Problem 1a:

$$\textsc{Not-T}\ \dfrac{\textsc{If-F}\ \dfrac{\textsc{Not-T}\ \dfrac{\texttt{true} \Downarrow \texttt{true}}{\texttt{not true} \Downarrow \texttt{false}} \qquad \textsc{Not-F}\ \dfrac{\texttt{false} \Downarrow \texttt{false}}{\texttt{not false} \Downarrow \texttt{true}}}{\texttt{if (not true) (not true) (not false)} \Downarrow \texttt{true}}}{\texttt{not (if (not true) (not true) (not false))} \Downarrow \texttt{false}}$$

Problem 1b:

$$\textsc{Trans}\ \dfrac{S_1 \qquad \textsc{Trans}\ \dfrac{S_2 \qquad \textsc{Trans}\ \dfrac{S_3 \qquad \textsc{Trans}\ \dfrac{\texttt{not true} \mapsto \texttt{false} \qquad \textsc{Refl}\ \dfrac{}{\texttt{false} \mapsto^* \texttt{false}}}{\texttt{not true} \mapsto^* \texttt{false}}}{\texttt{not (not false)} \mapsto^* \texttt{false}}}{\texttt{not (if false (not true) (not false))} \mapsto^* \texttt{false}}}{\texttt{not (if (not true) (not true) (not false))} \mapsto^* \texttt{false}}$$

$$S_1 = \textsc{Not}\ \dfrac{\textsc{If}\ \dfrac{\texttt{not true} \mapsto \texttt{false}}{\texttt{if (not true) (not true) (not false)} \mapsto \texttt{if false (not true) (not false)}}}{\texttt{not (if (not true) (not true) (not false))} \mapsto \texttt{not (if false (not true) (not false))}}$$

$$S_2 = \textsc{Not}\ \dfrac{\texttt{if false (not true) (not false)} \mapsto \texttt{not false}}{\texttt{not (if false (not true) (not false))} \mapsto \texttt{not (not false)}}$$

$$S_3 = \textsc{Not}\ \dfrac{\texttt{not false} \mapsto \texttt{true}}{\texttt{not (not false)} \mapsto \texttt{not true}}$$

Problem 1c:

$$\begin{aligned}
& \texttt{not (if (}\underline{\texttt{not true}}\texttt{) (not true) (not false))} \\
\mapsto\ & \texttt{not (}\underline{\texttt{if false (not true) (not false)}}\texttt{)} \\
\mapsto\ & \texttt{not (}\underline{\texttt{not false}}\texttt{)} \\
\mapsto\ & \texttt{not true} \\
\mapsto\ & \texttt{false}
\end{aligned}$$

Problem 2:

Claim: $\dfrac{e \Downarrow \texttt{false}}{\texttt{not (if (not } e\texttt{) } e \texttt{ (not } e\texttt{)) } \Downarrow \texttt{true}}$

Proof: $\textsc{Not-F}\ \dfrac{\textsc{If-T}\ \dfrac{\textsc{Not-F}\ \dfrac{e \Downarrow \texttt{false}}{\texttt{not } e \Downarrow \texttt{true}} \qquad e \Downarrow \texttt{false}}{\texttt{if (not } e\texttt{) } e \texttt{ (not } e\texttt{) } \Downarrow \texttt{false}}}{\texttt{not (if (not } e\texttt{) } e \texttt{ (not } e\texttt{)) } \Downarrow \texttt{true}}$

Problem 3a:

$$\textsc{And-T}\ \frac{e_1 \Downarrow \texttt{true} \qquad e_2 \Downarrow \texttt{true}}{\texttt{and}\ e_1\ e_2 \Downarrow \texttt{true}} \qquad \textsc{And-F-L}\ \frac{e_1 \Downarrow \texttt{false}}{\texttt{and}\ e_1\ e_2 \Downarrow \texttt{false}} \qquad \textsc{And-F-R}\ \frac{e_2 \Downarrow \texttt{false}}{\texttt{and}\ e_1\ e_2 \Downarrow \texttt{false}}$$

— OR —

$$\textsc{And-T}\ \frac{e_1 \Downarrow \texttt{true} \qquad e_2 \Downarrow b}{\texttt{and}\ e_1\ e_2 \Downarrow b} \qquad \textsc{And-F}\ \frac{e_1 \Downarrow \texttt{false}}{\texttt{and}\ e_1\ e_2 \Downarrow \texttt{false}}$$

Problem 3b:

$$\texttt{and true}\ e \mapsto e \qquad \texttt{and false}\ e \mapsto \texttt{false} \qquad \textsc{And}\ \frac{e_1 \mapsto e_1'}{\texttt{and}\ e_1\ e_2 \mapsto \texttt{and}\ e_1'\ e_2}$$