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CS321: Introduction to Theory of Computation
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CS 321 HW3

Submit a pdf in Canvas. Use a word processor and/or text editor. (25 pts, 5 pts each)

Determine whether or not the following languages are regular. If the language is regular then give an NFA or regular expression for the language. Otherwise, use the pumping lemma for regular languages or closure properties to prove the language is not regular.

1) $L = \{ a^n b^k : k \leq n \leq 2k\}$

Solution

We will show L is not Regular by using the pumping lemma.

- For a contradiction, assume L is regular.
- Since L is regular, there exists a pumping length m > 0
- Lets consider the string $w = a^{2m} b^m$ then $w \in L$
  $| w | \geq m$

  Then from pumping Lemma there exists $x,y,z \in \sum^*$ such that w = xyz with
  $|xy| \leq m$ -----------------------1
  and $|y| \geq 1$ -------------------2

  From 1 & 2 above , $y = a^j$ for $1 \leq j \leq m$

  Need to find an i such that $w_i = xy^iz$ is in L by the Pumping Lemma :

  When y is pumped up,
  $w_2 = xy^2z = xyyz = a^{2m+j} b^m \in L$ according to pumping lemma

  When J = 1, $a^{2m+1} b^m$
  2m+1 < 2m -> Contradicts the condition

  When J = m, $a^{2m+1} b^m$
  3m < 2m -> again contradicts the condition

The above is a contradiction of Pumping Lemma, Thus L cannot be a regular languag

2) $L = \{ b^n a^k : n > 0, k > 0 \}$ ▯ $\{ b^n a^k : k > 0, n > 0\}$

Solution :

The above given language is regular as it accepts strings of type

{ba,bba,baa….ab,aab,abb,..}

The regular expression for the given language is

r = (bb*aa* + aa*bb*)

3) $L = \{ a^n : n=3k \text{ for some } k \geq 0\}$

Solution :

The above given language is regular as it accepts strings of type

{λ, aaa,aaaaaa…}

The regular expression for the given language is

r = (aaa)*

4) $L = \{ a^n : n=k^3 \text{ for some } k \geq 0\}$

Solution :

We will show L is not Regular by using the pumping lemma.

- For a contradiction, assume L is regular.
- Since L is regular, there exists a pumping length $m > 0$
- Lets consider the string $w = a^{m^3}$ then $w \in L$
  $| w | \geq m$

  Then from pumping Lemma there exists $x,y,z \in \sum^*$ such that $w = xyz$ with
  $|xy| \leq m$ -----------------------1
  and $|y| \geq 1$ -------------------2

  From 1 & 2 above , $y = a^j$ for $1 \leq j \leq m$

  Need to find an i such that $w_i = xy^iz$ is in L by the Pumping Lemma :

When y is pumped up,

$w_2 = xy^2z = xyyz = a^{m^3+j} \in L$ according to pumping lemma

When J = 1, a^{m^3+1}

$m^3 + 1 < (m + 1)^3$

$< m^3 + 3m^2 + 3m + 1$

Hence $m^3 + 1 \neq (m + 1)^3$ -> Not a perfect cube, contradicts the condition and pumping lemma.

Also when J goes to maximum value , that is j = m, it becomes a^{m^3+m}

$m^3 + m < (m + 1)^3$

$m(m^2 + 1) < (m + 1)^3$

$< m^3 + 3m^2 + 3m + 1$

Hence $m^3 + m \neq (m + 1)^3$ -> Not a perfect cube, contradicts the condition and pumping lemma.

The above is a contradiction of Pumping Lemma, Thus L cannot be a regular language.

5) L = { w : $n^a(w) > nb(w)$, w ϵ {a, b}* }

Solution :

We will show L is not Regular by using the pumping lemma.

- For a contradiction, assume L is regular.
- Since L is regular, there exists a pumping length m > 0
- Lets consider the string w= $b^m a^{m+1}$ then w ∈ L

  | w | ≥ m

  Then from pumping Lemma there exists x,y,z ∈ ∑* such that w = xyz with

  |xy| ≤ m -----------------------1

  and |y| ≥ 1 -------------------2

  From 1 & 2 above , y = b^j for 1 ≤ j ≤ m

  Need to find an i such that $w_i = xy^iz$ is in L by the Pumping Lemma :

  When y is pumped up,

$w_2 = xy^2z = xyyz = b^{m+j}a^{m+1} \in L$ according to pumping lemma

Lets consider $w_3 = b^{m+2j}\, a^{m+1}$
When J = 1, $b^{m+2}\, a^{m+1}$
$m + 2 > m+1$, number of a's is less than number of b's
contradicts the condition and pumping lemma.

Also when J goes to maximum value , that is j = m, it becomes $b^{3m}\, a^{m+1}$
$3m > m+1$, number of a's is less than number of b's
contradicts the condition and pumping lemma.

Thus the above is a contradiction of Pumping Lemma, Thus L cannot be a regular language.