To generate ξ^{ij} with property

$$\langle \xi^{ij}(t,\vec{r})\, \xi^{mn}(t',\vec{r}\,') \rangle = \frac{2T}{\Delta t\, V_0} \kappa^{ijmn}\, \delta_{tt'}\, \delta_{\vec{r},\vec{r}\,'}$$

set $V_0 = 1$.

κ^{ijmn} is given by Eq.(44) of Teaney et. al.

(ignoring bulk viscosity) :

$$\kappa^{ijmn} = 2\eta\, \kappa^{ij}_{\mu\nu} \left(\Delta^{\mu\rho}\Delta^{\nu\sigma} - \frac{1}{d}\Delta^{\mu\nu}\Delta^{\rho\sigma} \right) \kappa^{mn}_{\rho\sigma}$$

$\kappa^{ij}_{\mu\nu}$ is given in Eq.(36a) and is <u>symmetric</u>

in indices μ & ν.

$$\Rightarrow \kappa^{ijmn} = 2\eta\, \kappa^{ij}_{\mu\nu} \left(\Delta^{(\mu\rho}\Delta^{\nu)\sigma} - \frac{1}{d}\Delta^{\mu\nu}\Delta^{\rho\sigma} \right) \kappa^{mn}_{\rho\sigma}$$

STEP I : To Generate 'local rest frame' random

numbers which are purely spatial :

$$\langle \lambda^{ij}_{LRF}\, \lambda^{kl}_{LRF} \rangle = \frac{4\eta T}{\Delta t} \left(\frac{\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}}{2} - \frac{1}{d}\delta^{ij}\delta^{kl} \right)$$

Note $\langle \lambda^{xx}_{LRF} \lambda^{yy}_{LRF} \rangle \neq 0$.

Generate independent random numbers γ^{ij},

$$\langle \gamma^{ij} \gamma^{kl} \rangle = \frac{4\eta T}{\Delta t} \left(\frac{\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}}{2} \right)$$

and subtract the trace,

$$\lambda^{ij}_{LRF} = \gamma^{ij} - \frac{\delta^{ij}}{d} \gamma^{k}_{k}$$

STEP 2: Boost by velocity $-\vec{v}$

$$\lambda^{\mu\nu} = \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} \lambda^{\alpha\beta}_{LRF},$$

(Note $\lambda^{0\alpha}_{LRF}$ is 0).

STEP 3: Obtain noise ξ^{ij} in density frame:

$$\xi^{ij} = \kappa^{ij}_{\mu\nu} \lambda^{\mu\nu}.$$