PHY293 Abridged

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Contents

0.1 Introduction

Prof. Grisouard has put up some pretty solid notes written in jupyter notebooks. Those offer a great comprehensive guide for the course contents. The goal here is to give a very concise overview of the things you need to know (NTK) to answer exam questions. Unlike some of our other courses, you don't need to be very intimately familiar with the derivations of everything in order to solve the problems (though it certainly doesn't hurt). Think of this as a really good cheat sheet.

Chapter 1

Simple Harmonic Oscillators (SHO's)

1.1 Set Up

Pretty much the same as covered last year.

$$F_{restorative} = -kx$$

$$F_{net} = ma$$

$$ma + kx = 0$$

We let $\omega_0^2 = k/m$ and rename a to $\ddot{x}$

1.2 ODE and Solution

ODE:

$$\ddot{x} + \omega_0^2 x = 0$$

Solution:

$$x(t) = acos(\omega_0 t + \theta)$$

Solve for 2 unknowns (usually a and θ) based on two initial conditions.

Chapter 2

Dampened Harmonic Oscillators (DHO's)

2.1 Set Up

We have all the same forces from last time plus a force of friction. We kind of have our hands tied with what models of friction we can use (remember how friction can be proportional to velocity, acceleration?) because this course only deals with linear systems. Therefore our one and only formula for friction is:

$$F_{friction} = F_f = -bv$$

2.2 ODE and Solutions

ODE We let $\gamma = b/m$ so that:

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

2.2.1 General Solution

And for underdampened and over dampened, the overall solution looks like:

$$x(t) = ae^{rt}$$

Where $a, r \in$, remembering that

$$e^{i\theta} = isin(\theta) + cos(\theta)$$

This is actually really cool, because the existence of a complex component in r is what enables oscillation.

If we populate the initial ODE with the derivatives of our $x(t)$, we get:

$$ar^2e^{rt} + \gamma are^{rt} + \omega ae^rt = 0$$

$$r^2 + \gamma r + \omega = 0$$

This results in a simple quadratic for which you can solve for r using the quadratic formula.

$$x = \frac{b + / - \sqrt{b^2 - 4ac}}{2a}$$

2.2.2 Regieme 1: Underdampened

If the dampening is weak enough, there will still be **some** oscillation before the oscillator comes to rest. In this case, the general solution is: