QP-2 : Quantum Teleportation

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1. Quantum Circuit creation

i) Imports :

In [1]:
```
# Importing the QuantumCircuit class from Qiskit
# The QuantumCircuit class is used to create quantum circuits
from qiskit import QuantumCircuit

# Importing the plot_histogram function from qiskit
# It used to visualize the simulation result.
from qiskit.visualization import plot_histogram

# Importing the numpy library
# Numpy is used for working with arrays and perform numerical operations
import numpy as np
```

ii) Quantum circuit creation and adding different components:

The secret state ***|Ψ> = cos(pi/5)|0> + i sin(pi/5)|1>***

- the probability of **|0>** is approx **65.5%**
- the probability of **|1>** is approx **34.5%**

In [2]:
```
# Creating a Quantum Circuit with 3 qubits and 1 classical bit
circuit = QuantumCircuit(3, 1)

# Preparing the state |Ψ> = cos(pi/5)|0> + i sin(pi/5)|1> on q0
circuit.ry(-2.0 * np.pi / 5.0, 0)

# Preparing Bell state |B00> on qubits q1 and q2
# H followed by CNOT produces entanglement
# Applying Hadamard gate to q1
circuit.h(1)
# Applying CNOT gate with q1 as control and q2 as target
circuit.cx(1, 2)

# 1st barrier : to separate state preparation from teleportation
circuit.barrier()
```

```python
# Performing Teleportation step
# Applying CNOT gate with q0 as control and q1 as target
circuit.cx(0, 1)
# Applying Hadamard gate to q0
circuit.h(0)

# 2nd Barrier : to separate teleportation from corrections
circuit.barrier()

# Applying the corrections to q2 to render the final state
# Applying X gate if q1 is 1
circuit.cx(1, 2)
# Applying Z gate if q0 is 1
circuit.cz(0, 2)

# 3rd barrier: to separate correction from final measurement
circuit.barrier()

# I am measuring q2 to verify the teleported state
# Measuring q2 and storing the result in classical bit 0
circuit.measure(2, 0)
```

Out[2]: <qiskit.circuit.instructionset.InstructionSet at 0x2af488701f0>

iii) Circuit diagram :

In [3]:
```python
# The draw method is used to visualize the quantum circuit.
# I am drawing the circuit using the 'mpl' output and 'iqp' style
circuit.draw(output='mpl', style='iqp')
```

Out[3]:

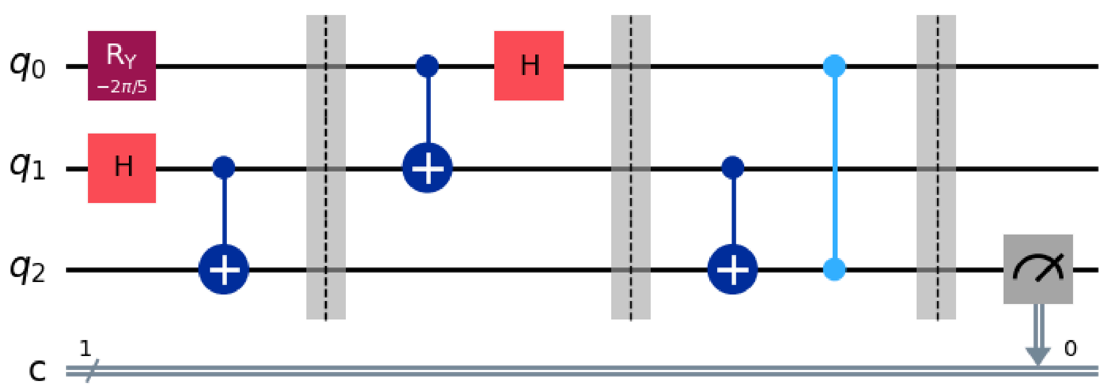


Fig 01 : Quantum Circuit for the Single Photon Interferometer

The above circuit diagram comprises of following notations and components :

Circuit Notations :

- **qo** is the 1st qubit represented by the solid horizontal line. (Alice's)
- **q1** is the 2nd qubit represented by the solid horizontal line. (Alice's)
- **q2** is the 3rd qubit represented by the solid horizontal line. (Bob's)

- **c** is the classical bit after measurement which is represented by the double lines.
- **1/** above the double lines represents the no. of classical data bits.

Circuit Components (left to right according to time step) :

- **Ry** in red box represents the rotation operator. It is used to apply rotation around the y-axis by -2π/5 rad. It is used for preparing the secret state |Ψ> = cos(pi/5)|0> + i sin(pi/5)|1> on q0.
- **H** in orange box represents the Hadamard gate on q1. It produces superposition.
- **CNOT** denoted by dark-blue line where **.** is the control(q1) and **+** is the target(q2). H followed by CNOT produces entanglement.
- **Barrier** denoted by dotted lines separates the initial state preparation from teleportation.
- **CNOT** denoted by dark-blue line where **.** is the control(q0) and **+** is the target(q1).
- **H** in orange box represents the Hadamard gate on q0. CNOT followed by H produces disentanglement.
- **Barrier** denoted by dotted lines separates to separate teleportation step from corrections.
- **CX** denoted by dark-blue line where **.** is the control(q1) and **+** is the target(q2).
- **CZ** denoted by sky-blue line where **.** is the control(q0) and **.** is the target(q2).
- **Meter** in gray box represents the Measurement operation.

2. Simulation code and output

i) Imports :

In [4]:
```
# The qiskit_aer library provides backend quantum simulators
# I am importing the Aer module which contains various type of simulators.
from qiskit_aer import Aer

# I am importing the transpile function from the qiskit library
# Transpile function is required to ensure that my circuit
# is able to run on the simulator.
from qiskit import transpile
```

ii) Getting the Simulator and running it

In [5]:
```
# The qasm simulator runs the circuit and its result is classical bits.
simulator = Aer.get_backend("qasm_simulator")

# Transpile transforms the circuit to something appropriate for the machine.
# I am transpiling my circuit for the backend qasm simulator
```

```
sim_circuit = transpile(circuit, backend = simulator)

# The run method in the simulator executes the transpiled circuit.
# I am running the trial 4096 times.
job_sim = simulator.run(sim_circuit, shots = 4096)
```

iii) Fetching the result and plotting histogram :

In [6]:
```
# I am fetching the results of the simulation job execution.
# This result contains the counts of each measurement outcome.
result_sim = job_sim.result()

# result.get_counts() method is used to find the count of different outcomes
# I am generating and displaying a histogram of the simulation outcomes.
plot_histogram(result_sim.get_counts(circuit))
```

Out[6]:

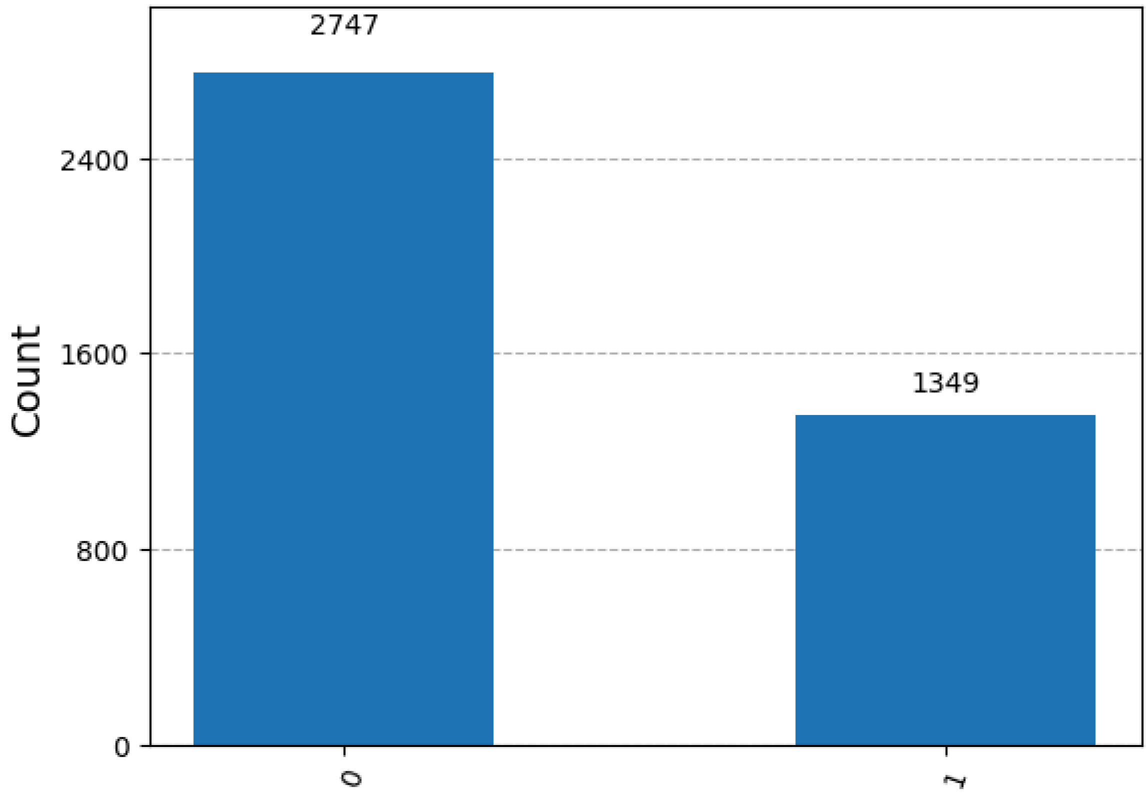


Fig 02 : Measured state Vs Counts

The histogram represents the probability of measuring the output states when my quantum circuit runs on the simulator.

In the above histogram :

- The **x-axis** represents the measured states.
- The **y-axis** represents the number of times each state was measured.

I ran my circuit for 4096 trials.

Here, on the x-axis, there is 0 with count 2720 and 1 with count 1376, which means that the measurement found the qubit q2 in :

- |0⟩ state 2747 times out of 4096 trials, so probability of |0⟩ state is 2747/4096 = **67.0%**
- |1⟩ state 1349 times out of 4096 trials, so probability of |1⟩ state is 1349/4096 = **33.0%**

These probability values of finding q2 in |0⟩ and |1⟩ is approx very close to that in intial state |Ψ>, |0> 65.5% and |1> 34.5%. This proves that the teleportation was successful.

3. IBM QC Hardware calculation

i) Imports :

In [7]:
```
# Importing the QiskitRuntimeService class from qiskit_ibm_runtime module
# The QiskitRuntimeService class is used to connect to IBMQ Services
# and run actual IBM QC hardware
from qiskit_ibm_runtime import QiskitRuntimeService

# Importing the SamplerV2 class from qiskit_ibm_runtime module
# The SamplerV2 class is used to find the probabilities of output states
from qiskit_ibm_runtime import SamplerV2 as Sampler
```

ii) Getting the Hardware and running it

In [8]:
```
# I am creating a new object of QiskitRuntimeService
# It is used to connect with my IBMQ account and use the sevices
service = QiskitRuntimeService()


# backends method is used to fetch list of all available quantum backends
mybackends = service.backends(operational = True, simulator = False,
                              min_num_qubits = 5)
mybackends
```

Out[8]:
```
[<IBMBackend('ibm_brisbane')>,
 <IBMBackend('ibm_sherbrooke')>,
 <IBMBackend('ibm_kyiv')>]
```

In [9]:
```
# least_busy method is used to pick the best available backend
device = service.least_busy(operational = True, simulator = False,
                            min_num_qubits = 5)
device
```

Out[9]:
```
<IBMBackend('ibm_kyiv')>
```

In [10]:
```
# Transpile transforms the circuit to something appropriate for the hardware
# seed is used to get the same transpiled circuit every time I run
```

```
transpiled_circuit = transpile(circuit, device, seed_transpiler = 13)

# SamplerV2 is used to find the probabilities of output states
# mode = device is used to select the least busy hardware I got above
sampler = Sampler(mode = device)

# The run method in the sampler executes the transpiled circuit
job_hardware = sampler.run([transpiled_circuit])
```

iii) Fetching the result and plotting histogram :

In [11]:
```
# I am fetching the results of the sampler job execution.
# This result contains the counts of each measurement outcome.
result_hardware = job_hardware.result()

# the 1st element at 0th index is the public result
pub_result = result_hardware[0]

# I am extracting the classical data part from the public result
# the values of c tells about the count of each outcome
classical_data = pub_result.data.c

# .get_counts() is used to measure the data in the classical bit 'c'
# I am generating and displaying a histogram of the execution outcomes
plot_histogram(classical_data.get_counts())
```

Out[11]:

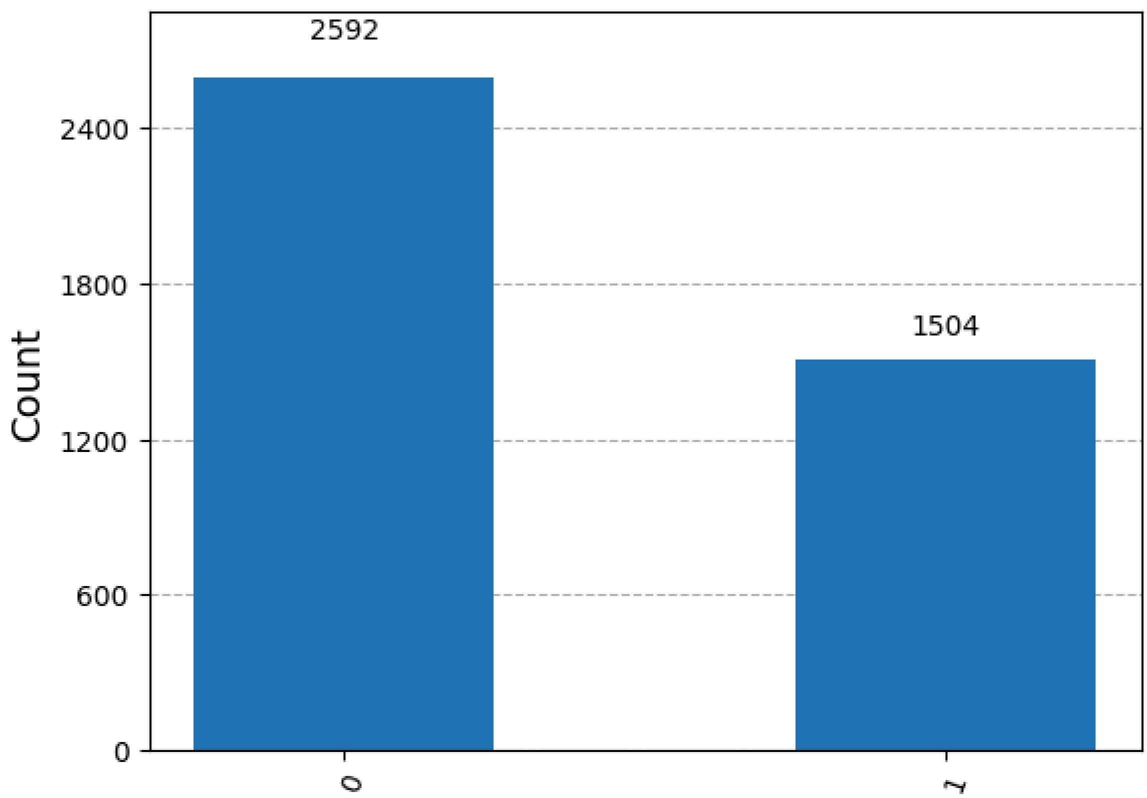


Fig 03 : Measured state Vs Counts

The histogram represents the probability of measuring the output states when my quantum circuit runs on the IBM QC Hardware.

In the above histogram :

- The **x-axis** represents the measured states.
- The **y-axis** represents the number of times each state was measured.

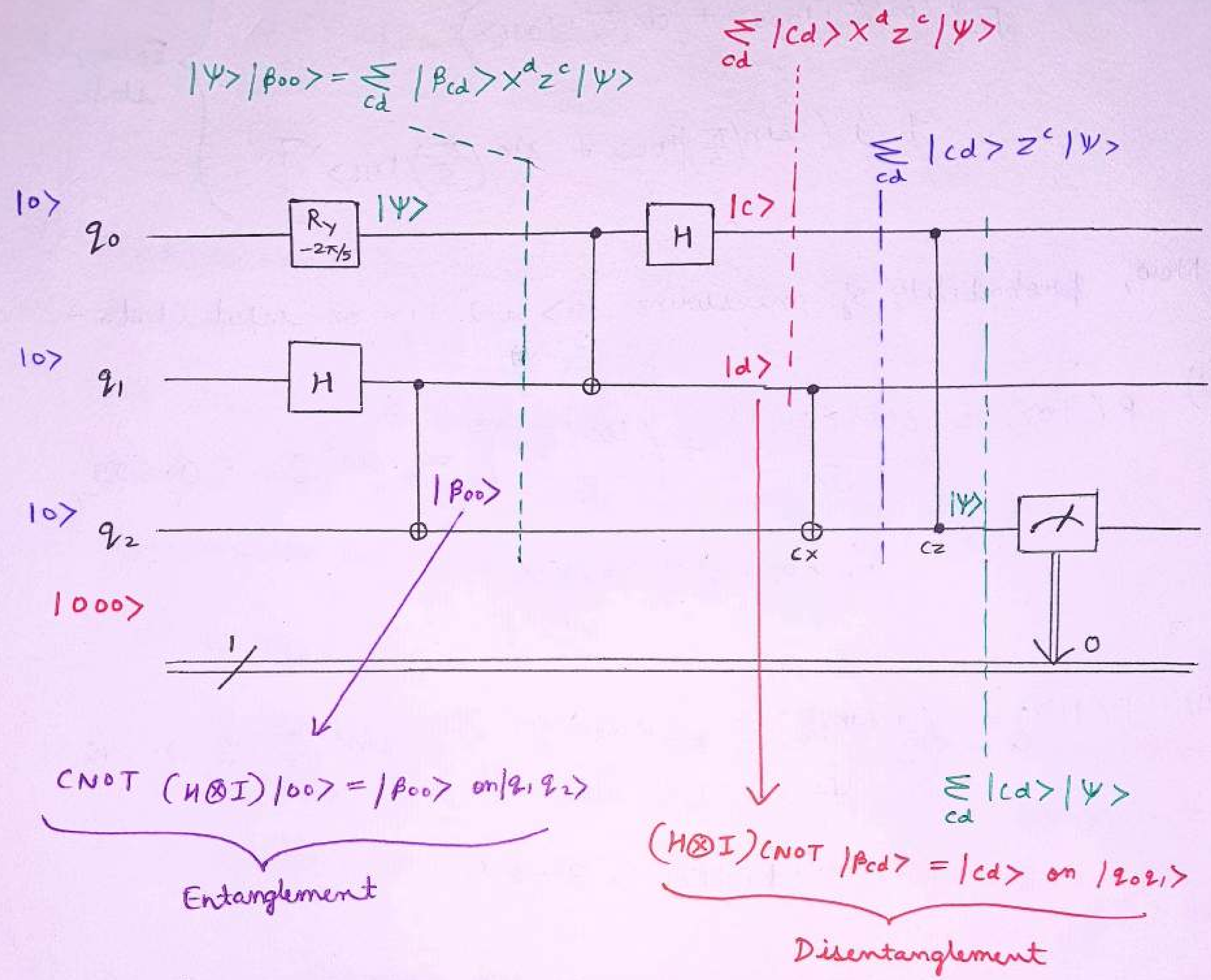
The circuit ran for 4096 trials on the IBM QC Hardware.

Here, on the x-axis, there is 0 with count 2720 and 1 with count 1376, which means that the measurement found the qubit q2 in :

- $|0\rangle$ state 2592 times out of 4096 trials, so probability of $|0\rangle$ state is 2592/4096 = **63.3%**
- $|1\rangle$ state 1504 times out of 4096 trials, so probability of $|1\rangle$ state is 1504/4096 = **36.7%**

These probability values of finding q2 in $|0\rangle$ and $|1\rangle$ is approx very close to that in intial state $|\Psi>$, $|0>$ 65.5% and $|1>$ 34.5%. This proves that the teleportation was successful.

Also the simulator and the IBM QC Hardware produced very similar results, which validates the teleportation.



State $|\psi\rangle = \cos\left(\frac{\pi}{5}\right)|0\rangle + i \sin\left(\frac{\pi}{5}\right)|1\rangle$

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\psi\rangle|\beta_{00}\rangle = \left[\cos\left(\frac{\pi}{5}\right)|0\rangle + i \sin\left(\frac{\pi}{5}\right)|1\rangle\right] \otimes \left[\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)\right]$$

$$\therefore |\psi\rangle|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{5}\right)|0\rangle\left(|00\rangle + |11\rangle\right) + i \sin\left(\frac{\pi}{5}\right)|1\rangle\left(|00\rangle + |11\rangle\right)\right)$$

$$\therefore |\psi\rangle|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\Big[\Big(\cos\left(\tfrac{\pi}{5}\right)|000\rangle + \cos\left(\tfrac{\pi}{5}\right)|011\rangle\Big) + i\left(\sin\left(\tfrac{\pi}{5}\right)|100\rangle + \sin\left(\tfrac{\pi}{5}\right)|111\rangle\right)\Big]$$

Entangled state

Now, probability of measuring $|0\rangle$ and $|1\rangle$ on input state -

i) $$P(|0\rangle) = \left(\frac{\cos\left(\frac{\pi}{5}\right)}{\sqrt{2}}\right)^2 + \left(\frac{\cos\left(\frac{\pi}{5}\right)}{\sqrt{2}}\right)^2 = \cos^2\left(\frac{\pi}{5}\right) \cong 0.655$$

$$\therefore P(|0\rangle) \cong 65.5\%.$$

ii) $$P(|1\rangle) = \left(\frac{i\sin\left(\frac{\pi}{5}\right)}{\sqrt{2}}\right)^2 + \left(\frac{i\sin\left(\frac{\pi}{5}\right)}{\sqrt{2}}\right)^2 = \sin^2\left(\frac{\pi}{5}\right) \cong 0.346$$

$$\therefore P(|1\rangle) \approx 34.6\%.$$

After teleportation, we must get approx. same probability value of the measured state q_2 for the teleportation to be successful.

c)

Alice starts with $q_1 q_2$ entangled Bell states.

$$2|\psi\rangle|\beta_{00}\rangle = \sum_{cd} \underbrace{|\beta_{cd}\rangle}_{\text{Alice}} \underbrace{X^d Z^c |\psi\rangle}_{\text{Bob}}$$

i) Alice measures her qubits in Bell basis states and finds the value of c and d ie $|\beta_{cd}\rangle$. When Alice informs Bob what c and d values are, Bob figures out his final state $X^d Z^c |\psi\rangle$ by performing one of the four operations on his EPR pair.

1) If Alice found $\overset{c}{0}, \overset{d}{0}$ $\longrightarrow$ Bob finds $|\psi\rangle$ $\rightarrow$ Bob applies $I \rightarrow |\psi\rangle$
2) If Alice found $\overset{c}{0}, \overset{d}{1}$ $\longrightarrow$ Bob finds $X|\psi\rangle$ $\rightarrow$ Bob applies $X \rightarrow |\psi\rangle$
3) If Alice found $\overset{c}{1}, \overset{d}{0}$ $\longrightarrow$ Bob finds $Z|\psi\rangle$ $\rightarrow$ Bob applies $Z \rightarrow |\psi\rangle$
4) If Alice found $\overset{c}{1}, \overset{d}{1}$ $\longrightarrow$ Bob finds $XZ|\psi\rangle$ $\rightarrow$ Bob applies X first then Z $\rightarrow |\psi\rangle$

ii) The above operations is equivalent to performing controlled operations from q_0 (control) to q_2 (target) $\Rightarrow$ CZ and from q_1 (control) to q_2 (target) $\Rightarrow$ CX

$$\text{so, } |\psi\rangle = \underset{\longleftarrow \text{ order of operation}}{Z^c X^d} X^d Z^c |\psi\rangle = \underset{\text{controlled } Z}{(CZ)}\underset{\text{controlled } X}{(CX)} X^d Z^c |\psi\rangle$$
$$= |\psi\rangle$$

Example $\rightarrow$ For $c=1, d=1$ case $\rightarrow$ $Z(X\,X)Z|\psi\rangle$

$$= Z\,I\,Z|\psi\rangle$$
$$= Z^2|\psi\rangle = I|\psi\rangle = |\psi\rangle$$

D)

i) Initial State →

$$|q_0 q_1 q_2\rangle = |000\rangle$$

ii) After applying $R_y\left(-\frac{2\pi}{5}\right)$ on q_0 →

$$|q_0 q_1 q_2\rangle = \left(\cos\left(\frac{\pi}{5}\right)|0\rangle + i \sin\left(\frac{\pi}{5}\right)|1\rangle\right) \otimes |00\rangle$$

iii) After creating the Bell state on $q_1 q_2$ →

$$|q_0 q_1 q_2\rangle = \left(\cos\left(\frac{\pi}{5}\right)|0\rangle + i \sin\left(\frac{\pi}{5}\right)|1\rangle\right) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

iv) After applying CX on q_0 (control) and q_1 (target) →

$$|q_0 q_1 q_2\rangle = \frac{1}{\sqrt{2}}\Big(\cos\left(\frac{\pi}{5}\right)|0\rangle \otimes (|00\rangle + |11\rangle$$

$$+ i \sin\left(\frac{\pi}{5}\right)|1\rangle \otimes (|10\rangle + |01\rangle)\Big)$$

v) After applying H on q_0 →

$$|q_0 q_1 q_2\rangle = \frac{1}{2}\Big(\cos\left(\frac{\pi}{5}\right)(|0\rangle + |1\rangle) \otimes (|00\rangle + |11\rangle)$$

$$+ i \sin\left(\frac{\pi}{5}\right)(|0\rangle - |1\rangle) \otimes |10\rangle + |01\rangle\Big)$$

vi) After applying X^d and Z^c on q_2 →

$$|q_2\rangle = |\psi\rangle = \cos\left(\frac{\pi}{5}\right)|0\rangle + i \sin\left(\frac{\pi}{5}\right)|1\rangle$$

vii) Final state →

$$|q_0 q_1 q_2\rangle = |\text{measurement } q_0 q_1\rangle \otimes |\psi\rangle$$