$$2H = \int d^3x \qquad \Pi^2 + (\nabla\phi)^2 + \mu^2\phi^2$$

$$\phi(x) = \int \frac{d^3p}{\tau^3 2E} \qquad \left(e^{ipx}a_p + e^{-ipx}a_p^*\right)$$

$$\implies \phi^2 = \int\int \frac{d^3p\, d^3k}{\tau^6 4EK} \qquad \left(e^{ix(p+k)}a_p a_k + e^{ix(p-k)}a_p a_k^* + e^{ix(k-p)}a_p^* a_k + e^{-ix(p+k)}a_p^* a_k^*\right)$$

$$\nabla\phi = \int \frac{d^3p\, pi}{\tau^3 2E} \qquad \left(e^{ipx}a_p + e^{-ipx}a_p^*\right)$$

$$\implies (\nabla\phi)^2 = \int\int \frac{d^3p\, d^3k\, i^2(p\cdot k)}{\tau^6 4EK} \qquad \left(e^{ix(p+k)}a_p a_k - e^{ix(p-k)}a_p a_k^* - e^{ix(k-p)}a_p^* a_k + e^{-ix(p+k)}a_p^* a_k^*\right)$$

$$\dot\phi = \Pi = \int \frac{d^3p\, iE}{\tau^3 2E} \qquad \left(e^{ipx}a_p - e^{-ipx}a_p^*\right)$$

$$\implies \Pi^2 = \int\int \frac{d^3p\, d^3k\, i^2 EK}{\tau^6 4EK} \qquad \left(e^{ix(p+k)}a_p a_k - e^{ix(p-k)}a_p a_k^* - e^{ix(k-p)}a_p^* a_k + e^{-ix(p+k)}a_p^* a_k^*\right)$$

Table 1: coefficents as far

	$a_p a_k$	$a_p a_k^*$	$a_p^* a_k$	$a_p^* a_k^*$
ϕ^2	$+e(p+k$	$+e(p-k$	$+e(k-p$	$+e(-p-k$
$(\nabla\phi)^2$	$+e(p+k$	$-e(p-k$	$-e(k-p$	$+e(-p-k$
Π^2	$+e(p+k$	$-e(p-k$	$-e(k-p$	$+e(-p-k$

$$[a_p, a_k^*] = 2E\tau^3\delta^3(p-k) = a_p a_k^* - a_k^* a_p$$
$$a_p a_k^* = a_k^* a_p + 2E\tau^3\delta^3(p-k)$$

$$\delta(a-b) = \frac{1}{\tau}\int dx e^{(a-b)ix}$$
$$f(y) = \int dx f(x)\delta(x-y)$$

$$H = \int d^3x \qquad \Pi^2 + (\nabla\phi)^2 + \mu^2\phi^2$$

$$[a_p, a_k^*] = 2E\tau^3\delta^3(p-k) \qquad = a_p a_k^* - a_k^* a_p$$

$$\mu^2\phi^2 = \mu^2 \int\int \frac{d^3p\, d^3k}{\tau^6 4EK} \qquad \left(e^{ix(p+k)}a_p a_k + e^{ix(p-k)}a_p a_k^* + e^{ix(k-p)}a_p^* a_k + e^{-ix(p+k)}a_p^* a_k^*\right)$$

$$\mu^2 \int d^3x\phi^2 = \mu^2 \int\int \frac{d^3p\, d^3k}{\tau^3 4EK} \qquad \left(\delta(p+k)a_p a_k + \delta(p-k)a_p a_k^* + \delta(k-p)a_p^* a_k + \delta(p+k)a_p^* a_k^*\right)$$
$$\mu^2 \int d^3x\phi^2 = \mu^2 \int \frac{d^3p}{\tau^3 4EE} \qquad \left(-a_p a_{-p} + a_p a_p^* + a_p^* a_p - a_p^* a_{-p}^*\right)$$

$$(\nabla\phi)^2 = \int\int \frac{d^3p\, d^3k\, i^2(p\cdot k)}{\tau^6 4EK} \qquad \left(e^{ix(p+k)}a_p a_k - e^{ix(p-k)}a_p a_k^* - e^{ix(k-p)}a_p^* a_k + e^{-ix(p+k)}a_p^* a_k^*\right)$$
$$\int d^3x(\nabla\phi)^2 = \int\int \frac{d^3p\, d^3k\, i^2(p\cdot k)}{\tau^3 4EK} \qquad \left(\delta(p+k)a_p a_k - \delta(p-k)a_p a_k^* - \delta(p-k)a_p^* a_k + \delta(p+k)a_p^* a_k^*\right)$$
$$\int d^3x(\nabla\phi)^2 = -\int \frac{d^3p\,(p\cdot -p)}{\tau^3 4EE} \qquad \left(-a_p a_{-p} - a_p a_p^* - a_p^* a_p - a_p^* a_{-p}^*\right)$$

$$\Pi^2 = \int\int \frac{d^3p\, d^3k\, i^2 EK}{\tau^6 4EK} \qquad \left(e^{ix(p+k)}a_p a_k - e^{ix(p-k)}a_p a_k^* - e^{ix(k-p)}a_p^* a_k + e^{-ix(p+k)}a_p^* a_k^*\right)$$
$$\int d^3x\Pi^2 = \int\int \frac{d^3p\, d^3k\, i^2 EK}{\tau^6 4EK} \qquad \left(\delta(p+k)a_p a_k - \delta(p-k)a_p a_k^* - \delta(p-k)a_p^* a_k + \delta(p+k)a_p^* a_k^*\right)$$
$$\int d^3x\Pi^2 = -\int \frac{d^3p\, EE}{\tau^3 4EE} \qquad \left(-a_p a_{-p} - a_p a_p^* - a_p^* a_p - a_p^* a_{-p}^*\right)$$

Table 2: coefficents

	ϕ^2	$(\nabla\phi^2)$	Π^2
$a_p a_{-p}$	$-\mu^2$	$-(p\cdot p)$	E^2
$a_p a_p^*$	μ^2	$-(p\cdot p)$	E^2
$a_p^* a_p$	μ^2	$-(p\cdot p)$	E^2
$a_p^* a_{-p}^*$	$-\mu^2$	$-(p\cdot p)$	E^2

$$E^2 = p^2 + m^2 - m^2 = -E^2 + p^2 I = \int f(x)dx$$
$$I^2 = \int f(x)dx \int f(x)dx$$
$$or$$
$$I^2 = \int f(x)dx \int f(x')dx'$$