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An iterative phase retrieval method is proposed, which uses a sequence of diffraction intensity patterns recorded at different wavelengths. This method has a rapid convergence, and a high immunity to noise and environmental disturbance. The wrap-free phase measurement range is also extended based on the principle of two-wavelength interferometry. Simulation and experimental results are presented to demonstrate the approach. © 2008 Optical Society of America

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Recovering the phase of radiation is of general interest in material and biomedical science, and in nanotechnology. So far, many techniques have been proposed, including holography [1,2] and different kinds of phase retrieval methods [3–5]. While many successful applications have been demonstrated, their wider uses still remain limited because of either high requirements on experiment conditions or the inapplicability to a general complex-valued field. To solve these problems, several experimental schemes for phase retrieval have been proposed. Rodenburg and Faulkner used a moving aperture, [6] or a moving illumination, in their diffraction patterns collection process [7]. In work by Zhang *et al.* and Almoro *et al.*, a set of diffraction patterns recorded in a volume was used to improve the convergence [8,9]. Recently, a new method based on aperture plane modulation was also proposed and experimentally demonstrated [10]. In 1999, Fienup *et al.* suggested the recording of diffraction patterns at a sequence of wavelengths to obtain a three-dimensional (3D) image analogous to radar, and that, along with the opacity of the illuminated object, also helped to estimate the object support for the Fienup algorithm [11]. These methods relax the requirements on camera performance and are effective in many applications, but they only allow for determination of the phase modulo 2π, and in some cases this is not sufficient.

In this Letter, instead of putting constraints on an object or acquiring a diffraction pattern at different planes, we acquire the necessary information for phase retrieval by tuning the illumination wavelength. The recording of a diffraction pattern needs a very simple experimental setup. As for transparent objects, first a beam from a tunable laser is expanded, collimated, and projected onto a camera normally. Then the object is simply inserted into the beam at a position Z away from camera; for reflecting objects, a beam splitter should be used. By changing the wavelengths $(\lambda_1, \lambda_2, \ldots, \lambda_M)$, a set of M diffraction intensity patterns is collected.

The flowchart, Fig. 1, illustrates the main procedures of our phase retrieval algorithm. The algorithm starts with a guessed phase (constant or random) of the diffracted wavefront at the recording plane, and proceeds as follows: (1) combine the guessed phase with the square root of the intensity I_{λ_1} to yield an estimate of the diffracted wavefront, (2) propagate this wavefront back to the object plane, (3) convert this phase for the next wavelength λ_2, (4) propagate the wavefront to the recording plane, (5) replace the calculated amplitude with the square root of recorded intensity at the new wavelength, and (6) repeat steps 2 to 5 until the difference between the calculated and recorded intensity is sufficiently small. In the iteration process, when the final illumination wavelength λ_M is reached, the wavelength sequence is then reversed; i.e., the last wavelength becomes the first one, the second last one becomes the second, etc. For the beam propagation in steps 2 and 5, the angular spectrum algorithm [12] is used since it works well for short recording distance.

To convert the phase for a different wavelength in step 3, a model of the object is required. For a trans-

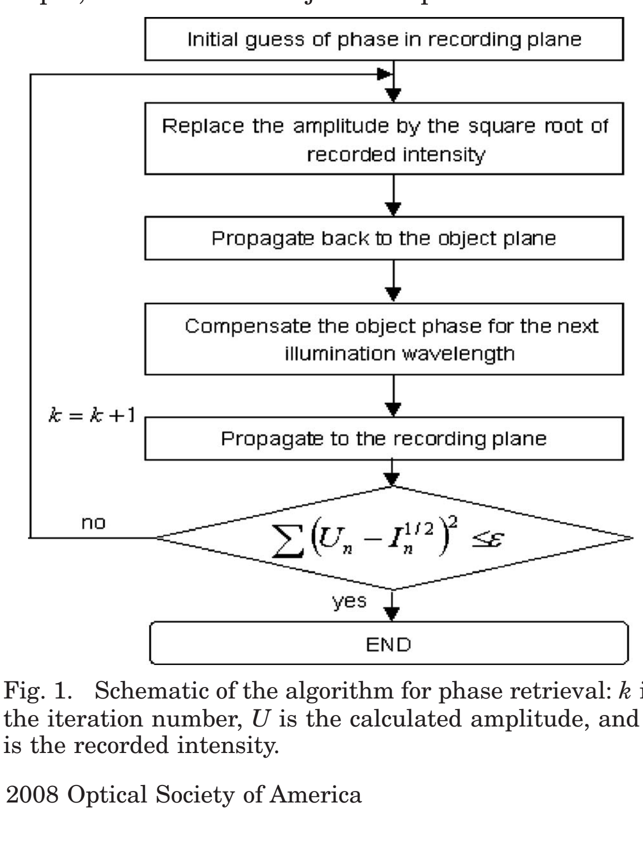

Fig. 1. Schematic of the algorithm for phase retrieval: k is the iteration number, U is the calculated amplitude, and I is the recorded intensity.

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mitting geometry, the phase retardance introduced by an object can be written as

$$\Delta\varphi_m(x) = 2\pi \frac{\Delta h(x)}{\lambda_m}(n_{\mathrm{obj}} - n_{\mathrm{air}}), \tag{1}$$

where $\Delta h(x)$ is the object thickness variation with respect to any specified reference plane, λ_m is the wavelength of the illumination light, and n_{obj} and n_{air} are the refractive indices of object and air, respectively. For a reflective geometry, a similar relationship holds, except that a factor of 2 is included to account for light passing through the object twice. For an object with a random phase distribution at a scale smaller than the system resolution that is determined by the recording numerical aperture and light wavelength, the model mentioned above cannot be used.

For a large $\Delta h(x)$, the phase calculated for a certain wavelength may be wrapped. In this case, a synthetic wavelength,

$$\Lambda = \frac{\lambda_m \lambda_n}{|\lambda_m - \lambda_n|}, \tag{2}$$

is used to calculate the wrap-free phase, as in two-wavelength interferometry [13], where λ_m and λ_n denote two different illumination wavelengths.

Figure 2 shows a typical simulation result obtained from 20 diffraction patterns recorded at wavelengths between 750 and 845 nm and with a step size of 5 nm. The size of the test object was 3.7 mm $\times$2.6 mm, and its amplitude [Fig. 2(a)] varied from 0 to 255. The phase introduced by the test object $\Delta\varphi$ varied between 0 and 10π for the wavelength 750 nm and between 0 and 8.88π for wavelength 845 nm. Some spatial variations in the object transmission are required so as to make the intensity difference of the diffraction pattern at different wavelengths resolvable by the camera. An object that has pure amplitude transmittance can also work in our simulation provided that its amplitude variation gives a resolvable difference in the diffraction patterns at different wavelengths. The propagation distance from the object to the camera was 30 mm. The system noise was assumed to be neglectable. One can see that the recovered wrap-free phase [Fig. 2(d)] is almost indistinguishable from the original except for a constant offset. Some features with a similar shape to the test phase are observable in the recovered amplitude [Fig. 2(c)]. However, the contrast of those features is quite low. To quantify the accuracy and convergence of this method, the sum-squared error (SSE), defined as

$$\mathrm{SSE} = \frac{\sum_{m,n} [|U(m,n)| - |\bar{U}(m,n)|]^2}{\sum_{m,n} |U(m,n)|^2}, \tag{3}$$

is used, where $U(m,n)$ and $\bar{U}(m,n)$ represent the original and the recovered object wavefronts. The result shown in Fig. 2 has an SSE value of 0.01. However, the absolute value of SSE could be strongly dependent on the object. In obtaining the convergence curves shown in Fig. 3(a), the same wavelength sequence as in Fig. 2 was used. When the number of recordings was reduced from 20 to 6, the convergence rate slowed by a factor of 6, and the SSE increased by a factor of 40. However, the convergence and the SSE showed only a slight improvement if more than 20 recordings were used. The reason is that the information provided by more than 20 recordings is redundant. Figure 3(b) shows the convergence dependence

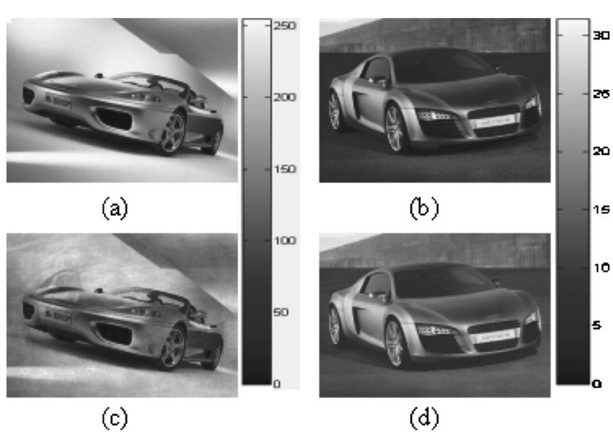

Fig. 2. Simulation result. Test (a) amplitude and (b) phase. Retrieved (c) amplitude and (d) phase.

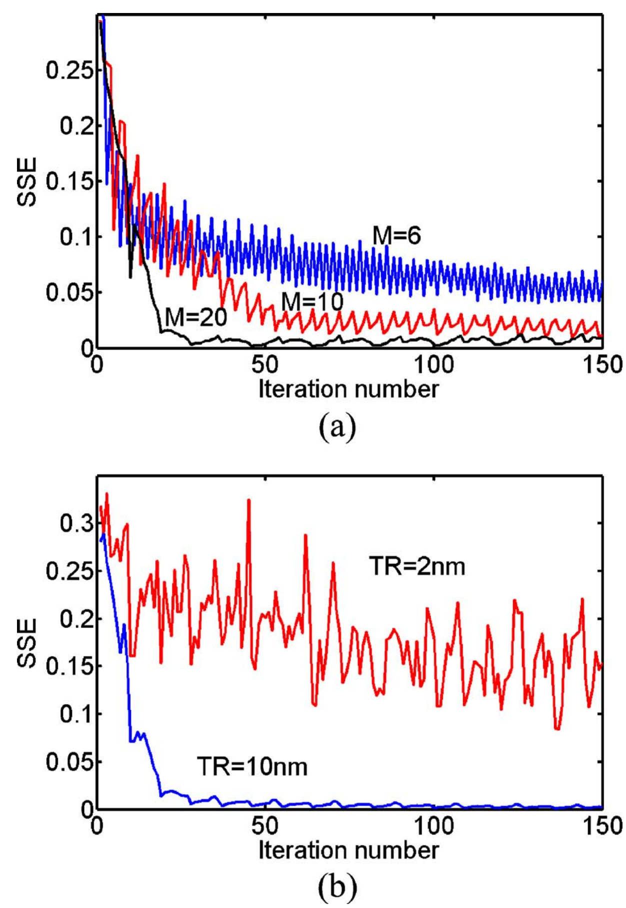

Fig. 3. (Color online) Convergence performance: (a) different numbers of intensity recordings $M=6$, 10, and 20; and (b) different tuning ranges (TR)=2 and 10 nm.

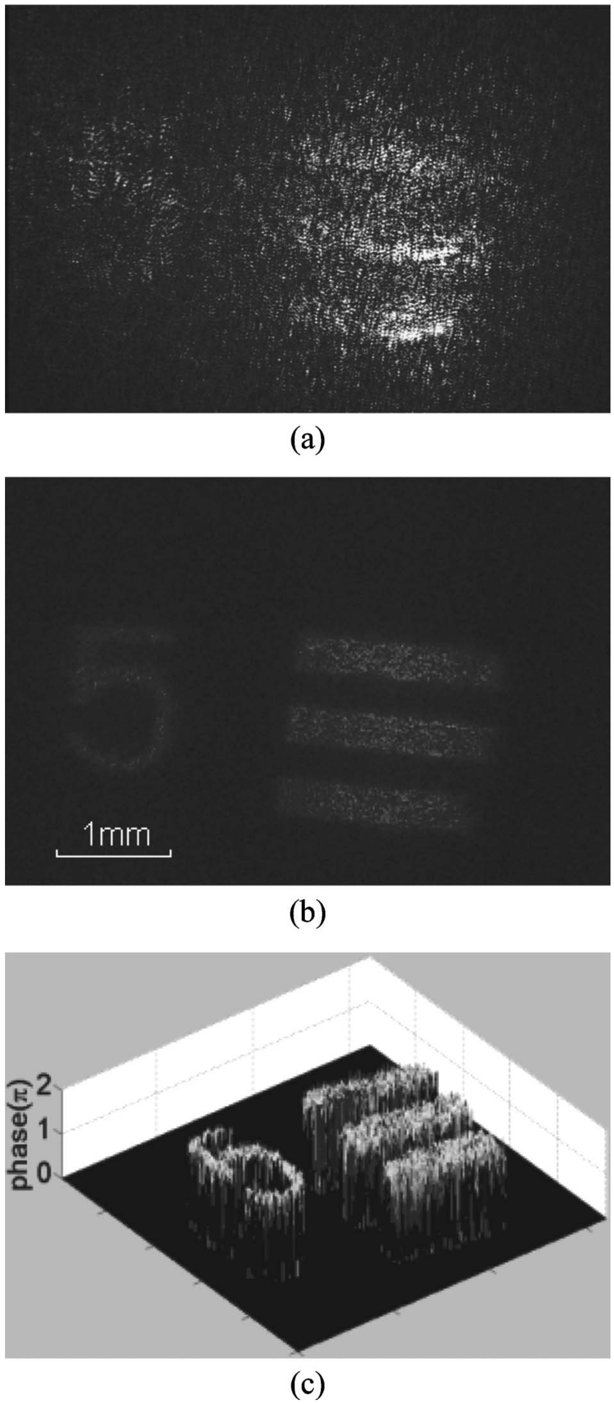

Fig. 4. Experiment result. (a) Recorded diffraction pattern with 750 nm illumination beam, retrieved (b) amplitude, and (c) phase.

on the wavelength tuning range for a fixed number of wavelengths. Here ten wavelengths were used. It was found that a tuning range of at least 10 nm was required to achieve an SSE lower than 0.02 for our test object. On the other hand, by selecting a proper wavelength step, the maximum range for a unique phase measurement and the convergence performance of algorithm can be tuned.

Experiments were also made to verify this method. The light source was a titanium–sapphire laser with a wavelength tuning range between 750 and 890 nm. The camera was a Sony XC-55 having 659×494 pixels with a size $7.4\ \mu\mathrm{m}\times 7.4\ \mu\mathrm{m}$. The test object, a U.S. Air Force resolution target attached to a random phase plate, was placed 20 mm in front of the camera. The phase plate had a height variation of about 0.5 μm (peak–peak value) and was used to produce the necessary speckle field. The signal-to-noise ratio (SNR) of the recorded data is about 40 dB. Figure 4(a) shows one of the recorded diffraction patterns. The speckle can be clearly seen. The recovered amplitude and phase after 160 iterations are shown in Figs. 4(b) and 4(c). In the iteration process, the amplitude pattern began to be recognizable after 40 iterations, and no obvious improvement was observed after 120 iterations. Although the details of the retrieved phase could not be observed, since the phase plate has a random phase pattern, the high contrast in the retrieved amplitude shows that the retrieved phase must be correct.

In conclusion, we have proposed a phase retrieval technique for complex-valued fields using multiple illumination wavelengths. In this method the only imposed object constraints are that the object should be nondispersive or that its dispersion is predictable. Based on the concept of the synthetic wavelength, the measurement range of the wrap-free phase has been extended. Collecting information at multiple wavelengths improves the convergence of the iterative algorithm. It was shown by both simulations and experiments that this method has a rapid convergence speed and a high immunity to noise. We believe that the proposed method could be a practical technique for 3D surface measurement. Work on extending the method to smooth objects is currently under way.

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