CS 321 HW 6 – 25 points

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1. Construct an NPDA M such that L(M) = L(G) where G = (V, T, S, P) with V = { S, A, B, C}, T = {a, b, c}

and P = {

S → aaAB | SBC

A → aa | aCC

B - > bb | BB

C → c

}

Let M be the NPDA, M= (Q, Σ, Γ, δ, q0, z, F)

where Q is a finite set of states,

$Q = \{q_0, q_1, q_2\}$

Σ is the input alphabet,

Σ = {a, b, c}

Γ is a finite set of symbols called the stack

alphabet,

Γ = {a, b, Z}

$\delta : Q \times (\Sigma \cup \{\lambda\}) \times \Gamma$

$\rightarrow$ set of finite subsets of Q

$\times \Gamma *$ is the transition

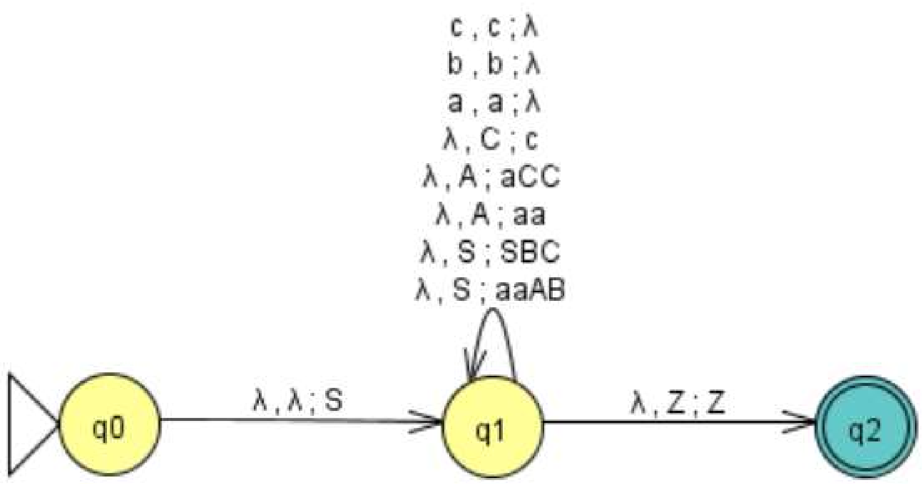
function,

$q_0 \in Q$ is the initial state of the control unit,

$Z \in \Gamma$ is the stack start symbol,

$F \subseteq Q$ is the set of final states

$F = \{q_2\}$



Another way:

We can reduce the production as below

S → TD | SU

A → aa | aC

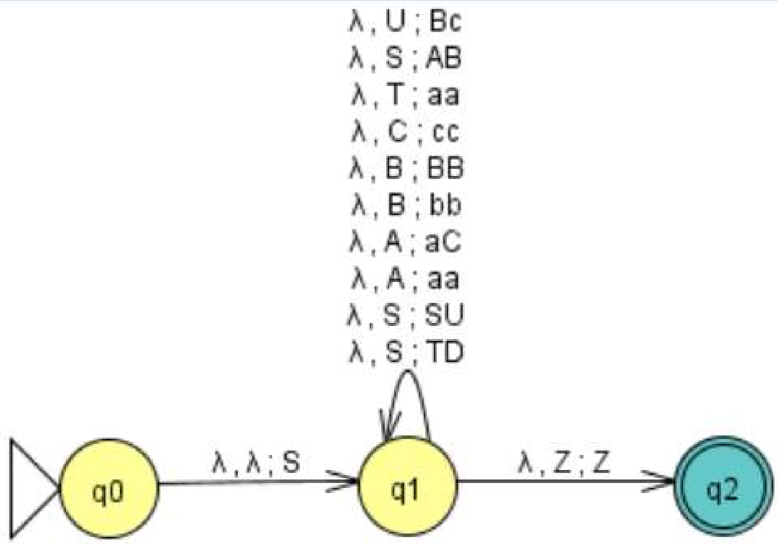
B - > bb | BB

C -> cc

T -> aa

D -> AB

U -> Bc



2. Show that the family of context-free languages is closed under reversal.

Let G be the Context free grammar for a language L. Let L^R be the language for the reversal of L and let G' be the grammar for L^R. We construct the G' from G as below :

For every production V -> x in G, we add the production V -> x^R in G' where x is a variable and x is a set of variables or terminals. That is we are reversing the body of every productions in L to generate the grammar forL^R.

We could see that a string w is generated in G if and only if the string w^R is generated by grammar G'.

Hence G' generates the language L^R and thus it is context free.

Example:

Let G be a CFL and has production as below

S -> aSb | ab

Then the reversal of L(G) is as follows :

S -> bSa | ba

For problems 2-5, use the pumping lemma for context-free languages to prove that L is not a CFL.

3. L1 = {$a^n b^{m}$; n= 2^m}.

Assume for a contradiction that L is context free.

Since L is context free and infinite, we can appl the pumping Lemma.

Let us pick the string w= $a^{2^m} b^m$, which is in the language.

We can write w = uvxyz with lengths |vxy| <=m and |vy|>=1

According to pumping lemma, $uv^ixy^iz \in L$ for all i >= 0

We examine all possible locations of string vxy in w

Case 1 - vy is a^k

For i=2, w_2= $a^{2^m+k} b^m$

$2^m < 2^m + k$

$< 2^m + m$

$< 2^{m+1}$

-> Does not belong to the language. It's a contradiction.

Case 2 - vy is b^k

For i=2, w_2= $a^{2^m} b^{m+k}$

$2^{m+k} = 2^m * 2^k$

$> 2^m$

-> Does not belong to the language. It's a contradiction.

Case 3 – v = a^k and y = b^p

For i=2, w_2=$a^{2^m+k} b^{m+p}$

2^m+k < 2^{m+1}

<= 2^{m+p}

-> Does not belong to the language. It's a contradiction.

Case 4 -

Overlapping either v or y to contain two different characters will generate strings out of sequence when pumped up.

Since all the cases are a contradiction, our initial assumption is wrong and L is not a CFL.

4. L2 = { $a^n b^n c^j$: n ≤ j }.

Assume for a contradiction that L is context free.

Since L is context free and infinite, we can appl the pumping Lemma.

Let us pick the string w= $a^m b^m c^m$, which is in the language.

We can write w = uvxyz with lengths |vxy| <=m and |vy|>=1

According to pumping lemma, $uv^i xy^i z \in L$ for all i >= 0

We examine all possible locations of string vxy in w

Case 1 - vy is a^k

For i=2, w_2= $a^{m+k} b^m c^m$

Number of a's and b's are not equal.

-> Does not belong to the language. It's a contradiction.

Case 2 - vy is b^k

For i=2, w_2= $a^m b^{m+k} c^m$

Number of a's and b's are not equal.

-> Does not belong to the language. It's a contradiction.

Case 3 - vy is c^k

For i=0, w_0= $a^m b^m c^{m+k}$

Number of a's and b's are greater than the number of c's

-> Does not belong to the language. It's a contradiction.

Case 4 – v = a^k and y = b^p

For i=2, w_2=$a^{m+k} b^{m+p} c^m$

- For k = p, w_2 would have number of a's and b's greater than the number of c's
- For k ≠ p, then number of a's wont be equal to number of b's.

-> Does not belong to the language. It's a contradiction.

Case 5 – v = b^k and y = c^p

For i=2, w_2=$a^m b^{m+k} c^{m+p}$

w_2 would have number of a's not equal to b's.

-> Does not belong to the language. It's a contradiction.

Case 6 -

Overlapping either v or y to contain two different characters will generate strings out of sequence when pumped up.

Since all the cases are a contradiction, our initial assumption is wrong, and L is not a CFL.

5. L3 = { w: w ∈{a,b,c}* and na(w) < nb(w) < nc(w) }

Assume for a contradiction that L is context free.

Since L is context free and infinite, we can appl the pumping Lemma.

Let us pick the string w= $a^{m}b^{m+1}c^{m+2}$, which is in the language.

We can write w = uvxyz with lengths |vxy| <=m and |vy|>=1

According to pumping lemma, $uv^{i}xy^{i}z \in L$ for all i >= 0

We examine all possible locations of string vxy in w

Case 1 - vy is a^{k}

For i=2, $w_2 = a^{m+k}b^{m+1}c^{m+2}$

Number of a's greater than or equal to number of b's.

-> Does not belong to the language. It's a contradiction.

Case 2 - vy is b^{k}

For i=2, $w_2 = a^{m}b^{m+1+k}c^{m+2}$

Number of b's greater than or equal to number of c's.

-> Does not belong to the language. It's a contradiction.

Case 3 - vy is c^{k}

For i=0, $w_0 = a^{m}b^{m+1}c^{m+2-k}$

Number of c's is lesser than or equal to number of b's.

-> Does not belong to the language. It's a contradiction.

Case 4 – v = a^{k} and y = b^{p}

k,p > = 1, then pumping up $w_2 = a^{m+k}b^{m+1+p}c^{m+2}$

Number of b's greater than or equal to number of c's.

-> Does not belong to the language. It's a contradiction.

Case 5 – v = b^k and y = c^p

k,p > = 1, then pumping down

For i=0, w_0= $a^m b^{m+1-k} c^{m+2-p}$

Number of b's less than or equal to number of a's.

-> Does not belong to the language. It's a contradiction.

Case 6 – v = $a^k b^p$ y = b^j

k,p,j > = 1, then pumping up w_2

Number of b's greater than or equal to number of c's.

-> Does not belong to the language. It's a contradiction.

Case 7 – v = a^k y = $b^p c^j$

k,p,j > = 1, then pumping down w_0

Number of b's less than or equal to number of a's.

-> Does not belong to the language. It's a contradiction.

Case 8 – v = $b^k c^p$ y = c^j

k,p,j > = 1, then pumping down, for w_0

Number of b's less than or equal to number of a's.

-> Does not belong to the language. It's a contradiction.

Case 8 – v = b^k y = $b^p c^j$

k,p,j > = 1, then pumping down, for w_0

Number of b's less than or equal to number of a's.

-> Does not belong to the language. It's a contradiction.

Since all the cases are a contradiction, our initial assumption is wrong, and L is not a CFL.