**NP-Complete Proof Template:**

Prove that Q is in NP-Complete

1) Show that $Q \in NP$.

Give a polynomial time algorithm that verifies an instance of Q in polynomial time. In other words if I give you an "answer" can you verify it polynomial time.

2) Show that $R \leq_P Q$ for some $R \in$ NP-Complete

a. Pick an instance, R, of your favorite NP-Complete problem. Usually the R you select has a structure similar to Q.
b. Show a polynomial algorithm to transform an arbitrary instance x of R into an instance of x' of Q.
c. Prove that R(x) = yes if and only if Q(x') = yes. That is
    a. If x is a "yes" solution of R then x' is a "yes" solution of Q. And
    b. If x' is a "yes" solution of Q then x is a "yes" solution of R.

If both 1) and 2) are true then Q is in NP-Complete

**NP-Complete Proof - Prove** that **ALMOST-HP** is in NP-Complete

**ALMOST-HP** is the problem of, given an undirected graph with n vertices, determining if that graph has a simple path that visits at least n-1 vertices.

1) Show that **ALMOST-HP** $\in$ NP.

Given a graph G=(V,E) with n vertices and a "certificate solution" ALMOST-HP, p =< $v_{(1)}$, $v_{(2)}$, … $v_{(n-1)}$>, we can verify in polynomial time that p is a simple path in G with n-1 vertices. We must check the adjacency list of each $v_{(i)}$ in p to verify that $v_{(i+1)}$ is adjacent in the graph G. Then we must also verify that there are (n-1) distinct vertices in p. This takes time at most O(V+E) = O($V^2$) which is polynomial time.

2) Show that R $\leq_P$ **ALMOST-HP** for some R$\in$NP-Complete

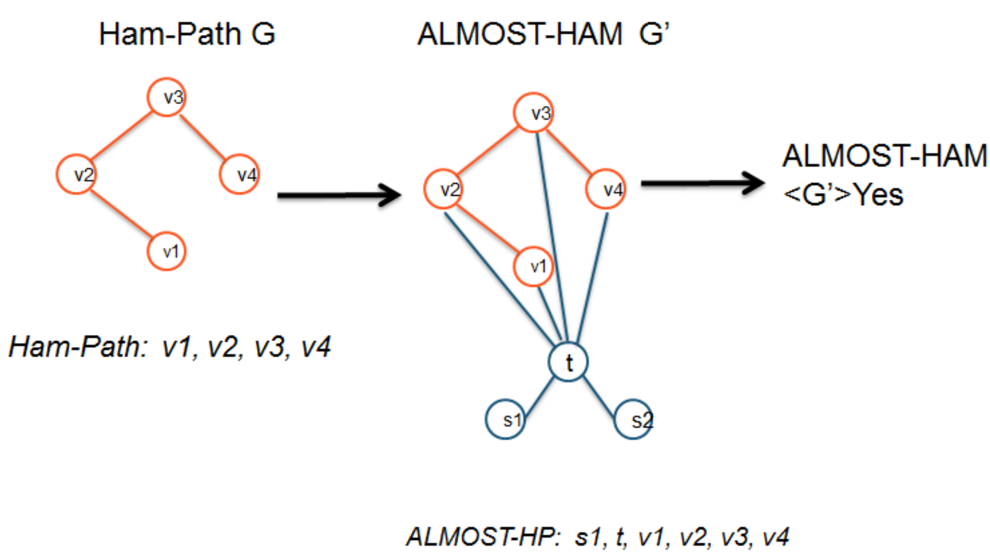
a. Select R=**HAM-Path** because it has a similar structure to **ALMOST-HP**. We know Ham-Path is in NP.

Show that HAM-Path $\leq_P$ **ALMOST-HP**

b. Show a polynomial algorithm to transform **HAM-Path** into an instance of **ALMOST-HP**.

Given a graph G we produce a new graph G' such that G has a Hamiltonian path if and only if G' has an **ALMOST-HP** path. G' is created by adding three new vertices $s_1$, $s_2$, and t with edges from both $s_1$ and $s_2$ to t and edges from t to every vertex in G. This transformation of G into G' can be done in polynomial time by adding 3 vertices and n+2 edges.

Below is an example of the transformation.



c. Prove you are able to "solve" HAM-Path by using ALMOST-HP. Therefore ALMOST-HP is as hard as Ham-Path.

   Show that the graph G has a HAM-Path if and only if graph G' has an ALMOST-HP path

   i) Now if G has a HAM-Path < $v_{(1)}, \ldots, v_{(n)}$>. G' has an ALMOST-HP path: <$s_1$, t, $v_{(1)}, \ldots, v_{(n)}$>.

   ii) If G' has an ALMOST-HP then one of the endpoints must be either $s_1$ or $s_2$, and the next vertex must be t and the path must then go into G, **skipping $s_2$ or $s_1$** (the one not already visited). Since we already have one vertex skipped, the rest of the path must visit all the vertices in G, so removing the first two edges from it produces a HAM-Path in G.

d. Since HAM-Path is in NP-Complete then **ALMOST-HP** must be in NP-Hard.

**Since 1) and 2) hold. ALMOST-HP is NP-Complete**