SHM path integral

Start with a hammy:

$$H\left(P,Q\right)=\frac{P^2}{2m}+\frac{m\omega^2Q^2}{2} \tag{1}$$

Interested in gound state to ground state, using:

$$\begin{aligned}\langle 0|0\rangle &= \langle 0|q_n\rangle\langle q_{n-1}|q_{n-2}\rangle\dots\langle q_1|0\rangle\\ &\Downarrow\\ \langle 0|0\rangle &= \int\mathcal{D}p\mathcal{D}q\,exp\left[i\int_{-\infty}^{\infty}dt\left(p\dot{q}-(1-i\epsilon)H+fq\right)\right]\end{aligned} \tag{2}$$

applying $(1-i\epsilon)$ on H leads to:

$$\begin{aligned}&m\omega^2\rightarrow(1-i\epsilon)m\omega^2\\ &\qquad and\\ &m^{-1}\rightarrow(1-i\epsilon)m^{-1}\\ &\quad\hookrightarrow\frac{(1-i\epsilon)(1+i\epsilon)}{m(1+i\epsilon)}=\frac{1+i\epsilon-i\epsilon+\mathcal{O}(2)}{m(1+i\epsilon)}\\ &\Rightarrow m\rightarrow(1+i\epsilon)m\end{aligned} \tag{3}$$

subbing back into 2

$$\langle 0|0\rangle=\int\mathcal{D}p\mathcal{D}q\,exp\left[i\int_{-\infty}^{\infty}dt\left(p\dot{q}-\frac{P^2}{(1+i\epsilon)2m}-\frac{1-i\epsilon}{2}m\omega^2Q^2+fq\right)\right] \tag{4}$$

$$\int\frac{d\omega}{\tau}\frac{e^{-i\omega(t-t')}}{-2i\omega\epsilon} \tag{5}$$