I.M.P

Asymptotic Notation :

$f(n)$ $f(n)$

→ Comparision of non-negative functions for large n value.

→ Asymptotic is a Greek word = Mean = Too Large.

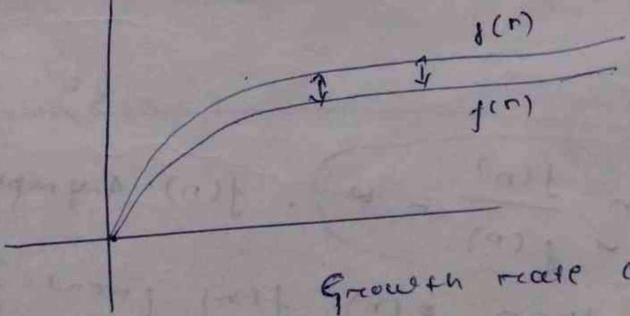
→ Growth rate comparision of $f(n)$, $g(n)$ for large input is called asymptotic comparision of $f(n)$, $g(n)$

→ $f(n)$ and $g(n)$ Asymptotically equal if a $\lim_{n \to \infty} \frac{f(n)}{g(n)}$ = constant (>0)

eg → $f(n) = 10n^2 + 20$ $g(n) = 5n^2 + 10n$.

$$\lim_{n \to \infty} \frac{n^2 \left[10 + \frac{20}{n^2}\right]}{n^2 \left[5 + \frac{10}{n}\right]} = \frac{10}{5} = 2 \text{ [constant]}$$

$f(n)$ and $g(n)$ are Asymptotically equal.



Both are growing parallelly.

Growth rate compare is equal.

$f(n)$ and $g(n)$ are asymptotically equal.

eg- $f(n) = 100n^2$ $g(n) = 2n^2$

$$\lim_{n \to \infty} \frac{100n^2}{2n^2} = 50 \text{ [const]}.$$

One func is ~~big~~ constant time (50) bigger than the other function

$f(n)$ and $g(n)$ Asymptotically not equal

if $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ or ∞

eg: $f(n) = 20n^2$ $\quad g(n) = 30n + 100$

$$\lim_{n \to \infty} \frac{20n^2}{n[30 + \frac{100}{n}]} = \frac{\infty}{30} = \infty$$

$f(n)$ & $g(n)$ are not equal. numerator is ∞. so $f(n)$ grows faster than $g(n)$

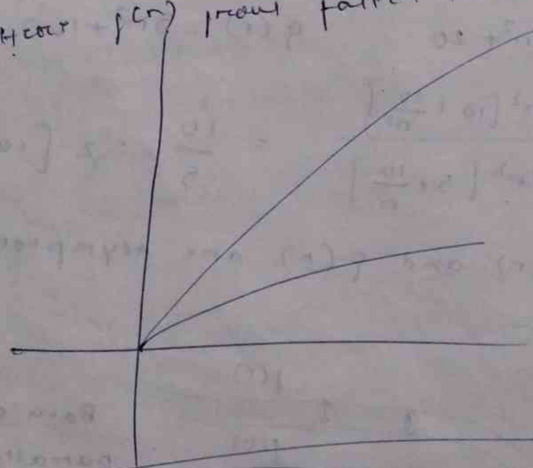
eg $f(n) = 10n^2 + 20n$

$g(n) = 5n^3 + 100$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{n^2[10 + \frac{20}{n}]}{n^2[5n + \frac{100}{n^2}]}$$

$$= \frac{10}{\infty} = 0$$

Hence $g(n)$ grows faster than $f(n)$.



$\lim \frac{\frac{2n+3}{n^3}}{\frac{n^3}{n^3}}$

If $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$. $f(n)$ Asymptotically bigger than $g(n)$. $f(n)$ grows faster than $g(n)$.

If $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$. $g(n)$ Asymptotically bigger than $\cancel{g(n)}$ $f(n)$. $g(n)$ grows faster than $f(n)$.

Big Oh Notation [O]

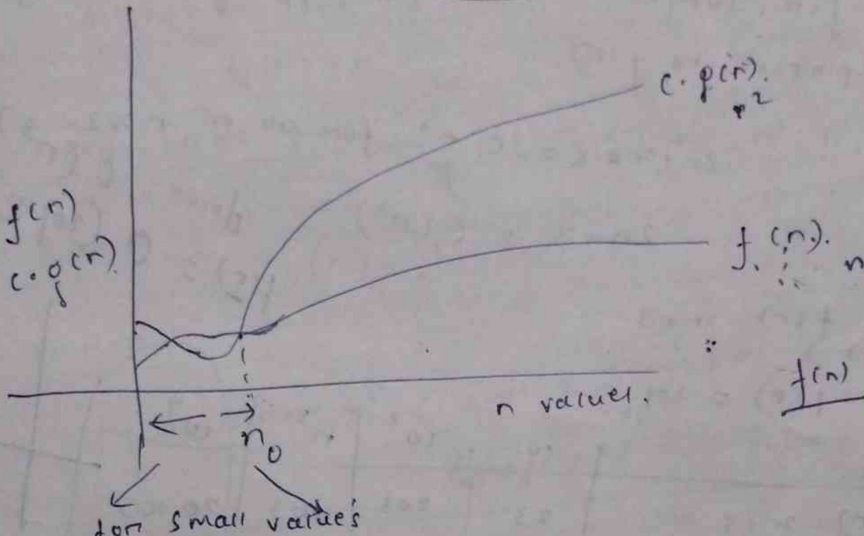
$f(n)$ and $g(n)$ are non-negative functions.

$f(n) = O(g(n))$ if $f(n) <= c \cdot g(n)$.

for all n values where $n \geq n_0$.

where c, n_0 are constants.

∴ non-negative fun eg. $n^2 - n$
Negative fun $n - n^2$



∴ $\underline{f(n)} = O$

$f(n) = O(g(n))$ mean's that iff $g(n)$ Asymptotically bigger or equal to $f(n)$.

∴ but $f(n)$ should not lesser than $f(n)$

$$f(n) = O(g(n)) \text{ iff } \lim_{n \to \infty} \frac{f(n)}{g(n)} = \text{Const (or) } 0$$

eg:

① $f(n) = 2n+3$ upper

$g(n) = 3n$

$2n+3 = 2n+3$

$2n+3 \leq 3n$

4

sol^n

$f(n) = 2n+3$

$c \cdot g(n) = 3 \cdot n$

n	0	1	2	3	4	5	6	7
$f(n) = 2n+3$	3	5	7	9	11	13	15	17
$c \cdot g(n) = 3 \cdot n$	0	3	6	9	12	15	18	21

growth rate bigger.

$f(n) = 2n+3 <= 3 \cdot g(n)$ for $n \geq 3$.

$\underline{2n+3 = O(n)}$.

$f(n) = n^2$

$f(n) = O(n^3)$

$f(n^2) = \Omega(n)$

ex-2

$f(n) = 2n+3$

$g(n) = n^3$

n values	0	1	2	3	4	5
$f(n) = 2n+3$	3	5	7	9	11	
$g(n) = n^3$	0	1	8	27	64	

for smaller i/p 0, 1 of f(n) is larger than f(n) but for larger value of or i/p g(n) is larger compare to f(n).

$$2n+3 <= c \cdot n^3 \text{ for all } n, n \geq 2$$

$$2n+3 = O(n^3)$$

$f(n) = O(n^3)$

$f(n) = O(\log n^2)$

ex-3.

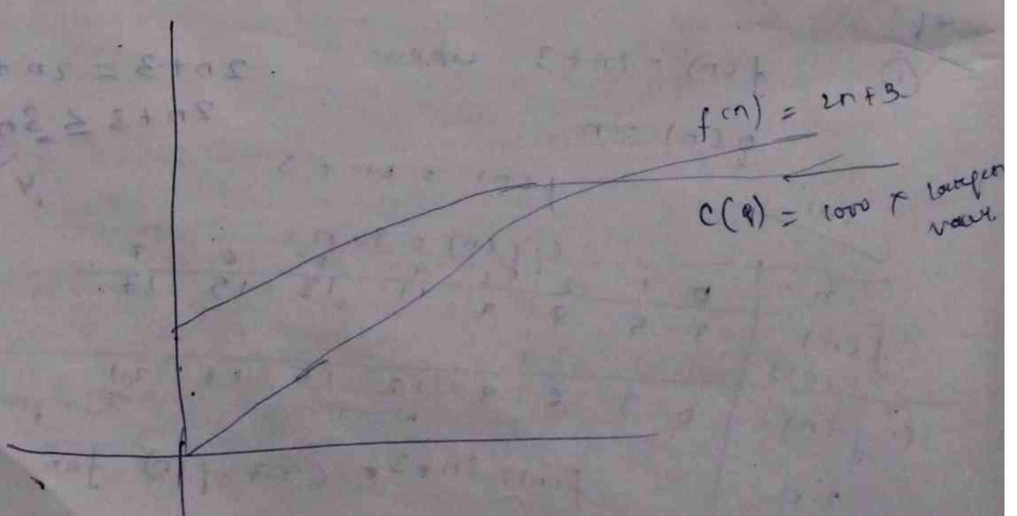
$f(n) = 2n+3$

$f(n) = \log_{10} n$

	10	10^2	n^3	10^4
$f(n) = 2n+3$	23	203	2003	20,003
$c \cdot g(n) = 1000 \cdot \log n$	1000	2000	13000	4200

$$2n+3 <= 1000 \cdot \log_{10} n$$ (false for all n values)

$$2n+3 \neq O(\log_2 n)$$



$f(n) \neq O(g(n))$.

$f(n) = O(f(n))$.

Decrement func :-

$f_1(n) = \frac{c}{n}$ where c = constant.

Here the constant's are bigger

$f_2(n) = a$ (constant)

log function.

$f_3(n) = \log_2 n$

polynomial function. :- $(n^k \ k)$

$f_4(n) = n^2$

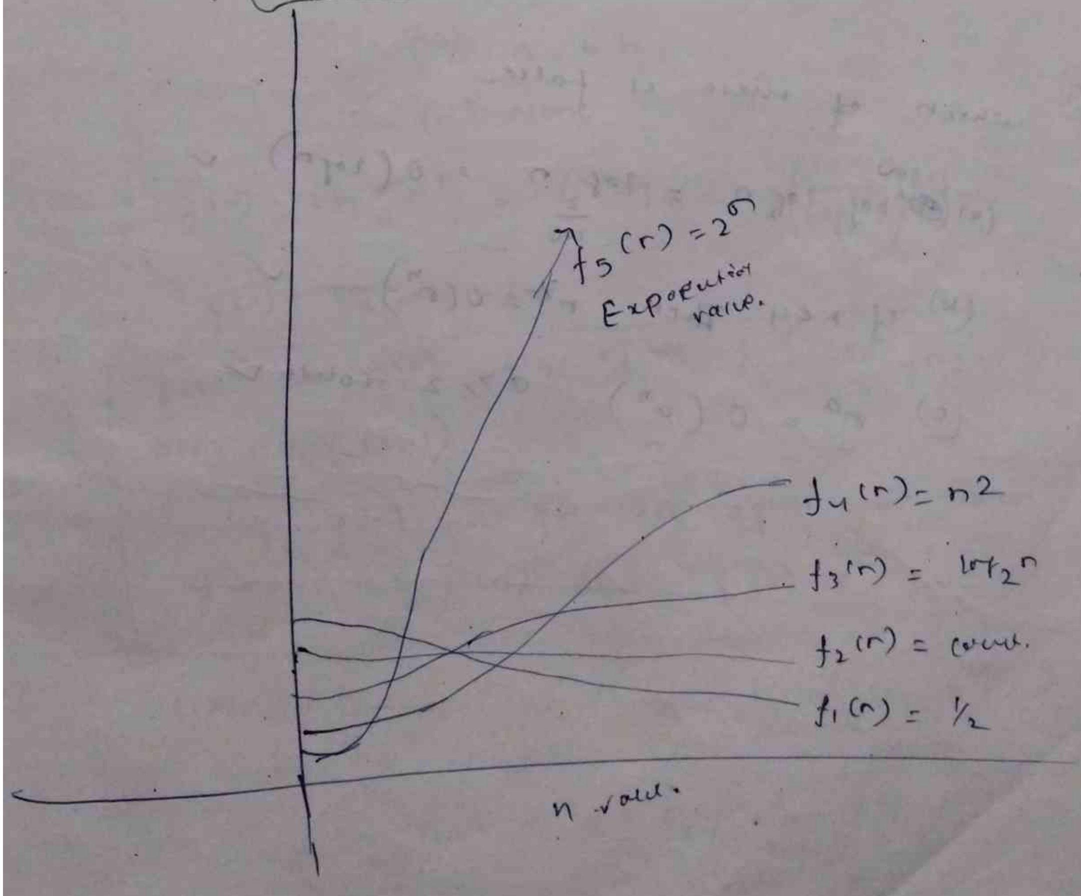
Expoential function (eg) $\{2^n, 3^n, n!, n!\}$

$f_5(n) = 2^n$

order

n.P

$\boxed{\text{decrement } f^n < \text{const.} < \text{log func} < \text{polynomial} < \text{Expoential} < \text{exp fun}^n}$



5 ew f $f(n) = n^{1000}$ (poly) $g(n) = 2^n$ (expo)

$f(n) = O(g(n))$

$f(n) + O(gw)$

6 eg. $f(n) = n^{0.001}$ (poly) $g(n) = \log n$ logarithm.

$g(n) = O(f(n))$

$f(n) = O(g(n))$.

	10^{10}	10^{100}	10^{1000}	$10^{10,000}$
$f(n) = n^{0.001}$	$(10^{10})^{1/1000} = 10^{1/100} =$	$(10^{10})^{\frac{1}{100}} = 1$	10^{10}	10^{100}
$g(n) = \log_{10}$	$\log 10^{10} = 10$	$\log 10 = 100$	1000	10,000. 10^5

$f(n)$ is bigger than $g(n)$.

eg 7 which of these is false.

(a) $100^{\wedge}$ 2 of $\log_2 n = \log_{\frac{2}{100}} n = O(\log n)$ ✓

(b) if $x < y$ then $n^x = O(n^y)$ ✓

(c) $n^a = O(a^n)$ $a \geq 2$ const ✓

(8)

Is $f(n) = \begin{cases} n^2 & 0 \le n \le 10, \text{and} \\ n & n > 10, \text{and} \end{cases}$

$q(n) = \begin{cases} n & 0 \le n \le 100 \\ n^3 & n \geq 100 \end{cases}$

which is true.

(i) $f(n) = O(q(n))$

(2) $q(n) = O(f(n))$

(3) a and b

(4) None

(9) $f(n) = \log_y n^x$ (x, y, const)

$q(x) = \log_{10} n$

what is true.

(a) $f(n) = O(q(n))$

(b) $q(n) = O(f(n))$

✓(c) a & b

(d) None

$f(n) = \log_y n^x = x \log_y n = \left(\frac{x}{\log_{10} y}\right) \log_{10} x$

$q(x) = \log_{10} n$

$f(n)$ and $q(n)$ Asy equal func. so $f(n) = O(f(n))$ and $q(n) = O(f(n))$

if $f(n)$ and $q(n)$ Asy equal func then $f(n) <= c \cdot f(n)$. and $q(n) <= c_2 \cdot f(n)$

$f(x)$ — $\frac{x}{\log_{10} y} \cdot \log_{10} n <= c_1 \cdot \log_{10} n$ — $f(x)$

$\log_{10} n <= c_2 \cdot \frac{x}{\log_{10} y} \log_{10} n$ — $f(x)$

$f(x)$

(10) $f(n) = n^{2.1}$ $\qquad g(n) = n^2 \log n$ $\qquad n^{0.1} \quad n(\log)$

which is true.

(a) $f(n) = O(g(n))$

(b) $g(n) = O(f(n))$

(c) a & b

(d) None.

$n^{2.1} \cancel{\to} n^2 \log n$

$n^2 . n^{0.1}$

(11) $f(n) = \begin{cases} 2^n & \text{for even } n \\ n & \text{" odd } n \end{cases}$

$g(n) = \begin{cases} 2^n & \text{for odd } n \\ n & \text{for even } n. \end{cases}$

which is true.

(a) $f(n) = O(g(n))$

(b) $g(n) = O(f(n))$

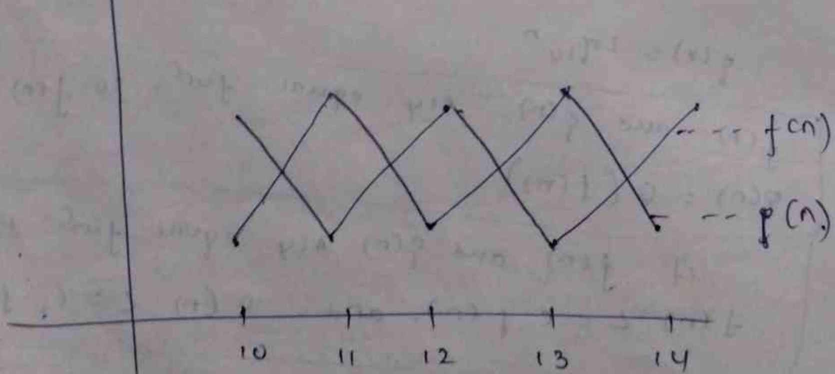
(c) a & b

(d) None.

for even $f(n)$ is bigger (2^n)

for odd $g(n)$ " " (2^n)

So both $f(n)$ and $g(n)$, combinely they are non comparable.



(12) $f(n) = n^{1+\sin n}$

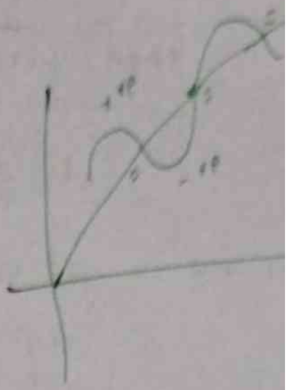
$g(n) = n.$

which is true.

(a) $f(n) = O(g(n))$ $\qquad$ (b) $g(n) = O(f(n))$

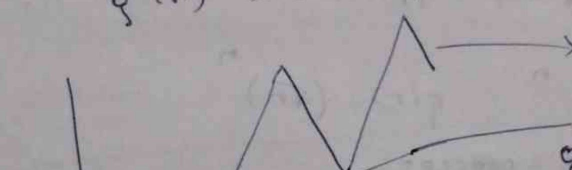
(c) a & b $\qquad$ (d) none.

	$n^{1+\sin n}$		n
0	n	=	n
90	$n^{1+1} = n^2$	>	n
180	$n^{1+0} = n$	=	n
270	$n^{1-1} = n^0$	<	n
360	n	=	n



Here $f(n)$ and $g(n)$ are non comparable. bcoz sometimes $f(n)$ bigger, ~~$g(n)$~~ other time $g(n)$. Though same $\sin n$ (it ranges between +ve to -ve)

(13) $f(n) = n^{2+\sin n}$, $g(n) = n$. Here $f(n)$ is bigger always



$g(n) = O(f(n))$

imp

(14) $n!,\ 2^n,\ n^n,\ n^{\sqrt{n}}$ → (all are exponential func)

$f(n) = 2^n$, $g(n) = n^{\sqrt{n}}$ } exponential func.

$2^n < 3^n$ (power same or base same $3 > 2$ so) $n^n > n^{\sqrt{n}}$

imp

If $\log f(x)$ asymptotically bigger than $\log(g(n))$ Then $f(n)$ asy bigger than $g(n)$

$f(n) = 2^n$ $g(n) = n^{\sqrt{n}}$

$\log f(n) = n \log 2 > \log g(n) = \sqrt{n} \log n$

2^n Asy bigger than $n^{\sqrt{n}}$

$n^{\sqrt{n}} = O(2^n)$

$2^n \neq O(n^{\sqrt{n}})$

(15) If $\log f(n)$ and $\log g(n)$ Asy equal then $f(x)$ and $g(x)$ may/ may not Asy equal

1. $2^n < 3^n \Rightarrow \log 2^n = \log 3^n$

~~2. $n^n > 2^n \Rightarrow n \log n > n \log$~~

2. $n^2 < n^3 \Rightarrow 2 \log n = 3 \log n$

3. $n^2 = n^2 \Rightarrow 2 \log n = 2 \log n.$

(16) if $f(n)$ & $g(n)$ asymptoti.. equal then $\log f(n)$ & $\log g(n)$ asymptotical equal.

(17) $f(n) = n^{\sqrt{n}}$ $\quad g(n) = (\sqrt{n})^{\log n}.$ $\quad \sqrt{n} \log n^{1/2}$ $\quad \log n \sqrt{n}$

$f(n)$ Asy bigger than $g(n)$. $\quad f(n)$ $\log n \times \log \sqrt{n}$

(18) $f(n) = n^{\log n}$ $\quad g(n) = (\sqrt{n})^n$

Apply log concept.

$\Rightarrow \log n^{\log n} = \log (\sqrt{n})^n$

$\Rightarrow \log n (\log n) = n \log \sqrt{n}$

$= (\log n)^2 = n \log n^{1/2}$ $\quad \frac{1}{2}$ constant

n is bigger, so.

$f(n) < g(n).$

(19) $f(n) = n \times n$ $\quad g(n) = n + n$

$f(n) > g(n).$

(20) $f(n) = (n-1)! * n$ $\quad g(n) = (n-1)! + n$

$f(n) > g(n).$

(21) $f(n) = n^n$ $\quad g(n) = n!$ (∵ Both are exponential)

$f(n) = n * n * n * \ldots * n > g(n) = n * (n-1) * (n-2) \ldots * 1$

$n^n > n!$ $\quad \boxed{\begin{matrix} n! = O(n^n) \\ n^n \neq O(n!) \end{matrix}}$

I.M.P

(22) $f(n) = n \log n \quad g(n) = \log n!$

$f(n) = \log n + \log n + \log n \dots \quad g(n) = \log n$

Sterling factorial formula.

$$n! = \sqrt{2\pi n e}\left(\frac{n}{e}\right)^n$$

Apply log on both side.

$$\log n! = \log\left[\sqrt{2\pi n e}\left(\frac{n}{e}\right)^n\right]$$

$$\log n! = \frac{1}{2}\log 2\pi e + \frac{1}{2}\log n$$

$$\log n! = \frac{1}{2}\log 2\pi e + \frac{1}{2}\log n + n\log n - n\log e$$

$\log n!$ ~~is equal~~ is Asy equal to $n \log n$

$f(n) = O(n \log n)$

$n \log n = O(\log n)$

If $f(n) > g(n)$ then

$\log f(n) > \log g(n)$

or

$\log f(n) = \log g(n)$

I.M.P. $f(n)$ function is polynomially bounded

iff $\log f(n) = O(\log n)$.

$f(n)$ is expoential fun

iff $\log f(n) \neq O(\log n)$.

Polynomial func

$f(n) = n^k$ (k any const)

$\log f(n) = k \log n$

I.M.P. if $\log f(n)$ Asy. smaller or equal to $\log n$.

then $f(n)$ polynomialy bond. bounded.

if $\log f(n)$ Asy bigger then $\log n$ then

$f(n)$ expoential func.

prove $f(n) = n!$ is expoential or polynomial

$\log n! = n \log n \neq O(\log n)$

$n!$ is Expoential function.

$n \log n$ is asy more

(2) $f(n) = n^{\log n}$

$\log f(n) = (\log n)^2 \neq O(\log n)$

so it is exponential. ✓

(3) $f(n) = (\log n)^{\log n}$

$\log f(n) = \log(\log n \log n) \neq O(\log n)$

~~= log~~ so it is exponential. ✓

(4) $f(n) = (\log n)^{\log \log n}$

$\log f(n) = \log \cancel{(\log \log n)}$

$= \log(\underbrace{\log n}_{m})^2 = O(\log n)$

$= (\log m)^2 = O(m)$ ✓ Big O is true.

so it is polynomially bounded.

(5) $f(n) = \log n!$ $n \log n$

$\log f(n) = \log \cancel{(\log n!)} = \log(n \log n)$

$\log n + \log \log n \leq O(\underbrace{\log n}_{m})$

✓ $\log n!$ is polynomially bounded. not exponential.

(6) $f(n) = (\log n)!$

$\log f(n) = \log(\underbrace{\log n}_{m})! = \log n * \log \log n$

$\log m! = m \log m = \log m * \log \log m$

it is exponential. $\neq O(\log n)$

(7) $f(n) = \log(\log n)!$

$\log n * \log \log n$ → polynomial.

$\log(\log n)! =$ polynomial bounded.

(8) $f(n) = (\log \underline{\log n})!$

$= (\log m)!$

$= m \log n$ — ~~polynomial~~ bounded

$= (\log \log n)(\log \log n) \leq O(\log n)$

$(\log \log n)!$ polynomial funcⁿ.

$$\boxed{\log\log n < (\log\log n)^k < (\log n) < \log n^k < n^\epsilon}$$

$k > 1$ consider $\log n = m$ $k > 1$

$n!$ — exponential

$(\log n)!$ — exponential.

$(\log\log n)!$ — Polynomial

$2^{2^{64}} \longrightarrow$ Biggest number which can imagine in the world.

if we apply 2 time's log on the value becomes polynomial value.

(9) Write following fun^c in asymptotic growth.

$n^{\sqrt{n}}$, $n^{\log n}$, $n\log n$, $\log n!$, $(\log n)!$

exp exp poly. poly exp

poly < exp

$\boxed{\text{ploynomials} < \text{exponential}}$

$n\log n = \log n!$

exponential fun^c:-

$n^{\log n} < n^{\sqrt{n}}$

2. $\log n$ $n^{\log n}$

Apply both side log

$\log m$ $\log n^m$

$\log n^2$

$\log(g(n)) > \log(f(n))$

$g(n) > f(n)$.

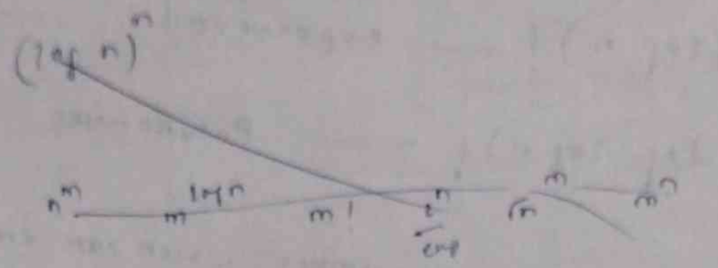
$$\boxed{(\log n)! < n^{\log n} < n^{\sqrt{n}}}$$

$$\boxed{n\log n = \log n < (\log n)! < n^{\log n} < n^{\sqrt{n}}}$$

Ans.

(10) $n^{\log n}, (\log n)^{\log n}, (\log n)!, 2^n, (\sqrt{n})^{\log n}, (\log n)^n$

exp   exp   exp   exp   exp exp poly   exp poly



$< 2^n$

$< 2^n$

(1) $n^{\log n} > (\sqrt{n})^{\log n} > (\log n)^{\log n}$

$(\log n)^{\log n} < (\sqrt{n})^{\log n} < n^{\log n}$

(2) $2^n < (\log n)^n$ (2 is constant & $2 < \log n$).

$[\because m! < m^n]$

(3) $(\log n)! < (\log n)^{\log n}$

(4) $(\log n)^n > n^{\log n}$ Apply log

$n \log(\log n) > (\log n)^2$

$(\log n)! < \log n^{\log n} < \sqrt{n}^{\log n} < n^{\log n} < \log n^n$

(5) 2^n $n^{\log n}$ Apply log both side

$2^n > n^{\log n}$

$(\log n)! < \log n^{\log n} < \sqrt{n}^{\log n} < n^{\log n} < 2^n < \log n^n$

(11) x^n, x^{2n}, x^{n+1} [x > 2 is const]

$2^{n/2}, 2^n, (\sqrt{n})^n, n^{\log_2 \sqrt{n}}$

write the func^n in Asymptotic increasing order.

$= x^n \pm x^{n+1} < x^{2n}$ - sol

$- \quad 2^{n/2} < 2^n$ - eq

$= n^{\log \sqrt{n}} < (\sqrt{n})^n$

$2^n \rightarrow n^{\log \sqrt{n}}$

$(\sqrt{n})^n \qquad (\sqrt{n})^n$

$\longrightarrow 2^{n/2} < 2^n < x^n = x^{n+1} < x^{2n}$

$2^n, x^n, x^{2n}$ compare with $(\sqrt{n})^n$

$f(n) = (\sqrt{n})^n$

$g(n) = x^{2n}$ x is constant.

Apply log.

$\boxed{n \log n > 2n \log x}$ constant.

$n \log n > n$ ✓

$n^{\log \sqrt{n}} = \sqrt{n} \log n$

$n^{\log \sqrt{n}} < 2^{n/2} < 2^n < x^n = x^{n+1} < x^{2n} < (\sqrt{n})^n$

(12) $f_1(n) = \log \log \log^k n$ $\qquad f_4 = \log \log \log^k n$

$f_2(n) = \log \log^k \log n$

$f_3(n) = \log^k \log \log n$ $\qquad f_1 > f_2 = f_3 = f_4$

write the Asymptotic order..?

$f_1(n) = \log . \log (k \log n) = \underline{\log \log k + \log . \log \log n}$

$f_4(n) = \log \log (\log n)^k = \log \log (k \dots \log \log n) = \log k + \log \log \log n$

$f_2(n) = \log (\log \log n)^k = k * \log \log \log n$

$f_3(n) = (\log \log \log n)^k$ $\qquad \boxed{f_1 = f_2 = f_4 < f_3}$

Omega notation

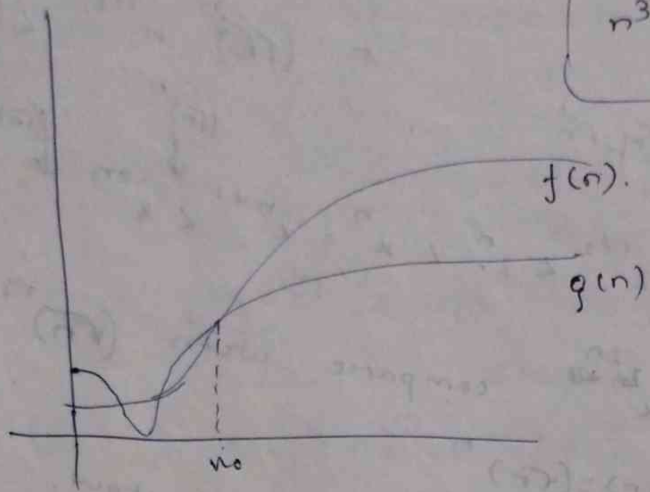
$f(n)$ and $g(n)$ non-neg function then

$f(n) = \Omega(g(n))$ iff $f(n) >= c(g(n))$

where $n \geq n_0$ & c, n, n_0 are constant

eg.

$$\boxed{\begin{array}{l} n^2 = O(n^3) \\ n^3 = \Omega(n^2 \end{array}}$$



	$n=1$
n^3+10n^2	11
$2n^3$	2

① → $f(n) = \Omega(g(n))$ iff

$g(n) \leq f(n)$ or $g(n) = f(n)$.

lesser, equal

② $f(n) = \Omega(g(n))$ iff

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \text{constant or } \infty$$

③ $f(n) = \Omega(g(n))$ iff any only if.

$g(n) = O(f(n))$

example

① find which is true/false ?

(i) $\log n = \Omega(n \log n)$ ✓ equal T

(2) $2^n = \Omega(2^{n/2})$ ✓ True.

(3) $2^n = \Omega(n!)$ False. $n! > 2^n$ (Apply log) $\log n! = n \lg n$

(4) $n! = \Omega(n^n)$ False. $n^n > n!$

(5) $n^2+n = \Omega(n)$ $n^2 > n$ True

(6) $n^2+n = \Omega(n^2)$ equal True.

(7) $n^2+n = \Omega(n^3)$ False.

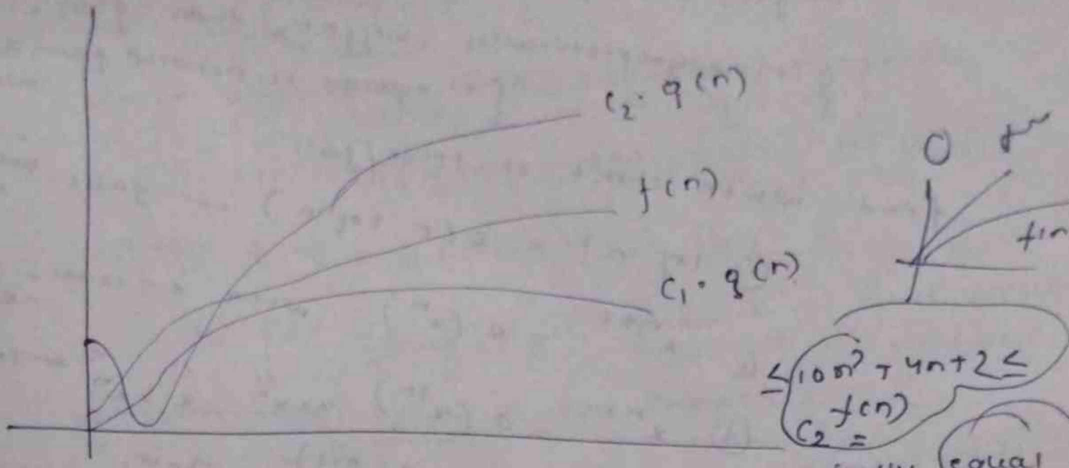
Theta Notation

$f(n)$ and $g(n)$ non negative function then

$f(n) = \theta(g(n))$ iff $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$

for all n values when $n \geq n_0$

c_1, c_2, n_0 are constant.



① Iff functions are asymptotically equal then only Theta can be applied.

eg- $c_1 n \leq 2n + 3 \leq c_2 n$ $\quad f(n) = 2n+3$

$g(n) = n$

so $2n+3 = \theta(n)$

② $f(n) = 2n+3$

$g(n) = \log_{10} n$

$c_1 \cdot \log n <= 2n+3 \leq c_2 \cdot \log n$

By taking constant it can be true.

But taking any constant it cant be true.

$(2n+3) \neq \theta(\log n)$

$\longrightarrow$ $f(n) = \theta(g(n))$ and $f(n)$ and $g(n)$ Asymptotical equal

$\longrightarrow$ $f(n) = \theta(g(n))$ ~~and~~ iff

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \text{constant.} (> 0)$$

$\longrightarrow$ $f(n) = \theta(g(n))$ iff.

$f(n) = O(g(n))$ and

$f(n) = \Omega(g(n))$.

Little Oh or small oh. (o).

f(n) and g(n) are   f(n) = o(g(n)) iff -

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$ g(n) asymptotically bigger than f(n)

→ f(n) = o(g(n)) iff

g(n) asymptotically bigger than f(n).

[∵ equality is removed from Big Oh notation.

find which stmt is true/false

① $\log n! = o(n \log n)$ — false both are equal

② $x^{n+1} = o(x^n)$ when x = const — false - equal

③ $x^{n+1} = o(x^{2n})$ x = x'' " x > x'' — True.

④ $n^2 \log n = n^2(n^{0.1})$ True.

⑤ $n^2 (\log n)^{10} = o(n^{2+0.1})$ True.

$(\log n)^k < n^\epsilon$

little omega (ω)

→ f(n) and g(n) are non-negative func. f(n) = ω g(n)

iff $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$ f(n) is asymptotically bigger g(n).

→ f(n) = ω(g(n)) iff g(n) asymptotically small than f(n).

→ f(n) = ω(g(n)) is true. then g(n) = o(f(n)) is also true.

$3^{\log 4} < n^{\log 4} \xrightarrow{\text{convert}} 4^{\log_2 n}$

$\sqrt{n} = 2^{\log \sqrt{n}} \longleftarrow \sqrt{n}^{\log_2 2}$

① which are the notation follows reflexive properties.

O, Ω, θ is true.

$$\left.\begin{aligned} f(n) &= O(f(n)) \\ &= \Omega(f(n)) \\ &= \theta(f(n)) \end{aligned}\right\} \text{follows reflexive.}$$

② which notation follow transitive properties

① $\overbrace{f(n) = O(g(n)}^{f(n) >= f(n)}$ and $\overbrace{g(n) = O(h(n))}^{f(n) >= f(n)}$

then $f(n) = O(h(n))$ is true.

② For $\Omega, \theta, o, \omega$ transitive rule follows are true.

③ ~~Anti-sy~~ Commutative properties

if $f(n) = \theta(g(n))$ then $\theta(g(n)) = \theta(f(n))$

For $\underline{\theta}$ notation it is only possible. for all other it is not possible.

④ Anti-symmetric :-

$f(n) = O(g(n))$ iff $g(n) = \Omega(f(n))$

$f(n) = o(g(n))$ iff $g(n) = \omega(f(n))$

so (O, Ω) are anti symmetric & (o, ω) are also anti symmetric.

$(\cancel{x + a})^3$

Binomial Expansion.

$$(x + a)^n = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0$$

$$= x^n \left[a_n + \frac{a_{n-1}}{x} + \cdots \frac{a_1}{x^{n-1}} + \frac{a_0}{x^n}\right]$$

$$= a_n x^n \leq (x + a)^n \leq (a_n + a_{n-1} + \cdots + a_0) x^n$$

$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n).$$

$$\boxed{(x + a)^n = \theta(x^n)}$$

$\boxed{(x + a)^n \text{ and } x^n \text{ both are asymptotically equal}}$

(2) $f(n) = O(h(n))$ and $g(n) = O(s(n))$

$f(n) + g(n) = ?$

which is true.

(a) $f(n) + g(n) = O(\max(h(n), s(n)))$ — true.

(b) $f(n) + g(n) = O(\min(h(n), s(n)))$ — false.

$\underline{2n+3} = O(n)$ $n^2 + n = O(n^2)$

$f(n)$ $h(n)$ $g(n)$ $s(n)$.

$f(n) + g(n) = 2n+3 + n^2 + n$

$= n^2 + 3n + 3 = O(\max(h(n), s(n)))$ n^2 n

$= O(n^2)$ is true.

(3) $f(n) = \Omega(h(n))$ $g(n) = \Omega(s(n))$

(a) $f(n) + g(n) = \Omega(\max(h(n), s(n)))$ - True

(b) $f(n) + g(n) = \Omega(\min(h(n), s(n)))$ - True.

(c) Both a and b

(d) None.

$2n+3 = \Omega(n)$ $n^2 + n = \Omega(n^2)$

$f(n)$ $h(n)$. $g(n)$ $s(n)$

$f(n) + g(n) = n^2 + 3n + 3 = \Omega(\max - -) = n^2$

$= \Omega(\min - - -) = n$

smaller or <u>equal</u>

so both are true.

(4) $f(n) = \Theta(h(n))$ $g(n) = \Theta(s(n))$

(a) $f(n) + g(n) = \Theta(\max(h(n), s(n)))$ - True

(b) $f(n) + g(n) = \Theta(\min h(n), s(n))$ - false.

$2n+3 = \Theta(n)$ $n^2 + n = \Theta(n^2)$

$f(n) + g(n) = n^2 + 3n + 3 = \Theta(\max - - -) = n^2$ true

(4) $f(n) = O(h(n))$ $g(n) = O(s(n))$

(a) $f(n) + g(n) = O(\max(h(n), s(n)))$ — True

(b) $f(n) + g(n) = O(\min(h(n), s(n)))$ — false

$2n + 3 = O(n^2)$ $n^2 + n = O(n^3)$

$f(n) + g(n) = n^2 + 3n + 3 = O(n^3)$ True. max.

(5) $f(n) = \omega(h(n))$ $g(n) = \omega(s(n))$

(a) $f(n) + g(n) = \omega(\max$ ——— $)$ — True

(b) $f(n) + g(n) = \omega(\min$ ——— $)$ — True.

$2n + 3 = n^{0.5}$ $n^2 + n = \omega(n)$

$2n + 3 + n^2 + n$ = max and min is true.

(6) $f(n) = O(g(n))$ then $2^{f(n)} = O(2^{g(n)})$ is true or false.

Stmt is false.

$2^{\cancel{\log n}} = \cancel{\log n}$ ~~$\log 2 = n$~~

$f(n) = 2n + 3$ $g(n) = n$

$= \cancel{O(2^{n})}$

but $2^{2n} = O(2^n)$ is false.

(7) $f(n) = O(f(n)^2)$

$f(n) = \frac{1}{2}$

$\frac{1}{n} = O\left(\frac{1}{n^2}\right)$ is false.

So $f(n) = O(f(n)^2)$ is false for all decrement function. This stmt is true if $f(n)$ is increment function. (Overall it is false bcoz of decrement fnⁿ)