Special kinds of Matrices

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Matrices - Introduction

Matrix algebra has at least two advantages:

- Reduces complicated systems of equations to simple expressions
- Adaptable to a systematic method of mathematical treatment and well-suited to computers

Definition:

A matrix is a set or group of numbers arranged in a square or rectangular array enclosed by two brackets

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 4 & 2 \\ -3 & 0 \end{bmatrix} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Matrices - Introduction

Properties:

- A specified number of rows and a specified number of columns
- Two numbers (rows x columns) describe the dimensions or size of the matrix.

Examples:

3x3 matrix

2x4 matrix

1x2 matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & -1 & 5 \\ 3 & 3 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 3 & -3 \\ 0 & 0 & 3 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 \end{bmatrix}$$

Matrices - Introduction

A matrix is denoted by a bold capital letter and the elements within the matrix are denoted by lowercase letters

e.g. matrix [**A**] with elements a_{ij}

$$\mathbf{A}_{mxn} = \quad {}_m\mathbf{A}^n \quad \begin{bmatrix} a_{11} & a_{12}\ldots & a_{ij} & a_{in} \\ a_{21} & a_{22}\ldots & a_{ij} & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{ij} & a_{mn} \end{bmatrix}$$

i goes from 1 to m

j goes from 1 to n

Matrices - Introduction

TYPES OF MATRICES

1. Column matrix or vector:

The number of rows may be any integer but the number of columns is always 1

$$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$

Matrices - Introduction

TYPES OF MATRICES

2. Row matrix or vector

Any number of columns but only one row

$$\begin{bmatrix} 1 & 1 & 6 \end{bmatrix} \qquad \begin{bmatrix} 0 & 3 & 5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \end{bmatrix}$$

Matrices - Introduction

TYPES OF MATRICES

3. Rectangular matrix

Contains more than one element and number of rows is not equal to the number of columns

$$\begin{bmatrix} 1 & 1 \\ 3 & 7 \\ 7 & -7 \\ 7 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 3 & 0 \end{bmatrix}$$

$$m \neq n$$

Matrices - Introduction

TYPES OF MATRICES

4. Square matrix

- The number of rows is equal to the number of columns (a square matrix A has an order of m)

m x m

$$\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 9 & 9 & 0 \\ 6 & 6 & 1 \end{bmatrix}$$

The principal or main diagonal of a square matrix is composed of all elements a_{ij} for which $i=j$

Matrices - Introduction

TYPES OF MATRICES

5. Diagonal matrix

A square matrix where all the elements are zero except those on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i \neq j$

$a_{ij} \neq 0$ for some or all $i = j$

Matrices - Introduction

TYPES OF MATRICES

6. Unit or Identity matrix - I

A diagonal matrix with ones on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} a_{ij} & 0 \\ 0 & a_{ij} \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i \neq j$

$a_{ij} = 1$ for some or all $i = j$

Matrices - Introduction

TYPES OF MATRICES

7. Null (zero) matrix - 0

All elements in the matrix are zero

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$a_{ij} = 0 \qquad \text{For all } i,j$$

Matrices - Introduction

TYPES OF MATRICES

8. Triangular matrix

A square matrix whose elements above or below the main diagonal are all zero

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 8 & 9 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

Matrices - Introduction

TYPES OF MATRICES

8a. Upper triangular matrix

- A square matrix whose elements below the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & a_{ij} & a_{ij} \\ 0 & a_{ij} & a_{ij} \\ 0 & 0 & a_{ij} \end{bmatrix} \quad \begin{bmatrix} 1 & 8 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 7 & 4 & 4 \\ 0 & 1 & 7 & 4 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i > j$

Matrices - Introduction

TYPES OF MATRICES

8b. Lower triangular matrix

A square matrix whose elements above the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ a_{ij} & a_{ij} & 0 \\ a_{ij} & a_{ij} & a_{ij} \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i < j$

Matrices – Introduction

TYPES OF MATRICES

9. Scalar matrix

A diagonal matrix whose main diagonal elements are equal to the same scalar

A scalar is defined as a single number or constant

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ 0 & a_{ij} & 0 \\ 0 & 0 & a_{ij} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i \neq j$

$a_{ij} = a$ for all $i = j$

Matrices - Operations

Commutative Law:
A + B = B + A

Associative Law:
A + (B + C) = (A + B) + C = A + B + C

$$\begin{bmatrix} 7 & 3 & -1 \\ 2 & -5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 6 \\ -4 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 8 & 5 \\ -2 & -7 & 9 \end{bmatrix}$$

A 2x3 **B** 2x3 **C** 2x3

Matrices - Operations

$\mathbf{A} + \mathbf{0} = \mathbf{0} + \mathbf{A} = \mathbf{A}$

$\mathbf{A} + (-\mathbf{A}) = \mathbf{0}$ (where $-\boldsymbol{A}$ is the matrix composed of $-a_{ij}$ as elements)

$$\begin{bmatrix} 6 & 4 & 2 \\ 3 & 2 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

Matrices - Operations

SCALAR MULTIPLICATION OF MATRICES

Matrices can be multiplied by a scalar (constant or single element)

Let k be a scalar quantity; then

kA = Ak

Ex. If k=4 and

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix}$$

Matrices - Operations

$$4 \times \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix} \times 4 = \begin{bmatrix} 12 & -4 \\ 8 & 4 \\ 8 & -12 \\ 16 & 4 \end{bmatrix}$$

Properties:

- k (**A** + **B**) = k**A** + k**B**
- (k + g)**A** = k**A** + g**A**
- k(**AB**) = (k**A**)**B** = **A**(k)**B**
- k(g**A**) = (kg)**A**

Matrices - Operations

MULTIPLICATION OF MATRICES

The product of two matrices is another matrix

Two matrices A and B must be conformable for multiplication to be possible

i.e. the number of columns of A must equal the number of rows of B

Example.

$$\mathbf{A} \quad \text{x} \quad \mathbf{B} = \mathbf{C}$$

$$(1\text{x}3) \quad (3\text{x}1) \quad (1\text{x}1)$$

Matrices - Operations

B x **A** = Not possible!

(2x1) (4x2)

A x **B** = Not possible!

(6x2) (6x3)

Example

A x **B** = **C**

(2x3) (3x2) (2x2)

Matrices - Operations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$(a_{11} \times b_{11}) + (a_{12} \times b_{21}) + (a_{13} \times b_{31}) = c_{11}$$

$$(a_{11} \times b_{12}) + (a_{12} \times b_{22}) + (a_{13} \times b_{32}) = c_{12}$$

$$(a_{21} \times b_{11}) + (a_{22} \times b_{21}) + (a_{23} \times b_{31}) = c_{21}$$

$$(a_{21} \times b_{12}) + (a_{22} \times b_{22}) + (a_{23} \times b_{32}) = c_{22}$$

Successive multiplication of row *i* of **A** with column *j* of **B** – row by column multiplication

Matrices - Operations

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 7 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ 6 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} (1\times 4)+(2\times 6)+(3\times 5) & (1\times 8)+(2\times 2)+(3\times 3) \\ (4\times 4)+(2\times 6)+(7\times 5) & (4\times 8)+(2\times 2)+(7\times 3) \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix}$$

Remember also:

IA = **A**

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix} = \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix}$$

Matrices - Operations

Assuming that matrices A, B and C are conformable for the operations indicated, the following are true:

1. $AI = IA = A$
2. $A(BC) = (AB)C = ABC$ - (associative law)
3. $A(B+C) = AB + AC$ - (first distributive law)
4. $(A+B)C = AC + BC$ - (second distributive law)

Caution!

1. AB not generally equal to BA, BA may not be conformable
2. If $AB = 0$, neither A nor B necessarily $= 0$
3. If $AB = AC$, B not necessarily $= C$

Matrices - Operations

AB not generally equal to BA, BA may not be conformable

$$T = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix}$$

$$TS = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 15 & 20 \end{bmatrix}$$

$$ST = \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 23 & 6 \\ 10 & 0 \end{bmatrix}$$

Matrices - Operations

If **AB** = **0**, neither **A** nor **B** necessarily = **0**

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Matrices - Operations

TRANSPOSE OF A MATRIX

If :

$$\underset{2\text{x}3}{A} = {}_2A^3 = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 3 & 1 \end{bmatrix}$$

Then transpose of A, denoted A^T is:

$$A^T = {}_2A^{3^T} = \begin{bmatrix} 2 & 5 \\ 4 & 3 \\ 7 & 1 \end{bmatrix}$$

$a_{ij} = a^T_{ji}$ For all i and j

Matrices - Operations

To transpose:

Interchange rows and columns

The dimensions of $\mathbf{A}^T$ are the reverse of the dimensions of $\mathbf{A}$

$$A = {}_2A^3 = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 3 & 1 \end{bmatrix} \qquad 2 \text{ x } 3$$

$$A^T = {}_3A^{T\,2} = \begin{bmatrix} 2 & 5 \\ 4 & 3 \\ 7 & 1 \end{bmatrix} \qquad 3 \text{ x } 2$$

Matrices and Linear Equations

Linear Equations

Linear Equations

- Linear equations are common and important for survey problems
- Matrices can be used to express these linear equations and aid in the computation of unknown values
- Example
- n equations in n unknowns, the a_{ij} are numerical coefficients, the b_i are constants and the x_j are unknowns

$$
\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\
&a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\
&\vdots \\
&a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n
\end{aligned}
$$

Linear Equations

The equations may be expressed in the form

AX = B

where

$$A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n1} \cdots & a_{nn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \text{ and } \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

n x n   n x 1   n x 1

Number of unknowns = number of equations = n

Linear Equations

If the determinant is nonzero, the equation can be solved to produce n numerical values for x that satisfy all the simultaneous equations

To solve, premultiply both sides of the equation by A^{-1} which exists because $|A| = 0$/

$$\mathbf{A}^{-1}\,\mathbf{AX} = \mathbf{A}^{-1}\,\mathbf{B}$$

Now since

$$\mathbf{A}^{-1}\,\mathbf{A} = \mathbf{I}$$

We get

$$\mathbf{X} = \mathbf{A}^{-1}\,\mathbf{B}$$

So if the inverse of the coefficient matrix is found, the unknowns, X would be determined

Linear Equations

Example

$$3x_1 - x_2 + x_3 = 2$$
$$2x_1 + x_2 = 1$$
$$x_1 + 2x_2 - x_3 = 3$$

The equations can be expressed as

$$\begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Linear Equations

When $\mathbf{A}^{-1}$ is computed the equation becomes

$$X = A^{-1}B = \begin{bmatrix} 0.5 & -0.5 & 0.5 \\ -1.0 & 2.0 & -1.0 \\ -1.5 & 3.5 & -2.5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix}$$

Therefore

$$x_1 = 2,$$
$$x_2 = -3,$$
$$x_3 = -7$$

Linear Equations

The values for the unknowns should be checked by substitution back into the initial equations

$$x_1 = 2, \quad x_2 = -3, \quad x_3 = -7$$

$$\begin{aligned} 3x_1 - x_2 + x_3 &= 2 \\ 2x_1 + x_2 &= 1 \\ x_1 + 2x_2 - x_3 &= 3 \end{aligned}$$

$$\begin{aligned} 3\times(2) - (-3) + (-7) &= 2 \\ 2\times(2) + (-3) &= 1 \\ (2) + 2\times(-3) - (-7) &= 3 \end{aligned}$$

Solve system of linear equations, using matrix method.

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Step 1

Write equation as AX = B

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

Hence $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Step 2

Calculate $|A|$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} - (-1) \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (1 + 3) + 1(2 + 3) + 1(2 - 1) = 1(4) + 1(5) + 1(1)$$

$$= 4 + 5 + 1 = 10$$

Since $|A| \neq 0$

$\therefore$ *The system of equation is consistent & has a unique solution*

Now, $AX = B$

$X = A^{-1} B$

Step 3

Calculate $X = A^{-1} B$

Calculating A^{-1}

Now,

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}' = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{11} = \begin{bmatrix} 1 & -3 \\ 1 & 1 \end{bmatrix} = 1 + 3 = 4$$

$$M_{12} = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 2 + 3 = 5$$

$$M_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1$$

$$M_{21} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -1 - 1 = -2$$

$$M_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$$

$$M_{23} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 + 1 = 2$$

$$M_{31} = \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 3 - 1 = 2$$

$$M_{32} = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -3 - 4 = -5$$

$$M_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3 + 2 = 3$$

$$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 . 4 = 4$$

$A_{12} = (-1)^{1+2}\ M_{12} = (-1)^3 . 5 = -5$

$A_{13} = (-1)^{1+3}\ M_{13} = (-1)^4 . (1) = 1$

$A_{21} = (-1)^{2+1}\ M_{21} = (-1)^3 . (-2) = 2$

$A_{22} = (-1)^{2+2}\ M_{22} = (-1)^4 . 0 = 0$

$A_{23} = (-1)^{2+3} . M_{23} = (-1)^5 . (2) = -2$

$A_{31} = (-1)^{3+1} . M_{31} = (-1)^4 . (2) = 2$

$A_{32} = (-1)^{3+2} . M_{32} = (-1)^5 . (-5) = 5$

$A_{33} = (-1)^{3+3} . M_{33} = (-1)^6 . 3 = 3$

Thus , adj A = $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

& $|A| = 10$

So, $A^{-1} = \frac{1}{|A|}$ adj A

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

& $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Now, solving

$X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & 1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4(4)+2(0)+2(2) \\ 0(4)+0(0)+5(2) \\ 2(4)+1(0)+3(2) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Hence, **x = 2 , y = −1, & z = 1**

Special vectors with examples

- Input vectors
- *Activation vectors*
- *Attention vectors*
- Embedding vectors
- Weight vectors
- Bias vectors
- Gradient vectors

- Example : *Attention vector*
- I need an English-to-French translation task, "*I love music*" to French. The attention vector could assign higher weights to the word "music" when generating the French translation, indicating that it has a strong influence on the translation of the corresponding French word.

Gradient vector-example

- Consider the function $f(x) = 3x^2 + 2x + 1$. We want to compute the gradient of this function.
- To find the gradient vector, we need to take the derivative of the function with respect to x. In this case, the derivative of $f(x)$ is:
- $f'(x) = 6x + 2$
- So, the gradient vector is $[6x + 2]$.
- Let's evaluate the gradient vector at a specific point, $x = 2$:
- $f'(2) = 6(2) + 2 = 12 + 2 = 14$
- Therefore, at $x = 2$, the gradient vector is $[14]$.
- The gradient vector represents the slope or rate of change of the function at a specific point.
- In this case, the gradient vector $[14]$ indicates that the function $f(x) = 3x^2 + 2x + 1$ has a slope of 14 at $x = 2$.

Embedding Vector

- An embedding vector is a series of numbers and can be considered as a matrix with only one row but multiple columns, such as [2,0,1,9,0,6,3,0].
- An embedding vector includes information representing the characteristics of an object, such as RGB (red-green-blue) color descriptions.
- A color can be described by the proportions of red, green, and blue.
- An embedding vector in RGB could be [R, G, B].

Special vectors with example

- Input vectors
- Activation vectors
- Attention vectors
- Embedding vectors
- Weight vectors
- Bias vectors
- Gradient vectors

1. Input Vectors

- **Definition**: Represent raw data fed into the model for processing.
- **Example**: In image recognition, an input vector could be the pixel intensity values of an image, such as $[255, 128, 64, 0]$ for grayscale.

2. Activation Vectors

- **Definition**: Output of a layer in a neural network after applying the activation function.
- **Example**: After passing data through a layer with ReLU activation, the vector might transform from $[-2, 3, -1]$ to $[0, 3, 0]$.

3. **Attention Vectors**

- **Definition**: Assign importance to different parts of the input during tasks like translation or image captioning.
- **Example**: In English-to-French translation, the sentence "I love music" might have an attention vector assigning higher weight to "music" to generate an accurate French translation.

4. **Embedding Vectors**

- **Definition**: Represent features or characteristics of objects in a lower-dimensional space.
- **Example**:
  - Word Embedding: "cat" might be represented as $[0.2, 0.8, 0.6]$ in a 3-dimensional space.
  - RGB Colors: Red can be represented as $[255, 0, 0]$.

5. **Weight Vectors**

- **Definition**: Parameters in a model that are learned during training to map input to output.
- **Example**: In a neural network layer, weights might be $[0.1, 0.5, -0.3]$, adjusting how much influence each input has.

6. **Bias Vectors**

- **Definition**: Additive constants to help the model fit data better by shifting activation functions.
- **Example**: If a weight vector is $[1.2, -0.5]$, a bias vector of $[0.3]$ shifts the result.

7. Gradient Vectors

- **Definition**: Represent the rate of change of a function with respect to its inputs or parameters, essential for optimization.
- **Example**:
    - For $f(x) = 3x^2 + 2x + 1$: The gradient vector at $x = 2$ is $[14]$, showing the slope of the function at that point.
    - In multi-dimensional spaces, the gradient guides weight updates during backpropagation.

Additional Examples:

1. **Input Vector**:
   - A speech input might be represented as a time-series vector: $[0.2, 0.5, -0.1, 0.3]$.
2. **Embedding Vector**:
   - A movie embedding could be $[0.6, 0.9, 0.2]$, representing its genre, rating, and popularity.
3. **Weight Vector**:
   - In logistic regression, weights might determine the contribution of features like $[1.5, -2.1, 0.7]$.
4. **Gradient Vector**:
   - For a loss function in a neural network, gradients like $[-0.03, 0.1, 0.05]$ indicate how to adjust weights to minimize error.