PnP

5. PnP

5.0 Introduction

5.0.1 What is the Perspective n Points (PnP) problem?

💡 **Given:** known 3D landmarks positions in the **world frame** and given their 2D image correspondences in the **camera frame**.

💡 **Determine:** 6DOF pose of the camera (or camera motion) in the world frame (including the intrinsic parameters if uncalibrated).

- However, if the 3D position of the feature points is known, then at least 3 point pairs (and at least one additional point verification result) are needed to estimate camera motion. (This is P3P)

5.0.2 The P3P/Spatial Resection Problem

Given:

- $3D$ coordinates of object points X_i
- 2D image coordinates x_i of corresponding object points
- K matrix, it is a **calibrated camera.**

Find:

- Extrinsic parameters R, X_O of the **calibrated** camera (unlike DLT)

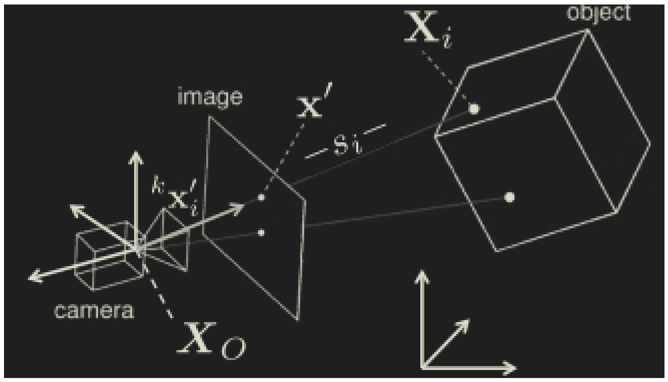
5.0.3 Difference between P3P and DLT

- P3P/Spatial Resection for calibrated cameras
    - 6 unknowns, so at least 3 points are needed

- DLT for uncalibrated cameras (seen)
  - 11 unknowns, so at least 6 points are needed

5.1 Solution to P3P

5.1.1 Revisiting normalized coordinates

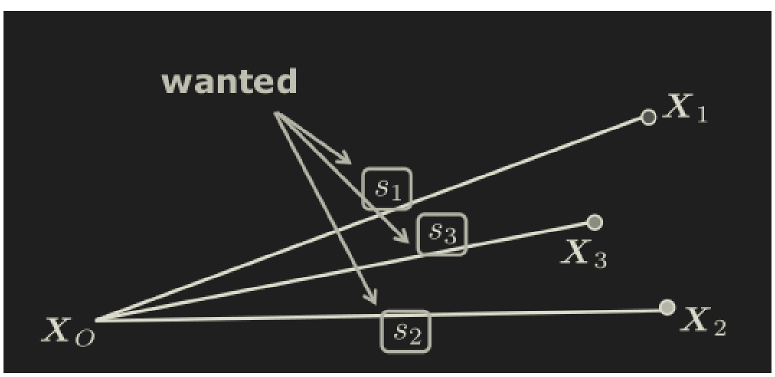


$$\mathrm{x} = \mathrm{K}R\,[I_3| - X_O]\,\mathrm{X}$$

$$^{k}\mathbf{x}'_i = \mathrm{K}^{-1}\mathbf{x}'_i$$

5.1.2 Two step process

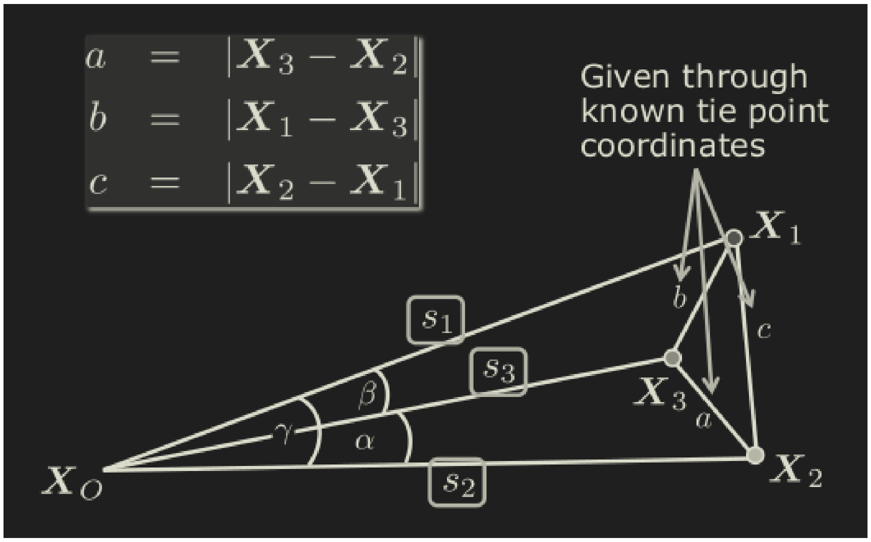
1. Length of projection rays
2. Orientation



💡 Clarity about camera frame and world frame: angles and distances between points

5.1.3 Length of projection rays

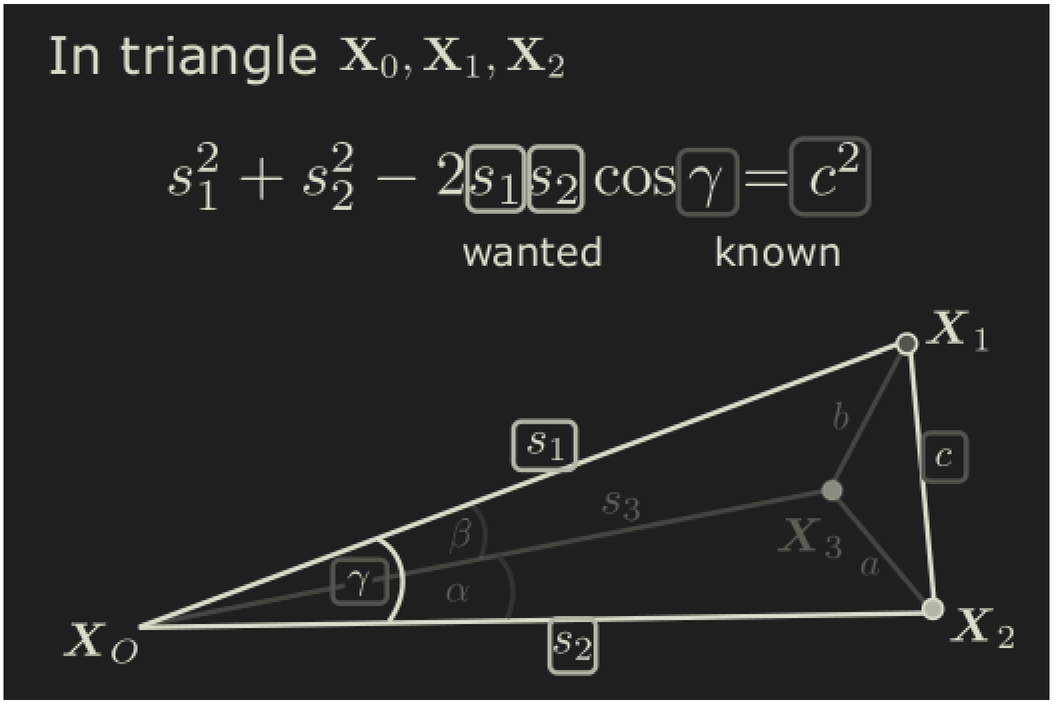
1. **Do we know a, b, c?**



2. Do we know angles?

$$\cos\gamma = \frac{(X_1 - X_0)\cdot(X_2 - X_0)}{\|X_1 - X_0\|\,\|X_2 - X_0\|}$$
$$\text{Clue: Normalized Coords}$$

Cosine rule:



$$
\begin{aligned}
a^2 &= s_2^2 + s_3^2 - 2s_2 s_3 \cos\alpha && -(1)\\
b^2 &= s_1^2 + s_3^2 - 2s_1 s_3 \cos\beta && -(2)\\
c^2 &= s_1^2 + s_2^2 - 2s_1 s_2 \cos\gamma && -(3)
\end{aligned}
$$

We have: $\quad a^2 = s_2^2 + s_3^2 - 2s_2 s_3 \cos\alpha$

Define: $\quad u = \frac{s_2}{s_1} \quad v = \frac{s_3}{s_1} \qquad -(4)$

$$\implies a^2 = s_1^2\left(u^2 + v^2 - 2uv\cos\alpha\right)$$

$$
\begin{aligned}
s_1^2 &= \frac{a^2}{u^2 + v^2 - 2uv\cos\alpha}\\
&= \frac{b^2}{1 + v^2 - 2v\cos\beta}\\
&= \frac{c^2}{1 + u^2 - 2u\cos\gamma}
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
b^2 &= s_1^2 + s_3^2 - 2s_1 s_3 \cos\beta\\
c^2 &= s_1^2 + s_2^2 - 2s_1 s_2 \cos\gamma
\end{aligned}
$$

Substitute ***u*** in other equation — **4th degree polynomial:**

$$A_4 v^4 + A_3 v^3 + A_2 v^2 + A_1 v + A_0 = 0$$

$$\begin{aligned}
A_4 &= \left(\tfrac{a^2-c^2}{b^2} - 1\right)^2 - \tfrac{4c^2}{b^2}\cos^2\alpha \\
A_3 &= 4\left[\frac{a^2-c^2}{b^2}\left(1-\frac{a^2-c^2}{b^2}\right)\cos\beta\right. \\
&\quad \left. - \left(1-\frac{a^2+c^2}{b^2}\right)\cos\alpha\cos\gamma + 2\frac{c^2}{b^2}\cos^2\alpha\cos\beta\right]
\end{aligned}$$

$$\begin{aligned}
A_2 = 2 &\left[\left(\frac{a^2-c^2}{b^2}\right)^2 - 1 + 2\left(\frac{a^2-c^2}{b^2}\right)^2\cos^2\beta\right. \\
&+ 2\left(\frac{b^2-c^2}{b^2}\right)\cos^2\alpha \\
&- 4\left(\frac{a^2+c^2}{b^2}\right)\cos\alpha\cos\beta\cos\gamma \\
&\left. + 2\left(\frac{b^2-a^2}{b^2}\right)\cos^2\gamma\right]
\end{aligned}$$

$$\begin{aligned}
A_1 = 4 &\left[-\left(\frac{a^2-c^2}{b^2}\right)\left(1+\frac{a^2-c^2}{b^2}\right)\cos\beta\right. \\
&+ \frac{2a^2}{b^2}\cos^2\gamma\cos\beta \\
&\left. - \left(1-\left(\frac{a^2+c^2}{b^2}\right)\right)\cos\alpha\cos\gamma\right]
\end{aligned}$$

$$A_0 = \left(1+\frac{a^2-c^2}{b^2}\right)^2 - \frac{4a^2}{b^2}\cos^2\gamma$$

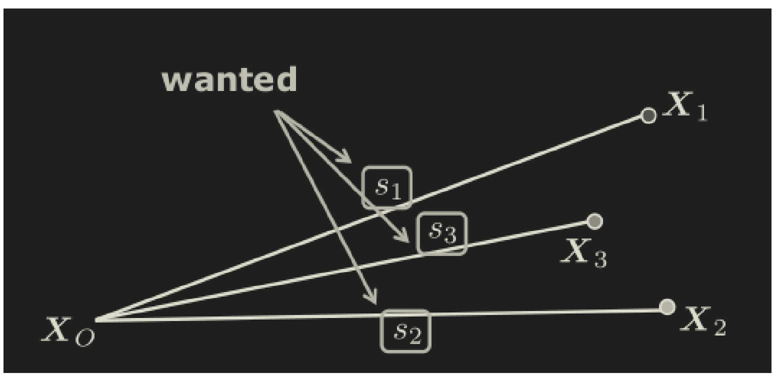
But upto 4 possible solutions possible. So we consider 4th point to confirm the right solution:

So say we know 2D-3D correspondence of (x, X) of 4th point, say (x_4, X_4). Just substitute X of 4th point (we know the K matrix) and the possible solutions of R, t in our camera equation and only one solution will give you the right (x_4).

5.1.4 Transformation between camera frame and world frame

$$
\begin{aligned}
{}^{c}X_1 &= s_1\, {}^{c}\hat{X}_1 \\
{}^{c}X_2 &= s_2\, {}^{c}\hat{X}_2 \\
{}^{c}X_3 &= s_3\, {}^{c}\hat{X}_3
\end{aligned}
$$

Now the question becomes: I have 3 points in one frame and same 3 points in another frame: I can use ICP now.



$$
P = {}^{w}X - {}^{w}\bar{X}, \quad Q = {}^{c}X - {}^{c}\bar{X}
$$

Let

$$
\begin{gathered}
\text{Covariance Matrix: } S = PQ^T \\
\mathbf{SVD}(S) = UDV^T \\
\text{Rotation : } R = VU^T \\
\text{Translation: } t = \bar{x} - R\bar{X}
\end{gathered}
$$