Min-Max normalization is a technique used to scale data within a specific range, usually between 0 and 1. The formula for min-max normalization is:

$$X' = \frac{X - X_{\min}}{X_{\max} - X_{\min}}$$

Where:

- X is the original value.
- $X_{\min}$ is the minimum value in the dataset.
- $X_{\max}$ is the maximum value in the dataset.
- X' is the normalized value.

Example:

Suppose we have the following data points representing the ages of a group of people:

$$\text{Ages} = [18, 20, 30, 40, 50]$$

We want to normalize these ages between 0 and 1.

Step 1: Identify the Minimum and Maximum Values

- $X_{\min} = 18$
- $X_{\max} = 50$

Step 2: Apply the Min-Max Normalization Formula

We'll apply the formula to each age:

1. **For Age 18**:

$$X' = \frac{18 - 18}{50 - 18} = \frac{0}{32} = 0$$

2. **For Age 20**:

$$X' = \frac{20 - 18}{50 - 18} = \frac{2}{32} \approx 0.0625$$

3. **For Age 30**:

$$X' = \frac{30 - 18}{50 - 18} = \frac{12}{32} = 0.375$$

4. **For Age 40**:

$$X' = \frac{40 - 18}{50 - 18} = \frac{22}{32} \approx 0.6875$$

5. **For Age 50**:

$$X' = \frac{50 - 18}{50 - 18} = \frac{32}{32} = 1$$

Step 3: The Normalized Data

After applying min-max normalization, the normalized ages are:

$$\text{Normalized Ages} = [0, 0.0625, 0.375, 0.6875, 1]$$

Conclusion:

The ages have been scaled to fit within the range of 0 to 1, where 0 corresponds to the minimum age (18) and 1 corresponds to the maximum age (50). This technique is useful for ensuring that all features in a dataset contribute equally to models that are sensitive to the scale of the input data.

Z-score normalization (also known as standardization) transforms data to have a mean of 0 and a standard deviation of 1. This is particularly useful when features have different scales but need to be compared or combined. The formula for Z-score normalization is:

$$Z = \frac{X - \mu}{\sigma}$$

Where:

- X is the original value.
- μ is the mean of the dataset.
- σ is the standard deviation of the dataset.
- Z is the normalized value.

Example:

Suppose we have the following data points representing the heights (in inches) of a group of people:

$$\text{Heights} = [60, 62, 67, 70, 72]$$

Step 1: Calculate the Mean (μ)

The mean height is calculated as:

$$\mu = \frac{60 + 62 + 67 + 70 + 72}{5} = \frac{331}{5} = 66.2 \text{ inches}$$

Step 2: Calculate the Standard Deviation (σ)

The standard deviation is calculated as:

$$\sigma = \sqrt{\frac{(60 - 66.2)^2 + (62 - 66.2)^2 + (67 - 66.2)^2 + (70 - 66.2)^2 + (72 - 66.2)^2}{5}}$$

Breaking this down:

$$\sigma = \sqrt{\frac{(-6.2)^2 + (-4.2)^2 + (0.8)^2 + (3.8)^2 + (5.8)^2}{5}}$$

$$\sigma = \sqrt{\frac{38.44 + 17.64 + 0.64 + 14.44 + 33.64}{5}}$$

$$\sigma = \sqrt{\frac{104.8}{5}} = \sqrt{20.96} \approx 4.58 \text{ inches}$$

Step 3: Apply the Z-Score Normalization Formula

We'll apply the formula to each height:

1. **For Height 60 inches:**

$$Z = \frac{60 - 66.2}{4.58} \approx \frac{-6.2}{4.58} \approx -1.35$$

2. **For Height 62 inches:**

$$Z = \frac{62 - 66.2}{4.58} \approx \frac{-4.2}{4.58} \approx -0.92$$

3. **For Height 67 inches:**

$$Z = \frac{67 - 66.2}{4.58} \approx \frac{0.8}{4.58} \approx 0.17$$

4. **For Height 70 inches:**

$$Z = \frac{70 - 66.2}{4.58} \approx \frac{3.8}{4.58} \approx 0.83$$

5. **For Height 72 inches:**

$$Z = \frac{72 - 66.2}{4.58} \approx \frac{5.8}{4.58} \approx 1.27$$

Step 4: The Normalized Data

After applying Z-score normalization, the normalized heights are:

$$\text{Normalized Heights} = [-1.35, -0.92, 0.17, 0.83, 1.27]$$