Finite-energy GKP state:

$$|0_\Delta\rangle = \mathcal{N}_{\Delta,0}\,\hat{E}_\Delta \sum_{j\in\mathbb{Z}} |j2\sqrt{\pi}\rangle_x$$

position basis

normalization envelope operator

$$|0_\Delta\rangle = \mathcal{N}_{\Delta,0} \sum_{j\in\mathbb{Z}} \sum_{n=0}^{\infty} |n\rangle \times \langle n|\hat{E}_\Delta|j2\sqrt{\pi}\rangle$$

$$= \mathcal{N}_{\Delta,0} \sum_{n=0}^{\infty} c_n |n\rangle$$

where $c_n = \sum_{j\in\mathbb{Z}} \langle n|\hat{E}_\Delta|j2\sqrt{\pi}\rangle$

Taking a Gaussian envelope for concreteness, $\hat{E}_\Delta = e^{-\Delta^2\hat{n}}$

$$c_n = e^{-\Delta^2 n} \sum_{j\in\mathbb{Z}} \langle n|j2\sqrt{\pi}\rangle$$

Looking back into QM class, we have

$$\langle n|x\rangle = (2^n n!\sqrt{\pi})^{-1/2} e^{-x^2/2} H_n(x)$$

$$\langle n|j2\sqrt{\pi}\rangle = H_n(j2\sqrt{\pi})\, e^{-j^2 2\pi} (2^n n!\sqrt{\pi})^{-1/2}$$

In practice you can truncate the j sum after finite number of peaks, $j\in[-10,10]$

Because of symmetry, we know that $c_n = 0$ for n odd, so that saves time. Also, $H_{2n}(x) = H_{2n}(-x)$ for even Hermite functions, so that

$$|0_\Delta\rangle = \left(\sum_{n=0}^{\infty} c_{2n} |2n\rangle \right).\mathrm{unit}()$$

$$c_{2n} = \frac{e^{-\Delta^2 2n}}{2^n \sqrt{(2n)!}} \times \left[H_{2n}(0) + 2 \sum_{j=1}^{j_{max}} e^{-j^2 2\pi} H_{2n}(j 2\sqrt{\pi}) \right]$$

You just need to be careful with numerical issues in the factorial