We want to generate noise $\xi^{ij}(t, \vec{r})$ with the following property:

(Eq. 106b) of Teaney et. al.

$$\langle \xi^{ij}(t,\vec{r})\, \xi^{mn}(t',\vec{r}') \rangle = \frac{2T}{\Delta t\, V_0}\, \kappa^{ijmn}\, \delta_{tt'}\, \delta_{\vec{r}\vec{r}'} \qquad \text{— (1)}$$

κ^{ijmn} is a complicated object (Eq. (44)) which is defined as (considering only shear)

$$\kappa^{ijmn} \equiv \underbrace{\kappa^{ij}_{\mu\nu}}_{\text{Known}}\, 2\eta \left(\Delta^{\mu\rho} \Delta^{\nu\sigma} - \frac{1}{d} \Delta^{\mu\nu} \Delta^{\rho\sigma} \right) \underbrace{\kappa^{mn}_{\rho\sigma}}_{\text{Known}}$$

Let us define new stochastic variables $\lambda^{\mu\nu}$:

$$\xi^{ij}(t,\vec{r}) \equiv \kappa^{ij}_{\mu\nu}\, \lambda^{\mu\nu}(t,\vec{r})$$

① is ensured if $\lambda^{\mu\nu}$ satisfies.

$$\langle \lambda^{\mu\nu}(t,\vec{r})\, \lambda^{\rho\sigma}(t',\vec{r}')\rangle = \frac{2T}{\Delta t\, V_0}\, 2\eta \left(\Delta^{\mu\rho}\Delta^{\nu\sigma} - \frac{1}{d}\Delta^{\mu\nu}\Delta^{\rho\sigma} \right)$$

$$\times\ \delta_{tt'}\, \delta_{\vec{r}\vec{r}'}$$

The above equation is tensorial; holds in any frame. So, compute in local rest frame, where $\lambda^{\mu\nu}_{LRF}$ is purely spatial:

$$\left\langle \lambda^{ij}_{LRF}(t,\vec{r})\, \lambda^{kl}_{LRF}(t',\vec{r}')\right\rangle = \frac{4T\eta}{\Delta t\, V_0} \left(\delta^{ik}\delta^{jl} - \frac{1}{d}\delta^{ij}\delta^{kl} \right)$$

$$\delta_{tt'}\, \delta_{\vec{r},\vec{r}'}$$

Step 1: Generate λ^{ij}_{LRF} according to above. There are non-vanishing correlations between different random numbers; eg, $\langle \lambda^{xx}_{LRF}\ \lambda^{yy}_{LRF} \rangle \neq 0$. They can be handled by generating uncorrelated random numbers followed by rotation.

Step 2: Using λ^{ij}_{LRF}, compute $\lambda^{\mu\nu}$ by boosting by velocity $-\vec{v}$.

$$\lambda^{\mu\nu} = \Lambda^{\mu}{}_{\alpha} \, \Lambda^{\nu}{}_{\beta} \, \lambda^{\alpha\beta}_{LRF}$$

Step 3: Using $\lambda^{\mu\nu}$, compute the noise terms of interest:

$$\xi^{ij} = \kappa^{ij}_{\mu\nu} \lambda^{\mu\nu}$$