The Overtaking Procedure of the Autonomous Vehicle using Bezier Curves

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ABSTRACT

This research assists in determining the path trajectory of an autonomous vehicle using Bezier curves. This research uses the MATLAB scripting language to find the optimal distance between control points of a fifth degree Bezier curve. This was found by finding the integral of the second derivative of the Bezier curve's curvature. The results of this research indicate an increasing linear dependency between the distance of the six control points and the length of the vehicle's trajectory, assuming that the width remains constant. The approximated correlation derived from the generated program will assist in finding a precise relationship between the two variables and will reduce the time in which the autonomous vehicles plans its path.

Keywords: Path Planning, Curve Theory, Bezier Curves, 5^{th} Degree Bezier Curves, Curvature, Parametric Curves

INTRODUCTION

Research conducted on the development of the autonomous vehicle has placed a strong importance on the efficiency, comfort level, and overall safety of road transportation systems. In order to account for the safety of a vehicle, trajectory generation algorithms are expected to be of a parametric form and of varying with a continuous curvature. The smoothness is a necessary characteristic in path trajectory generation for car-like vehicles because non-smooth movements can cause slippage or overactuation [1, 2]. The Bezier curve has been identified as a viable candidate for a trajectory generation algorithm due to its capability as a solution for curvature continuity and computational efficiency in real-time applications [3, 4].

Past attempts have included the development of path-smoothing algorithms using cubic Bezier curves that would be implemented after the initial path has been detected [2]. Bezier curves, too, have been implemented in local planner trajectory algorithms for turn stretches and intersections [5]. Authors have also seen success with Bezier curves in path planners with obstacle avoidance capabilities in urban scenarios [6].

Applications of the Bezier curve and parametric curves have been implemented by taking the derivative of the curve at the junction point [2, 7]. Optimization methods have been proposed to examine Bezier curves that minimizes path length by constraining the maximum curvature, however this proves to be a difficult feat to accomplish while simultaneously accounting for curvature continuity [8-10].

The purpose of this research is to identify the formula that will initially generate a continuous-curvature path without the need of a path-smoothing algorithm so as to minimize the reaction time of the vehicle. The formula will use the Bezier curve as its trajectory generation algorithm and develop a relationship between the distance between the control points of the Bezier curve and the length of the path of the autonomous vehicle.

Mathematically, a Bezier curve is a parametric curve, of a degree n, expressed in (Eq. 1)

$$B(t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} P_i \qquad \text{(Eq1)}$$

$$b_{i,n}(t) = \binom{n}{i} (1-t)^{n-i} P_i \qquad \text{(Eq2)}$$

Where the degree, n, indicates the existence of n+1 control points, P_i is the control point, a coordinate value, at the index, I, and the polynomial in (Eq 2) is the Bernstein basis polynomial. The Bezier curve, it is important to note, is an interpolation function, where the control points, save for the first and last control point, do not reside on the actual path of the curve.

In the scope of this research, the Bezier curve, of fifth degree, represents the path trajectory, with certain length, l, and width, w, of an autonomous vehicle during the overtaking procedure as seen in Image 1.

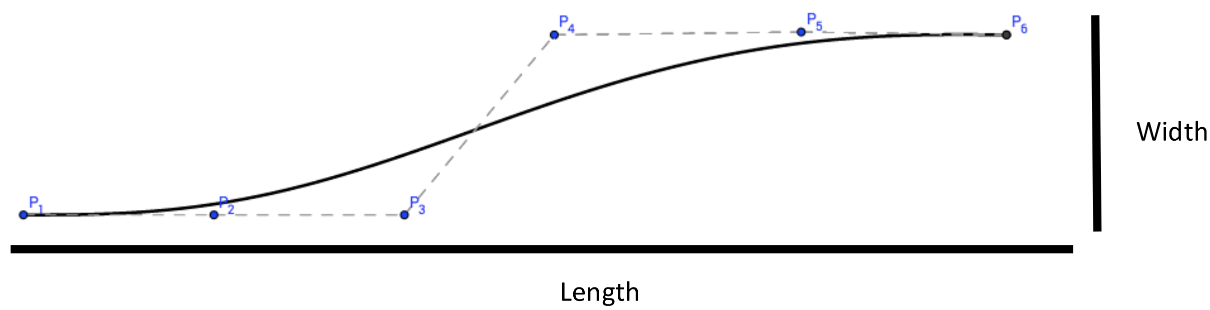

IMAGE 1

The focus of this research is to determine a relationship between the distance between the 6 control points of the Bezier curve, with minimum curvature, and the length and width of the path. At the onset of the research project, it was predicted that there would be a relationship that could be expressed in terms of a calculable function. It is expected that this relationship, though not explicitly linear, would be an increasing correlation between the two variables. Logic corroborates this claim as the longer the length of the Bezier curve along the x-axis, the longer the distance between the x-components of the Bezier curve control points.

MATERIALS AND METHODOLOGY

ASSUMPTIONS

For the purpose and scope of the research a number of assumptions are taken into account: It is assumed that the path is being planned on a straight road with no curves. The width of the path is fixed to resemble the standard width of the road, approximately 3.2 meters. The length of the path is initially fixed to 30 meters. However, this does eventually change to find a correlation between the distance between the control points and a range of lengths.

The Bezier curve is assumed be a fifth degree curve that starts and ends on the same degree relative to the road, that value is assumed to be zero degrees. To ensure that the Bezier curve begins at zero degrees in relation to the direction of the road, control points P1, P2, and P3 are assumed to lie on the same y-axis, where y = 0 meters. Similarly, to ensure that the Bezier curve ends at zero degrees in relation to the road, control points P4, P5, and P6 are assumed to like on the same y-axis, where y = 30 meters. The Bezier curve begins and ends on the same degree.

The path is assumed to be symmetrical. For that reason, the distance between P1 and P2 and the distance between P5 and P6 are assumed equal. The distance between P2 and P3 and the distance between P4 and P5 are also assumed equal. This is expressed in (Eq 3) and (Eq 4)

$$|P2 - P1| = |P6 - P5| \qquad \text{(Eq3)}$$

$$|P3 - P2| = |P5 - P4| \qquad \text{(Eq4)}$$

Whenever the distance between the control points is referred to in the scope of this paper, it is referring to the distances between the control points along the same axes; the difference between P2 and P3 is negligible and will not be regarded in the scope of this paper. The distance value is only referring to the x-component in each of the control points as the y-components are fixed by the width of the path.

Initially, the distances between the control points, are assumed equal. This will then later be replaced by the assumption the two above equations are only true for the sake of a more optimal relationship.

The ideal path for the autonomous vehicle, for the sake of the program, is understood to be the path with the minimum curvature. This is to be regarded as the most important constraint as it will guide the methodology behind the programs. The minimum curvature is selected as the key parameter to its ability to give the most direct path for the autonomous vehicle while still in compliance of the above constraints.

MATERIALS

All programs were generated using the MATLAB scripting language.

METHODOLOGY

The objective is to create a series of programs that determined the distances between the control points corresponding to a Bezier curve that adheres to the above constraints and has the minimum curvature.

First, the first, second, and third derivatives of the Bezier curve are derived with purpose of being used in calculating the curvature of the Bezier curve. The curvature of a parametric curve is expressed in (Eq 5) using the first and second derivatives of the Bezier curve, represented by (‘); where the x is the equation of the Bezier curve in relation to the x-components and similarly for y.

$$K = \frac{|x'y'' - y'x''|}{((x')^2 + (y')^2)^{3/2}} \qquad \text{(Eq5)}$$

The curvature, its derivative, and its second derivative are then calculated for both the x and y components. The minimum curvature is determined by the least variation in the curvature’s derivative. The area of the second derivative is calculated using trapezoidal sums. This numerical value is taken and stored for each iteration of the recursive function changing the distance values between control points. The Bezier curve with the small numerical value is the ideal curve for the given length and width.

RESULTS AND DISCUSSION

A program was created that generates the distance between the control points, expect P2 and P3, based upon the width and the length inputted into the program. This program was then expanded to calculate the distances for a range of lengths and widths. The result was an increasing, linear correlation between the distance of the control points and the length of the path given a fixed width.

Upon the completion of the first program, a second program was created where only the assumptions in (Eq 3) and (Eq 4), in regards to the length and width of the path, were made. The resulting program generated two distance values; one for the distance expressed in (Eq3) and one for the distance expressed in (Eq 4). The results showed a Bezier curve with a smaller curvature than the program where all distance between control points were assumed to be equal.

The methodology and result of this program was then compared to another existing program that determined the distance of the control points examining the height of the first derivative of the curvature of the Bezier curve. The two results corroborated each other.

While the means exist to determine the distance between control points to form a Bezier curve of least curvature from a given length and fixed width, there is a factor that hinders the relationship the implementation of this method in an autonomous vehicle: time. The time that it takes for the program to determine the ideal Bezier curve is not yet at an acceptable time interval for the autonomous vehicle to plan its path in practical situations.

Future applications of this research will focus on finding a faster methodology to determine the ideal Bezier curve and then to mathematically prove it. The culmination of this and further research will be applied to the overall path planning logic and controller of an autonomous vehicle during the overtaking procedure. This research may also serve as the basis of research in determining the path trajectory of an autonomous vehicle during an overtaking procedure on a curved road.

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REFERENCES

[1] E. Magid, D. Keren, E. Rivlin, and I. Yavneh, “Spline-based robot navi- gation,” in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, Beijing, China, Oct. 2006, pp. 2296–2301.

[2] Y. Kwangjin and S. Sukkarieh "An analytical continuous-curvature path-smoothing algorithm", *IEEE Trans. Robot.*, vol. 26, no. 3, pp.561 -568 2010

[3] L. Han, H. Yashiro, H. Nejad, Q. H. Do, and S. Mita. Bezier curve based path planning for autonomous vehicle in urban environment. In Intelligent Vehicles Symposium (IV). 2010 IEEE, pages 1036-1042, University of California, San Diego, CA, USA, June 21-24 2010.

[4] J. Hwang, R. Arkin, and D. Kwon, “Mobile robots at your fingertip: Bezier curve on-line trajectory generation for supervisory control,” in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, Las Vegas, NV, Oct. 2003, pp. 1444–1449.

[5] J. Perez, R. Lattarulo, and F. Nashashibi, “Dynamic trajectory generation using continuous-curvature algorithms for door to door assistance vehicles,” in Intelligent Vehicles Symposium (IV), 2014 IEEE, 2014.

[6] Gonzalez, D.; Perez, J.; Lattarulo, R.; Milanes, V.; Nashashibi, F., "Continuous curvature planning with obstacle avoidance capabilities in urban scenarios," in *Intelligent Transportation Systems (ITSC), 2014 IEEE 17th International Conference on* , vol., no., pp.1430-1435, 8-11 Oct. 2014 doi: 10.1109/ITSC.2014.6957887

[7] D. J. Walton, D. S. Meek, and J. M. Ali. 2003. Planar G^2 transition curves composed of cubic Bézier spiral segments. *J. Comput. Appl. Math.* 157, 2 (August 2003), 453-476. DOI=http://dx.doi.org/10.1016/S0377-0427(03)00435-7

[8] K. Nagatani, Y. Iwai, and Y. Tanaka, “Sensor-based navigation for car- like mobile robots based on a generalized Voronoi graph,” *Adv. Robot.*, vol. 17, no. 5, pp. 385–401, 2003.

[9] M. Khatib, H. Jaouni, R. Chatila, and J. P. Laumond, “Dynamic path modification for car-like nonholonomic mobile robots,” in *Proc. IEEE Int. Conf. Robot. Autom.*, Albuquerque, New Mexico, Apr. 1997, pp. 2920– 2925.

[10] D. J. Walton and D. S. Meek, “Curvature extrema of planar parametric polynomial cubic curves,” *J. Comput. Appl. Math.*, vol. 134, no. 1–2, pp. 69–83, Sep. 2001.