CS 325 HW 2 – 30pts

Problem 1: (6 pts) Give the asymptotic bounds for T(n) in each of the following recurrences. Make your bounds as tight as possible and justify your answers.

a) $T(n) = T(n-2) + n$

Solution

Lets consider n=(n-2), then our T(n) becomes

T(n) = T(n-4) + (n-2) + n

for n=(n-4), then our T(n) becomes

T(n) = T(n-6) + (n-4) + (n-2) + n

Therefore T(n) =T(n-2*k) + (n*k) –(2 * $\sum_{i=1}^{level-1} i$)

If we take the first part of the equation and get T(0), then

n-2*k = 0

therefore, Value of k can be n/2 . If we substitute this in the second part of the above equation n*k

n*k = n*(n/2) = $n^2/2$

Hence T(n) = θ(n^2)

b) $T(n) = 4T(n/2) + n^3$

Solution

Using Master theorem
a=4, b= 2
n^ $\log_b a = n^2$; f(n) = n^3

Case 3 : f(n) = Ω($n^{2+\epsilon}$) for ϵ = 1 and

$4(n/2)^3 <= C\ n^3$ for c=1/2

Therefore T(n) = θ(n^3)

c) $T(n) = 9T(n/3) + n^2$

Solution

Using Master theorem

a=9, b= 3

n^ $\log_b a = n^2$; $f(n) = n^2$

Case 2 : $f(n) = \theta(n^2)$

Therefore $T(n) = \theta(n^2 \lg n)$

Problem 2: (4 pts) How many times as a function of n (in θ form), does the following PHP function echo "Print"? Write a recurrence and solve it.

```
function Algo1( $n )  {
 if ($n > 1) {
Algo1($n/3);
Algo1($n/3);
Algo1($n/3);
Algo1($n/3);
for ($i = 1; $i <= $n; $i += 1)
{  echo " Print  ".$i." <br> ";     }
echo " <br>";  }
else {
return 1;
 } }
```

Solution :

Here the problem is divided into 4 parts of n/3 size. The Algo1 function is called recursively 4 times and in each call the echo is called n times.

Hence the Recurrence can be written as

$T(n) = 4T(n/3) + O(n)$

We can solve the above recurrence using Master Theorem

Using Master theorem

a=4, b= 3

n^ $\log_b a$ = $n^{\log_3 4}$ will be greater than 1

f(n) = n

Case 1 : f(n) = $O(n^{\log_b a - \epsilon})$ for ϵ = 1 and

Therefore T(n) = $\theta(n^{\log_3 4})$

Problem 3: (5 pts) The ternary search algorithm is a modification of the binary search algorithm that splits the input not into two sets of almost-equal sizes, but into three sets of sizes approximately onethird.

a) Write pseudo-code a recursive ternary search algorithm

Solution

Pseudocode for Ternary search :

```
Function int ternarySearch( low, high, x)
   If(high > = 1)
     {
        mid1 = low + ( high – low)/3;
        mid2 = high – (high – low)/3;
        If(a[mid1] == x)
                return mid1;
        If(a[mid1] == x)
                return mid1;
        If(x < a[Mid1] )
                return ternarySearch(low, mid1-1, x);
        else If(x > a[Mid2] )
                return ternarySearch(mid2+1, high, x);
        else return ternarySearch(mid1+1, mid2-1, x);
        }
return 0; }
```

b) Let T(n) denote the running time of ternary on an array of size n. Write a recurrence relation for T(n).
Solution - As we can see from the above pseudocode, the array of n size is divided into size n/3. That is the ternary search function is called only once based on the conditions in order to implement this. Also there is a constant time for the comparison.

Hence the Recurrence relation can be written as
$T(n) = n/3 + C$

Worst case it can be $2n/3 + C$

c) Solve the recurrence relation to obtain the asymptotic running time.

$T(n) = T(n/3) + C$

We can solve this using Master Theorem

$a=1, b = 3$

$n^{(\log_b a)} = n^0$; $f(n) = C$

$f(n) = \theta (n^{(\log_b a)}) = \theta(n^0) = \theta(1)$ -> Case 2

Therefore $T(n) = \theta (\lg n)$

Problem 4: (5 pts) The Mergesort3 algorithm is a variation of Mergesort that instead of splitting the list into two halves, splits the list into three thirds. Mergesort3 recursively sorts each third and then merges the thirds together into a sorted list by calling a function named Merge3.

a) Write pseudo-code for Mergesort3 and Merge3

Mergesort3

```
function mergeSort3(a[], low, high, f[])
    {
        // If array size is 1 then do nothing
        if (high - low < 2)
            return;
        // Splitting array into 3 parts
```

```
        int mid1 = low + ((high - low) / 3);
        int mid2 = low + 2 * ((high - low) / 3) + 1;
        // Sorting 3 arrays recursively
        mergeSort3(f, low, mid1, a);
        mergeSort3(f, mid1, mid2, a);
        mergeSort3(f, mid2, high, a);
        // Calling the merge function to merge the sorted arrays
        merge(f, low, mid1, mid2, high, a);
    }
```

Merge3

```
function merge(a[],low,m1,m2,high,f[])
{
int i = low, j = m1, p = m2, l = low;
        while ((i < m1) && (j < m2) && (p < high))
        {
                if (a[i] < (a[j])) {

                        if (a[i] < a[p])
                                f[l++] = a[i++];

                        else  f[l++] = a[p++];
        }
        else   {

                if (a[j] < a[p]))
                        f[l++] = a[j++];

                        else f[l++] = a[p++];

                }

        }

        // comparing first and second ranges
```

```
while ((i < m1) && (j < m2))

{
        if (a[i]<a[j]))
                f[l++] = a[i++];

        else    f[l++] = a[j++];

}

// comparing second and third ranges

while ((j < m2) && (p < high))

{
        if (a[j]<a[p]))
                f[l++] = a[j++];

        else    f[l++] = a[p++];

}

// comparing first and third range

while ((i < m1) && (p < high))

{
        if (a[i]<a[p]))
                f[l++] = a[i++];
        else
                f[l++] = a[p++];

}

// copying from first range

while (i < m1)
        f[l++] = a[i++];

// copying from second range

while (j < m2)
        f[l++] = a[j++];
```

```
        // copying from the third range

        while (p < high)
                f[l++] = a[p++];

}
```

b) Let T(n) denote the running time of Mergesort3 on an array of size n. Write a recurrence relation for T(n).

As we can see from the pseudo code, we can say that the array is splitted into 3 parts of size (n/3). And the merge function takes n times a constant.

Therefore the recurrence for merge 3 sort can be written as below :

$T(n) = 3T(n/3) + Cn$

c) Solve the recurrence relation to obtain the asymptotic running time.

$T(n) = 3T(n/3) + Cn$

We can solve the above recurrence relation using Master Theorem

a=3, b = 3

$n^{(\log_b a)} = n^1$; $f(n) = Cn$

$f(n) = \theta (n^{(\log_b a)}) = \theta(n^1) = \theta(n)$ -> Case 2

Therefore $T(n) = \theta (n \lg n)$

Problem 5: (10 pts)

a) Implement the Mergesort3 algorithm to sort an array/vector of integers. You must implement the algorithm in same language you used in homework 1. Name the program "Mergesort3". Your program should be able to read inputs from a file called "data.txt" where the first value of each line is the number of integers that need to be sorted, followed by the integers. Example values for data.txt: 4 19 2 5 11 8 1 2 3 4 5 6 1 2 The output will be written to files called "merge3.txt". For the above example the output would be: 2 5 11 19 1 1 2 2 3 4 5 6

Solution – Code has been submitted in Teach and is running in Flip Server.

Path in Flip Server – jayapats/CS325/HW2/Merge3File.java

b) Modify code- Now that you have verified that your code runs correctly using the data.txt input file, you can modify the code to collect running time data. Instead of reading arrays from the file data.txt and sorting, you will now generate arrays of size n containing random integer values from 0 to 10,000 to sort. Use the system clock to record the running times of each algorithm for ten different values of n for example: n = 5000, 10000, 15000, 20,000, …, 50,000. You may need to modify the values of n if an algorithm runs too fast or too slow to collect the running time data (do not collect times over a minute). Output the array size n and time to the terminal. Name these new program merge3Time.

Solution – Code has been submitted in Teach and is running in Flip Server.

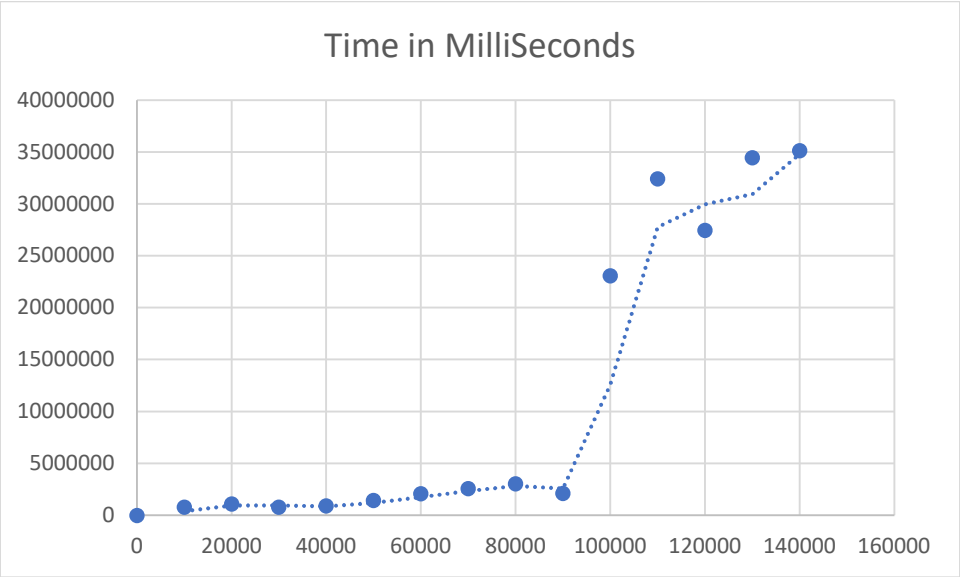
Path in Flip Server – jayapats/CS325/HW2/Merge3Time.java

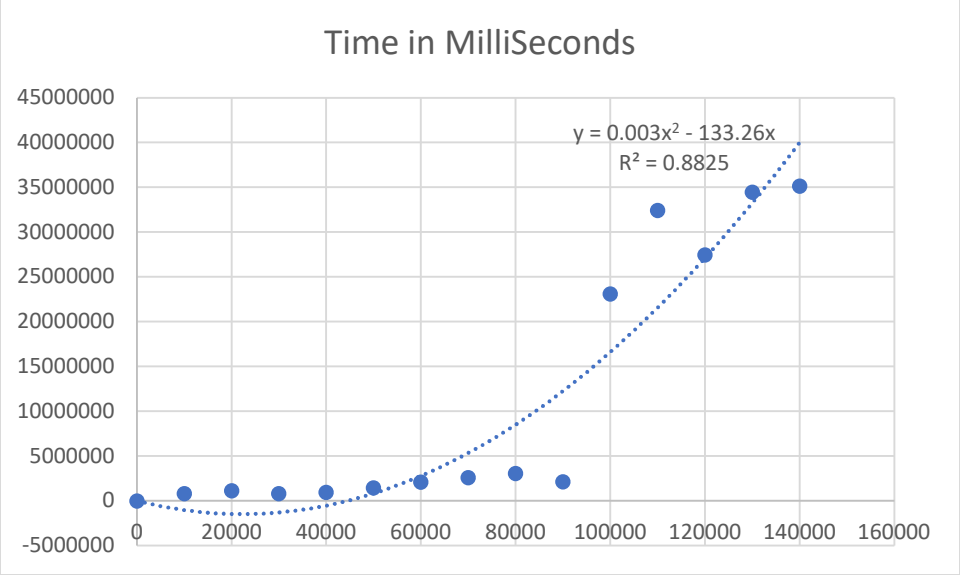
c) Collect running times - Collect your timing data on the engineering server. You will need at least eight values of t (time) greater than 0. Create a table of running times for each algorithm.

Value of N	Excecution time			
	1	2	3	Average
0	760	0	0	253.3333
10000	1410000	510000	510000	810000
20000	1020000	1020000	1320000	1120000
30000	830000	730000	830000	796666.7
40000	940000	940000	940000	940000
50000	1450000	1350000	1550000	1450000
60000	2060000	2060000	2160000	2093333
70000	2570000	2470000	2770000	2603333
80000	2880000	3280000	2980000	3046667
90000	2090000	2190000	2090000	2123333
100000	23100000	23100000	23100000	23100000
110000	36110000	35110000	26110000	32443333
120000	27120000	28120000	27120000	27453333
130000	31130000	30130000	42130000	34463333
140000	39140000	33140000	33140000	35140000

c) Plot data and fit a curve - For each algorithm plot the running time data you collected on a graph with n on the x-axis and time on the y-axis. You may use Excel, Matlab, R or any

other software. What type of curve best fits each data set? Give the equation of the curves that best "fits" the data and draw that curves on the graphs.



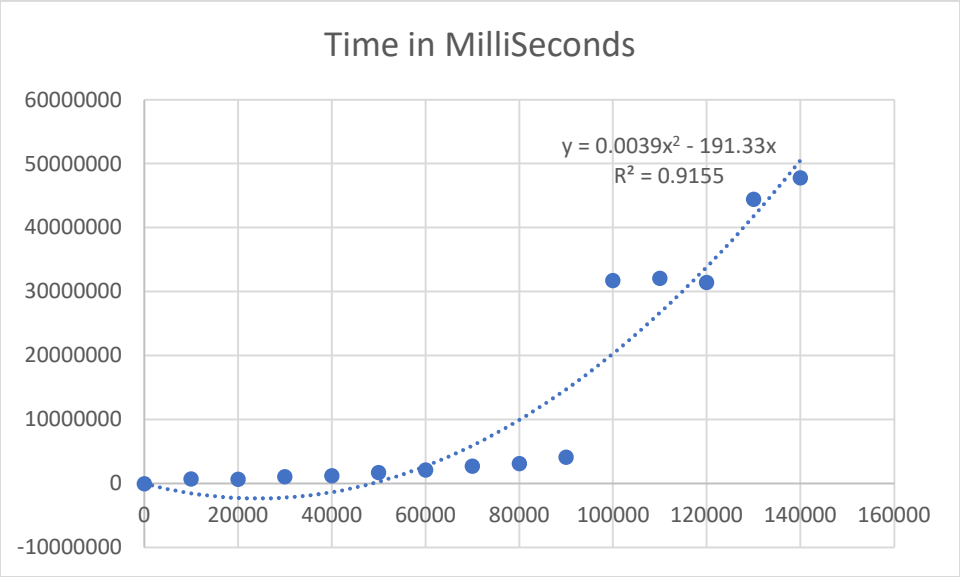


d) Compare - Plot the data from Mergesort3 and Mergesort (from HW 1) together on a combined graph. Which algorithm runs faster? How does your experimental running times compare to the theoretical running times of the algorithms?

Merge 2 way Run time :

Value of N	Excecution time			
	1	2	3	Average
0	800	0	0	266.6667
10000	910000	910000	410000	743333.3
20000	720000	720000	620000	686666.7
30000	1330000	1030000	930000	1096667
40000	1240000	1440000	1040000	1240000
50000	1750000	1750000	1750000	1750000
60000	2160000	2160000	2160000	2160000
70000	2770000	2870000	2570000	2736667
80000	2880000	3280000	3280000	3146667
90000	3490000	4590000	4390000	4156667
100000	39100000	28100000	28100000	31766667
110000	28110000	29110000	39110000	32110000
120000	31120000	32120000	31120000	31453333
130000	44130000	43130000	46130000	44463333
140000	45140000	50140000	48140000	47806667

Merge 2 way sort Graph :



Merge3 Runtime recurrence:

$T(n) = 3T(n/3) + Cn$

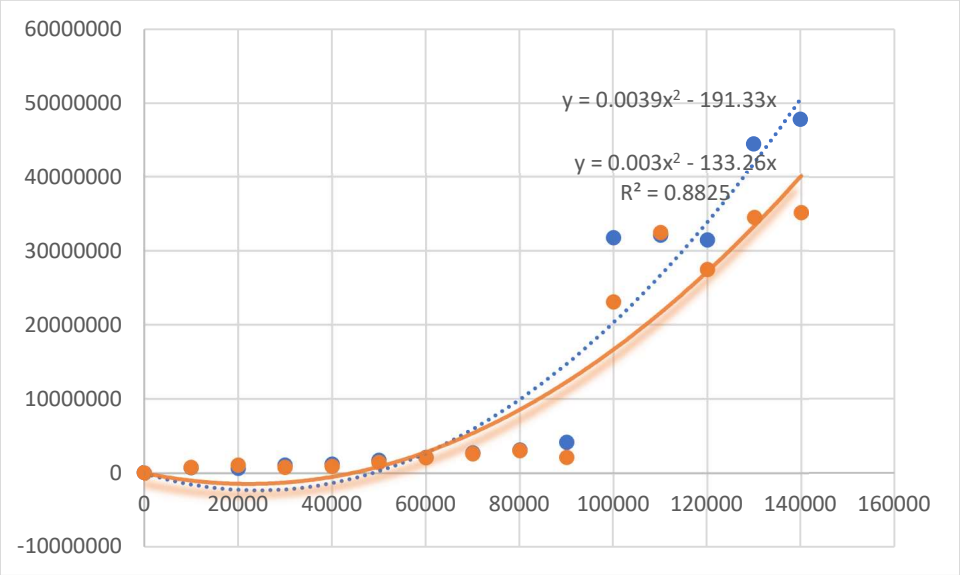
Merge 2 Runtime recurrence:

$T(n) = 2T(n/2) + Cn$

Combined raph :

Orange – Merge 3 Algorithm

Blue – 2 way merge Algorithm



Comparing the normal merge and merge 3, both runs for the same runtime. Both fits the run time data perfectly.