PHY293 Abridged

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Contents

0.1 Introduction

Prof. Grisouard has put up some pretty solid notes written in jupyter notebooks. Those offer a great comprehensive guide for the course contents. The goal here is to give a very concise overview of the things you need to know (NTK) to answer exam questions. Unlike some of our other courses, you don't need to be very intimately familiar with the derivations of everything in order to solve the problems (though it certainly doesn't hurt). Think of this as a really good cheat sheet.

Chapter 1

Simple Harmonic Oscillators (SHO's)

1.1 Set Up

Pretty much the same as covered last year.

$$F_{restorative} = -kx$$

$$F_{net} = ma$$

$$ma + kx = 0$$

We let $\omega_0^2 = k/m$ and rename a to $\ddot{x}$

1.2 ODE and Solution

ODE:

$$\ddot{x} + \omega_0^2 x = 0$$

Solution:

$$x(t) = a\cos(\omega_0 t + \theta)$$

Solve for 2 unknowns (usually a and θ) based on two initial conditions.

Chapter 2

Dampened Harmonic Oscillators (DHO's)

2.1 Set Up

We have all the same forces from last time plus a force of friction. We kind of have our hands tied with what models of friction we can use (remember how friction can be proportional to velocity, acceleration?) because this course only deals with linear systems. Therefore our one and only formula for friction is:

$$F_{friction} = F_f = -bv$$

2.2 ODE and Solutions

ODE We let $\gamma = b/m$ so that:

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

2.2.1 General Solution

And for underdampened and over dampened, the overall solution looks like:

$$x(t) = ae^{rt}$$

Where $a, r \in \mathbb{C}$, remembering that

$$e^{i\theta} = isin(\theta) + cos(\theta)$$

This is actually really cool, because the existence of a complex component in r is what enables oscillation.

If we populate the initial ODE with the derivatives of our $x(t)$, we get:

$$ar^2 e^{rt} + \gamma a r e^{rt} + \omega a e^r t = 0$$

$$r^2 + \gamma r + \omega = 0$$

This results in a simple quadratic for which you can solve for r using the quadratic formula.

$$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

There will be up to 2 r values. Since the solution space is a 2-D vector space, any linear combination of vectors (i.e. values of r) is still a valid vector.

You might be wondering how a figures into this whole thing. That's just based on initial conditions, really.

2.2.2   Regieme 1: Underdampened

If the dampening is weak enough, there will still be **some** oscillation before the oscillator comes to rest. In this case, the general solution is:

$$x(t) = A_0 e^{-\gamma t/2} cos(w_d t + \phi)$$

Where $w_d^2 = w_d ampened^2 = \omega_0^2 - \frac{\gamma^2}{4}$

You get this by solving for r with the quadratic formula, plugging the two imaginary roots in, then taking just the real components because you know that $x(t)$ is real.

Logarithmic Decrement

This quantity is the ln of the ratio of 1 peak to the next.

$$\frac{\gamma T_d}{2}$$

Where T_d is the period based on w_d.

2.2.3   Regieme 2: Heavily Dampened

In this case both roots are real. This occurs when $\gamma^2 - 4\omega_0^2 > 0$

Let r_p, r_m be the two real roots. Then the solution is:

$$x(t) = a_p e^{r_p t} + a_m e^{r_m t}$$

You can solve for the a values at leisure with two initial conditions.

2.2.4   Regieme 3: Critically Dampened

When the discriminant of the quadratic is zero (i.e. $\gamma^2 = 4\omega_0^2 =$), you get an overconstrained problem because you have fewer unknowns than you have constraints. Therefore, the new solution for this exact case is:

$$x(t) = (A + Bt)e^{-\gamma t/2}$$

And it decays the fastest and is pretty cool. Yay.

2.3   Energy

$$E = K + U$$
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$
$$\frac{dE}{dt} = -bv^2$$

2.3.1   Underdampened Energy

The equations get really messy so we assume that $\omega_0 >> \gamma/2$ and $\omega_0 = \omega_d$. After you sub everything in and use the pythagorean identity to cancel out the squared sine/cosine, you get:

$$E = \frac{1}{2}kA_0^2 e^{-\gamma/t}$$

2.3.2   Critical and Overdampened Energy

These ones are really easy, they just decay and that's it. Just remember the equations for energy and solve for the velocity equation.

2.4   Q-Factor

Q is a measure of the tendency to oscillate dividied by the tendency to dampen.

$$Q = frac\omega_0\gamma$$

Let $\tau = \frac{1}{gamma}$ be proportional to the lifetime of the oscillator. Then $n = \frac{\tau}{T_0}$ is the number of cycles in a lifetime.

$$Q = 2\pi n$$

Q is also proportional to the rate at which dampening removes energy from the system. Then

$$E_n = E_0 e^{-\gamma t}$$

Represents the amount of mechanical energy in the system at time t.

$$\frac{E_n - E_{n+1}}{E_n} = ... = \frac{2\pi}{Q}$$

Therefore Q is the energy in oscillator divided by the amount of energy lost at the following **radian**.