- Handle pruning: If β is a handle and $A \rightarrow \beta$ is a production then replace β by A
- A right most derivation in reverse can be obtained by handle pruning.

Handles …

- Handles always appear at the top of the stack and never inside it
- This makes stack a suitable data structure
- Consider two cases of right most derivation to verify the fact that handle appears on the top of the stack
  - $S \rightarrow \alpha Az \rightarrow \alpha\beta Byz \rightarrow \alpha\beta\gamma yz$
  - $S \rightarrow \alpha BxAz \rightarrow \alpha Bxyz \rightarrow \alpha\gamma xyz$
- Bottom up parsing is based on recognizing handles

Handle always appears on the top

Case I: $S \rightarrow \alpha Az \rightarrow \alpha\beta Byz \rightarrow \alpha\beta\gamma yz$

stack	input	action
$\alpha\beta\gamma$	yz	reduce by $B \rightarrow \gamma$
$\alpha\beta B$	yz	shift y
$\alpha\beta By$	z	reduce by $A \rightarrow \beta By$
αA	z	

Case II: $S \rightarrow \alpha BxAz \rightarrow \alpha Bxyz \rightarrow \alpha\gamma xyz$

stack	input	action
$\alpha\gamma$	xyz	reduce by $B \rightarrow \gamma$
αB	xyz	shift x
αBx	yz	shift y
αBxy	z	reduce $A \rightarrow y$
αBxA	z	

Conflicts

- The general shift-reduce technique is:
  - if there is no handle on the stack then shift
  - If there is a handle then reduce
- However, what happens when there is a choice
  - What action to take in case both shift and reduce are valid?
    shift-reduce conflict
  - Which rule to use for reduction if reduction is possible by more than one rule?
    reduce-reduce conflict
- Conflicts come either because of ambiguous grammars or parsing method is not powerful enough

Shift reduce conflict

Consider the grammar E → E+E | E*E | id
and input id+id*id

stack	input	action
E+E	*id	reduce by E→E+E
E	*id	shift
E*	id	shift
E*id		reduce by E→id
E*E		reduce byE→E*E
E		

stack	input	action
E+E	*id	shift
E+E*	id	shift
E+E*id		reduce by E→id
E+E*E		reduce by E→E*E
E+E		reduce by E→E+E
E		

Reduce reduce conflict

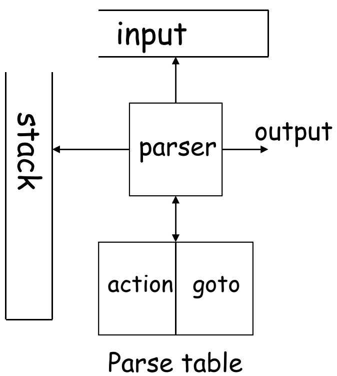
Consider the grammar M → R+R | R+c | R
R → c

and input c+c

Stack	input	action
	c+c	shift
c	+c	reduce by R→c
R	+c	shift
R+	c	shift
R+c		reduce by R→c
R+R		reduce by M→R+R
M		

Stack	input	action
	c+c	shift
c	+c	reduce by R→c
R	+c	shift
R+	c	shift
R+c		reduce by M→R+c
M		

LR parsing



- Input contains the input string.
- Stack contains a string of the form $S_0X_1S_1X_2\ldots\ldots X_nS_n$ where each X_i is a grammar symbol and each S_i is a state.
- Tables contain action and goto parts.
- action table is indexed by state and terminal symbols.
- goto table is indexed by state and non terminal symbols.

Example

Consider the grammar
And its parse table

E → E + T | T
T → T * F | F
F → (E) | id

State	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Parse id + id * id

Stack	Input	Action
0	id+id*id$	shift 5
0 id 5	+id*id$	reduce by F→id
0 F 3	+id*id$	reduce by T→F
0 T 2	+id*id$	reduce by E→T
0 E 1	+id*id$	shift 6
0 E 1 + 6	id*id$	shift 5
0 E 1 + 6 id 5	*id$	reduce by F→id
0 E 1 + 6 F 3	*id$	reduce by T→F
0 E 1 + 6 T 9	*id$	shift 7
0 E 1 + 6 T 9 * 7	id$	shift 5
0 E 1 + 6 T 9 * 7 id 5	$	reduce by F→id
0 E 1 + 6 T 9 * 7 F 10	$	reduce by T→T*F
0 E 1 + 6 T 9	$	reduce by E→E+T
0 E 1	$	ACCEPT

Actions in an LR (shift reduce) parser

- Assume S_i is top of stack and a_i is current input symbol
- Action $[S_i, a_i]$ can have four values

1. shift a_i to the stack and goto state S_j
2. reduce by a rule
3. Accept
4. error

Configurations in LR parser

Stack: $S_0X_1S_1X_2 \ldots X_mS_m$ Input: $a_ia_{i+1} \ldots a_n\$$

- If action$[S_m, a_i]$ = shift S
  Then the configuration becomes
  Stack: $S_0X_1S_1 \ldots\ldots X_mS_ma_iS$ Input: $a_{i+1} \ldots a_n\$$
- If action$[S_m, a_i]$ = reduce $A \rightarrow \beta$
  Then the configuration becomes
  Stack: $S_0X_1S_1 \ldots X_{m-r}S_{m-r}\ AS$ Input: $a_ia_{i+1} \ldots a_n\$$
  Where r = $|\beta|$ and S = goto$[S_{m-r}, A]$
- If action$[S_m, a_i]$ = accept
  Then parsing is completed. HALT
- If action$[S_m, a_i]$ = error
  Then invoke error recovery routine.

LR parsing Algorithm

Initial state: Stack: S_0 Input: w$

```
Loop{
   if action[S,a] = shift S'
      then push(a); push(S'); ip++
      else if action[S,a] = reduce A→β
              then pop (2*|β|) symbols;
                  push(A); push (goto[S'',A])
                  (S'' is the state after popping symbols)
              else if action[S,a] = accept
                  then exit
                  else error
   }
```

Example

Consider the grammar
And its parse table

$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid id$

State	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Parser states

- Goal is to know the valid reductions at any given point
- Summarize all possible stack prefixes α as a parser state
- Parser state is defined by a DFA state that reads in the stack α
- Accept states of DFA are unique reductions

Constructing parse table

Augment the grammar

- G is a grammar with start symbol S
- The augmented grammar G' for G has a new start symbol S' and an additional production $S' \rightarrow S$
- When the parser reduces by this rule it will stop with accept

Viable prefixes

- α is a viable prefix of the grammar if
  - There is a w such that αw is a right sentential form
  - α.w is a configuration of the shift reduce parser
- As long as the parser has viable prefixes on the stack no parser error has been seen
- The set of viable prefixes is a regular language (not obvious)
- Construct an automaton that accepts viable prefixes

LR(0) items

- An LR(0) item of a grammar G is a production of G with a special symbol "." at some position of the right side
- Thus production A→XYZ gives four LR(0) items
  A → .XYZ
  A → X.YZ
  A → XY.Z
  A → XYZ.
- An item indicates how much of a production has been seen at a point in the process of parsing
  - Symbols on the left of "." are already on the stacks
  - Symbols on the right of "." are expected in the input

Start state

- Start state of DFA is an empty stack corresponding to S'→.S item
  - This means no input has been seen
  - The parser expects to see a string derived from S
- **Closure** of a state adds items for all productions whose LHS occurs in an item in the state, just after "."
  - Set of possible productions to be reduced next
  - Added items have "." located at the beginning
  - No symbol of these items is on the stack as yet

Closure operation

- If I is a set of items for a grammar G then closure(I) is a set constructed as follows:
  - Every item in I is in closure (I)
  - If A → α.Bβ is in closure(I) and B → γ is a production then B → .γ is in closure(I)
- Intuitively A →α.Bβ indicates that we might see a string derivable from Bβ as input
- If input B → γ is a production then we might see a string derivable from γ at this point

Example

Consider the grammar

E' → E
E → E + T | T
T → T * F | F
F → (E) | id

If I is { E' → .E } then closure(I) is

E' → .E
E → .E + T
E → .T
T → .T * F
T → .F
F → .id
F → .(E)

Applying symbols in a state

- In the new state include all the items that have appropriate input symbol just after the "."
- Advance "." in those items and take closure

Goto operation

- Goto(I,X) , where I is a set of items and X is a grammar symbol,
  - is closure of set of item A →αX.β
  - such that A → α.Xβ is in I
- Intuitively if I is a set of items for some valid prefix α then goto(I,X) is set of valid items for prefix αX
- If I is { E'→E. , E→E. + T } then goto(I,+) is

  E → E + .T
  T → .T * F
  T → .F
  F → .(E)
  F → .id

Sets of items

C : Collection of sets of LR(0) items for grammar G'

C = { closure ({ S' → .S }) }
repeat
  for each set of items I in C
    and each grammar symbol X
      such that goto (I,X) is not empty and not in C
        ADD goto(I,X) to C
until no more additions

Example

Grammar:
  E' → E
  E → E+T | T
  T → T*F | F
  F → (E) | id

I_0: closure(E'→.E)
  E' → .E
  E → .E + T
  E → .T
  T → .T * F
  T → .F
  F → .(E)
  F → .id

I_1: goto(I_0,E)
  E' → E.
  E → E. + T

I_2: goto(I_0,T)
  E → T.
  T → T. *F

I_3: goto(I_0,F)
  T → F.

I_4: goto(I_0,( )
  F → (.E)
  E → .E + T
  E → .T
  T → .T * F
  T → .F
  F → .(E)
  F → .id

I_5: goto(I_0,id)
  F → id.

I_6: goto(I_1,+)
E → E + .T
T → .T * F
T → .F
F → .(E)
F → .id

I_7: goto(I_2,*)
T → T * .F
F →.(E)
F → .id

I_8: goto(I_4,E)
F → (E.)
E → E. + T

goto(I_4,T) is I_2
goto(I_4,F) is I_3
goto(I_4,( ) is I_4
goto(I_4,id) is I_5

I_9: goto(I_6,T)
E → E + T.
T → T. * F

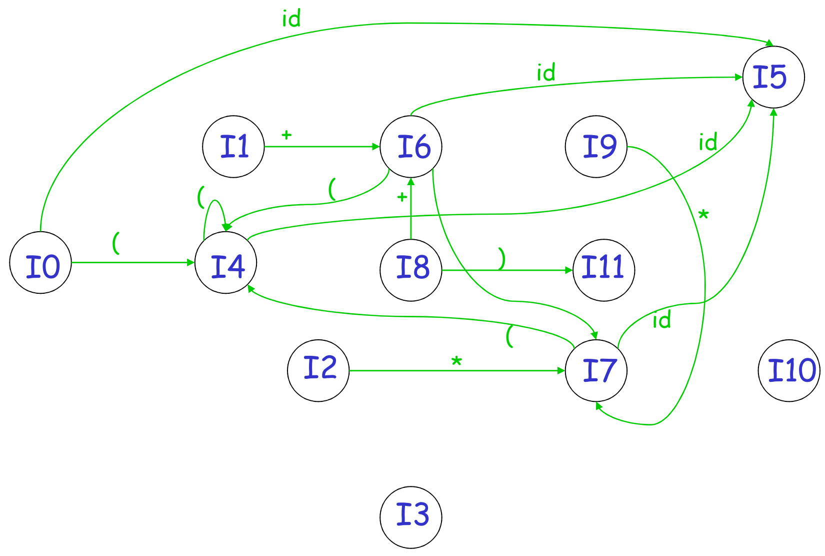
goto(I_6,F) is I_3
goto(I_6,( ) is I_4
goto(I_6,id) is I_5

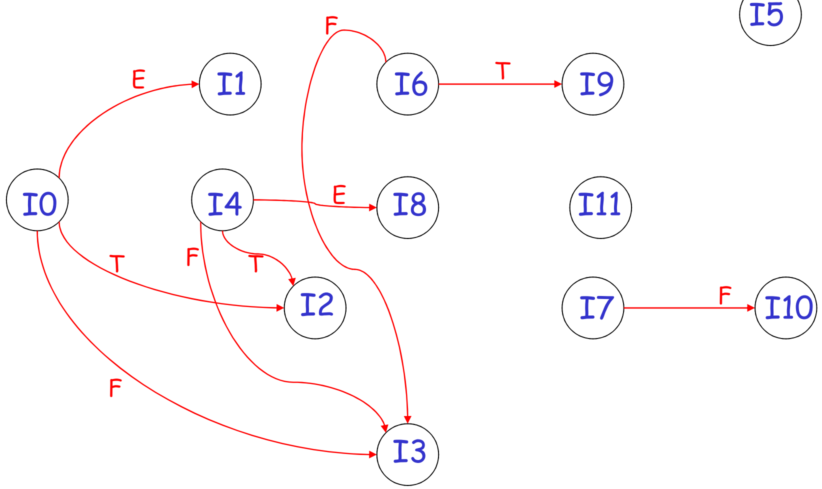
I_{10}: goto(I_7,F)
T → T * F.

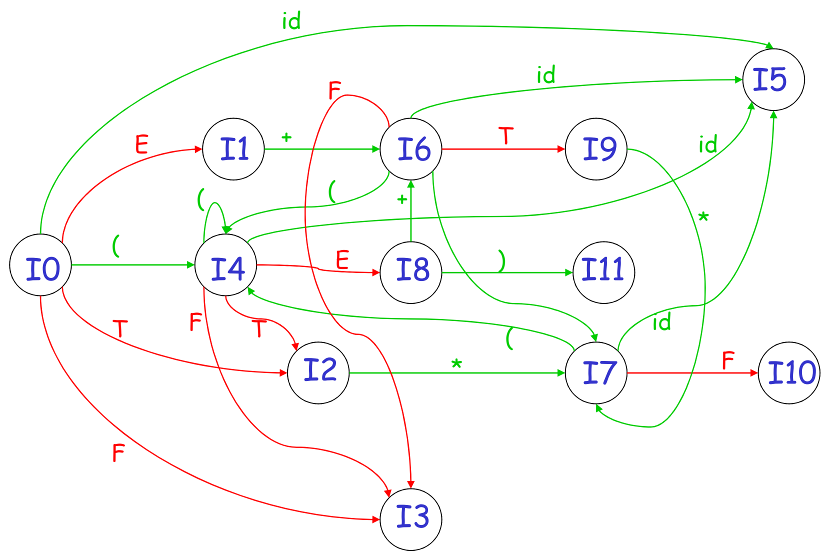
goto(I_7,( ) is I_4
goto(I_7,id) is I_5

I_{11}: goto(I_8,))
F → (E).

goto(I_8,+) is I_6
goto(I_9,*) is I_7







Construct SLR parse table

- Construct $C=\{I_0, \ldots, I_n\}$ the collection of sets of LR(0) items
- If $A \rightarrow \alpha.a\beta$ is in I_i and $goto(I_i, a) = I_j$ then action[i,a] = shift j
- If $A \rightarrow \alpha.$ is in I_i then action[i,a] = reduce $A \rightarrow \alpha$ for all a in follow(A)
- If $S' \rightarrow S.$ is in I_i then action[i,$] = accept
- If $goto(I_i, A) = I_j$ then goto[i,A]=j for all non terminals A
- All entries not defined are errors

Notes

- This method of parsing is called SLR (Simple LR)
- LR parsers accept LR(k) languages
  - L stands for left to right scan of input
  - R stands for rightmost derivation
  - k stands for number of lookahead token
- SLR is the simplest of the LR parsing methods. SLR is too weak to handle most languages!
- If an SLR parse table for a grammar does not have multiple entries in any cell then the grammar is unambiguous
- All SLR grammars are unambiguous
- Are all unambiguous grammars in SLR?

Example

- Consider following grammar and its SLR parse table:

$S' \rightarrow S$
$S \rightarrow L = R$
$S \rightarrow R$
$L \rightarrow *R$
$L \rightarrow id$
$R \rightarrow L$

I_0: $S' \rightarrow .S$
$S \rightarrow .L=R$
$S \rightarrow .R$
$L \rightarrow .*R$
$L \rightarrow .id$
$R \rightarrow .L$

I_1: $goto(I_0, S)$
$S' \rightarrow S.$

I_2: $goto(I_0, L)$
$S \rightarrow L.=R$
$R \rightarrow L.$

Assignment (not to be submitted): Construct rest of the items and the parse table.

SLR parse table for the grammar

	=	*	id	$	S	L	R
0		s4	s5		1	2	3
1				acc			
2	s6,r6			r6			
3				r3			
4		s4	s5			8	7
5	r5			r5			
6		s4	s5			8	9
7	r4			r4			
8	r6			r6			
9				r2			

The table has multiple entries in action[2,=]

- There is both a shift and a reduce entry in action[2,=]. Therefore state 2 has a shift-reduce conflict on symbol "=", However, the grammar is not ambiguous.

- Parse id=id assuming reduce action is taken in [2,=]

Stack	input	action
0	id=id	shift 5
0 id 5	=id	reduce by L→id
0 L 2	=id	reduce by R→L
0 R 3	=id	error

- if shift action is taken in [2,=]

Stack	input	action
0	id=id$	shift 5
0 id 5	=id$	reduce by L→id
0 L 2	=id$	shift 6
0 L 2 = 6	id$	shift 5
0 L 2 = 6 id 5	$	reduce by L→id
0 L 2 = 6 L 8	$	reduce by R→L
0 L 2 = 6 R 9	$	reduce by S→L=R
0 S 1	$	ACCEPT

Problems in SLR parsing

- No sentential form of this grammar can start with R=…
- However, the reduce action in action[2,=] generates a sentential form starting with R=
- Therefore, the reduce action is incorrect
- In SLR parsing method state i calls for reduction on symbol "a", by rule A→α if I_i contains [A→α.] and "a" is in follow(A)
- However, when state I appears on the top of the stack, the viable prefix βα on the stack may be such that βA can not be followed by symbol "a" in any right sentential form
- Thus, the reduction by the rule A→α on symbol "a" is invalid
- SLR parsers cannot remember the left context

Canonical LR Parsing

- Carry extra information in the state so that wrong reductions by A → α will be ruled out
- Redefine LR items to include a terminal symbol as a second component (look ahead symbol)
- The general form of the item becomes [A → α.β, a] which is called LR(1) item.
- Item [A → α., a] calls for reduction only if next input is a. The set of symbols "a"s will be a subset of Follow(A).

Closure(I)

```
repeat
   for each item [A → α.Bβ, a] in I
      for each production B → γ in G'
      and for each terminal b in First(βa)
            add item [B → .γ, b] to I
until no more additions to I
```

Example

Consider the following grammar

$S' \rightarrow S$
$S \rightarrow CC$
$C \rightarrow cC \mid d$

Compute closure(I) where I={[S' → .S, $]}

$S' \rightarrow .S,$	$
$S \rightarrow .CC,$	$
$C \rightarrow .cC,$	c
$C \rightarrow .cC,$	d
$C \rightarrow .d,$	c
$C \rightarrow .d,$	d

Example

Construct sets of LR(1) items for the grammar on previous slide

I_0: $S' \rightarrow .S,$ $
$S \rightarrow .CC,$ $
$C \rightarrow .cC,$ c/d
$C \rightarrow .d,$ c/d

I_1: goto(I_0,S)
$S' \rightarrow S.,$ $

I_2: goto(I_0,C)
$S \rightarrow C.C,$ $
$C \rightarrow .cC,$ $
$C \rightarrow .d,$ $

I_3: goto(I_0,c)
$C \rightarrow c.C,$ c/d
$C \rightarrow .cC,$ c/d
$C \rightarrow .d,$ c/d

I_4: goto(I_0,d)
$C \rightarrow d.,$ c/d

I_5: goto(I_2,C)
$S \rightarrow CC.,$ $

I_6: goto(I_2,c)
$C \rightarrow c.C,$ $
$C \rightarrow .cC,$ $
$C \rightarrow .d,$ $

I_7: goto(I_2,d)
$C \rightarrow d.,$ $

I_8: goto(I_3,C)
$C \rightarrow cC.,$ c/d

I_9: goto(I_6,C)
$C \rightarrow cC.,$ $

Construction of Canonical LR parse table

- Construct $C=\{I_0, \ldots, I_n\}$ the sets of LR(1) items.
- If $[A \rightarrow \alpha.a\beta, b]$ is in I_i and goto(I_i, a)=I_j then action[i,a]=shift j
- If $[A \rightarrow \alpha., a]$ is in I_i then action[i,a] reduce $A \rightarrow \alpha$
- If $[S' \rightarrow S., \$]$ is in I_i then action[i,$] = accept
- If goto(I_i, A) = I_j then goto[i,A] = j for all non terminals A

Parse table

State	c	d	$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

Notes on Canonical LR Parser

- Consider the grammar discussed in the previous two slides. The language specified by the grammar is c*dc*d.
- When reading input cc…dcc…d the parser shifts cs into stack and then goes into state 4 after reading d. It then calls for reduction by C→d if following symbol is c or d.
- IF $ follows the first d then input string is c*d which is not in the language; parser declares an error
- On an error canonical LR parser never makes a wrong shift/reduce move. It immediately declares an error
- Problem: Canonical LR parse table has a large number of states

LALR Parse table

- Look Ahead LR parsers
- Consider a pair of similar looking states (same kernel and different lookaheads) in the set of LR(1) items
  I_4: $C \rightarrow$ d. , c/d $\qquad$ I_7: $C \rightarrow$ d., $
- Replace I_4 and I_7 by a new state I_{47} consisting of ($C \rightarrow$ d., c/d/$)
- Similarly I_3 & I_6 and I_8 & I_9 form pairs
- Merge LR(1) items having the same core

Construct LALR parse table

- Construct $C=\{I_0,\ldots\ldots,I_n\}$ set of LR(1) items
- For each core present in LR(1) items find all sets having the same core and replace these sets by their union
- Let C' = $\{J_0,\ldots\ldots.,J_m\}$ be the resulting set of items
- Construct action table as was done earlier
- Let $J = I_1 \cup I_2 \ldots\ldots. \cup I_k$

  since I_1 , I_2……., I_k have same core, goto(J,X) will have he same core

  Let K=goto(I_1,X) U goto(I_2,X)……goto(I_k,X) the goto(J,X)=K

LALR parse table …

State	c	d	$	S	C
0	s36	s47		1	2
1			acc		
2	s36	s47			5
36	s36	s47			89
47	r3	r3	r3		
5			r1		
89	r2	r2	r2		

Notes on LALR parse table

- Modified parser behaves as original except that it will reduce C→d on inputs like ccd. The error will eventually be caught before any more symbols are shifted.
- In general core is a set of LR(0) items and LR(1) grammar may produce more than one set of items with the same core.
- Merging items never produces shift/reduce conflicts but may produce reduce/reduce conflicts.
- SLR and LALR parse tables have same number of states.

Notes on LALR parse table…

- Merging items may result into conflicts in LALR parsers which did not exist in LR parsers
- New conflicts can not be of shift reduce kind:
  - Assume there is a shift reduce conflict in some state of LALR parser with items
    {[X→α.,a],[Y→γ.aβ,b]}
  - Then there must have been a state in the LR parser with the same core
  - Contradiction; because LR parser did not have conflicts
- LALR parser can have new reduce-reduce conflicts
  - Assume states
    {[X→α., a], [Y→α., b]} and {[X→α., b], [Y→α., a]}
  - Merging the two states produces
    {[X→α., a/b], [Y→α., a/b]}

Notes on LALR parse table…

- LALR parsers are not built by first making canonical LR parse tables
- There are direct, complicated but efficient algorithms to develop LALR parsers
- Relative power of various classes
  - SLR(1) ≤ LALR(1) ≤ LR(1)
  - SLR(k) ≤ LALR(k) ≤ LR(k)
  - LL(k) ≤ LR(k)

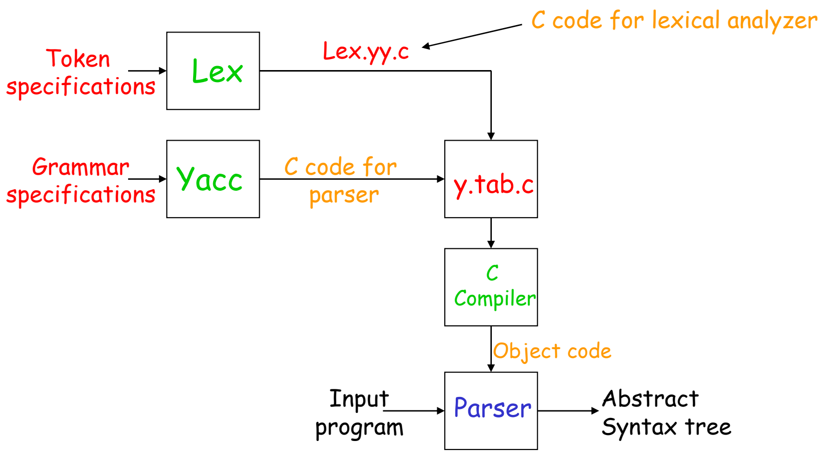
Error Recovery

- An error is detected when an entry in the action table is found to be empty.
- Panic mode error recovery can be implemented as follows:
  - scan down the stack until a state S with a goto on a particular nonterminal A is found.
  - discard zero or more input symbols until a symbol a is found that can legitimately follow A.
  - stack the state goto[S,A] and resume parsing.
- **Choice of A:** Normally these are non terminals representing major program pieces such as an expression, statement or a block. For example if A is the nonterminal stmt, a might be semicolon or end.

Parser Generator

- Some common LR parser generators
  - YACC: **Y**et **A**nother **C**ompiler **C**ompiler
  - Bison: GNU Software
- Yacc/Bison source program specification (accept LALR grammars)
  declaration
  %%
  translation rules
  %%
  supporting C routines

Yacc and Lex schema



Refer to YACC Manual