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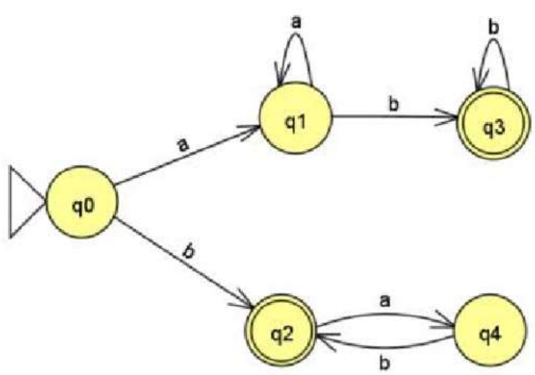
CS321: Introduction to Theory of Computation
January 21, 2020

CS 321 HW2

Submit typed solutions created using a word processor or text editor as a pdf file to Canvas.
For any solutions involving an NFA or DFA also submit the .jff JFLAP files.

1) (*8 pts*) Given an NFA M with the transition graph shown below.



a) Give a regular expression r such that L(r) = L(M)

r = b(ab)* + a(a)* b(b)*

L(r) = {b,bab,babab.., ab, aab,abb,aabbb,}

b) Construct a regular grammar G such that L(G) = L(M)

Let G = (V, T, S, P) be the grammar for the above mentioned NFA, where V = { S, Q1, Q2, Q3, Q4} and T = {a,b }

The production P is defined as below:

S -> a Q1 | b Q2
Q1 -> a Q1 | b Q3
Q3 -> b Q3 | λ
Q2 -> a Q4 | λ
Q4 -> b Q2

2) *(8 pts)* Given the regular grammar G = (V, S, T, P) where V = { A, B, C, S}, T = { 0, 1} and productions P
defined below,

$S \to 00A \mid 1B$
$A \to 0A \mid \lambda$
$B \to 11C \mid 1$
$C \to 0B$

a. Construct an NFA M such that L(M) = L(G)

Let M be the NFA for the above defined Grammar.

M = (Q, ∑, δ, q0, F) where

Q = {S, A, B, C, D, E, F}

∑ = {0, 1}

F = {A, D}

Transition Function :

δ (S,0)= E

δ (S,1)= B

δ (E,0)= A

δ (E,1)= φ

δ (A,0)= A

δ (A,1)= φ

δ (B,0)= φ

δ (B,1)= {F,D}

δ (D,1)= φ

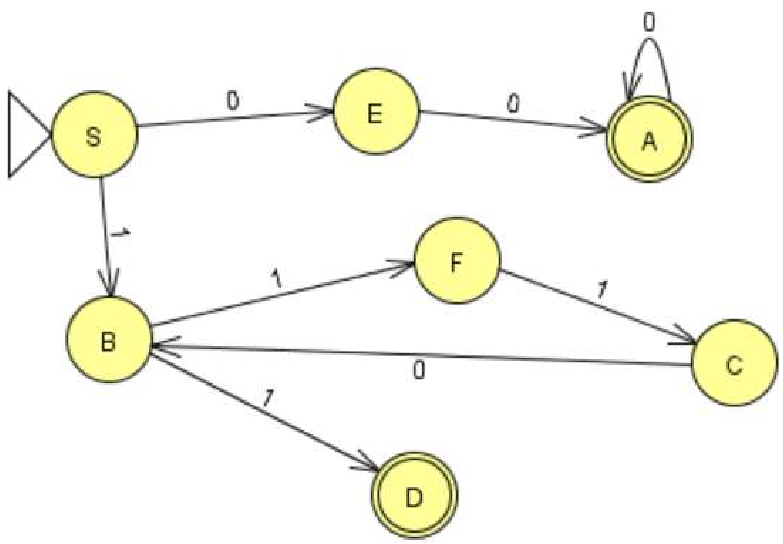
δ (D,0)= φ

δ (F,0)= φ

δ (F,1)= C

δ (C,0)= B

δ (C,1)= φ



Another way :

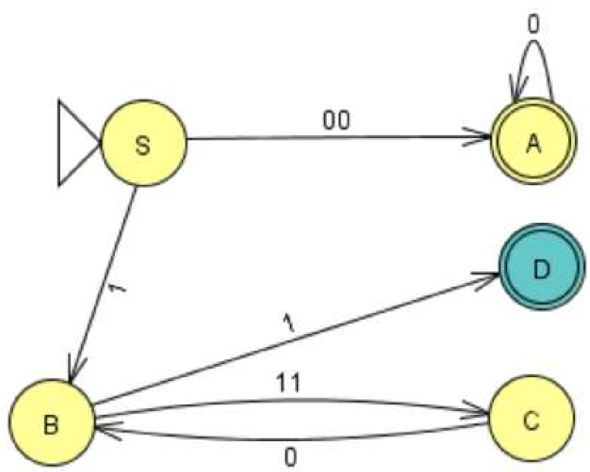
Let M be the NFA for the above defined Grammar.

M = (Q, ∑, δ, q0, F) where

Q = {S, A, B, C, D}

∑ = {0, 1}

F = {A, D}



Transition function :

δ (S,00)= A

δ (S,1)= B

δ (A,0)= A

δ (B,1)= D

δ (B,11)= C

δ (C, 0)= B

b) Give a regular expression r such that L(r) = L(G)

Regular expression r = 000* + 1(110)*1

3) *(4 pts)* Suppose that a bank only permits passwords that are strings from the alphabet = {a, b, c, d,
1, 2, 3, 4} that follow the rules:

- The length is at least five characters
- It begins with a letter {a, b, c, d}
- It ends with two digits {1, 2, 3, 4}

The set of legal passwords forms a regular language L. Construct a regular expression r such that L(r) = L.

Regular Expression r = (a + b + c + d + 1 + 2 + 3 + 4) (a + b + c + d + 1 + 2 + 3 + 4) (a + b + c + d + 1 + 2 + 3 + 4)(a + b + c + d + 1 + 2 + 3 + 4)*(1 + 2 + 3 + 4) (1 + 2 + 3 + 4)

4) (*5 pts*) Prove that if L is regular language then L^R is a regular language.

Given that L is a Regular language, then there exists a NFA M = (Q, ∑, δ, q0, F). Let w be the set of strings in Language L.

There also exists a regular grammar G = (V, S , T , P) such that L = L(G)

Let L^R be the reverse of language L.. To show L^R is a regular language, lets construct an NFA M' = (Q ∪ {q0'}, ∑, δ', q0', F'), such that

L(M') = L^R.

δ'(q0' , ϵ) = F

δ' (q0, α) = φ where α ∈ ∑

δ' (p, α) = { q | δ(q, a)= p } where q ∈ Q, α ∈ ∑ --------------------------eq1

Now to prove L(M') = L^R:

Since w ∈ L(R), we know that w = a1,a2,a3…an and there exists states m0, m1, m2…mn such that m0 = q0 and mn = F.

M' will accept w^R, which can be rewritten from the above as w^R = ϵ a_n, a_{n-1},a_{n-2} …..a_1

Which has state sequence as q0', m_n, m_{n-1},m_{n-2}……..m1

q0' is the initial state of M' and m_1 is the Final state of M'.

The transition function for M' can be written as below :

1st transition function - δ'(q0' , ϵ) = m_n, hence m_n ∈ F.

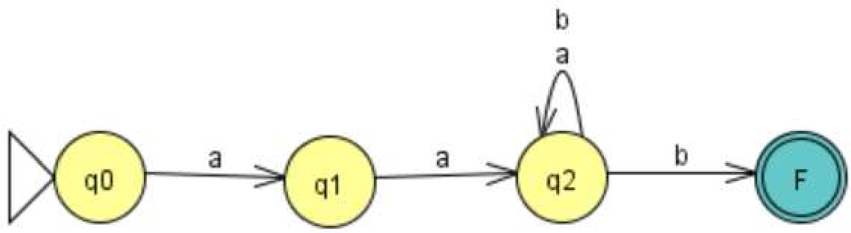
The next transitions in general can be written as

$M_{i-1} \in \delta'(m_i , a_i)$

Which can be written as $m_{i-1} \in \{ q \mid \delta(q, a_i)= m_i \}$ according to eq1 and also as $\delta(m_{i-1}, a_i) = m_i$ which was derived from w ∈ L(M).

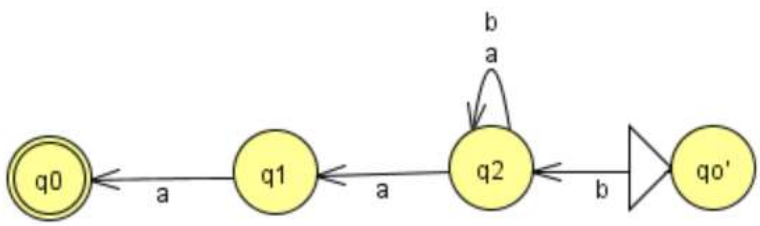
Example – Let us consider the below NFA M

NFA M



The reverse of the above NFA would be

M’



$L^R = \{ w^R : w \in L \}$

As we can see the reverse of the language L^R consists of the same set of strings accepted by Language L in a reverse order.

Q’ = Q -> Set of states in Q and Q’ are the same.

∑’ = ∑ -> Set of all possible strings are same

F’ = q0 -> The initial state of the original Language is the final state for the Reversed Language.

δ’(q , α) = δ1 (q , α) where α ϵ ∑, q ϵ Q

If $w^R \epsilon L(M')$ then,

$\delta' {}^*(q0 , w) = qf$ where $qf \epsilon F'$ and $qf \epsilon F$

From the above it can be said that

$\delta' {}^*(q0' , w^R) = F'$

therefore $w^R \epsilon L(M')$ and is a regular language.