* Conversion of FA to RE!

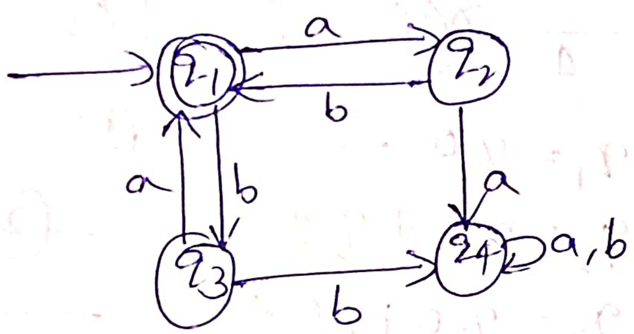
- → State Elimination Method
- * → Arden's Theorem
- → Generalized NFA (GNFA)
- → Brzozowski Algebraic Method.

1) **Arden's Theorem**:- If P and Q are two RE's over Σ and if P doesnot contain ε, then the following equation in R given by $\boxed{R = Q + RP}$ has a unique solution i.e;

$\boxed{R = QP^*}$

$$R = Q + RP$$
$$= Q + QP^*P$$
$$= Q(\varepsilon + P^*P) \qquad (\because \varepsilon + R^*R = R^*)$$
$$= QP^* \Rightarrow R \quad \text{Hence Proved.}$$

(1Q) Find the RE for the following FA:-



Sol:-

Algorithm:-

Step-1:- Construct state equation for all the states, based on incoming edges.

Step-2:- Add "ε" to the equation of initial state.

Step-3:- Simplify the equation using Arden's Theorem, and find regular expression.

From fig:

Step-1:- $q_1 = q_2 b + q_3 a$ ——— (1)

$q_2 = q_1 a$ ——— (2)

$q_3 = q_1 b$ ——— (3)

$q_4 = q_2 a + q_3 b + q_4 a + q_4 b$ ——— (4)

Step-2:- $q_1 = \varepsilon + q_2 b + q_3 a$

Step-3 :- final state is q_1

$$q_1 = \varepsilon + q_2 b + q_3 a$$

$$q_1 = \varepsilon + q_1 ab + q_1 ba \quad (\because \text{from } 2 \,\&\, 3)$$

$$= \varepsilon + q_1(ab+ba)$$

from Arden's Theorem $R = Q + RP$

Here $Q = \varepsilon$, $R = q_1$, $P = ab+ba$ $\quad R = QP^*$ $\quad (\because \varepsilon R = R)$

$$\boxed{\therefore q_1 = \varepsilon(ab+ba)^* = (ab+ba)^*}$$

Therefore the required regular expression is $(ab+ba)^*$ for the given automata.

2Q) Find the regular expression for the given FA :-

Sol :-

Step-1 :- $q_1 = q_1 0$ ——— (1)

$q_2 = q_1 1 + q_2 1$ ——— (2)

$q_3 = q_2 0 + q_3 0 + q_3 1$ ——— (3)

Step-2 :- $q_1 = \varepsilon + q_1 0$;

Step-3 :- final states are q_1 and q_2

(a) Final state $q_1 = \varepsilon + q_1 0$

from Ardens Theorem → Here $Q = \varepsilon$, $R = q_1$, $P = 0$

then $q_1 = \varepsilon 0^*$ $\quad (\because \varepsilon R^* = R^*)$

$$\boxed{q_1 = 0^*}$$

(b) Final state $q_2 = q_1 1 + q_2 1$

$q_2 = 0^* 1 + q_2 1$ $\quad$ (from $q_1 = 0^*$)

from Ardens Theorem Here $Q = 0^* 1$, $R = q_2$, $P = 1$ $\quad R = Q + RP$

$$\boxed{q_2 = 0^* 1 1^*}$$

$R = QP^*$

If we have 2 or more final states then the RE will be union of both final states.

$= q_1 + q_2$

$= 0^* + 0^*11^*$

$= 0^*(\varepsilon + 11^*) \qquad (\because \varepsilon + RR^* = R^*)$

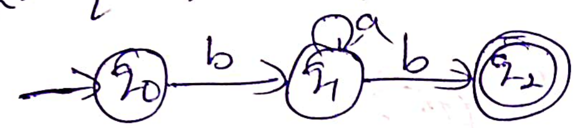
$\boxed{RE = 0^*1^*}$

Hence the finite automata is 0^*1^*.

* Conversion of RE to FA:

1) ba^*b

$L = \{bb, bab, baab \ldots\}$



2) $(a+b)c$

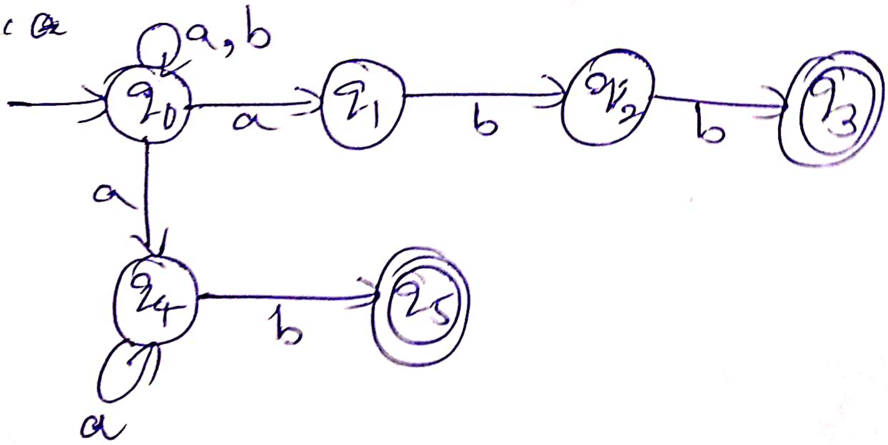
$L = \{ac, bc\}$

3) $a(bc)^*$

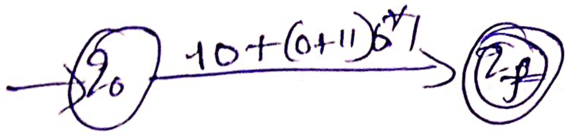
$L = \{a, abc, abcbc \ldots\}$

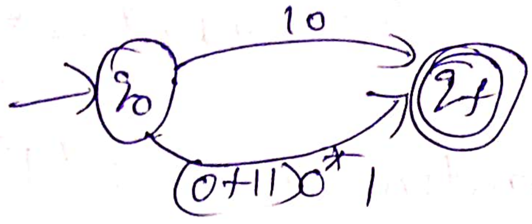
4) $(a|b)^*(abb|a^*b)$

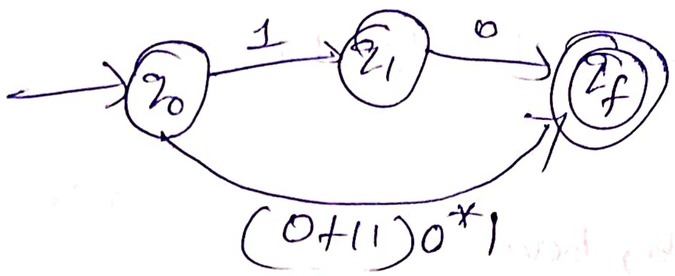
$= (a+b)^*(abb + a^*b)$

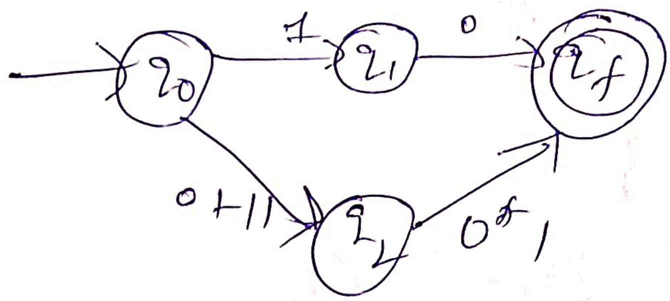


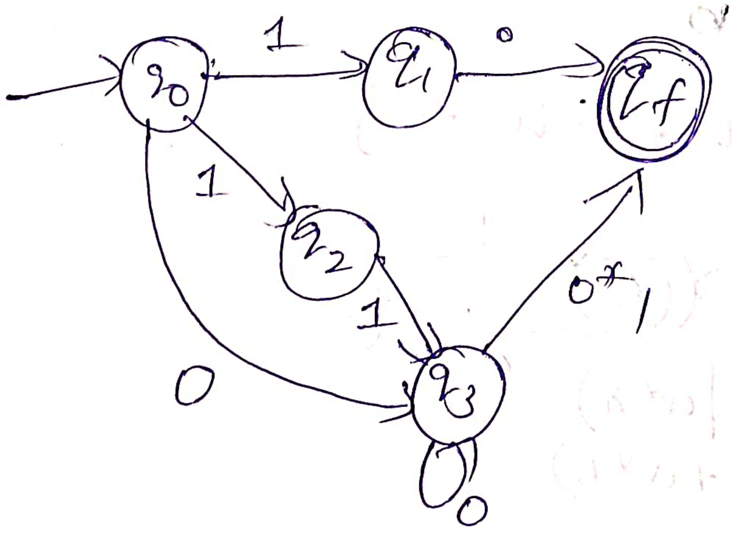
Q) Design a FA from given RE $10+(0+11)0^*1$











Q) Construct NFA for the regular expression $b+ba^*$

$r = b+ba^*$

$r = r_1 + r_2$

$r_1 = b$

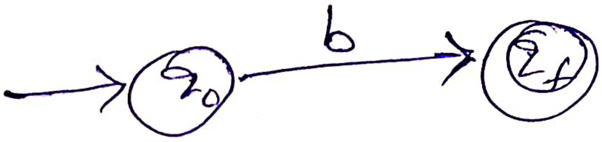
$r_2 = ba^*$

$r_2 = r_3 \cdot r_4$

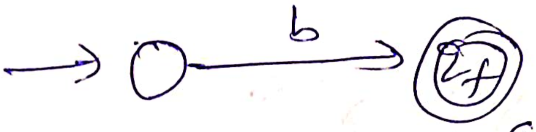
$r_3 = b$

$r_4 = a^*$

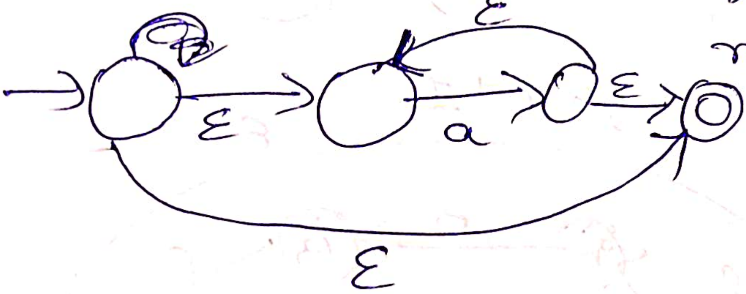
r_1



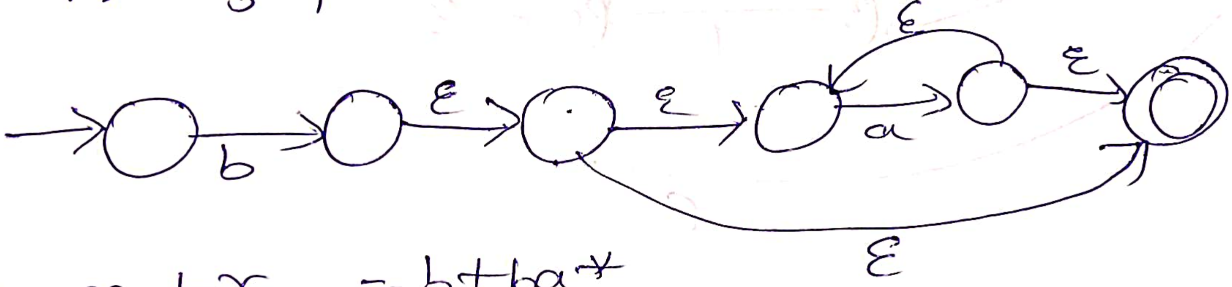
r_3



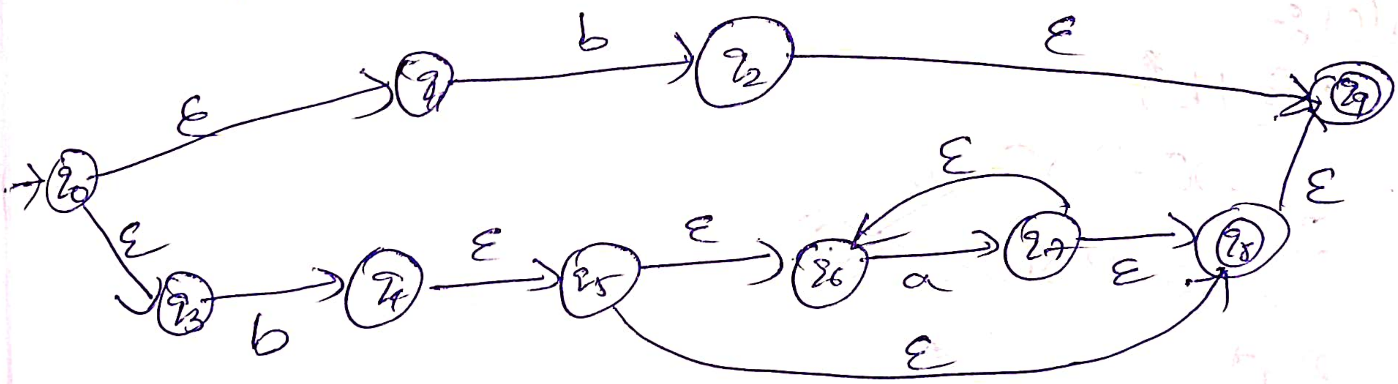
r_4



$r_2 = r_3 r_4$



$r = r_1 + r_2 = b+ba^*$

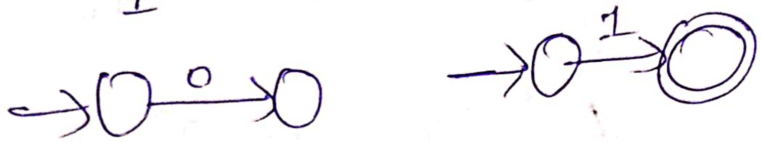


9) Construct NFA with ε moves $(0+1)^*$

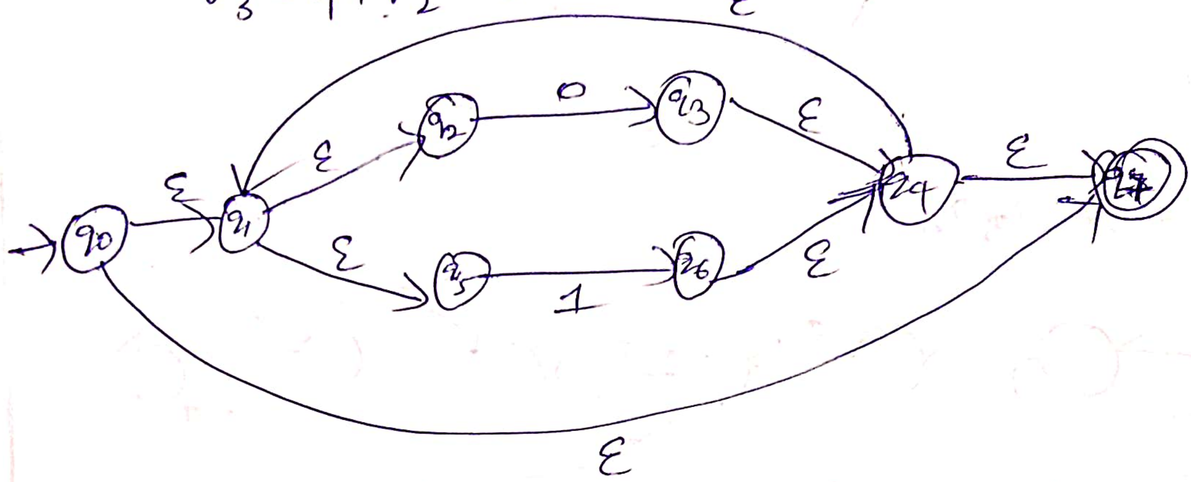
$r_3 = r_1 + r_2$

$r = r_3^*$

$r_1 = 0 \qquad r_2 = 1$

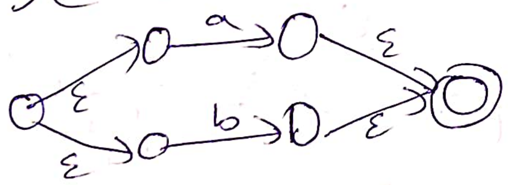


$r_3 = r_1 + r_2 = 0+1$

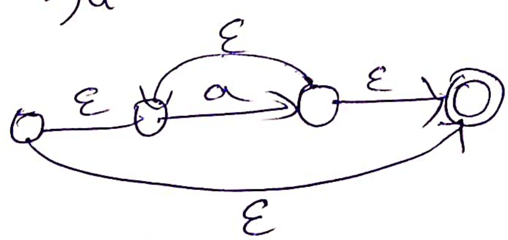


<u>Rules :-</u>

1) $(a+b)^*$



2) a^*



3) ab