



PATH TRAJECTORY OF AUTONOMOUS VEHICLES USING BEZIER CURVES



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ABSTRACT

This research explores the nature of the parametric Bezier Curves and its application in determining the path trajectory of an autonomous vehicle using Bezier curves. The research uses the MATLAB scripting language to find the optimal distance between control points of a fifth degree Bezier curve. This was found by finding the integral of the second derivative of the Bezier curve's curvature. The results of this research indicate an increasing linear dependency between the distance of the six control points and the length of the vehicle's trajectory, assuming that the width remains constant. The approximated correlation derived from the generated program will assist in finding a precise relationship between the two variables and will reduce the time in which the autonomous vehicles plans its path.

INTRODUCTION

Research conducted on the development of the autonomous vehicle has placed a strong importance on the efficiency, comfort level, and overall safety of road transportation systems. In order to account for the safety of a vehicle, trajectory generation algorithms are expected to be of a parametric form and of varying with a continuous curvature. The Bezier curve has been identified as a viable candidate for a trajectory generation algorithm due to its capability as a solution for curvature continuity and computational efficiency in real-time applications.

Mathematically, a Bezier curve is a parametric curve, of a degree n, expressed as:

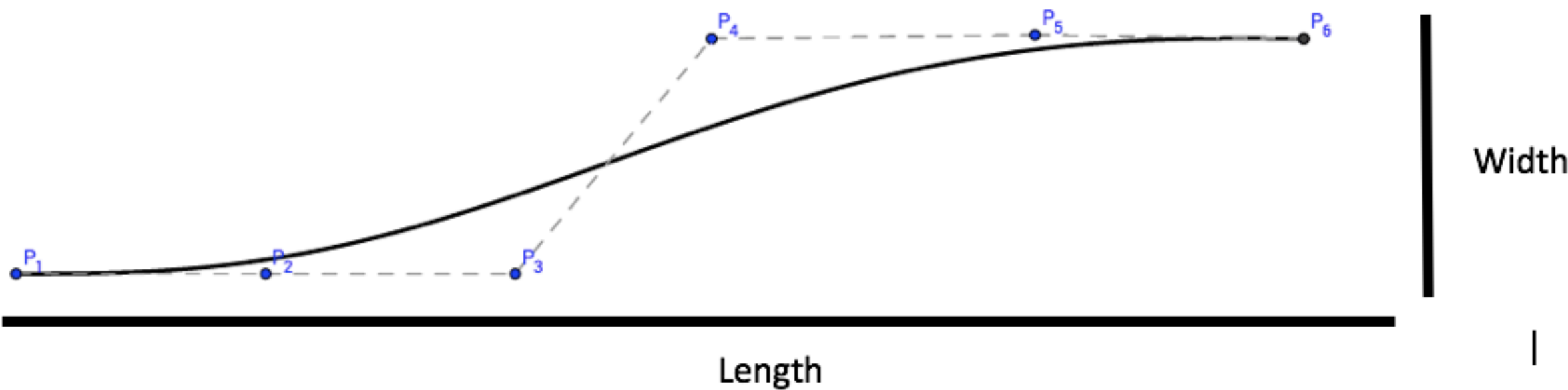
$$B(t) = \sum_{i=0}^n \binom{n}{i} (1-t)^{n-i} P_i$$

In the scope of this research, the Bezier curve, of fifth degree, represents the path trajectory, with certain length, l, and width, w, of an autonomous vehicle during the overtaking procedure as seen in the image.

RESEARCH QUESTION

The focus of this research is to determine a relationship between the distance between the 6 control points of the Bezier curve, with minimum curvature, and the length and width of the path.

METHODOLOGY



ASSUMPTIONS

- It is assumed that the path is being planned on a straight road with no curves.
- The width of the path is fixed to resemble the standard width of the road, approximately 3.2 meters.
- The distance between the following control points are assumed equal:

$$\begin{aligned} |P2 - P1| &= |P6 - P5| \\ |P3 - P2| &= |P5 - P4| \end{aligned}$$

MATERIALS

All programs were generated using the MATLAB scripting language.

METHOD

$$K = \frac{|x'y'' - y'x''|}{((x')^2 + (y')^2)^{3/2}}$$

The curvature of a Bezier is calculated using the above equation. The Bezier curve with the smallest variation in the curvature's integral is the ideal curve for the given length and width.

DATA & ANALYSIS

Two programs were created to find the ideal Bezier curve for a given length and width. The first assumed that all distances between control points were equal, save |P4 – P3|. The second follows the assumptions above. Both calculated an ideal Bezier curve that corroborated each other.

DISCUSSION

The time that it takes for the program to determine the ideal Bezier curve is not yet at an acceptable time interval for the autonomous vehicle to plan its path in practical situations. Future applications of this research will focus on finding a faster methodology to determine the ideal Bezier curve and then to mathematically prove it. The culmination of this and further research will be applied to the overall path planning logic and controller of an autonomous vehicle during the overtaking procedure. This research may also serve as the basis of research in determining the path trajectory of an autonomous vehicle during an overtaking procedure on a curved road.

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