CS 321 HW 4 – 25 points

1. (9 pts) Give context-free grammars that generate the following languages.

a. L1 = { w ∈ {0, 1}$*$ | w contains at least three 1s }

Let the Grammar G = (V,∑,P,S), where

V = {S, A}

∑ = { 0,1}

Productions P:

S -> A1A1A1A

A -> 0A | 1A | λ

b. L2 = { w ∈ {0, 1}* | w = w^R and |w| is even }

Let the Grammar G – (V,∑,P,S) , where

V = {S}

∑ = { 0,1}

Productions P:

S -> 0S0 | 1S1 | λ

c. L3 = { a^i b^j c^k | i, j, k ≥ 0, and i = j or i = k

Let the Grammar G – (V,∑,P,S) , where

V = {S, A, C, T, B}

∑ = { a,b,c}

Productions P:

S -> AC| T

A -> aAb | λ

C -> cC | λ

T -> aTc | B

B -> bB | λ

2. (5 pts) Consider the following grammar G = ({S, A}, {a, b}, S, P} where P is defined below

S -> SS | AAA | λ

A -> aA | Aa | b

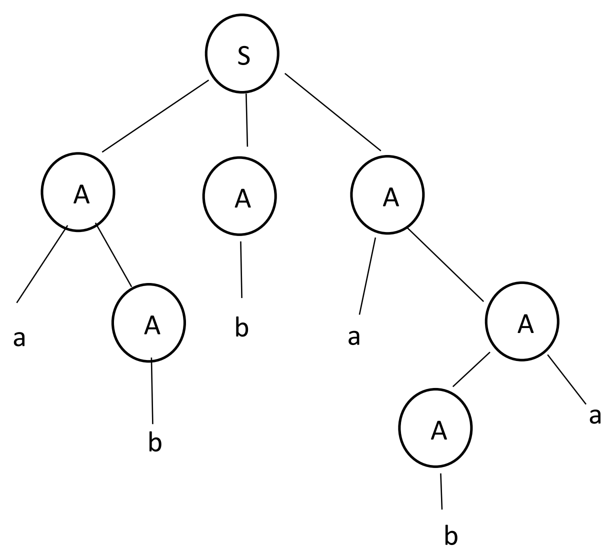
a. Give a left-most derivation for the string abbaba.

S -> AAA -> aAAA -> abAA -> abAaA -> abbaA -> abbaAa -> abbaba

b. Show that the grammar is ambiguous by exhibiting two distinct derivation trees for some terminal string.

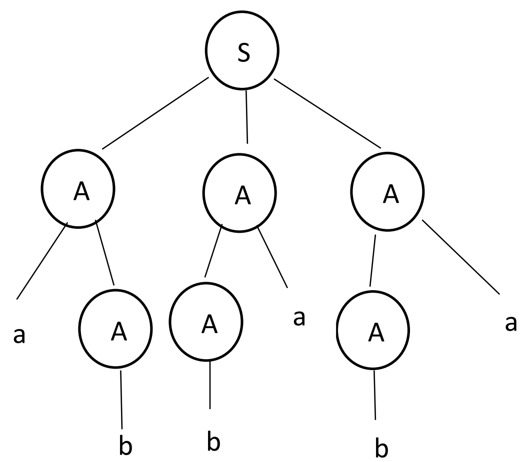
Lets consider the string abbaba, it has 2 left most derivation

Leftmost derivation 1:



Leftmost derivation 2:

String = abbaba can also derived by the below in left most derivation



Therefore, this grammar is ambiguous.

3. (3 pts) Find an s-grammar for $L = \{a^n b^{n+1}: n >= 1 \}$.

Let the Grammar G = (V,∑,P,S), where

V = {S, A, B}

∑ = { a,b}

Productions P:

S -> aAB

A-> aAB | b

B -> b

4. (4 pts) Let $L = \{ a^n b^{n+1} : n >= 0 \}$

a. Show that L^2 is a context-free language.

To show L^2 is a context free language, we can define a context free grammar as below :

Let the Grammar G = (V,∑,P,S), where

V = {S, A}

$\sum = \{ a,b\}$

Productions P:

S -> AA

A -> aAb | λ

b. Show that L^* is a context-free language.

To show L^* is a context free language, we can define a context free grammar as below :

Let the Grammar G = (V,∑,P,S), where

V = {S, A}

$\sum = \{ a,b\}$

Productions P:

S -> AA| λ

A -> aAb | λ

Also, according to the closure properties of context free languages, they are closed under Kleen star.

Thus L^* is context free.

5. (4 pts) Prove that context-free languages are closed under star-closure (*).

Let the Language be represented by

$L = \{ a^n b^n \mid n >= 0 \}$

If L is context free, then L^* is also context free

$L^* = \{ a^n b^n \mid n >= 0 \}^*$

Let the Grammar for L be G – (V,∑,P, S)

The grammar for L^* can be derived by adding a new start symbol S_1 and the following productions

$$S_1 \text{ -> } SS_1 \mid \lambda$$

The rightmost derivation from S_1 generates a sequence of zero or more S's, each of which generates some string in L. Therefore, CFL is closed under Kleene Closure.