

Bit Manipulation

Bitwise operators and their application

Note: Time complexity of all the bitwise operator are approx $O(1)$

& => bitwise AND
| => bitwise OR
^ => bitwise XOR
>> => bitwise Right shift
<< => bitwise Left shift
~ => bitwise NOT

Note that & (bitwise AND) is different from && (logical AND) , | (bitwise OR) is different from || (logical OR) and ~ (bitwise NOT) is different from ! (logical NOT)

Rules->

AND

0&0=0
0&1=0
1&0=0
1&1=1

OR

0&0=0
0&1=1
1&0=1
1&1=1

XOR

0&0=0
0&1=1

1&0=1
1&1=0

Properties of XOR:

$a \wedge a = 0$
 $0 \wedge a = a$

NOT

$\sim 0 = 1$
 $\sim 1 = 0$

```
int a=5 (101), b=7 (111);  
cout << a&b ; // (101) & (111)=(101)=5  
cout << a|b ; // (101) | (111)=(111)=7  
cout << a^b ; // (101) ^ (111)=(010)=2  
cout << ~a ; // ~(101)=(010)=2
```

Left shift operator(<<)

a -> 0000001110
a<<1 -> 000011100

Right shift operator(>>)

a -> 0000001110
a>>1 -> 0000000111

E.g

```
int a=5;  
a=a<<1; // then a? = 10 (2*a);  
a=5;  
a=a>>1; // then a? = 2 (a/2) e.g (5/2)=2;
```

Sol

a=5(00101);
a<<1 -> (01010) = 10;

Q.) You are given a number and find the value of (a<<b)?

Ans: $a \cdot (2^b)$

Q.) You are given a number and find the value of (a>>b)?

Ans: $a/(2^b)$

Why?	2^3	2^2	2^1	2^0 ($2^2+2^0=5$)
$a=5 \Rightarrow$	0	1	0	1
$a<<1 \Rightarrow$	1	0	1	0

$$(2^3+2^1)=2(2^2+2^0)=2*a;$$

Thus, $(1<<n)$ is equivalent to 2^n

Q.) You are given an array of N numbers in which all the numbers are repeated twice except one number which is present exactly once then find out that number?

Link: <https://www.hackerrank.com/challenges/lonely-integer/problem>

e.g-> {2,3,4,4,2} so Answer=3;

Hint-> use XOR property

$$a^a=0;$$

$$a=5 \Rightarrow (101) \text{ then } a^a (101)^{(101)} = (000)=0;$$

If we use xor of all the numbers present in the array i.e

$$2^3^4^4^2 = 3$$

If And-> $(1\&2\&4\&4\&2) \Rightarrow (1\&2\&3\&4)$ No Need.

```
int main()
{
    int n;
    cin>>n;
    vector<int> arr(n);
    for(int i=0;i<n;i++) cin>>arr[i];
    int ans = 0;
    for(int i=0;i<n;i++){
        ans=ans^arr[i];
    }
```

```

}
cout<<ans;
}

```

Q.) You have to check whether the given number is odd or even but you are not allowed to use % operator then how do you do that?

a=5-> (0101)
 b=10-> (1010)
 a&1=>(0101)&(0001)=(0001)=1
 b&1=>(1010)&(0001)=(0000)=0

```

if((a&1)==1){
cout<<"ODD"<<endl;
}
else {
cout<<"EVEN"<<endl;
}

```

Q.) How to check whether i^{th} bit (from right) is 1 or 0 for the given input number n.

001010101010
 43210 (index)
 0->0
 1->1
 (1<<i) -> 0000000100000000
 i

Answer:

```

if(n&(1<<i))
{
    cout<<"i-th bit is set"<<endl;
}
else

```

```
{
    cout<<"i-th bit is not set"<<endl;
}
```

Q.) How to set the ith bit to 1 for the given input number n;

Ans:

$n = n | (1 \ll i);$

Q.) How to set the ith bit to 0 for the given input number n;

Ans:

$n = n \& (\sim(1 \ll i))$

a-> 0000010101010100000
 i
 (1<<i) 0000000010000000000
 ~(1<<i) 1111111110111111111

Q.) How to calculate the number of setbits(1) in the given number.

$0 \leq n \leq 2^{63}-1$

Sol:-

```
long long n; // 0<=n<=2^63-1
cin>>n;
long long ans=0;
// ans stores the number of set bits
for(int i=0;i<64;i++){
    // 1<<i -> int
    // 1LL<<i -> long long int
    if(n&(1LL<<i)){
        ans++;
    }
}
cout<<ans;
```

Q.) How to swap two numbers using the XOR operator.

Sol: $(x \oplus y) \oplus y \rightarrow x$

$(x \oplus y) \oplus x \rightarrow y$

$x = x \oplus y; // x = x \oplus y, y = y$

$y = x \oplus y; // y = (x \oplus y) \oplus y \rightarrow x // x = x \oplus y, y = x$

$x = x \oplus y; // x \oplus y \oplus x = y // x = y, y = x$

Q.) How to generate all possible non empty subsequences of the given string.

e.g-> (abc) has following subsequences:

a,b,c,ab,bc,ac,abc

Subsequence -> delete some elements from anywhere in the string and concatenate the remaining.

Eg abcdefgh -> del c,f,h -> abdeg

Substring -> delete some elements from the beginning and some from the end.

Eg abcdefgh -> del a,b from begin and g,h from end

-> cdef

Sol :-

Assume that length of string is n.

Represent any subsequence of this string as a binary number of length n.

Eg abc -> binary number of length 3

If binary digit is 1 -> then that char is present in this sequence.

Else it is deleted.

abc

110 -> ab 1 to 7 -> 001,010,011,100,101,110,111

-> c, b, bc, a, ac, ab, abc

101 -> ac

N -> 1 to $2^n - 1$

```
vector<string> seq;  
// stores all non empty subsequences  
string s;  
cin>>s;
```

```

int n = s.length();
//n<=15
for(int i=1;i<(1<<n);i++){ // 1 to 2^n-1
    string temp="";
    for(int j=0;j<n;j++){
        if(i&(1<<j)){ // if jth bit in i is set
            temp+=s[j];
        }
    }
    seq.push_back(temp);
}
for(int i=0;i<seq.size();i++){
    cout<<seq[i]<<" ";
}

```

Q. <https://www.hackerrank.com/challenges/and-product/problem>

N queries -> a and b

Output $a \& (a+1) \& (a+2) \dots b$

12 and 15 -> 1100 and 1111 -> 1100 -> 12

14 15 -> 1110 and 1111 -> 1110 -> 14

1001 and 1101 -> 1000 -> 8

1001&1010&1011&1100&1101 -> 1000 -> 8

01111 and 10000 -> 0

```

string x;
for(int i=31;i>=0;i--){
    if(a&(1<<i)){
        x+='1';
    }
    else{
        x+='0';
    }
}

```

```

while(a){
    int x = a%2;
}

```

```

    x+=('0'+x);
    x/=2;
}
reverse(x.begin(),x.end());

```

Q. <https://www.hackerrank.com/challenges/sansa-and-xor/problem>

3 4 5

3

4

5

3,4

4,5

3,4,5

Answer=> $(3)^{(4)}(4)^{(5)}(3^4)^{(4^5)}(3^4^5)$

-> If occurrence is even then don't take it.

4 -> $a^a a^a a^a = 0$

-> If occurrence is odd then include it in your answer.

5 -> $a^a a^a a^a a^a = a$

```

int sansaXor(vector<int> arr) {
    int ans=0;
    int n=arr.size();
    for(int i=0;i<n;i++){
        long occ=(i+1)*(n-i);
        if(occ&1) ans^=arr[i];
    }
    return ans;
}

```