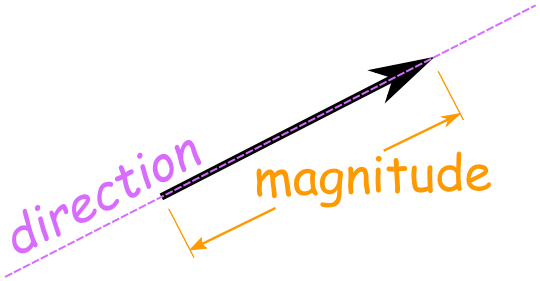
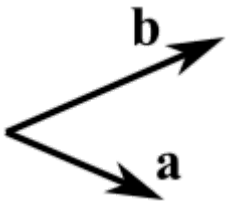
Cross Product

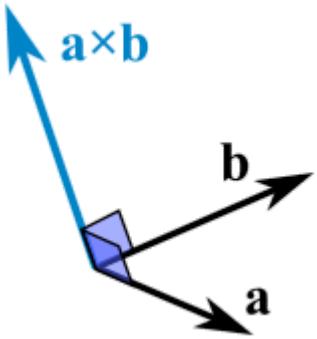
A vector has **magnitude** (how long it is) and **direction**:



Two vectors can be **multiplied** using the "**Cross Product**" *(also see Dot Product)*

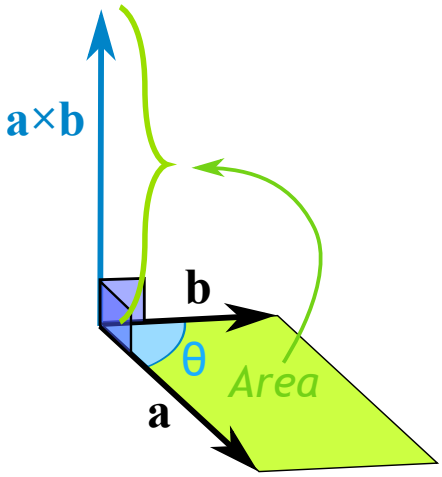


The Cross Product **a × b** of two vectors is **another vector** that is at right angles to both:

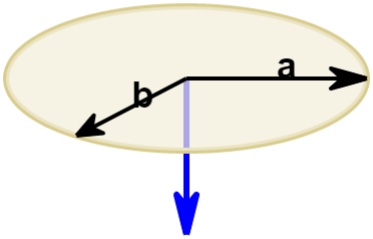


And it all happens in 3 dimensions!

The magnitude (length) of the cross product equals the area of a parallelogram with vectors **a** and **b** for sides:



See how it changes for different angles:



The cross product (*blue*) is:

- zero in length when vectors **a** and **b** point in the same, or opposite, direction
- reaches maximum length when vectors **a** and **b** are at right angles

And it can point one way or the other!

So how do we calculate it?

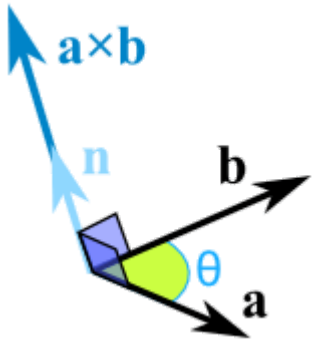
Calculating

We can calculate the Cross Product this way:

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}|\ |\mathbf{b}|\ \sin(\theta)\ \mathbf{n}$$

- $|\mathbf{a}|$ is the magnitude (length) of vector **a**
- $|\mathbf{b}|$ is the magnitude (length) of vector **b**

- θ is the angle between **a** and **b**
- **n** is the unit vector at right angles to both **a** and **b**



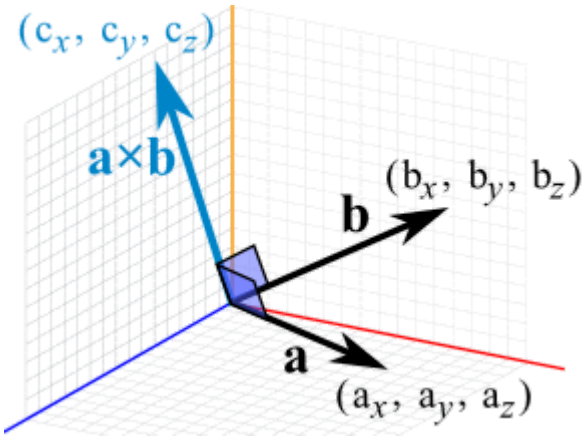
So the **length** is: the length of **a** times the length of **b** times the sine of the angle between **a** and **b**,

Then we multiply by the vector **n** so it heads in the correct **direction** (at right angles to both **a** and **b**).

OR WE CAN CALCULATE IT THIS WAY:

When **a** and **b** start at the origin point (0,0,0), the Cross Product will end at:

- $\mathbf{c_x = a_y b_z - a_z b_y}$
- $\mathbf{c_y = a_z b_x - a_x b_z}$
- $\mathbf{c_z = a_x b_y - a_y b_x}$



Example: The cross product of **a** = (2,3,4) and **b** = (5,6,7)

- $c_x = a_y b_z - a_z b_y = 3\times7 - 4\times6 = -3$
- $c_y = a_z b_x - a_x b_z = 4\times5 - 2\times7 = 6$
- $c_z = a_x b_y - a_y b_x = 2\times6 - 3\times5 = -3$

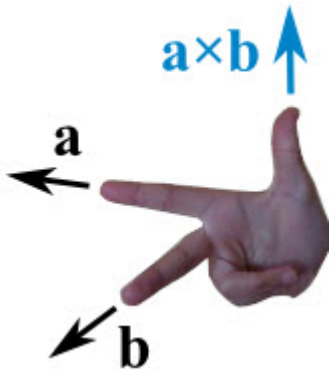
Answer: **a** × **b** = (−3,6,−3)

Which Direction?

The cross product could point in the completely opposite direction and still be at right angles to the two other vectors, so we have the:

"Right Hand Rule"

With your right-hand, point your index finger along vector **a**, and point your middle finger along vector **b**: the cross product goes in the direction of your thumb.



Dot Product

The Cross Product gives a **vector** answer, and is sometimes called the **vector product**.

But there is also the Dot Product which gives a **scalar** (ordinary number) answer, and is sometimes called the **scalar product**.

Question: What do you get when you cross an elephant with a banana?

Answer: |**elephant**| |**banana**| sin(θ) **n**

Question 1 Question 2 Question 3 Question 4 Question 5
Question 6 Question 7 Question 8 Question 9 Question 10