MAT292 Abridged

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0.1 Introduction

The textbook and lectures for this course offer a great comprehensive guide for the methods of solving ODE's. The goal here is to give a very concise overview of the things you need to know (NTK) to answer exam questions. Unlike some of our other courses, you don't need to be very intimately familiar with the derivations of everything in order to solve the problems (though it certainly doesn't hurt). Think of this as a really good cheat sheet.

Chapter 1

Qualitative Things and Definitions

1.1 Definitions

1. **Differential Equation:** Any equation that contains a differential of dependent variable(s) with respect to any independent variable(s)
2. **Order:** The order of the highest derivative present.
3. **Autonomous:** When the definition of the $\frac{dy}{dt}$ doesn't contain t
4. **ODE and PDE:** Ordinary derivatives or partial derivatives.
5. **Linear Differential Equations:** nth order Linear ODE is of the form:
$$\sum a_i(t)y^{(i)} = 0$$
6. **Homogenous:** if the 0th element of the above sum has $a_0(t) = 0$ for all t.

1.2 Qualitative Analytic Methods to Know

1. Phase lines
2. Slope fields

1.3 Types of Equilibrium

1. Asymptotic stable equilibrium
2. Unstable equilibrium
3. Semistable equilibibirium

Chapter 2

1st Order ODE's

2.1 Separable 1st Order ODE's

If you can write the ODE as:

$$\frac{dy}{dx} = p(x)q(y)$$

Then you can put $p(x)$ with dx on one side and $q(y)$ with dy on the other and integrate them both so solve the ODE.

2.2 Method of Integrating Factors

This is used to solve ODE's that can be put into the form

$$\frac{dy}{dt} + p(t) * y = g(t)$$

The chain rule can be written as: $\int (f'(x)g(x) + f(x)g'(x))dx = f(x)g(x)$

We can use an **integrating factor** equivalent to $e^{\int p(t)dt}$ to multiply both sides and arrive at a form that can be integrated with ease using the reverse chain rule.

2.3 Exact Equations

If the equation is of the form

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

and

$$M_y(x,y) = N_x(x,y)$$

then $\exists$ a function f satistfying

$$f_x(x,y) = M(x,y); f_y(x,y) = N(x,y)$$

The solution: $f(x,y) = C$ where C is an arbitrary constant.

Chapter 3

Systems of Two 1st Order DE's

3.1 Set Up

Your first goal is to get the system in the form

$$\frac{d\boldsymbol{u}}{dt} = \boldsymbol{K}\boldsymbol{u} + \boldsymbol{b}$$

Where $\boldsymbol{K}$ is a 2 by 2 matrix, $\boldsymbol{u}$ is your vector of values you want to predict, and $\boldsymbol{b}$ is a 2-long vector of constants.

More generally, the equation is of the type

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{P}(t)x + \boldsymbol{g}(t)$$

Called a **first order linear system of two dimensions**. If $\boldsymbol{g}(t) = \mathbf{0} \forall t$ then it is called **homogenous**, else **non-homogenous**. We let x be composed of values

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

3.2 Existence and Uniqueness of Solutions

Theorem: $\exists$ unique solution to

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{P}(t)x + \boldsymbol{g}(t)$$

so long as the functions $\boldsymbol{P}(t)$ exist and are continuous on the interval I in equestion.

3.2.1 Linear Autonomous Systems

If the right side doesn't depend on t, it's autonomous. In this case, the autonomous version looks (familiarly) like:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{A}x + \boldsymbol{b}$$

Equilibrium points arise when $\boldsymbol{A}x = -\boldsymbol{b}$

3.3 Solving