Monte Carlo in Derivative Pricing

Derivatives are financial products which derive their price from simpler financial instruments (called underlying) such as stocks. The price of a derivative is usually a complicated function of the future price of the underlying instrument. To find the price of the derivative, we need to find the future distribution of the price of the underlying instrument.

Product

Let us assume we have a derivative called an option whose underlying is a stock. This derivative pays the following amount at a future fixed time T (which is called expiration time):

$V = 1\ if\ S(T) > K$

$V = 0\ otherwise$

where K is called the strike. This is a specific kind of option called "binary option". Note that S(T) is a random variable – we do not know its future value.

Model

To find the expected value of the option in the future, we need to know the distribution of the stock in the future. Let us assume that the price of the stock evolves in the following way:

The initial value of the stock is S(0). The value of the stock at any future time t is given by,

$S(t) = S(0)\exp(X(t))$

where X(t) is a auxiliary time dependent variable whose initial value X(0) = 0.

$dX(t) = \epsilon_t v dt$

where $\epsilon_t = \epsilon_{t-dt}\ with\ probability\ 1 - \lambda dt$

or $\epsilon_t = -\epsilon_{t-dt}\ with\ probability\ \lambda dt$

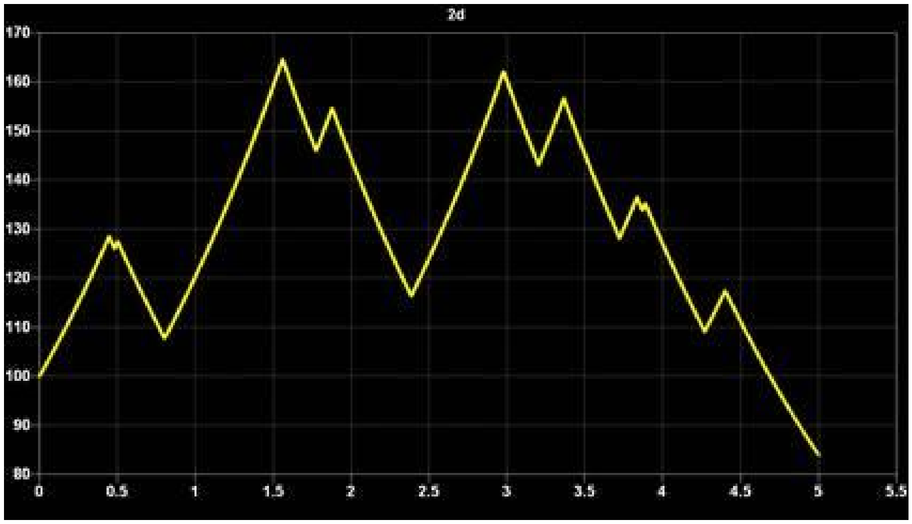
Basically the direction of "motion" changes at every time with probability λdt. Initial condition for dX(t) is as follows:

$dX(0) = vdt\ or - vdt\ with\ equal\ probability$

This means $\varepsilon_0 = \pm 1$ with probability ½ each.

Example of the process

Consider the process for $X(t)$. X(t) can be thought of as position of a particle with initial velocity ±V with equal probability and flipping its direction of motion at any time t with probability λdt.



Suppose initial value of $\epsilon_t = 1$. Lets say that the first flip happens after n time steps. The value of $X(ndt)$ will be $v * ndt$. The flip happens with probability λdt and the process $X(t)$ has speed $-v$. It will continue with $-v$ after each dt with probability $1 - \lambda dt$ till a flip happens again with probability λdt, and so on. A sample "path" is shown in the above figure.

Assume that initial value of S(t) is S0 = 100. Note that initial values for X(t) and dX(t) are specified in the model description above.

Given this information, answer the following questions:

1. Given values of v and λ, find the mean, std dev, skew and excess kurtosis of the distribution of X(t) at time t=1. Also find the expected value of the binary option with strike K and expiration time T=1 in this case.
2. Given values of v and λ, find the mean, std dev, skew and excess kurtosis of the distribution of X(t) at time t=1, given that S(t) < B for all 0 < t < 1. Also find the expected value of the binary option with strike K and expiration time T=1 in this case.
3. You are given stock_data.csv containing stock prices for a fictitious stock for 5 years. Find values of v and λ with which you can fit the data to the above process.

Inputs to part 1 and 2 are given in inputs.csv
[Note: You can use Monte Carlo technique to find the above values.]

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Input

Initial Values:

- S(0) = 100
- X(0) = 0
- dX(0) = ±v with equal probability

For part 1 and 2, the inputs will be v, λ, K, B. You have to ignore B for part 1. The inputs will be contained in the following files:

Filename	Fields	Available From
stock_data.csv	Time (yrs), Stock Price	9pm, Sep 28, 2013
input_A.csv	v, λ, K, B	9pm, Sep 28, 2013
input_B.csv	v, λ, K, B	6pm, Spe 30, 2013

A sample output file (sample_output_A.csv) will also be provided for first 6 inputs in input_A.csv.

Output

Zipped file to be submitted by **9pm, Sep 30, 2013** containing:

Filename*	**Fields**
ModelDocumentation	Format as described below
SourceCode	(zipped folder with all relevant files)
output_A.csv	ID, v, λ, K, B, Mean, Std Dev, Skewness, Kurtosis, Expected Value
output_B.csv	ID, v, λ, K, B, Mean, Std dev, Skewness, Kurtosis, Expected Value

*First line of **output_A.csv** should contain 3,v, λ where v and λ are the calculated parameters for part 3.

- ID should be 1 or 2 corresponding to output for parts 1 and 2 respectively. You have to submit an output for part 1 and 2 for each set of v, λ, K, B.

- Partial submissions, without one or more outputs will be accepted. Model documentation & SourceCode is mandatory.

Model Documentation Format

The model documentation should include the following parts:

1. Description of the approach
2. Analysis
3. Assumptions (if any)

Definitions

- Mean:
$$\bar{X} = \sum_{i=1}^{N} \frac{X_i}{N}$$
- Std Dev σ:
$$\sigma^2 = \sum_{i=1}^{N} \frac{(X_i - \bar{X})^2}{N}$$
- Skew:
$$Skewness = \sum_{i=1}^{N} \frac{(X_i - \bar{X})^3}{N\sigma^3}$$
- Kurtosis:
$$Kurtosis = \sum_{i} \frac{(X_i - \bar{X})^4}{N\sigma^4} - 3$$

References

[1]http://en.wikipedia.org/wiki/Derivative_(finance)

[2]http://en.wikipedia.org/wiki/Monte_Carlo_methods_in_finance