IEE 574 PROJECT

Drone Routing Optimization Problem

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CONTENTS

1. INTRODUCTION

The Centers for Disease Control and Prevention is looking for the most optimal method of delivering vaccines to the 20 nearby city hospitals with the help of drones.The CDC is located in Oroville, California and it has fulfill the hourly demands of the hospital in each of the nearby 20 cities. Lab production takes place from 8:00 AM to 12:00 PM, but vaccine orders are collected from 7:00 AM to 11:00 AM. A make-to-order strategy for meeting the demand is implemented.Our goal is to minimize the cost of distributing the vaccines which depends on costs of scheduling drones and transportation cost.

Assumptions:

1) Scheduled drones are capable of satisfying each hourly demand they are assigned.
2) Drones are charged overnight, and they do not break down.
3) If a drone is scheduled in a particular hour it will stop only after its battery is exhausted and not before that.

2. DATA DESCRIPTION

Keyword	Description
Drone Capacity	The maximum number of vaccines a type of drone can carry
Battery life	The number of hours up to which a drone can serve between lab and cities.
Demand	The required demand for vaccines for different cities depending upon the population of a city.
Distance Matrix	The distance from city 'i' to city 'j' (with one lab included).
Variable Costs	The costs incurred by the company depending on the distance from i to j. (Ignored as it is proportional to the distance i.e we are trying to minimise the distance which would also minimise the costs).

3. DESCRIPTION OF MODELS

3.1 Minimize the Drone Scheduling Costs

The objective is to find the required number of type 1 and type 2 drones such that the total cost of drone scheduling is minimised while satisfying the constraints given in the problem.

The below table gives the details on the constraints:

Drone	Capacity per hour (vaccine units)	Cost in $1000 (paid once)	Battery life	Min# to schedule (In total, not per hour)
Type-I	450	350	3 hours	3
Type-II	1000	550	2 hours	2

As shown above, the drones can carry only a limited number of vaccines and that they have a limited battery life. Also, the cost associated with assigning each type of drone is given in the table. The problem also specifies to assign at least 3 type 1 drones and 2 type 2 drones.

With this information, we will proceed to the formulation part of this problem

3.1.2 Formulation and model with Mathematical programming:

The formulations are as below:

Decision variable:

Xij - represents number of type j drones assigned in hour i

Objective function :

Minimize Z : $350(X_{11}+X_{12}+X_{13}+X_{14}) + 550(X_{21}+X_{22}+X_{23}+X_{24})$

(We are minimizing the total cost spent in scheduling drones)

Subject to:

For 1st hour demand constraint: $450(X_{11}) + 1000(X_{21}) >= 7767$

For 2nd hour demand constraint: $450(X_{11}+X_{12}) + 1000(X_{21}+X_{22}) >= 4851$

For 3rd hour demand constraint: $450(X_{11}+X_{12}+X_{13}) + 1000(X_{22}+X_{23}) >= 3499$

For 4th hour demand constraint: $450(X_{12}+X_{13}+X_{14}) + 1000(X_{23}+X_{24}) >= 2733$

Model file for the above formulation:

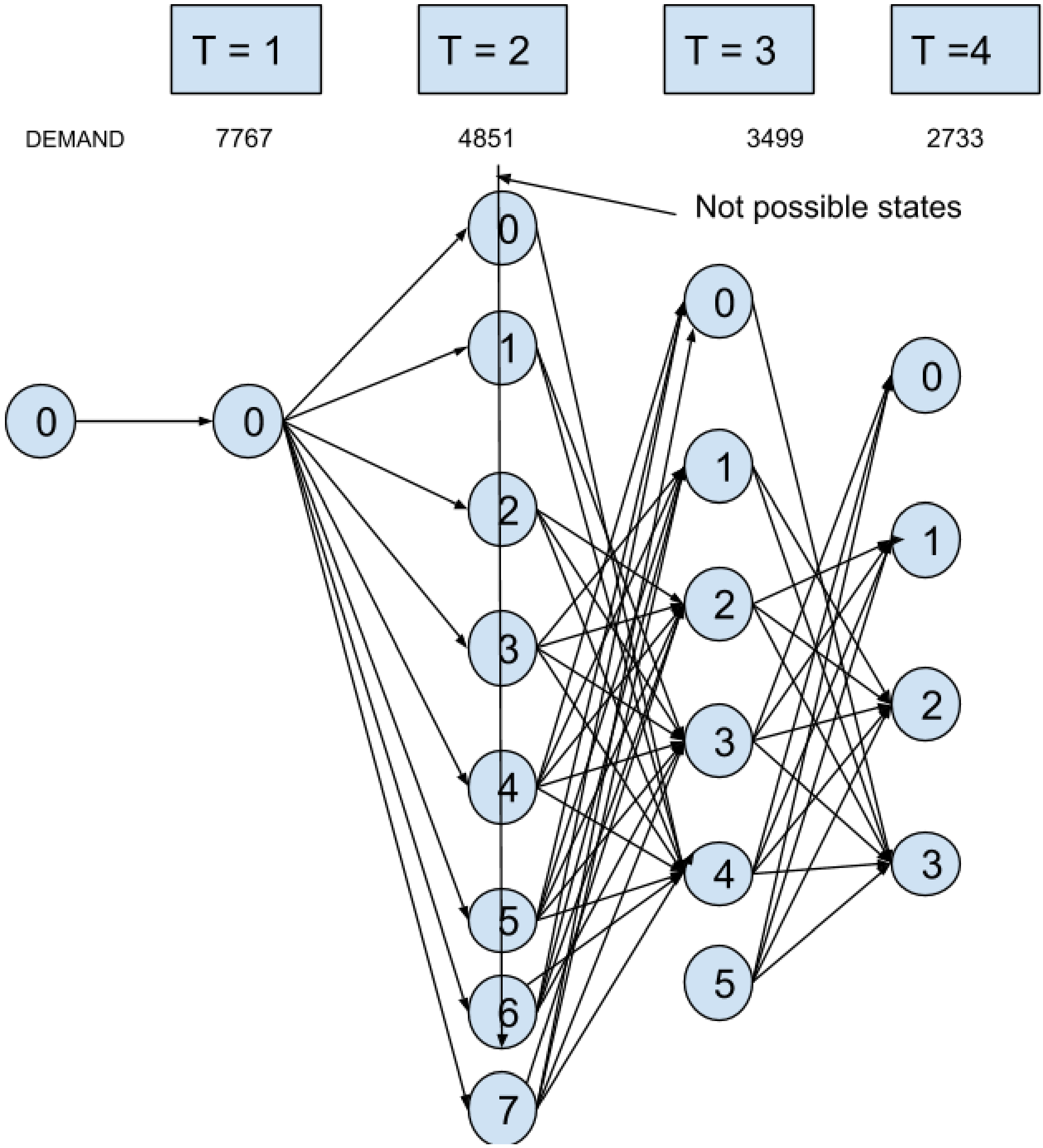
```
param cost{1..2}; # cost associated with each drone type
param c{1..2};#
param a{1..4,1..2}; #
param typ1;# Minimum number of type 1 drones needed
param typ2;# Minimum number of type 2 drones needed
param d{1..4};# aggregated demand for 4 hours

var x{i in 1..2, j in 1..4} integer>=0;

minimize ScheduleCost : sum{i in 1..2, j in 1..4} cost[i]*x[i,j];

subject to C1: c[1]*x[1,1] + c[2]*x[2,1]>=d[1];
subject to C2: c[1]*(x[1,1]+x[1,2]) + c[2]*(x[2,1]+x[2,2])>=d[2];
subject to C3: c[1]*(x[1,1]+x[1,2]+x[1,3])+c[2]*(x[2,2]+x[2,3])>=d[3];
subject to C4: c[1]*(x[1,2]+x[1,3]+x[1,4]) + c[2]*(x[2,3]+x[2,4])>=d[4];
subject to C5{ i in 1..1}: sum{j in 1..4} x[i,j]>=typ1;
subject to C6{ i in 2..2}: sum{j in 1..4} x[i,j]>=typ2;
```

The above formulation was solved using ampl and the results obtained are recorded in the results section of the report

3.1.3 Formulation and model With Dynamic programming

3.1.3.1 Formulation for Type 2 drone

The formulations are as below:



Dynamic programming network for Type 2 drones

Our aim at each stage

State 1(s) : The shift that we are at currently

State 2(v) :number of drones already loaded

X_t= New number of drones taken at stage t

D_t = Demand of Vaccines at stage

Stage : No. of drones that have one hour of battery left after that state

$F_t(s,v) = \text{minimum}_{dt <= 1000Xt + v}(X_t + F_{t+1}(X_t, 1000X_t)$

Objective Function

$F_1(0,0)$

Calculations for the type 2 drone

Stage 4

s			X_t		X_4^*	$F^*(X_4^*)$
	0	1	2	3		
0,0	-	-	-	1650	3	1650
1,1000	-	-	1100	1650	2	1100
2,2000	-	550	1100	1650	1	550
3,3000	0	550	1100	1650	0	0
4,4000	0	550	1100	1650	0	0

Stage 3

s				X_t		X_3^*	$F^*(X^*_3)$
	0	1	2	3	4		
0,0	-	-	-	-	2200+0	4	2200

1,1000	-	-	-	1650+0	2200	3	1650
2,2000	-	-	1100+550	1650+0	2200+0	3,4	1650
3,3000	-	550+1100	1100+550	1650+0	2200+0	1,2,3	1650
4,4000	0+1650	550+1100	1100+550	1650+0	2200+0	1,2,3	1650
5,5000	0+1650	550+1100	1100+550	1650+0	2200+0	1,2,3,4	1650

Stage 2

S				X_t			X_2^*	$F^*(X^*_2)$
	0	1	2	3	4	5		
0,0	-	-	-	-	-	2750+ 1650	5	4400
1,1000	-	-	-	-	2200 +1650	2750 +1650	4	3850
2,2000	-	-	-	1650+ 1650	2200+ 1650	2750 +1650	3	3300
3,3000	-	-	1100+ 1650	1650+ 1650	2200+ 1650	2750+ 1650	2	2750
4,4000	-	550+ 1650	1100+ 1650	1650+ 1650	2200+ 1650	2750+ 1650	1	2200
5,5000	0+ 2200	550+ 1650	1100+ 1650	1650+ 1650	2200+ 1650	2750+ 1650	0	2200
6,6000	2200	2200	1100+ 1650	1650+ 1650	2200+ 1650	2750+ 1650	0,1	2200
7,7000	2200	2200	1100+ 1650	1650+ 1650	2200+ 1650	2750+ 1650	0,1	2200

Stage 1

s	$X_t = 7$	X_2^*	$f^*(X^*_2)$
0	3850+2200	7	6050

Hence there are various optimal values for this

1)At stage 1 we have to buy 7 type 2 drone

2)At stage 2 we can buy either 0 drones or 1 drone of type 2

3)At stage three

(i)If we bought 0 drones at stage 2

we have to buy 4 drones at stage 3

(ii)If we bought 1 drone at stage 2

We have to buy 3 drones at stage 3

4)At stage 4

(i)If we bought 4 drones at stage 3 or 3 drones

We need not buy any drones for stage 4

Hence our objective function is to minimize cost

which comes out to be 3850+2200 = 6050

3.1.3.2 Formulation for Type 1 drone

The formulations are as below:

Stage(t) - The Shift we are at currently

State 1(s_1) - Number of drones with 1 hr of battery left

State 2(s_2) - Number of drone with 2 hr battery left

State 3(v) - Number of vaccines already loaded

Our objective at each stage

$F_t(s_1,s_2,v) = \text{Minimize}_{dt<=450Xt+450X(t-1)}(X_t+F_{t+1}(X_{t-1},X_t,450X_t+450X_{t-1})$

Objective function

$F_1(0,0,0)$

3.2 Minimize Transportation Costs (Problem 2):

From the optimal number of drones that are obtained from 3.1 section, the CDC lab now requires to determine the optimal routes for the drones to deliver the vaccines to each city as per the demand. So this time, in this problem we are interested in the individual demands for each city in hour 1 and not the aggregated demands as in the previous case.

To solve this vehicle routing problem (VRP), we would like to use exact formulations and heuristics as explained in the following sections:

3.2.1 Exact Formulation: In this, we would like to use two types of subtour elimination constraints : MTZ and Subtour Elimination constraints.

3.2.1.1 Exact Formulation 1 using MTZ (Miller Tucker Zemlin) Subtour elimination constraints:

The formulation is as follows:

Decision Variable:

Xijk - represents the path of drone of type k from city i to city j

r_i - represents flow variable

param n; #number of cities or hospital including the lab

param T; #number of drones of each type

param a{k in 1..T}; #capacity for each drone type

param d{j in 1..n}; #vaccine demand from each city or hospital

param cost{i in 1..n,j in 1..n}; #distance between cities to cities and lab to cities

(More information is can be seen properly labeled in the model file below)

Objective Function:

Minimize distance $\sum_{i=1}^{i=12} \sum_{j=1}^{j=21} \sum_{k=1}^{k=2} C_{i,j}X_{i,j,k}$

Constraints

1)Each hospital is visited once and by only one drone

$$\sum_{i=1}^{i=21} \sum_{k=1}^{k=2} X_{i,j,k} = 1 \qquad \forall\ j = 2....21$$

2) a drone that enters a city will leave the city

$$\sum_{i=1}^{i=21} X_{i,p,k} - \sum_{j=1}^{j=21} X_{p,j,k} = 0 \qquad \forall\ k = 1,2\ ,\ \forall\ p = 2...21$$

3)The cdc lab should not be included as it has no demand, it is th

supplier

$r_i = 0$

4)Capacity constraint for each drone type

$$r_j - r_i > (d_j + a_T) * \sum_{k=1}^{k=2} X_{i,j,k} - a_t \qquad \forall\ i= 1......21\ ,\ \forall\ j = 2...21$$

5) Capacity constraint for each drone type

$$r_j <= \sum_{i=1}^{i=21} \sum_{k=1}^{k=2} a_k X_{i,j,k} \qquad \forall\ j = 2...21$$

6)Number of type 1 and type 2 drones

$$\sum_{j=2}^{j=21} X_{1,j,1} = 8$$

$$\sum_{j=2}^{j=21} X_{1,j,2} = 6$$

Model file is as follows:

```
#Exact formulation 1
param n; #number of cities or hospital including the lab
param T; #number of drones of each type
param a{k in 1..T}; #capacity for each drone type
param d{j in 1..n}; #vaccine demand from each city or hospital
param cost{i in 1..n,j in 1..n}; #distance between cities to cities and lab
to cities

var x{i in 1..n,j in 1..n,k in 1..T}binary; # represents path of drone type k
from city i to city j

var r{i in 1..n}; #flow variable

minimize Distance: sum{i in 1..n, j in 1..n,k in 1..T} cost[i,j]*x[i,j,k];

subject to c1{j in 2..n}: sum{i in 1..n,k in 1..T} x[i,j,k] = 1; #each
hospital is visited once and by only one drone

subject to c2{k in 1..T,p in 2..n}: sum{i in 1..n:i<>p} x[i,p,k] - sum{j in
1..n:p<>j} x[p,j,k] = 0; #a drone that enters a city will leave the city
subject to c3{i in 1..n}: r[1] = 0; #the cdc lab should not be included as it
has no demand, it is the supplier

subject to c4{i in 1..n, j in 2..n}: r[j]-r[i] >= (d[j]+a[T])*sum{k in
1..T}x[i,j,k] - a[T]; #Subtour elimination constraints
```

```
subject to c6{ j in 2..n}: r[j] <= sum{i in 1..n,k in 1..T}
a[k]*x[i,j,k];#Capacity constraint for each drone type

subject to c7{k in 1..T}: sum{j in 2..n} x[1,j,1] = 8; # Number of type 1
drones

subject to c8{k in 1..T}: sum{j in 2..n} x[1,j,2] = 6; # Number of type 2
drones
```

The initial results for the formulation turned out to be infeasible as the capacity of drones was not sufficient enough to fulfill the demand hence we add type 1 drones till we get an optimal solution. We added 4 type 1 drones as we get 4 extra Type 1 drones added in the Greedy heuristics and the results came out to be as follows:

3.2.1.2 Exact Formulation 2 using Subtour elimination constraints using Cardinality:

The formulations are as follows:

(Variables annotations can be seen in model file below)

```
set D;                    #Order of drones

set H ordered;                    # Hospital

set C ordered:= 2..21;

param n := CARDINALITY{C}; #Number of Cities or Hospitals

set m := 0..(2**n - 1);  #Total number of possible subsets

set POWER{p in m} := {q in C: (p div 2**(ord(q)-1)) mod 2 = 1}; #Power set

param CAP12{j in D};                    #Capacities for drone type 1 and 2
```

param de{i in H};   #demand of hospital H

param di{i in H, k in H}; #distance of hospital d from hospital f

param d1;   #Total Number of drones

Minimize Total Distance:

$$\sum_{i \in H} \sum_{k \in H} \sum_{j \in D} C_{i,j} X_{i,k,j} * d_{i,k}$$

Constraints

$$\sum_{i \in H} \sum_{j \in D, i \neq k} X_{i,k,j} = 1 \qquad \forall \text{ k in H, } k \neq 1$$

$$\sum_{k \in H} \sum_{j \in D, i \neq k} X_{i,k,j} = 1 \qquad \forall \text{ i in H, } i \neq 1$$

$$\sum_{i \in H} \sum_{k \in H, i=1, k \neq 1} X_{i,k,j} = 1 \qquad \forall \text{ j in D}$$

$$\sum_{i \in H} \sum_{k \in H} X_{i,k,j} * de_i <= CAP_J \qquad \forall \text{ j in D}$$

$$\sum_{i \in H} \sum_{k \in H, i \neq 1, k=1} X_{i,k,j} = 1 \qquad \forall \text{ j in D}$$

$$\sum_{i \in H} X_{i,f,j} - \sum_{k \in H} X_{f,k,j} = 0 \qquad \forall \text{ j in D,f in H}$$

$$\sum_{j \in D} \sum_{s \in Power(r)} \sum_{t \in Power(r), s \neq t, Cardinality(Power(r) \leq 3)} X_{s,t,j} \leq \text{Cardinality(Power(r)) - 1}$$

$$\forall \text{ r in m,} r \neq 0$$

The model file for this problem is as follows:

Model file:

```
var x{i in H, k in H, j in D} binary;      #1 if drone j of type visits arc
i, 0 otherwise
```

```
var u{i in H}>=0 integer;


set D;             #Order of drones

set H ordered;                  # Hospitals

set C ordered:= 2..21;


param n := CARDINALITY{C}; #Number of Cities or Hospitals

set m := 0..(2**n - 1);  #Total number of possible subsets

set POWER{p in m} := {q in C: (p div 2**(ord(q)-1)) mod 2 = 1}; #Power set


param CAP12{j in D};                  #Capacities for drone type 1 and 2


param de{i in H};              #demand of hospital H


param di{i in H, k in H};      #distance of hospital d from hospital f


param d1;                             #Total Number of drones


minimize totaldistance: sum{i in H, k in H, j in D} x[i,k,j] * di[i, k];



subject to  c1{k in H:k<>1}: sum{i in H,j in D:i<>k} x[i, k, j] = 1;

subject to  c2{i in H:i<>1}: sum{k in H,j in D:i<>k} x[i, k, j] = 1;

subject to c3{j in D}: sum{i in H, k in H:i=1 and k<>1} x[i, k, j] = 1;

subject to c4{j in D}:sum{i in H, k in H} x[i, k, j] * de[i] <= CAP12[j];

subject to c5{j in D}: sum{i in H, k in H:k=1 and i<>1} x[i, k, j] = 1;
```

```
subject to c6{j in D, f in H}:sum{i in H} x[i, f, j] - sum{k in H} x[f, k, j]
= 0;

subject to c7{r in m: r != 0}: sum{j in D, s in POWER[r], t in POWER[r]: s<>t
and CARDINALITY{POWER[r]} <= 3} x[s,t,j] <= CARDINALITY {POWER[r]} - 1;
```

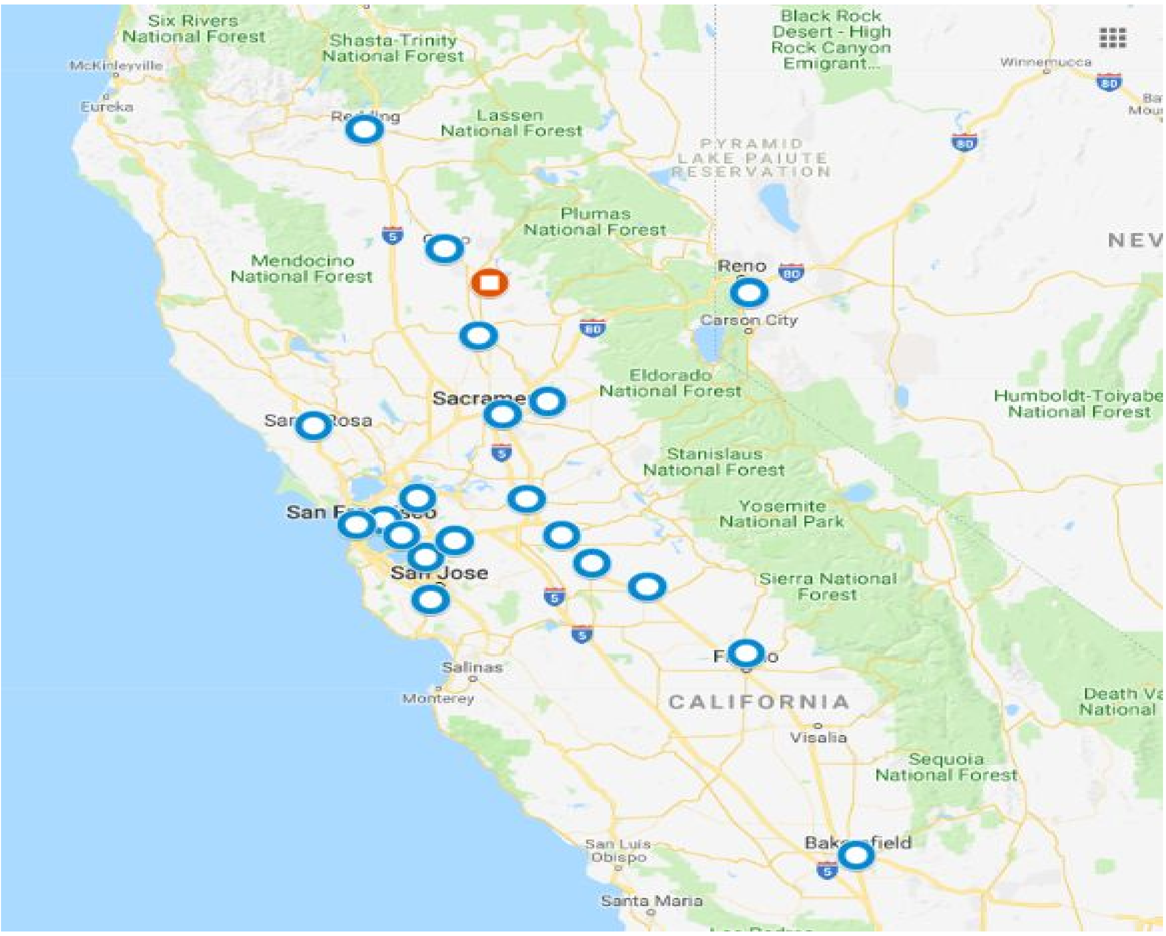
3.3 Heuristics

3.3.1 Greedy Heuristic

As our laptops are miserable and low performance computers, we develop the Greedy heuristic for our problem as hinted by our lecturer. We take our CDC lab as the starting and ending node, and obtain an optimal route which minimizes the distance to cover each city's demands for each of the Type 1 and Type 2 drones. To carry out this heuristic, we first need to have a matrix of distances which contains all the distances from each of the cities to all other cities. The distance matrix is computed as follows:

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	cities	Lab	Sacrame	Reno	Stocktc	Oaklan	San Fra	Modest	Fremont	San Jose	Fresno	Bakersfi	Yuba Ci	Santa Rc	Livermor	Folsom	Chico	San Lear	Redding	Merced	Concord	Turlock
1	Lab	0	64.67	95.36	106.76	122.66	128.47	126.78	136.46	156.96	205.91	316.25	26.06	95.45	70.93	126.76	23.22	127.86	86.4	160.34	108.3	142.94
2	Sacramento	64.67	0	140	47	83	93	70	107	128	168	276	46	104	92	23	95	96	168	116	71	88
3	Reno	95.36	140	0	171	221	231	193	238	258	291	399	133	242	214	131	173	252	206	217	208	227
4	Stockton	106.76	47	171	0	73	87	27	67	87	125	233	89	114	43	56	139	63	211	73	53	45
5	Oakland	122.66	83	221	73	0	15	76	29	51	174	272	118	66	34	103	168	12	212	122	22	93
6	San Francisco	128.47	93	231	87	15	0	90	42	53	188	285	128	61	47	113	178	25	222	136	34	106
7	Modesto	126.78	70	193	27	76	90	0	70	90	100	208	112	138	47	79	161	66	234	48	76	23
8	Fremont	136.46	107	238	67	29	42	70	0	27	167	265	142	104	18	135	193	18	236	115	41	86
9	San Jose	156.96	128	258	87	51	53	90	27	0	148	240	172	111	38	155	212	39	256	118	60	106
10	Fresno	205.91	168	291	125	174	188	100	167	148	0	117	210	236	145	177	259	164	332	58	174	81
11	Bakersfield	316.25	276	399	233	272	285	208	265	240	117	0	318	334	243	285	367	262	439	166	272	189
12	Yuba City	26.06	46	133	89	118	128	112	142	172	210	318	0	134	130	51	47	132	118	159	100	133
13	Santa Rosa	95.45	104	242	114	66	61	138	104	111	236	334	134	0	95	125	151	74	194	184	68	154
14	Livermore	70.93	92	214	43	34	47	47	18	38	145	243	130	95	0	112	178	23	228	92	33	63
15	Folsom	126.76	23	131	56	103	113	79	135	155	177	285	51	125	112	0	99	115	187	135	89	98
16	Chico	23.22	95	173	139	168	178	161	193	212	259	367	47	151	178	99	0	170	73	204	146	179
17	San Leandro	127.86	96	252	63	12	25	66	18	39	164	262	132	74	23	115	170	0	224	112	29	82
18	Redding	86.4	168	206	211	212	222	234	236	256	332	439	118	194	228	187	73	224	0	279	199	252
19	Merced	160.34	116	217	73	122	136	48	115	118	58	166	159	184	92	135	204	112	279	0	124	29
20	Concord	108.3	71	208	53	22	34	76	41	60	174	272	100	68	33	89	146	29	199	124	0	88
21	Turlock	142.94	88	227	45	93	106	23	86	106	81	189	133	154	63	98	179	82	252	29	88	0

The 20 authentic cities together with our CDC lab at Oroville is marked and depicted on Google Maps whose image is as below:



We first find out the optimal route which minimizes the distance according to greedy heuristic and start assigning Type 2 drones at first to satisfy each of the city's demand according to the route. **We assume that if the vaccines are still remaining on the drone and if it can not satisfy the next city's demand in whole, the drone returns back to the lab**. After this, we send out another drone to the next city on the route which the previous drone did not satisfy. We take the number of Type 2 drones required in the 1st hour and assign them first. after all the 6 Type 2 drones are used, we then start assigning Type 1 drone until all the city's demands are met with considering the above mentioned assumption.

The results are as follows:

We need:

Type 2 Drones : 6

Type 1 Drones: 8

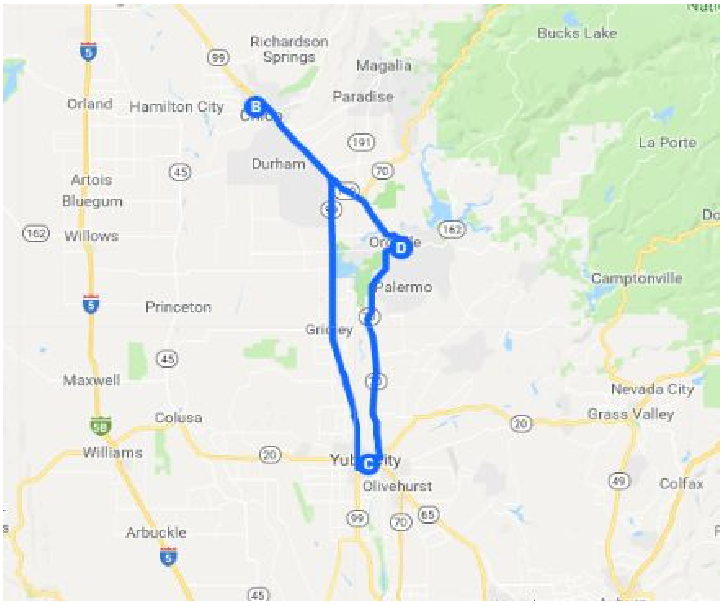
20 cities are numbered from 2 to 21 for the ease of calculation and route formation and we take 1 as our CDC lab at Oroville, CA.

The routes and the minimum distance together with the map for each drones of Type 1 and Type 2 are:

As the starting and ending cities are the same, the notation is one below the other.

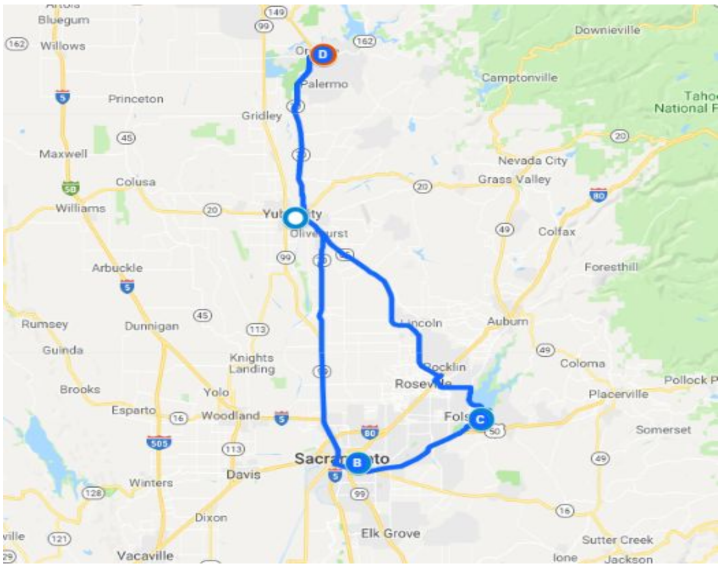
Type 2 Drone 1

1 → 16 → 12 → 1 ----- 96.28 miles



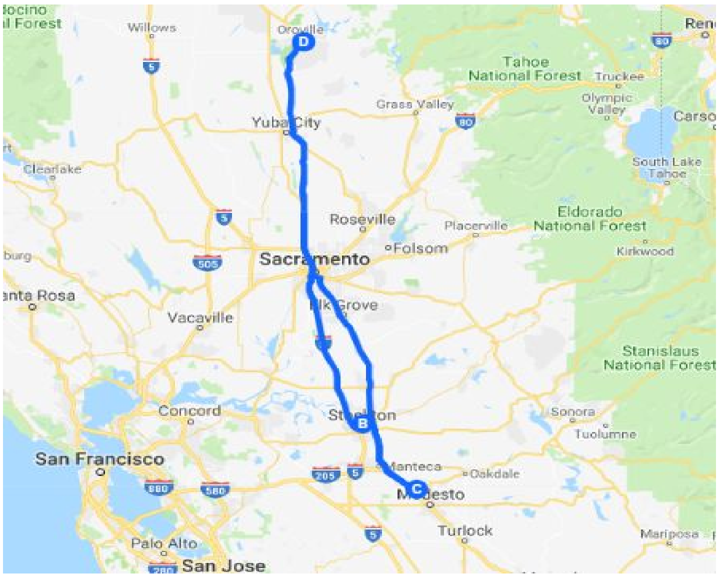
Type 2 Drone 2

1 → 2 → 15 → 1 ----- 214.43 miles



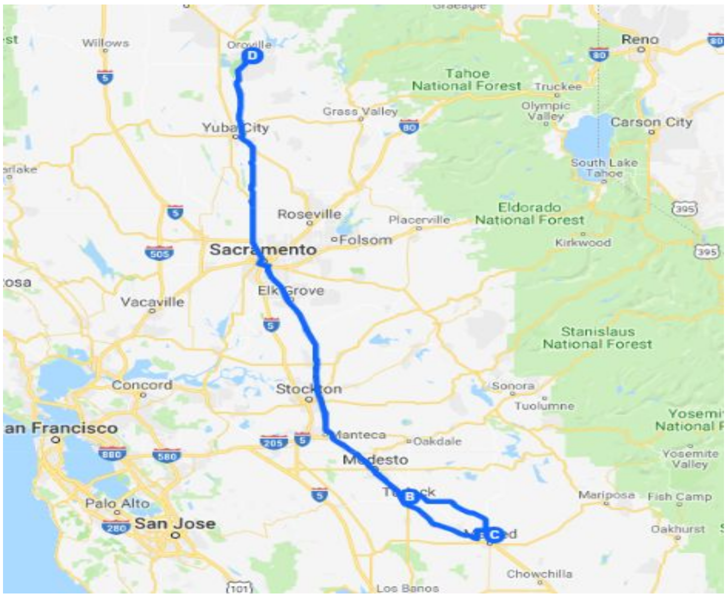
Type 2 Drone 3

1 → 4 → 7 → 1 ----- 260.54 miles



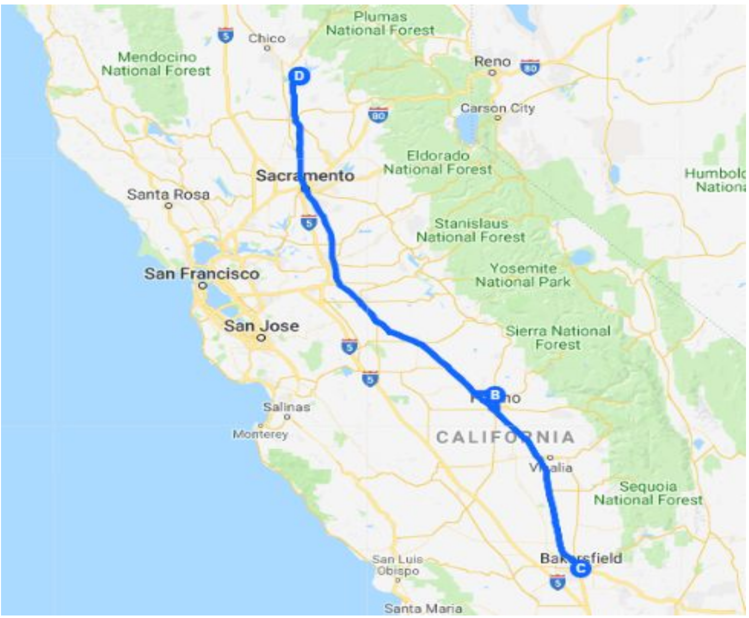
Type 2 Drone 4

1 → 21 → 19 → 1 ----- 332.28 miles



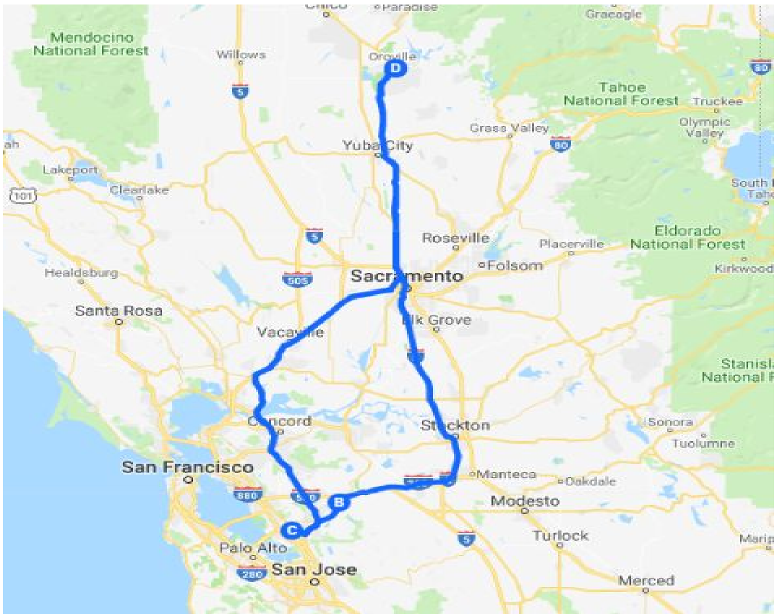
Type 2 Drone 5

1 → 10 → 11 → 1 ----- 639.16 miles



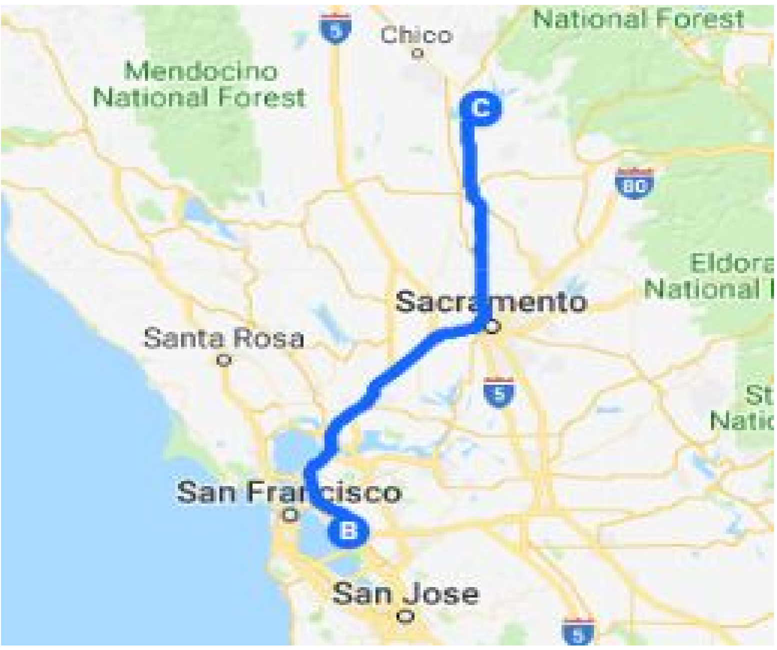
Type 2 Drone 6

1 → 14 → 8 → 1 ----- 225.39 miles



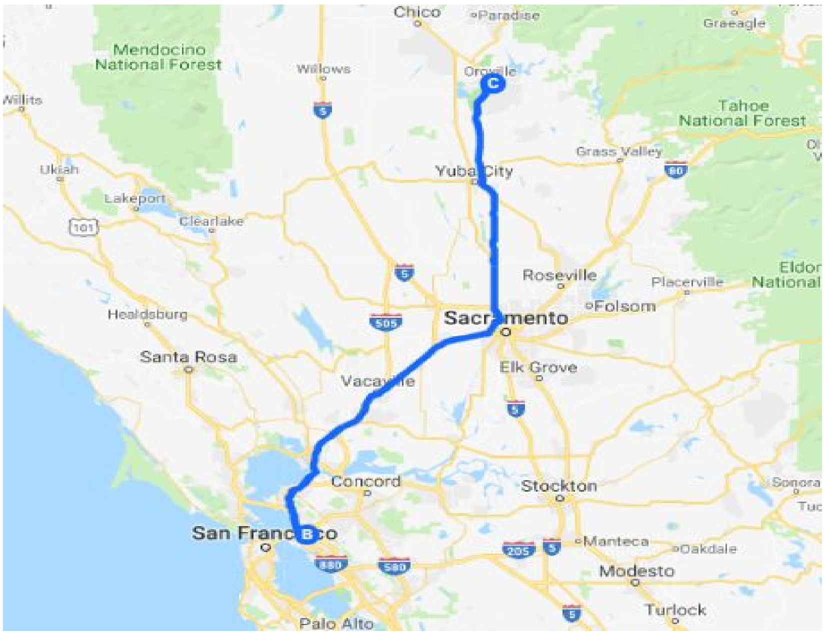
Type 1 Drone 1

1 → 17 → 1 ----- 255.72 miles



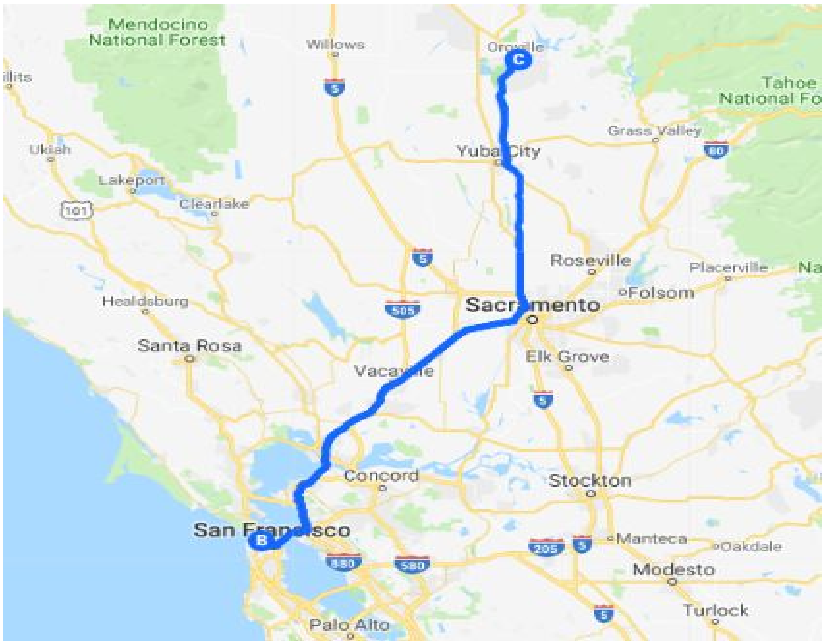
Type 1 Drone 2

1 → 5 → 1 ----- 245.32 miles



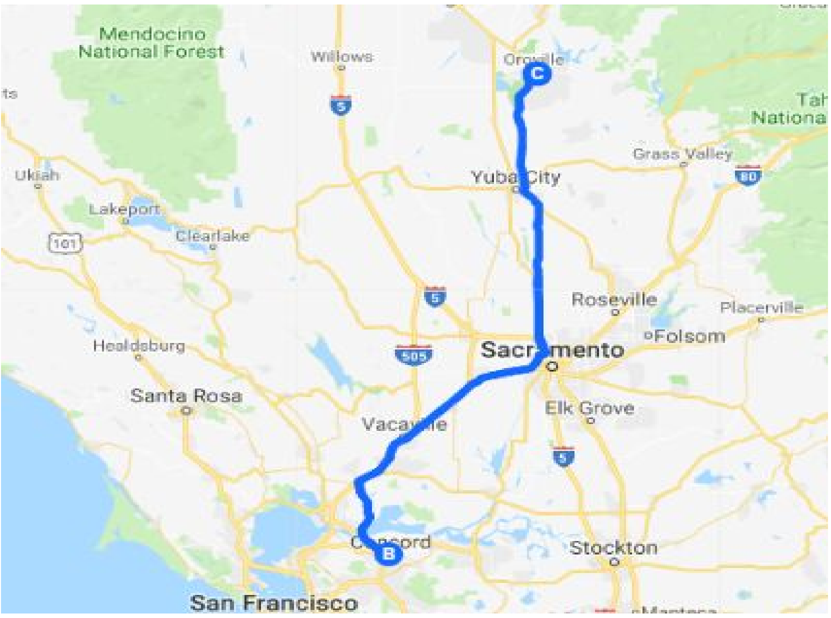
Type 1 Drone 3

1 → 6 → 1 ----- 256.94 miles



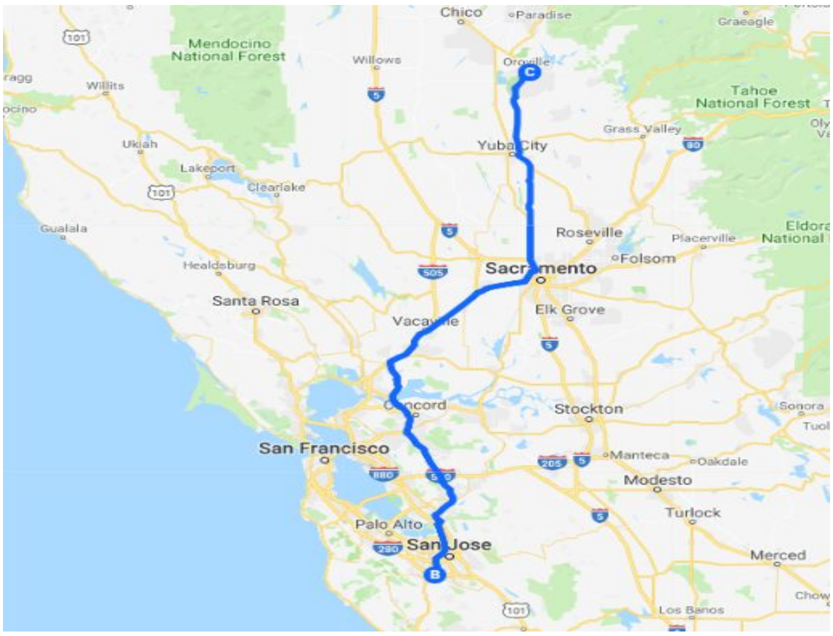
Type 1 Drone 4

1 → 20 → 1 ----- 216.6 miles



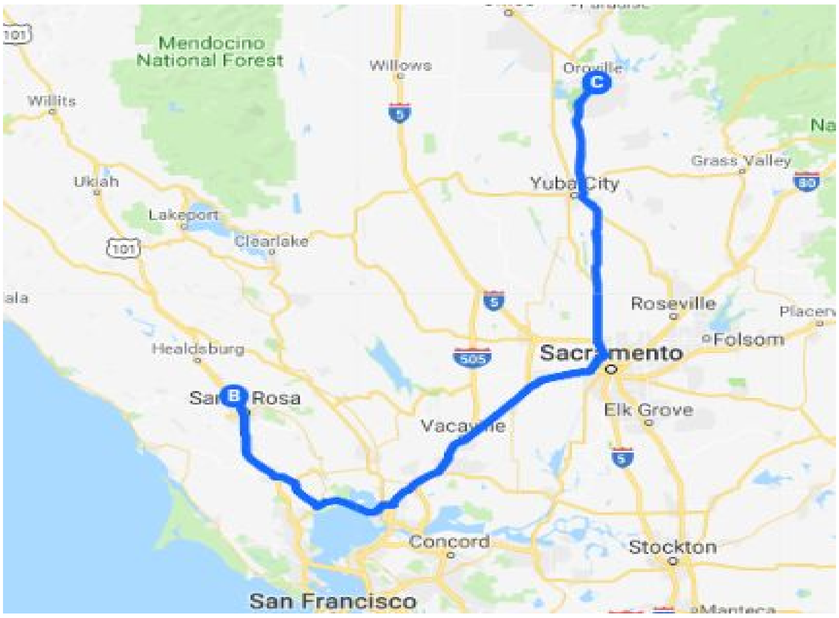
Type 1 Drone 5

1 → 9 → 1 ----- 313.92 miles



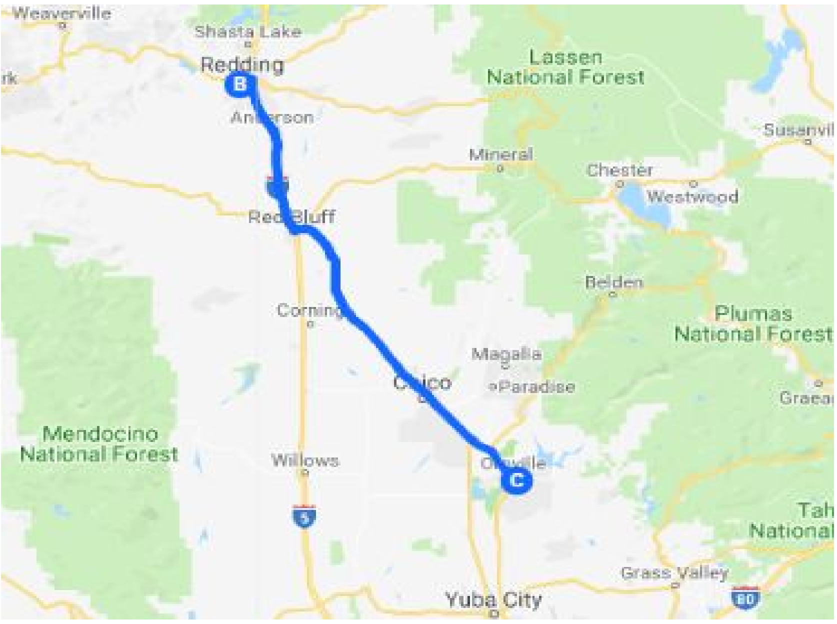
Type 1 Drone 6

1 → 13 → 1 ----- 190.9 miles



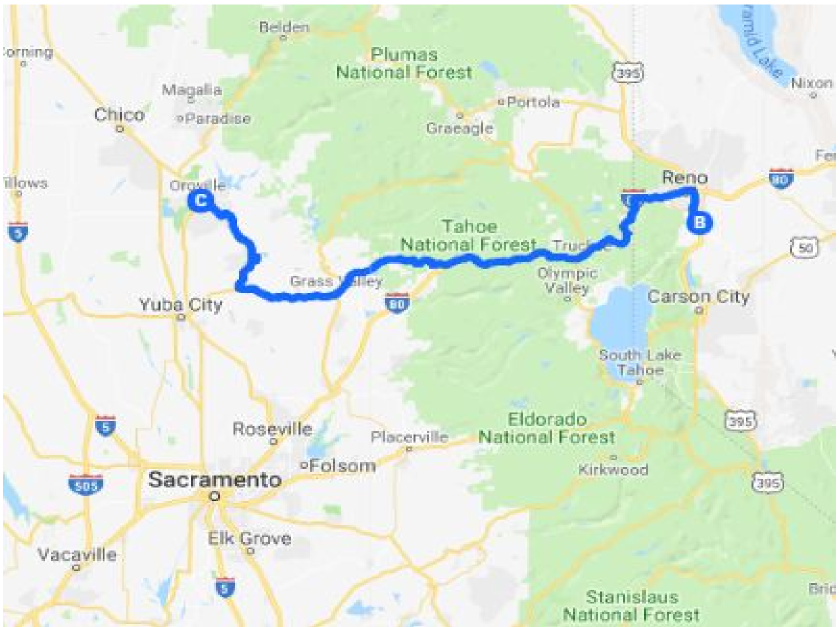
Type 1 Drone 7

1 → 18 → 1 ----- 172.8 miles



Type 1 Drone 8

1 → 3 → 1 ----- 190.72 miles



3.3.2 Local-search heuristic:

As we have got one optimal minimum route for each drone, we try to see if there is any improvement in the distance if we swap between the cities in each drone routes for Type 2 Drones:

Swapping cities within a particular route

Swapping cities 16 ↔ 12 of Type 2 Drone 1

1 → 12 → 16 → 1 ----- 96.28 miles

Swapping cities 2 ↔ 15 of Type 2 Drone 2

1 → 15 → 2 → 1 ----- 214.43 miles

Swapping cities 4 ↔ 7 of Type 2 Drone 3

1 → 7 → 4 → 1 ----- 260.54 miles

Swapping cities 21 ↔ 19 of Type 2 Drone 4

1 → 19 → 21 → 1 ----- 332.28 miles

Swapping cities 10 ↔ 11 of Type 2 Drone 5

1 → 11 → 10 → 1 ----- 639.16 miles

Swapping cities 14 ↔ 8 of Type 2 drone 6

1 → 8 → 14 → 1 ----- 225.39 miles

Swapping cities between Drones:

Swapping cities 16 and 2 between drone 1 and 2 of Type 2

1 → 2 → 12 → 1 ----- 136.73 miles

1 → 16 → 15 → 1 ----- 248.98 miles

Swapping cities 15 and 4 between drones 2 and 3 of Type 2

1 → 2 → 4 → 1 ----- 218.43 miles

1 → 15 → 7 → 1 ----- 332.54 miles

Swapping 10 and 8 between drones 5 and 6 of Type 2

1 → 8 → 11 → 1 ----- 717.71 miles

1 → 14 → 10 → 1 ----- 421.84 miles

4. Results:

4.1 Drone Scheduling Cost Results:

The problem was solved using ampl and the following solutions were obtained:

$X_{11} = 2$ $X_{21} = 6$

$X_{12} = 1$ $X_{22} = 0$

$X_{13} = 0$ $X_{23} = 2$

$X_{14} = 0$ $X_{24} = 0$

Conclusion:

Thus, we need the number of type 1 drones and type 2 drones as mentioned above and the objective function values was obtained to be $5450 which is a reasonable cost.

4.2 Dynamic Programming results

The problem is solved separately for type 1 and type 2 drones

For the type 1 drone

It is observed that there are 2 optimal ways in which the price required is same

1)At stage 1 we have to buy 7 type 2 drone

2)At stage 2 we can buy either 0 drones or 1 drone of type 2

3)At stage three

(i)If we bought 0 drones at stage 2

we have to buy 4 drones at stage 3

(ii)If we bought 1 drone at stage 2

We have to buy 3 drones at stage 3

4)At stage 4

(i)If we bought 4 drones at stage 3 or 3 drones

We need not buy any drones for stage 4

Conclusion : The minimum price that we have to pay when scheduling only type 2 drones is 3850+2200 = 6050

4.3 Exact formulation 1 results:

The routes for each type of drone through different cities is as follows:

Type 1 drones: 8 drones travel in the following routes respectively:

$1 \rightarrow 2 \rightarrow 1$

$1 \rightarrow 3 \rightarrow 1$

$1 \rightarrow 12 \rightarrow 1$

$1 \rightarrow 13 \rightarrow 1$

$1 \rightarrow 14 \rightarrow 1$

1 → 15 → 1

1 →16 → 1

1 → 18 →1

Type 2 drones: 6 drones travel in the following routes respectively:

1 → 4 → 7 → 1

1 → 6 → 5 →1

1 → 8 →9 → 1

1 →11 →10 →1

1 → 17 → 20 → 1

1 → 19 → 21 → 1

Computational Results :

solve_system_time = 0.46875

solve_user_time = 4.01562

solve_time = 4.48438

solve_elapsed_time = 0.844

24997 MIP simplex iterations

3665 branch-and-bound nodes.

Conclusion:

Thus, we got the optimal routes which cover all the cities fulfilling their required demand with the objective function values of 3665 miles. The formulation was stated theoretically and was solved using ampl to obtain the optimal results.

4.4 Exact Formulation 2 results:

The results are as follows:

Type 1 drones: 8 drones travel in the following routes respectively:

1 → 14 → 1

1 → 12 → 1

1 → 13 → 1

1 → 3 → 1

1 → 18 → 1

1 → 15 → 1

1 → 2 → 1

1 → 16 → 1

Type 2 drones: 6 drones travel in the following routes respectively:

1 → 8 → 9 → 1

1 → 6 → 5 → 1

1 → 19 → 21 → 1

1 → 10 → 11 → 1

1 → 7 → 4 → 1

1 → 17 → 20 → 1

Computational Results :

solve_system_time = 3.09375

solve_user_time = 24.2031

solve_time = 27.2969

solve_elapsed_time = 5.719

91916 MIP simplex iterations

5146 branch-and-bound nodes

Conclusion:

Thus, we got the optimal routes which cover all the cities fulfilling their required demand with the objective function values of 3258 miles.

4.5 Comparison between the two exact formulations:

S.no	Measure	Formulation 1	Formulation 2
1	Objective Function Value	3665 (miles)	3258 (miles)
2	System time	0.468	3.093
3	User time	4.015	24.203
4	Total time	4.48	27.2969
5	MIP Iterations	24997	91916
6	Branch and Bound Nodes	3665	5146

Thus, it is clear that formulation 2 gave better routes for the drones keeping the travel distance at its minimum.

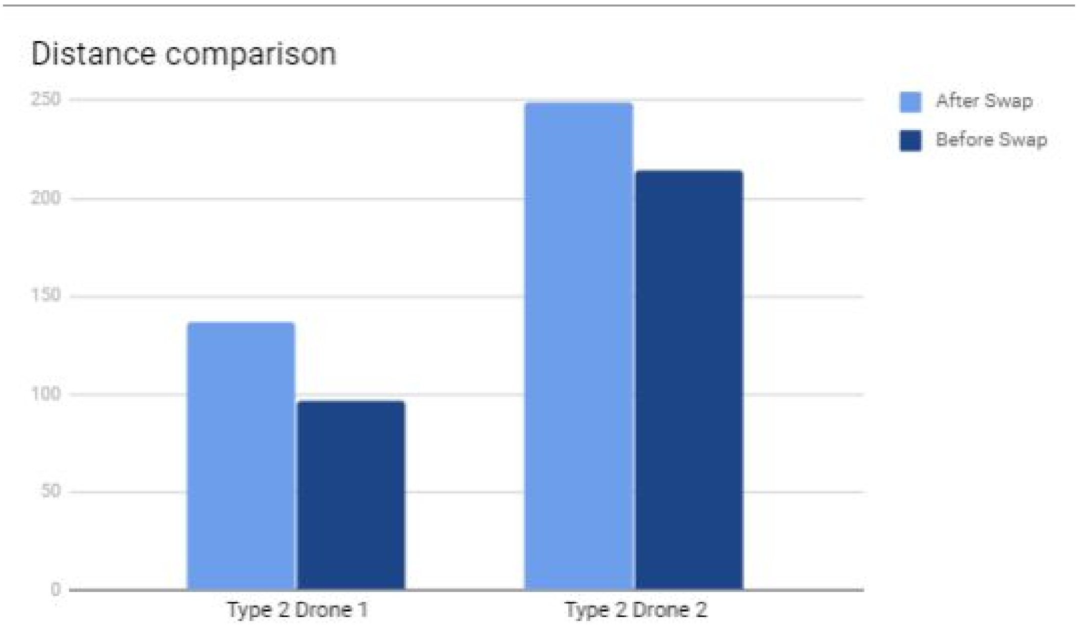
4.6 Heuristics Results

Swapping cities within a particular route of a drone:

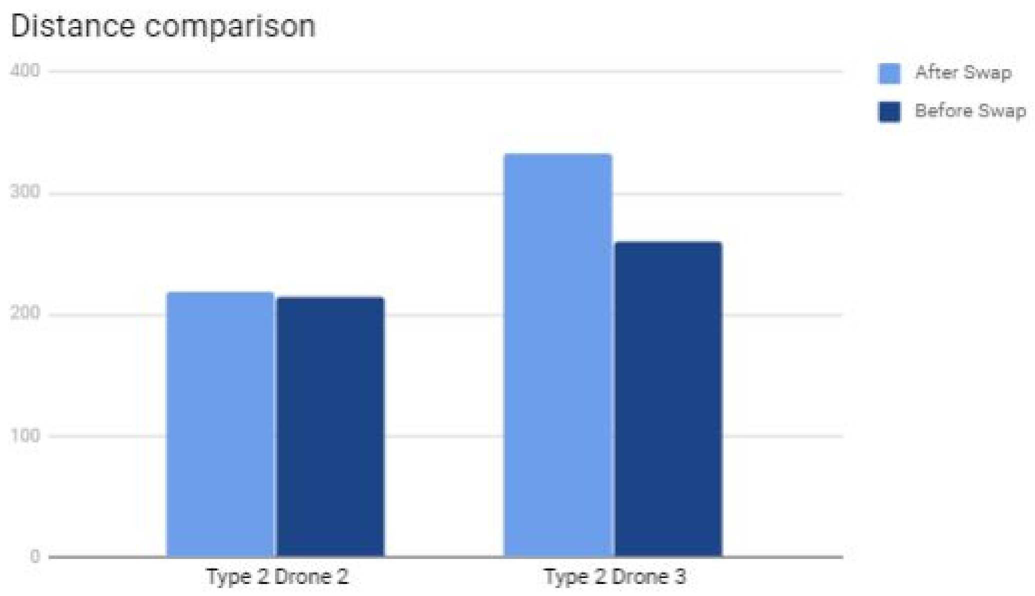
As we can see, for each of the swaps for the respective drone, the distance obtained by swapping is same as before swaps. This occured due to the fact that we only have 2 intermediate cities between the same starting and destination cities.

Swapping cities between the drones:

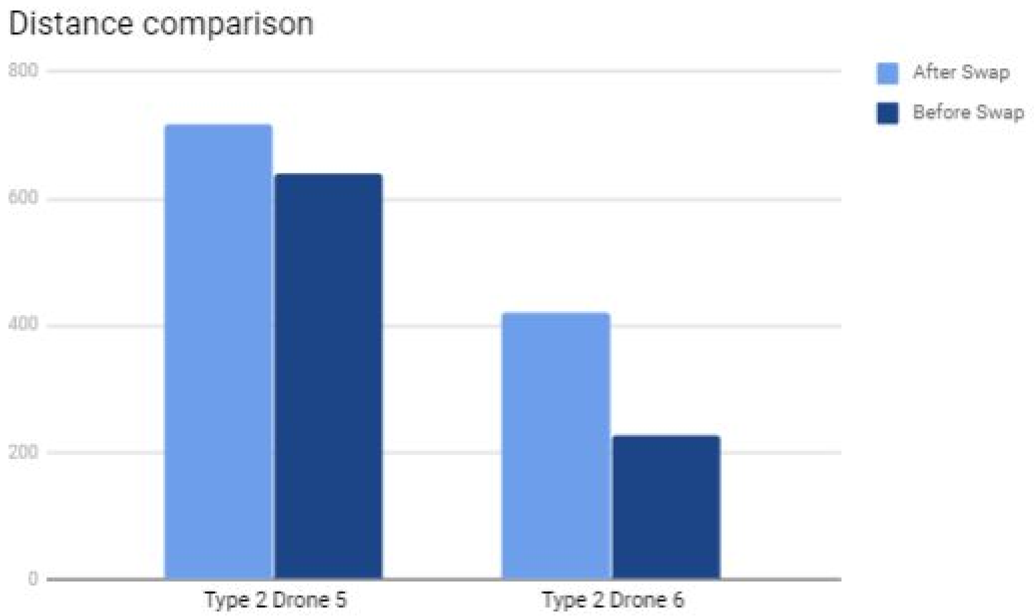
Swapping cities 16 and 2 between drone 1 and 2 of Type 2



Swapping cities 15 and 4 between drones 2 and 3 of Type 2:



Swapping 10 and 8 between drones 5 and 6 of Type 2



We can clearly see that all of the above routes are worse than the original routes before the Swaps. Thus we keep all the routes of each drones, concluding that neither the swapping the cities in a particular route nor the swapping cities between the drones gives us an improved distance.

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