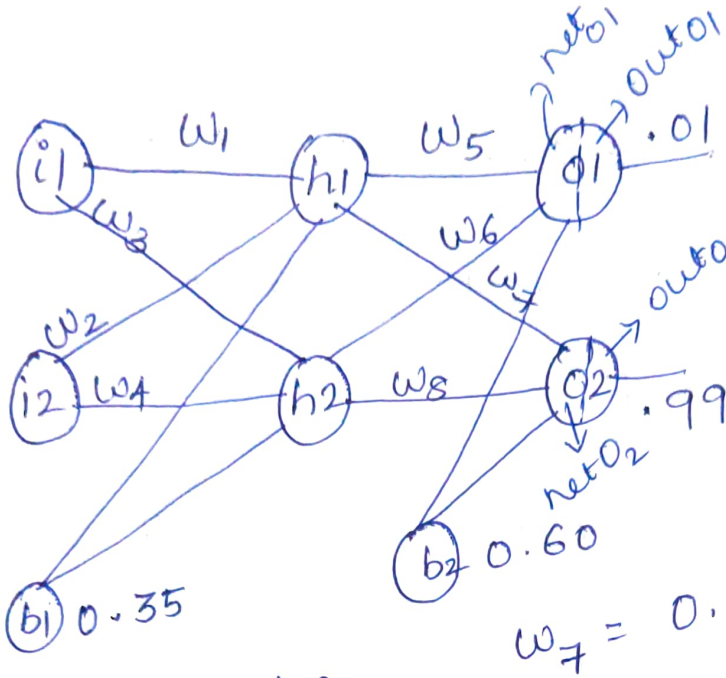
Backward propagation



$i1 = .05$
$i2 = .10$
$w_1 = .15$
$w_2 = 0.20$
$w_3 = 0.25$
$w_4 = 0.30$
$w_7 = 0.50$
$w_8 = 0.55$
$01 = .01$
$02 = 0.99$
$w_5 = 0.40$
$w_6 = 0.45$

Hidden layer 1

$$net_{h_1} = w_1 \times i_1 + w_2 \times i_2 + b_1 \times 1$$
$$= 0.15 \times 0.05 + 0.20 \times 0.10 + 0.35 \times 1$$
$$= 0.3775$$

$$Out_{h1} = \frac{1}{1 + e^{-net_{h_1}}} = \frac{1}{1 + e^{-0.3775}} = 0.5933$$

Hidden layer 2

$$net_{h_2} = w_3 \times i_3 + w_4 \times i_2 + b_1 \times 1$$
$$(0.0125 + 0.03 + 0.35)$$
$$= 0.25 \times 0.05 + 0.30 \times 0.10 + 0.35 \times 1$$
$$= 0.~~0125~~ ~~0.675366~~ \; 0.3925$$

$$out_{h_2} = 1/1 + e^{-net_{h_2}} = 1/1 + e^{~~0.675366~~ \; 0.3925}$$

$out_{h2} = 0.59688$

Output $(O_1) = w_5 \times Out_{h1} + w_6 \times Out_{h2} + b_2 \times 1$

$= 0.4 \times 0.59327 + 0.45 \times 0.59688 + 0.6 \times 1$

$= 1.05905$

$= \frac{1}{1 + e^{-1.05905}} = 0.75317$

Output $(O_2) = w_7 \times Out_{h1} + w_8 \times Out_{h2} + b_2 \times 1$

$= 0.50 \times 0.59327 + 0.55 \times 0.59688 + 0.6 \times 1$

$= 1.224919$

$\frac{1}{1 + e^{-1.224919}} = 0.772928$

Compute the total Error.

$E_{tot} = \sum \frac{1}{2} (target - Output)^2$

for $E_{01} = \frac{1}{2} (0.01 - 0.75317)$

$= 0.274811$

$$E_{O2} = \frac{1}{2}(0.99 - 0.772928)$$

$$= 0.023547$$

$$E_{tot} = E_{O1} + E_{O2} = 0.274811 + 0.023547$$

$$= 0.298358$$

Backward propagation [pass]

consider w_5

$$\frac{\partial E_{tot}}{\partial w_5} = \frac{\partial E_{tot}}{\partial out_{O1}} \times \frac{\partial out_{O1}}{\partial net_{O1}} \times \frac{\partial net_{O1}}{\partial w_5}$$

$$E_{O1} = \frac{1}{2}(\text{target} - \text{Output})^2$$

$$\frac{\partial E_{total}}{\partial Out_{O1}} = \frac{1}{2}(target_{O1} - O/P_{out_1})^2 + \frac{1}{2}(target_{O2} - O/P_{out_2})^2$$

$$= 2 \cdot \frac{1}{2}(target_{output_1} - Output_{output_2})^{2-1} \times -1 + 0$$

$$= -(target_{O1} - Output_{O1})$$

$$= (Out_{O1} - target_{O1})$$

$$= (0.75317 - 0.01)$$

$$= 0.74136$$

$$Out_{o1} = \frac{1}{1 + e^{-net_{o1}}}$$

$$\frac{\partial Out_{o1}}{\partial net_{o1}} = Out_{o1}(1 - Out_{o1})$$

$$= 0.75317(1 - 0.75317)$$

$$= 0.18590$$

How much the total net i/p of O_1 change w.r.t w_5?

$$net_{o1} = w_5 \times Out_{h1} + w_6 \times Out_{h2} + b_2 \times 1$$

$$\frac{\partial net_{o1}}{\partial w_5} = 1 \times Out_{h1} \times w_5^{(1-1)} + 0 + 0$$

$$= Out_{h1} + 0 = 0.5933$$

Putting all together

$$\frac{\partial E_{tot}}{\partial w_5} = \frac{\partial E_{tot}}{\partial Out_{o1}} \times \frac{\partial Out_{o1}}{\partial net_{o1}} \times \frac{\partial net_{o1}}{\partial w_5}$$

$$= 0.74136 \times 0.18590 \times 0.5933$$

$$= 0.082$$

$\frac{\partial E_{tot}}{\partial w_5}$ can be written as

$$-(target_{o1} - Out_{o1}) \times Out_{o1}(1 - Out_{o1}) \times Out_{o1}$$

Alternatively

$$\frac{\partial E_{tot}}{\partial O_1} \text{ and } \frac{\partial O_1}{\partial net_{O_1}} \Rightarrow \frac{\partial E_{tot}}{\partial net_{O_1}}$$

$$\delta_{O_1} = \frac{\partial E_{tot}}{\partial Out_{O_1}} \times \frac{\partial Out_{O_1}}{\partial net_{O_1}} = \frac{\partial E_{tot}}{\partial net_{O_1}}$$

$$= -(target_{O_1} - Out_{O_1}) \times Out_{O_1}(1 - Out_{O_1})$$

$$\frac{\partial E_{tot}}{\partial w_5} = -\delta_{O_1} \times Out_{h_1}$$

$$w_5^+ = w_5 - \eta \times \frac{\partial E_{tot}}{\partial w_5}$$

$$= 0.4 - 0.5 \times 0.082$$

$$= 0.359$$

$$w_6^+ = w_6 - \eta \times \frac{\partial E_{tot}}{\partial w_6}$$

$$= 0.45 - 0.5 \times 0.082$$

$$= 0.409$$

$$w_7^+ = w_7 - \eta \times \frac{\partial E_{tot}}{\partial w_7}$$

$$= 0.50 - 0.5 \times 0.079923$$

$$= 0.46003$$