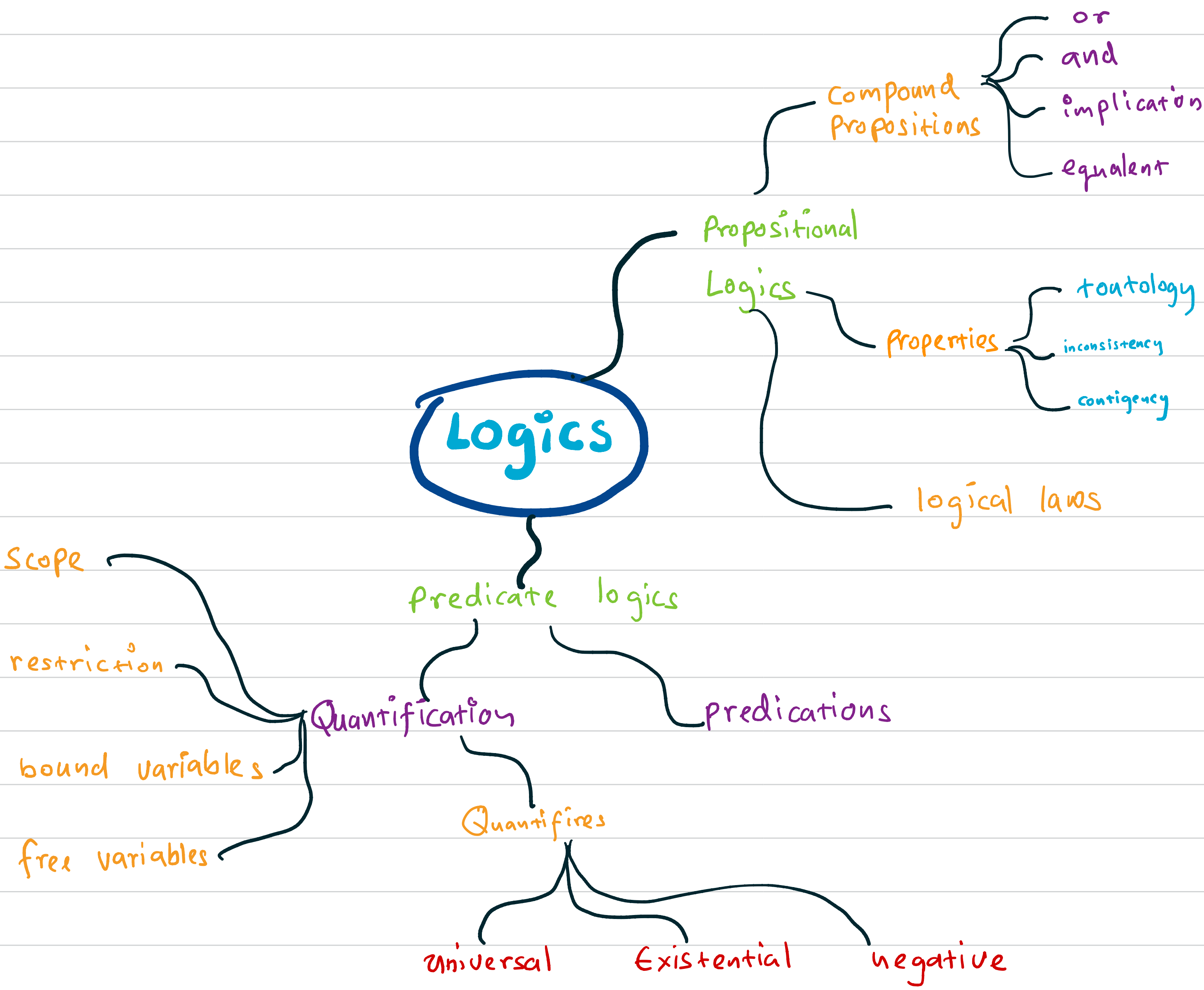
Logics



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– 2020 –

PROPOSITIONAL LOGICS

> " A Proposition is any sentence written down in mathematics or English which can only be **TRUE** or **FALSE** "

Example :-

01. Today is wednesday
02. It is raining

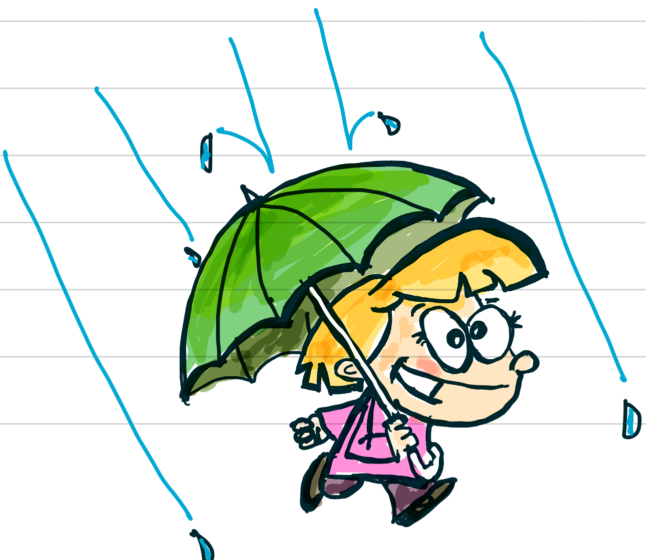
Theese propositions have only TRUE or FALSE answers; No other values. We call it "**LAW OF EXCLUDED MIDDLE**"

We use "propositional symbols" to represent a proposition

$P \equiv$ It is raining :)

The opposit of a proposition represented by the logical not ("$\neg$") symbol (negation)

$\neg P \equiv$ It is not raining :(

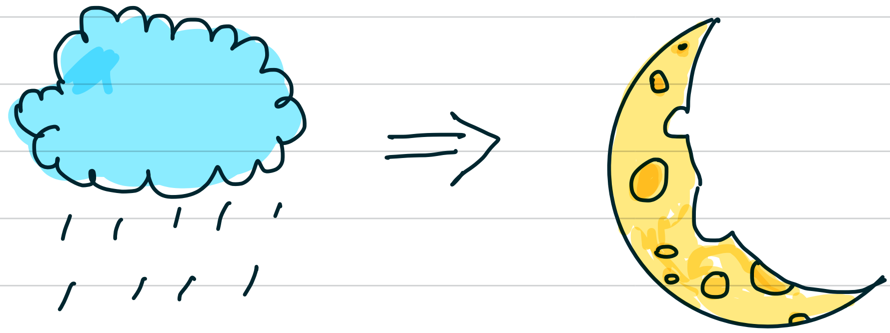


Compound Propositions

" Compound propositions made up simple propositions combined using logical connectives. "

$\vee$	or
$\wedge$	and
$\Rightarrow$	if… then / implies
$\Leftrightarrow$	is equivalent to

Logical Implication ($\Rightarrow$)



If it's raining then the moon is made of cheese

or

It is raining implies the moon is made of cheese

This doesn't mean if it is not raining, the moon should not made of chees.
But, if it is raining, then the moon is made of cheese for sure!

It's raining **and** the moon is made of cheese TRUE
It's **not** raining **and** the moon is made of cheese TRUE
It's **not** raining **and** the moon is **not** made of cheese TRUE
It's raining **and** the moon is **not** made of cheese FALSE

This can happen, who knows!

The truth table :-

P 	Q 	$P \Rightarrow Q$
true	true	true
true	false	false
false	true	true
false	false	true

$P \Rightarrow Q$ is true only if (P is false) OR (Q is true)

Another example :-

If Jimmy is a Dog, then Jimmy has 4 legs

P = Jimmy is a Dog

Q = Jimmy has 4 legs

P 	Q 	$P \Rightarrow Q$
true	true	true
true	false	false
false	true	true
false	false	true

Jimmy can be a cat

Jimmy can be a bird

LOGICAL EQUIVALANCE ($\Leftrightarrow$)

Equivalance states whether two logical statements have the same truth value.

This is Jayantha.
Jayantha is an animal.

We need to know whether he is a mamel or not!

If Jayantha drinks milk then he is amamel.
If Jayantha is a mamel, he is drinking milk.

A = Jayantha drinks milk
B = Jayantha is a mamel

A	B	$A \Leftrightarrow B$
true	true	true
true	false	false
false	true	false
false	false	true

$A \Leftrightarrow B$ is true if and only if $A \Rightarrow B$ AND $B \Rightarrow A$ are true

Bracketed Propositions

To ensure that a complex proposition has the intended meaning, we must use brackets.

ex :- $P \wedge (Q \Rightarrow R)$

Contingencies, Inconsistencies & Tautologies

Tautology → a logical statement which is true in every interpretation

ex :- $\neg p \vee p$

Inconsistency → a logical statement which is false in every interpretation

ex :- $\neg p \wedge p$

Contingency → a logical statement which is
- true in at least one interpretation
- false in at least one interpretation

ex :- $p \Rightarrow Q$

Logical Laws

Here's a list of standard logical laws :-

$P \wedge P = P$	[∧ Idempotence]
$P \vee P = P$	[∨ Idempotence]
$P \wedge \neg P = false$	[Contradiction]
$P \vee \neg P = true$	[Excluded Middle]
$(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$	[∧ Associativity]
$(P \vee Q) \vee R = P \vee (Q \vee R)$	[∨ Associativity]
$P \wedge Q = Q \wedge P$	[∧ Commutativity]
$P \vee Q = Q \vee P$	[∨ Commutativity]
$P \wedge true = P$	[∧ Identity]
$P \vee false = P$	[∨ Identity]
$P \wedge false = false$	[∧ Zero]
$P \vee true = true$	[∨ Zero]

$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$	[$\wedge$ Distribution over $\vee$]
$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$	[$\vee$ Distribution over $\wedge$]
$P \wedge (P \vee Q) = P$	[Absorption I]
$P \vee (P \wedge Q) = P$	[Absorption II]
$\neg(P \wedge Q) = \neg P \vee \neg Q$	[de Morgan I]
$\neg(P \vee Q) = \neg P \wedge \neg Q$	[de Morgan II]
$\neg(\neg P) = P$	[Double Negation]
$P \Rightarrow Q = \neg P \vee Q$	[Implication]
$P \Rightarrow Q = \neg P \Rightarrow \neg Q$	[Contrapositive]
$P \vee Q = (\neg P) \Rightarrow Q$	[$\vee$ Definition]
$P \Leftrightarrow Q = (P \Rightarrow Q) \wedge (Q \Rightarrow P)$	[$\Leftrightarrow$ Definition]

PREDICATE Logic

any proposition can be defined as a Predication

proposition → predication

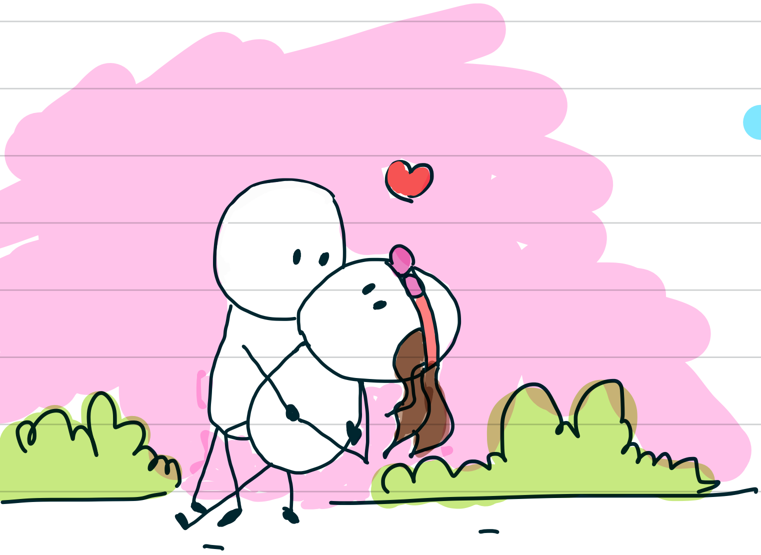
Predicate (arg1, arg2, ..., argn)

P(x) can be read as x is a P

Ex:-

Sri Lanka is a country

Country (sri_lanka)



Upali loves Sumana

Love (upali, sumana)

constants

or

x = 'upali', y = 'sumana'

Love (x, y)

variables

Rathne sends Jayanthi a letter

Send (rathne, jayanthi, letter)

A predicate with $1 \leq n$ arguments is called n place predicate

ex :- Love (upali, samantha) is 2 place predicate

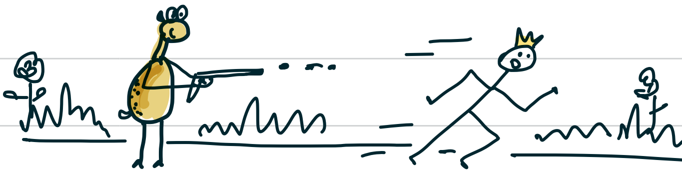
ARGUMENT STRUCTURE MATTERS!

Thissa shot Muwa

Shoot (thissa, muwa)

Shoot (muwa, thissa)

Muwa shot Thissa



Same predicate, diferent meanings!

COMPLEX PREDICATES

The teacher saw that the children were reading a book.

proposition 1 : The teacher saw something

proposition 2 : The children were reading a book

See (teacher, Read (children, book))

Read more : youtu.be/IhodKMPWShc

QUANTIFICATION

Consider the following examples:

p: All girls love Tharindu

Love (girls, tharindu)

q: Some girls love Tharindu

Love (girls, tharindu)

r: Girls don't love Tharindu

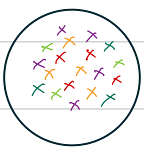
Love (girls, tharindu)

This predicates doesn't reflect the different meanings. :(We need an addition machanism to formalize the relationships between sets...

That's where the **Quantifiers** comes to the picture!

$\forall x$

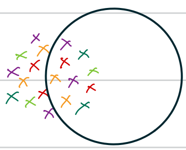
universal quanifier



For all x it holds that....

$\exists x$

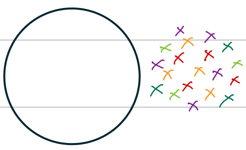
existential quantifier



There exists at least one x such that ...

$\neg x$ $\sim x$

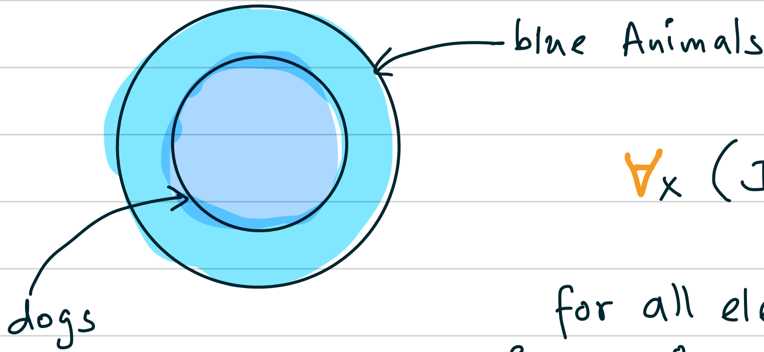
negative quantifier



For no x it holds that...

UNIVERSAL QUANTIFIER

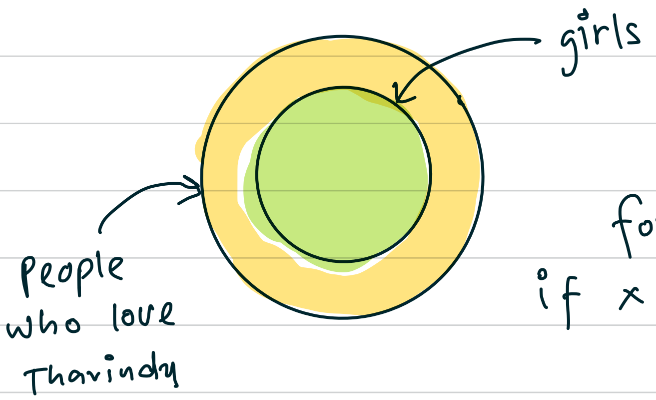
p: All dogs are blue



$\forall x\,(Dog(x) \Rightarrow Blue(x))$

for all element x, it holds that if x is a dog, then x is blue.

q: All girls love Tharindu

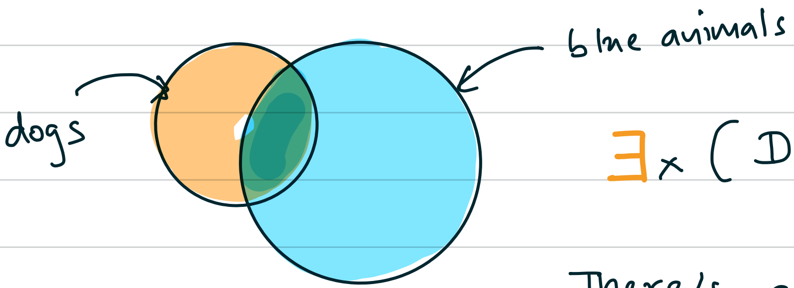


$\forall x\,(Girl(x) \Rightarrow Love(x, tharindu))$

for all element x, it holds that if x is a girl, then x loves tharindu.

EXISTENTIAL QUANTIFIER

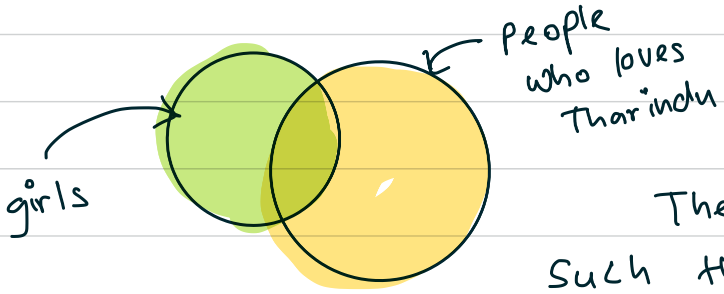
p: Some dogs are blue



$\exists x\,(Dog(x) \land Blue(x))$

There's at least one element such that x is a dog and x is blue.

q: Some girls love Tharindu

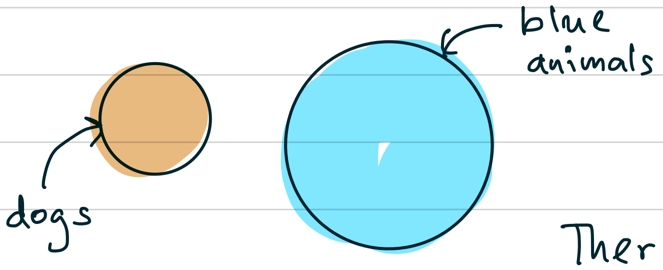


$\exists x \left(Girl(x) \land Love(x, tharindu) \right)$

There's at least one element x, such that x is a girl and also x loves Tharindu.

NEGATIVE QUANTIFIER ($\neg$, ~)

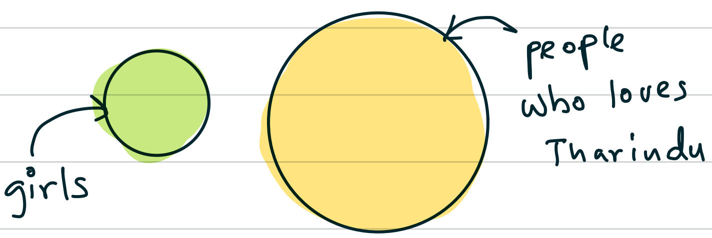
p: No dog is blue



$\neg x \left(Dog(x) \Rightarrow Blue(x) \right)$

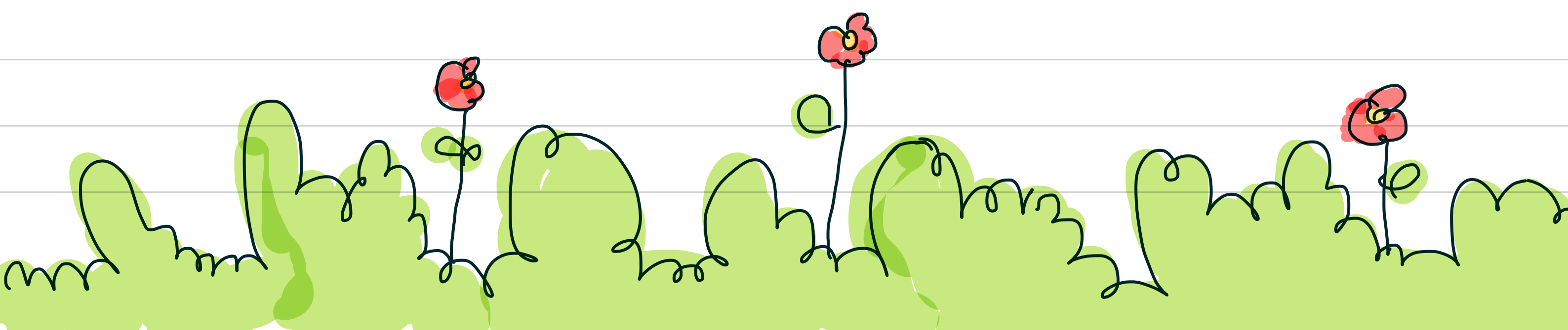
There's no element x, such that if x is a dog; then x is also blue.

q: No girl loves Tharindu



$\neg x \left(Girl(x) \Rightarrow Love(x, Tharindu) \right)$

There's no element x, such that if x is a girl; then x is also loves Tharindu.



"Quantifiers cannot refer to anything by themselfs. It's alway's necessary to specify what is being Quantified. The **RESTRICTION** establishes the set of entities that are efected (restricted)."

— Bhathiya & Santhush

ex :-

$$\forall_x \left(\underbrace{\boxed{Dog(x)}}_{\text{restriction}} \Rightarrow \underbrace{\boxed{Blue(x)}}_{\text{scope}} \right)$$

The **Scope** of the quantifire is a set of entities denoted by the main predication of the proposition in which the quantifire appears.

Further restrictions can be defined by adding adjectives.

ex :-

All smoking dogs are blue.

$$\forall_x \left(\underbrace{Dog(x) \land Smoke(x)}_{\text{restriction}} \Longrightarrow \underbrace{Blue(x)}_{\text{scope}} \right)$$

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01. Everybody is happy

$\forall_x (Person(x) \Rightarrow Happy(x))$

02. Kumara likes some girls

$\exists_x (Girl(x) \& Like(Kumara, x))$

03. Nobody shoot Thissa

$\neg_x (Person(x) \Rightarrow Shot(x, thissa))$

$\neg\exists_x (person(x) \& Shot(x, thissa))$

Read as:
"It is not the case that there's at least one element x such that x is a person and also x shoot Thissa"

BOUND VARIBLES AND FREE VARIABLES

"A variable in a formula is defined as "bound" if it is named by the quantifier & within the scope."

– Kanaka Herath

ex :-

$$\forall_x \left(Dog(x) \Rightarrow Drunk(x) \right)$$

x is a bound variable!

"A variable that occures in a fomula is said to be "free" if it is not a bound variable"

– Shashika Nisansala

ex :-

$$\forall_x \left(Person(x) \Rightarrow Love(x, y) \right)$$

y is a free variable!
x is a bound variable...

note :- A single variable may occur both bound and free in the same formula.

ex :-

$$\left(\forall_x \left(A(x) \Rightarrow B(x) \right) \right) \wedge Q(x)$$

x = bound
x = free

$$\forall_x \left(Dog(x) \Rightarrow Drunk(x) \right)$$

can be also written as:-

$$\forall(x) \cdot \left(x \in DOG \Rightarrow Drunk(x) \right)$$

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$$\exists(n) \left(Natural\_number(n) \wedge Odd(n) \right)$$

$$\downarrow$$

$$\exists(n) \cdot \left(n \in \mathbb{N} \wedge Odd(n) \right)$$