

CS 325 - Homework 5

1. (7 points) Let X and Y be two decision problems. Suppose we know that X reduces to Y in polynomial time. Which of the following statements are true? Explain

- a. If Y is NP-complete then so is X .
- b. If X is NP-complete then so is Y .
- c. If Y is NP-complete and X is in NP then X is NP-complete.
- d. If X is NP-complete and Y is in NP then Y is NP-complete.
- e. If X is in P, then Y is in P.
- f. If Y is in P, then X is in P.
- g. X and Y can't both be in NP.

2. (8 points) A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. Show that $\text{HAM-PATH} = \{ (G, u, v) : \text{there is a Hamiltonian path from } u \text{ to } v \text{ in } G \}$ is NP-complete. You may use the fact that HAM-CYCLE is NP-complete.

3. (15 points) K-COLOR. Given a graph $G = (V, E)$, a k -coloring is a function $c: V \rightarrow \{1, 2, \dots, k\}$ such that $c(u) \neq c(v)$ for every edge $(u, v) \in E$. In other words the number 1, 2, ..., k represent the k colors and adjacent vertices must have different colors. The decision problems K-COLOR asks if a graph can be colored with at most K colors.

- a. The 2-COLOR decision problem is in P. Describe an efficient algorithm to determine if a graph has a 2-coloring. What is the running time of your algorithm?
- b. The 3-COLOR decision problem is NP-complete by using a reduction from SAT. Use the fact that 3-COLOR is NP-complete to prove that 4-COLOR is NP-complete.