Metropolis Hastings Tests

Importing libraries

```
import numpy as np
from scipy.stats import binom, beta
import pandas as pd
import matplotlib.pyplot as plt
import os

script_dir = os.path.dirname(os.path.realpath('__file__'))
```

Setting initial seed/values

```
np.random.seed(3)

tosses = 10
prob_heads = np.random.rand()
heads = np.random.binomial(tosses, prob_heads)


def rho_accept(x, y, heads, tosses):
    if y < 0 or y > 1:
        return 0
    return min(binom.pmf(heads, tosses, y) / binom.pmf(heads, tosses, x), 1)
```

Normal proposal distribution

```
no_samples = 30000
init_sample = 0.5

norm_mh_samples = [init_sample]
```

```
for i in range(1, no_samples):
    last_sample = norm_mh_samples[i - 1]
    new_guess = np.random.normal(last_sample, np.sqrt(1 / 12))
    accept_prob = rho_accept(last_sample, new_guess, heads, tosses)
    norm_mh_samples.append(
        np.random.choice([last_sample, new_guess], p=[1 - accept_prob, accept_prob])
    )
```

Uniform proposal distribution

```
unif_mh_samples = [init_sample]

for i in range(1, no_samples):
    last_sample = unif_mh_samples[i - 1]
    new_guess = np.random.uniform(last_sample - 1 / 2, last_sample + 1 / 2)
    accept_prob = rho_accept(last_sample, new_guess, heads, tosses)
    unif_mh_samples.append(
        np.random.choice([last_sample, new_guess], p=[1 - accept_prob, accept_prob])
    )
```

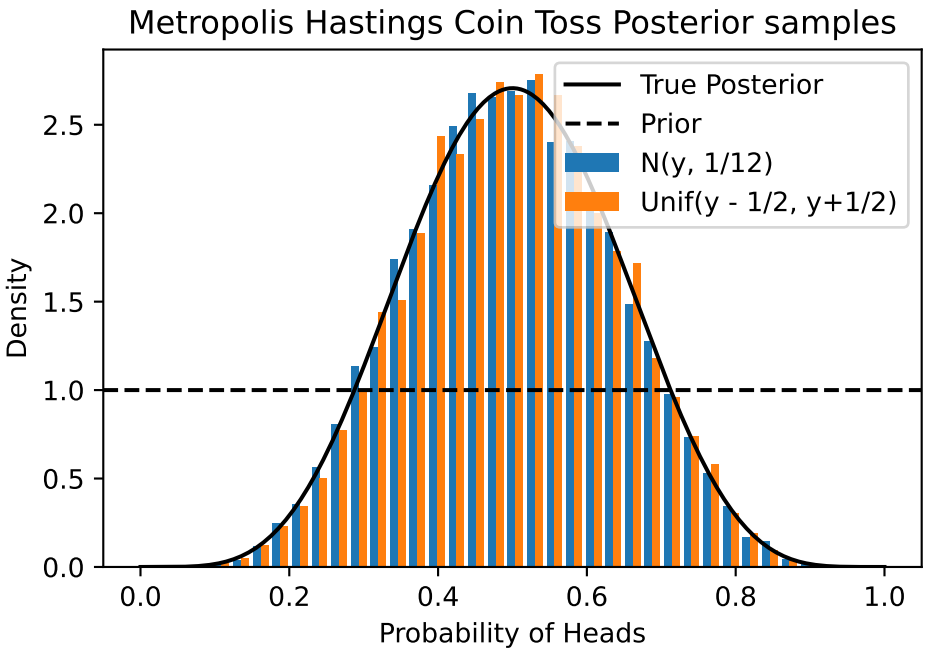
Combining into dataframe

```
mh_df = pd.DataFrame(
    {"N(y, 1/12)": norm_mh_samples, "Unif(y - 1/2, y+1/2)": unif_mh_samples}
)
```

Plotting

```
x_vals = np.linspace(0, 1, 400)
true_posterior = beta.pdf(x_vals, 1 + heads, 1 + tosses - heads)

plt.plot(x_vals, true_posterior, label="True Posterior", color="black")
plt.axhline(y=1, color="black", linestyle="dashed", label="Prior")
plt.hist(mh_df, bins=33, density=True, label=list(mh_df.columns))
plt.title("Metropolis Hastings Coin Toss Posterior samples")
plt.xlabel("Probability of Heads")
plt.ylabel("Density")
plt.legend()
plt.savefig(
    os.path.join(script_dir, "write_up/images/coin_MH_Python.pdf")
)
plt.show()
```


Metropolis Hastings Coin Toss Posterior samples
True Posterior
Prior
N(y, 1/12)
Unif(y - 1/2, y+1/2)
Density
0.0
0.5
1.0
1.5
2.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0
Probability of Heads