

# AER210 Abridged

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# Chapter 1

## Review: Stuff to Have Memorized

### 1.1 Trig Functions and Derivatives

$$\frac{d}{dx} \sin(x) = \cos(x) \quad \frac{d}{dx} \csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x) \quad \frac{d}{dx} \sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x) \quad \frac{d}{dx} \cot(x) = -\csc^2(x)$$

### 1.2 Inverse Trig Derivatives

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

### 1.3 How to complete the Square

1. Put  $ax^2 + bx$  in brackets and forcefully factor out the  $a$
2. Add  $(\frac{b}{2})^2$  to the inside of the brackets and subtract it from the outside (you got it)
3. Factor and be happy that you've completed the square;

## 1.4 Trig Angle Sums

1.  $\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

2.  $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

3.  $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

4.  $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

## 1.5 Identities Assorted

$$\sin^2(x) = 1/2(1 - \cos(2x)) \quad (1.1)$$

$$\cos^2(x) = 1/2(1 + \cos(2x)) \quad (1.2)$$

$$\sin x \cos x = 1/2 \sin 2x \quad (1.3)$$

$$\sin A \cos B = 1/2[\sin(A - B) + \sin(A + B)] \quad (1.4)$$

$$\sin A \sin B = 1/2[\cos(A - B) - \cos(A + B)] \quad (1.5)$$

$$\cos A \cos B = 1/2[\cos(A - B) + \cos(A + B)] \quad (1.6)$$

$$\frac{d}{dx} \csc(x) = -\csc(x) \cot(x) \quad (1.7)$$

$$\cot^2(x) = \csc^2(x) - 1 \quad (1.8)$$

$$\frac{d}{dx} \cot(x) = -\csc^2(x) \quad (1.9)$$

# Chapter 2

## In-Class Review

### 2.1 Vectors & Vector Functions

- Vector = magnitude + direction
- If the origin of the vector is the origin of the coordinate system, it's a position vector.
- Dot product:  $\vec{a} \cdot \vec{b} = \vec{a}_1 \cdot \vec{b}_1 + \dots + \vec{a}_n \cdot \vec{b}_n$
- Cross product:  $\vec{a} \times \vec{b} = \det(i, j, k; \vec{a}^T; \vec{b}^T)$
- Cross product is the area of the parallelogram traced out by the two vectors.
- Scalar triple product:  $\vec{a} \cdot (\vec{b} \times \vec{c})$ , produces a scalar, represents the volume of the parallelepiped formed by the three vectors.
- To get the derivative of a vector function, simply take the derivative of each of the internal functions and package them into a new vector function.

### 2.2 Arc Length

#### 2.2.1 One-Variable Functions

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

## 2.2.2 Parametric Functions

Let  $y(t)$  and  $x(t)$  describe a parametric function in 2 dimensions. Then the arc length would be:

$$L = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt$$

## 2.2.3 Vector Functions

Let  $\vec{r}(t)$  describe a vector function that converts a scalar  $t$  into a vector. Then the arclength function would be:

$$L = \int_a^b |\vec{r}'(t)| dt$$

## 2.2.4 Reparameterizing with respect to Arc Length

**What is this?** Let there be a vector function  $\vec{r}(t)$  and its corresponding arc length function  $s(t)$ . Since  $s$  is strictly increasing, we can safely **reparameterize**  $\vec{r}(t)$  to be  $\vec{r}(s(t)) \rightarrow \vec{r}(s)$ .

**Why would you want to do this?** This type of reparameterization is useful because now we do not have to rely on any particular coordinate system.

### Steps to Reparameterizing

1. Find  $s(t) = \int_a^t |\vec{r}'(u)| du$ .
2. Put  $s$  in terms of  $t$ .
3. Substitute the expression found in part 2 in the original  $\vec{r}(t)$ .

## 2.3 Partial Derivatives

### 2.3.1 Functions of Several Variables

**A function of two variables** transforms each pair of Reals  $(x, y)$  in a given set to a single real number. The given set is the domain, and the set of reals that the pair is transformed to is the range.

**Level functions** are functions that have  $f(x, y) = k$  for given ranges of  $(x, y)$

**Functions of 3 or more variables** are pretty easy to extrapolate from functions of two variables, tbh.

## 2.3.2 Limits and Continuity with Functions of Several Variables

### Limits

**Definition of limit** with many variables:

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y) = L$$

if for every number  $\epsilon > 0$  there is a corresponding number  $\delta > 0$  s.t. if  $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$  then  $|f(x, y) - L| < \epsilon$

**How to find:** Regard the non-mentioned variable in the notation as a constant and differentiate with respect to the mentioned variable.

## 2.3.3 Higher Partial Derivatives

$$(f_x)_y = f_{xy} = f_{12} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial z}{\partial y \partial x}$$

### Clairaut's Theorem

If  $f_{xy}$  and  $f_{yx}$  are both **defined** and **continuous** on disk  $D$  then:

$$f_{xy}(a, b) = f_{yx}(a, b)$$

## 2.4 Gradient

Think of the gradient like an operator that applies to functions of many variables (functions of vectors). The  $\nabla$  just calculates the partial derivative of the function with respect to each of its input variables and puts it into a vector.

$$\nabla f(x, y) = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

Or, more generally for a function  $f(\vec{x})$ ,

$$\nabla f(\vec{x}) = [\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_n}]$$

## 2.5 Chain Rule with Many Variables

Let there be a function  $f(\vec{x})$ . Let  $\vec{x}$  of length  $n$  be a function of  $\vec{t}$  of length  $m$ . We take the partial derivative of  $f$  with respect to  $t_i$  by the following:

$$\frac{\partial f}{\partial t_i} = \frac{\partial f}{\partial x_1} \frac{\partial x_1}{\partial t_i} + \dots + \frac{\partial f}{\partial x_n} \frac{\partial x_n}{\partial t_i}$$



## Chapter 3

# Multiple Integrals