SHM path integral

Start with a hammy:

$$H\left(P,Q\right)=\frac{P^2}{2m}+\frac{m\omega^2Q^2}{2} \tag{1}$$

In path integrals, operators are functions so $P\rightarrow p$ and $Q\rightarrow q$. Here we're interested in gound state to ground state, because of reasons. Using:

$$\begin{aligned}
\langle 0|0\rangle &= \langle 0|q_n\rangle \langle q_{n-1}|q_{n-2}\rangle \ldots \langle q_1|0\rangle && (2)\\
&\Downarrow \\
\langle 0|0\rangle &= \int \mathcal{D}p\mathcal{D}q\, exp\left[i\int_{-\infty}^{\infty} dt\left(p\dot{q}-(1-i\epsilon)H+fq\right)\right] && (3)
\end{aligned}$$

Where H is Weyl-ordered (average of normal and anti-normal ordering).
Applying $(1-i\epsilon)$ on H will pick out the ground states in $\pm\infty$ time, leads to the following transforms:

$$\begin{aligned}
&\frac{1}{2}m\omega^2 q \rightarrow \frac{1}{2}(1-i\epsilon)m\omega^2 && (4)\\
&\qquad and && (5)\\
&\frac{1}{2m}p^2 \rightarrow \frac{1}{2(1-i\epsilon)m}p^2 = \frac{(1-i\epsilon)(1+i\epsilon)}{2m(1+i\epsilon)}p^2 = \frac{1+i\epsilon-i\epsilon+\mathcal{O}(2)}{2m(1+i\epsilon)}p^2 && (6)\\
\Rightarrow\ &\frac{1}{2m}p^2 \rightarrow \frac{1}{2(1+i\epsilon)m}p^2 && (7)
\end{aligned}$$

subbing back into 3:

$$\langle 0|0\rangle = \int \mathcal{D}p\mathcal{D}q\, exp\left[i\int_{-\infty}^{\infty} dt\left(p\dot{q}-\frac{p^2}{(1+i\epsilon)2m}-\frac{1-i\epsilon}{2}m\omega^2q^2+fq\right)\right] \tag{8}$$

now the sweet insides can be integrated out over $\mathcal{D}p$ to turn it into a laggy, using:

$$\partial_p\mathcal{H}=\dot{q} \tag{9}$$