

# AER210 Homework Problems

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## 1 What is this?

I am slightly faster at writing LaTeX equations than writing by hand. I hope to get even faster and practice math even better by keeping a running formatted document of my homework problems here.

## 2 Pre-Midterm 1 Practice

## 3 Applications of Double Integrals

From section 15.4 in Stewart Calculus.

**15.4.7:** Find  $\bar{x}, \bar{y}$  and mass for  $D$ , bounded by  $y = 1 - x^2$  and  $y = 0$ .

$\rho(x, y) = ky$

Finding mass:

$$\begin{aligned} m &= \int_{-1}^1 \int_0^{1-x^2} ky \, dy \, dx \\ m &= \int_{-1}^1 \frac{k}{2} (1 - x^2)^2 \, dx \\ m &= \frac{k}{2} \int_{-1}^1 (1 - 2x^2 + x^4) \, dx \\ m &= \frac{k}{2} \left[ x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \right]_{-1 \rightarrow 1} \\ m &= \frac{k}{2} [16/15] \end{aligned}$$

$$m = k \frac{8}{15}$$

Finding  $M_y$ :

$$\begin{aligned}
 M_y &= \iint_D x\rho(x, y) dA \\
 &= \int_{-1}^1 \int_0^{1-x^2} kxy dy dx \\
 &= \dots \frac{kx}{2} [y^2]_{0 \rightarrow 1-x^2} \\
 &= \dots \frac{k}{2} [x(1-x^2)^2] \\
 &= \int_{-1}^1 k/2 (x - 2x^3 + x^5) dx \\
 &= k/2 [x^2/2 - 2/4 x^4 + x^6/6]_{(-1 \rightarrow 1)} \\
 &= k/2 [1/2 - 1/2 + 1/6] - [1/2 - 1/2 + 1/6] \\
 M_y &= 0
 \end{aligned}$$

Finding  $M_x$

$$\begin{aligned}
 M_x &= \iint_D y\rho(x, y) dA \\
 &= \int_{-1}^1 \int_0^{1-x^2} ky^2 dy dx \\
 &= \int_{-1}^1 [ky^3/3]_{0 \rightarrow 1-x^2} dx \\
 &= \int_{-1}^1 k/3 [(1-x^2)^3]_{0 \rightarrow 1-x^2} dx \\
 &= k/3 \int_{-1}^1 (1 - 2x^2 + x^4)(1 - x^2) dx \\
 &= k/3 \int_{-1}^1 [1 - x^2 - 2x^2 + 2x^3 + x^4 - x^6] dx \\
 &= k/3 \int_{-1}^1 [1 - 3x^2 + 2x^3 + x^4 - x^6] dx \\
 &= k/3 [x - x^3 + x^4/2 + x^5/5 - x^7/7]_{-1 \rightarrow 1} \\
 &= k/3 ([1 - 1 + 1/2 + 1/5 - 1/7] - [-1 + 1 + 1/2 - 1/5 + 1/7])
 \end{aligned}$$