CS 321 HW 7

1. (10 pts) Design single-tape Turing machines that accept the following languages using JFLAP

a) L2 = { w : na(w) = nb(w) : w ϵ {a, b}+ }.

Let M be the Turing Machine for the given language.

$M = (Q, \Sigma, \Gamma, \delta, q_0, \quad , F) =$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9\}$

q_0 - Start state

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, X, Y, A, \square\}$,

$\mathrm{F} = \{q_9\}$

δ transition is *as per the* below *transition diagram*,

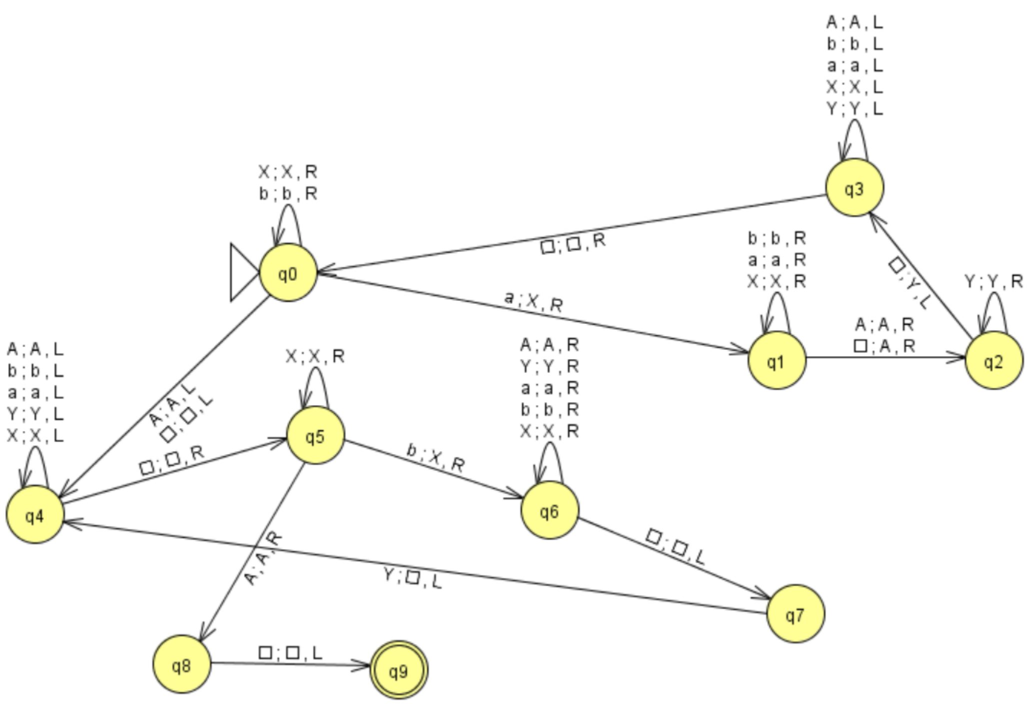


Table Text Size

Input	Result
abbaba	Accept
aaabbb	Accept
aaaaaabbbbbb	Accept
ba	Accept
a	Reject
abb	Reject
bbaab	Reject

b) L3 = {ww : w ϵ {a, b}+ }.

Let M be the Turing Machine for the given language.

$M = (Q, \Sigma, \Gamma, \delta, q_0, \quad , F) =$

$Q \ =$

$\{q_0, q_1, q_2, q_{3,} \ q_4, q_5, q_6, q_7, q_8, q_{9,}, q_{10,}, q_{11,}, q_{12,}, q_{13,}, q_{14,}, q_{15,}, q_{16,}, q_{17,}, q_{18,}, q_{19,}, q_{20,}, q_{21,}, q_{22,}, q_{23,}\}$

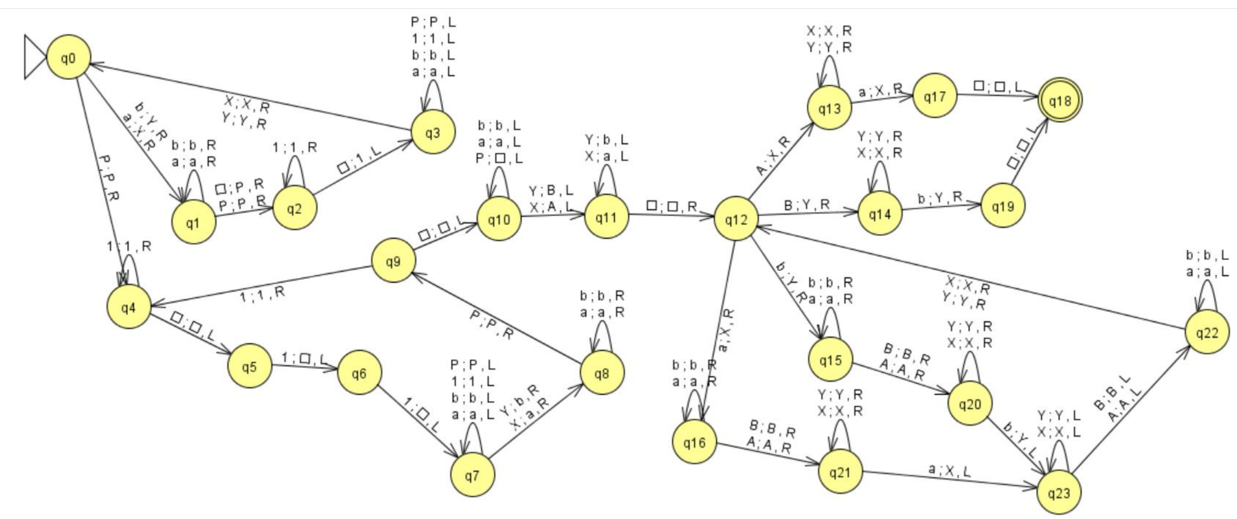
q_0 - Start state

$\Sigma \ = \ \{a, b\}$

$\Gamma \ = \ \{a, b, X, Y, A, \mathrm{B}, \mathrm{P}, 1, \square\},$

$\mathrm{F} \ = \ \{q_{18}\}$

δ transition is *as per the* below *transition diagram*



Input	Result
abaaba	Accept
bbbbbb	Accept
aabbaabb	Accept
a	Reject
aabb	Reject
bbb	Reject

2. (10 pts) Design Turing Machines using JFLAP to compute the following functions for x and y positive integers represented in unary. The value f(x) represented in unary should be on the tape surrounded by blanks after the calculation.

a) $f(x) = \{ x - y,$

$x > y$ **0, otherwise**

Let M be the Turing Machine for the given language.

$M = (Q, \Sigma, \Gamma, \delta, q_0, \quad , F) =$

$Q =$

$\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}, q_{13}\}$

q_0 - Start state

$\Sigma = \{1, \text{-}\}$

$\Gamma = \{0, 1, \text{-}\, X, Y, \square\},$

$\mathrm{F} = \{q_7, q_{11}\}$

δ transition is *as per the* below *transition diagram*

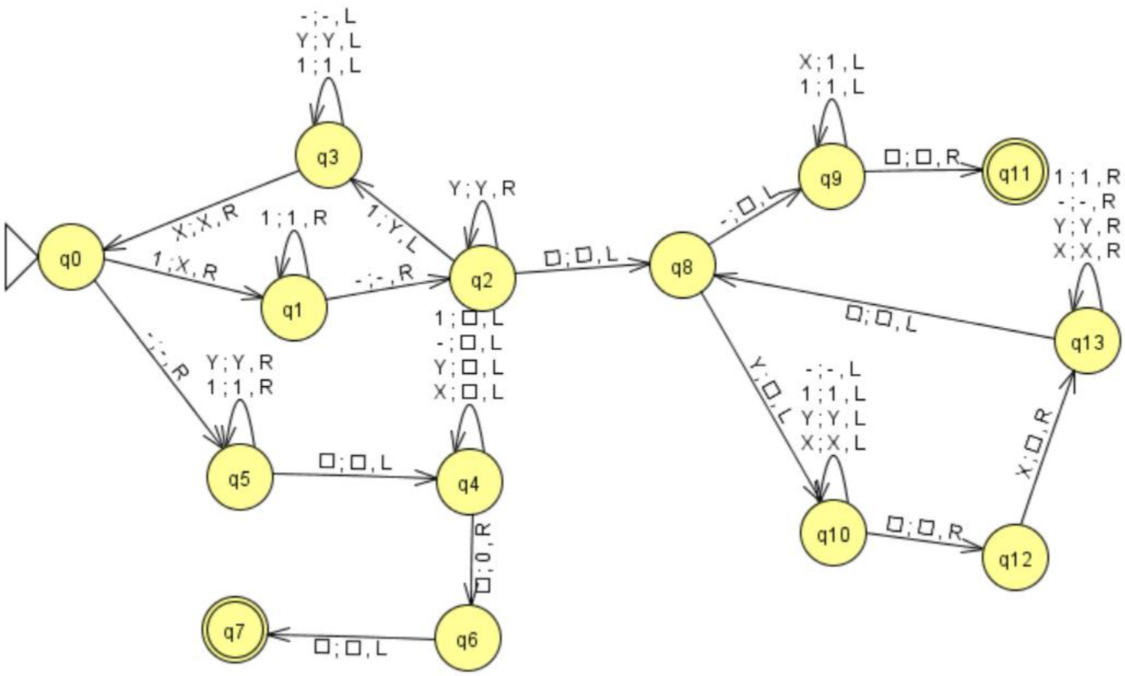


Table Text Size

Input	Output	Result
11-1	1	Accept
1-1	0	Accept
111-1	11	Accept
1-1111	0	Accept
1111-11	11	Accept

b) $f(x)$ **=** $x \bmod 5$

Let M be the Turing Machine for the given language.

$M = (Q, \Sigma, \Gamma, \delta, q_0, \quad , F) =$

$Q \ =$

$\{q_0, q_1, q_2, q_{3,}\ q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\}$

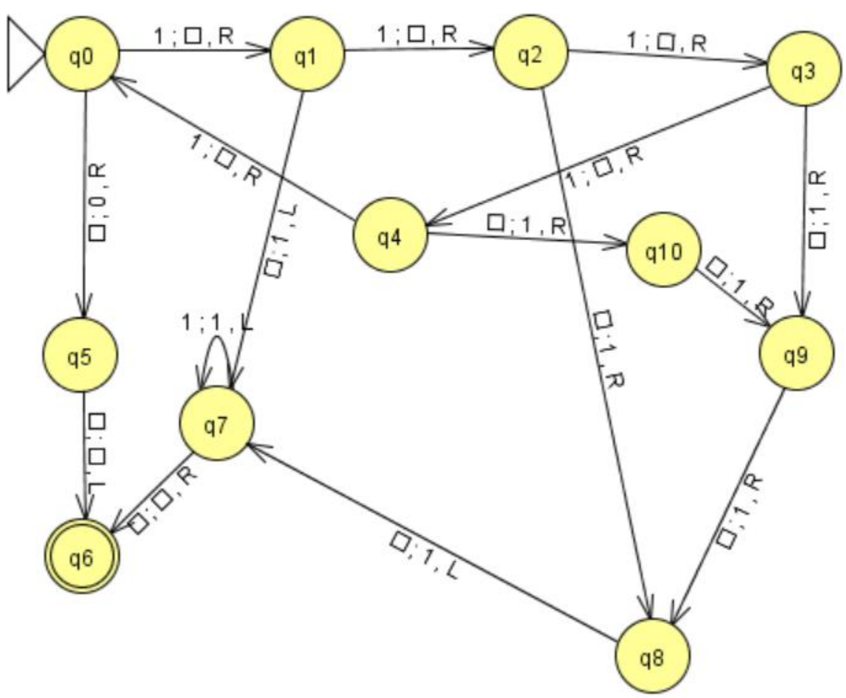
q_0 - Start state

$\Sigma = \{1\}$

$\Gamma = \{0,1,\square\}$,

$F = \{q_6\}$

δ transition is *as per the* below *transition diagram*



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Input	Output	Result
1	1	Accept
11111	0	Accept
1111111	11	Accept
1111111111	0	Accept
11111111111	1	Accept

3. (5 pts) The nor of two languages is defined below:

nor(L1, L2) = { w: w ∈L1 and w ∈ L2}.

Prove that recursive languages are closed under the nor operation.

Proof :

Let L1 and L2 be recursive languages. Therefore by definition of recursive languages, there exists Turing machines M1 & M2 such that

L(M1) = L1

L(M2) = L2

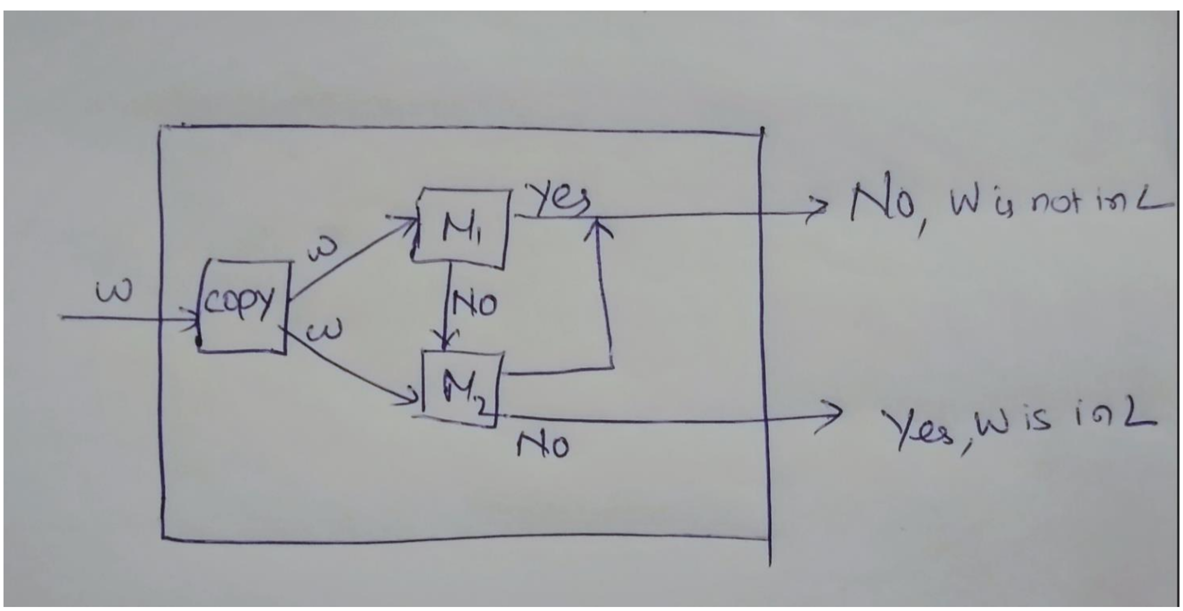
Since L1 and L2 are recursive languages, the Turing Machines M1 & M2 always halts.

Let L be the nor of L1 and L2 :

L = nor(L1,L2)

We need to show that L is recursive. In order to prove this let us construct a Turing Machine M that accepts L and halts on all the inputs.

Machine M : In both cases, the machine Halts.



The above constructed machine M accepts the language L and halts. Hence recursive languages are closed under nor.