Introduction to Automata Theory

Reading: Chapter 1

What is Automata Theory?

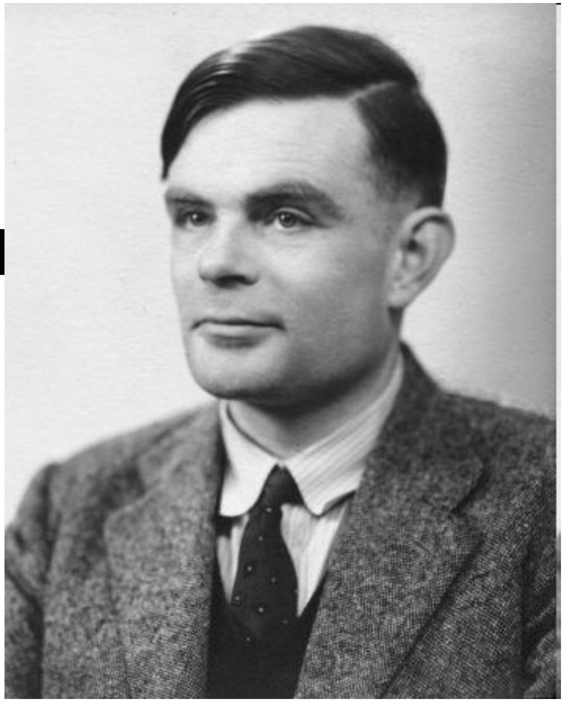
- *Study of abstract computing devices, or "machines"*
- Automaton = an abstract computing device
  - Note: A "device" need not even be a physical hardware!
- A fundamental question in computer science:
  - Find out what different models of machines can do and cannot do
  - The *theory of computation*
- Computability vs. Complexity

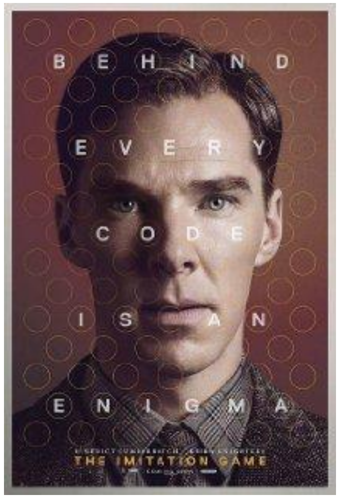
(A pioneer of automata theory)

Alan Turing (1912-1954)

- Father of Modern Computer Science
- English mathematician
- Studied abstract machines called ***Turing machines*** even before computers existed

Heard of the Turing test?





Theory of Computation: A Historical Perspective

1930s	• Alan Turing studies Turing machines • Decidability • Halting problem
1940-1950s	• "Finite automata" machines studied • Noam Chomsky proposes the "Chomsky Hierarchy" for formal languages
1969	Cook introduces "intractable" problems or "NP-Hard" problems
1970-	Modern computer science: compilers, computational & complexity theory evolve

Languages & Grammars

Or “**words**”

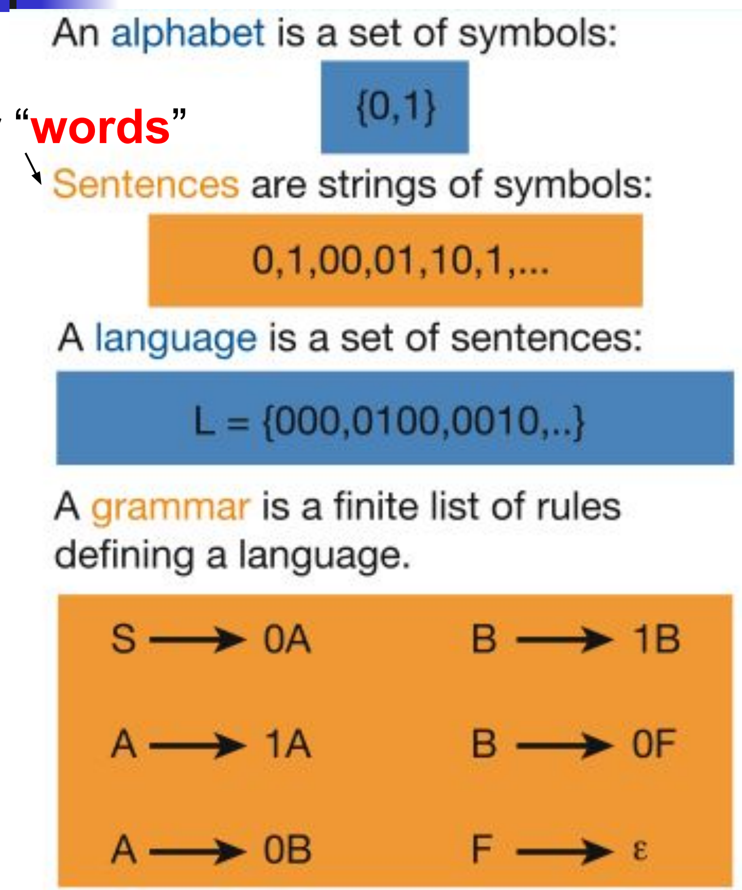
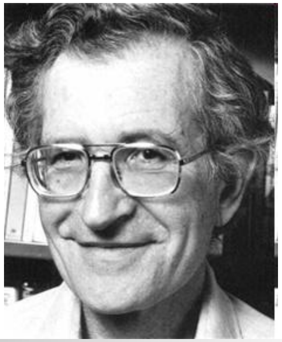


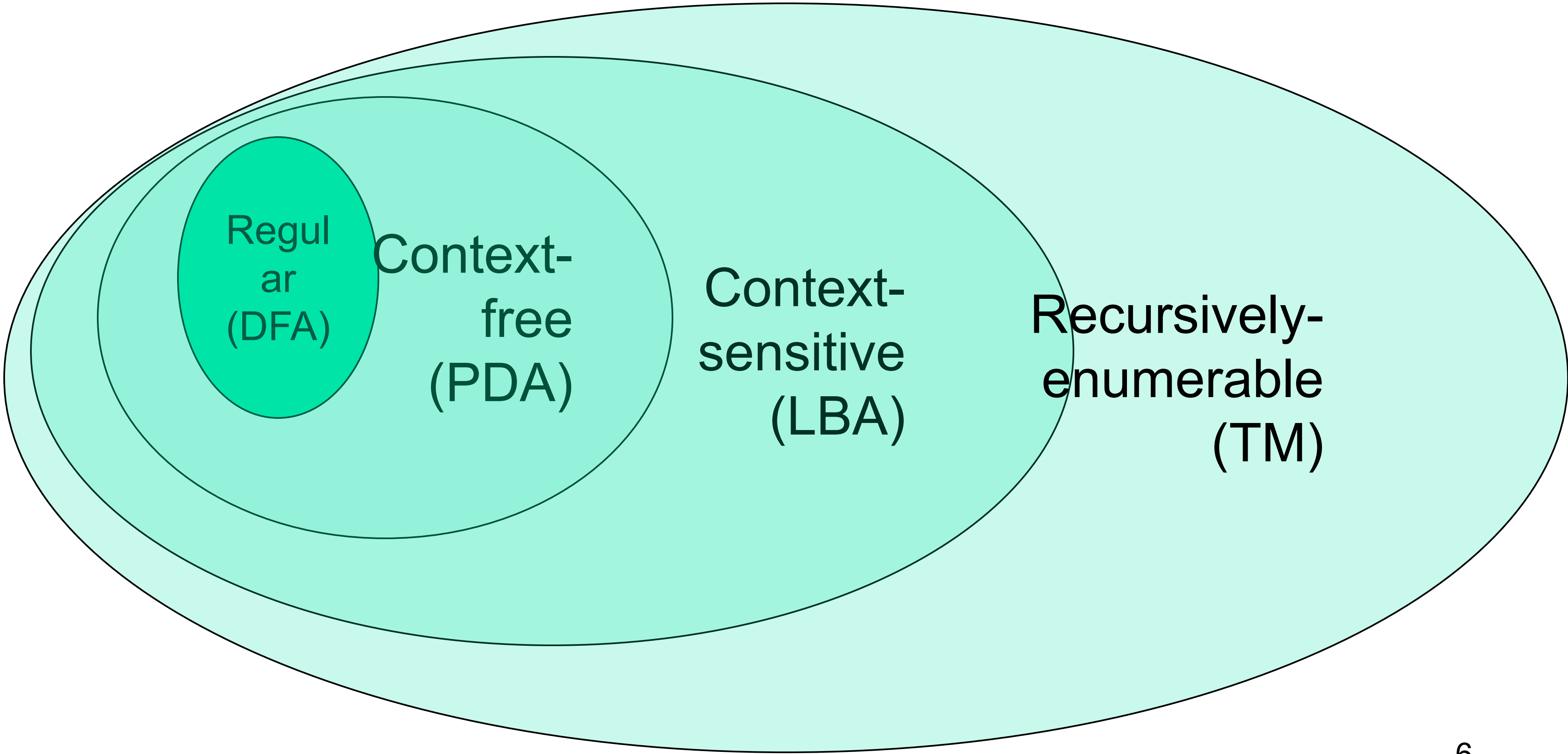
Image source: Nowak et al. Nature, vol 417, 2002

- <u>Languages</u>: “*A language is a collection of sentences of finite length all constructed from a finite alphabet of symbols*”
- <u>Grammars</u>: “*A grammar can be regarded as a device that enumerates the sentences of a language*” - nothing more, nothing less
- *N. Chomsky, Information and Control, Vol 2, 1959*

The Chomsky Hierachy



- A containment hierarchy of classes of formal languages



The Central Concepts of Automata Theory

Alphabet

An alphabet is a finite, non-empty set of symbols

- We use the symbol ∑ (sigma) to denote an alphabet
- Examples:
  - Binary: ∑ = {0,1}
  - All lower case letters: ∑ = {a,b,c,..z}
  - Alphanumeric: ∑ = {a-z, A-Z, 0-9}
  - DNA molecule letters: ∑ = {a,c,g,t}
  - …

Strings

A string or word is a finite sequence of symbols chosen from ∑

- ***Empty string is ε (or "epsilon")***

- Length of a string *w*, denoted by "|*w*|", is equal to the *number of (non- ε) characters in the string*
  - *E.g., x = 010100* |*x*| = *6*
  - *x = 01 ε 0 ε 1 ε 00 ε* |*x*| = *?*

- *xy* = concatentation of two strings *x* and *y*

Powers of an alphabet

Let Σ be an alphabet.

- Σ^k = the set of all strings of length k
- $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \ldots$
- $\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \ldots$

Languages

L is a said to be a language over alphabet $\sum$, only if L $\subseteq \sum^*$

- this is because $\sum^*$ is the set of all strings (of all possible length including 0) over the given alphabet $\sum$

Examples:

1. Let L be *the* language of all strings consisting of *n* 0's followed by *n* 1's:
   L = {ε, 01, 0011, 000111,…}
2. Let L be *the* language of all strings of with equal number of 0's and 1's:
   L = {ε, 01, 10, 0011, 1100, 0101, 1010, 1001,…}

Canonical ordering of strings in the language

Definition: Ø denotes the Empty language

- Let L = {ε}; Is L=Ø? NO

The Membership Problem

Given a string $w \in \Sigma^*$ and a language L over Σ, decide whether or not $w \in L$.

<u>Example:</u>

Let w = 100011

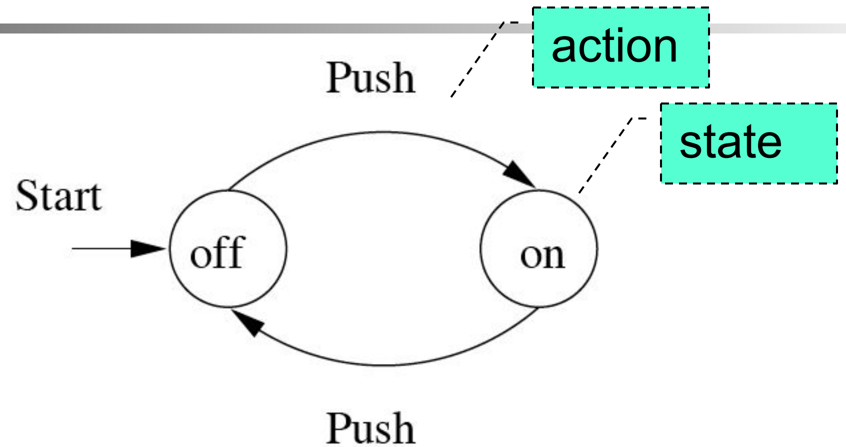
Q) Is $w \in$ the language of strings with equal number of 0s and 1s?

Finite Automata

- Some Applications
  - Software for designing and checking the behavior of digital circuits
  - Lexical analyzer of a typical compiler
  - Software for scanning large bodies of text (e.g., web pages) for pattern finding
  - Software for verifying systems of all types that have a finite number of states (e.g., stock market transaction, communication/network protocol)

Finite Automata : Examples

- On/Off switch



- Modeling recognition of the word "*then*"