Course title : **CSE2001**

Course title : **Data Structures and Algorithms**

Module : **6**

Topic : **3**

Minimum Spanning Tree

Objectives

This session will give the knowledge about

- Minimum Spanning Tree
- Prim’s Algorithm
- Kruskal’s Algorithm

Introduction to MST

A Minimum Spanning Tree (MST) is a sub-graph of an undirected connected graph, which includes all the vertices of the graph with a minimum possible number of edges (n-1 edges).

Minimum spanning tree has direct application in the design of networks. It is used in algorithms approximating the travelling salesman problem, multi-terminal minimum cut problem and minimum-cost weighted perfect matching. Other practical applications are:

- Cluster Analysis
- Handwriting recognition
- Image segmentation

Constrains on MST

1. All vertices must be connected
2. N-1 edges should be there
3. Cost of Spanning tree (Addition of all edges' cost) must be minimum
4. No Cycle

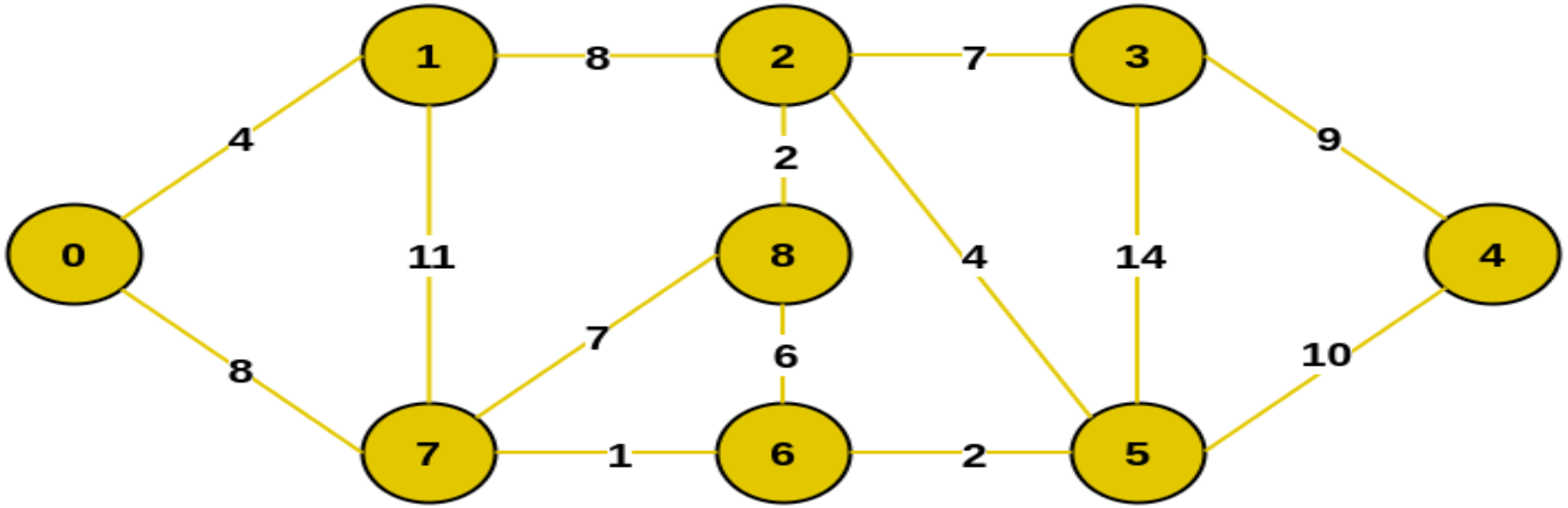
Algorithms:

- Prim's Algorithm
- Kruskal's Algorithm

Rules for Prim's Algorithm

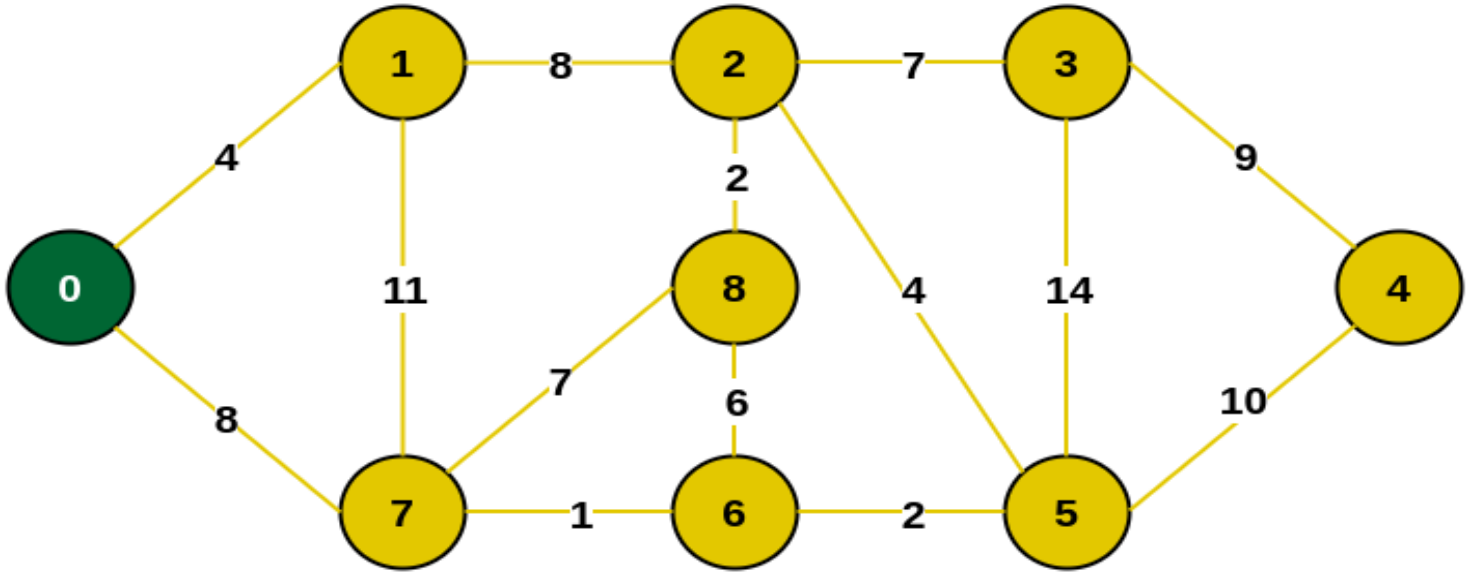
1. Remove self loops
2. Remove parallel edges
3. Select any vertices (smallest vertices)
4. Select the smallest connected next vertices

Prim’s Algorithm



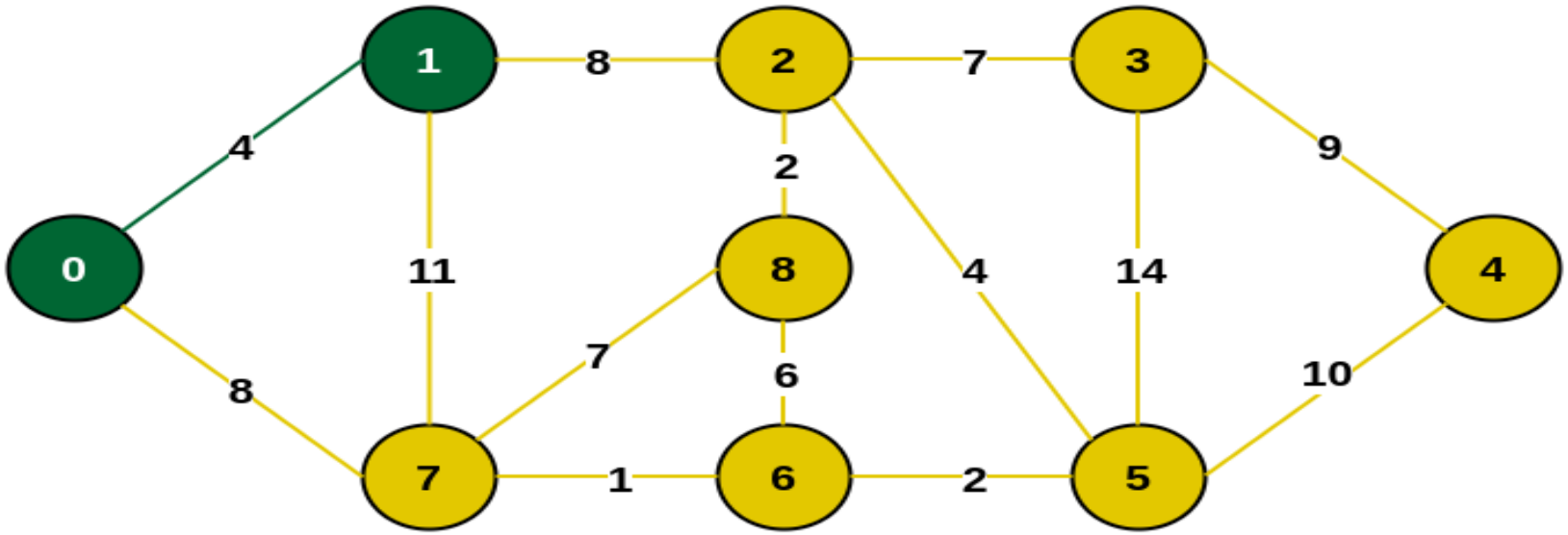
Example of a Graph

Prim’s Algorithm



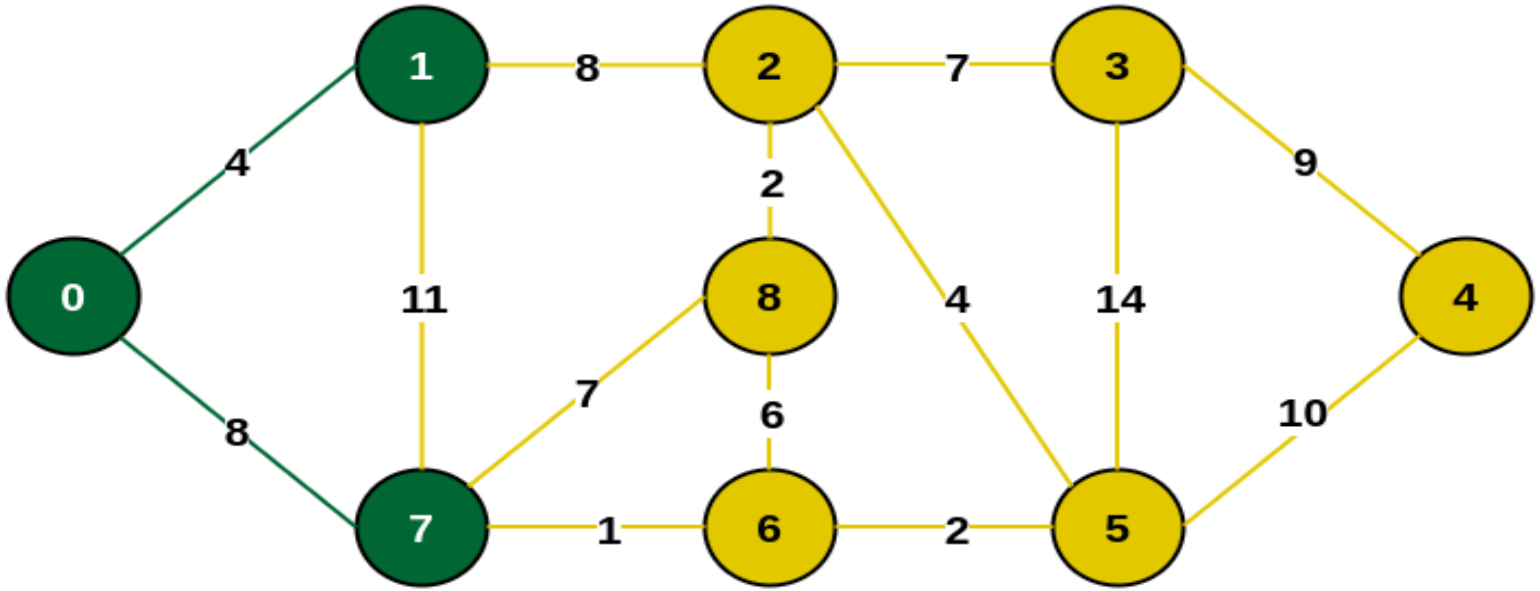
Select an arbitrary starting vertex. Here we have selected 0

Prim’s Algorithm



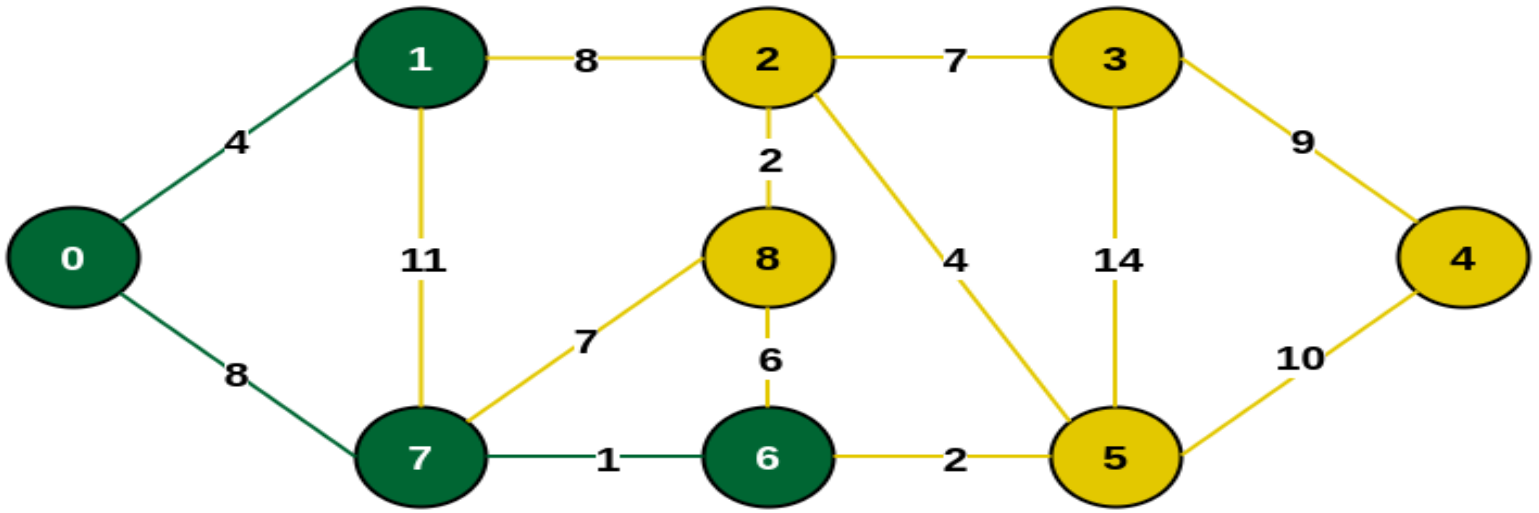
Minimum weighted edge from MST to other vertices is 0-1 with weight 4

Prim’s Algorithm



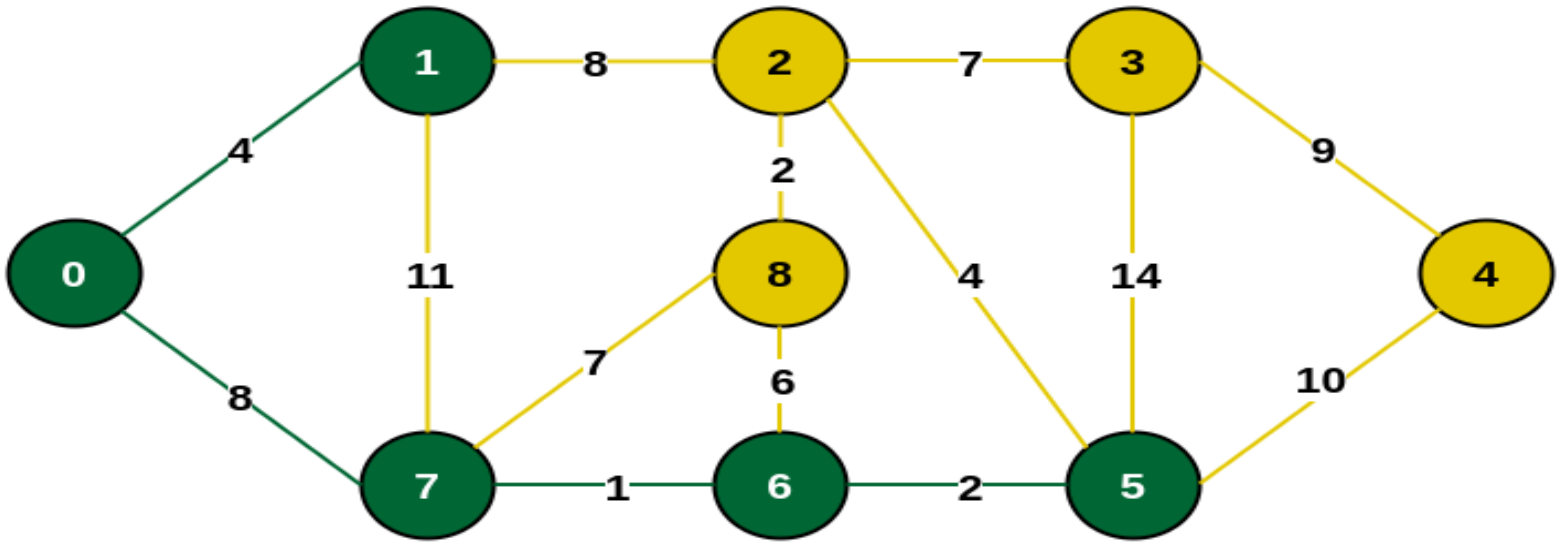
Minimum weighted edge from MST to other vertices is 0-7 with weight 8

Prim’s Algorithm

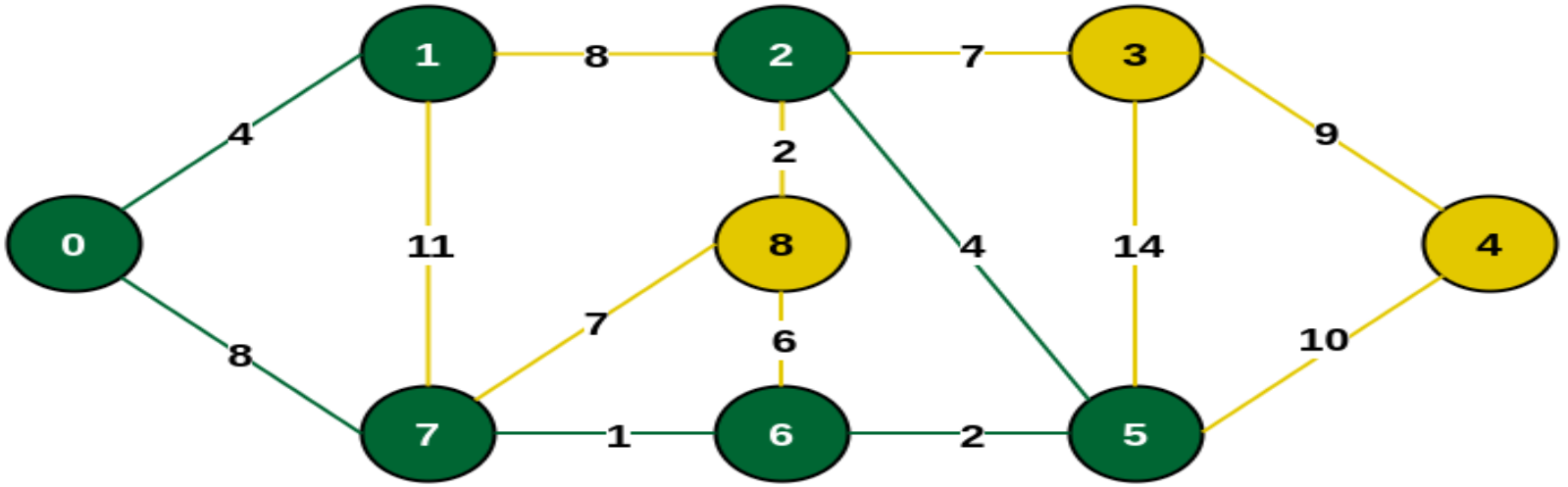


Minimum weighted edge from MST to other vertices is 7-6 with weight 1

Prim’s Algorithm

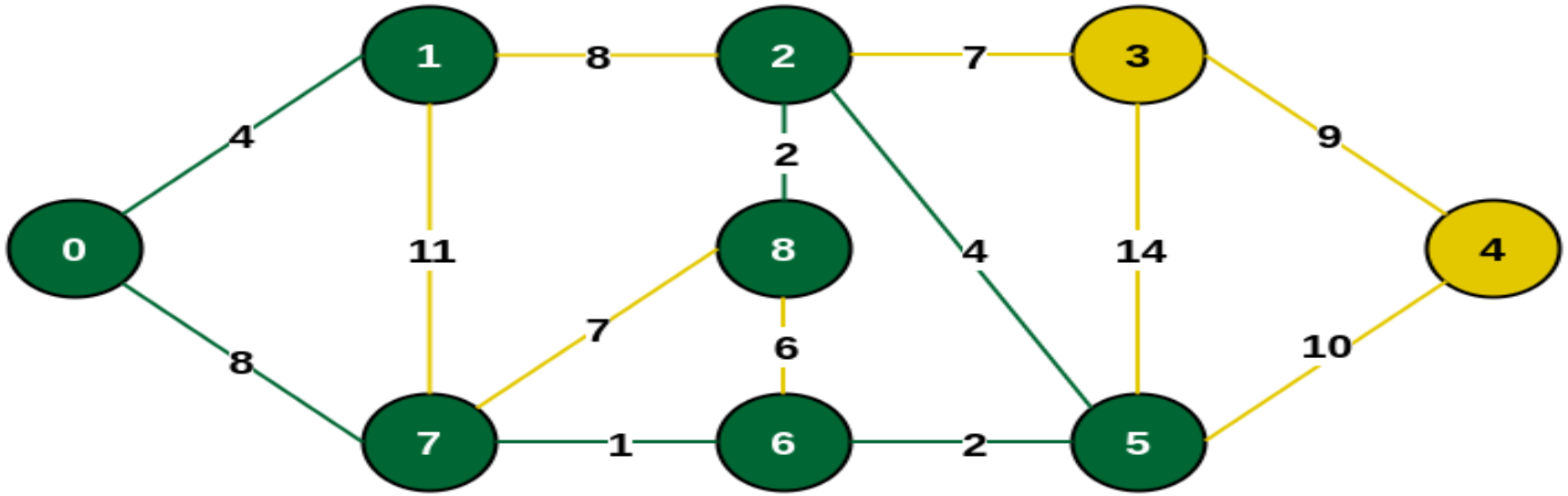


Prim’s Algorithm



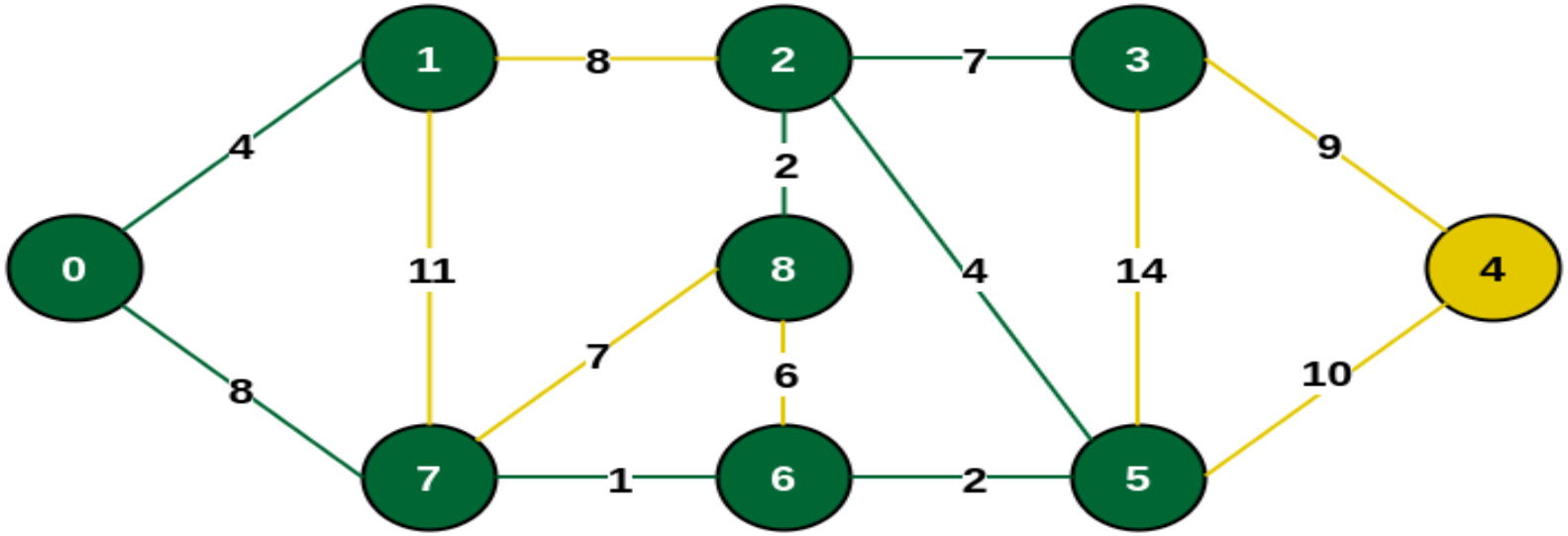
Minimum weighted edge from MST to other vertices is 5-2 with weight 4

Prim’s Algorithm



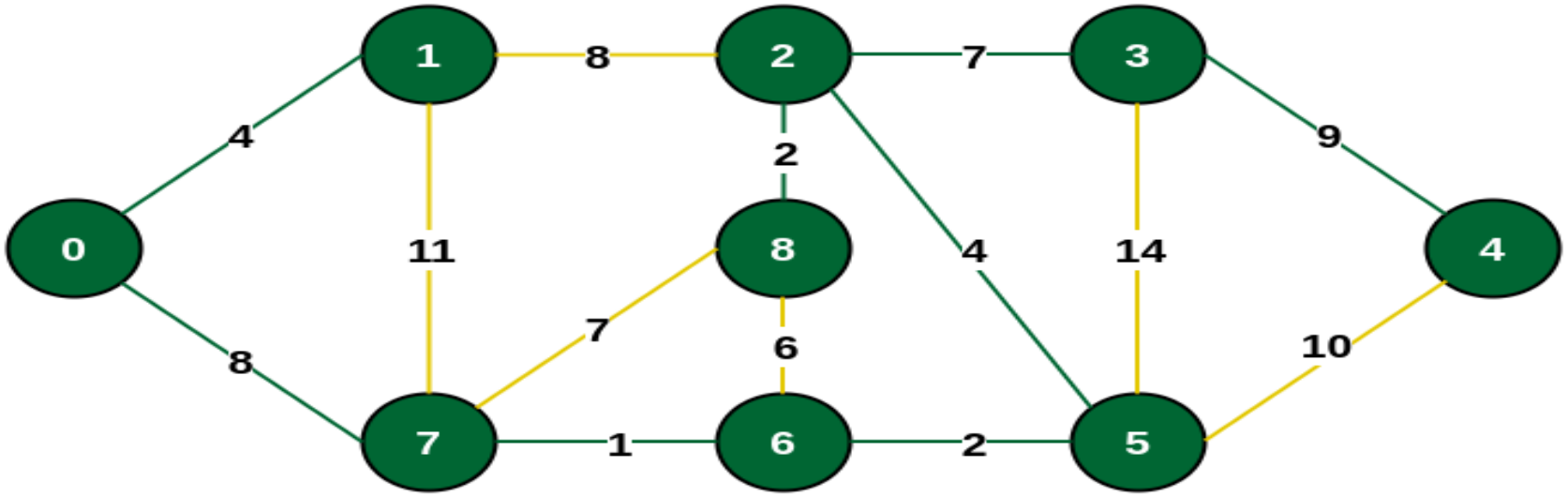
Minimum weighted edge from MST to other vertices is 2-8 with weight 2

Prim’s Algorithm



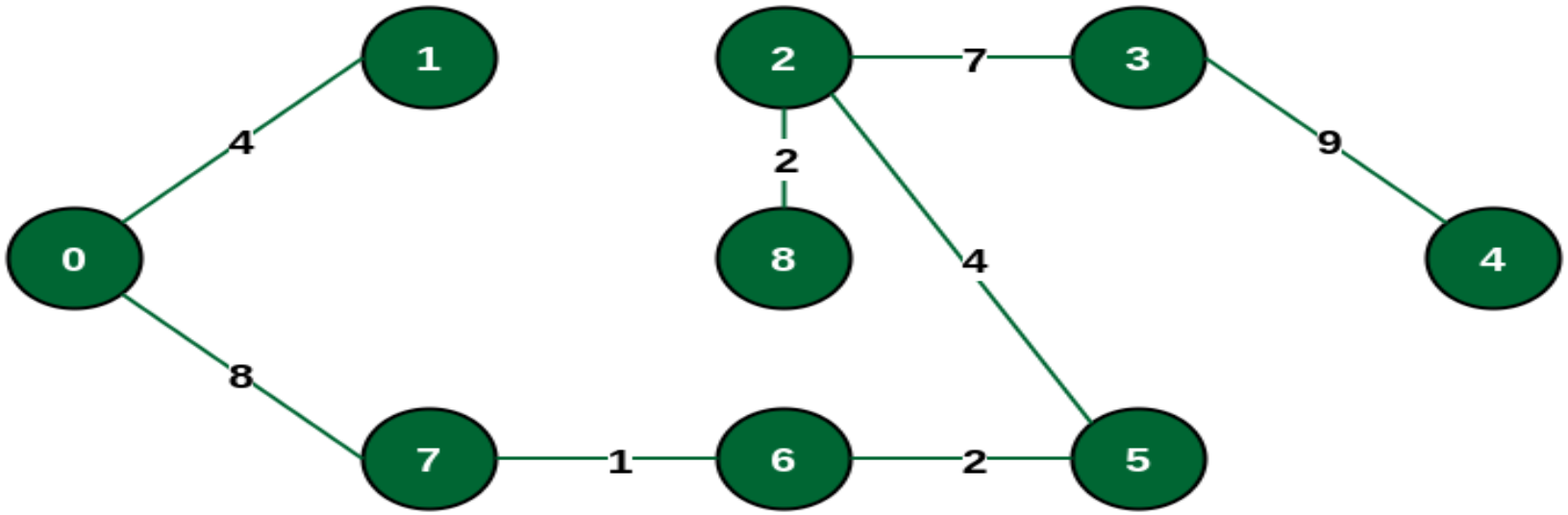
Minimum weighted edge from MST to other vertices is 2-3 with weight 7

Prim’s Algorithm



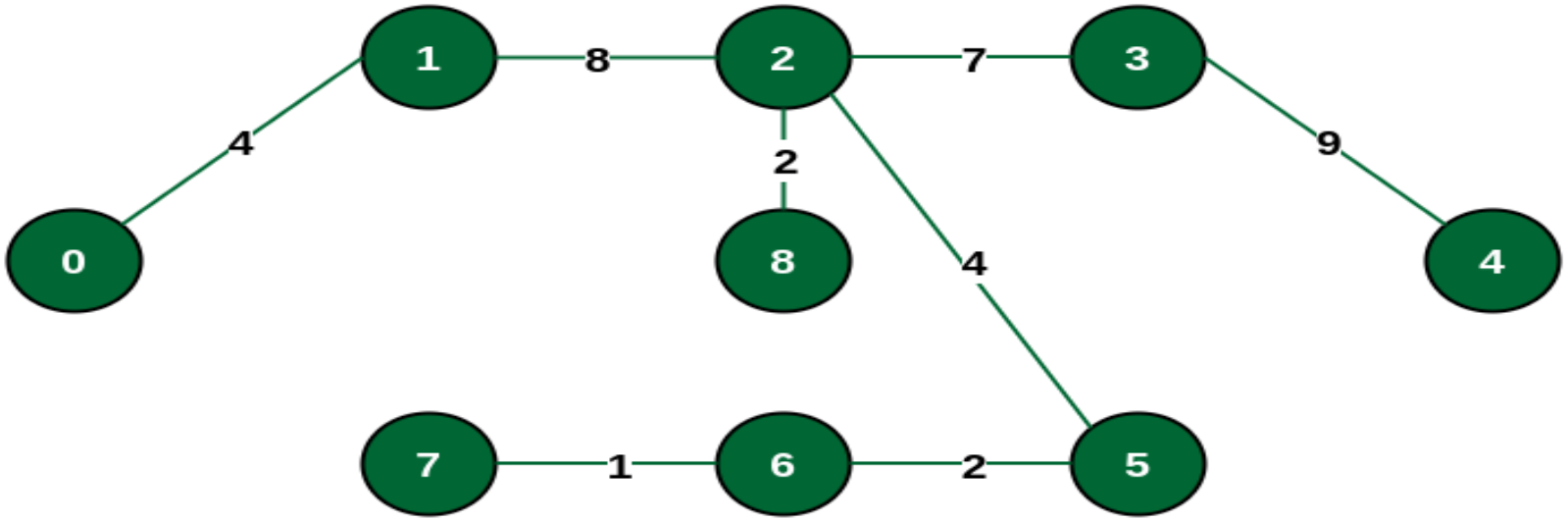
Minimum weighted edge from MST to other vertices is 3-4 with weight 9

Prim’s Algorithm



The final structure of MST

Prim’s Algorithm



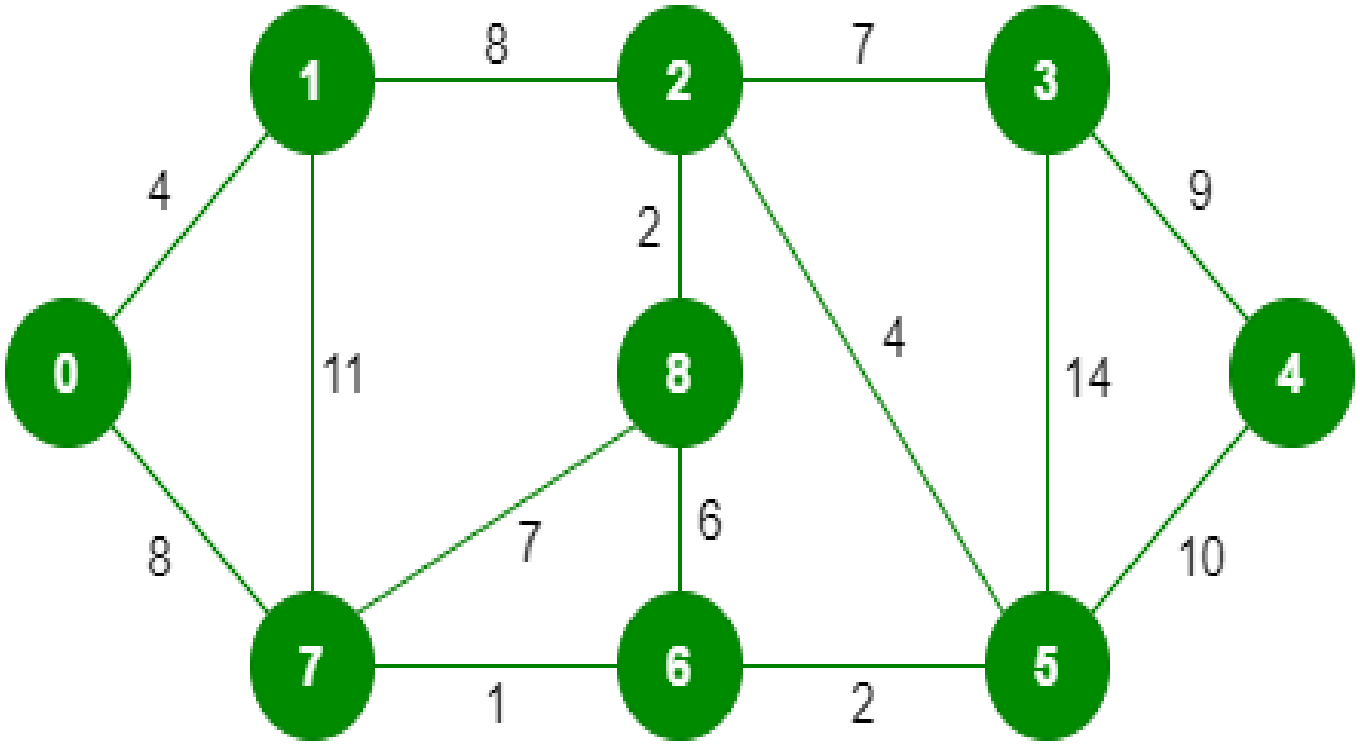
Alternative MST structure

Structure of the alternate MST if we had selected edge {1, 2} in the MST

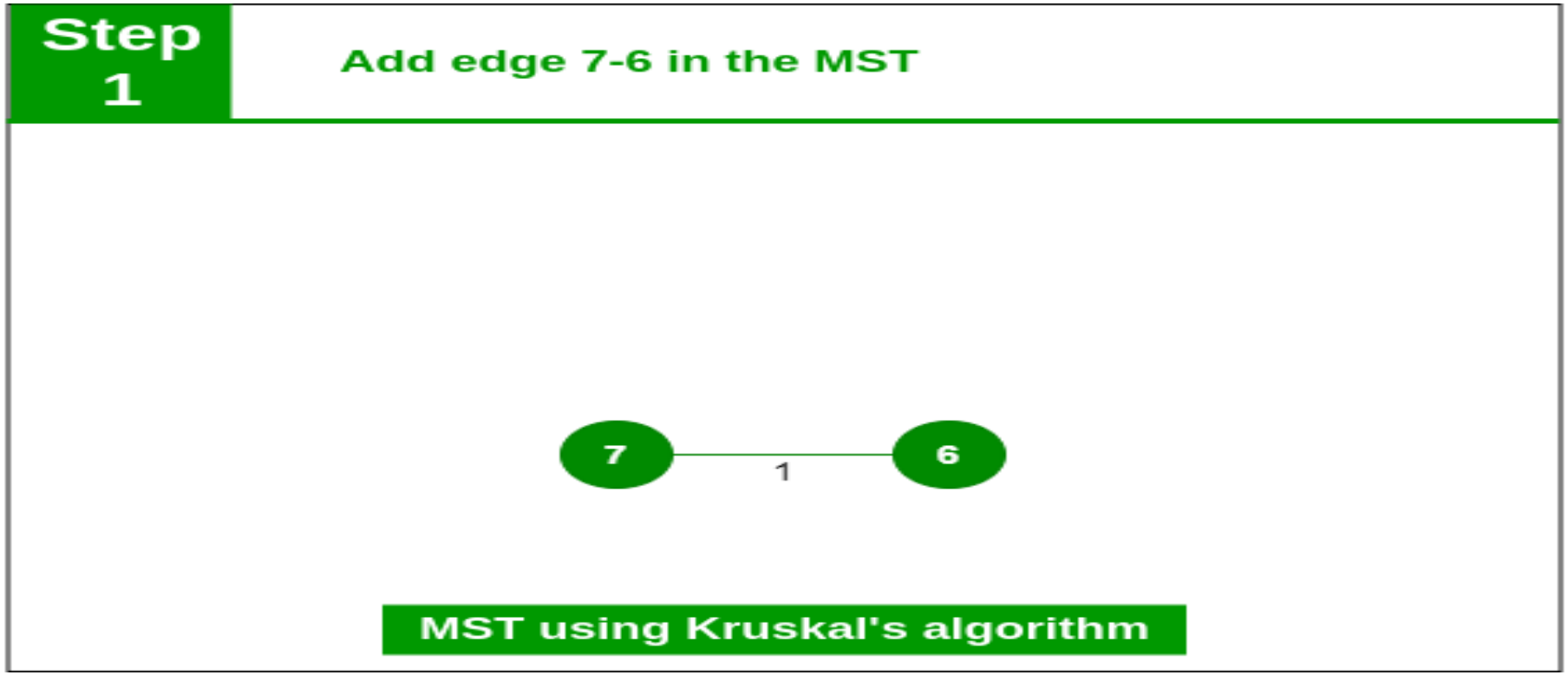
Rules for Kruskal's Algorithm

1. Remove self loops
2. Remove parallel edges
3. Select the smallest edge
4. Select the next smallest edge
5. Avoid if cycle creates
6. Repeat step 3, 4 and 5

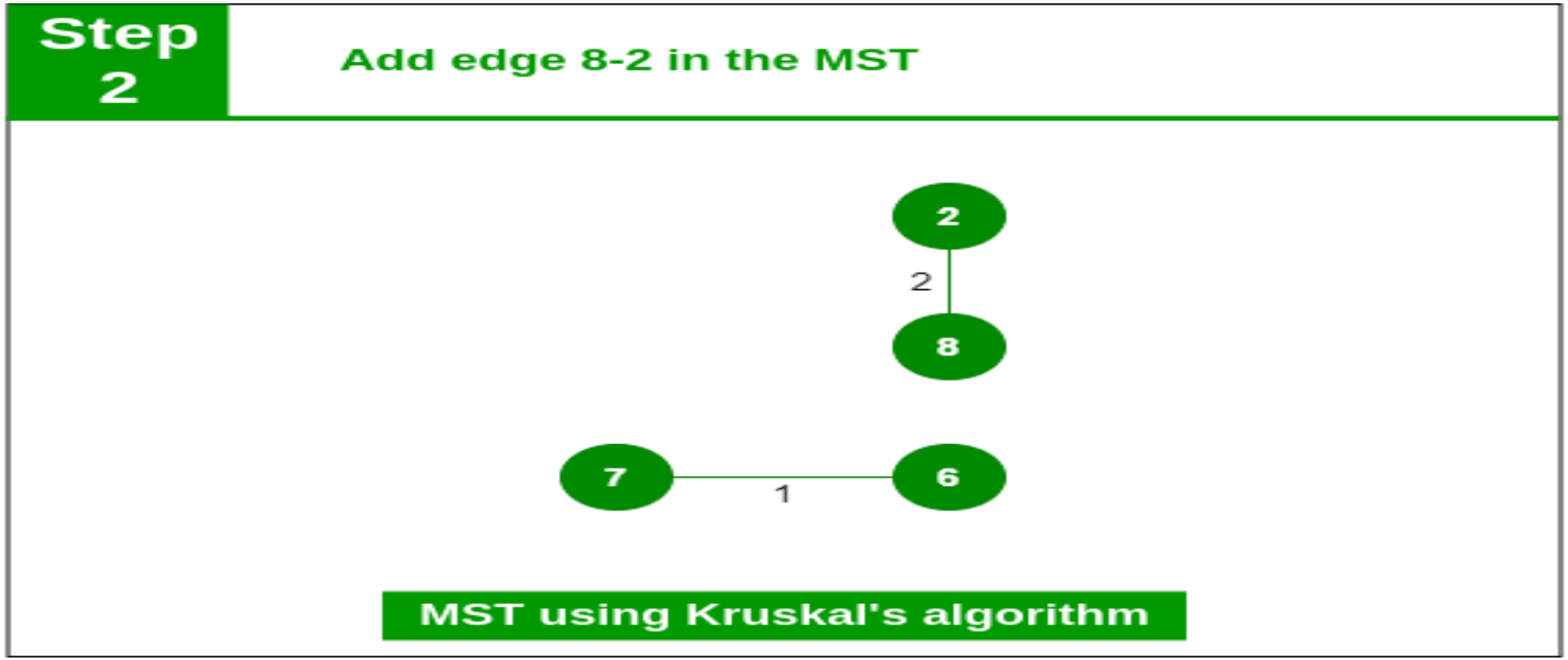
Kruskal's Algorithm



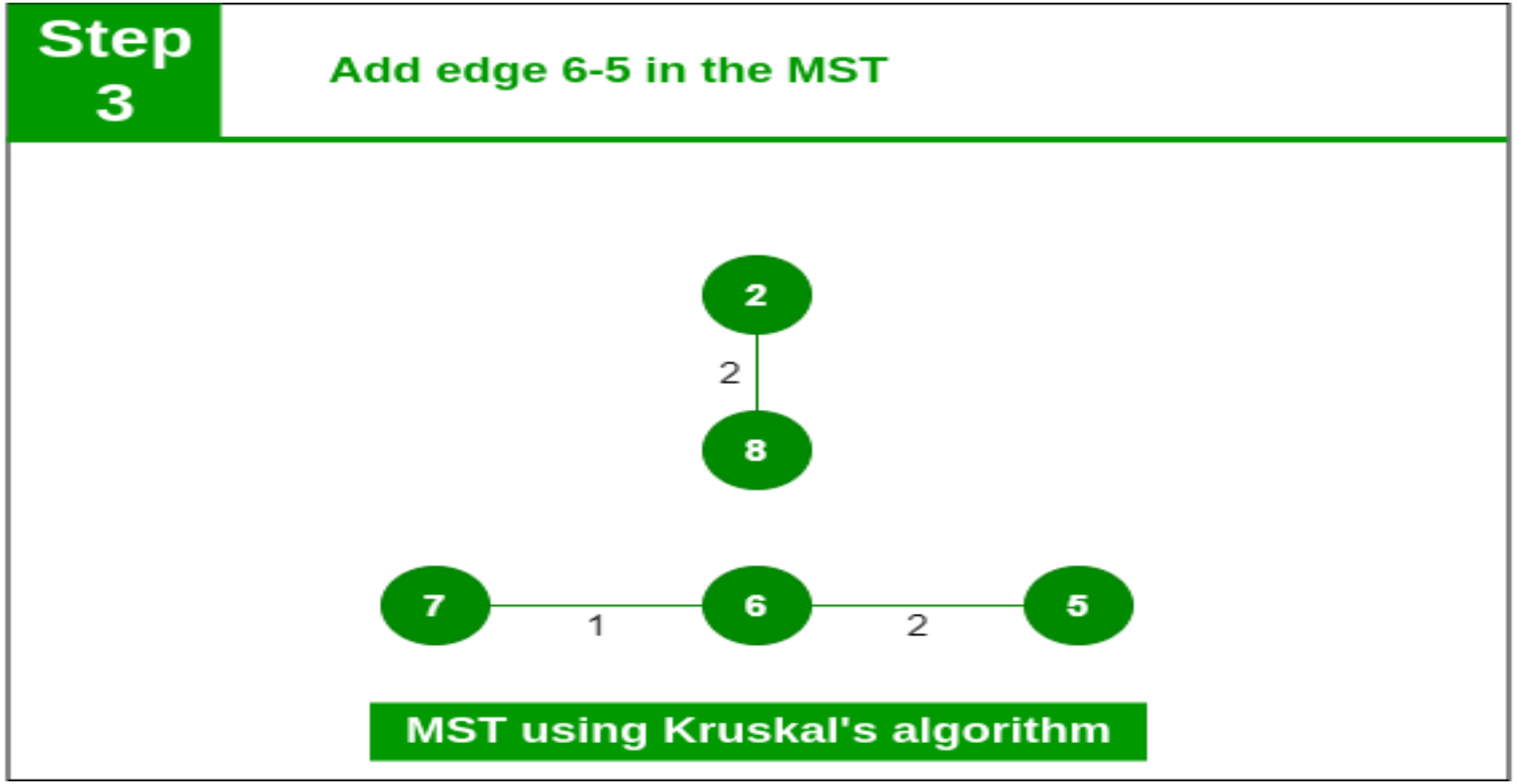
Kruskal’s Algorithm



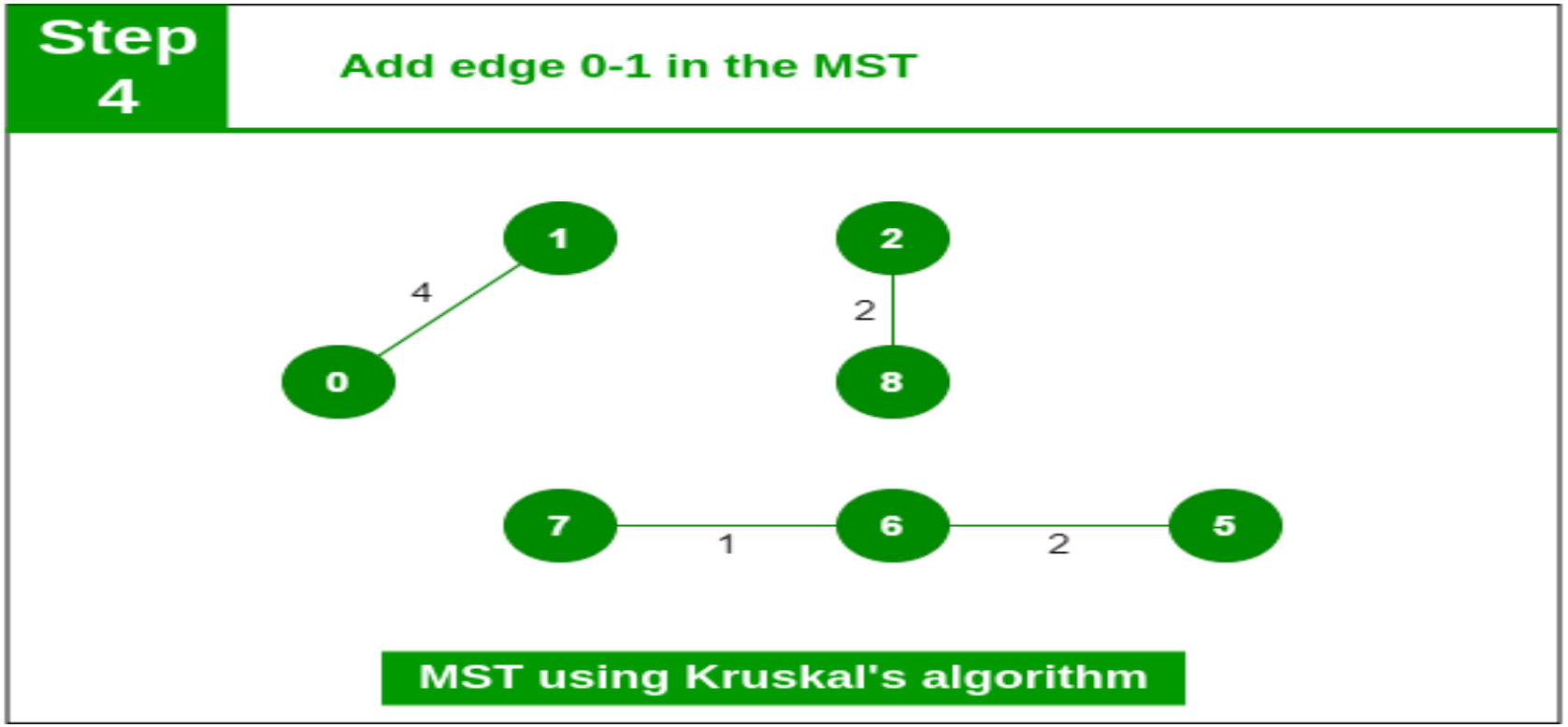
Kruskal’s Algorithm



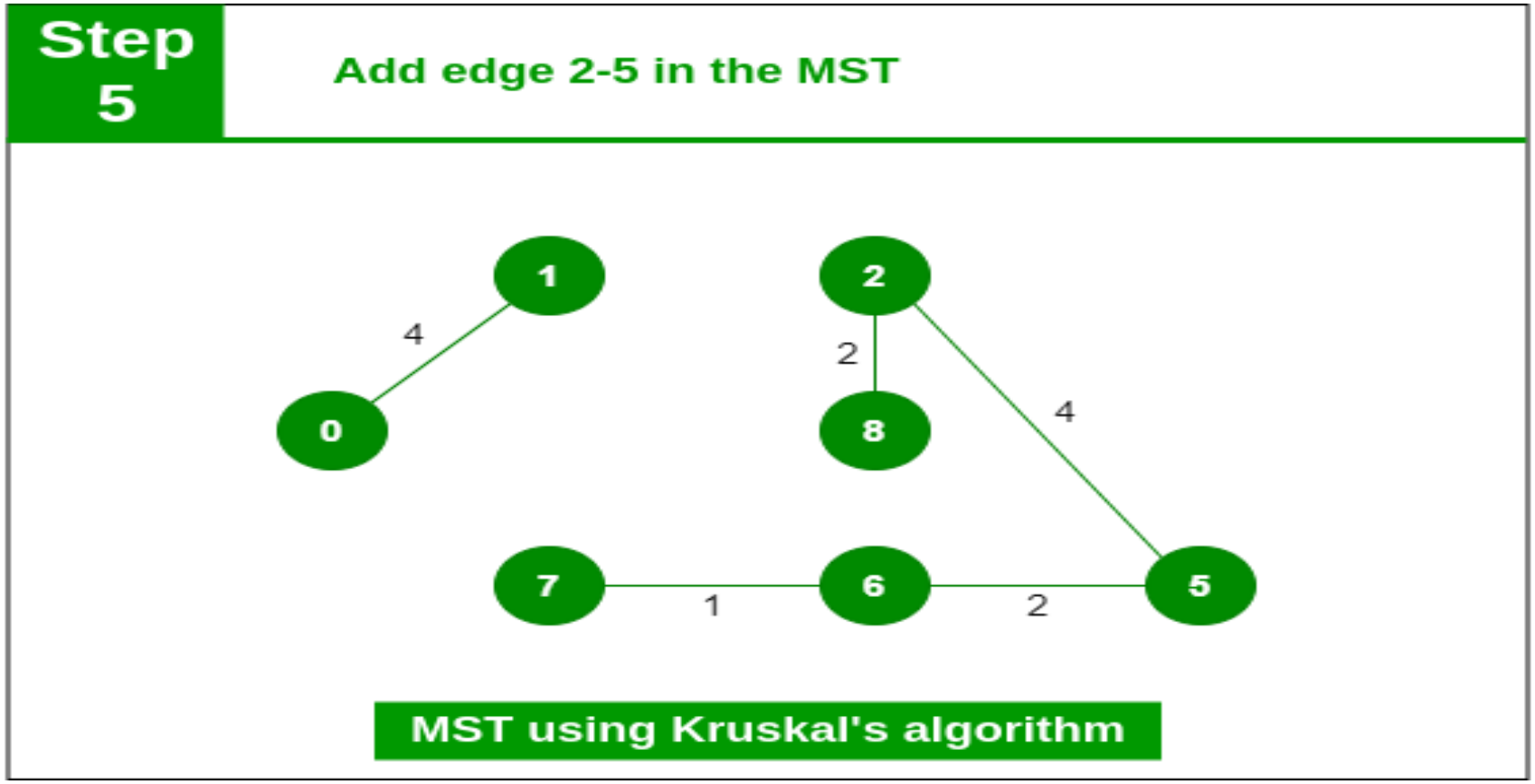
Kruskal’s Algorithm



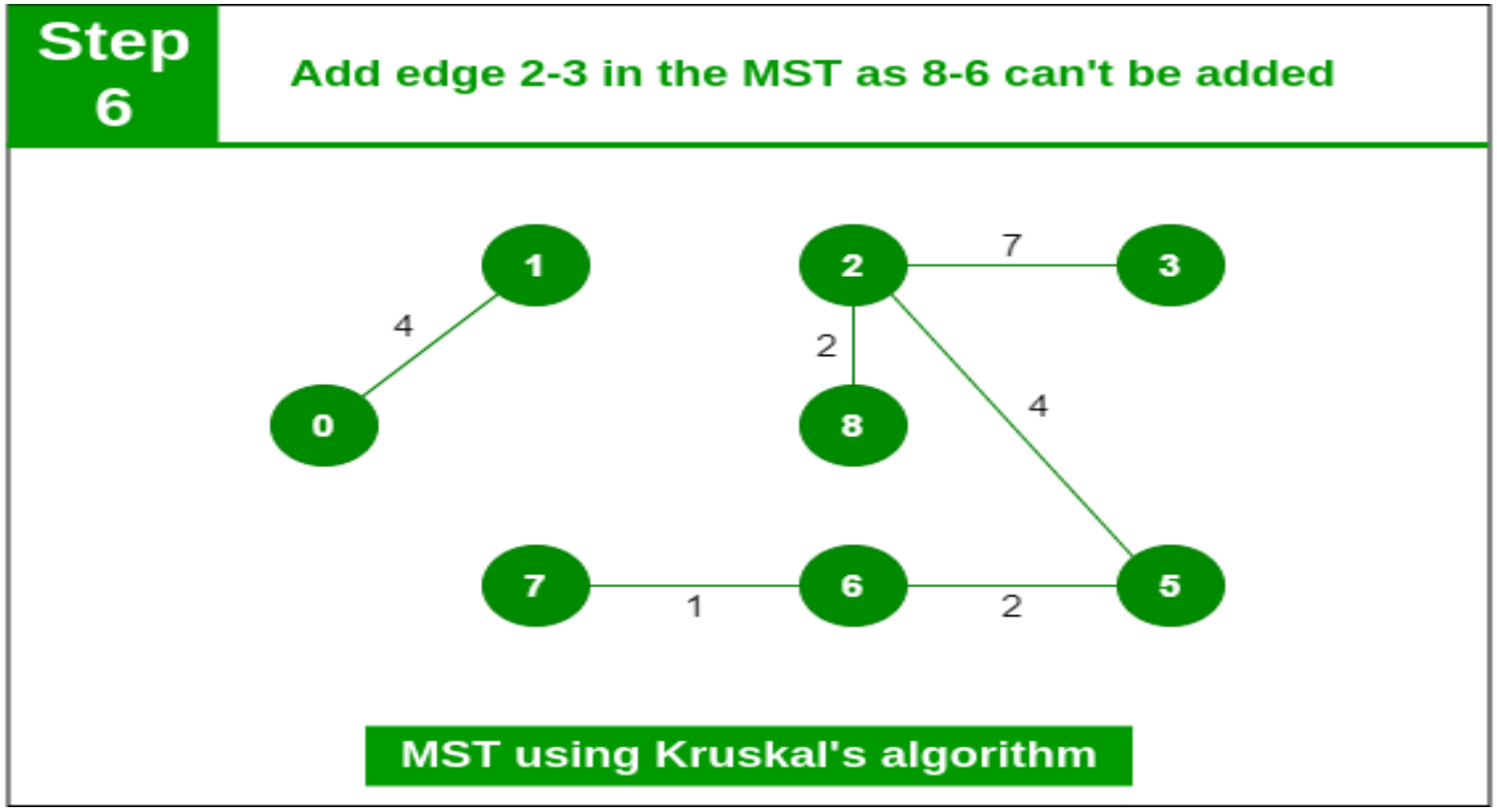
Kruskal’s Algorithm



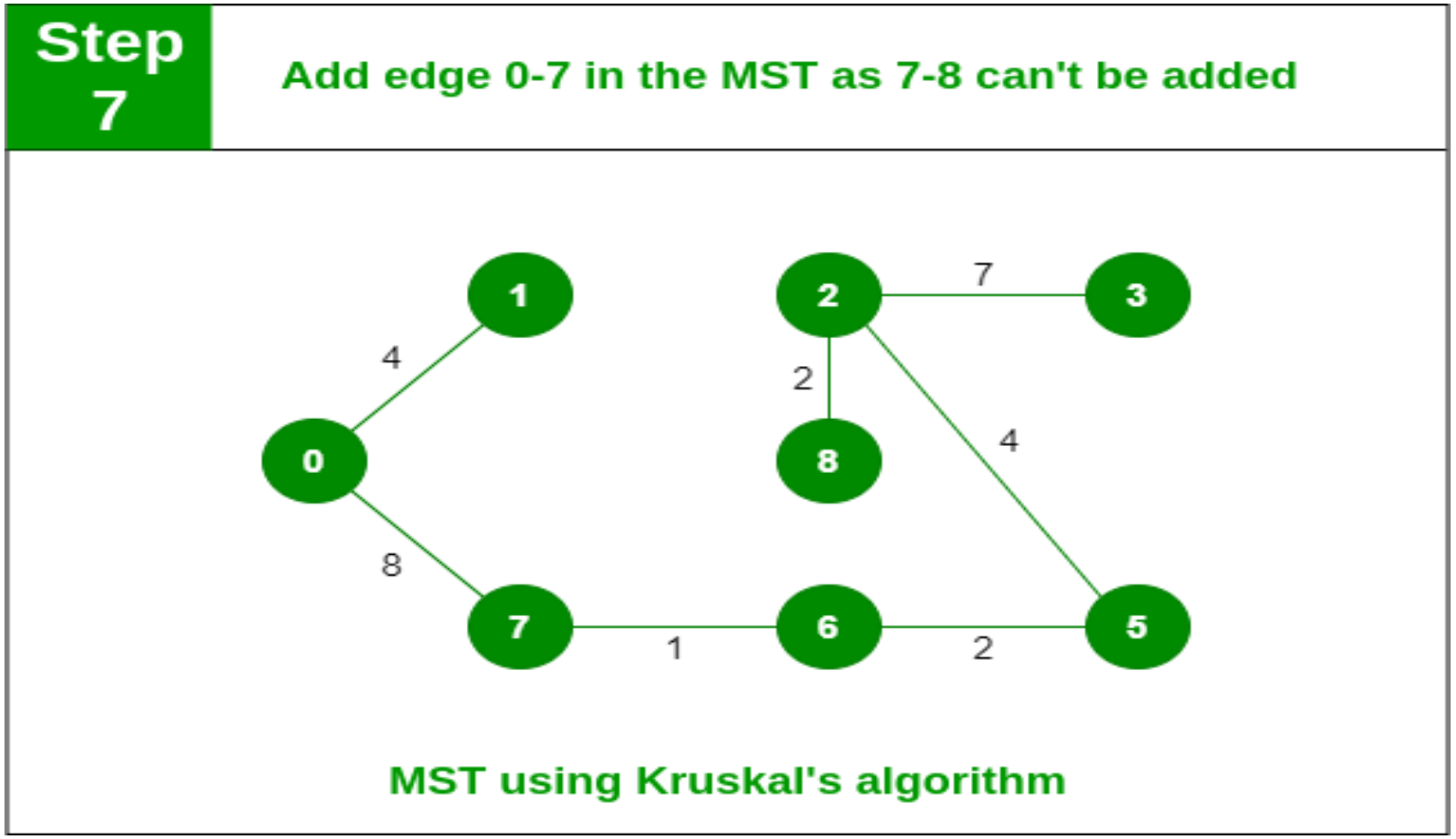
Kruskal’s Algorithm



Kruskal’s Algorithm



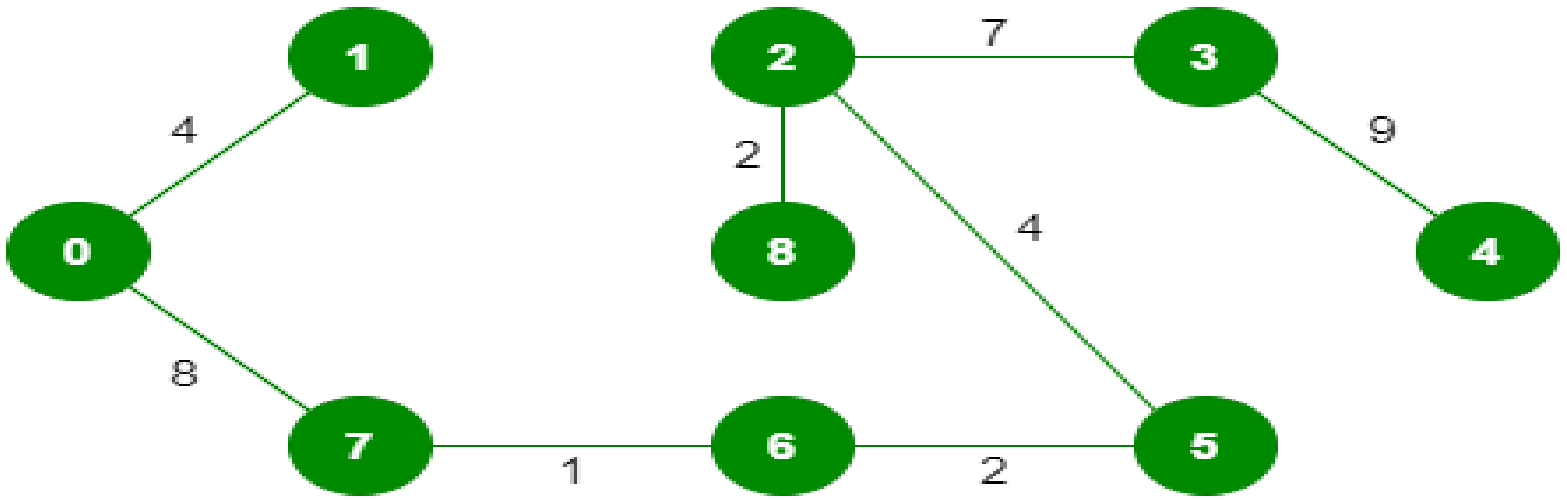
Kruskal’s Algorithm



Kruskal’s Algorithm

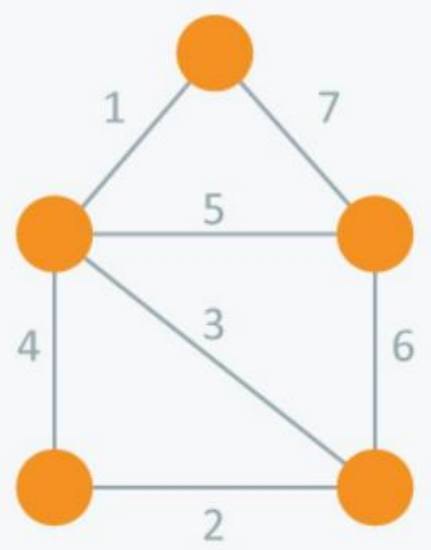
Step 8

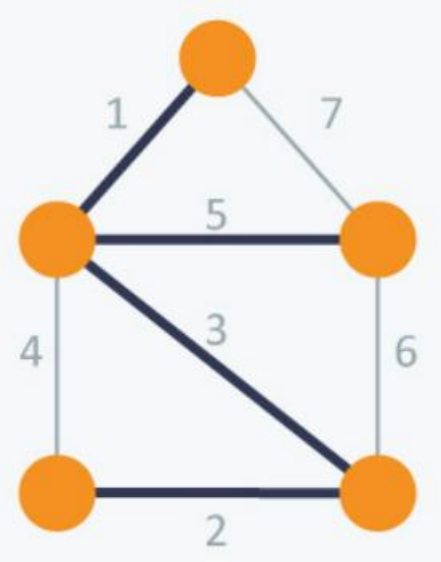
Add edge 3-4 in the MST. It completes the MST

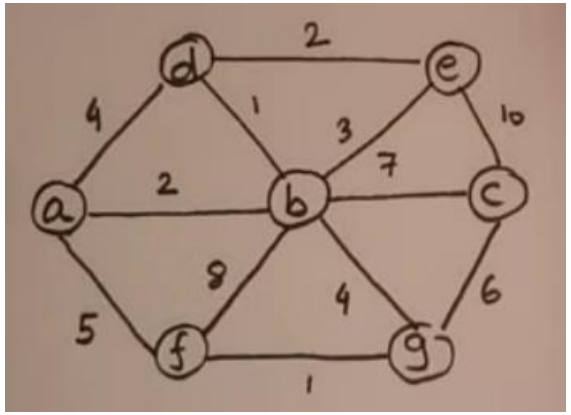


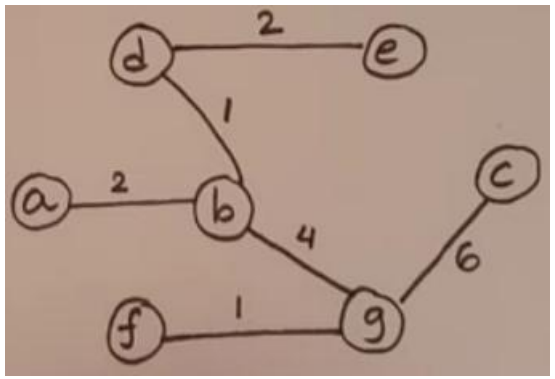
MST using Kruskal's algorithm

Practice Problems

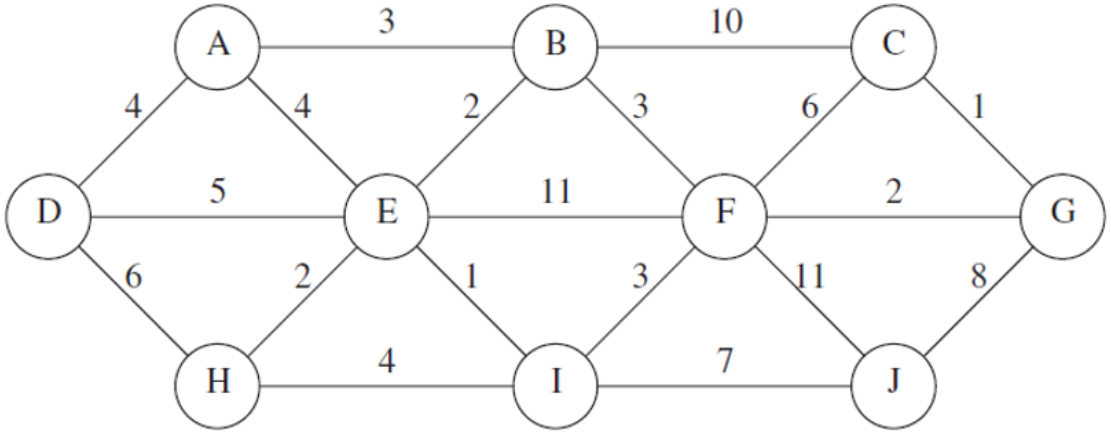


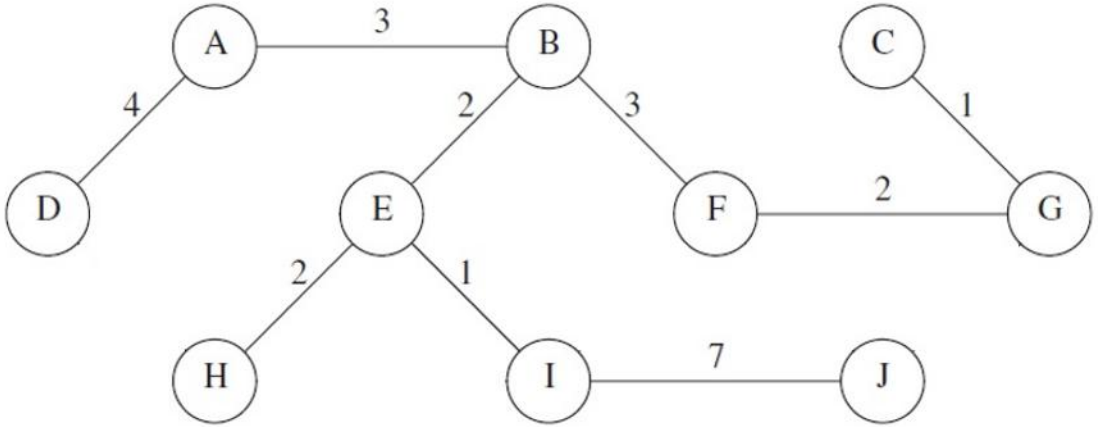




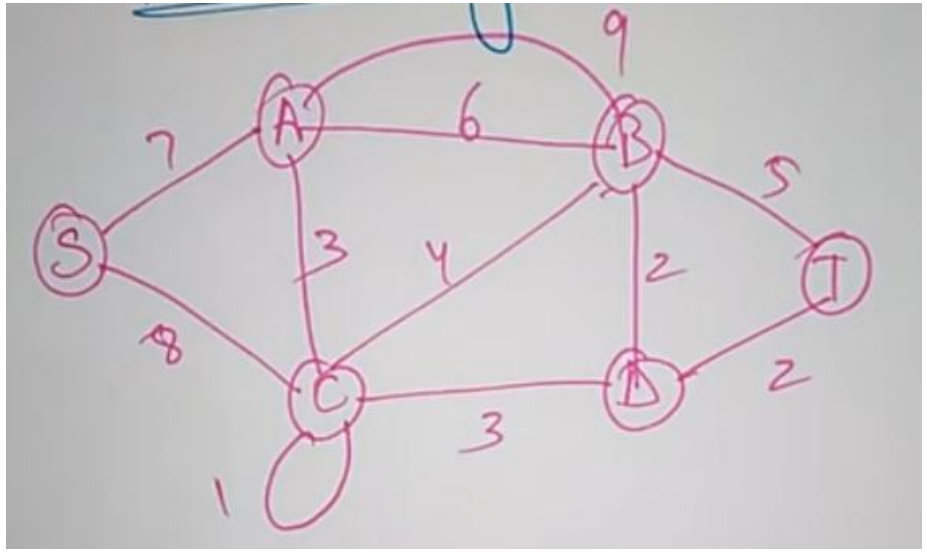


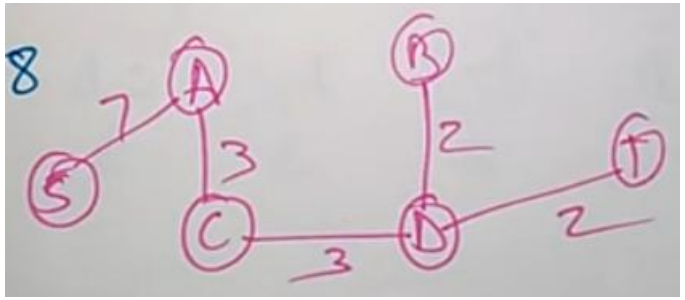
Practice Problems

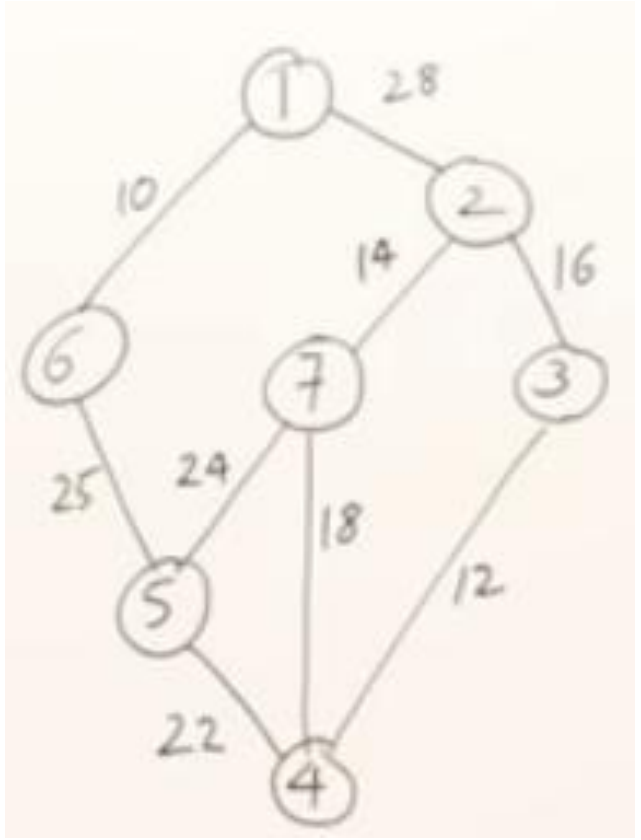


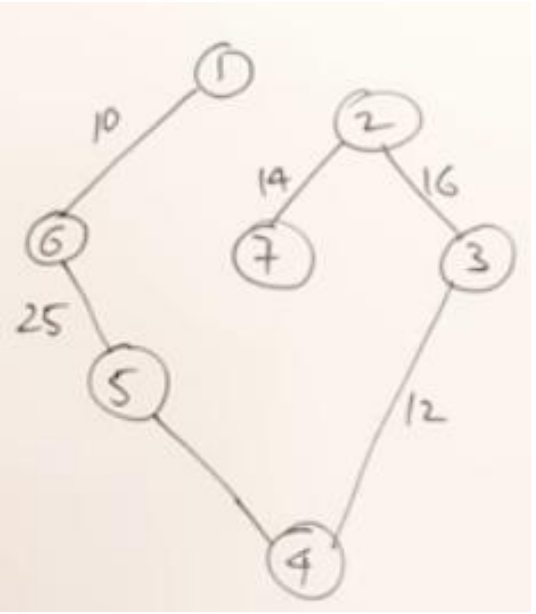


Practice Problems









Practice Problems