Inference on the Champagne Model using a Gaussian Process

TODO

- Change outputs

Setting up the Champagne Model

Imports

```
import pandas as pd
import numpy as np
from typing import Any
import matplotlib.pyplot as plt
import multiprocessing as mp
import pickle
import random

from scipy.stats import qmc
from scipy.stats import norm

import tensorflow as tf
import tensorflow_probability as tfp
from tensorflow_probability.python.distributions import normal

tfb = tfp.bijectors
tfd = tfp.distributions
tfk = tfp.math.psd_kernels
tfp_acq = tfp.experimental.bayesopt.acquisition
```

```
gpu_devices = tf.config.experimental.list_physical_devices("GPU")
for device in gpu_devices:
    tf.config.experimental.set_memory_growth(device, True)
```

```
2024-06-04 22:50:50.189796: I tensorflow/core/platform/cpu_feature_guard.cc:210] This Tensorl
To enable the following instructions: AVX2 FMA, in other operations, rebuild TensorFlow with
2024-06-04 22:50:50.779761: W tensorflow/compiler/tf2tensorrt/utils/py_utils.cc:38] TF-TRT Wa
2024-06-04 22:50:51.539463: I external/local_xla/xla/stream_executor/cuda/cuda_executor.cc:98
2024-06-04 22:50:51.572644: W tensorflow/core/common_runtime/gpu/gpu_device.cc:2251] Cannot d
Skipping registering GPU devices...
```

Model itself

```
np.random.seed(590154)

population = 1000
initial_infecteds = 10
epidemic_length = 1000
number_of_events = 30000

pv_champ_alpha = 0.4  # prop of effective care
pv_champ_beta = 0.4  # prop of radical cure
pv_champ_gamma_L = 1 / 223  # liver stage clearance rate
pv_champ_delta = 0.05  # prop of imported cases
pv_champ_lambda = 0.04  # transmission rate
pv_champ_f = 1 / 72  # relapse frequency
pv_champ_r = 1 / 60  # blood stage clearance rate

gamma_L_max = 1/30
lambda_max = 0.1
f_max = 1/14
r_max = 1/14

num_lhc_samples = 50
initial_repeats = 1
```

```
def champagne_stochastic(
    alpha_,
    beta_,
```

```
    gamma_L,
    lambda_,
    f,
    r,
    N=population,
    I_L=initial_infecteds,
    I_0=0,
    S_L=0,
    delta_=0,
    end_time=epidemic_length,
    num_events=number_of_events,
    seed=12,
):
    np.random.seed(seed)
    if (0 > (alpha_ or beta_)) or (1 < (alpha_ or beta_)):
        return "Alpha or Beta out of bounds"
    if 0 > (gamma_L or lambda_ or f or r):
        return "Gamma, lambda, f or r out of bounds"

    t = 0
    S_0 = N - I_L - I_0 - S_L
    inc_counter = 0

    list_of_outcomes = [
        {"t": 0, "S_0": S_0, "S_L": S_L, "I_0": I_0, "I_L": I_L, "inc_counter": 0}
    ]

    prop_new = alpha_ * beta_ * f / (alpha_ * beta_ * f + gamma_L)
    i = 0

    while (i < num_events) or (t < 30):
        i += 1
        if S_0 == N:
            while t < 31:
                t += 1
                new_stages = {
                    "t": t,
                    "S_0": N,
                    "S_L": 0,
                    "I_0": 0,
                    "I_L": 0,
                    "inc_counter": inc_counter,
```

```
            }
            list_of_outcomes.append(new_stages)
        break

    S_0_to_I_L = (1 - alpha_) * lambda_ * (I_L + I_0) / N * S_0
    S_0_to_S_L = alpha_ * (1 - beta_) * lambda_ * (I_0 + I_L) / N * S_0
    I_0_to_S_0 = r * I_0 / N
    I_0_to_I_L = lambda_ * (I_L + I_0) / N * I_0
    I_L_to_I_0 = gamma_L * I_L
    I_L_to_S_L = r * I_L
    S_L_to_S_0 = (gamma_L + (f + lambda_ * (I_0 + I_L) / N) * alpha_ * beta_) * S_L
    S_L_to_I_L = (f + lambda_ * (I_0 + I_L) / N) * (1 - alpha_) * S_L

    total_rate = (
        S_0_to_I_L
        + S_0_to_S_L
        + I_0_to_S_0
        + I_0_to_I_L
        + I_L_to_I_0
        + I_L_to_S_L
        + S_L_to_S_0
        + S_L_to_I_L
    )

    delta_t = np.random.exponential(1 / total_rate)
    new_stages_prob = [
        S_0_to_I_L / total_rate,
        S_0_to_S_L / total_rate,
        I_0_to_S_0 / total_rate,
        I_0_to_I_L / total_rate,
        I_L_to_I_0 / total_rate,
        I_L_to_S_L / total_rate,
        S_L_to_S_0 / total_rate,
        S_L_to_I_L / total_rate,
    ]
    t += delta_t
    silent_incidences = np.random.poisson(
        delta_t * alpha_ * beta_ * lambda_ * (I_L + I_0) * S_0 / N
    )

    new_stages = np.random.choice(
        [
```

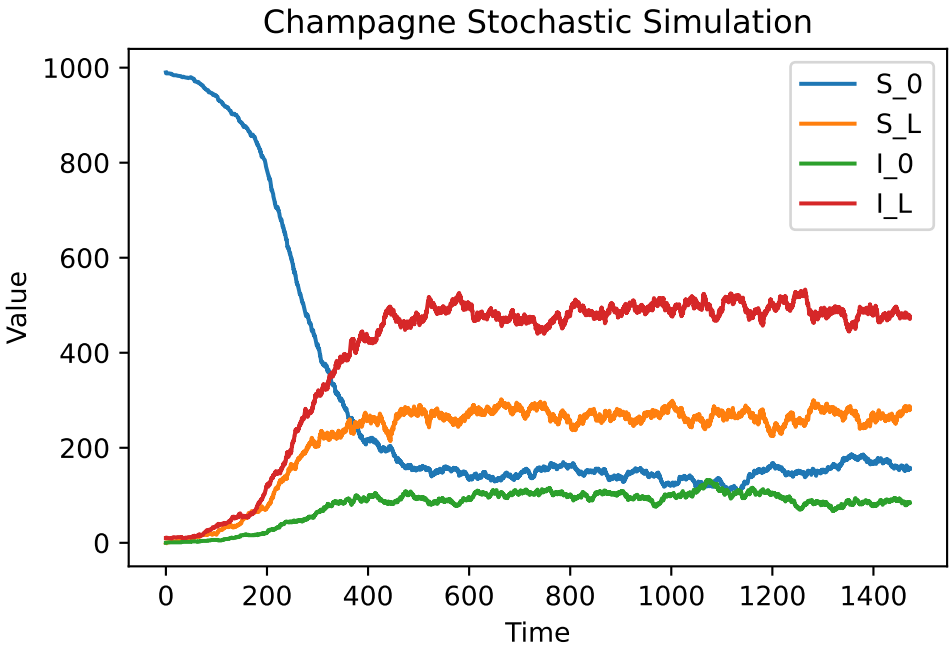
```
                {
                    "t": t,
                    "S_0": S_0 - 1,
                    "S_L": S_L,
                    "I_0": I_0,
                    "I_L": I_L + 1,
                    "inc_counter": inc_counter + silent_incidences + 1,
                },
                {
                    "t": t,
                    "S_0": S_0 - 1,
                    "S_L": S_L + 1,
                    "I_0": I_0,
                    "I_L": I_L,
                    "inc_counter": inc_counter + silent_incidences + 1,
                },
                {
                    "t": t,
                    "S_0": S_0 + 1,
                    "S_L": S_L,
                    "I_0": I_0 - 1,
                    "I_L": I_L,
                    "inc_counter": inc_counter + silent_incidences,
                },
                {
                    "t": t,
                    "S_0": S_0,
                    "S_L": S_L,
                    "I_0": I_0 - 1,
                    "I_L": I_L + 1,
                    "inc_counter": inc_counter + silent_incidences,
                },
                {
                    "t": t,
                    "S_0": S_0,
                    "S_L": S_L,
                    "I_0": I_0 + 1,
                    "I_L": I_L - 1,
                    "inc_counter": inc_counter + silent_incidences,
                },
                {
                    "t": t,
```

```
                    "S_0": S_0,
                    "S_L": S_L + 1,
                    "I_0": I_0,
                    "I_L": I_L - 1,
                    "inc_counter": inc_counter + silent_incidences,
                },
                {
                    "t": t,
                    "S_0": S_0 + 1,
                    "S_L": S_L - 1,
                    "I_0": I_0,
                    "I_L": I_L,
                    "inc_counter": inc_counter
                    + silent_incidences
                    + np.random.binomial(1, prop_new),
                },
                {
                    "t": t,
                    "S_0": S_0,
                    "S_L": S_L - 1,
                    "I_0": I_0,
                    "I_L": I_L + 1,
                    "inc_counter": inc_counter + silent_incidences + 1,
                },
            ],
            p=new_stages_prob,
        )

        list_of_outcomes.append(new_stages)

        S_0 = new_stages["S_0"]
        I_0 = new_stages["I_0"]
        I_L = new_stages["I_L"]
        S_L = new_stages["S_L"]
        inc_counter = new_stages["inc_counter"]

    outcome_df = pd.DataFrame(list_of_outcomes)
    return outcome_df


champ_samp = champagne_stochastic(
    pv_champ_alpha,
```

```
    pv_champ_beta,
    pv_champ_gamma_L,
    pv_champ_lambda,
    pv_champ_f,
    pv_champ_r,
)  # .melt(id_vars='t')
```

Plotting outcome

```
champ_samp.drop("inc_counter", axis=1).plot(x="t", legend=True)
plt.xlabel("Time")
plt.ylabel("Value")
plt.title("Champagne Stochastic Simulation")
plt.savefig("champagne_GP_images/champagne_simulation.pdf")
plt.show()
```



Function that Outputs Final Prevalence

```
def incidence(df, start, days):
    start_ind = df[df["t"].le(start)].index[-1]
    end_ind = df[df["t"].le(start + days)].index[-1]
    incidence_week = df.iloc[end_ind]["inc_counter"] - df.iloc[start_ind]["inc_counter"]
    return incidence_week


def champ_sum_stats(alpha_, beta_, gamma_L, lambda_, f, r, seed=12301923):
    champ_df_ = champagne_stochastic(alpha_, beta_, gamma_L, lambda_, f, r, seed=seed)
    fin_t = champ_df_.iloc[-1]["t"]
    first_month_inc = incidence(champ_df_, 0, 30)
    fin_t = champ_df_.iloc[-1]["t"]
    fin_week_inc = incidence(champ_df_, fin_t - 7, 7)
    fin_prev = champ_df_.iloc[-1]["I_0"] + champ_df_.iloc[-1]["I_L"]

    return np.array([fin_prev, first_month_inc, fin_week_inc])


observed_sum_stats = champ_sum_stats(
    pv_champ_alpha,
    pv_champ_beta,
    pv_champ_gamma_L,
    pv_champ_lambda,
    pv_champ_f,
    pv_champ_r,
)

print(observed_sum_stats)


def single_discrepency(alpha_, beta_, gamma_L, lambda_, f, r, seed=12301923):
    x = champ_sum_stats(alpha_, beta_, gamma_L, lambda_, f, r, seed=seed)
    return np.log(np.linalg.norm((x - observed_sum_stats) / observed_sum_stats))


def discrepency_fn(alpha_, beta_, gamma_L, lambda_, f, r, mean_of=30):
    seed = int(np.random.uniform() * 1000000)
    with mp.Pool(processes=mp.cpu_count()) as pool:
        args = [
            (alpha_, beta_, gamma_L, lambda_, f, r, seed * i) for i in range(mean_of)
        ]
```

```
        results = pool.starmap(single_discrepency, args)
    mean_obs = np.mean(results)
    return mean_obs
```

```
[613.  24.  65.]
```

Gaussian Process Regression on Final Prevalence Discrepency

```
my_seed = np.random.default_rng(seed=1795)  # For replicability

variables_names = ["alpha", "beta", "gamma_L", "lambda", "f", "r"]

LHC_sampler = qmc.LatinHypercube(d=6, seed=my_seed)
LHC_samples = LHC_sampler.random(n=num_lhc_samples)

# Using Champagne Initialisation table 2
LHC_samples[:, 2] = gamma_L_max * LHC_samples[:, 2]
LHC_samples[:, 3] = lambda_max * LHC_samples[:, 3]
LHC_samples[:, 4] = f_max * LHC_samples[:, 4]
LHC_samples[:, 5] = r_max * LHC_samples[:, 5]


# LHC_samples[:, 2] = 1/50* LHC_samples[:, 2]
# LHC_samples[:, 3] = 0.2 * LHC_samples[:, 3]
# LHC_samples[:, 4] = 1/10 * LHC_samples[:, 4]
# LHC_samples[:, 5] = 1/10 * LHC_samples[:, 5]
# LHC_samples[:, 2] = -pv_champ_gamma_L * np.log(LHC_samples[:, 2])
# LHC_samples[:, 3] = -pv_champ_lambda * np.log(LHC_samples[:, 3])
# LHC_samples[:, 4] = -pv_champ_f * np.log(LHC_samples[:, 4])
# LHC_samples[:, 5] = -pv_champ_r * np.log(LHC_samples[:, 5])

LHC_samples = np.repeat(LHC_samples, initial_repeats, axis = 0)

LHC_indices_df = pd.DataFrame(LHC_samples, columns=variables_names)

print(LHC_indices_df.head())
```

```
      alpha      beta   gamma_L    lambda         f         r
0  0.100008  0.122349  0.009668  0.030752  0.024172  0.017094
```

```
1  0.659225  0.590955  0.001070  0.077894  0.010618  0.003555
2  0.503558  0.005003  0.031832  0.054106  0.002897  0.021146
3  0.011840  0.630562  0.023631  0.066976  0.050888  0.037636
4  0.271011  0.942434  0.014052  0.060275  0.044476  0.055432
```

Generate Discrepencies

```
random_discrepencies = LHC_indices_df.apply(
    lambda x: discrepency_fn(
        x["alpha"], x["beta"], x["gamma_L"], x["lambda"], x["f"], x["r"]
    ),
    axis=1,
)

print(random_discrepencies.head())
```

```
0   -0.757989
1   -0.402862
2   -1.136214
3    0.376783
4    0.480001
dtype: float64
```

Differing Methods to Iterate Function

```
# import timeit

# def function1():
#     np.vectorize(champ_sum_stats)(random_indices_df['alpha'],
#     random_indices_df['beta'], random_indices_df['gamma_L'],
#     random_indices_df['lambda'], random_indices_df['f'], random_indices_df['r'])
#     pass

# def function2():
#     random_indices_df.apply(
#         lambda x: champ_sum_stats(
#             x['alpha'], x['beta'], x['gamma_L'], x['lambda'], x['f'], x['r']),
#             axis = 1)
```

```
#     pass

# # Time function1
# time_taken_function1 = timeit.timeit(
#     "function1()", globals=globals(), number=100)

# # Time function2
# time_taken_function2 = timeit.timeit(
#     "function2()", globals=globals(), number=100)

# print("Time taken for function1:", time_taken_function1)
# print("Time taken for function2:", time_taken_function2)
```

Time taken for function1: 187.48960775700016 Time taken for function2: 204.06618941299985

Constrain Variables to be Positive

```
constrain_positive = tfb.Shift(np.finfo(np.float64).tiny)(tfb.Exp())
```

Custom Quadratic Mean Function

```
class quad_mean_fn(tf.Module):
    def __init__(self):
        super(quad_mean_fn, self).__init__()
        # self.amp_alpha_mean = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=1.0,
        #     dtype=np.float64,
        #     name="amp_alpha_mean",
        # )
        # self.alpha_tp = tf.Variable(pv_champ_alpha, dtype=np.float64, name="alpha_tp")
        # self.amp_beta_mean = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=0.5,
        #     dtype=np.float64,
        #     name="amp_beta_mean",
        # )
        # self.beta_tp = tf.Variable(pv_champ_beta, dtype=np.float64, name="beta_tp")
```

```
        self.amp_gamma_L_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_gamma_L_mean",
        )
        # self.gamma_L_tp = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=1.0,
        #     dtype=np.float64,
        #     name="gamma_L_tp",
        # )
        self.amp_lambda_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_lambda_mean",
        )
        # self.lambda_tp = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=1.0,
        #     dtype=np.float64,
        #     name="lambda_tp",
        # )
        self.amp_f_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_f_mean",
        )
        # self.f_tp = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=1.0,
        #     dtype=np.float64,
        #     name="f_tp",
        # )
        self.amp_r_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_r_mean",
        )
```

```
        # self.r_tp = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=1.0,
        #     dtype=np.float64,
        #     name="r_tp",
        # )
        # self.bias_mean = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=1.0,
        #     dtype=np.float64,
        #     name="bias_mean",
        # )
        self.bias_mean = tf.Variable(-1.5, dtype=np.float64, name="bias_mean")

    def __call__(self, x):
        return (
            self.bias_mean
            # + self.amp_alpha_mean * (x[..., 0] - self.alpha_tp) ** 2
            # + self.amp_beta_mean * (x[..., 1] - self.beta_tp) ** 2
            # + self.amp_gamma_L_mean * (x[..., 2] - self.gamma_L_tp) ** 2
            # + self.amp_lambda_mean * (x[..., 3] - self.lambda_tp) ** 2
            # + self.amp_f_mean * (x[..., 4] - self.f_tp) ** 2
            # + self.amp_r_mean * (x[..., 5] - self.r_tp) ** 2
            + self.amp_gamma_L_mean * (x[..., 2]) ** 2
            + self.amp_lambda_mean * (x[..., 3]) ** 2
            + self.amp_f_mean * (x[..., 4]) ** 2
            + self.amp_r_mean * (x[..., 5]) ** 2
        )


quad_mean_fn().__call__(x=np.array([[1.0, 1.0, 1.0, 1.0, 1.0, 1.0]]))  # should return 1
```

```
<tf.Tensor: shape=(1,), dtype=float64, numpy=array([2.5])>
```

Custom Linear Mean Function

```
class lin_mean_fn(tf.Module):
    def __init__(self):
        super(lin_mean_fn, self).__init__()
        # self.amp_alpha_lin = tfp.util.TransformedVariable(
```

```
        #     bijector=constrain_positive,
        #     initial_value=1.0,
        #     dtype=np.float64,
        #     name="amp_alpha_lin",
        # )
        # self.amp_beta_lin = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=0.5,
        #     dtype=np.float64,
        #     name="amp_beta_lin",
        # )
        self.amp_gamma_L_lin = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_gamma_L_lin",
        )
        self.amp_lambda_lin = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_lambda_lin",
        )
        self.amp_f_lin = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_f_lin",
        )
        self.amp_r_lin = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_r_lin",
        )
        # self.bias_lin = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=1.0,
        #     dtype=np.float64,
        #     name="bias_lin",
        # )
        self.bias_lin = tf.Variable(0.0, dtype=np.float64, name="bias_mean")
```

```
    def __call__(self, x):
        return (
            self.bias_lin
            # + self.amp_alpha_lin * (x[..., 0])
            # + self.amp_beta_lin * (x[..., 1])
            + self.amp_gamma_L_lin * (x[..., 2])
            + self.amp_lambda_lin * (x[..., 3])
            + self.amp_f_lin * (x[..., 4])
            + self.amp_r_lin * (x[..., 5])
        )
```

```
class const_mean_fn(tf.Module):
    def __init__(self):
        super(const_mean_fn, self).__init__()
        self.bias_lin = tf.Variable(0.0, dtype=np.float64, name="bias_mean")

    def __call__(self, x):
        return self.bias_lin
```

Making the ARD Kernel

```
index_vals = LHC_indices_df.values
obs_vals = random_discrepencies.values

amplitude_champ = tfp.util.TransformedVariable(
    bijector=constrain_positive,
    initial_value=4.0,
    dtype=np.float64,
    name="amplitude_champ",
)

observation_noise_variance_champ = tfp.util.TransformedVariable(
    bijector=constrain_positive,
    initial_value=1.,
    dtype=np.float64,
    name="observation_noise_variance_champ",
)
```

```
length_scales_champ = tfp.util.TransformedVariable(
    bijector=tfb.Sigmoid(
        np.float64(0.0),
        [1.0 / 2, 1.0 / 2, gamma_L_max / 2, lambda_max / 2, f_max / 2, r_max / 2],
    ),
    initial_value=[1 / 8, 1 / 8, gamma_L_max / 8, lambda_max / 8, f_max / 8, r_max / 8],
    dtype=np.float64,
    name="length_scales_champ",
)
```

```
kernel_champ = tfk.FeatureScaled(
    tfk.MaternFiveHalves(amplitude=amplitude_champ),
    scale_diag=length_scales_champ,
)
```

Define the Gaussian Process with Quadratic Mean Function and ARD Kernel

```
# Define Gaussian Process with the custom kernel
champ_GP = tfd.GaussianProcess(
    kernel=kernel_champ,
    observation_noise_variance=observation_noise_variance_champ,
    index_points=index_vals,
    mean_fn=const_mean_fn(),
)

print(champ_GP.trainable_variables)

Adam_optim = tf.keras.optimizers.Adam(learning_rate=0.01)
```

```
(<tf.Variable 'amplitude_champ:0' shape=() dtype=float64, numpy=1.3862943611198906>, <tf.Vari
array([-1.09861229, -1.09861229, -1.09861229, -1.09861229, -1.09861229,
       -1.09861229])>, <tf.Variable 'observation_noise_variance_champ:0' shape=() dtype=float
```

Train the Hyperparameters

Leave One Out Predictive Log-likelihood

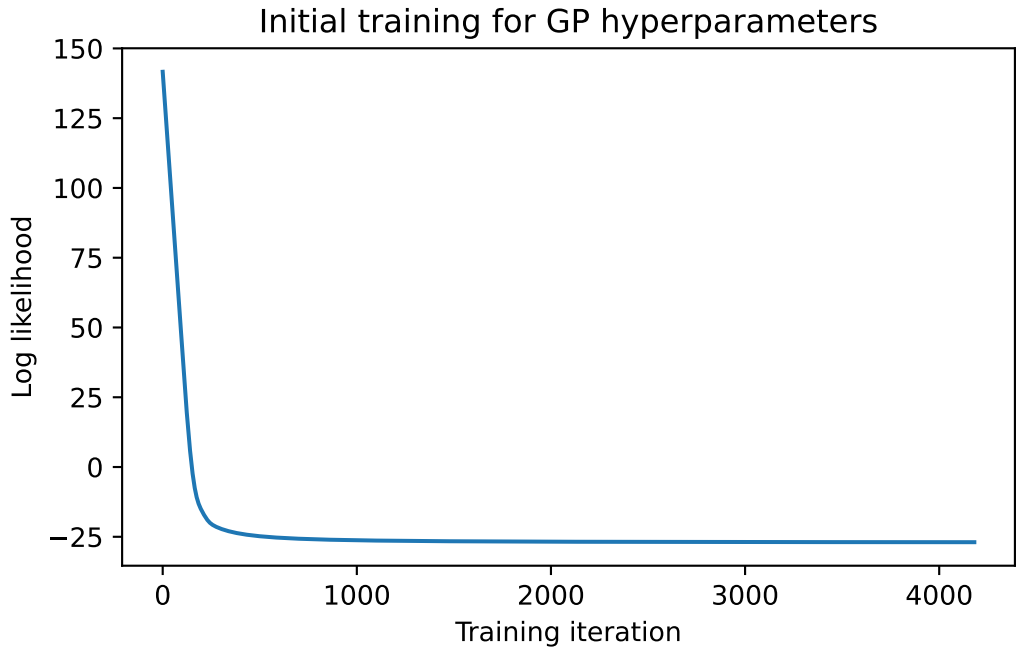
```
# predictive log stuff
@tf.function(autograph=False, jit_compile=False)
def optimize():
    with tf.GradientTape() as tape:
        K = (
            champ_GP.kernel.matrix(index_vals, index_vals)
            + tf.eye(index_vals.shape[0], dtype=np.float64)
            * observation_noise_variance_champ
        )
        means = champ_GP.mean_fn(index_vals)
        K_inv = tf.linalg.inv(K)
        K_inv_y = K_inv @ tf.reshape(obs_vals - means, shape=[obs_vals.shape[0], 1])
        K_inv_diag = tf.linalg.diag_part(K_inv)
        log_var = tf.math.log(K_inv_diag)
        log_mu = tf.reshape(K_inv_y, shape=[-1]) ** 2
        loss = -tf.math.reduce_sum(log_var - log_mu)
    grads = tape.gradient(loss, champ_GP.trainable_variables)
    Adam_optim.apply_gradients(zip(grads, champ_GP.trainable_variables))
    return loss


num_iters = 10000

lls_ = np.zeros(num_iters, np.float64)
tolerance = 1e-6  # Set your desired tolerance level
previous_loss = float("inf")

for i in range(num_iters):
    loss = optimize()
    lls_[i] = loss

    # Check if change in loss is less than tolerance
    if abs(loss - previous_loss) < tolerance:
        print(f"Hyperparameter convergence reached at iteration {i+1}.")
        lls_ = lls_[range(i + 1)]
        break

    previous_loss = loss
```

```
Hyperparameter convergence reached at iteration 4182.
```

```
print("Trained parameters:")
for var in champ_GP.trainable_variables:
    if "bias" in var.name:
        print("{} is {}\n".format(var.name, var.numpy().round(3)))
    else:
        if "length" in var.name:
            print(
                "{} is {}\n".format(
                    var.name,
                    tfb.Sigmoid(
                        np.float64(0.0),
                        [
                            1.0 / 2,
                            1.0 / 2,
                            gamma_L_max / 2,
                            lambda_max / 2,
                            f_max / 2,
                            r_max / 2,
                        ],
                    )
                    .forward(var)
                    .numpy()
                    .round(3),
                )
            )
        else:
            print(
                "{} is {}\n".format(
                    var.name, constrain_positive.forward(var).numpy().round(3)
                )
            )

initial_losses_LOOCV = lls_
```

```
Trained parameters:
amplitude_champ:0 is 0.768

length_scales_champ:0 is [0.424 0.5   0.017 0.022 0.036 0.036]

observation_noise_variance_champ:0 is 0.001

bias_mean:0 is 0.089
```

```
plt.figure(figsize=(6, 3.5))
plt.plot(lls_)
plt.title("Initial training for GP hyperparameters")
plt.xlabel("Training iteration")
plt.ylabel("Log likelihood")
plt.savefig("champagne_GP_images/hyperparam_loss_log_discrep.pdf")
plt.show()
```



Creating slices across one variable dimension

```
plot_samp_no = 21
plot_samp_times = 10
plot_gp_no = 100
gp_samp_no = 30
```

```
slice_samples_dict = {
    "alpha_slice_samples": np.repeat(np.concatenate(
        (
            np.linspace(0, 1, plot_samp_no, dtype=np.float64).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
```

```
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), plot_samp_times,  axis = 0),
    "alpha_gp_samples": np.concatenate(
        (
            np.linspace(0, 1, plot_gp_no, dtype=np.float64).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ),
    "beta_slice_samples": np.repeat(np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.linspace(0, 1, plot_samp_no, dtype=np.float64).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), plot_samp_times,  axis = 0),
    "beta_gp_samples": np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.linspace(0, 1, plot_gp_no, dtype=np.float64).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ),
    "gamma_L_slice_samples": np.repeat(np.concatenate(
        (
```

```
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
            np.linspace(0, gamma_L_max, plot_samp_no, dtype=np.float64).reshape(-1, 1),  # ga
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), plot_samp_times,  axis = 0),
    "gamma_L_gp_samples": np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.linspace(0, gamma_L_max, plot_gp_no, dtype=np.float64).reshape(-1, 1),  # gamm
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ),
    "lambda_slice_samples": np.repeat(np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
            np.linspace(0, lambda_max, plot_samp_no, dtype=np.float64).reshape(-1, 1), # lamb
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), plot_samp_times,  axis = 0),
    "lambda_gp_samples": np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.linspace(0, lambda_max, plot_gp_no, dtype=np.float64).reshape(-1, 1), # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ),
```

```
    "f_slice_samples": np.repeat(np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            np.linspace(0, f_max, plot_samp_no, dtype=np.float64).reshape(-1, 1), # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), plot_samp_times,  axis = 0),
    "f_gp_samples": np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.linspace(0, f_max, plot_gp_no, dtype=np.float64).reshape(-1, 1), # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ),
    "r_slice_samples": np.repeat(np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            np.linspace(0, r_max, plot_samp_no, dtype=np.float64).reshape(-1, 1), # r
        ),
        axis=1,
    ), plot_samp_times,  axis = 0),
    "r_gp_samples": np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.linspace(0, r_max, plot_gp_no, dtype=np.float64).reshape(-1, 1), # r
        ),
```

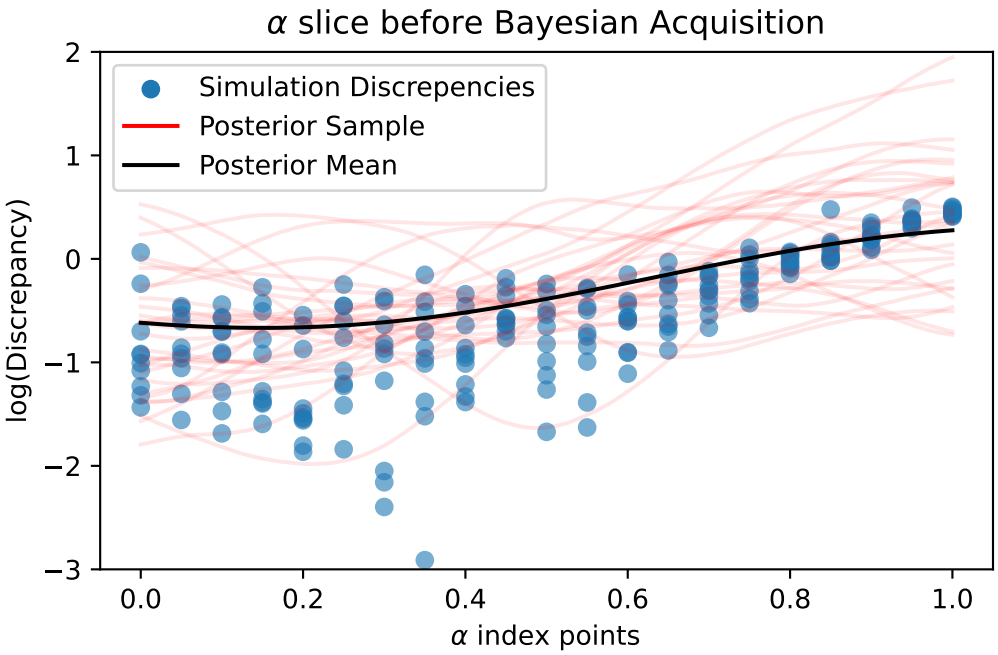
```
        axis=1,
    ),
}
```

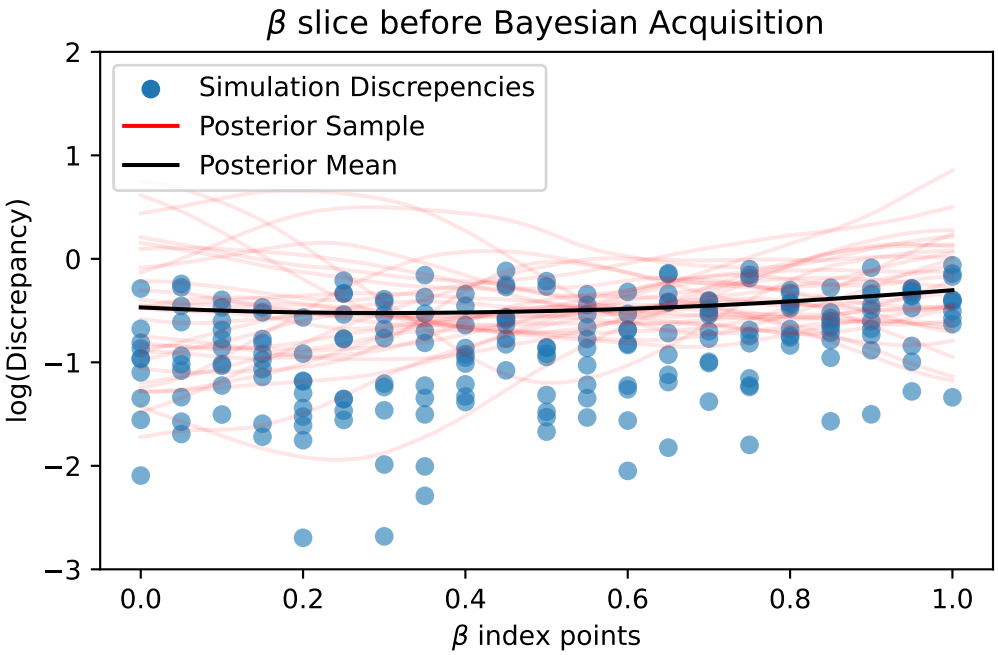
Plotting the GPs across different slices

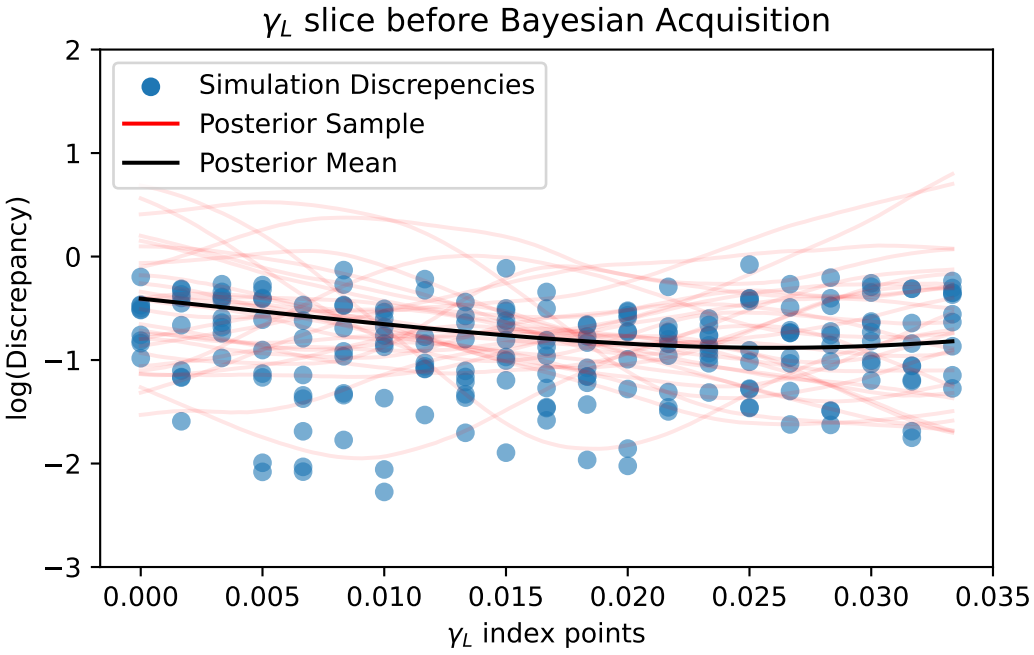
```
GP_seed = tfp.random.sanitize_seed(4362)
vars = ["alpha", "beta", "gamma_L", "lambda", "f", "r"]
slice_indices_dfs_dict = {}
slice_index_vals_dict = {}
slice_discrepencies_dict = {}
gp_samples_dict = {}

for var in vars:
    val_df = pd.DataFrame(
        slice_samples_dict[var + "_slice_samples"], columns=variables_names
    )
    slice_indices_dfs_dict[var + "_slice_indices_df"] = val_df
    slice_index_vals_dict[var + "_slice_index_vals"] = val_df.values

    df_temp = val_df.assign(seed=range(val_df.shape[0]))
    seed = int(np.random.uniform() * 1000000)
    with mp.Pool(processes=mp.cpu_count()) as pool:
        args = list(df_temp.itertuples(index=False, name=None))
        results = pool.starmap(single_discrepency, args)

    discreps = results
    slice_discrepencies_dict[var + "_slice_discrepencies"] = discreps

    gp_samples_df = pd.DataFrame(
        slice_samples_dict[var + "_gp_samples"], columns=variables_names
    )
    slice_indices_dfs_dict[var + "_gp_indices_df"] = gp_samples_df
    slice_index_vals_dict[var + "_gp_index_vals"] = gp_samples_df.values

    champ_GP_reg_plot = tfd.GaussianProcessRegressionModel(
        kernel=kernel_champ,
        index_points=gp_samples_df.values,
        observation_index_points=index_vals,
        observations=obs_vals,
```

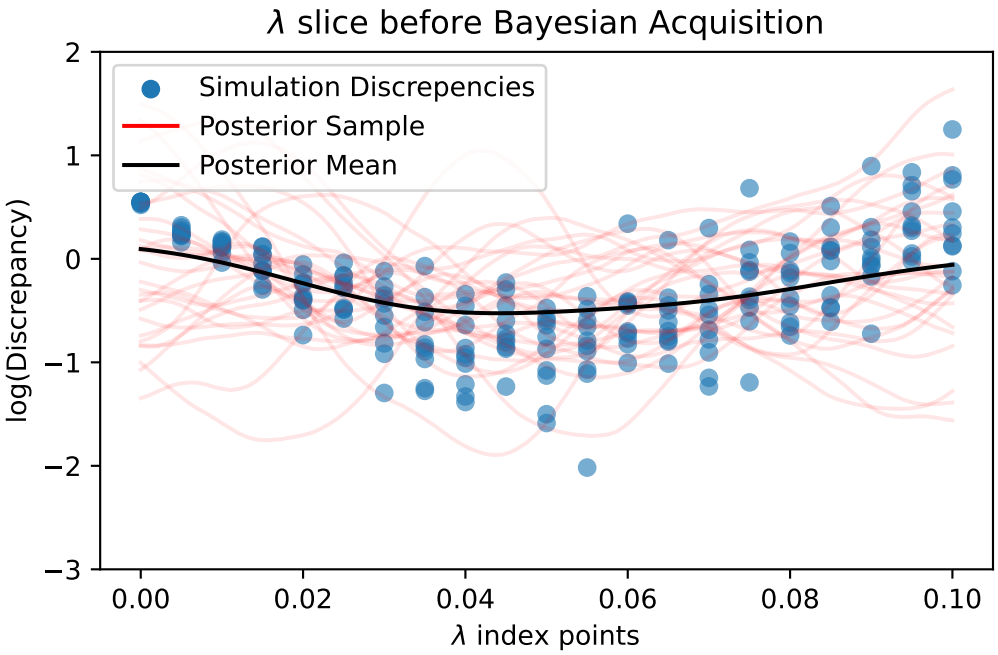
```
    observation_noise_variance=observation_noise_variance_champ,
    predictive_noise_variance=0.0,
    mean_fn=const_mean_fn(),
)
GP_samples = champ_GP_reg_plot.sample(gp_samp_no, seed=GP_seed)

gp_samples_dict[var + "initial_gp_samps"] = GP_samples

plt.figure(figsize=(6, 3.5))
plt.scatter(
    val_df[var].values,
    discreps,
    label="Simulation Discrepencies",
    alpha=0.6,
)
for i in range(gp_samp_no):
    plt.plot(
        gp_samples_df[var].values,
        GP_samples[i, :],
        c="r",
        alpha=0.1,
        label="Posterior Sample" if i == 0 else None,
    )
plt.plot(
    slice_indices_dfs_dict[var + "_gp_indices_df"][var].values,
    champ_GP_reg_plot.mean_fn(
        slice_indices_dfs_dict[var + "_gp_indices_df"].values
    ),
    c="black",
    alpha=1,
    label="Posterior Mean",
)
leg = plt.legend(loc="upper left")
for lh in leg.legend_handles:
    lh.set_alpha(1)
if var in ["f", "r"]:
    plt.xlabel("$" + var + "$ index points")
    plt.title("$" + var + "$ slice before Bayesian Acquisition")
else:
    plt.xlabel("$\\" + var + "$ index points")
    plt.title("$\\" + var + "$ slice before Bayesian Acquisition")
# if var not in ["alpha", "beta"]:
```

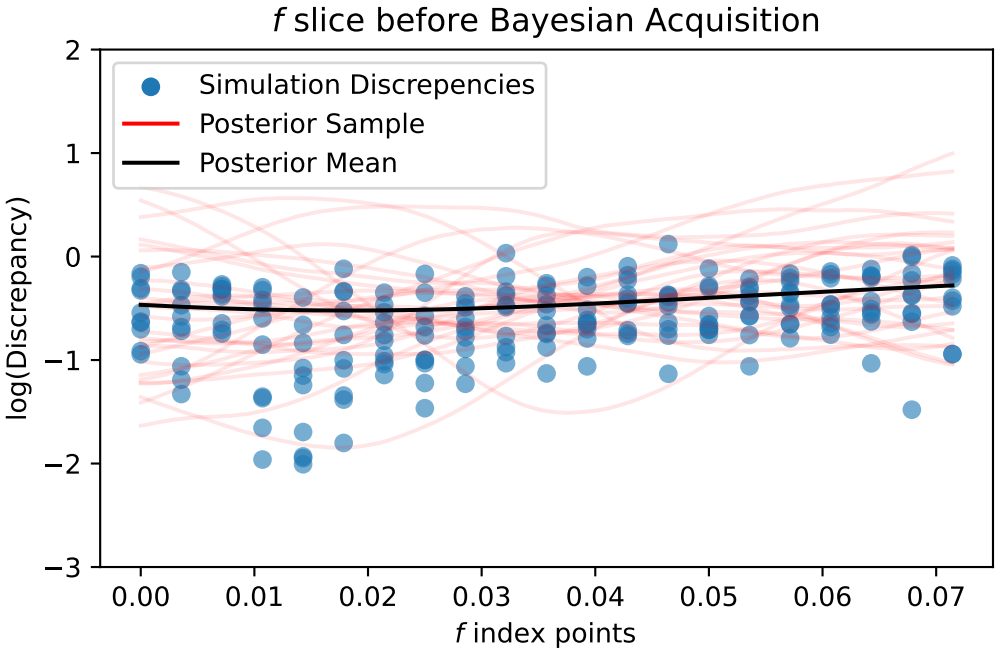
```
    #     plt.xscale("log", base=np.e)
    plt.ylabel("log(Discrepancy)")
    plt.ylim((-3, 2))
    plt.savefig("champagne_GP_images/initial_" + var + "_slice_log_discrep.pdf")
    plt.show()
```

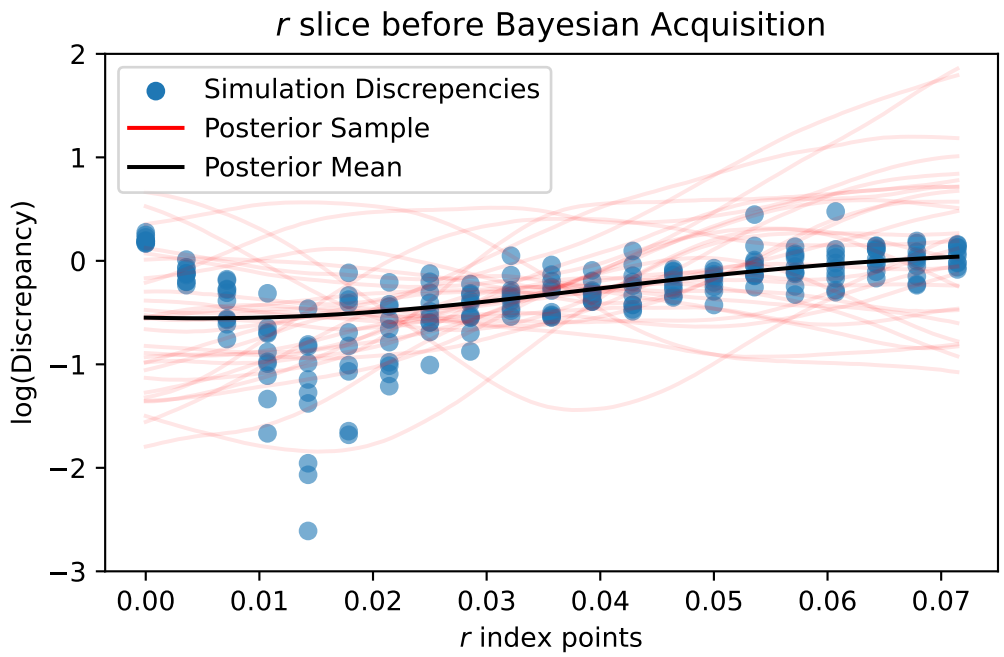



β slice before Bayesian Acquisition
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points


γ_L slice before Bayesian Acquisition
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points


λ slice before Bayesian Acquisition
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
λ index points
2
1
0
−1
−2
−3
0.00
0.02
0.04
0.06
0.08
0.10


f slice before Bayesian Acquisition
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
f index points
2
1
0
−1
−2
−3
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07



Acquiring the next datapoint to test

Proof that .variance returns what we need in acquisition function

```
champ_GP_reg = tfd.GaussianProcessRegressionModel(
    kernel=kernel_champ,
    observation_index_points=index_vals,
    observations=obs_vals,
    observation_noise_variance=observation_noise_variance_champ,
    mean_fn=const_mean_fn(),
)

new_guess = np.array([0.4, 0.4, 0.004, 0.04, 0.01, 0.17])
mean_t = champ_GP_reg.mean_fn(new_guess)
variance_t = champ_GP_reg.variance(index_points=[new_guess])

kernel_self = kernel_champ.apply(new_guess, new_guess)
kernel_others = kernel_champ.apply(new_guess, index_vals)
K = kernel_champ.matrix(
    index_vals, index_vals
```

```
) + observation_noise_variance_champ * np.identity(index_vals.shape[0])
inv_K = np.linalg.inv(K)
print("Self Kernel is {}".format(kernel_self.numpy().round(3)))
print("Others Kernel is {}".format(kernel_others.numpy().round(3)))
print(inv_K)
my_var_t = kernel_self - kernel_others.numpy() @ inv_K @ kernel_others.numpy()

print("Variance function is {}".format(variance_t.numpy().round(3)))
print("Variance function is {}".format(my_var_t.numpy().round(3)))
```

```
Self Kernel is 0.589
Others Kernel is [0.001 0.    0.001 0.001 0.005 0.007 0.002 0.001 0.    0.    0.002 0.003
 0.001 0.003 0.004 0.003 0.002 0.005 0.002 0.002 0.002 0.003 0.005 0.001
 0.    0.    0.002 0.001 0.    0.    0.001 0.005 0.003 0.001 0.003 0.
 0.004 0.005 0.001 0.003 0.003 0.    0.001 0.003 0.006 0.002 0.001 0.004
 0.    0.002]
[[ 2.29672988e+00  3.79129165e-02 -4.30502178e-02 ... -2.99825298e-02
   7.55257514e-03 -6.12922240e-01]
 [ 3.79129165e-02  2.34648869e+00  9.28325111e-03 ... -2.40140861e-02
  -6.39703097e-01  4.31965331e-04]
 [-4.30502178e-02  9.28325111e-03  2.08014737e+00 ... -2.82404170e-01
   1.54528474e-02 -3.19334868e-01]
 ...
 [-2.99825298e-02 -2.40140861e-02 -2.82404170e-01 ...  2.26133797e+00
  -5.88522492e-02  4.40383061e-02]
 [ 7.55257514e-03 -6.39703097e-01  1.54528474e-02 ... -5.88522492e-02
   2.02771243e+00  1.65371871e-02]
 [-6.12922240e-01  4.31965331e-04 -3.19334868e-01 ...  4.40383061e-02
   1.65371871e-02  2.54180856e+00]]
Variance function is [0.59]
Variance function is 0.589
```

Loss function

```
next_alpha = tfp.util.TransformedVariable(
    initial_value=0.5,
    bijector=tfb.Sigmoid(),
    dtype=np.float64,
    name="next_alpha",
)
```

```
next_beta = tfp.util.TransformedVariable(
    initial_value=0.5,
    bijector=tfb.Sigmoid(),
    dtype=np.float64,
    name="next_beta",
)

next_gamma_L = tfp.util.TransformedVariable(
    initial_value=gamma_L_max/2,
    bijector=tfb.Sigmoid(np.float64(0.), gamma_L_max),
    dtype=np.float64,
    name="next_gamma_L",
)

next_lambda = tfp.util.TransformedVariable(
    initial_value=lambda_max/2,
    bijector=tfb.Sigmoid(np.float64(0.), lambda_max),
    dtype=np.float64,
    name="next_lambda",
)

next_f = tfp.util.TransformedVariable(
    initial_value=f_max/2,
    bijector=tfb.Sigmoid(np.float64(0.), f_max),
    dtype=np.float64,
    name="next_f",
)

next_r = tfp.util.TransformedVariable(
    initial_value=r_max/2,
    bijector=tfb.Sigmoid(np.float64(0.), r_max),
    dtype=np.float64,
    name="next_r",
)

next_vars = (
    (next_alpha.trainable_variables[0],
    next_beta.trainable_variables[0],
    next_gamma_L.trainable_variables[0],
    next_lambda.trainable_variables[0],
    next_f.trainable_variables[0],
    next_r.trainable_variables[0],)
```

```
)

next_vars
```

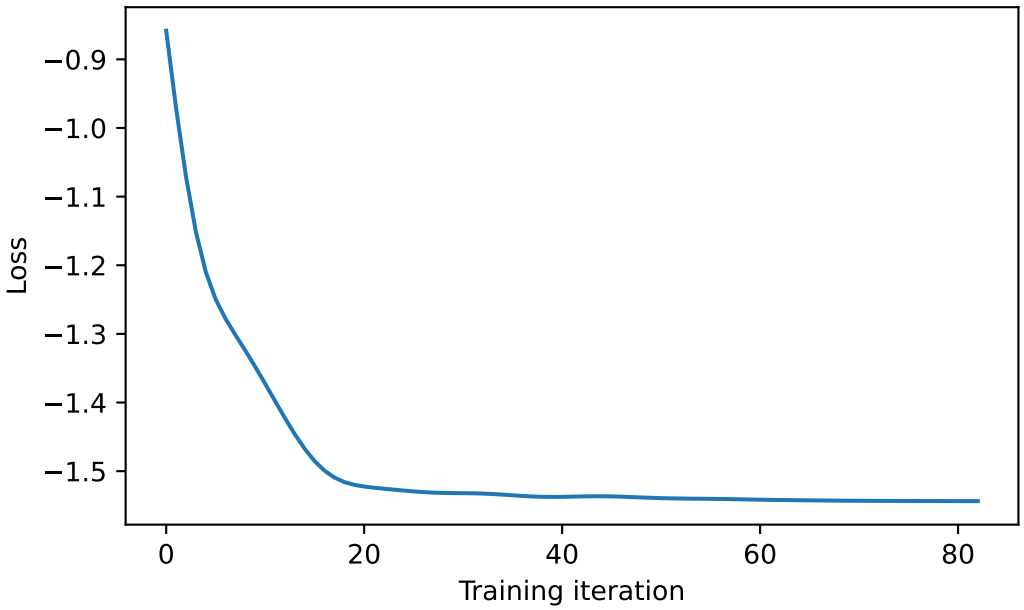
```
(<tf.Variable 'next_alpha:0' shape=() dtype=float64, numpy=0.0>,
 <tf.Variable 'next_beta:0' shape=() dtype=float64, numpy=0.0>,
 <tf.Variable 'next_gamma_L:0' shape=() dtype=float64, numpy=0.0>,
 <tf.Variable 'next_lambda:0' shape=() dtype=float64, numpy=0.0>,
 <tf.Variable 'next_f:0' shape=() dtype=float64, numpy=0.0>,
 <tf.Variable 'next_r:0' shape=() dtype=float64, numpy=0.0>)
```

```
eta_t = tf.constant(1.0, dtype=np.float64)

def UCB_loss(champ_GP_reg):
    next_guess = tf.reshape(
        tf.stack([next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]),
        [1, 6],
    )
    mean_t = champ_GP_reg.mean_fn(next_guess)
    std_t = tf.math.sqrt(
        champ_GP_reg.variance(index_points=next_guess)
        - observation_noise_variance_champ
    )
    return tf.squeeze(mean_t - std_t)


optimizer_fast = tf.keras.optimizers.Adam(learning_rate=0.1)

@tf.function(autograph=False, jit_compile=False)
def opt_var():
    with tf.GradientTape() as tape:
        loss = UCB_loss(champ_GP_reg)
    grads = tape.gradient(loss, next_vars)
    optimizer_fast.apply_gradients(zip(grads, next_vars))
    return loss

num_iters = 10000

lls_ = np.zeros(num_iters, np.float64)
tolerance = 1e-6  # Set your desired tolerance level
previous_loss = float("inf")
```

```
for i in range(num_iters):
    loss = opt_var()
    lls_[i] = loss

    # Check if change in loss is less than tolerance
    if abs(loss - previous_loss) < tolerance:
        print(f"Acquisition function convergence reached at iteration {i+1}.")
        lls_ = lls_[range(i + 1)]
        break

    previous_loss = loss

print("Trained parameters:")
for var in [next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]:
    print("{} is {}".format(var.name, (var.bijector.forward(var).numpy().round(3))))
```

```
Acquisition function convergence reached at iteration 83.
Trained parameters:
next_alpha is 0.606
next_beta is 0.554
next_gamma_L is 0.017
next_lambda is 0.051
next_f is 0.036
next_r is 0.036
```

```
plt.figure(figsize=(6, 3.5))
plt.plot(lls_)
plt.xlabel("Training iteration")
plt.ylabel("Loss")
plt.savefig("champagne_GP_images/bolfi_optim_loss_log_discrep.pdf")
plt.show()
```




```
def update_GP_LOO(champ_GP, index_vals, obs_vals, observation_noise_variance_champ):

    def LOO_loss(champ_GP, index_vals, obs_vals, observation_noise_variance_champ):
        K = (
            champ_GP.kernel.matrix(index_vals, index_vals)
            + tf.eye(index_vals.shape[0], dtype=np.float64)
            * observation_noise_variance_champ
        )
        means = champ_GP.mean_fn(index_vals)
        K_inv = tf.linalg.inv(K)
        K_inv_y = K_inv @ tf.reshape(obs_vals - means, shape=[obs_vals.shape[0], 1])
        K_inv_diag = tf.linalg.diag_part(K_inv)
        log_var = tf.math.log(K_inv_diag)
        log_mu = tf.reshape(K_inv_y, shape=[-1]) ** 2
        return -tf.math.reduce_sum(log_var - log_mu)


    @tf.function(autograph=False, jit_compile=False)
    def opt_GP():
        with tf.GradientTape() as tape:
            loss = LOO_loss(champ_GP, index_vals, obs_vals, observation_noise_variance_champ
        grads = tape.gradient(loss, champ_GP.trainable_variables)
        optimizer_slow.apply_gradients(zip(grads, champ_GP.trainable_variables))
        return loss
```

```
    num_iters = 10000

    lls_ = np.zeros(num_iters, np.float64)
    tolerance = 1e-6  # Set your desired tolerance level
    previous_loss = float("inf")

    for i in range(num_iters):
        loss = opt_GP()

        # Check if change in loss is less than tolerance
        if abs(loss - previous_loss) < tolerance:
            print(f"Hyperparameter convergence reached at iteration {i+1}.")
            break

        previous_loss = loss
    for var in optimizer_slow.variables:
        var.assign(tf.zeros_like(var))


def update_GP_MLE(champ_GP):
    @tf.function(autograph=False, jit_compile=False)
    def train_model():
        with tf.GradientTape() as tape:
            loss = -champ_GP.log_prob(obs_vals)
        grads = tape.gradient(loss, champ_GP.trainable_variables)
        optimizer_slow.apply_gradients(zip(grads, champ_GP.trainable_variables))
        return loss

    num_iters = 10000

    lls_ = np.zeros(num_iters, np.float64)
    tolerance = 1e-6  # Set your desired tolerance level
    previous_loss = float("inf")

    for i in range(num_iters):
        loss = train_model()

        # Check if change in loss is less than tolerance
        if abs(loss - previous_loss) < tolerance:
            print(f"Hyperparameter convergence reached at iteration {i+1}.")
            break
```

```
        previous_loss = loss
    for var in optimizer_slow.variables:
        var.assign(tf.zeros_like(var))


# def UCB_loss(eta_t, champ_GP_reg):
#     next_guess = tf.reshape(
#         tf.stack([next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]),
#         [1, 6],
#     )
#     mean_t = champ_GP_reg.mean_fn(next_guess)
#     std_t = champ_GP_reg.stddev(index_points=next_guess)
#     return tf.squeeze(mean_t - eta_t * std_t)


def update_var_UCB(eta_t, champ_GP_reg, next_vars):
    optimizer_fast = tf.keras.optimizers.Adam(learning_rate=0.1)

    @tf.function(autograph=False, jit_compile=False)
    def opt_var():
        with tf.GradientTape() as tape:
            loss = UCB_loss(eta_t, champ_GP_reg)
        grads = tape.gradient(loss, next_vars)
        optimizer_fast.apply_gradients(zip(grads, next_vars))
        return loss

    num_iters = 10000

    lls_ = np.zeros(num_iters, np.float64)
    tolerance = 1e-3  # Set your desired tolerance level
    previous_loss = float("inf")

    for i in range(num_iters):
        loss = opt_var()
        lls_[i] = loss

        # Check if change in loss is less than tolerance
        if abs(loss - previous_loss) < tolerance:
            print(f"Acquisition function convergence reached at iteration {i+1}.")
            break

        previous_loss = loss
```

```
    next_guess = tf.reshape(
        tf.stack([next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]),
        [1, 6],
    )
    print(
        "The final UCB loss was {}".format(loss.numpy().round(3))
        + " with predicted mean of {}".format(
            champ_GP_reg.mean_fn(next_guess).numpy().round(3)
        )
    )
    for var in optimizer_fast.variables:
        var.assign(tf.zeros_like(var))


def update_var_EI(GP_reg, alpha, beta, gamma_L, lambda_, f, r, min_obs):
    def EI_loss(alpha, beta, gamma_L, lambda_, f, r, min_obs):
        next_guess = tf.reshape(
            tf.stack([alpha, beta, gamma_L, lambda_, f, r]),
            [1, 6],
        )
        mean_t = GP_reg.mean_fn(next_guess)
        std_t = GP_reg.stddev(index_points=next_guess)
        delt = min_obs - mean_t
        return -tf.squeeze(
            delt * tfd.Normal(0, std_t).cdf(delt)
            + std_t * GP_reg.prob(delt, index_points=next_guess)
        )

    optimizer_fast = tf.keras.optimizers.Adam(learning_rate=0.1)

    @tf.function(autograph=False, jit_compile=False)
    def opt_var():
        with tf.GradientTape() as tape:
            loss = EI_loss(alpha, beta, gamma_L, lambda_, f, r, min_obs)
        grads = tape.gradient(loss, next_vars)
        optimizer_fast.apply_gradients(zip(grads, next_vars))
        return loss

    num_iters = 10000

    lls_ = np.zeros(num_iters, np.float64)
    tolerance = 1e-6  # Set your desired tolerance level
```

```
    previous_loss = np.float64("inf")

    for i in range(num_iters):
        loss = opt_var()
        lls_[i] = loss

        # Check if change in loss is less than tolerance
        if abs(loss - previous_loss) < tolerance:
            print(f"Acquisition function convergence reached at iteration {i+1}.")
            lls_ = lls_[range(i + 1)]
            break

        previous_loss = loss

    next_guess = tf.reshape(
        tf.stack([next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]),
        [1, 6],
    )
    print(
        "The final EI loss was {}".format(loss.numpy().round(3))
        + " with predicted mean of {}".format(
            champ_GP_reg.mean_fn(next_guess).numpy().round(3)
        )
    )


# update_var_EI(
#     champ_GP_reg, next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r
# )
# EI = tfp_acq.GaussianProcessExpectedImprovement(champ_GP_reg, obs_vals)


def new_eta_t(t, d, exploration_rate):
    # return np.log((t + 1) ** (d * 2 + 2) * np.pi**2 / (3 * exploration_rate))
    return np.sqrt(np.log((t + 1) ** (d * 2 + 2) * np.pi**2 / (3 * exploration_rate)))
```

```
# optimizer_fast = tf.keras.optimizers.Adam(learning_rate=1.)
# update_var_EI()
# plt.figure(figsize=(6, 3.5))
# plt.plot(lls_)
# plt.xlabel("Training iteration")
# plt.ylabel("Loss")
```

```
# plt.show()
```

```
num_slice_updates = 11

all_slices = [np.linspace(0, 1, num_slice_updates, dtype=np.float64),  # alpha
        np.linspace(0, 1, num_slice_updates, dtype=np.float64),  # beta
        np.linspace(0, gamma_L_max, num_slice_updates, dtype=np.float64),  # gamma_L
        np.linspace(0, lambda_max, num_slice_updates, dtype=np.float64),  # lambda
        np.linspace(0, f_max, num_slice_updates, dtype=np.float64),  # f
        np.linspace(0, r_max, num_slice_updates, dtype=np.float64),  # r
]
```

```
exploration_rate = 1
d = 6
update_GP_hp_freq = 20  # how many iterations before updating GP hyperparams
eta_t = tf.Variable(0, dtype=np.float64, name="eta_t")
min_obs = tf.Variable(100, dtype=np.float64, name="min_obs", shape=())
min_index = index_vals[
    champ_GP_reg.mean_fn(index_vals) == min(champ_GP_reg.mean_fn(index_vals))
][0]
simulation_reps = 20

for t in range(501):
    min_index = index_vals[
        champ_GP_reg.mean_fn(index_vals) == min(champ_GP_reg.mean_fn(index_vals))
    ][
        0,
    ]
    optimizer_slow = tf.keras.optimizers.Adam()
    eta_t.assign(new_eta_t(t, d, exploration_rate))
    min_obs.assign(min(champ_GP_reg.mean_fn(index_vals)))
    print("Iteration " + str(t))
    # print(eta_t)

    ########################################################################

    # for var in [next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]:
    #     var.assign(
    #         var.bijector.forward(np.float64(100000000.0))
    #         * np.float64(np.random.uniform())
    #     )
```

```
    index_update = 0
    for var in [next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]:
        if np.random.uniform() > 0.5:
            var.assign(min_index[index_update])
        else:
            var.assign(
                var.bijector.forward(np.float64(100000000.0))
                * np.float64(np.random.uniform())
            )
        index_update += 1

    # update_var_UCB(eta_t, champ_GP_reg)
    update_var_EI(
        champ_GP_reg,
        next_alpha,
        next_beta,
        next_gamma_L,
        next_lambda,
        next_f,
        next_r,
        min_obs,
    )

    new_params = np.array(
        [
            next_alpha.numpy(),
            next_beta.numpy(),
            next_gamma_L.numpy(),
            next_lambda.numpy(),
            next_f.numpy(),
            next_r.numpy(),
        ]
    ).reshape(1, -1)
    print("The next parameters to simulate from are {}".format(new_params.round(3)))

    new_discrepency = discrepency_fn(
        next_alpha.numpy(),
        next_beta.numpy(),
        next_gamma_L.numpy(),
        next_lambda.numpy(),
        next_f.numpy(),
        next_r.numpy(),
```

```
    )

    index_vals = np.append(index_vals, new_params, axis=0)
    obs_vals = np.append(obs_vals, new_discrepency)

    print("The mean of the samples was {}".format(new_discrepency.round(3)))

    slice_var = [next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r][
        t % 6
    ]
    for val in all_slices[t % 6]:
        if np.random.uniform() < 1 / 5 + np.exp(1 - t / 4):
            slice_var.assign(val)

            new_params = np.array(
                [
                    next_alpha.numpy(),
                    next_beta.numpy(),
                    next_gamma_L.numpy(),
                    next_lambda.numpy(),
                    next_f.numpy(),
                    next_r.numpy(),
                ]
            ).reshape(1, -1)

            new_discrepency = discrepency_fn(
                next_alpha.numpy(),
                next_beta.numpy(),
                next_gamma_L.numpy(),
                next_lambda.numpy(),
                next_f.numpy(),
                next_r.numpy(),
            )

            index_vals = np.append(index_vals, new_params, axis=0)
            obs_vals = np.append(obs_vals, new_discrepency)

    ##########################################################################

    champ_GP_reg = tfd.GaussianProcessRegressionModel(
        kernel=kernel_champ,
        observation_index_points=index_vals,
```

```
        observations=obs_vals,
        observation_noise_variance=observation_noise_variance_champ,
        predictive_noise_variance=0.0,
        mean_fn=const_mean_fn(),
    )

    if t % update_GP_hp_freq == 0:
        champ_GP = tfd.GaussianProcess(
            kernel=kernel_champ,
            observation_noise_variance=observation_noise_variance_champ,
            index_points=index_vals,
            mean_fn=const_mean_fn(),
        )
        update_GP_LOO(champ_GP, index_vals, obs_vals, observation_noise_variance_champ)
        # update_GP_MLE(champ_GP)
        min_value = min(champ_GP_reg.mean_fn(index_vals))
        min_index = index_vals[champ_GP_reg.mean_fn(index_vals) == min_value][0,]
        print(
            "The minimum predicted mean of the observed indices is {}".format(
                min_value.numpy().round(3)
            )
            + " at the point \n{}".format(min_index.round(3))
        )

    if (t > 0) & (t % 50 == 0):
        print("Trained parameters:")
        for train_var in champ_GP.trainable_variables:
            if "bias" in train_var.name:
                print("{} is {}\n".format(train_var.name, train_var.numpy().round(3)))
            else:
                if "length" in train_var.name:
                    print(
                        "{} is {}\n".format(
                            train_var.name,
                            tfb.Sigmoid(
                                np.float64(0.0),
                                [
                                    1.0 / 2,
                                    1.0 / 2,
                                    gamma_L_max / 2,
                                    lambda_max / 2,
                                    f_max / 2,
```

```
                                r_max / 2,
                            ],
                        )
                        .forward(train_var)
                        .numpy()
                        .round(3),
                    )
                )
            else:
                print(
                    "{} is {}\n".format(
                        train_var.name,
                        constrain_positive.forward(train_var).numpy().round(3),
                    )
                )

    for var in vars:
        champ_GP_reg_plot = tfd.GaussianProcessRegressionModel(
            kernel=kernel_champ,
            index_points=slice_indices_dfs_dict[var + "_gp_indices_df"].values,
            observation_index_points=index_vals,
            observations=obs_vals,
            observation_noise_variance=observation_noise_variance_champ,
            predictive_noise_variance=0.0,
            mean_fn=const_mean_fn(),
        )
        GP_samples = champ_GP_reg_plot.sample(gp_samp_no, seed=GP_seed)
        gp_samples_dict[var + "_gp_samps" + str(t) + "iters"] = GP_samples

        plt.figure(figsize=(6, 3.5))
        plt.scatter(
            slice_indices_dfs_dict[var + "_slice_indices_df"][var].values,
            slice_discrepencies_dict[var + "_slice_discrepencies"],
            label="Simulation Discrepencies",
        )
        for i in range(gp_samp_no):
            plt.plot(
                slice_indices_dfs_dict[var + "_gp_indices_df"][var].values,
                GP_samples[i, :],
                c="r",
                alpha=0.1,
                label="Posterior Sample" if i == 0 else None,
```

```
            )
        plt.plot(
            slice_indices_dfs_dict[var + "_gp_indices_df"][var].values,
            champ_GP_reg_plot.mean_fn(
                slice_indices_dfs_dict[var + "_gp_indices_df"].values
            ),
            c="black",
            alpha=1,
            label="Posterior Mean",
        )
        leg = plt.legend(loc="upper left")
        for lh in leg.legend_handles:
            lh.set_alpha(1)
        if var in ["f", "r"]:
            plt.xlabel("$" + var + "$ index points")
            plt.title(
                "$" + var + "$ slice after " + str(t) + " Bayesian acquisitions"
            )
        else:
            plt.xlabel("$\\" + var + "$ index points")
            plt.title(
                "$\\" + var + "$ slice after " + str(t) + " Bayesian acquisitions"
            )
        plt.ylabel("log(Discrepancy)")
        plt.ylim((-3, 2))
        plt.savefig(
            "champagne_GP_images/"
            + var
            + "_slice_"
            + str(t)
            + "_bolfi_updates_log_discrep.pdf"
        )
        plt.show()
```

```
Iteration 0
Acquisition function convergence reached at iteration 71.
The final EI loss was -0.398 with predicted mean of [-0.569]
The next parameters to simulate from are [[0.087 0.767 0.022 0.056 0.011 0.022]]
The mean of the samples was -0.801
The minimum predicted mean of the observed indices is -1.133 at the point
[0.504 0.005 0.032 0.054 0.003 0.021]
Iteration 1
```

```
Acquisition function convergence reached at iteration 76.
The final EI loss was -0.389 with predicted mean of [-0.577]
The next parameters to simulate from are [[0.675 0.47  0.03  0.07  0.067 0.013]]
The mean of the samples was -0.314
Iteration 2
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.33 with predicted mean of [-0.526]
The next parameters to simulate from are [[0.371 0.269 0.018 0.063 0.013 0.022]]
The mean of the samples was -0.728
Iteration 3
Acquisition function convergence reached at iteration 426.
The final EI loss was -0.399 with predicted mean of [-0.567]
The next parameters to simulate from are [[0.369 0.282 0.022 0.067 0.014 0.026]]
The mean of the samples was -0.467
Iteration 4
Acquisition function convergence reached at iteration 146.
The final EI loss was -0.399 with predicted mean of [-0.567]
The next parameters to simulate from are [[0.671 0.517 0.03  0.059 0.066 0.016]]
The mean of the samples was -0.368
Iteration 5
Acquisition function convergence reached at iteration 261.
The final EI loss was -0.399 with predicted mean of [-0.567]
The next parameters to simulate from are [[0.605 0.765 0.023 0.056 0.01  0.018]]
The mean of the samples was -0.628
Iteration 6
Acquisition function convergence reached at iteration 132.
The final EI loss was -0.399 with predicted mean of [-0.568]
The next parameters to simulate from are [[0.369 0.193 0.011 0.062 0.013 0.023]]
The mean of the samples was -0.838
Iteration 7
Acquisition function convergence reached at iteration 92.
The final EI loss was -0.399 with predicted mean of [-0.565]
The next parameters to simulate from are [[0.598 0.731 0.024 0.057 0.007 0.028]]
The mean of the samples was -0.476
Iteration 8
Acquisition function convergence reached at iteration 75.
The final EI loss was -0.399 with predicted mean of [-0.567]
The next parameters to simulate from are [[0.101 0.222 0.025 0.042 0.037 0.029]]
The mean of the samples was -0.694
Iteration 9
Acquisition function convergence reached at iteration 99.
The final EI loss was -0.373 with predicted mean of [-0.533]
The next parameters to simulate from are [[0.668 0.47  0.003 0.078 0.009 0.011]]
```

```
The mean of the samples was -0.522
Iteration 10
Acquisition function convergence reached at iteration 78.
The final EI loss was -0.399 with predicted mean of [-0.569]
The next parameters to simulate from are [[0.484 0.207 0.011 0.064 0.014 0.024]]
The mean of the samples was -0.563
Iteration 11
Acquisition function convergence reached at iteration 491.
The final EI loss was -0.399 with predicted mean of [-0.568]
The next parameters to simulate from are [[0.091 0.218 0.032 0.042 0.041 0.028]]
The mean of the samples was -0.742
Iteration 12
Acquisition function convergence reached at iteration 105.
The final EI loss was -0.399 with predicted mean of [-0.566]
The next parameters to simulate from are [[0.711 0.172 0.011 0.063 0.015 0.02 ]]
The mean of the samples was -0.789
Iteration 13
Acquisition function convergence reached at iteration 248.
The final EI loss was -0.399 with predicted mean of [-0.57]
The next parameters to simulate from are [[0.304 0.277 0.01  0.063 0.012 0.022]]
The mean of the samples was -0.574
Iteration 14
Acquisition function convergence reached at iteration 158.
The final EI loss was -0.399 with predicted mean of [-0.568]
The next parameters to simulate from are [[0.368 0.276 0.022 0.069 0.009 0.023]]
The mean of the samples was -0.644
Iteration 15
Acquisition function convergence reached at iteration 78.
The final EI loss was -0.387 with predicted mean of [-0.582]
The next parameters to simulate from are [[0.142 0.741 0.023 0.059 0.006 0.012]]
The mean of the samples was -0.276
Iteration 16
Acquisition function convergence reached at iteration 374.
The final EI loss was -0.399 with predicted mean of [-0.567]
The next parameters to simulate from are [[0.68  0.53  0.031 0.062 0.034 0.014]]
The mean of the samples was -0.631
Iteration 17
Acquisition function convergence reached at iteration 74.
The final EI loss was -0.321 with predicted mean of [-1.531]
The next parameters to simulate from are [[0.488 0.003 0.014 0.062 0.009 0.007]]
The mean of the samples was -0.5
Iteration 18
Acquisition function convergence reached at iteration 107.
```

```
The final EI loss was -0.399 with predicted mean of [-0.585]
The next parameters to simulate from are [[0.292 0.099 0.011 0.063 0.015 0.022]]
The mean of the samples was -0.489
Iteration 19
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.586]
The next parameters to simulate from are [[0.    0.808 0.023 0.054 0.013 0.022]]
The mean of the samples was -0.659
Iteration 20
Acquisition function convergence reached at iteration 164.
The final EI loss was -0.399 with predicted mean of [-0.587]
The next parameters to simulate from are [[0.684 0.001 0.03  0.076 0.068 0.014]]
The mean of the samples was -0.573
The minimum predicted mean of the observed indices is -1.147 at the point
[0.488 0.003 0.014 0.062 0.009 0.014]
Iteration 21
Acquisition function convergence reached at iteration 244.
The final EI loss was -0.399 with predicted mean of [-0.572]
The next parameters to simulate from are [[0.349 0.313 0.012 0.065 0.01  0.022]]
The mean of the samples was -0.472
Iteration 22
Acquisition function convergence reached at iteration 246.
The final EI loss was -0.399 with predicted mean of [-0.574]
The next parameters to simulate from are [[0.367 0.283 0.022 0.03  0.014 0.026]]
The mean of the samples was -0.479
Iteration 23
Acquisition function convergence reached at iteration 131.
The final EI loss was -0.397 with predicted mean of [-0.574]
The next parameters to simulate from are [[0.486 0.004 0.014 0.062 0.009 0.007]]
The mean of the samples was -0.423
Iteration 24
Acquisition function convergence reached at iteration 166.
The final EI loss was -0.399 with predicted mean of [-0.568]
The next parameters to simulate from are [[0.1   0.223 0.02  0.042 0.037 0.029]]
The mean of the samples was -0.557
Iteration 25
Acquisition function convergence reached at iteration 66.
The final EI loss was -0.379 with predicted mean of [-0.556]
The next parameters to simulate from are [[0.103 0.164 0.014 0.058 0.013 0.021]]
The mean of the samples was -0.537
Iteration 26
Acquisition function convergence reached at iteration 118.
The final EI loss was -0.399 with predicted mean of [-0.569]
```

```
The next parameters to simulate from are [[0.067 0.222 0.019 0.042 0.036 0.029]]
The mean of the samples was -0.47
Iteration 27
Acquisition function convergence reached at iteration 76.
The final EI loss was -0.399 with predicted mean of [-0.569]
The next parameters to simulate from are [[0.68  0.163 0.03  0.071 0.069 0.013]]
The mean of the samples was -0.549
Iteration 28
Acquisition function convergence reached at iteration 43.
The final EI loss was -0.389 with predicted mean of [-0.579]
The next parameters to simulate from are [[0.676 0.013 0.01  0.078 0.069 0.015]]
The mean of the samples was -0.408
Iteration 29
Acquisition function convergence reached at iteration 202.
The final EI loss was -0.399 with predicted mean of [-0.569]
The next parameters to simulate from are [[0.107 0.352 0.014 0.057 0.013 0.021]]
The mean of the samples was -0.447
Iteration 30
Acquisition function convergence reached at iteration 124.
The final EI loss was -0.399 with predicted mean of [-0.57]
The next parameters to simulate from are [[0.487 0.002 0.014 0.062 0.009 0.008]]
The mean of the samples was -0.48
Iteration 31
Acquisition function convergence reached at iteration 91.
The final EI loss was -0.395 with predicted mean of [-0.563]
The next parameters to simulate from are [[0.869 0.101 0.004 0.092 0.016 0.011]]
The mean of the samples was -0.758
Iteration 32
Acquisition function convergence reached at iteration 126.
The final EI loss was -0.398 with predicted mean of [-0.564]
The next parameters to simulate from are [[0.667 0.481 0.002 0.078 0.009 0.01 ]]
The mean of the samples was -0.576
Iteration 33
Acquisition function convergence reached at iteration 350.
The final EI loss was -0.399 with predicted mean of [-0.564]
The next parameters to simulate from are [[0.088 0.337 0.014 0.056 0.013 0.021]]
The mean of the samples was -0.548
Iteration 34
Acquisition function convergence reached at iteration 181.
The final EI loss was -0.399 with predicted mean of [-0.565]
The next parameters to simulate from are [[0.095 0.222 0.018 0.041 0.036 0.028]]
The mean of the samples was -0.53
Iteration 35
```

```
Acquisition function convergence reached at iteration 164.
The final EI loss was -0.396 with predicted mean of [-0.564]
The next parameters to simulate from are [[0.69  0.191 0.03  0.028 0.063 0.018]]
The mean of the samples was -0.667
Iteration 36
Acquisition function convergence reached at iteration 359.
The final EI loss was -0.399 with predicted mean of [-0.564]
The next parameters to simulate from are [[0.763 0.189 0.011 0.063 0.014 0.021]]
The mean of the samples was -0.491
Iteration 37
Acquisition function convergence reached at iteration 94.
The final EI loss was -0.396 with predicted mean of [-0.567]
The next parameters to simulate from are [[0.897 0.119 0.004 0.094 0.015 0.011]]
The mean of the samples was -0.543
Iteration 38
Acquisition function convergence reached at iteration 62.
The final EI loss was -0.221 with predicted mean of [-0.331]
The next parameters to simulate from are [[0.503 0.121 0.017 0.059 0.002 0.059]]
The mean of the samples was -0.151
Iteration 39
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.564]
The next parameters to simulate from are [[0.385 0.266 0.015 0.066 0.01  0.024]]
The mean of the samples was -0.822
Iteration 40
Acquisition function convergence reached at iteration 122.
The final EI loss was -0.336 with predicted mean of [-0.518]
The next parameters to simulate from are [[0.774 0.402 0.02  0.035 0.007 0.01 ]]
The mean of the samples was -0.432
Hyperparameter convergence reached at iteration 3819.
The minimum predicted mean of the observed indices is -1.077 at the point
[0.488 0.003 0.014 0.062 0.009 0.014]
Iteration 41
Acquisition function convergence reached at iteration 74.
The final EI loss was -0.391 with predicted mean of [-0.538]
The next parameters to simulate from are [[0.472 0.023 0.014 0.064 0.011 0.035]]
The mean of the samples was -0.51
Iteration 42
Acquisition function convergence reached at iteration 85.
The final EI loss was -0.396 with predicted mean of [-0.526]
The next parameters to simulate from are [[0.756 0.415 0.02  0.038 0.007 0.01 ]]
The mean of the samples was -0.549
Iteration 43
```

```
Acquisition function convergence reached at iteration 114.
The final EI loss was -0.399 with predicted mean of [-0.53]
The next parameters to simulate from are [[0.374 0.216 0.018 0.066 0.014 0.024]]
The mean of the samples was -0.392
Iteration 44
Acquisition function convergence reached at iteration 251.
The final EI loss was -0.399 with predicted mean of [-0.53]
The next parameters to simulate from are [[0.361 0.257 0.005 0.065 0.009 0.022]]
The mean of the samples was -0.554
Iteration 45
Acquisition function convergence reached at iteration 321.
The final EI loss was -0.399 with predicted mean of [-0.529]
The next parameters to simulate from are [[0.072 0.221 0.022 0.043 0.042 0.03 ]]
The mean of the samples was -0.418
Iteration 46
Acquisition function convergence reached at iteration 128.
The final EI loss was -0.398 with predicted mean of [-0.528]
The next parameters to simulate from are [[0.69  0.186 0.03  0.026 0.064 0.016]]
The mean of the samples was -0.371
Iteration 47
Acquisition function convergence reached at iteration 168.
The final EI loss was -0.399 with predicted mean of [-0.528]
The next parameters to simulate from are [[0.678 0.187 0.03  0.072 0.069 0.013]]
The mean of the samples was -0.501
Iteration 48
Acquisition function convergence reached at iteration 88.
The final EI loss was -0.399 with predicted mean of [-0.528]
The next parameters to simulate from are [[0.678 0.136 0.031 0.073 0.069 0.012]]
The mean of the samples was -0.558
Iteration 49
Acquisition function convergence reached at iteration 151.
The final EI loss was -0.398 with predicted mean of [-0.523]
The next parameters to simulate from are [[0.372 0.285 0.022 0.029 0.015 0.026]]
The mean of the samples was -0.472
Iteration 50
Acquisition function convergence reached at iteration 9.
The final EI loss was 0.0 with predicted mean of [0.571]
The next parameters to simulate from are [[0.314 0.434 0.018 0.084 0.067 0.034]]
The mean of the samples was 0.793
Trained parameters:
amplitude_champ:0 is 0.754

length_scales_champ:0 is [0.289 0.5   0.017 0.024 0.036 0.011]
```

```
observation_noise_variance_champ:0 is 0.082

bias_mean:0 is 0.374

Iteration 51
Acquisition function convergence reached at iteration 151.
The final EI loss was -0.399 with predicted mean of [-0.529]
The next parameters to simulate from are [[0.588 0.726 0.024 0.057 0.008 0.027]]
The mean of the samples was -0.454
Iteration 52
Acquisition function convergence reached at iteration 121.
The final EI loss was -0.399 with predicted mean of [-0.529]
The next parameters to simulate from are [[0.683 0.108 0.03  0.075 0.069 0.013]]
The mean of the samples was -0.476
Iteration 53
Acquisition function convergence reached at iteration 85.
The final EI loss was -0.398 with predicted mean of [-0.534]
The next parameters to simulate from are [[0.098 0.871 0.016 0.057 0.012 0.021]]
The mean of the samples was -0.629
Iteration 54
Acquisition function convergence reached at iteration 450.
The final EI loss was -0.399 with predicted mean of [-0.532]
The next parameters to simulate from are [[0.346 0.259 0.004 0.063 0.009 0.021]]
The mean of the samples was -0.634
Iteration 55
Acquisition function convergence reached at iteration 90.
The final EI loss was -0.397 with predicted mean of [-0.532]
The next parameters to simulate from are [[0.143 0.74  0.023 0.054 0.006 0.013]]
The mean of the samples was -0.529
Iteration 56
Acquisition function convergence reached at iteration 503.
The final EI loss was -0.399 with predicted mean of [-0.535]
The next parameters to simulate from are [[0.36  0.316 0.024 0.066 0.012 0.025]]
The mean of the samples was -0.591
Iteration 57
Acquisition function convergence reached at iteration 113.
The final EI loss was -0.399 with predicted mean of [-0.533]
The next parameters to simulate from are [[0.14  0.715 0.024 0.053 0.006 0.013]]
The mean of the samples was -0.587
Iteration 58
Acquisition function convergence reached at iteration 87.
The final EI loss was -0.399 with predicted mean of [-0.535]
```

```
The next parameters to simulate from are [[0.674 0.524 0.031 0.06  0.041 0.015]]
The mean of the samples was -0.459
Iteration 59
Acquisition function convergence reached at iteration 192.
The final EI loss was -0.399 with predicted mean of [-0.539]
The next parameters to simulate from are [[0.127 0.726 0.023 0.054 0.006 0.013]]
The mean of the samples was -0.599
Iteration 60
Acquisition function convergence reached at iteration 59.
The final EI loss was -0.112 with predicted mean of [-1.234]
The next parameters to simulate from are [[0.575 0.182 0.022 0.059 0.005 0.016]]
The mean of the samples was -1.223
Hyperparameter convergence reached at iteration 1318.
The minimum predicted mean of the observed indices is -1.19 at the point
[0.575 0.182 0.022 0.059 0.005 0.016]
Iteration 61
Acquisition function convergence reached at iteration 245.
The final EI loss was -0.399 with predicted mean of [-0.596]
The next parameters to simulate from are [[0.438 0.135 0.011 0.063 0.02  0.023]]
The mean of the samples was -0.667
Iteration 62
Acquisition function convergence reached at iteration 213.
The final EI loss was -0.399 with predicted mean of [-0.596]
The next parameters to simulate from are [[0.763 0.186 0.011 0.064 0.015 0.02 ]]
The mean of the samples was -0.613
Iteration 63
Acquisition function convergence reached at iteration 129.
The final EI loss was -0.399 with predicted mean of [-0.595]
The next parameters to simulate from are [[0.406 0.212 0.003 0.064 0.008 0.022]]
The mean of the samples was -0.707
Iteration 64
Acquisition function convergence reached at iteration 223.
The final EI loss was -0.399 with predicted mean of [-0.594]
The next parameters to simulate from are [[0.362 0.276 0.021 0.031 0.018 0.026]]
The mean of the samples was -0.552
Iteration 65
Acquisition function convergence reached at iteration 105.
The final EI loss was -0.399 with predicted mean of [-0.597]
The next parameters to simulate from are [[0.356 0.259 0.02  0.031 0.017 0.026]]
The mean of the samples was -0.506
Iteration 66
Acquisition function convergence reached at iteration 123.
The final EI loss was -0.399 with predicted mean of [-0.594]
```

```
The next parameters to simulate from are [[0.672 0.422 0.03  0.057 0.062 0.015]]
The mean of the samples was -0.504
Iteration 67
Acquisition function convergence reached at iteration 133.
The final EI loss was -0.399 with predicted mean of [-0.594]
The next parameters to simulate from are [[0.086 0.223 0.016 0.042 0.028 0.029]]
The mean of the samples was -0.532
Iteration 68
Acquisition function convergence reached at iteration 191.
The final EI loss was -0.399 with predicted mean of [-0.595]
The next parameters to simulate from are [[0.462 0.256 0.008 0.067 0.01  0.022]]
The mean of the samples was -0.677
Iteration 69
Acquisition function convergence reached at iteration 5.
The final EI loss was 0.0 with predicted mean of [0.708]
The next parameters to simulate from are [[0.121 0.499 0.019 0.092 0.006 0.02 ]]
The mean of the samples was 0.807
Iteration 70
Acquisition function convergence reached at iteration 75.
The final EI loss was -0.396 with predicted mean of [-0.592]
The next parameters to simulate from are [[0.619 0.331 0.022 0.059 0.011 0.029]]
The mean of the samples was -0.466
Iteration 71
Acquisition function convergence reached at iteration 235.
The final EI loss was -0.399 with predicted mean of [-0.595]
The next parameters to simulate from are [[0.617 0.309 0.023 0.058 0.01  0.029]]
The mean of the samples was -0.556
Iteration 72
Acquisition function convergence reached at iteration 125.
The final EI loss was -0.397 with predicted mean of [-0.595]
The next parameters to simulate from are [[0.606 0.749 0.023 0.056 0.01  0.006]]
The mean of the samples was -0.599
Iteration 73
Acquisition function convergence reached at iteration 120.
The final EI loss was -0.399 with predicted mean of [-0.595]
The next parameters to simulate from are [[0.69  0.499 0.032 0.061 0.031 0.015]]
The mean of the samples was -0.635
Iteration 74
Acquisition function convergence reached at iteration 102.
The final EI loss was -0.399 with predicted mean of [-0.596]
The next parameters to simulate from are [[0.654 0.098 0.03  0.071 0.068 0.012]]
The mean of the samples was -0.606
Iteration 75
```

```
Acquisition function convergence reached at iteration 92.
The final EI loss was -0.396 with predicted mean of [-0.594]
The next parameters to simulate from are [[0.487 0.232 0.02  0.043 0.036 0.029]]
The mean of the samples was -0.593
Iteration 76
Acquisition function convergence reached at iteration 219.
The final EI loss was -0.399 with predicted mean of [-0.597]
The next parameters to simulate from are [[0.625 0.227 0.023 0.058 0.009 0.029]]
The mean of the samples was -0.636
Iteration 77
Acquisition function convergence reached at iteration 62.
The final EI loss was -0.013 with predicted mean of [-1.217]
The next parameters to simulate from are [[0.589 0.07  0.027 0.059 0.014 0.016]]
The mean of the samples was -1.425
Iteration 78
Acquisition function convergence reached at iteration 75.
The final EI loss was -0.399 with predicted mean of [-0.651]
The next parameters to simulate from are [[0.676 0.165 0.029 0.03  0.06  0.02 ]]
The mean of the samples was -0.614
Iteration 79
Acquisition function convergence reached at iteration 80.
The final EI loss was -0.399 with predicted mean of [-0.652]
The next parameters to simulate from are [[0.651 0.401 0.028 0.058 0.065 0.014]]
The mean of the samples was -0.734
Iteration 80
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.654]
The next parameters to simulate from are [[0.326 0.185 0.006 0.06  0.013 0.021]]
The mean of the samples was -0.623
Hyperparameter convergence reached at iteration 1241.
The minimum predicted mean of the observed indices is -1.304 at the point
[0.589 0.07  0.027 0.059 0.014 0.016]
Iteration 81
Acquisition function convergence reached at iteration 225.
The final EI loss was -0.399 with predicted mean of [-0.651]
The next parameters to simulate from are [[0.324 0.255 0.022 0.032 0.021 0.026]]
The mean of the samples was -0.75
Iteration 82
Acquisition function convergence reached at iteration 214.
The final EI loss was -0.399 with predicted mean of [-0.652]
The next parameters to simulate from are [[0.328 0.315 0.021 0.032 0.013 0.027]]
The mean of the samples was -0.452
Iteration 83
```

```
Acquisition function convergence reached at iteration 106.
The final EI loss was -0.399 with predicted mean of [-0.653]
The next parameters to simulate from are [[0.441 0.333 0.011 0.065 0.005 0.024]]
The mean of the samples was -0.593
Iteration 84
Acquisition function convergence reached at iteration 83.
The final EI loss was -0.399 with predicted mean of [-0.652]
The next parameters to simulate from are [[0.654 0.633 0.029 0.058 0.03  0.015]]
The mean of the samples was -0.562
Iteration 85
Acquisition function convergence reached at iteration 129.
The final EI loss was -0.399 with predicted mean of [-0.652]
The next parameters to simulate from are [[0.342 0.226 0.021 0.032 0.023 0.027]]
The mean of the samples was -0.659
Iteration 86
Acquisition function convergence reached at iteration 145.
The final EI loss was -0.399 with predicted mean of [-0.652]
The next parameters to simulate from are [[0.77  0.187 0.014 0.063 0.013 0.019]]
The mean of the samples was -0.604
Iteration 87
Acquisition function convergence reached at iteration 92.
The final EI loss was -0.398 with predicted mean of [-0.653]
The next parameters to simulate from are [[0.686 0.157 0.03  0.076 0.039 0.013]]
The mean of the samples was -0.614
Iteration 88
Acquisition function convergence reached at iteration 278.
The final EI loss was -0.399 with predicted mean of [-0.655]
The next parameters to simulate from are [[0.105 0.701 0.022 0.055 0.005 0.016]]
The mean of the samples was -0.719
Iteration 89
Acquisition function convergence reached at iteration 118.
The final EI loss was -0.399 with predicted mean of [-0.655]
The next parameters to simulate from are [[0.52  0.09  0.015 0.062 0.009 0.008]]
The mean of the samples was -0.512
Iteration 90
Acquisition function convergence reached at iteration 132.
The final EI loss was -0.399 with predicted mean of [-0.657]
The next parameters to simulate from are [[0.663 0.474 0.028 0.055 0.064 0.013]]
The mean of the samples was -0.706
Iteration 91
Acquisition function convergence reached at iteration 163.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.63  0.208 0.021 0.059 0.013 0.027]]
```

```
The mean of the samples was -0.634
Iteration 92
Acquisition function convergence reached at iteration 93.
The final EI loss was -0.397 with predicted mean of [-0.654]
The next parameters to simulate from are [[0.766 0.378 0.022 0.035 0.07  0.01 ]]
The mean of the samples was -0.71
Iteration 93
Acquisition function convergence reached at iteration 155.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.629 0.452 0.027 0.059 0.059 0.014]]
The mean of the samples was -0.579
Iteration 94
Acquisition function convergence reached at iteration 110.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.372 0.292 0.022 0.027 0.048 0.026]]
The mean of the samples was -0.628
Iteration 95
Acquisition function convergence reached at iteration 139.
The final EI loss was -0.399 with predicted mean of [-0.655]
The next parameters to simulate from are [[0.366 0.275 0.021 0.034 0.011 0.025]]
The mean of the samples was -0.693
Iteration 96
Acquisition function convergence reached at iteration 238.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.22  0.21  0.021 0.06  0.006 0.016]]
The mean of the samples was -0.779
Iteration 97
Acquisition function convergence reached at iteration 142.
The final EI loss was -0.399 with predicted mean of [-0.659]
The next parameters to simulate from are [[0.341 0.259 0.032 0.066 0.009 0.023]]
The mean of the samples was -0.617
Iteration 98
Acquisition function convergence reached at iteration 117.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.089 0.152 0.025 0.042 0.041 0.029]]
The mean of the samples was -0.581
Iteration 99
Acquisition function convergence reached at iteration 211.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.665 0.479 0.028 0.055 0.062 0.012]]
The mean of the samples was -0.819
Iteration 100
Acquisition function convergence reached at iteration 131.
```

```
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.365 0.231 0.019 0.034 0.016 0.025]]
The mean of the samples was -0.782
Hyperparameter convergence reached at iteration 1220.
The minimum predicted mean of the observed indices is -1.313 at the point
[0.589 0.07  0.027 0.059 0.014 0.016]
Trained parameters:
amplitude_champ:0 is 0.702

length_scales_champ:0 is [0.294 0.5   0.017 0.024 0.036 0.012]

observation_noise_variance_champ:0 is 0.081

bias_mean:0 is 0.443

Iteration 101
Acquisition function convergence reached at iteration 49.
The final EI loss was -0.002 with predicted mean of [-1.317]
The next parameters to simulate from are [[0.591 0.06  0.028 0.059 0.016 0.016]]
The mean of the samples was -1.077
Iteration 102
Acquisition function convergence reached at iteration 133.
The final EI loss was -0.399 with predicted mean of [-0.62]
The next parameters to simulate from are [[0.619 0.149 0.023 0.058 0.012 0.03 ]]
The mean of the samples was -0.63
Iteration 103
Acquisition function convergence reached at iteration 4.
The final EI loss was 0.0 with predicted mean of [0.847]
The next parameters to simulate from are [[0.068 0.636 0.026 0.093 0.012 0.015]]
The mean of the samples was 1.192
Iteration 104
Acquisition function convergence reached at iteration 119.
The final EI loss was -0.399 with predicted mean of [-0.62]
The next parameters to simulate from are [[0.856 0.119 0.004 0.091 0.017 0.011]]
The mean of the samples was -0.742
Iteration 105
Acquisition function convergence reached at iteration 87.
The final EI loss was -0.397 with predicted mean of [-0.618]
The next parameters to simulate from are [[0.792 0.218 0.027 0.062 0.012 0.019]]
The mean of the samples was -0.56
Iteration 106
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.618]
```

```
The next parameters to simulate from are [[0.633 0.133 0.022 0.058 0.008 0.03 ]]
The mean of the samples was -0.505
Iteration 107
Acquisition function convergence reached at iteration 71.
The final EI loss was -0.001 with predicted mean of [-1.238]
The next parameters to simulate from are [[0.574 0.093 0.027 0.059 0.012 0.017]]
The mean of the samples was -1.321
Iteration 108
Acquisition function convergence reached at iteration 87.
The final EI loss was -0.396 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.104 0.181 0.029 0.033 0.055 0.022]]
The mean of the samples was -0.686
Iteration 109
Acquisition function convergence reached at iteration 85.
The final EI loss was -0.399 with predicted mean of [-0.629]
The next parameters to simulate from are [[0.685 0.195 0.03  0.03  0.065 0.02 ]]
The mean of the samples was -0.721
Iteration 110
Acquisition function convergence reached at iteration 97.
The final EI loss was -0.399 with predicted mean of [-0.629]
The next parameters to simulate from are [[0.483 0.176 0.008 0.069 0.014 0.022]]
The mean of the samples was -0.495
Iteration 111
Acquisition function convergence reached at iteration 66.
The final EI loss was -0.398 with predicted mean of [-0.628]
The next parameters to simulate from are [[0.738 0.432 0.021 0.041 0.007 0.011]]
The mean of the samples was -0.77
Iteration 112
Acquisition function convergence reached at iteration 101.
The final EI loss was -0.399 with predicted mean of [-0.627]
The next parameters to simulate from are [[0.631 0.06  0.029 0.071 0.066 0.013]]
The mean of the samples was -0.37
Iteration 113
Acquisition function convergence reached at iteration 164.
The final EI loss was -0.399 with predicted mean of [-0.63]
The next parameters to simulate from are [[0.644 0.771 0.025 0.056 0.015 0.019]]
The mean of the samples was -0.479
Iteration 114
Acquisition function convergence reached at iteration 44.
The final EI loss was -0.398 with predicted mean of [-0.627]
The next parameters to simulate from are [[0.627 0.15  0.029 0.065 0.071 0.011]]
The mean of the samples was -0.545
Iteration 115
```

```
Acquisition function convergence reached at iteration 99.
The final EI loss was -0.399 with predicted mean of [-0.628]
The next parameters to simulate from are [[0.514 0.235 0.002 0.064 0.012 0.021]]
The mean of the samples was -0.697
Iteration 116
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.628]
The next parameters to simulate from are [[0.748 0.459 0.02  0.038 0.01  0.01 ]]
The mean of the samples was -0.771
Iteration 117
Acquisition function convergence reached at iteration 99.
The final EI loss was -0.399 with predicted mean of [-0.627]
The next parameters to simulate from are [[0.674 0.063 0.03  0.071 0.062 0.014]]
The mean of the samples was -0.695
Iteration 118
Acquisition function convergence reached at iteration 109.
The final EI loss was -0.399 with predicted mean of [-0.627]
The next parameters to simulate from are [[0.661 0.666 0.028 0.058 0.028 0.015]]
The mean of the samples was -0.633
Iteration 119
Acquisition function convergence reached at iteration 88.
The final EI loss was -0.399 with predicted mean of [-0.627]
The next parameters to simulate from are [[0.619 0.092 0.026 0.058 0.022 0.03 ]]
The mean of the samples was -0.584
Iteration 120
Acquisition function convergence reached at iteration 90.
The final EI loss was -0.399 with predicted mean of [-0.627]
The next parameters to simulate from are [[0.585 0.649 0.025 0.056 0.011 0.024]]
The mean of the samples was -0.589
Hyperparameter convergence reached at iteration 1017.
The minimum predicted mean of the observed indices is -1.254 at the point
[0.574 0.093 0.027 0.059 0.012 0.017]
Iteration 121
Acquisition function convergence reached at iteration 68.
The final EI loss was -0.399 with predicted mean of [-0.627]
The next parameters to simulate from are [[0.522 0.062 0.016 0.061 0.009 0.032]]
The mean of the samples was -0.595
Iteration 122
Acquisition function convergence reached at iteration 84.
The final EI loss was -0.399 with predicted mean of [-0.627]
The next parameters to simulate from are [[0.335 0.187 0.004 0.06  0.013 0.021]]
The mean of the samples was -0.815
Iteration 123
```

```
Acquisition function convergence reached at iteration 68.
The final EI loss was -0.398 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.335 0.911 0.019 0.033 0.022 0.027]]
The mean of the samples was -0.497
Iteration 124
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.541 0.76  0.025 0.053 0.009 0.025]]
The mean of the samples was -0.609
Iteration 125
Acquisition function convergence reached at iteration 253.
The final EI loss was -0.399 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.071 0.16  0.029 0.042 0.044 0.03 ]]
The mean of the samples was -0.626
Iteration 126
Acquisition function convergence reached at iteration 93.
The final EI loss was -0.399 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.665 0.448 0.004 0.077 0.009 0.011]]
The mean of the samples was -0.669
Iteration 127
Acquisition function convergence reached at iteration 106.
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.048 0.138 0.019 0.056 0.013 0.021]]
The mean of the samples was -0.599
Iteration 128
Acquisition function convergence reached at iteration 20.
The final EI loss was 0.0 with predicted mean of [0.484]
The next parameters to simulate from are [[0.909 0.322 0.016 0.004 0.017 0.02 ]]
The mean of the samples was 0.456
Iteration 129
Acquisition function convergence reached at iteration 25.
The final EI loss was 0.0 with predicted mean of [0.377]
The next parameters to simulate from are [[0.311 0.894 0.023 0.    0.023 0.029]]
The mean of the samples was 0.345
Iteration 130
Acquisition function convergence reached at iteration 102.
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.775 0.413 0.02  0.04  0.011 0.009]]
The mean of the samples was -0.643
Iteration 131
Acquisition function convergence reached at iteration 68.
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.126 0.152 0.018 0.059 0.012 0.019]]
```

```
The mean of the samples was -0.664
Iteration 132
Acquisition function convergence reached at iteration 167.
The final EI loss was -0.399 with predicted mean of [-0.627]
The next parameters to simulate from are [[0.64  0.811 0.026 0.057 0.017 0.017]]
The mean of the samples was -0.541
Iteration 133
Acquisition function convergence reached at iteration 94.
The final EI loss was -0.399 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.543 0.252 0.002 0.065 0.01  0.021]]
The mean of the samples was -0.625
Iteration 134
Acquisition function convergence reached at iteration 11.
The final EI loss was 0.0 with predicted mean of [0.768]
The next parameters to simulate from are [[0.136 0.19  0.022 0.093 0.021 0.02 ]]
The mean of the samples was 0.772
Iteration 135
Acquisition function convergence reached at iteration 173.
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.385 0.207 0.021 0.034 0.012 0.024]]
The mean of the samples was -0.601
Iteration 136
Acquisition function convergence reached at iteration 81.
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.649 0.519 0.027 0.057 0.069 0.012]]
The mean of the samples was -0.689
Iteration 137
Acquisition function convergence reached at iteration 127.
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.073 0.208 0.027 0.041 0.039 0.031]]
The mean of the samples was -0.727
Iteration 138
Acquisition function convergence reached at iteration 2.
The final EI loss was -0.0 with predicted mean of [-0.005]
The next parameters to simulate from are [[0.942 0.863 0.027 0.057 0.012 0.019]]
The mean of the samples was 0.04
Iteration 139
Acquisition function convergence reached at iteration 218.
The final EI loss was -0.399 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.043 0.297 0.018 0.043 0.027 0.028]]
The mean of the samples was -0.619
Iteration 140
Acquisition function convergence reached at iteration 41.
```

```
The final EI loss was -0.004 with predicted mean of [-1.26]
The next parameters to simulate from are [[0.571 0.114 0.027 0.059 0.008 0.017]]
The mean of the samples was -1.22
Hyperparameter convergence reached at iteration 1036.
The minimum predicted mean of the observed indices is -1.251 at the point
[0.571 0.114 0.027 0.059 0.008 0.017]
Iteration 141
Acquisition function convergence reached at iteration 124.
The final EI loss was -0.399 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.342 0.932 0.031 0.065 0.009 0.023]]
The mean of the samples was -0.61
Iteration 142
Acquisition function convergence reached at iteration 18.
The final EI loss was 0.0 with predicted mean of [0.508]
The next parameters to simulate from are [[0.743 0.317 0.019 0.004 0.014 0.021]]
The mean of the samples was 0.353
Iteration 143
Acquisition function convergence reached at iteration 186.
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.279 0.291 0.005 0.061 0.007 0.021]]
The mean of the samples was -0.625
Iteration 144
Acquisition function convergence reached at iteration 115.
The final EI loss was -0.399 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.036 0.219 0.024 0.043 0.034 0.03 ]]
The mean of the samples was -0.73
Iteration 145
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.398 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.449 0.25  0.019 0.055 0.023 0.027]]
The mean of the samples was -0.591
Iteration 146
Acquisition function convergence reached at iteration 75.
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.125 0.184 0.022 0.059 0.007 0.017]]
The mean of the samples was -0.64
Iteration 147
Acquisition function convergence reached at iteration 86.
The final EI loss was -0.399 with predicted mean of [-0.624]
The next parameters to simulate from are [[0.569 0.907 0.026 0.056 0.014 0.019]]
The mean of the samples was -0.603
Iteration 148
Acquisition function convergence reached at iteration 126.
```

```
The final EI loss was -0.399 with predicted mean of [-0.626]
The next parameters to simulate from are [[0.067 0.803 0.024 0.058 0.01  0.021]]
The mean of the samples was -0.455
Iteration 149
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.899]
The next parameters to simulate from are [[0.138 0.269 0.027 0.099 0.008 0.017]]
The mean of the samples was 0.842
Iteration 150
Acquisition function convergence reached at iteration 92.
The final EI loss was -0.399 with predicted mean of [-0.624]
The next parameters to simulate from are [[0.215 0.202 0.011 0.061 0.007 0.022]]
The mean of the samples was -0.544
Trained parameters:
amplitude_champ:0 is 0.7

length_scales_champ:0 is [0.27  0.5   0.017 0.024 0.036 0.012]

observation_noise_variance_champ:0 is 0.081

bias_mean:0 is 0.376

Iteration 151
Acquisition function convergence reached at iteration 144.
The final EI loss was -0.399 with predicted mean of [-0.624]
The next parameters to simulate from are [[0.364 0.284 0.023 0.066 0.007 0.026]]
The mean of the samples was -0.569
Iteration 152
Acquisition function convergence reached at iteration 132.
The final EI loss was -0.399 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.547 0.162 0.018 0.062 0.009 0.028]]
The mean of the samples was -0.662
Iteration 153
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.826]
The next parameters to simulate from are [[0.207 0.504 0.027 0.092 0.008 0.017]]
The mean of the samples was 0.71
Iteration 154
Acquisition function convergence reached at iteration 300.
The final EI loss was -0.399 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.36  0.293 0.022 0.034 0.008 0.025]]
The mean of the samples was -0.513
Iteration 155
```

```
Acquisition function convergence reached at iteration 120.
The final EI loss was -0.399 with predicted mean of [-0.624]
The next parameters to simulate from are [[0.642 0.818 0.029 0.058 0.012 0.018]]
The mean of the samples was -0.672
Iteration 156
Acquisition function convergence reached at iteration 120.
The final EI loss was -0.399 with predicted mean of [-0.624]
The next parameters to simulate from are [[0.75  0.391 0.022 0.039 0.007 0.009]]
The mean of the samples was -0.62
Iteration 157
Acquisition function convergence reached at iteration 190.
The final EI loss was -0.399 with predicted mean of [-0.624]
The next parameters to simulate from are [[0.784 0.422 0.02  0.037 0.02  0.009]]
The mean of the samples was -0.635
Iteration 158
Acquisition function convergence reached at iteration 179.
The final EI loss was -0.399 with predicted mean of [-0.625]
The next parameters to simulate from are [[0.777 0.499 0.022 0.037 0.017 0.01 ]]
The mean of the samples was -0.669
Iteration 159
Acquisition function convergence reached at iteration 160.
The final EI loss was -0.399 with predicted mean of [-0.624]
The next parameters to simulate from are [[0.601 0.057 0.023 0.06  0.016 0.03 ]]
The mean of the samples was -0.743
Iteration 160
Acquisition function convergence reached at iteration 41.
The final EI loss was -0.003 with predicted mean of [-1.254]
The next parameters to simulate from are [[0.567 0.094 0.025 0.058 0.008 0.017]]
The mean of the samples was -1.499
Hyperparameter convergence reached at iteration 994.
The minimum predicted mean of the observed indices is -1.303 at the point
[0.567 0.094 0.025 0.058 0.008 0.017]
Iteration 161
Acquisition function convergence reached at iteration 75.
The final EI loss was -0.399 with predicted mean of [-0.653]
The next parameters to simulate from are [[0.23  0.244 0.006 0.061 0.009 0.016]]
The mean of the samples was -0.506
Iteration 162
Acquisition function convergence reached at iteration 103.
The final EI loss was -0.399 with predicted mean of [-0.651]
The next parameters to simulate from are [[0.78  0.192 0.016 0.063 0.011 0.018]]
The mean of the samples was -0.66
Iteration 163
```

```
Acquisition function convergence reached at iteration 142.
The final EI loss was -0.399 with predicted mean of [-0.65]
The next parameters to simulate from are [[0.562 0.06  0.02  0.06  0.007 0.032]]
The mean of the samples was -0.646
Iteration 164
Acquisition function convergence reached at iteration 98.
The final EI loss was -0.399 with predicted mean of [-0.65]
The next parameters to simulate from are [[0.519 0.11  0.019 0.063 0.006 0.031]]
The mean of the samples was -0.74
Iteration 165
Acquisition function convergence reached at iteration 98.
The final EI loss was -0.399 with predicted mean of [-0.649]
The next parameters to simulate from are [[0.795 0.168 0.027 0.062 0.012 0.019]]
The mean of the samples was -0.711
Iteration 166
Acquisition function convergence reached at iteration 120.
The final EI loss was -0.399 with predicted mean of [-0.65]
The next parameters to simulate from are [[0.394 0.243 0.022 0.061 0.011 0.028]]
The mean of the samples was -0.577
Iteration 167
Acquisition function convergence reached at iteration 122.
The final EI loss was -0.399 with predicted mean of [-0.65]
The next parameters to simulate from are [[0.564 0.151 0.025 0.058 0.01  0.032]]
The mean of the samples was -0.639
Iteration 168
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.399 with predicted mean of [-0.657]
The next parameters to simulate from are [[0.306 0.232 0.021 0.059 0.01  0.027]]
The mean of the samples was -0.708
Iteration 169
Acquisition function convergence reached at iteration 96.
The final EI loss was -0.399 with predicted mean of [-0.653]
The next parameters to simulate from are [[0.556 0.862 0.022 0.056 0.011 0.019]]
The mean of the samples was -0.683
Iteration 170
Acquisition function convergence reached at iteration 75.
The final EI loss was -0.002 with predicted mean of [-1.313]
The next parameters to simulate from are [[0.567 0.086 0.024 0.058 0.008 0.017]]
The mean of the samples was -1.276
Iteration 171
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.654]
The next parameters to simulate from are [[0.696 0.173 0.029 0.031 0.067 0.02 ]]
```

```
The mean of the samples was -0.588
Iteration 172
Acquisition function convergence reached at iteration 80.
The final EI loss was -0.397 with predicted mean of [-0.652]
The next parameters to simulate from are [[0.401 0.368 0.021 0.042 0.007 0.011]]
The mean of the samples was -0.743
Iteration 173
Acquisition function convergence reached at iteration 61.
The final EI loss was -0.0 with predicted mean of [-1.307]
The next parameters to simulate from are [[0.567 0.093 0.024 0.058 0.008 0.017]]
The mean of the samples was -1.203
Iteration 174
Acquisition function convergence reached at iteration 99.
The final EI loss was -0.399 with predicted mean of [-0.647]
The next parameters to simulate from are [[0.21  0.18  0.022 0.057 0.012 0.027]]
The mean of the samples was -0.599
Iteration 175
Acquisition function convergence reached at iteration 84.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.335 0.213 0.018 0.034 0.014 0.025]]
The mean of the samples was -0.573
Iteration 176
Acquisition function convergence reached at iteration 163.
The final EI loss was -0.399 with predicted mean of [-0.647]
The next parameters to simulate from are [[0.562 0.05  0.019 0.061 0.003 0.033]]
The mean of the samples was -0.657
Iteration 177
Acquisition function convergence reached at iteration 8.
The final EI loss was 0.0 with predicted mean of [0.37]
The next parameters to simulate from are [[0.891 0.428 0.017 0.009 0.014 0.019]]
The mean of the samples was 0.402
Iteration 178
Acquisition function convergence reached at iteration 82.
The final EI loss was -0.399 with predicted mean of [-0.647]
The next parameters to simulate from are [[0.153 0.131 0.01  0.057 0.007 0.019]]
The mean of the samples was -0.772
Iteration 179
Acquisition function convergence reached at iteration 89.
The final EI loss was -0.398 with predicted mean of [-0.644]
The next parameters to simulate from are [[0.47  0.087 0.01  0.071 0.009 0.023]]
The mean of the samples was -0.571
Iteration 180
Acquisition function convergence reached at iteration 96.
```

```
The final EI loss was -0.399 with predicted mean of [-0.645]
The next parameters to simulate from are [[0.661 0.67  0.026 0.061 0.006 0.02 ]]
The mean of the samples was -0.605
Hyperparameter convergence reached at iteration 1019.
The minimum predicted mean of the observed indices is -1.291 at the point
[0.567 0.093 0.024 0.058 0.008 0.017]
Iteration 181
Acquisition function convergence reached at iteration 66.
The final EI loss was -0.392 with predicted mean of [-0.642]
The next parameters to simulate from are [[0.637 0.43  0.001 0.07  0.01  0.016]]
The mean of the samples was -0.329
Iteration 182
Acquisition function convergence reached at iteration 178.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.503 0.134 0.013 0.061 0.026 0.023]]
The mean of the samples was -0.683
Iteration 183
Acquisition function convergence reached at iteration 12.
The final EI loss was 0.0 with predicted mean of [0.893]
The next parameters to simulate from are [[0.171 0.826 0.013 0.076 0.066 0.059]]
The mean of the samples was 0.964
Iteration 184
Acquisition function convergence reached at iteration 7.
The final EI loss was 0.0 with predicted mean of [0.548]
The next parameters to simulate from are [[0.13  0.145 0.027 0.078 0.064 0.011]]
The mean of the samples was 0.629
Iteration 185
Acquisition function convergence reached at iteration 169.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.453 0.187 0.014 0.071 0.008 0.021]]
The mean of the samples was -0.68
Iteration 186
Acquisition function convergence reached at iteration 97.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.356 0.261 0.02  0.033 0.068 0.026]]
The mean of the samples was -0.61
Iteration 187
Acquisition function convergence reached at iteration 459.
The final EI loss was -0.399 with predicted mean of [-0.647]
The next parameters to simulate from are [[0.338 0.207 0.027 0.066 0.008 0.024]]
The mean of the samples was -0.737
Iteration 188
Acquisition function convergence reached at iteration 187.
```

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The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.564 0.117 0.023 0.056 0.03  0.027]]
The mean of the samples was -0.757
Iteration 189
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.645]
The next parameters to simulate from are [[0.358 0.46  0.024 0.062 0.018 0.023]]
The mean of the samples was -0.458
Iteration 190
Acquisition function convergence reached at iteration 183.
The final EI loss was -0.399 with predicted mean of [-0.647]
The next parameters to simulate from are [[0.354 0.24  0.015 0.035 0.014 0.025]]
The mean of the samples was -0.599
Iteration 191
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.256 0.13  0.022 0.061 0.012 0.018]]
The mean of the samples was -0.713
Iteration 192
Acquisition function convergence reached at iteration 91.
The final EI loss was -0.399 with predicted mean of [-0.645]
The next parameters to simulate from are [[0.674 0.192 0.029 0.031 0.048 0.017]]
The mean of the samples was -0.524
Iteration 193
Acquisition function convergence reached at iteration 79.
The final EI loss was -0.399 with predicted mean of [-0.644]
The next parameters to simulate from are [[0.087 0.34  0.017 0.058 0.004 0.02 ]]
The mean of the samples was -0.654
Iteration 194
Acquisition function convergence reached at iteration 133.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.576 0.773 0.024 0.058 0.002 0.022]]
The mean of the samples was -0.692
Iteration 195
Acquisition function convergence reached at iteration 5.
The final EI loss was 0.0 with predicted mean of [0.493]
The next parameters to simulate from are [[0.485 0.733 0.02  0.003 0.011 0.027]]
The mean of the samples was 0.352
Iteration 196
Acquisition function convergence reached at iteration 11.
The final EI loss was 0.0 with predicted mean of [0.395]
The next parameters to simulate from are [[0.905 0.466 0.012 0.006 0.029 0.019]]
The mean of the samples was 0.441
```

```
Iteration 197
Acquisition function convergence reached at iteration 189.
The final EI loss was -0.399 with predicted mean of [-0.645]
The next parameters to simulate from are [[0.671 0.251 0.029 0.06  0.063 0.018]]
The mean of the samples was -0.588
Iteration 198
Acquisition function convergence reached at iteration 171.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.549 0.018 0.018 0.06  0.004 0.034]]
The mean of the samples was -0.536
Iteration 199
Acquisition function convergence reached at iteration 206.
The final EI loss was -0.399 with predicted mean of [-0.645]
The next parameters to simulate from are [[0.583 0.767 0.029 0.061 0.01  0.019]]
The mean of the samples was -0.732
Iteration 200
Acquisition function convergence reached at iteration 157.
The final EI loss was -0.399 with predicted mean of [-0.648]
The next parameters to simulate from are [[0.239 0.11  0.021 0.056 0.013 0.027]]
The mean of the samples was -0.687
Hyperparameter convergence reached at iteration 1045.
The minimum predicted mean of the observed indices is -1.293 at the point
[0.567 0.093 0.024 0.058 0.008 0.017]
Trained parameters:
amplitude_champ:0 is 0.682

length_scales_champ:0 is [0.285 0.5   0.017 0.023 0.036 0.012]

observation_noise_variance_champ:0 is 0.081

bias_mean:0 is 0.376

Iteration 201
Acquisition function convergence reached at iteration 184.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.638 0.584 0.028 0.055 0.056 0.013]]
The mean of the samples was -0.729
Iteration 202
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.572 0.728 0.024 0.057 0.022 0.018]]
The mean of the samples was -0.608
Iteration 203
```

```
Acquisition function convergence reached at iteration 103.
The final EI loss was -0.399 with predicted mean of [-0.646]
The next parameters to simulate from are [[0.491 0.091 0.01  0.058 0.029 0.023]]
The mean of the samples was -0.704
Iteration 204
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.647]
The next parameters to simulate from are [[0.131 0.155 0.028 0.034 0.053 0.022]]
The mean of the samples was -0.643
Iteration 205
Acquisition function convergence reached at iteration 63.
The final EI loss was -0.001 with predicted mean of [-1.295]
The next parameters to simulate from are [[0.572 0.085 0.025 0.058 0.009 0.017]]
The mean of the samples was -1.513
Iteration 206
Acquisition function convergence reached at iteration 3.
The final EI loss was 0.0 with predicted mean of [0.446]
The next parameters to simulate from are [[0.184 0.116 0.024 0.081 0.016 0.016]]
The mean of the samples was 0.339
Iteration 207
Acquisition function convergence reached at iteration 148.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.746 0.338 0.02  0.04  0.011 0.009]]
The mean of the samples was -0.77
Iteration 208
Acquisition function convergence reached at iteration 99.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.667 0.156 0.03  0.077 0.034 0.013]]
The mean of the samples was -0.571
Iteration 209
Acquisition function convergence reached at iteration 112.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.599 0.161 0.023 0.059 0.013 0.007]]
The mean of the samples was -0.548
Iteration 210
Acquisition function convergence reached at iteration 107.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.313 0.151 0.019 0.034 0.015 0.027]]
The mean of the samples was -0.604
Iteration 211
Acquisition function convergence reached at iteration 250.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.622 0.121 0.029 0.075 0.031 0.011]]
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The mean of the samples was -0.674
Iteration 212
Acquisition function convergence reached at iteration 149.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.555 0.821 0.024 0.051 0.013 0.022]]
The mean of the samples was -0.702
Iteration 213
Acquisition function convergence reached at iteration 153.
The final EI loss was -0.042 with predicted mean of [-0.095]
The next parameters to simulate from are [[0.844 0.003 0.033 0.061 0.001 0.071]]
The mean of the samples was 0.173
Iteration 214
Acquisition function convergence reached at iteration 105.
The final EI loss was -0.399 with predicted mean of [-0.655]
The next parameters to simulate from are [[0.401 0.351 0.021 0.043 0.026 0.011]]
The mean of the samples was -0.654
Iteration 215
Acquisition function convergence reached at iteration 112.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.259 0.12  0.026 0.057 0.011 0.029]]
The mean of the samples was -0.764
Iteration 216
Acquisition function convergence reached at iteration 114.
The final EI loss was -0.399 with predicted mean of [-0.655]
The next parameters to simulate from are [[0.804 0.511 0.022 0.038 0.023 0.009]]
The mean of the samples was -0.668
Iteration 217
Acquisition function convergence reached at iteration 91.
The final EI loss was -0.398 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.694 0.257 0.028 0.033 0.062 0.01 ]]
The mean of the samples was -0.671
Iteration 218
Acquisition function convergence reached at iteration 4.
The final EI loss was 0.0 with predicted mean of [0.475]
The next parameters to simulate from are [[0.504 0.088 0.024 0.087 0.046 0.06 ]]
The mean of the samples was 0.514
Iteration 219
Acquisition function convergence reached at iteration 184.
The final EI loss was -0.399 with predicted mean of [-0.656]
The next parameters to simulate from are [[0.338 0.108 0.025 0.06  0.02  0.024]]
The mean of the samples was -0.732
Iteration 220
Acquisition function convergence reached at iteration 134.
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The final EI loss was -0.399 with predicted mean of [-0.655]
The next parameters to simulate from are [[0.04  0.213 0.031 0.042 0.039 0.031]]
The mean of the samples was -0.689
Hyperparameter convergence reached at iteration 1008.
The minimum predicted mean of the observed indices is -1.311 at the point
[0.572 0.1   0.025 0.058 0.009 0.017]
Iteration 221
Acquisition function convergence reached at iteration 75.
The final EI loss was -0.399 with predicted mean of [-0.654]
The next parameters to simulate from are [[0.889 0.049 0.005 0.092 0.016 0.01 ]]
The mean of the samples was -0.617
Iteration 222
Acquisition function convergence reached at iteration 56.
The final EI loss was -0.0 with predicted mean of [-1.311]
The next parameters to simulate from are [[0.57  0.111 0.026 0.057 0.009 0.017]]
The mean of the samples was -1.386
Iteration 223
Acquisition function convergence reached at iteration 145.
The final EI loss was -0.399 with predicted mean of [-0.659]
The next parameters to simulate from are [[0.537 0.09  0.025 0.06  0.    0.033]]
The mean of the samples was -0.69
Iteration 224
Acquisition function convergence reached at iteration 103.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.61  0.696 0.027 0.066 0.009 0.018]]
The mean of the samples was -0.645
Iteration 225
Acquisition function convergence reached at iteration 139.
The final EI loss was -0.398 with predicted mean of [-0.659]
The next parameters to simulate from are [[0.835 0.16  0.021 0.092 0.016 0.011]]
The mean of the samples was -0.637
Iteration 226
Acquisition function convergence reached at iteration 87.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.545 0.473 0.025 0.055 0.011 0.028]]
The mean of the samples was -0.652
Iteration 227
Acquisition function convergence reached at iteration 137.
The final EI loss was -0.399 with predicted mean of [-0.659]
The next parameters to simulate from are [[0.511 0.001 0.023 0.059 0.005 0.034]]
The mean of the samples was -0.6
Iteration 228
Acquisition function convergence reached at iteration 82.
```

```
The final EI loss was -0.398 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.615 0.091 0.027 0.053 0.034 0.03 ]]
The mean of the samples was -0.627
Iteration 229
Acquisition function convergence reached at iteration 112.
The final EI loss was -0.399 with predicted mean of [-0.659]
The next parameters to simulate from are [[0.638 0.723 0.023 0.056 0.012 0.006]]
The mean of the samples was -0.675
Iteration 230
Acquisition function convergence reached at iteration 137.
The final EI loss was -0.399 with predicted mean of [-0.657]
The next parameters to simulate from are [[0.865 0.188 0.022 0.06  0.012 0.013]]
The mean of the samples was -0.606
Iteration 231
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.659]
The next parameters to simulate from are [[0.623 0.191 0.026 0.052 0.033 0.028]]
The mean of the samples was -0.609
Iteration 232
Acquisition function convergence reached at iteration 136.
The final EI loss was -0.399 with predicted mean of [-0.659]
The next parameters to simulate from are [[0.427 0.029 0.017 0.061 0.001 0.033]]
The mean of the samples was -0.692
Iteration 233
Acquisition function convergence reached at iteration 62.
The final EI loss was -0.397 with predicted mean of [-0.655]
The next parameters to simulate from are [[0.788 0.094 0.027 0.054 0.021 0.02 ]]
The mean of the samples was -0.643
Iteration 234
Acquisition function convergence reached at iteration 65.
The final EI loss was -0.0 with predicted mean of [-1.317]
The next parameters to simulate from are [[0.574 0.112 0.025 0.057 0.009 0.017]]
The mean of the samples was -1.56
Iteration 235
Acquisition function convergence reached at iteration 151.
The final EI loss was -0.399 with predicted mean of [-0.665]
The next parameters to simulate from are [[0.61  0.159 0.021 0.059 0.008 0.007]]
The mean of the samples was -0.661
Iteration 236
Acquisition function convergence reached at iteration 77.
The final EI loss was -0.398 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.301 0.224 0.006 0.061 0.01  0.014]]
The mean of the samples was -0.837
```

```
Iteration 237
Acquisition function convergence reached at iteration 99.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.859 0.029 0.005 0.093 0.015 0.011]]
The mean of the samples was -0.702
Iteration 238
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.398 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.688 0.624 0.029 0.031 0.061 0.013]]
The mean of the samples was -0.608
Iteration 239
Acquisition function convergence reached at iteration 3.
The final EI loss was 0.0 with predicted mean of [0.427]
The next parameters to simulate from are [[0.154 0.21  0.026 0.074 0.057 0.014]]
The mean of the samples was 0.167
Iteration 240
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.647 0.427 0.003 0.069 0.007 0.014]]
The mean of the samples was -0.641
Hyperparameter convergence reached at iteration 1049.
The minimum predicted mean of the observed indices is -1.336 at the point
[0.574 0.112 0.025 0.057 0.009 0.017]
Iteration 241
Acquisition function convergence reached at iteration 90.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.89  0.073 0.007 0.089 0.017 0.011]]
The mean of the samples was -0.585
Iteration 242
Acquisition function convergence reached at iteration 197.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.45  0.134 0.002 0.064 0.015 0.021]]
The mean of the samples was -0.72
Iteration 243
Acquisition function convergence reached at iteration 292.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.679 0.134 0.012 0.062 0.011 0.024]]
The mean of the samples was -0.688
Iteration 244
Acquisition function convergence reached at iteration 8.
The final EI loss was 0.0 with predicted mean of [0.314]
The next parameters to simulate from are [[0.623 0.679 0.025 0.093 0.026 0.02 ]]
The mean of the samples was 0.289
```

```
Iteration 245
Acquisition function convergence reached at iteration 89.
The final EI loss was -0.398 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.626 0.789 0.029 0.057 0.013 0.006]]
The mean of the samples was -0.714
Iteration 246
Acquisition function convergence reached at iteration 2.
The final EI loss was -0.0 with predicted mean of [0.033]
The next parameters to simulate from are [[0.972 0.111 0.03  0.058 0.009 0.016]]
The mean of the samples was 0.173
Iteration 247
Acquisition function convergence reached at iteration 128.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.482 0.004 0.023 0.058 0.002 0.034]]
The mean of the samples was -0.683
Iteration 248
Acquisition function convergence reached at iteration 92.
The final EI loss was -0.398 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.18  0.484 0.022 0.032 0.009 0.02 ]]
The mean of the samples was -0.533
Iteration 249
Acquisition function convergence reached at iteration 114.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.274 0.412 0.02  0.034 0.011 0.023]]
The mean of the samples was -0.759
Iteration 250
Acquisition function convergence reached at iteration 5.
The final EI loss was 0.0 with predicted mean of [0.386]
The next parameters to simulate from are [[0.284 0.374 0.023 0.004 0.013 0.022]]
The mean of the samples was 0.247
Trained parameters:
amplitude_champ:0 is 0.662

length_scales_champ:0 is [0.254 0.5   0.017 0.025 0.036 0.011]

observation_noise_variance_champ:0 is 0.082

bias_mean:0 is 0.379

Iteration 251
Acquisition function convergence reached at iteration 131.
The final EI loss was -0.012 with predicted mean of [0.048]
The next parameters to simulate from are [[0.997 0.002 0.033 0.043 0.071 0.071]]
```

```
The mean of the samples was 0.457
Iteration 252
Acquisition function convergence reached at iteration 82.
The final EI loss was -0.399 with predicted mean of [-0.665]
The next parameters to simulate from are [[0.83  0.115 0.028 0.055 0.021 0.016]]
The mean of the samples was -0.681
Iteration 253
Acquisition function convergence reached at iteration 78.
The final EI loss was -0.398 with predicted mean of [-0.663]
The next parameters to simulate from are [[0.615 0.615 0.016 0.063 0.013 0.018]]
The mean of the samples was -0.536
Iteration 254
Acquisition function convergence reached at iteration 102.
The final EI loss was -0.399 with predicted mean of [-0.665]
The next parameters to simulate from are [[0.713 0.202 0.03  0.035 0.051 0.02 ]]
The mean of the samples was -0.763
Iteration 255
Acquisition function convergence reached at iteration 13.
The final EI loss was 0.0 with predicted mean of [0.156]
The next parameters to simulate from are [[0.89  0.209 0.015 0.026 0.015 0.016]]
The mean of the samples was 0.136
Iteration 256
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.39  0.275 0.022 0.038 0.021 0.013]]
The mean of the samples was -0.661
Iteration 257
Acquisition function convergence reached at iteration 113.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.18  0.178 0.03  0.034 0.054 0.023]]
The mean of the samples was -0.716
Iteration 258
Acquisition function convergence reached at iteration 101.
The final EI loss was -0.398 with predicted mean of [-0.665]
The next parameters to simulate from are [[0.788 0.144 0.026 0.082 0.011 0.017]]
The mean of the samples was -0.485
Iteration 259
Acquisition function convergence reached at iteration 105.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.799 0.148 0.023 0.089 0.018 0.012]]
The mean of the samples was -0.664
Iteration 260
Acquisition function convergence reached at iteration 134.
```

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The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.298 0.11  0.028 0.064 0.005 0.025]]
The mean of the samples was -0.753
Hyperparameter convergence reached at iteration 891.
The minimum predicted mean of the observed indices is -1.336 at the point
[0.567 0.086 0.024 0.058 0.008 0.017]
Iteration 261
Acquisition function convergence reached at iteration 97.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.128 0.114 0.028 0.043 0.042 0.029]]
The mean of the samples was -0.709
Iteration 262
Acquisition function convergence reached at iteration 70.
The final EI loss was -0.398 with predicted mean of [-0.673]
The next parameters to simulate from are [[0.547 0.111 0.016 0.063 0.007 0.008]]
The mean of the samples was -0.671
Iteration 263
Acquisition function convergence reached at iteration 170.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.579 0.986 0.021 0.059 0.011 0.016]]
The mean of the samples was -0.671
Iteration 264
Acquisition function convergence reached at iteration 186.
The final EI loss was -0.014 with predicted mean of [0.041]
The next parameters to simulate from are [[0.996 0.002 0.033 0.1   0.071 0.046]]
The mean of the samples was 0.32
Iteration 265
Acquisition function convergence reached at iteration 130.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.741 0.214 0.029 0.034 0.054 0.019]]
The mean of the samples was -0.629
Iteration 266
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.886]
The next parameters to simulate from are [[0.048 0.278 0.024 0.09  0.025 0.017]]
The mean of the samples was 0.879
Iteration 267
Acquisition function convergence reached at iteration 85.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.283 0.094 0.021 0.056 0.007 0.031]]
The mean of the samples was -0.691
Iteration 268
Acquisition function convergence reached at iteration 119.
```

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The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.274 0.082 0.023 0.055 0.012 0.031]]
The mean of the samples was -0.637
Iteration 269
Acquisition function convergence reached at iteration 48.
The final EI loss was -0.004 with predicted mean of [-1.343]
The next parameters to simulate from are [[0.585 0.074 0.023 0.058 0.009 0.017]]
The mean of the samples was -1.261
Iteration 270
Acquisition function convergence reached at iteration 96.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.284 0.034 0.019 0.056 0.003 0.032]]
The mean of the samples was -0.836
Iteration 271
Acquisition function convergence reached at iteration 8.
The final EI loss was 0.0 with predicted mean of [0.46]
The next parameters to simulate from are [[0.62  0.368 0.029 0.094 0.063 0.021]]
The mean of the samples was 0.458
Iteration 272
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.63  0.051 0.027 0.049 0.033 0.029]]
The mean of the samples was -0.727
Iteration 273
Acquisition function convergence reached at iteration 75.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.589 0.002 0.025 0.06  0.015 0.008]]
The mean of the samples was -0.634
Iteration 274
Acquisition function convergence reached at iteration 164.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.574 0.797 0.025 0.048 0.017 0.021]]
The mean of the samples was -0.67
Iteration 275
Acquisition function convergence reached at iteration 100.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.581 0.082 0.025 0.063 0.069 0.013]]
The mean of the samples was -0.712
Iteration 276
Acquisition function convergence reached at iteration 123.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.874 0.097 0.009 0.086 0.015 0.011]]
The mean of the samples was -0.723
```

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Iteration 277
Acquisition function convergence reached at iteration 87.
The final EI loss was -0.398 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.584 0.118 0.018 0.062 0.055 0.011]]
The mean of the samples was -0.635
Iteration 278
Acquisition function convergence reached at iteration 112.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.757 0.112 0.025 0.089 0.014 0.014]]
The mean of the samples was -0.674
Iteration 279
Acquisition function convergence reached at iteration 80.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.13  0.744 0.025 0.057 0.016 0.021]]
The mean of the samples was -0.689
Iteration 280
Acquisition function convergence reached at iteration 96.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.641 0.762 0.025 0.056 0.016 0.006]]
The mean of the samples was -0.641
Hyperparameter convergence reached at iteration 1006.
The minimum predicted mean of the observed indices is -1.333 at the point
[0.585 0.074 0.023 0.058 0.009 0.017]
Iteration 281
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.648 0.145 0.03  0.031 0.054 0.018]]
The mean of the samples was -0.672
Iteration 282
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.044 0.069 0.025 0.055 0.007 0.019]]
The mean of the samples was -0.683
Iteration 283
Acquisition function convergence reached at iteration 59.
The final EI loss was -0.399 with predicted mean of [-0.665]
The next parameters to simulate from are [[0.779 0.322 0.023 0.037 0.065 0.009]]
The mean of the samples was -0.855
Iteration 284
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.819 0.178 0.021 0.091 0.013 0.011]]
The mean of the samples was -0.679
```

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Iteration 285
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.222 0.13  0.027 0.057 0.02  0.025]]
The mean of the samples was -0.73
Iteration 286
Acquisition function convergence reached at iteration 179.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.424 0.102 0.016 0.07  0.014 0.019]]
The mean of the samples was -0.593
Iteration 287
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.313 0.811 0.021 0.035 0.023 0.027]]
The mean of the samples was -0.801
Iteration 288
Acquisition function convergence reached at iteration 122.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.588 0.632 0.024 0.067 0.008 0.017]]
The mean of the samples was -0.67
Iteration 289
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.416 0.29  0.021 0.036 0.015 0.027]]
The mean of the samples was -0.721
Iteration 290
Acquisition function convergence reached at iteration 155.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.496 0.102 0.003 0.065 0.014 0.022]]
The mean of the samples was -0.654
Iteration 291
Acquisition function convergence reached at iteration 3.
The final EI loss was 0.0 with predicted mean of [0.376]
The next parameters to simulate from are [[0.725 0.497 0.023 0.004 0.01  0.019]]
The mean of the samples was 0.337
Iteration 292
Acquisition function convergence reached at iteration 5.
The final EI loss was 0.0 with predicted mean of [0.359]
The next parameters to simulate from are [[0.667 0.652 0.022 0.004 0.013 0.022]]
The mean of the samples was 0.337
Iteration 293
Acquisition function convergence reached at iteration 86.
The final EI loss was -0.399 with predicted mean of [-0.667]
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The next parameters to simulate from are [[0.333 0.04  0.021 0.033 0.022 0.027]]
The mean of the samples was -0.818
Iteration 294
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.665]
The next parameters to simulate from are [[0.086 0.972 0.018 0.054 0.013 0.022]]
The mean of the samples was -0.797
Iteration 295
Acquisition function convergence reached at iteration 6.
The final EI loss was 0.0 with predicted mean of [0.592]
The next parameters to simulate from are [[0.091 0.377 0.021 0.068 0.041 0.065]]
The mean of the samples was 0.634
Iteration 296
Acquisition function convergence reached at iteration 144.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.573 0.098 0.028 0.058 0.014 0.032]]
The mean of the samples was -0.59
Iteration 297
Acquisition function convergence reached at iteration 106.
The final EI loss was -0.399 with predicted mean of [-0.665]
The next parameters to simulate from are [[0.22  0.083 0.023 0.052 0.014 0.031]]
The mean of the samples was -0.744
Iteration 298
Acquisition function convergence reached at iteration 109.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.324 0.058 0.019 0.035 0.013 0.027]]
The mean of the samples was -0.709
Iteration 299
Acquisition function convergence reached at iteration 93.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.695 0.574 0.019 0.059 0.008 0.018]]
The mean of the samples was -0.636
Iteration 300
Acquisition function convergence reached at iteration 3.
The final EI loss was 0.0 with predicted mean of [0.158]
The next parameters to simulate from are [[0.634 0.642 0.024 0.099 0.009 0.02 ]]
The mean of the samples was 0.296
Hyperparameter convergence reached at iteration 1089.
The minimum predicted mean of the observed indices is -1.332 at the point
[0.585 0.074 0.023 0.058 0.009 0.017]
Trained parameters:
amplitude_champ:0 is 0.667
```

```
length_scales_champ:0 is [0.237 0.5   0.017 0.025 0.036 0.011]

observation_noise_variance_champ:0 is 0.08

bias_mean:0 is 0.395

Iteration 301
Acquisition function convergence reached at iteration 112.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.521 0.217 0.02  0.046 0.026 0.027]]
The mean of the samples was -0.629
Iteration 302
Acquisition function convergence reached at iteration 102.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.457 0.174 0.019 0.037 0.028 0.028]]
The mean of the samples was -0.633
Iteration 303
Acquisition function convergence reached at iteration 102.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.254 0.094 0.026 0.055 0.012 0.031]]
The mean of the samples was -0.532
Iteration 304
Acquisition function convergence reached at iteration 131.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.331 0.03  0.02  0.03  0.021 0.025]]
The mean of the samples was -0.545
Iteration 305
Acquisition function convergence reached at iteration 105.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.734 0.489 0.023 0.036 0.014 0.011]]
The mean of the samples was -0.669
Iteration 306
Acquisition function convergence reached at iteration 110.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.541 0.187 0.026 0.061 0.001 0.032]]
The mean of the samples was -0.641
Iteration 307
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.328 0.127 0.02  0.057 0.007 0.033]]
The mean of the samples was -0.624
Iteration 308
Acquisition function convergence reached at iteration 96.
```

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The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.326 0.15  0.023 0.035 0.009 0.027]]
The mean of the samples was -0.645
Iteration 309
Acquisition function convergence reached at iteration 76.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.437 0.058 0.021 0.056 0.007 0.035]]
The mean of the samples was -0.708
Iteration 310
Acquisition function convergence reached at iteration 8.
The final EI loss was 0.0 with predicted mean of [0.347]
The next parameters to simulate from are [[0.988 0.13  0.021 0.042 0.04  0.024]]
The mean of the samples was 0.344
Iteration 311
Acquisition function convergence reached at iteration 119.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.052 0.181 0.017 0.043 0.023 0.028]]
The mean of the samples was -0.629
Iteration 312
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.389 0.02  0.015 0.058 0.005 0.034]]
The mean of the samples was -0.68
Iteration 313
Acquisition function convergence reached at iteration 114.
The final EI loss was -0.398 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.251 0.144 0.01  0.06  0.001 0.027]]
The mean of the samples was -0.782
Iteration 314
Acquisition function convergence reached at iteration 130.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.581 0.217 0.024 0.063 0.001 0.03 ]]
The mean of the samples was -0.648
Iteration 315
Acquisition function convergence reached at iteration 90.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.255 0.403 0.02  0.062 0.009 0.018]]
The mean of the samples was -0.671
Iteration 316
Acquisition function convergence reached at iteration 171.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.638 0.946 0.027 0.06  0.008 0.016]]
The mean of the samples was -0.673
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Iteration 317
Acquisition function convergence reached at iteration 63.
The final EI loss was -0.396 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.421 0.262 0.022 0.041 0.06  0.013]]
The mean of the samples was -0.698
Iteration 318
Acquisition function convergence reached at iteration 117.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.821 0.095 0.029 0.089 0.012 0.012]]
The mean of the samples was -0.764
Iteration 319
Acquisition function convergence reached at iteration 61.
The final EI loss was -0.0 with predicted mean of [-1.333]
The next parameters to simulate from are [[0.584 0.082 0.024 0.058 0.009 0.017]]
The mean of the samples was -1.11
Iteration 320
Acquisition function convergence reached at iteration 123.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.738 0.433 0.021 0.04  0.014 0.007]]
The mean of the samples was -0.735
Hyperparameter convergence reached at iteration 1228.
The minimum predicted mean of the observed indices is -1.316 at the point
[0.574 0.112 0.025 0.057 0.009 0.017]
Iteration 321
Acquisition function convergence reached at iteration 167.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.091 0.764 0.023 0.054 0.024 0.021]]
The mean of the samples was -0.623
Iteration 322
Acquisition function convergence reached at iteration 214.
The final EI loss was -0.399 with predicted mean of [-0.661]
The next parameters to simulate from are [[0.78  0.415 0.017 0.042 0.005 0.009]]
The mean of the samples was -0.681
Iteration 323
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.686 0.254 0.029 0.033 0.044 0.016]]
The mean of the samples was -0.757
Iteration 324
Acquisition function convergence reached at iteration 145.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.549 0.011 0.026 0.06  0.    0.033]]
The mean of the samples was -0.688
```

```
Iteration 325
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.535 0.078 0.029 0.062 0.007 0.033]]
The mean of the samples was -0.649
Iteration 326
Acquisition function convergence reached at iteration 229.
The final EI loss was -0.399 with predicted mean of [-0.659]
The next parameters to simulate from are [[0.794 0.158 0.026 0.079 0.006 0.016]]
The mean of the samples was -0.765
Iteration 327
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.307]
The next parameters to simulate from are [[0.211 0.115 0.025 0.078 0.04  0.016]]
The mean of the samples was 0.388
Iteration 328
Acquisition function convergence reached at iteration 123.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.06  0.231 0.031 0.04  0.049 0.03 ]]
The mean of the samples was -0.736
Iteration 329
Acquisition function convergence reached at iteration 173.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.255 0.123 0.027 0.057 0.024 0.017]]
The mean of the samples was -0.753
Iteration 330
Acquisition function convergence reached at iteration 73.
The final EI loss was -0.399 with predicted mean of [-0.657]
The next parameters to simulate from are [[0.739 0.257 0.029 0.032 0.068 0.018]]
The mean of the samples was -0.635
Iteration 331
Acquisition function convergence reached at iteration 153.
The final EI loss was -0.399 with predicted mean of [-0.658]
The next parameters to simulate from are [[0.611 0.112 0.026 0.061 0.022 0.008]]
The mean of the samples was -0.544
Iteration 332
Acquisition function convergence reached at iteration 55.
The final EI loss was -0.001 with predicted mean of [-1.319]
The next parameters to simulate from are [[0.579 0.101 0.025 0.058 0.008 0.017]]
The mean of the samples was -1.388
Iteration 333
Acquisition function convergence reached at iteration 6.
The final EI loss was -0.0 with predicted mean of [-0.161]
```

```
The next parameters to simulate from are [[0.658 0.93  0.021 0.065 0.015 0.021]]
The mean of the samples was -0.238
Iteration 334
Acquisition function convergence reached at iteration 221.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.611 0.818 0.019 0.055 0.001 0.021]]
The mean of the samples was -0.64
Iteration 335
Acquisition function convergence reached at iteration 63.
The final EI loss was -0.398 with predicted mean of [-0.663]
The next parameters to simulate from are [[0.626 0.068 0.022 0.069 0.01  0.027]]
The mean of the samples was -0.646
Iteration 336
Acquisition function convergence reached at iteration 242.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.083 0.253 0.013 0.054 0.009 0.019]]
The mean of the samples was -0.809
Iteration 337
Acquisition function convergence reached at iteration 115.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.273 0.02  0.028 0.063 0.004 0.028]]
The mean of the samples was -0.797
Iteration 338
Acquisition function convergence reached at iteration 125.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.054 0.986 0.017 0.054 0.013 0.022]]
The mean of the samples was -0.632
Iteration 339
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.196]
The next parameters to simulate from are [[0.602 0.622 0.023 0.099 0.009 0.018]]
The mean of the samples was 0.197
Iteration 340
Acquisition function convergence reached at iteration 81.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.58  0.998 0.019 0.054 0.008 0.017]]
The mean of the samples was -0.713
Hyperparameter convergence reached at iteration 1186.
The minimum predicted mean of the observed indices is -1.334 at the point
[0.585 0.074 0.023 0.058 0.009 0.017]
Iteration 341
Acquisition function convergence reached at iteration 69.
The final EI loss was -0.398 with predicted mean of [-0.664]
```

```
The next parameters to simulate from are [[0.682 0.726 0.02  0.067 0.011 0.013]]
The mean of the samples was -0.513
Iteration 342
Acquisition function convergence reached at iteration 4.
The final EI loss was 0.0 with predicted mean of [0.273]
The next parameters to simulate from are [[0.654 0.097 0.025 0.095 0.065 0.021]]
The mean of the samples was 0.224
Iteration 343
Acquisition function convergence reached at iteration 2.
The final EI loss was -0.0 with predicted mean of [0.012]
The next parameters to simulate from are [[0.935 0.468 0.012 0.059 0.009 0.016]]
The mean of the samples was 0.09
Iteration 344
Acquisition function convergence reached at iteration 89.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.175 0.179 0.029 0.032 0.047 0.021]]
The mean of the samples was -0.61
Iteration 345
Acquisition function convergence reached at iteration 142.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.364 0.266 0.023 0.038 0.023 0.014]]
The mean of the samples was -0.65
Iteration 346
Acquisition function convergence reached at iteration 182.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.605 0.756 0.014 0.055 0.007 0.019]]
The mean of the samples was -0.709
Iteration 347
Acquisition function convergence reached at iteration 118.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.635 0.929 0.028 0.054 0.02  0.015]]
The mean of the samples was -0.643
Iteration 348
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.548 0.827 0.024 0.043 0.015 0.022]]
The mean of the samples was -0.706
Iteration 349
Acquisition function convergence reached at iteration 190.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.585 0.514 0.024 0.071 0.01  0.017]]
The mean of the samples was -0.583
Iteration 350
```

```
Acquisition function convergence reached at iteration 202.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.442 0.06  0.021 0.049 0.003 0.034]]
The mean of the samples was -0.594
Trained parameters:
amplitude_champ:0 is 0.673

length_scales_champ:0 is [0.22  0.5   0.017 0.027 0.036 0.012]

observation_noise_variance_champ:0 is 0.081

bias_mean:0 is 0.435

Iteration 351
Acquisition function convergence reached at iteration 91.
The final EI loss was -0.399 with predicted mean of [-0.665]
The next parameters to simulate from are [[0.104 0.189 0.008 0.054 0.008 0.02 ]]
The mean of the samples was -0.804
Iteration 352
Acquisition function convergence reached at iteration 12.
The final EI loss was 0.0 with predicted mean of [0.693]
The next parameters to simulate from are [[0.135 0.162 0.016 0.094 0.019 0.064]]
The mean of the samples was 0.855
Iteration 353
Acquisition function convergence reached at iteration 85.
The final EI loss was -0.398 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.544 0.361 0.03  0.061 0.003 0.031]]
The mean of the samples was -0.671
Iteration 354
Acquisition function convergence reached at iteration 105.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.553 0.042 0.023 0.063 0.064 0.014]]
The mean of the samples was -0.7
Iteration 355
Acquisition function convergence reached at iteration 103.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.595 0.92  0.02  0.062 0.005 0.016]]
The mean of the samples was -0.698
Iteration 356
Acquisition function convergence reached at iteration 3.
The final EI loss was 0.0 with predicted mean of [0.233]
The next parameters to simulate from are [[0.976 0.726 0.021 0.054 0.022 0.02 ]]
The mean of the samples was 0.28
```

```
Iteration 357
Acquisition function convergence reached at iteration 96.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.798 0.216 0.022 0.09  0.016 0.01 ]]
The mean of the samples was -0.681
Iteration 358
Acquisition function convergence reached at iteration 100.
The final EI loss was -0.399 with predicted mean of [-0.666]
The next parameters to simulate from are [[0.323 0.092 0.026 0.056 0.011 0.033]]
The mean of the samples was -0.62
Iteration 359
Acquisition function convergence reached at iteration 72.
The final EI loss was -0.002 with predicted mean of [-1.336]
The next parameters to simulate from are [[0.58  0.071 0.02  0.057 0.009 0.017]]
The mean of the samples was -1.441
Iteration 360
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.674]
The next parameters to simulate from are [[0.121 0.136 0.028 0.041 0.051 0.027]]
The mean of the samples was -0.708
Hyperparameter convergence reached at iteration 1235.
The minimum predicted mean of the observed indices is -1.35 at the point
[0.58  0.071 0.02  0.057 0.009 0.017]
Iteration 361
Acquisition function convergence reached at iteration 94.
The final EI loss was -0.399 with predicted mean of [-0.675]
The next parameters to simulate from are [[0.274 0.146 0.021 0.058 0.025 0.02 ]]
The mean of the samples was -0.617
Iteration 362
Acquisition function convergence reached at iteration 69.
The final EI loss was -0.001 with predicted mean of [-1.352]
The next parameters to simulate from are [[0.577 0.055 0.019 0.057 0.009 0.017]]
The mean of the samples was -1.212
Iteration 363
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.333]
The next parameters to simulate from are [[0.776 0.058 0.019 0.01  0.01  0.018]]
The mean of the samples was 0.258
Iteration 364
Acquisition function convergence reached at iteration 103.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.014 0.163 0.023 0.056 0.008 0.022]]
The mean of the samples was -0.564
```

```
Iteration 365
Acquisition function convergence reached at iteration 74.
The final EI loss was -0.398 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.645 0.199 0.022 0.06  0.058 0.008]]
The mean of the samples was -0.54
Iteration 366
Acquisition function convergence reached at iteration 200.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.321 0.811 0.022 0.057 0.017 0.025]]
The mean of the samples was -0.592
Iteration 367
Acquisition function convergence reached at iteration 87.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.344 0.324 0.022 0.034 0.015 0.019]]
The mean of the samples was -0.612
Iteration 368
Acquisition function convergence reached at iteration 102.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.352 0.989 0.03  0.062 0.009 0.022]]
The mean of the samples was -0.682
Iteration 369
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.829 0.243 0.025 0.061 0.005 0.015]]
The mean of the samples was -0.626
Iteration 370
Acquisition function convergence reached at iteration 99.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.71  0.717 0.022 0.064 0.011 0.013]]
The mean of the samples was -0.659
Iteration 371
Acquisition function convergence reached at iteration 183.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.567 0.993 0.029 0.06  0.006 0.018]]
The mean of the samples was -0.644
Iteration 372
Acquisition function convergence reached at iteration 103.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.272 0.012 0.028 0.061 0.016 0.026]]
The mean of the samples was -0.596
Iteration 373
Acquisition function convergence reached at iteration 109.
The final EI loss was -0.399 with predicted mean of [-0.669]
```

```
The next parameters to simulate from are [[0.26  0.029 0.03  0.057 0.017 0.029]]
The mean of the samples was -0.575
Iteration 374
Acquisition function convergence reached at iteration 91.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.27  0.139 0.025 0.056 0.03  0.017]]
The mean of the samples was -0.701
Iteration 375
Acquisition function convergence reached at iteration 88.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.589 0.278 0.014 0.059 0.006 0.025]]
The mean of the samples was -0.705
Iteration 376
Acquisition function convergence reached at iteration 102.
The final EI loss was -0.398 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.671 0.401 0.026 0.065 0.007 0.022]]
The mean of the samples was -0.552
Iteration 377
Acquisition function convergence reached at iteration 93.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.872 0.02  0.004 0.089 0.013 0.01 ]]
The mean of the samples was -0.533
Iteration 378
Acquisition function convergence reached at iteration 210.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.341 0.115 0.027 0.062 0.01  0.015]]
The mean of the samples was -0.603
Iteration 379
Acquisition function convergence reached at iteration 89.
The final EI loss was -0.398 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.156 0.049 0.027 0.056 0.006 0.017]]
The mean of the samples was -0.754
Iteration 380
Acquisition function convergence reached at iteration 4.
The final EI loss was 0.0 with predicted mean of [0.067]
The next parameters to simulate from are [[0.267 0.127 0.024 0.008 0.04  0.021]]
The mean of the samples was 0.077
Hyperparameter convergence reached at iteration 1227.
The minimum predicted mean of the observed indices is -1.34 at the point
[0.579 0.101 0.025 0.058 0.008 0.017]
Iteration 381
Acquisition function convergence reached at iteration 205.
The final EI loss was -0.399 with predicted mean of [-0.669]
```

```
The next parameters to simulate from are [[0.69  0.382 0.001 0.07  0.009 0.013]]
The mean of the samples was -0.557
Iteration 382
Acquisition function convergence reached at iteration 129.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.159 0.555 0.023 0.049 0.002 0.013]]
The mean of the samples was -0.682
Iteration 383
Acquisition function convergence reached at iteration 165.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.448 0.078 0.024 0.049 0.    0.035]]
The mean of the samples was -0.562
Iteration 384
Acquisition function convergence reached at iteration 78.
The final EI loss was -0.398 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.736 0.105 0.025 0.053 0.006 0.022]]
The mean of the samples was -0.66
Iteration 385
Acquisition function convergence reached at iteration 5.
The final EI loss was 0.0 with predicted mean of [0.559]
The next parameters to simulate from are [[0.13  0.138 0.004 0.08  0.03  0.021]]
The mean of the samples was 0.427
Iteration 386
Acquisition function convergence reached at iteration 97.
The final EI loss was -0.398 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.22  0.09  0.024 0.044 0.045 0.028]]
The mean of the samples was -0.635
Iteration 387
Acquisition function convergence reached at iteration 106.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.799 0.009 0.026 0.093 0.012 0.014]]
The mean of the samples was -0.716
Iteration 388
Acquisition function convergence reached at iteration 100.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.67  0.168 0.03  0.04  0.044 0.026]]
The mean of the samples was -0.719
Iteration 389
Acquisition function convergence reached at iteration 112.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.259 0.034 0.031 0.056 0.02  0.028]]
The mean of the samples was -0.681
Iteration 390
```

```
Acquisition function convergence reached at iteration 89.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.614 0.717 0.033 0.061 0.008 0.02 ]]
The mean of the samples was -0.766
Iteration 391
Acquisition function convergence reached at iteration 90.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.327 0.137 0.019 0.042 0.054 0.027]]
The mean of the samples was -0.554
Iteration 392
Acquisition function convergence reached at iteration 100.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.642 0.135 0.007 0.068 0.01  0.023]]
The mean of the samples was -0.717
Iteration 393
Acquisition function convergence reached at iteration 103.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.788 0.234 0.028 0.081 0.004 0.014]]
The mean of the samples was -0.693
Iteration 394
Acquisition function convergence reached at iteration 96.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.532 0.075 0.012 0.062 0.006 0.008]]
The mean of the samples was -0.544
Iteration 395
Acquisition function convergence reached at iteration 59.
The final EI loss was -0.002 with predicted mean of [-1.342]
The next parameters to simulate from are [[0.572 0.111 0.025 0.057 0.006 0.017]]
The mean of the samples was -1.337
Iteration 396
Acquisition function convergence reached at iteration 250.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.716 0.159 0.023 0.078 0.01  0.019]]
The mean of the samples was -0.725
Iteration 397
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.537 0.871 0.024 0.04  0.013 0.022]]
The mean of the samples was -0.58
Iteration 398
Acquisition function convergence reached at iteration 90.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.292 0.007 0.03  0.058 0.025 0.025]]
```

```
The mean of the samples was -0.762
Iteration 399
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.715 0.077 0.03  0.033 0.058 0.02 ]]
The mean of the samples was -0.539
Iteration 400
Acquisition function convergence reached at iteration 6.
The final EI loss was 0.0 with predicted mean of [0.273]
The next parameters to simulate from are [[0.145 0.166 0.011 0.002 0.012 0.023]]
The mean of the samples was 0.312
Hyperparameter convergence reached at iteration 1199.
The minimum predicted mean of the observed indices is -1.342 at the point
[0.572 0.111 0.025 0.057 0.006 0.017]
Trained parameters:
amplitude_champ:0 is 0.672

length_scales_champ:0 is [0.208 0.5   0.017 0.026 0.036 0.012]

observation_noise_variance_champ:0 is 0.078

bias_mean:0 is 0.441

Iteration 401
Acquisition function convergence reached at iteration 50.
The final EI loss was 0.0 with predicted mean of [-1.342]
The next parameters to simulate from are [[0.573 0.113 0.025 0.057 0.007 0.017]]
The mean of the samples was -1.226
Iteration 402
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.573 0.371 0.027 0.065 0.004 0.027]]
The mean of the samples was -0.686
Iteration 403
Acquisition function convergence reached at iteration 82.
The final EI loss was -0.398 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.046 0.044 0.027 0.05  0.019 0.018]]
The mean of the samples was -0.71
Iteration 404
Acquisition function convergence reached at iteration 49.
The final EI loss was -0.394 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.51  0.609 0.014 0.05  0.006 0.025]]
The mean of the samples was -0.692
```

```
Iteration 405
Acquisition function convergence reached at iteration 59.
The final EI loss was -0.0 with predicted mean of [-1.334]
The next parameters to simulate from are [[0.575 0.104 0.025 0.057 0.007 0.017]]
The mean of the samples was -1.264
Iteration 406
Acquisition function convergence reached at iteration 135.
The final EI loss was -0.399 with predicted mean of [-0.661]
The next parameters to simulate from are [[0.758 0.417 0.013 0.04  0.005 0.009]]
The mean of the samples was -0.688
Iteration 407
Acquisition function convergence reached at iteration 106.
The final EI loss was -0.399 with predicted mean of [-0.66]
The next parameters to simulate from are [[0.637 0.235 0.028 0.03  0.057 0.021]]
The mean of the samples was -0.705
Iteration 408
Acquisition function convergence reached at iteration 145.
The final EI loss was -0.399 with predicted mean of [-0.66]
The next parameters to simulate from are [[0.246 0.027 0.023 0.057 0.002 0.033]]
The mean of the samples was -0.762
Iteration 409
Acquisition function convergence reached at iteration 68.
The final EI loss was -0.002 with predicted mean of [-1.325]
The next parameters to simulate from are [[0.58  0.052 0.019 0.059 0.009 0.017]]
The mean of the samples was -1.453
Iteration 410
Acquisition function convergence reached at iteration 110.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.334 0.14  0.021 0.033 0.069 0.026]]
The mean of the samples was -0.732
Iteration 411
Acquisition function convergence reached at iteration 94.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.556 0.049 0.02  0.062 0.065 0.013]]
The mean of the samples was -0.601
Iteration 412
Acquisition function convergence reached at iteration 89.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.509 0.037 0.018 0.053 0.024 0.03 ]]
The mean of the samples was -0.649
Iteration 413
Acquisition function convergence reached at iteration 110.
The final EI loss was -0.399 with predicted mean of [-0.672]
```

```
The next parameters to simulate from are [[0.305 0.062 0.023 0.034 0.067 0.026]]
The mean of the samples was -0.604
Iteration 414
Acquisition function convergence reached at iteration 93.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.406 0.347 0.022 0.042 0.058 0.012]]
The mean of the samples was -0.658
Iteration 415
Acquisition function convergence reached at iteration 98.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.561 0.051 0.021 0.06  0.07  0.014]]
The mean of the samples was -0.564
Iteration 416
Acquisition function convergence reached at iteration 106.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.694 0.13  0.029 0.034 0.041 0.017]]
The mean of the samples was -0.648
Iteration 417
Acquisition function convergence reached at iteration 225.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.681 0.571 0.014 0.06  0.008 0.017]]
The mean of the samples was -0.607
Iteration 418
Acquisition function convergence reached at iteration 78.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.741 0.336 0.02  0.057 0.004 0.018]]
The mean of the samples was -0.633
Iteration 419
Acquisition function convergence reached at iteration 100.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.538 0.164 0.025 0.059 0.069 0.014]]
The mean of the samples was -0.729
Iteration 420
Acquisition function convergence reached at iteration 116.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.626 0.929 0.022 0.061 0.011 0.008]]
The mean of the samples was -0.866
Hyperparameter convergence reached at iteration 1180.
The minimum predicted mean of the observed indices is -1.344 at the point
[0.58  0.052 0.019 0.059 0.009 0.017]
Iteration 421
Acquisition function convergence reached at iteration 97.
The final EI loss was -0.399 with predicted mean of [-0.673]
```

```
The next parameters to simulate from are [[0.496 0.037 0.026 0.054 0.015 0.034]]
The mean of the samples was -0.754
Iteration 422
Acquisition function convergence reached at iteration 95.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.106 0.84  0.014 0.058 0.003 0.021]]
The mean of the samples was -0.699
Iteration 423
Acquisition function convergence reached at iteration 105.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.169 0.162 0.028 0.033 0.043 0.021]]
The mean of the samples was -0.602
Iteration 424
Acquisition function convergence reached at iteration 152.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.63  0.044 0.024 0.046 0.032 0.029]]
The mean of the samples was -0.721
Iteration 425
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.673]
The next parameters to simulate from are [[0.33  0.005 0.022 0.035 0.068 0.026]]
The mean of the samples was -0.678
Iteration 426
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.543 0.098 0.032 0.059 0.003 0.034]]
The mean of the samples was -0.682
Iteration 427
Acquisition function convergence reached at iteration 155.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.196 0.028 0.028 0.056 0.009 0.032]]
The mean of the samples was -0.77
Iteration 428
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.346 0.076 0.014 0.038 0.019 0.026]]
The mean of the samples was -0.727
Iteration 429
Acquisition function convergence reached at iteration 162.
The final EI loss was -0.399 with predicted mean of [-0.674]
The next parameters to simulate from are [[0.562 0.151 0.013 0.06  0.012 0.008]]
The mean of the samples was -0.621
Iteration 430
```

```
Acquisition function convergence reached at iteration 46.
The final EI loss was -0.027 with predicted mean of [-0.14]
The next parameters to simulate from are [[0.564 0.452 0.032 0.077 0.    0.07 ]]
The mean of the samples was -0.227
Iteration 431
Acquisition function convergence reached at iteration 126.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.088 0.541 0.017 0.043 0.024 0.027]]
The mean of the samples was -0.592
Iteration 432
Acquisition function convergence reached at iteration 92.
The final EI loss was -0.399 with predicted mean of [-0.673]
The next parameters to simulate from are [[0.658 0.123 0.026 0.032 0.041 0.018]]
The mean of the samples was -0.667
Iteration 433
Acquisition function convergence reached at iteration 105.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.32  0.117 0.015 0.053 0.028 0.023]]
The mean of the samples was -0.622
Iteration 434
Acquisition function convergence reached at iteration 235.
The final EI loss was -0.399 with predicted mean of [-0.673]
The next parameters to simulate from are [[0.299 0.748 0.029 0.065 0.005 0.021]]
The mean of the samples was -0.673
Iteration 435
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.144]
The next parameters to simulate from are [[0.925 0.987 0.011 0.059 0.009 0.017]]
The mean of the samples was 0.132
Iteration 436
Acquisition function convergence reached at iteration 69.
The final EI loss was -0.397 with predicted mean of [-0.682]
The next parameters to simulate from are [[0.287 0.235 0.001 0.062 0.002 0.015]]
The mean of the samples was -0.967
Iteration 437
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.846 0.144 0.006 0.088 0.015 0.01 ]]
The mean of the samples was -1.03
Iteration 438
Acquisition function convergence reached at iteration 92.
The final EI loss was -0.398 with predicted mean of [-0.673]
The next parameters to simulate from are [[0.422 0.769 0.017 0.061 0.006 0.021]]
```

```
The mean of the samples was -0.654
Iteration 439
Acquisition function convergence reached at iteration 146.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.141 0.081 0.031 0.056 0.002 0.033]]
The mean of the samples was -0.697
Iteration 440
Acquisition function convergence reached at iteration 144.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.817 0.123 0.028 0.089 0.019 0.013]]
The mean of the samples was -0.62
Hyperparameter convergence reached at iteration 1177.
The minimum predicted mean of the observed indices is -1.344 at the point
[0.58  0.052 0.019 0.059 0.009 0.017]
Iteration 441
Acquisition function convergence reached at iteration 89.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.243 0.601 0.028 0.057 0.013 0.03 ]]
The mean of the samples was -0.569
Iteration 442
Acquisition function convergence reached at iteration 90.
The final EI loss was -0.399 with predicted mean of [-0.673]
The next parameters to simulate from are [[0.178 0.824 0.029 0.059 0.004 0.031]]
The mean of the samples was -0.674
Iteration 443
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.452 0.005 0.021 0.063 0.001 0.034]]
The mean of the samples was -0.583
Iteration 444
Acquisition function convergence reached at iteration 83.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.558 0.032 0.028 0.053 0.002 0.033]]
The mean of the samples was -0.613
Iteration 445
Acquisition function convergence reached at iteration 197.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.411 0.3   0.022 0.045 0.06  0.012]]
The mean of the samples was -0.71
Iteration 446
Acquisition function convergence reached at iteration 200.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.856 0.216 0.012 0.08  0.014 0.012]]
```

```
The mean of the samples was -0.659
Iteration 447
Acquisition function convergence reached at iteration 91.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.427 0.005 0.025 0.053 0.001 0.037]]
The mean of the samples was -0.751
Iteration 448
Acquisition function convergence reached at iteration 107.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.819 0.207 0.002 0.088 0.016 0.012]]
The mean of the samples was -0.621
Iteration 449
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.27]
The next parameters to simulate from are [[0.581 0.782 0.028 0.096 0.025 0.017]]
The mean of the samples was 0.189
Iteration 450
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.943]
The next parameters to simulate from are [[0.179 0.052 0.019 0.1   0.066 0.017]]
The mean of the samples was 1.265
Trained parameters:
amplitude_champ:0 is 0.664

length_scales_champ:0 is [0.208 0.5   0.017 0.025 0.036 0.012]

observation_noise_variance_champ:0 is 0.079

bias_mean:0 is 0.424

Iteration 451
Acquisition function convergence reached at iteration 105.
The final EI loss was -0.398 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.585 0.115 0.025 0.037 0.008 0.017]]
The mean of the samples was -0.534
Iteration 452
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.366 0.52  0.016 0.064 0.009 0.02 ]]
The mean of the samples was -0.554
Iteration 453
Acquisition function convergence reached at iteration 110.
The final EI loss was -0.399 with predicted mean of [-0.672]
```

```
The next parameters to simulate from are [[0.848 0.04  0.001 0.089 0.014 0.011]]
The mean of the samples was -0.522
Iteration 454
Acquisition function convergence reached at iteration 62.
The final EI loss was -0.0 with predicted mean of [-1.345]
The next parameters to simulate from are [[0.579 0.061 0.019 0.059 0.008 0.017]]
The mean of the samples was -1.187
Iteration 455
Acquisition function convergence reached at iteration 118.
The final EI loss was -0.399 with predicted mean of [-0.664]
The next parameters to simulate from are [[0.383 0.324 0.021 0.039 0.005 0.013]]
The mean of the samples was -0.644
Iteration 456
Acquisition function convergence reached at iteration 50.
The final EI loss was -0.397 with predicted mean of [-0.657]
The next parameters to simulate from are [[0.307 0.019 0.018 0.063 0.009 0.015]]
The mean of the samples was -0.636
Iteration 457
Acquisition function convergence reached at iteration 93.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.265 0.011 0.015 0.063 0.004 0.026]]
The mean of the samples was -0.524
Iteration 458
Acquisition function convergence reached at iteration 86.
The final EI loss was -0.399 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.827 0.07  0.027 0.055 0.024 0.018]]
The mean of the samples was -0.696
Iteration 459
Acquisition function convergence reached at iteration 88.
The final EI loss was -0.399 with predicted mean of [-0.671]
The next parameters to simulate from are [[0.729 0.376 0.027 0.035 0.063 0.009]]
The mean of the samples was -0.572
Iteration 460
Acquisition function convergence reached at iteration 75.
The final EI loss was -0.002 with predicted mean of [-1.346]
The next parameters to simulate from are [[0.594 0.029 0.018 0.06  0.009 0.016]]
The mean of the samples was -1.36
Hyperparameter convergence reached at iteration 1193.
The minimum predicted mean of the observed indices is -1.367 at the point
[0.594 0.029 0.018 0.06  0.009 0.016]
Iteration 461
Acquisition function convergence reached at iteration 68.
The final EI loss was -0.003 with predicted mean of [-1.373]
```

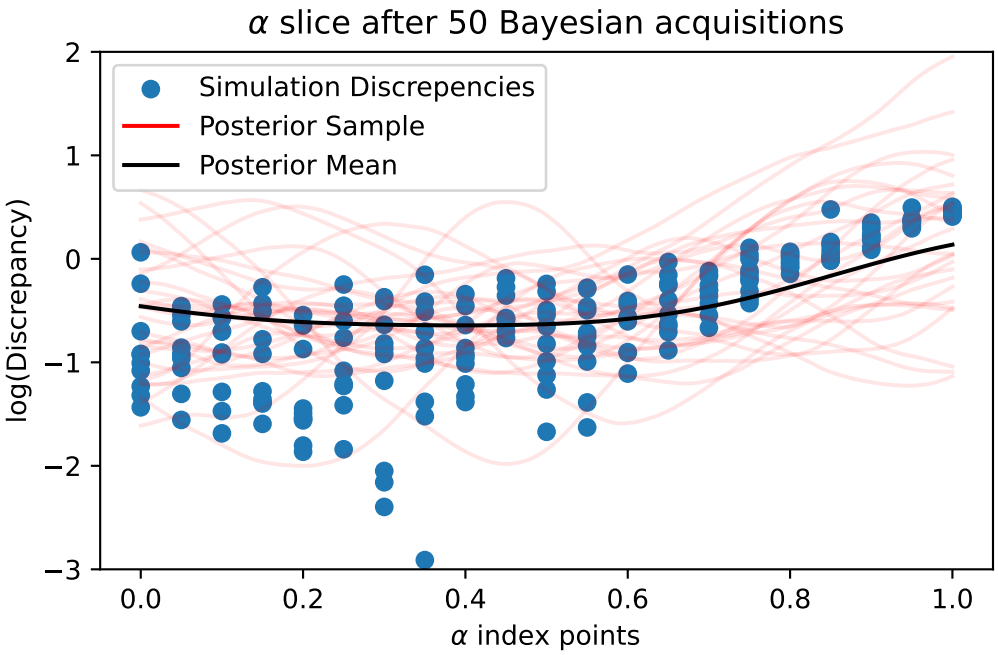
```
The next parameters to simulate from are [[0.606 0.008 0.018 0.061 0.012 0.016]]
The mean of the samples was -1.409
Iteration 462
Acquisition function convergence reached at iteration 99.
The final EI loss was -0.399 with predicted mean of [-0.684]
The next parameters to simulate from are [[0.69  0.009 0.009 0.075 0.06  0.014]]
The mean of the samples was -0.606
Iteration 463
Acquisition function convergence reached at iteration 82.
The final EI loss was -0.398 with predicted mean of [-0.683]
The next parameters to simulate from are [[0.553 0.204 0.021 0.066 0.04  0.016]]
The mean of the samples was -0.658
Iteration 464
Acquisition function convergence reached at iteration 108.
The final EI loss was -0.399 with predicted mean of [-0.684]
The next parameters to simulate from are [[0.711 0.004 0.027 0.075 0.067 0.014]]
The mean of the samples was -0.592
Iteration 465
Acquisition function convergence reached at iteration 196.
The final EI loss was -0.399 with predicted mean of [-0.684]
The next parameters to simulate from are [[0.147 0.097 0.031 0.056 0.011 0.018]]
The mean of the samples was -0.578
Iteration 466
Acquisition function convergence reached at iteration 89.
The final EI loss was -0.392 with predicted mean of [-0.684]
The next parameters to simulate from are [[0.499 0.009 0.033 0.062 0.015 0.011]]
The mean of the samples was -0.636
Iteration 467
Acquisition function convergence reached at iteration 87.
The final EI loss was -0.397 with predicted mean of [-0.683]
The next parameters to simulate from are [[0.507 0.006 0.018 0.075 0.015 0.018]]
The mean of the samples was -0.798
Iteration 468
Acquisition function convergence reached at iteration 106.
The final EI loss was -0.399 with predicted mean of [-0.685]
The next parameters to simulate from are [[0.856 0.023 0.006 0.091 0.01  0.01 ]]
The mean of the samples was -0.672
Iteration 469
Acquisition function convergence reached at iteration 116.
The final EI loss was -0.399 with predicted mean of [-0.685]
The next parameters to simulate from are [[0.329 0.01  0.021 0.033 0.03  0.029]]
The mean of the samples was -0.704
Iteration 470
```

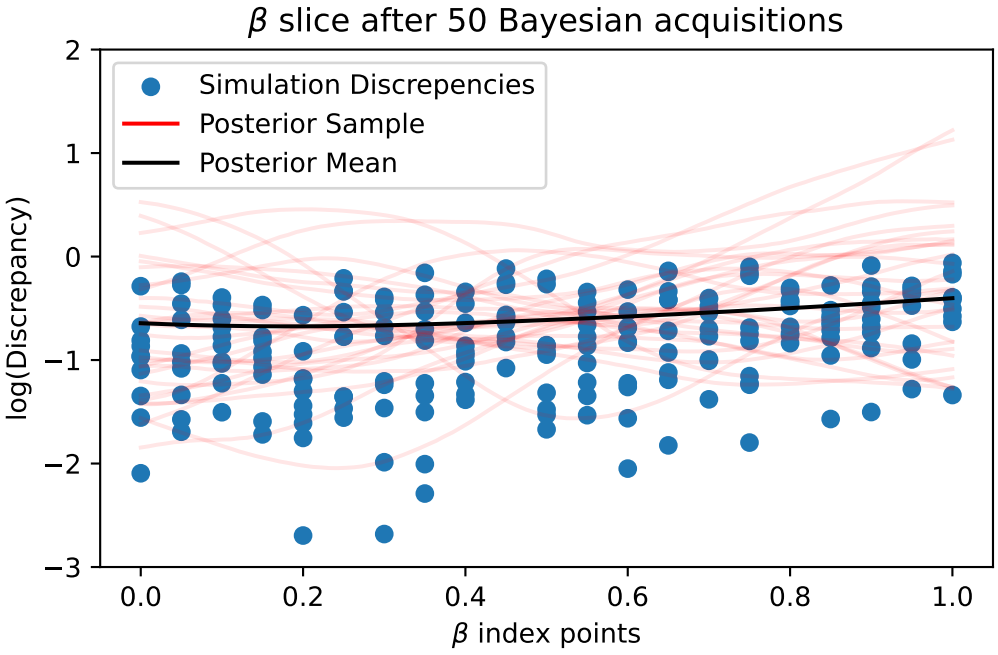
```
Acquisition function convergence reached at iteration 99.
The final EI loss was -0.398 with predicted mean of [-0.684]
The next parameters to simulate from are [[0.826 0.003 0.017 0.064 0.009 0.016]]
The mean of the samples was -0.546
Iteration 471
Acquisition function convergence reached at iteration 74.
The final EI loss was -0.001 with predicted mean of [-1.37]
The next parameters to simulate from are [[0.599 0.009 0.018 0.061 0.012 0.017]]
The mean of the samples was -1.267
Iteration 472
Acquisition function convergence reached at iteration 5.
The final EI loss was 0.0 with predicted mean of [0.407]
The next parameters to simulate from are [[0.873 0.614 0.015 0.004 0.034 0.021]]
The mean of the samples was 0.435
Iteration 473
Acquisition function convergence reached at iteration 79.
The final EI loss was -0.387 with predicted mean of [-0.688]
The next parameters to simulate from are [[0.602 0.913 0.033 0.045 0.    0.02 ]]
The mean of the samples was -0.846
Iteration 474
Acquisition function convergence reached at iteration 93.
The final EI loss was -0.399 with predicted mean of [-0.679]
The next parameters to simulate from are [[0.857 0.005 0.032 0.062 0.012 0.015]]
The mean of the samples was -0.728
Iteration 475
Acquisition function convergence reached at iteration 69.
The final EI loss was -0.0 with predicted mean of [-1.36]
The next parameters to simulate from are [[0.597 0.018 0.018 0.061 0.011 0.017]]
The mean of the samples was -1.091
Iteration 476
Acquisition function convergence reached at iteration 101.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.599 0.073 0.025 0.038 0.01  0.017]]
The mean of the samples was -0.677
Iteration 477
Acquisition function convergence reached at iteration 101.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.285 0.387 0.019 0.034 0.005 0.022]]
The mean of the samples was -0.571
Iteration 478
Acquisition function convergence reached at iteration 6.
The final EI loss was 0.0 with predicted mean of [0.183]
The next parameters to simulate from are [[0.991 0.246 0.019 0.086 0.015 0.017]]
```

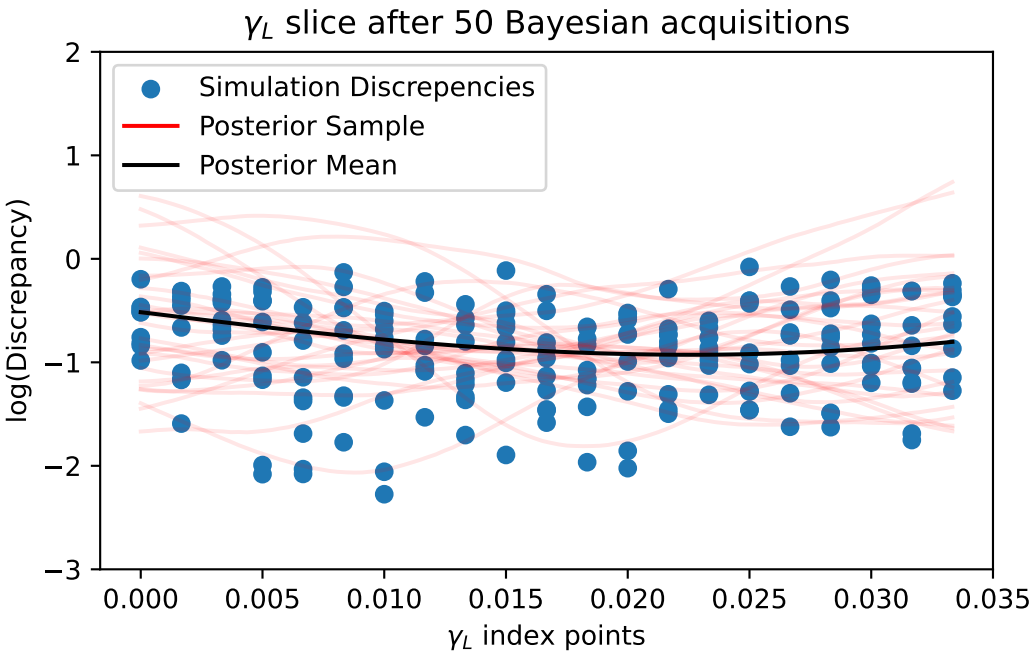
```
The mean of the samples was 0.272
Iteration 479
Acquisition function convergence reached at iteration 98.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.797 0.547 0.018 0.038 0.029 0.009]]
The mean of the samples was -0.737
Iteration 480
Acquisition function convergence reached at iteration 107.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.575 0.28  0.022 0.071 0.01  0.022]]
The mean of the samples was -0.643
Hyperparameter convergence reached at iteration 1209.
The minimum predicted mean of the observed indices is -1.339 at the point
[0.597 0.018 0.018 0.061 0.011 0.017]
Iteration 481
Acquisition function convergence reached at iteration 118.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.088 0.17  0.03  0.049 0.028 0.031]]
The mean of the samples was -0.639
Iteration 482
Acquisition function convergence reached at iteration 196.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.357 0.187 0.022 0.038 0.003 0.027]]
The mean of the samples was -0.719
Iteration 483
Acquisition function convergence reached at iteration 60.
The final EI loss was -0.398 with predicted mean of [-0.672]
The next parameters to simulate from are [[0.632 0.974 0.026 0.043 0.011 0.016]]
The mean of the samples was -0.779
Iteration 484
Acquisition function convergence reached at iteration 114.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.55  0.004 0.032 0.061 0.    0.033]]
The mean of the samples was -0.706
Iteration 485
Acquisition function convergence reached at iteration 111.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.561 0.017 0.017 0.064 0.059 0.014]]
The mean of the samples was -0.633
Iteration 486
Acquisition function convergence reached at iteration 169.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.602 0.556 0.024 0.056 0.058 0.013]]
```

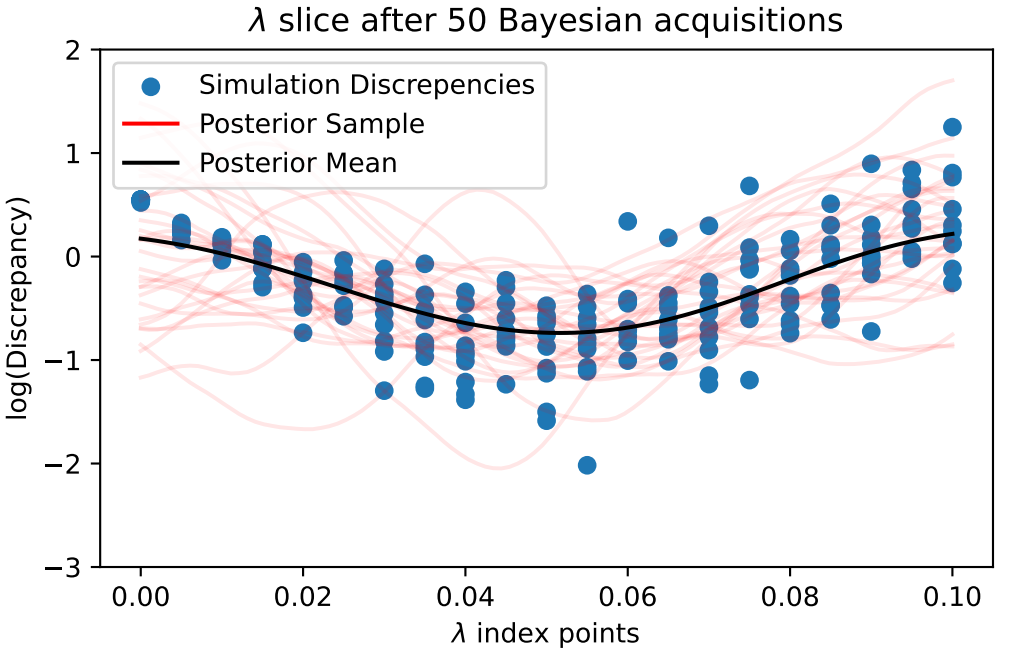
```
The mean of the samples was -0.818
Iteration 487
Acquisition function convergence reached at iteration 98.
The final EI loss was -0.399 with predicted mean of [-0.67]
The next parameters to simulate from are [[0.554 0.003 0.033 0.064 0.009 0.033]]
The mean of the samples was -0.689
Iteration 488
Acquisition function convergence reached at iteration 62.
The final EI loss was -0.001 with predicted mean of [-1.341]
The next parameters to simulate from are [[0.591 0.039 0.019 0.061 0.01  0.016]]
The mean of the samples was -1.279
Iteration 489
Acquisition function convergence reached at iteration 97.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.569 0.685 0.022 0.057 0.03  0.015]]
The mean of the samples was -0.697
Iteration 490
Acquisition function convergence reached at iteration 112.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.538 0.003 0.033 0.063 0.001 0.033]]
The mean of the samples was -0.772
Iteration 491
Acquisition function convergence reached at iteration 11.
The final EI loss was 0.0 with predicted mean of [0.931]
The next parameters to simulate from are [[0.218 0.312 0.021 0.079 0.068 0.065]]
The mean of the samples was 0.946
Iteration 492
Acquisition function convergence reached at iteration 85.
The final EI loss was -0.399 with predicted mean of [-0.669]
The next parameters to simulate from are [[0.573 0.026 0.018 0.068 0.    0.029]]
The mean of the samples was -0.582
Iteration 493
Acquisition function convergence reached at iteration 92.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.501 0.066 0.007 0.063 0.031 0.021]]
The mean of the samples was -0.551
Iteration 494
Acquisition function convergence reached at iteration 69.
The final EI loss was -0.397 with predicted mean of [-0.667]
The next parameters to simulate from are [[0.202 0.464 0.014 0.032 0.015 0.025]]
The mean of the samples was -0.615
Iteration 495
Acquisition function convergence reached at iteration 99.
```

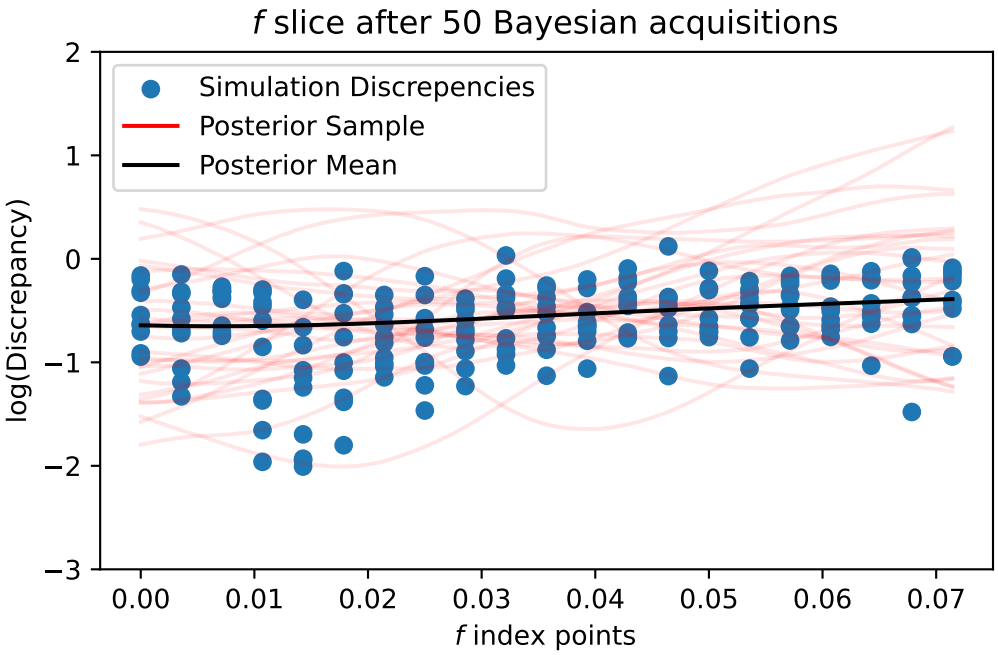
```
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.698 0.073 0.007 0.074 0.06  0.014]]
The mean of the samples was -0.71
Iteration 496
Acquisition function convergence reached at iteration 102.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.471 0.    0.025 0.057 0.019 0.035]]
The mean of the samples was -0.646
Iteration 497
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.171]
The next parameters to simulate from are [[0.967 0.699 0.019 0.062 0.006 0.017]]
The mean of the samples was 0.224
Iteration 498
Acquisition function convergence reached at iteration 104.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.838 0.246 0.007 0.088 0.013 0.01 ]]
The mean of the samples was -0.698
Iteration 499
Acquisition function convergence reached at iteration 2.
The final EI loss was 0.0 with predicted mean of [0.306]
The next parameters to simulate from are [[0.591 0.747 0.024 0.093 0.01  0.029]]
The mean of the samples was 0.287
Iteration 500
Acquisition function convergence reached at iteration 106.
The final EI loss was -0.399 with predicted mean of [-0.668]
The next parameters to simulate from are [[0.124 0.154 0.016 0.059 0.002 0.026]]
The mean of the samples was -0.638
Hyperparameter convergence reached at iteration 1183.
The minimum predicted mean of the observed indices is -1.337 at the point
[0.591 0.039 0.019 0.061 0.01  0.016]
Trained parameters:
amplitude_champ:0 is 0.653

length_scales_champ:0 is [0.206 0.5   0.017 0.026 0.036 0.011]

observation_noise_variance_champ:0 is 0.081

bias_mean:0 is 0.427
```

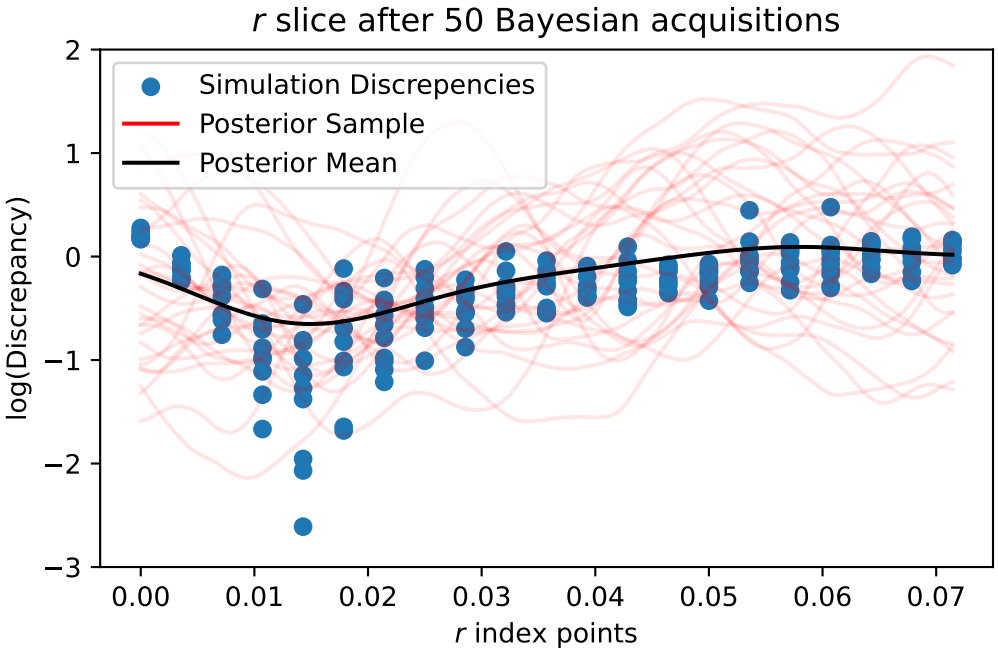

α slice after 50 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points
2
1
0
−1
−2
−3
0.0
0.2
0.4
0.6
0.8
1.0


β slice after 50 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points
2
1
0
−1
−2
−3
0.0
0.2
0.4
0.6
0.8
1.0


γ_L slice after 50 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
2
1
0
−1
−2
−3
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035


λ slice after 50 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
λ index points
2
1
0
−1
−2
−3
0.00
0.02
0.04
0.06
0.08
0.10


f slice after 50 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
f index points
2
1
0
−1
−2
−3
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07


r slice after 50 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
r index points
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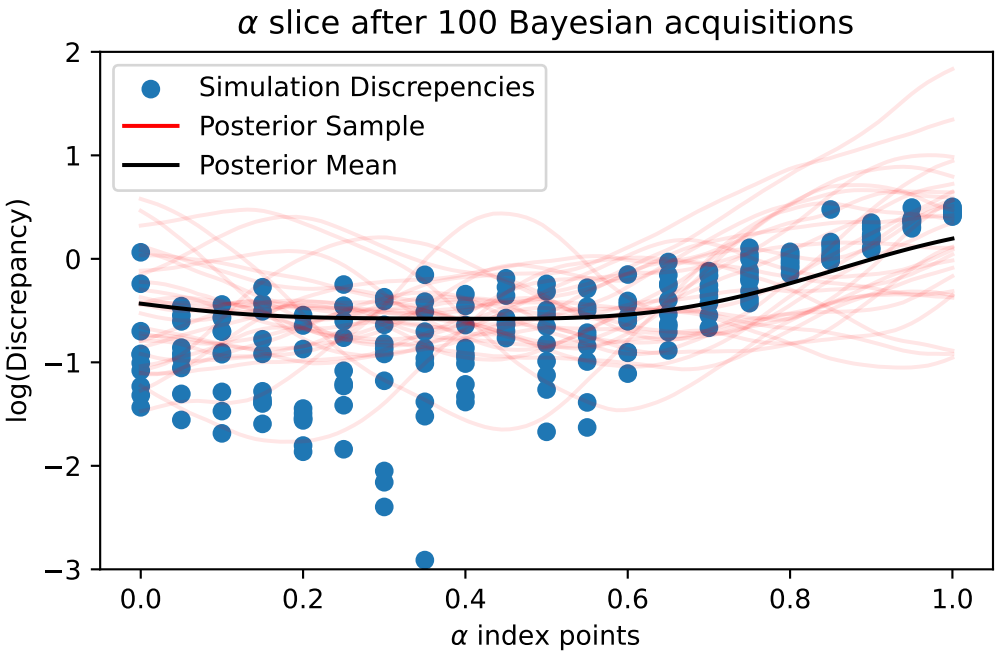
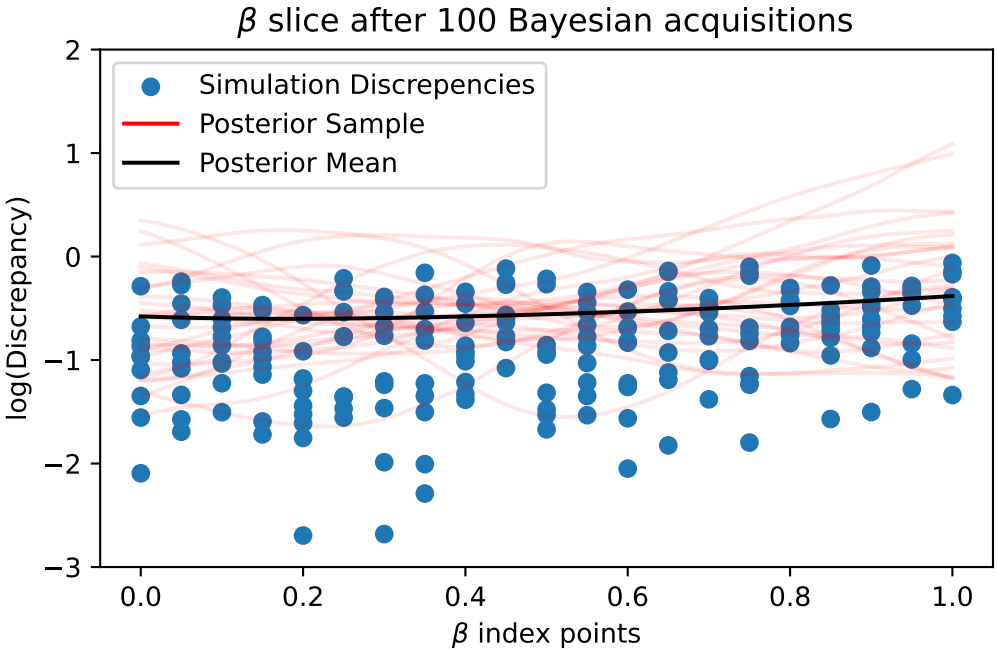
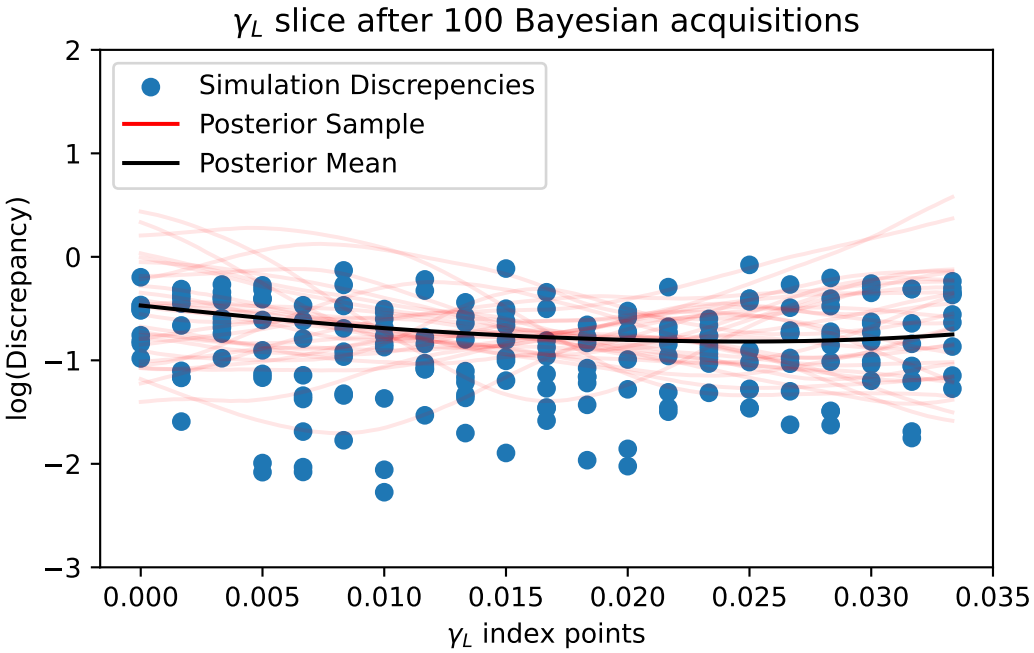

α slice after 100 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points


β slice after 100 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points


γ_L slice after 100 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
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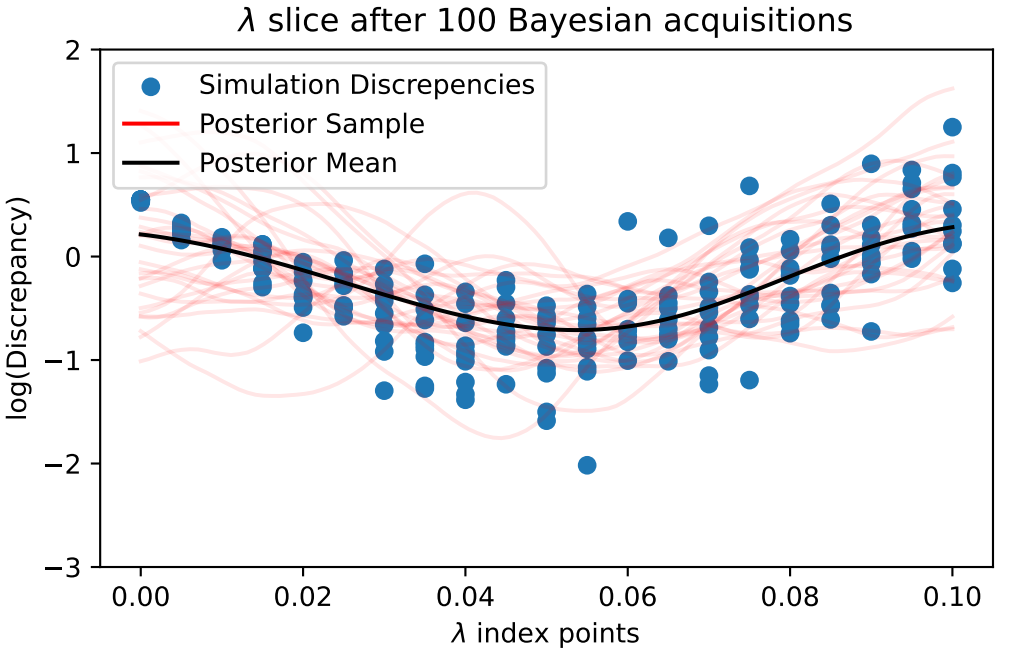
λ slice after 100 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
λ index points
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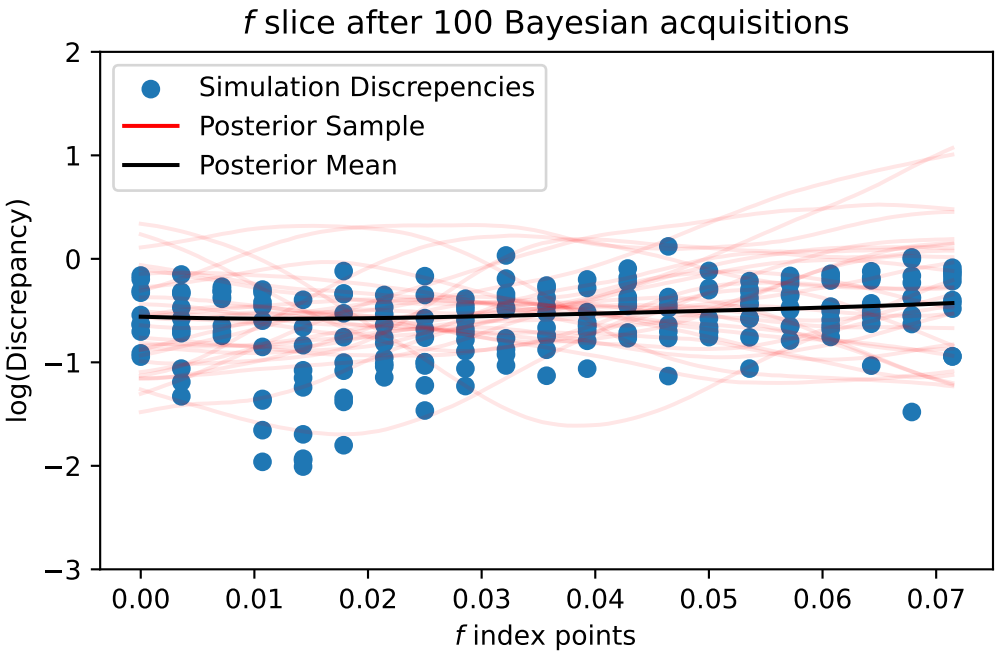
f slice after 100 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
f index points
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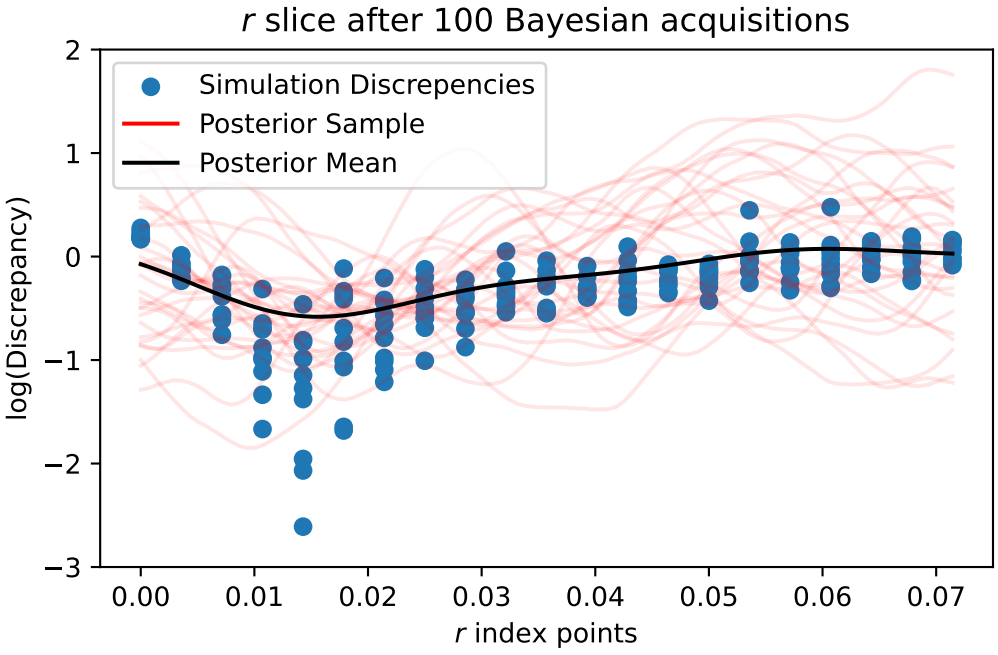
r slice after 100 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
r index points
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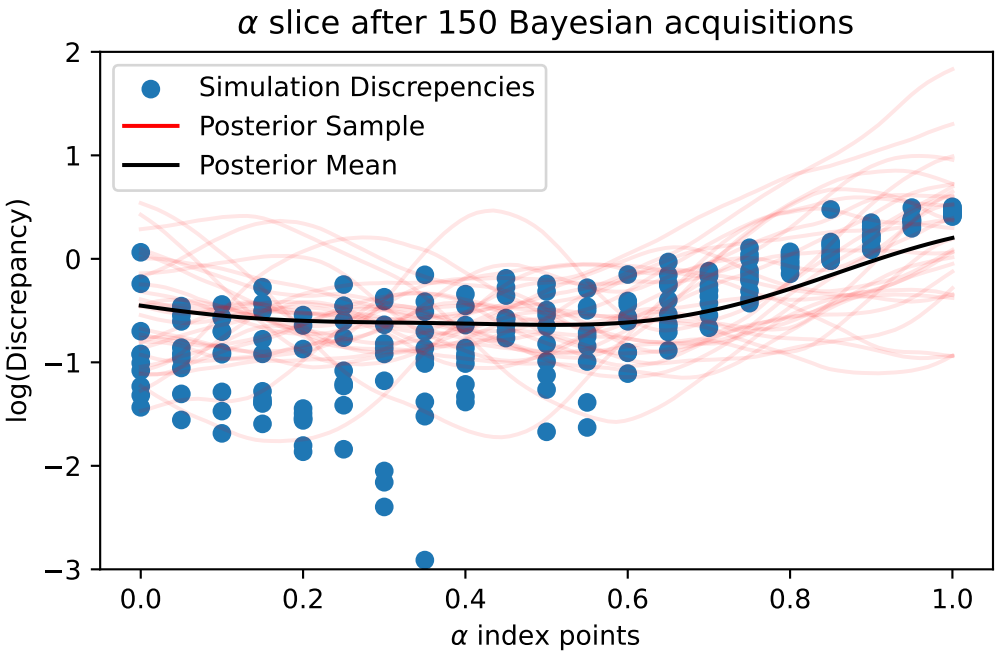
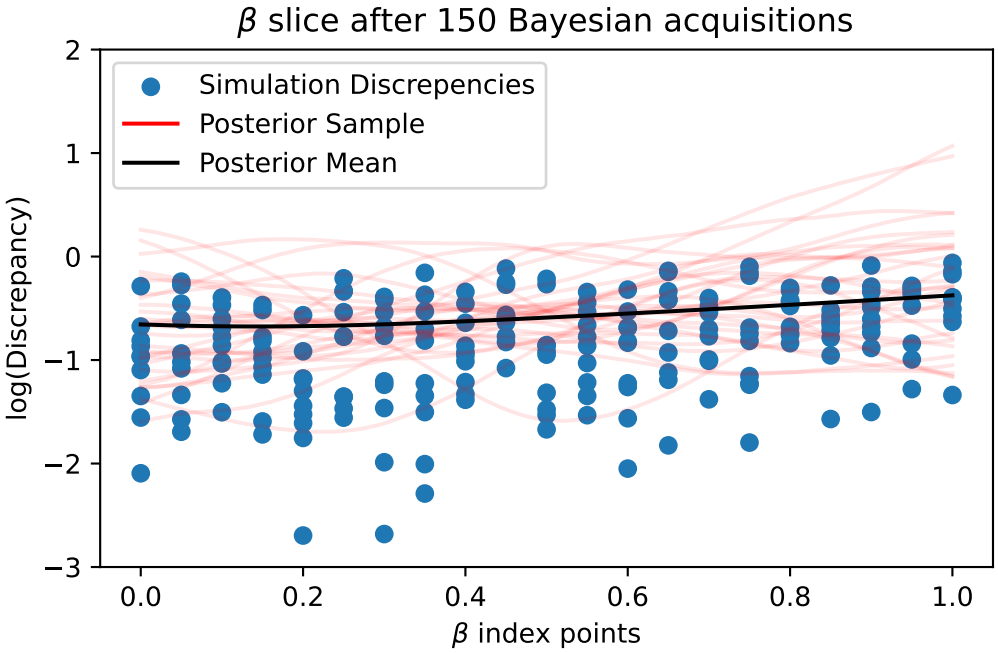
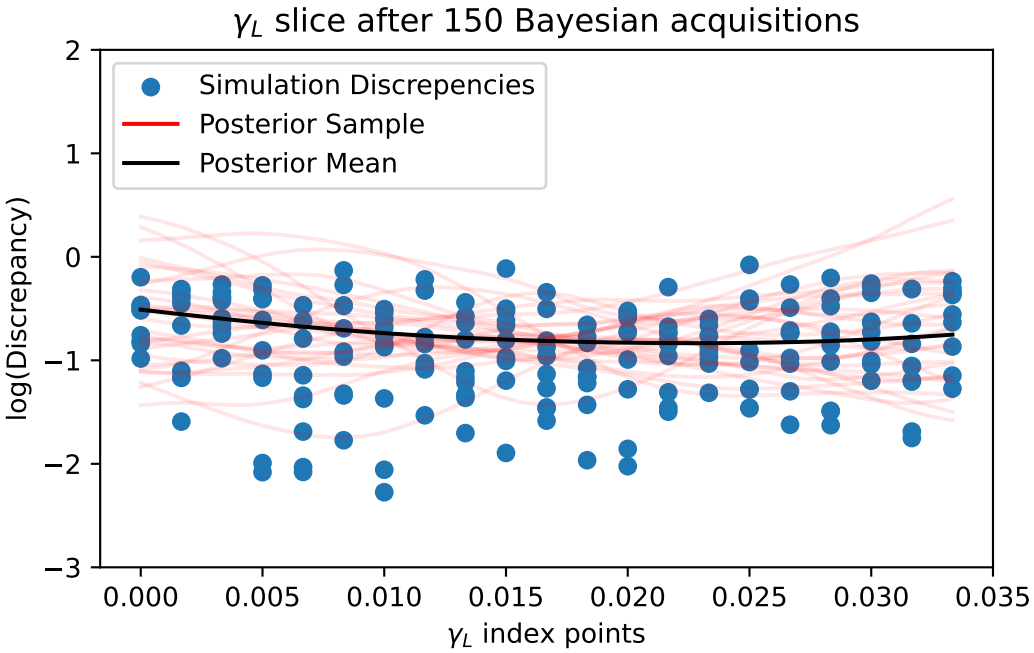

α slice after 150 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points


β slice after 150 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points


γ_L slice after 150 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
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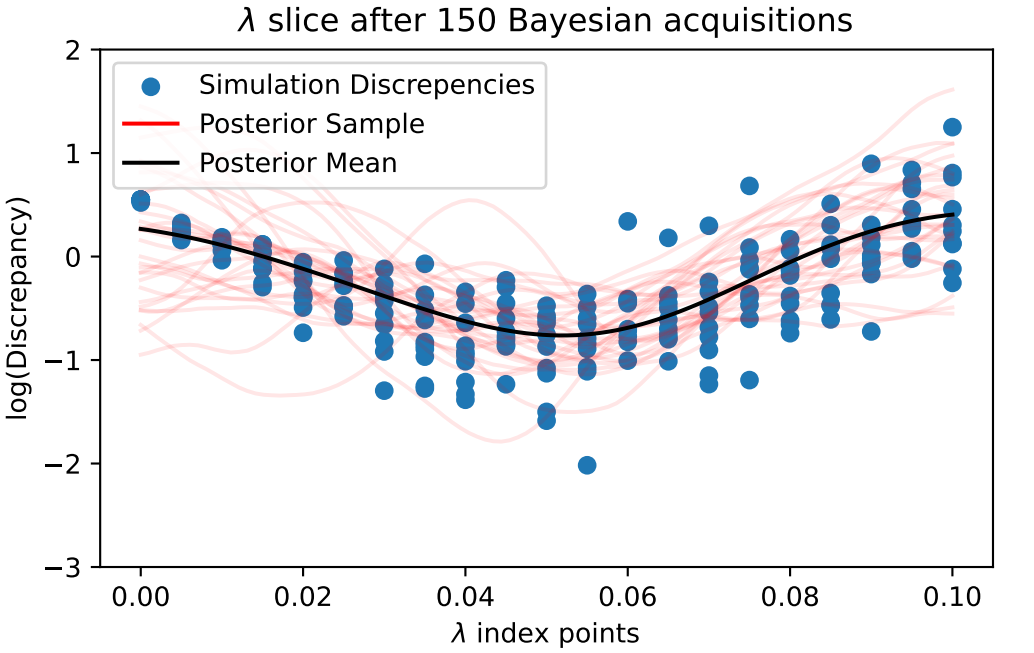
λ slice after 150 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
λ index points
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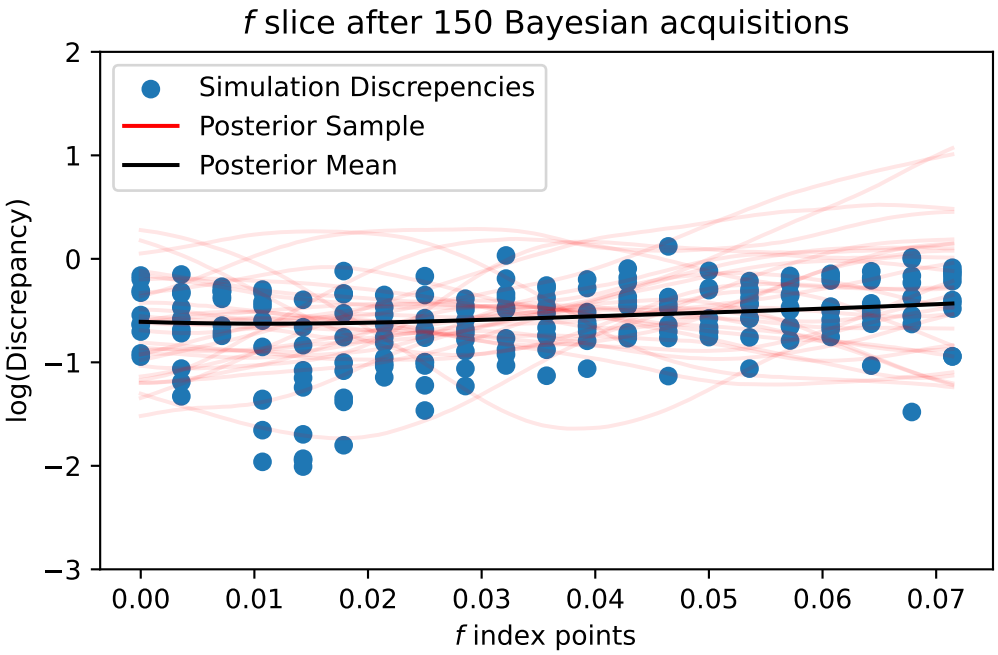
f slice after 150 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
f index points
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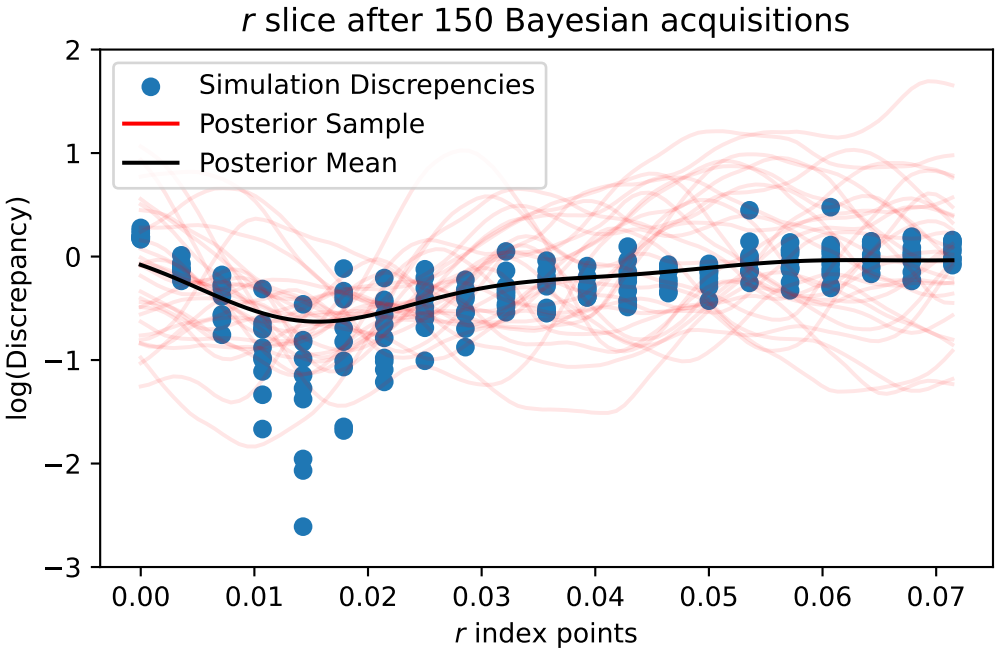
r slice after 150 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
r index points
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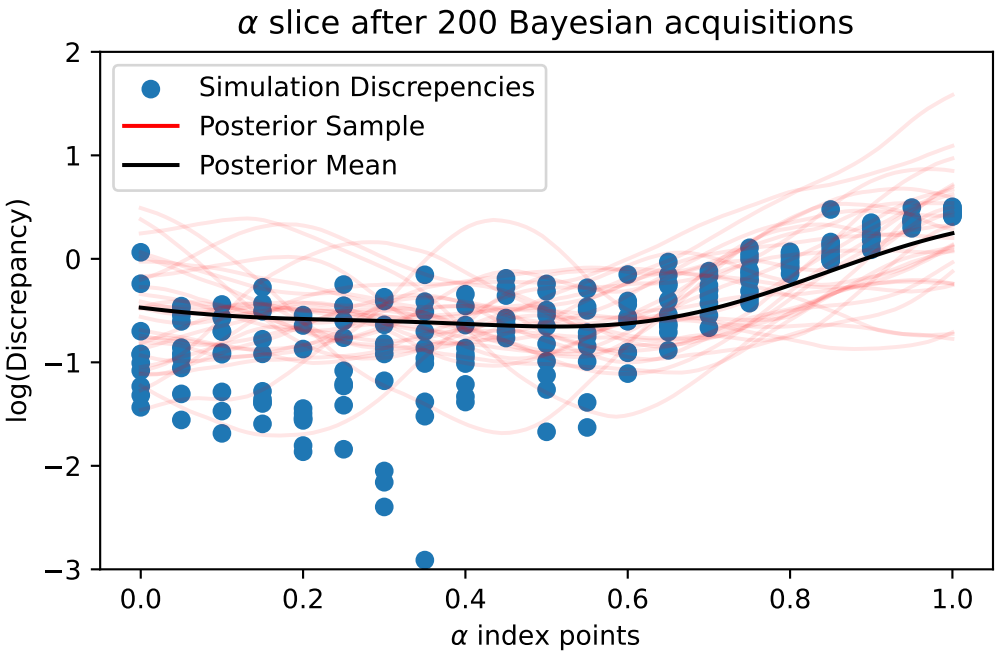
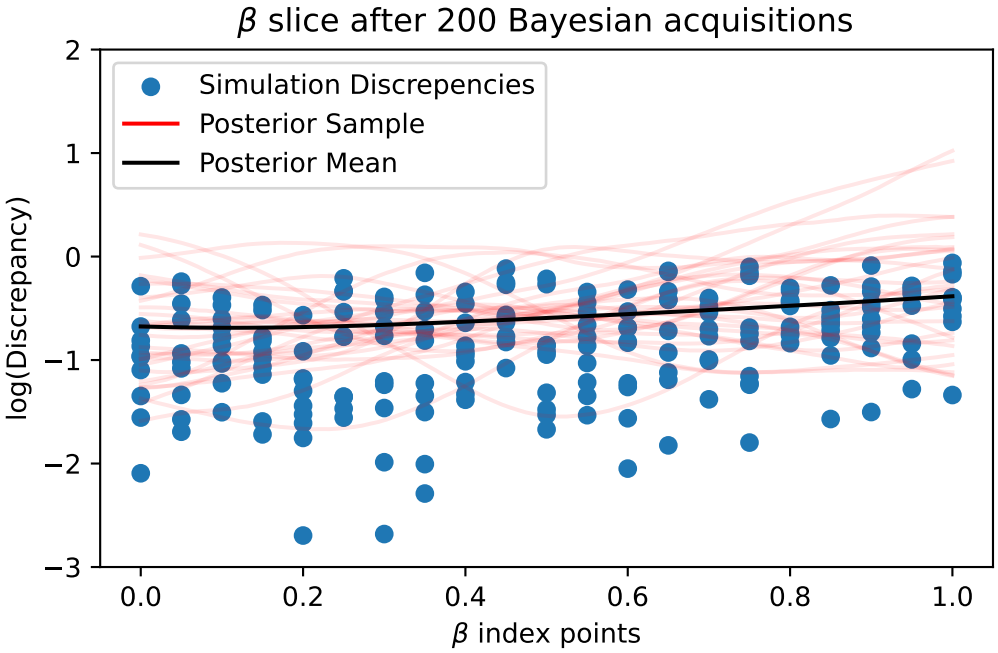
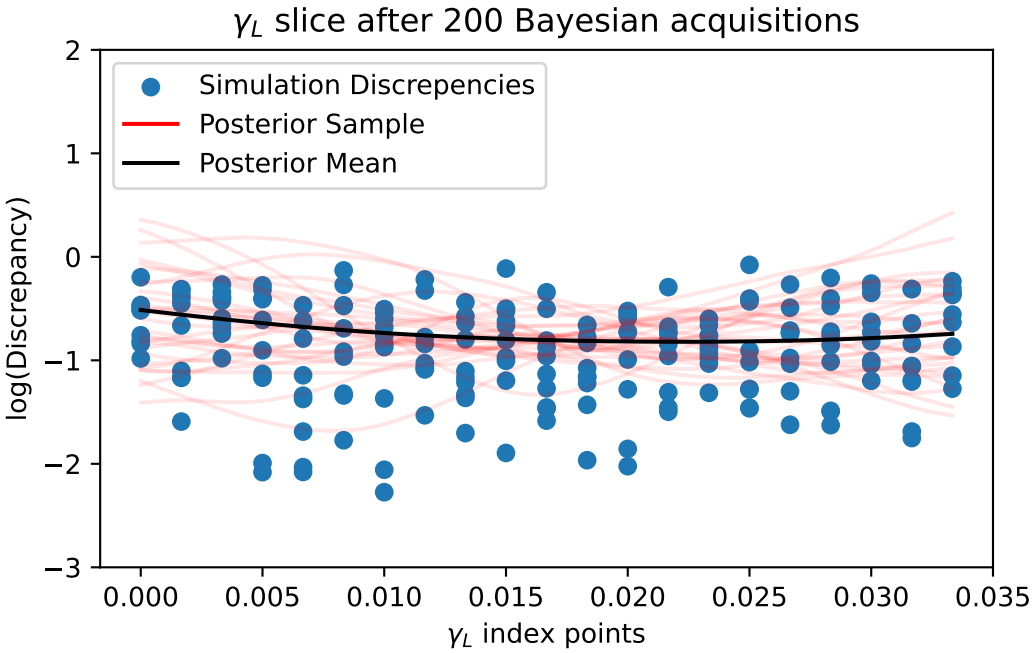

α slice after 200 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points


β slice after 200 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points


γ_L slice after 200 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
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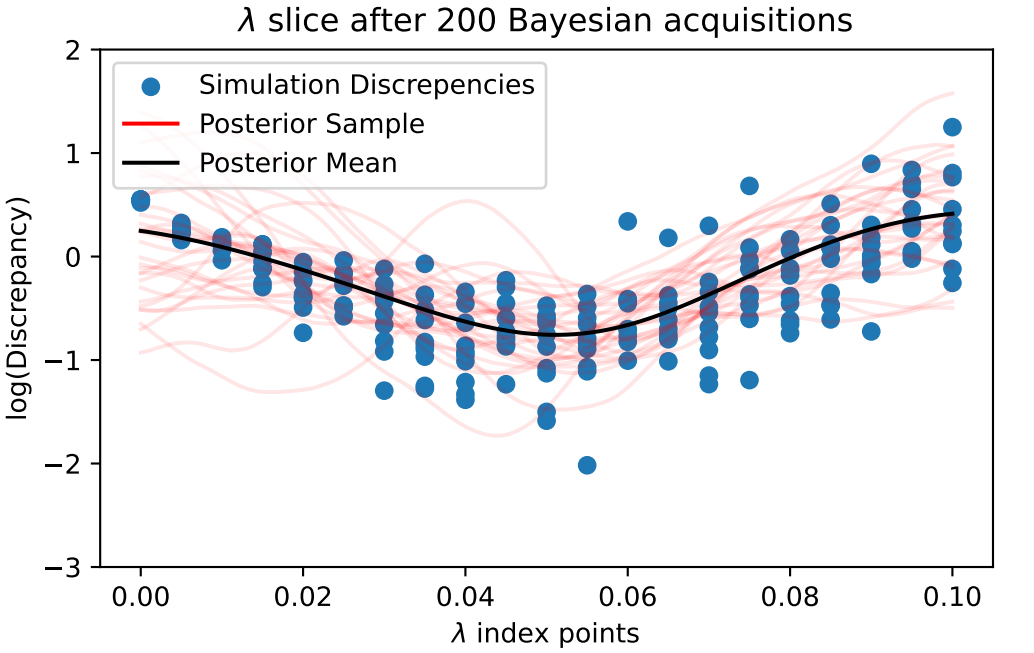
λ slice after 200 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
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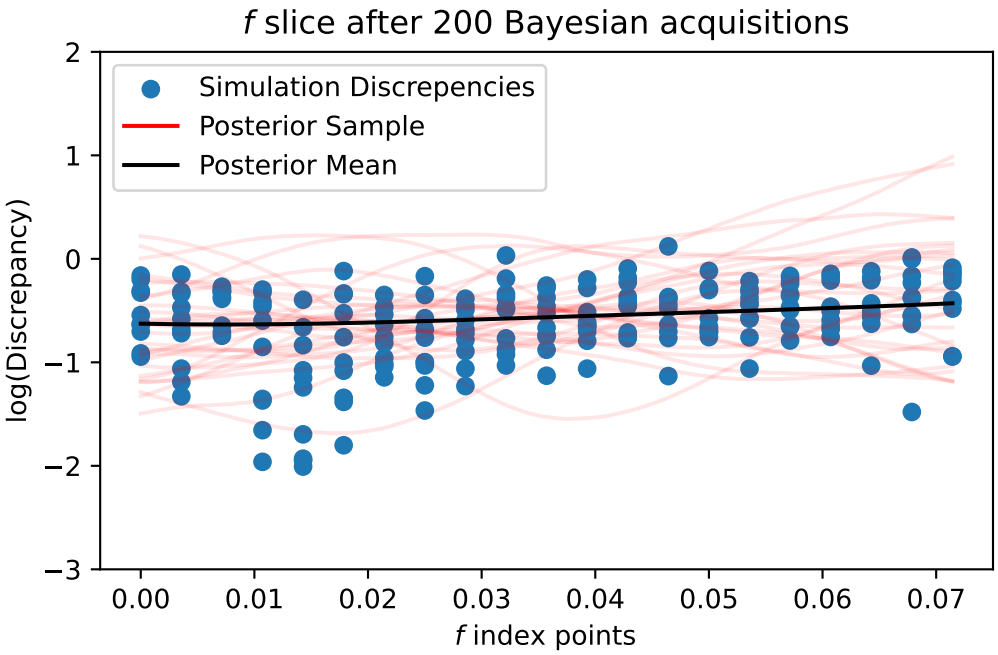
f slice after 200 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
f index points
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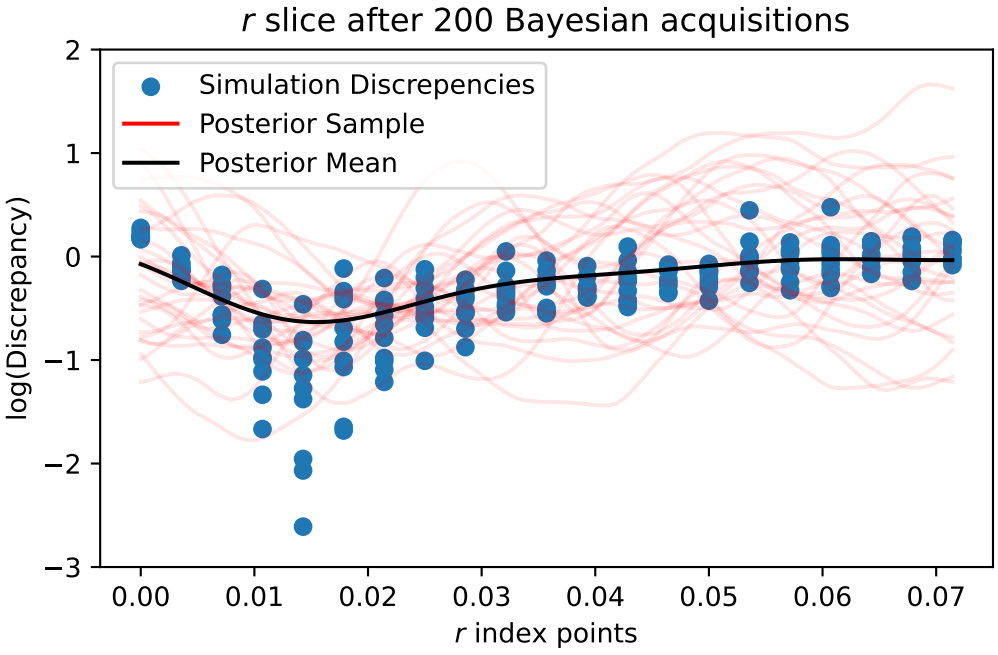
r slice after 200 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
r index points
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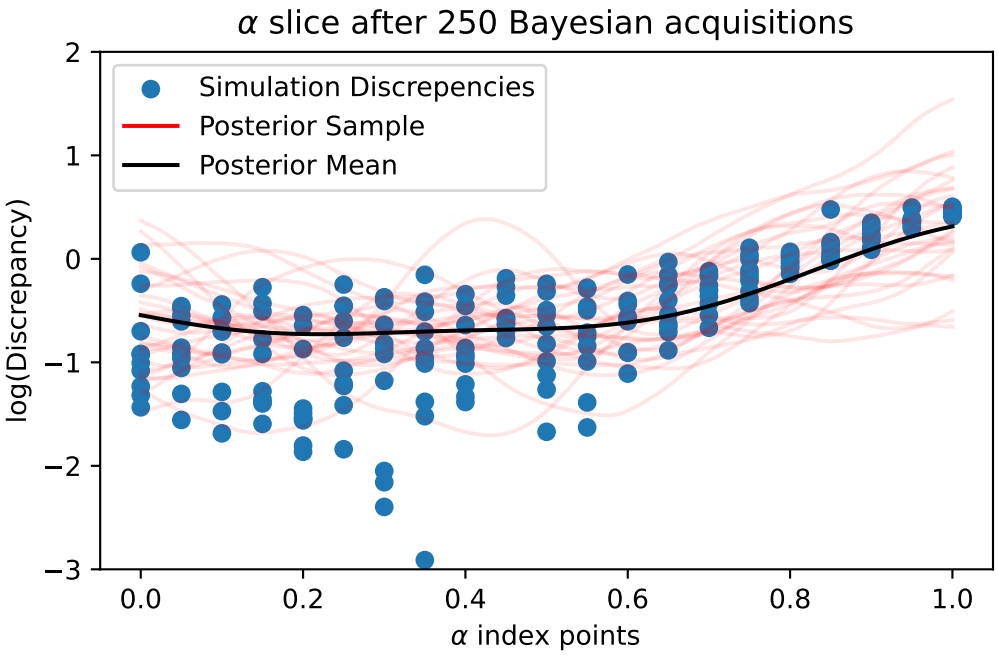
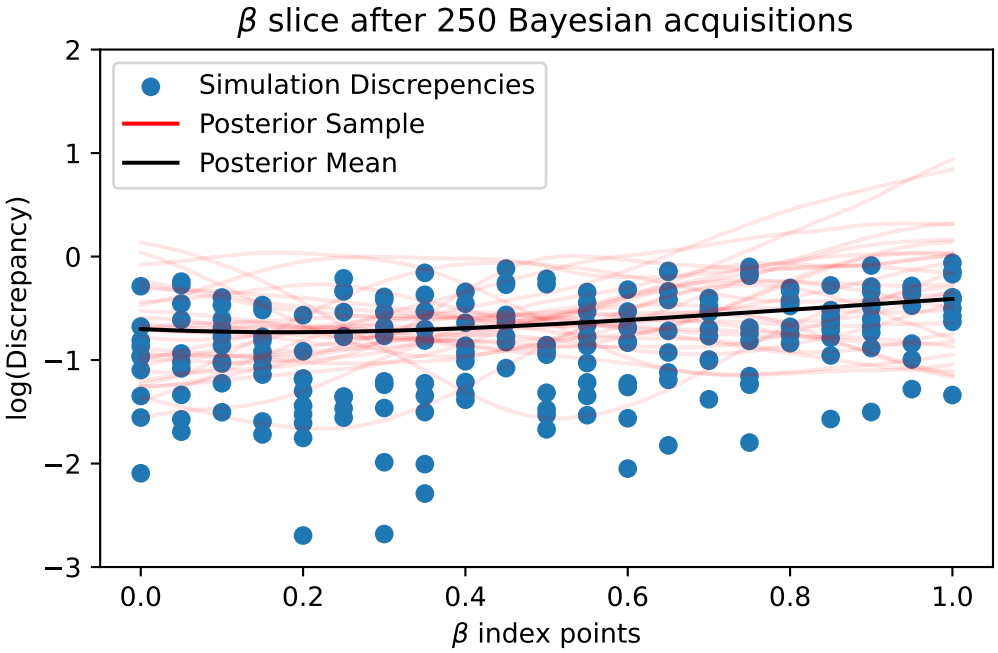
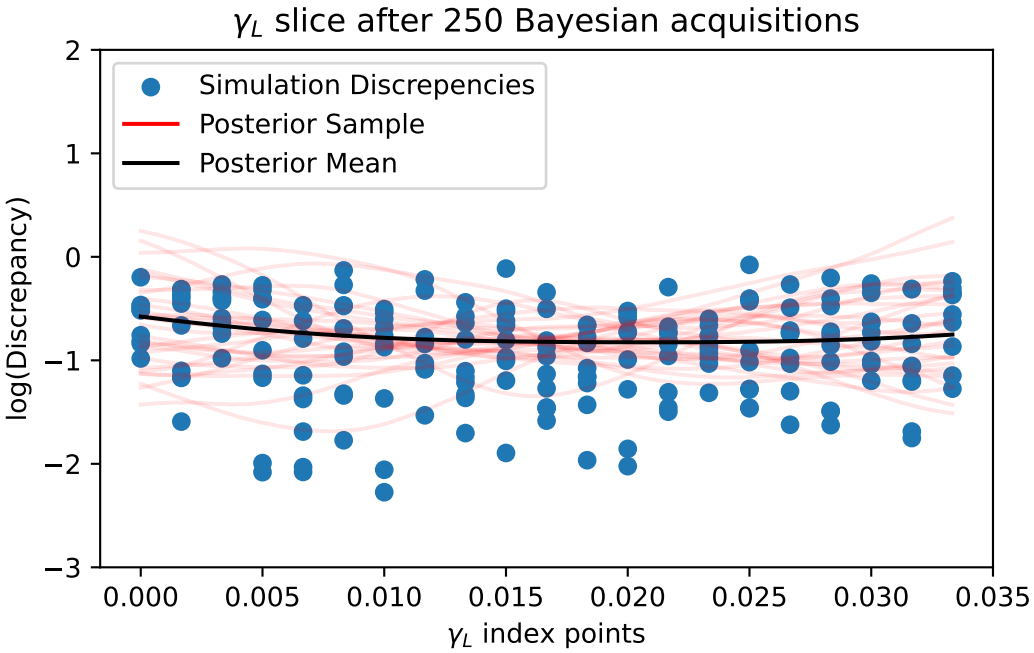

α slice after 250 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points


β slice after 250 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points


γ_L slice after 250 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
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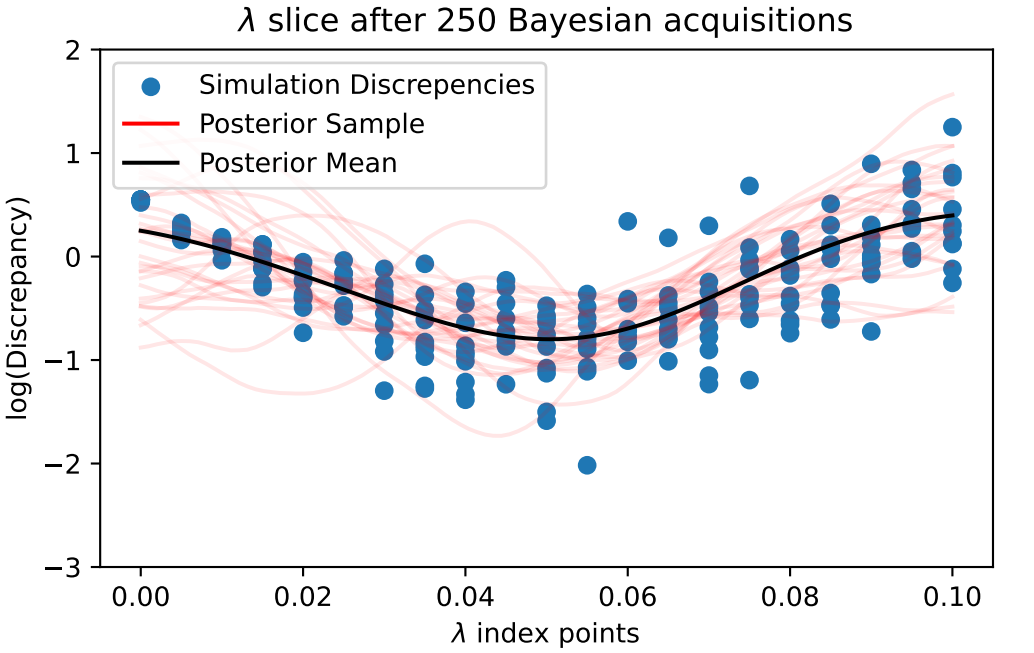
λ slice after 250 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
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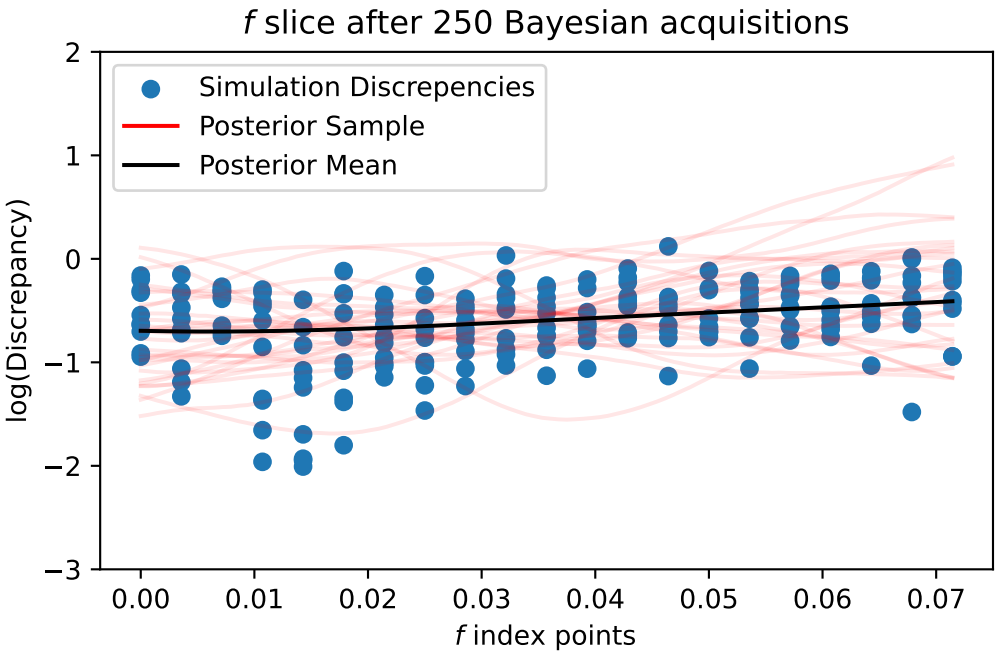
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Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
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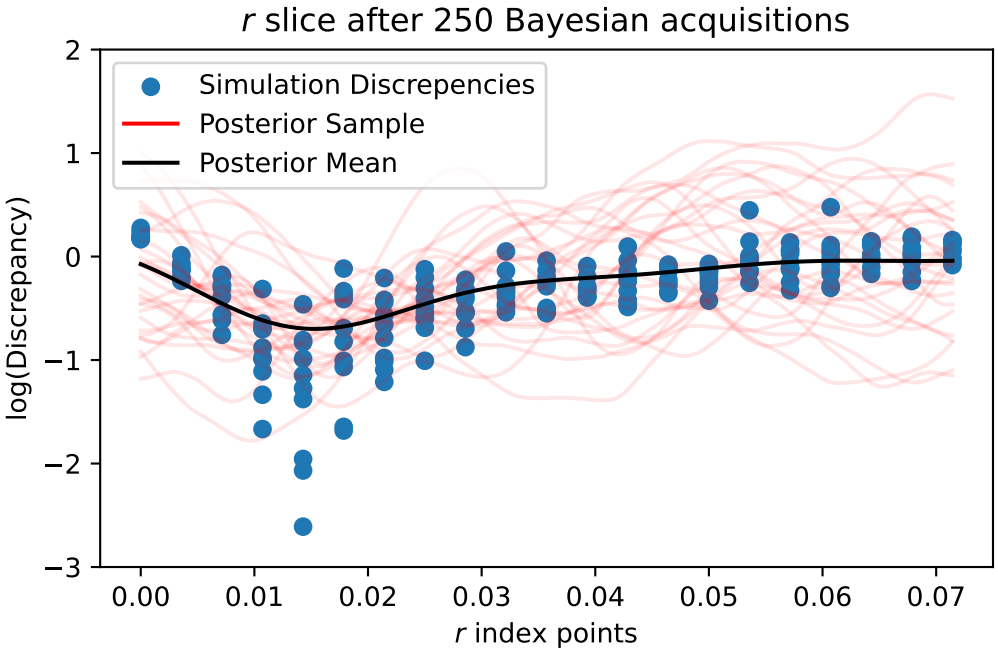
r slice after 250 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
r index points
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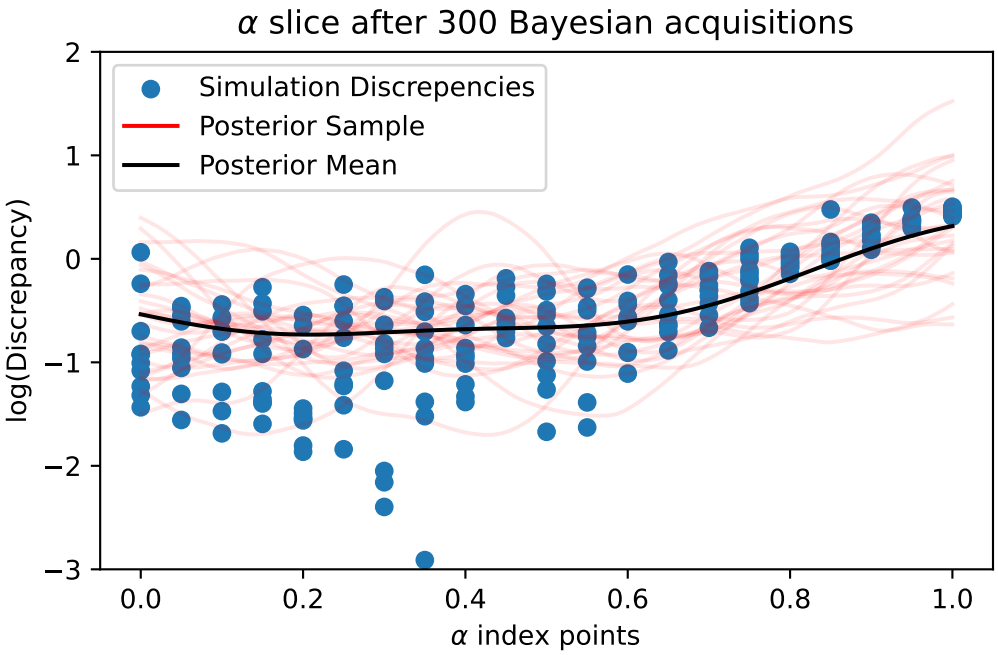
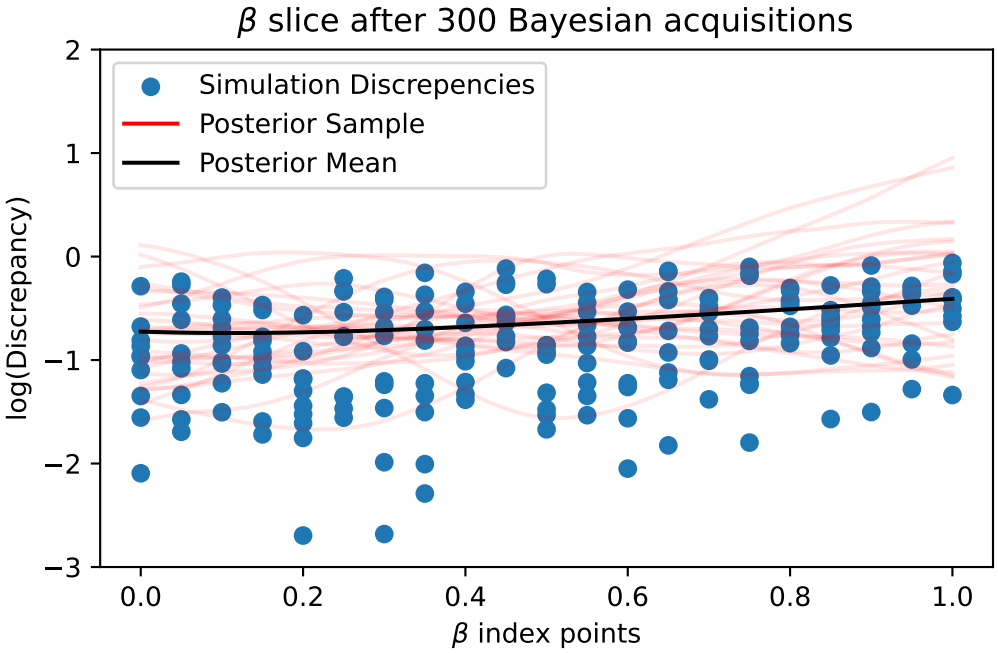

α slice after 300 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points


β slice after 300 Bayesian acquisitions
Simulation Discrepencies
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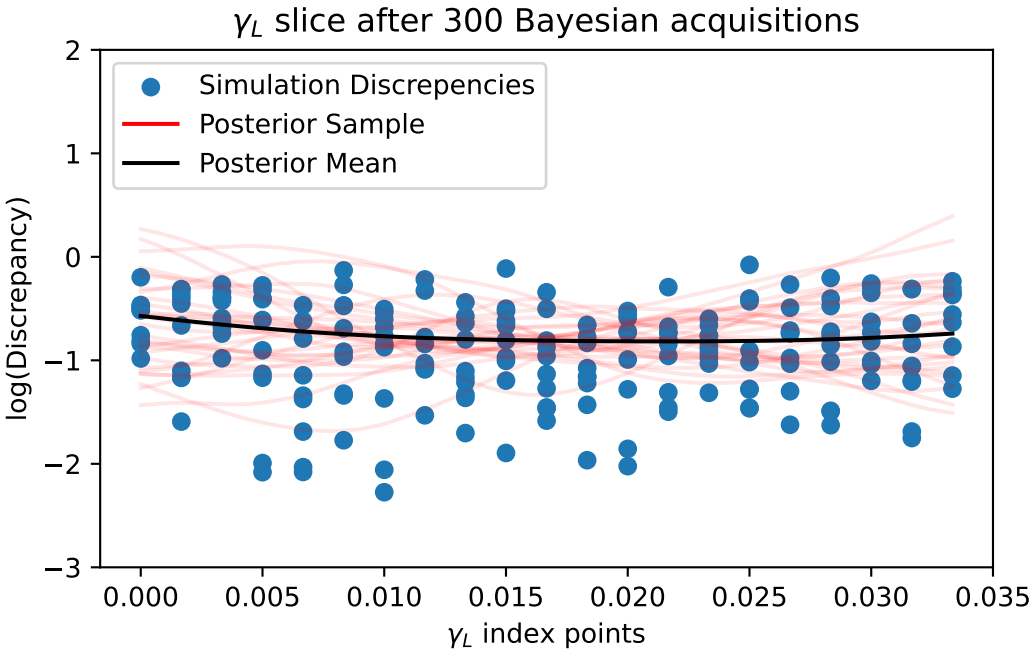
Posterior Mean
log(Discrepancy)
β index points


γ_L slice after 300 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
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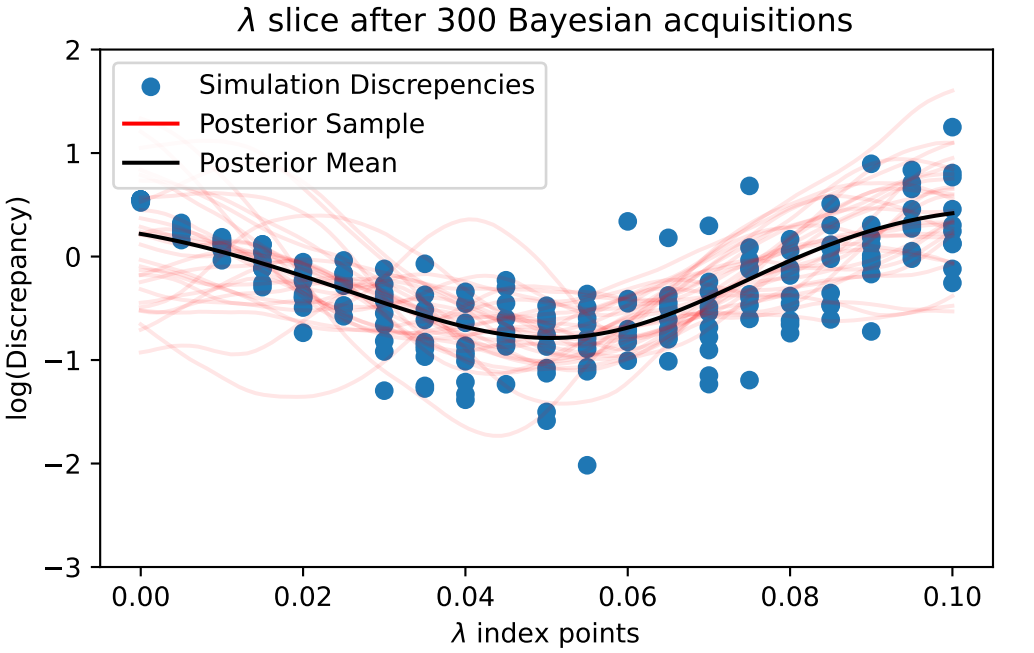
λ slice after 300 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
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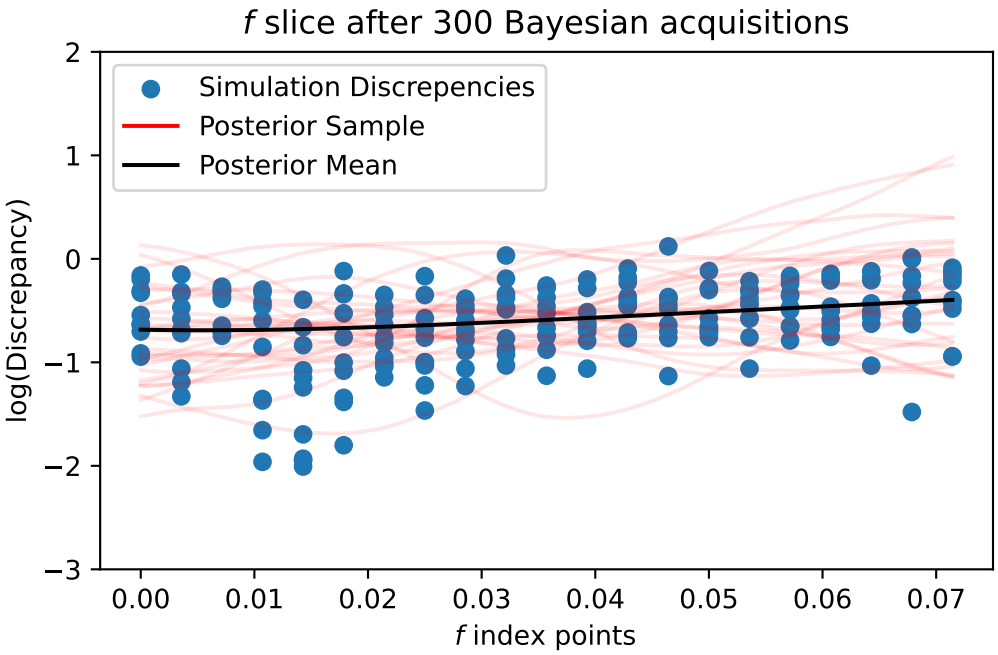
f slice after 300 Bayesian acquisitions
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Posterior Sample
Posterior Mean
log(Discrepancy)
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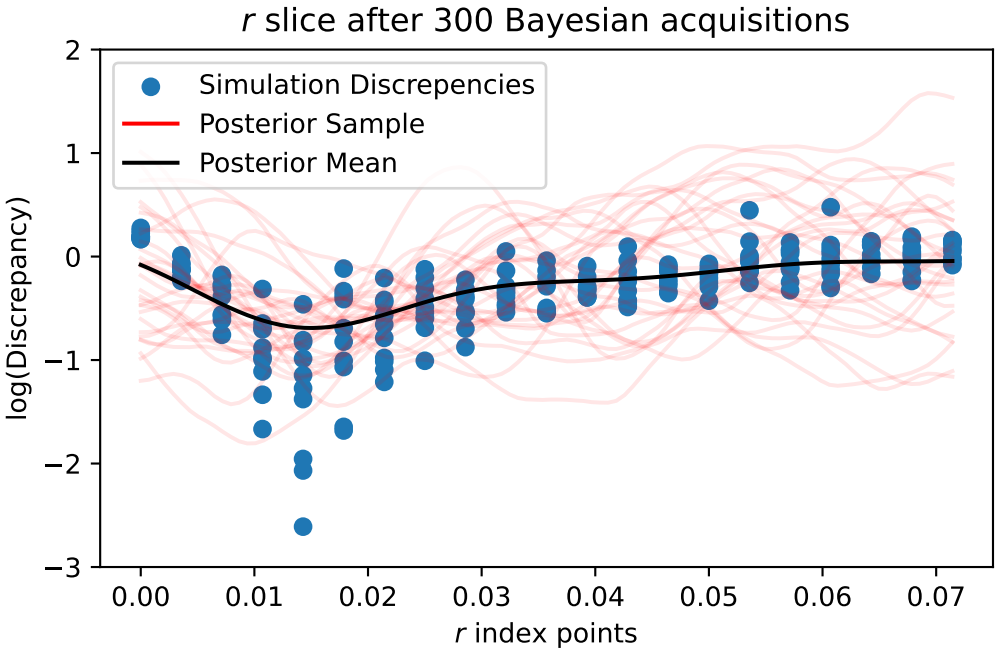
r slice after 300 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
r index points
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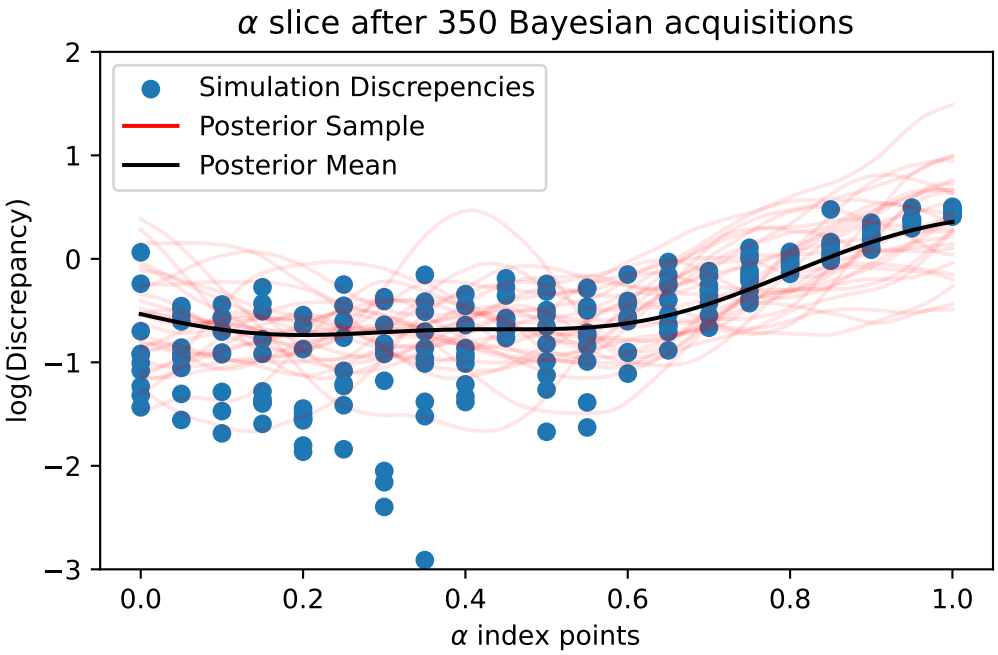
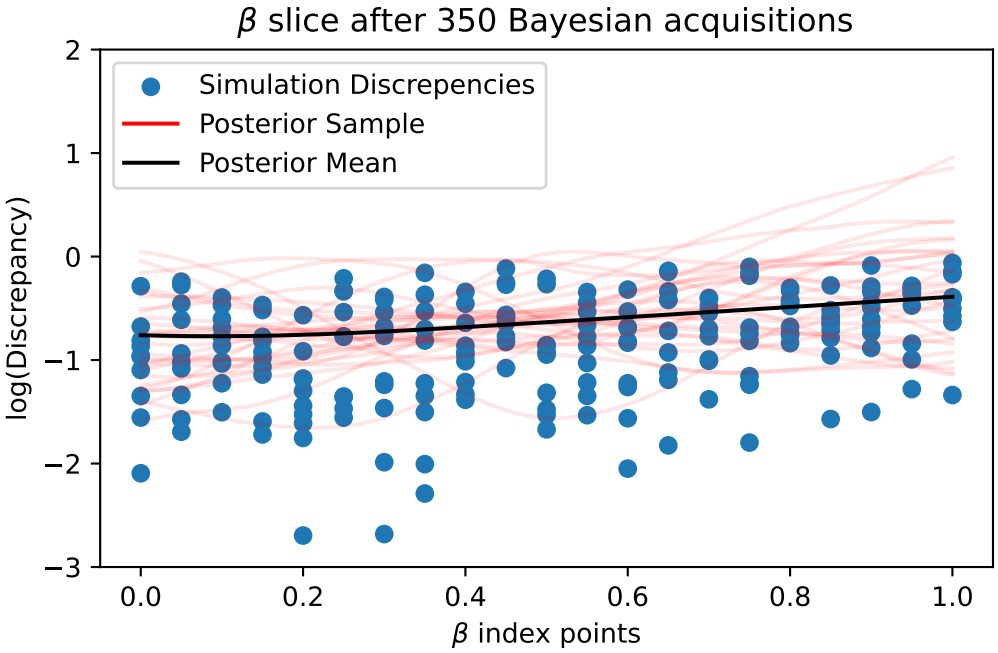
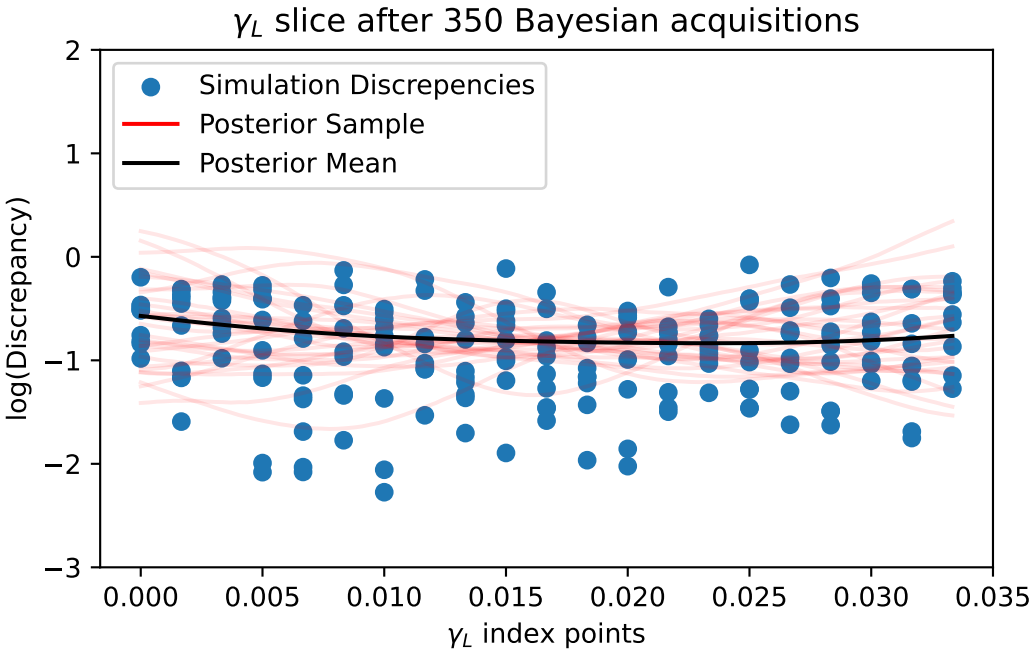

α slice after 350 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points


β slice after 350 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points


γ_L slice after 350 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
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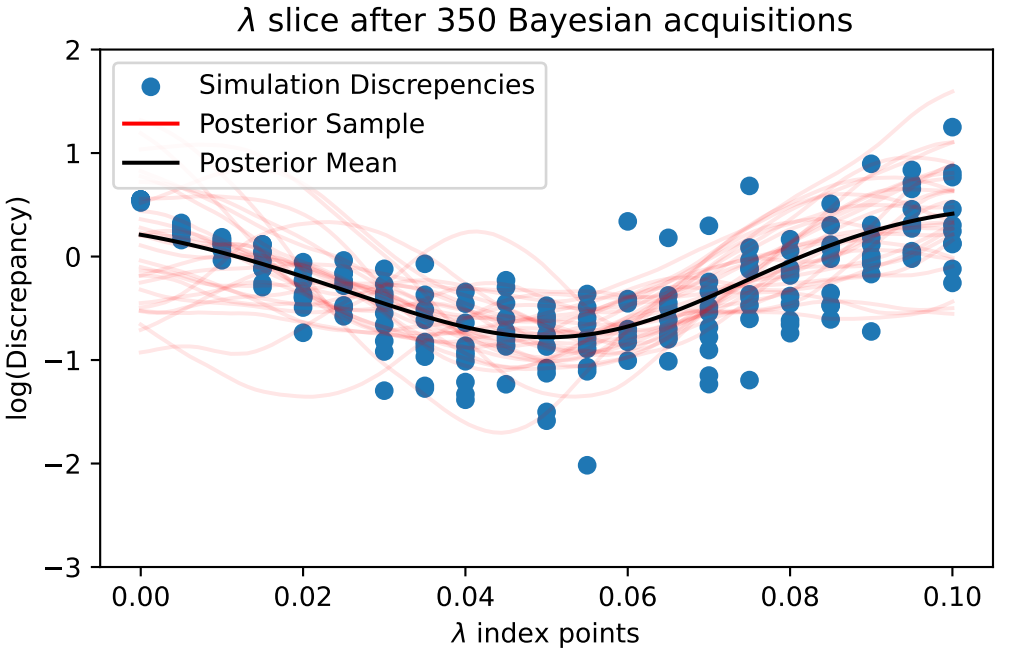
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Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
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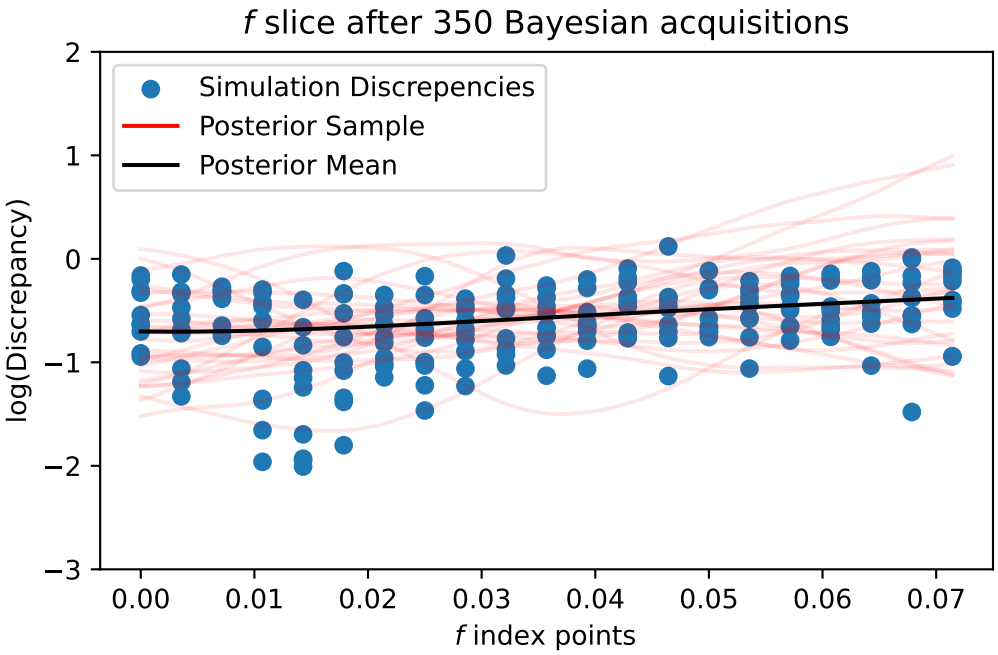
f slice after 350 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
f index points
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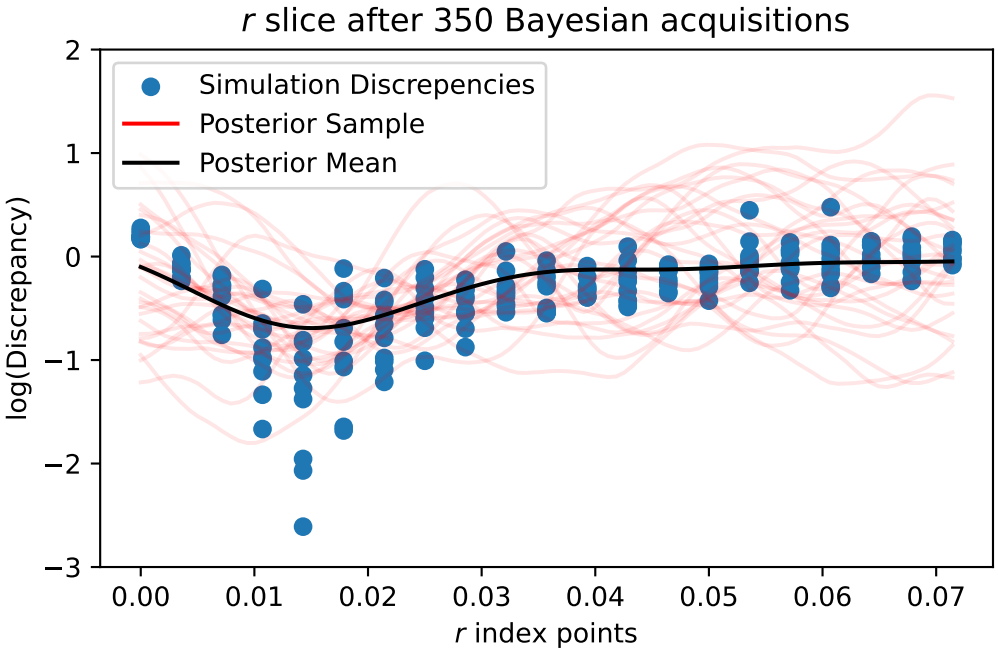
r slice after 350 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
r index points
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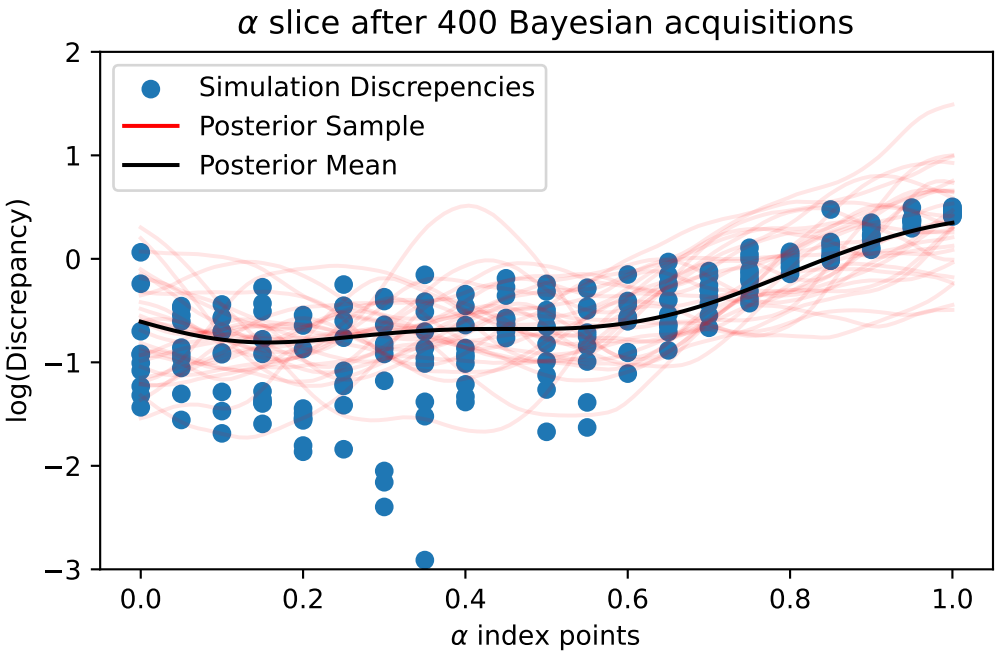
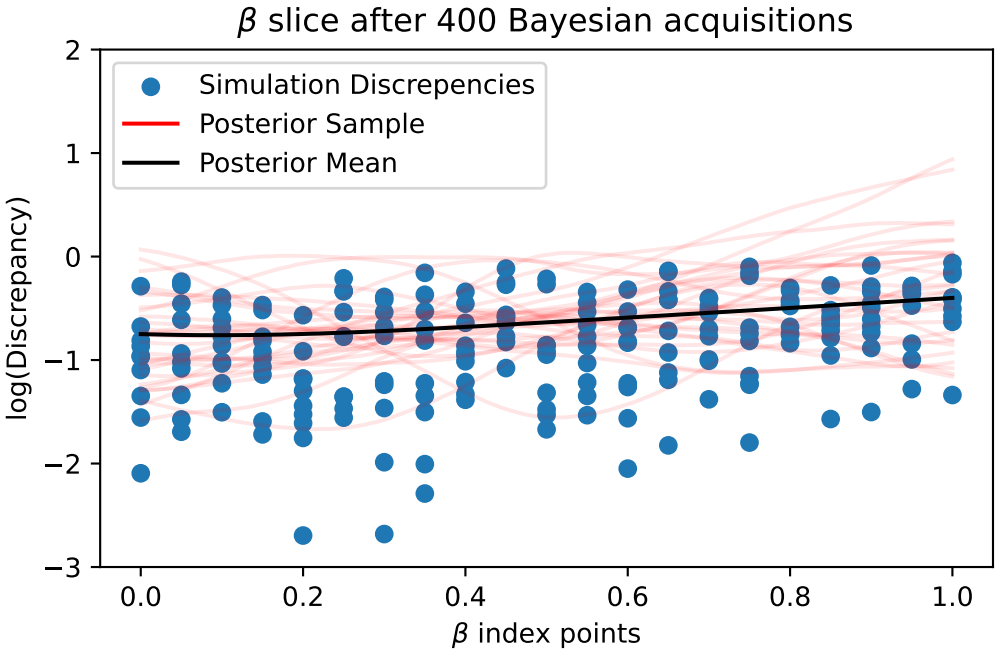
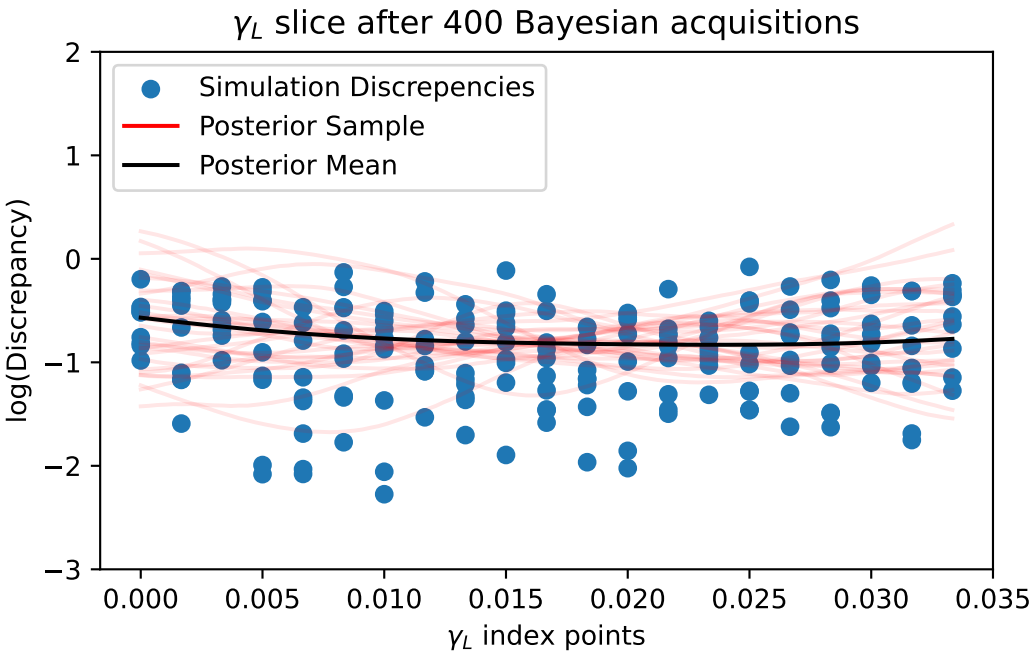
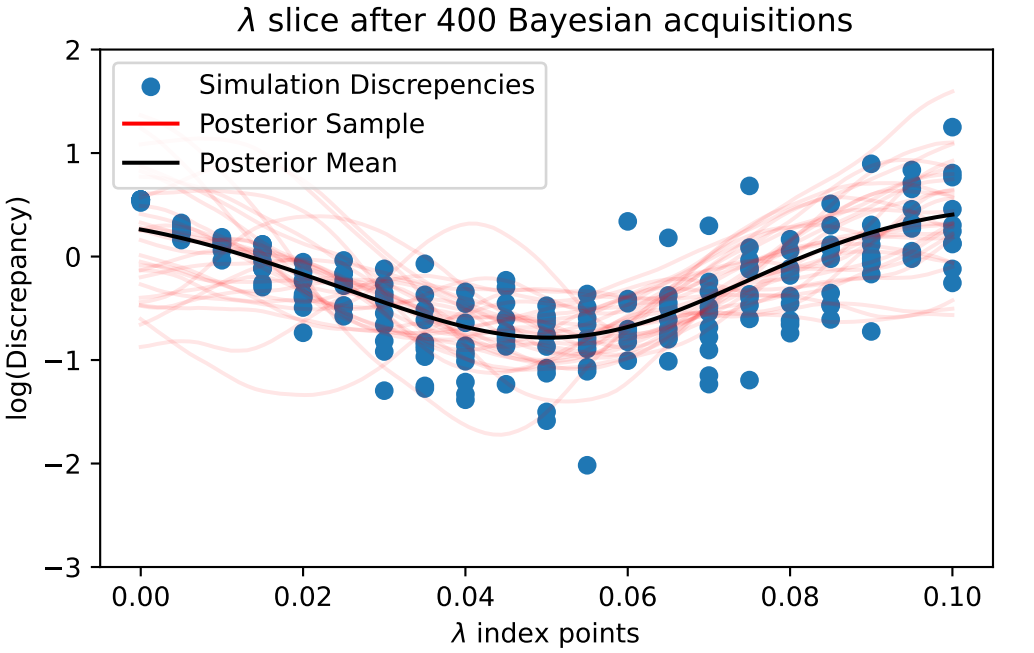
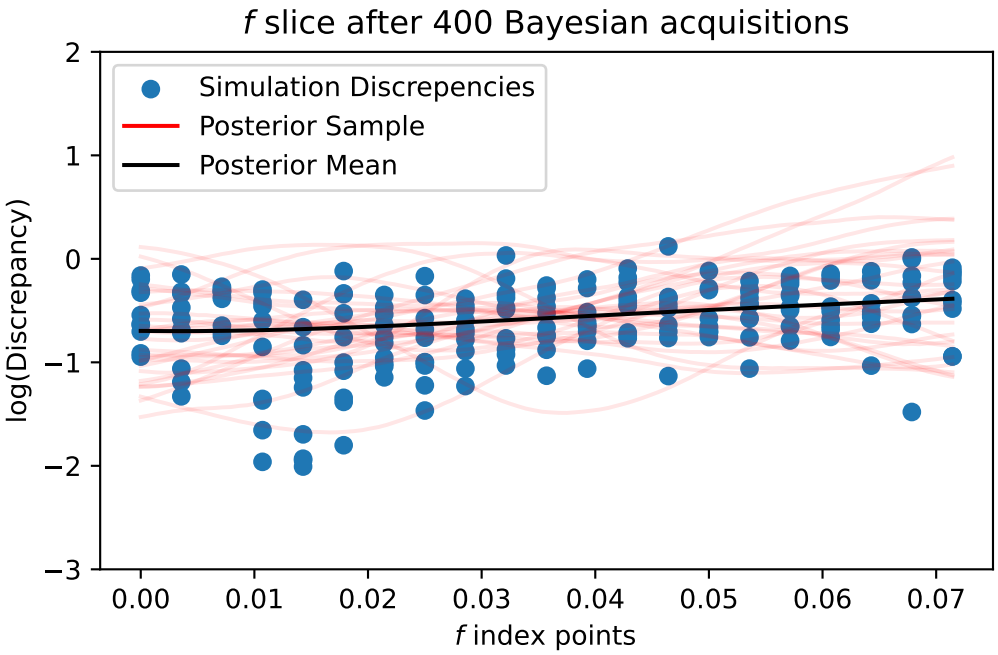
0
−1
−2
−3
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07

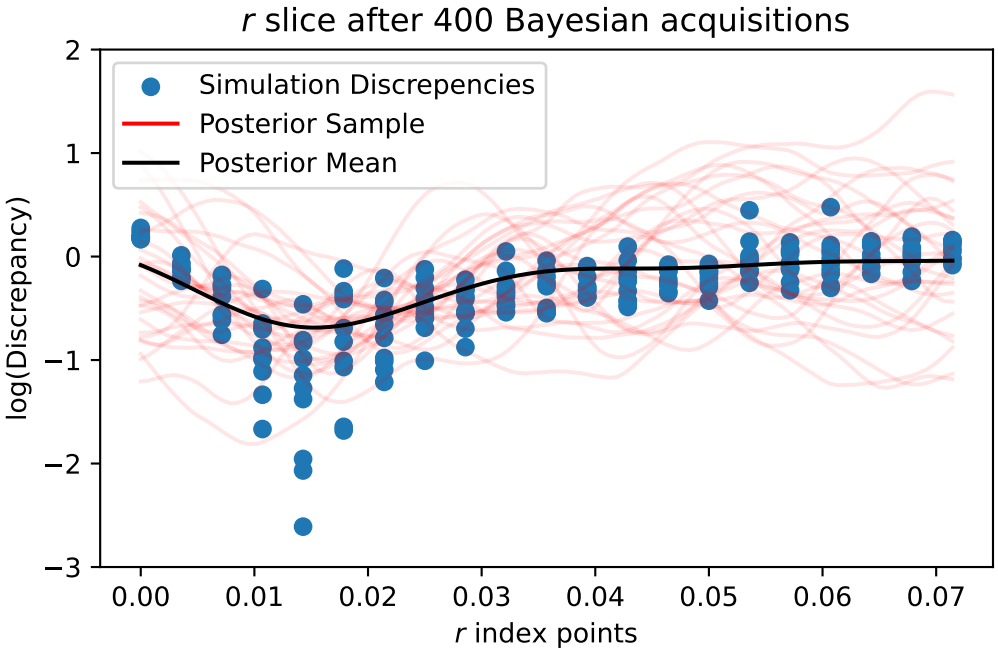

α slice after 400 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points

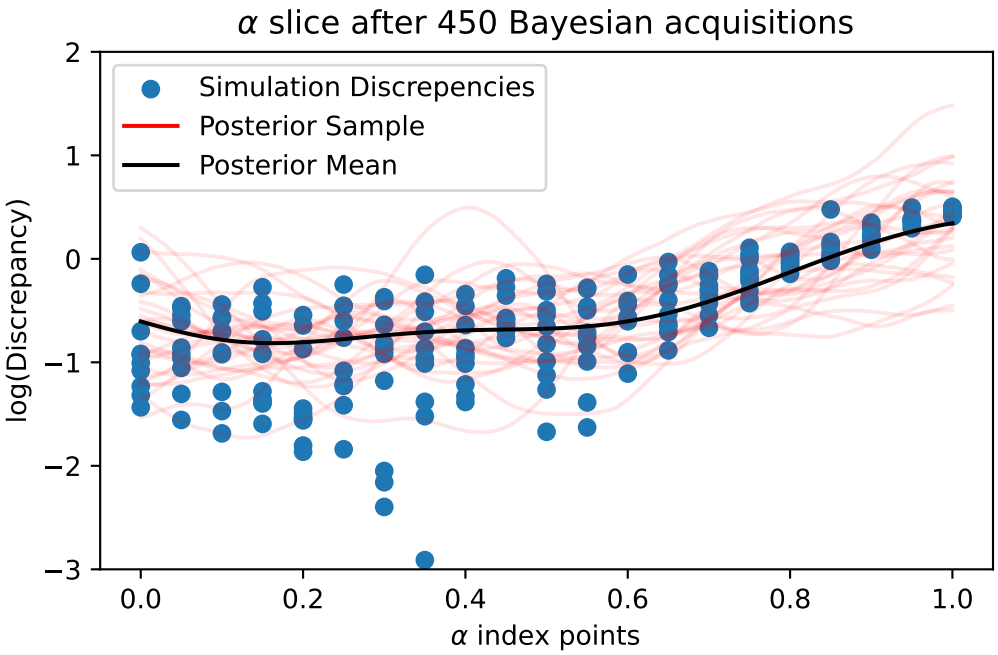

β slice after 400 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points

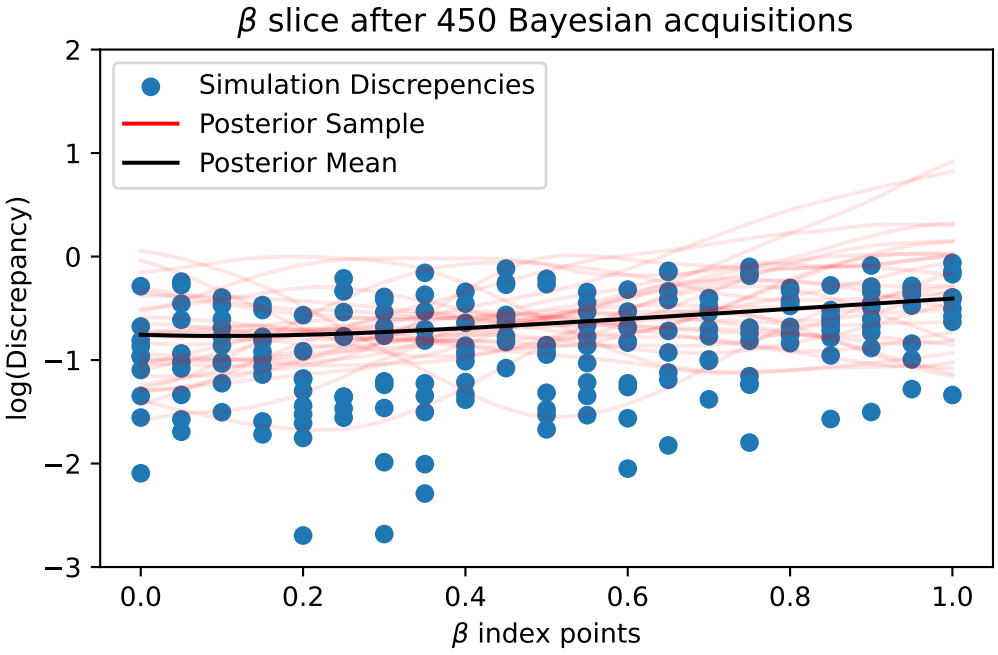

γ_L slice after 400 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
2
1
0
−1
−2
−3
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035

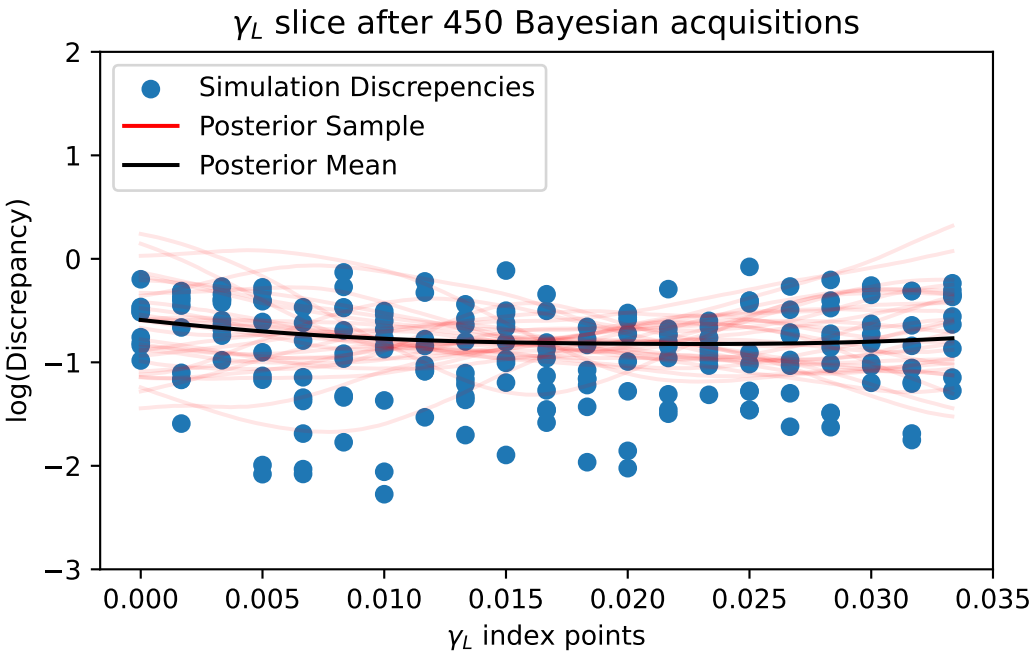

λ slice after 400 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
λ index points
2
1
0
−1
−2
−3
0.00
0.02
0.04
0.06
0.08
0.10

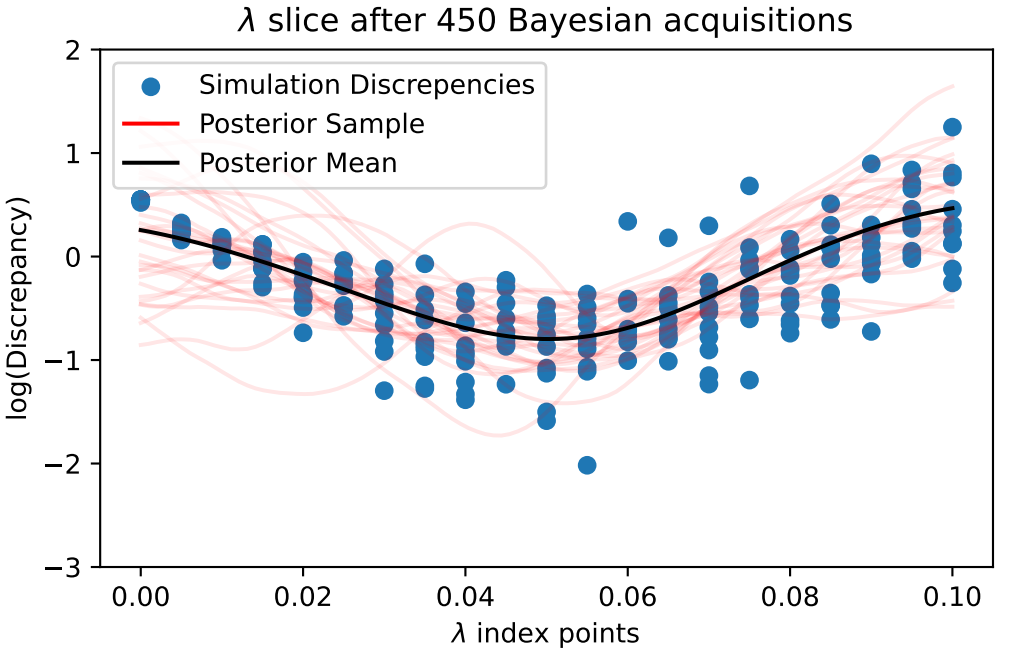

f slice after 400 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
f index points
2
1
0
−1
−2
−3
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07

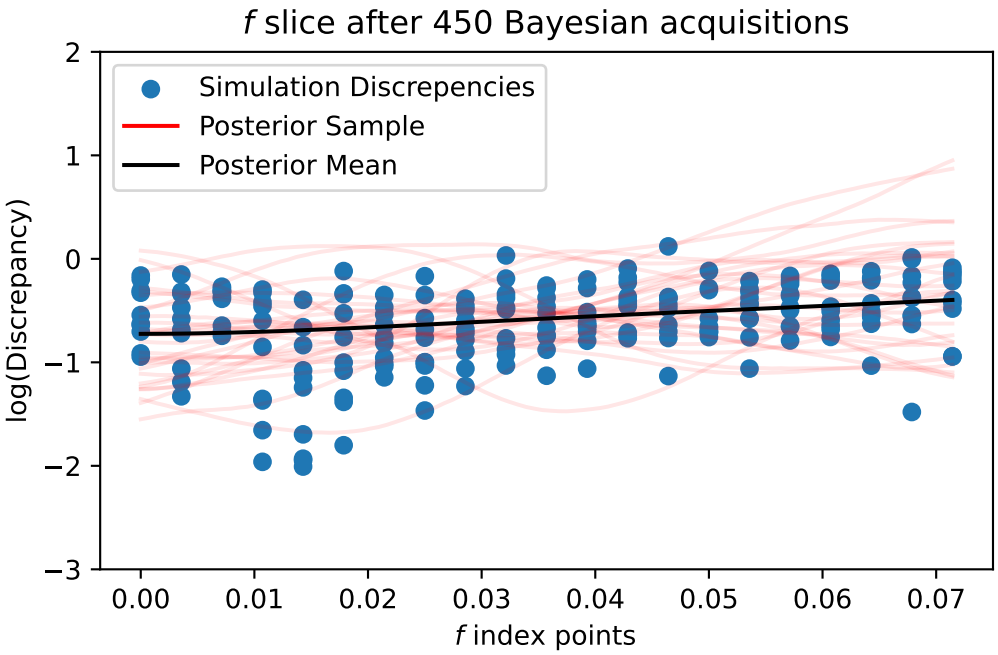

r slice after 400 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
r index points
2
1
0
−1
−2
−3
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07

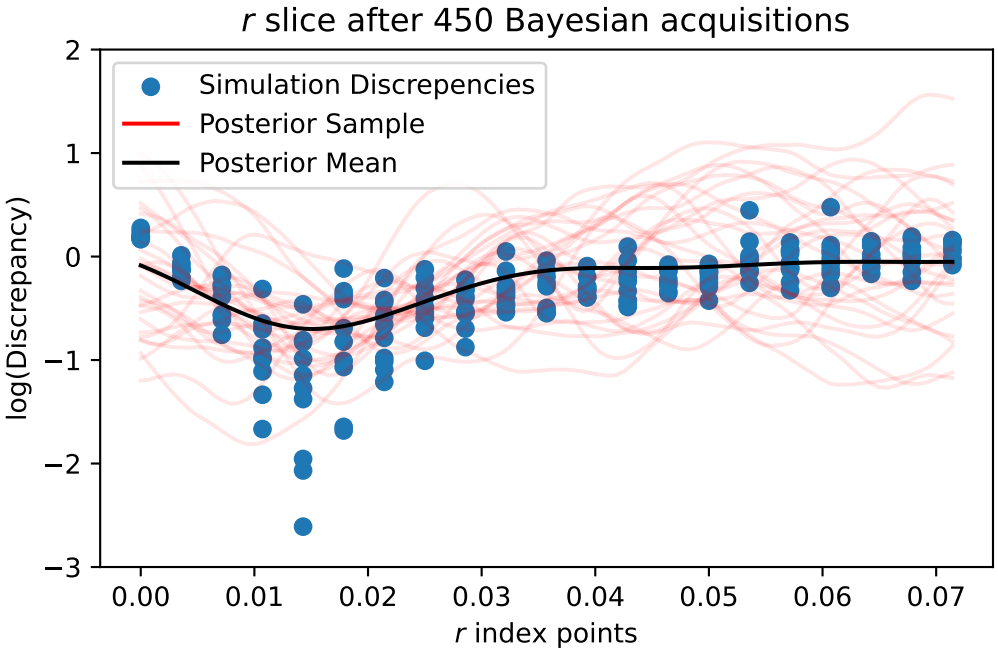

α slice after 450 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points
2
1
0
−1
−2
−3
0.0
0.2
0.4
0.6
0.8
1.0

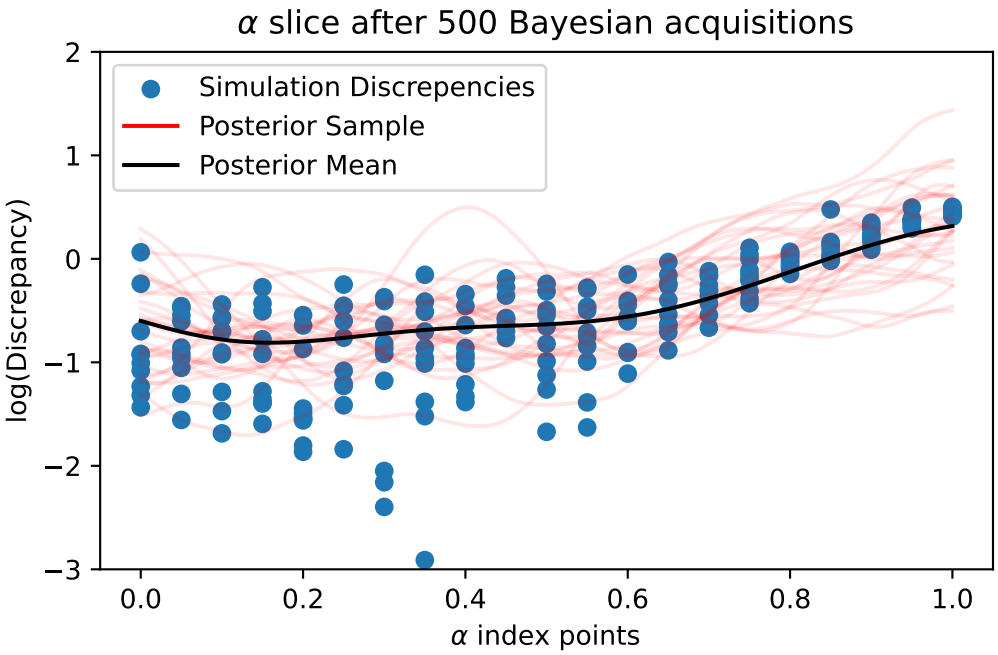

β slice after 450 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points
2
1
0
−1
−2
−3
0.0
0.2
0.4
0.6
0.8
1.0

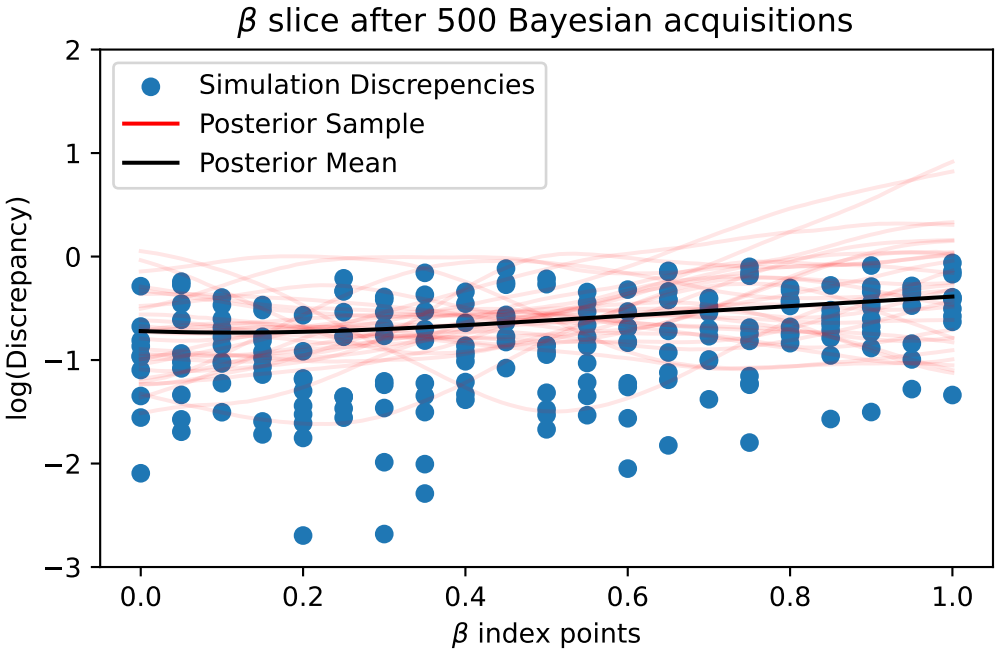

γ_L slice after 450 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
2
1
0
−1
−2
−3
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035

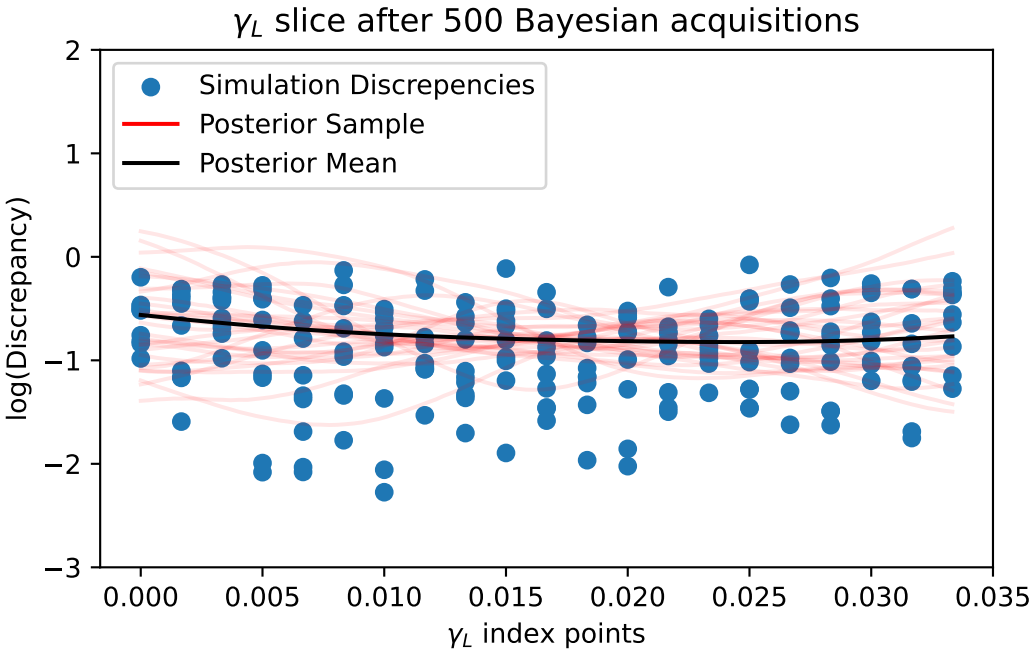

λ slice after 450 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
λ index points
2
1
0
−1
−2
−3
0.00
0.02
0.04
0.06
0.08
0.10

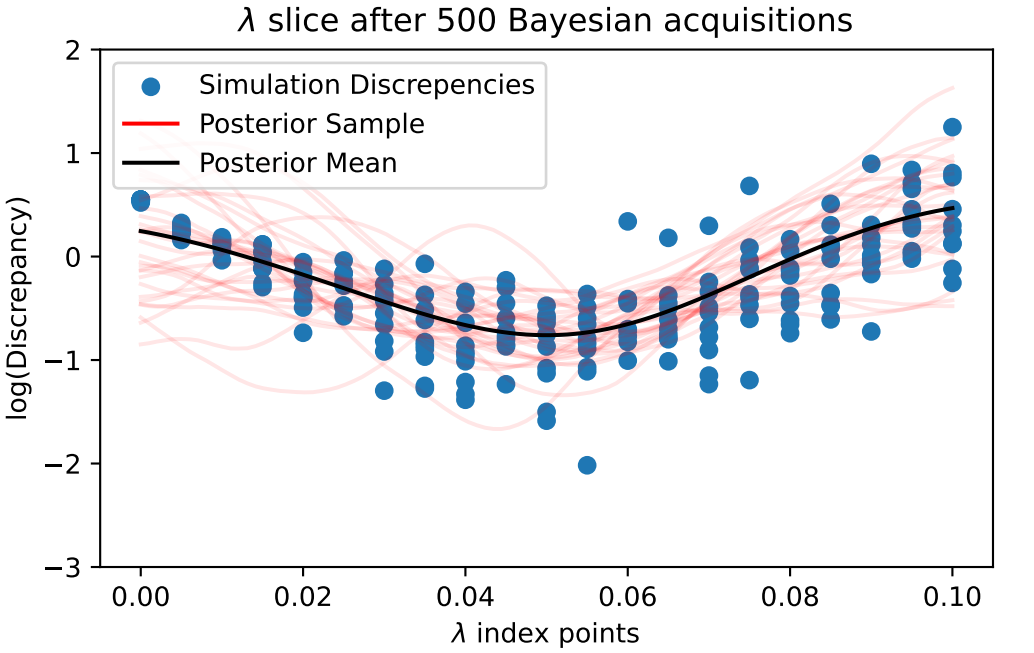

f slice after 450 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
f index points
2
1
0
−1
−2
−3
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07

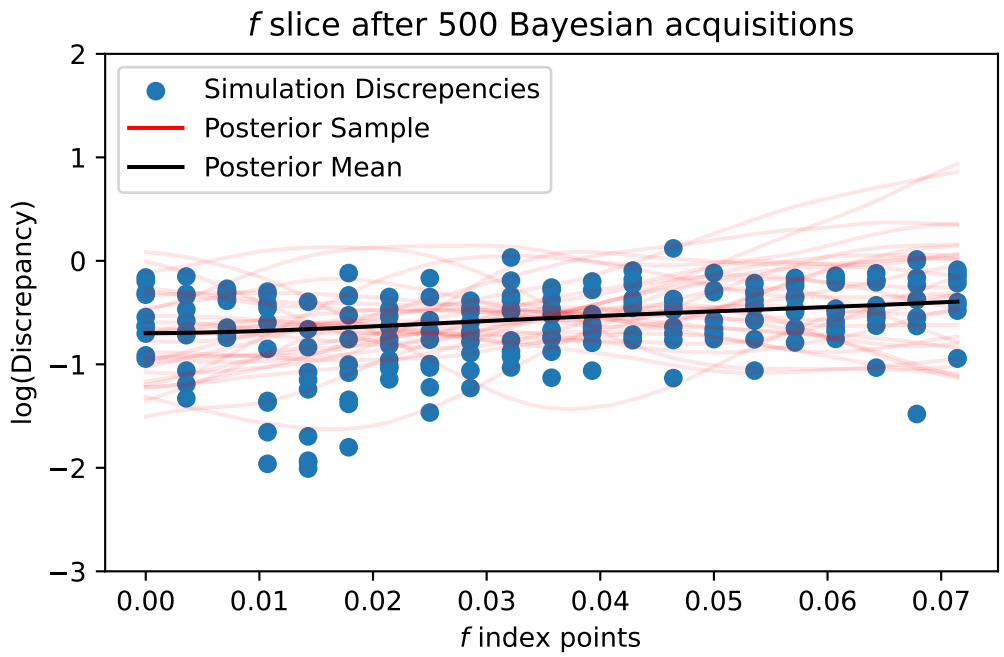

r slice after 450 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
r index points
2
1
0
−1
−2
−3
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07

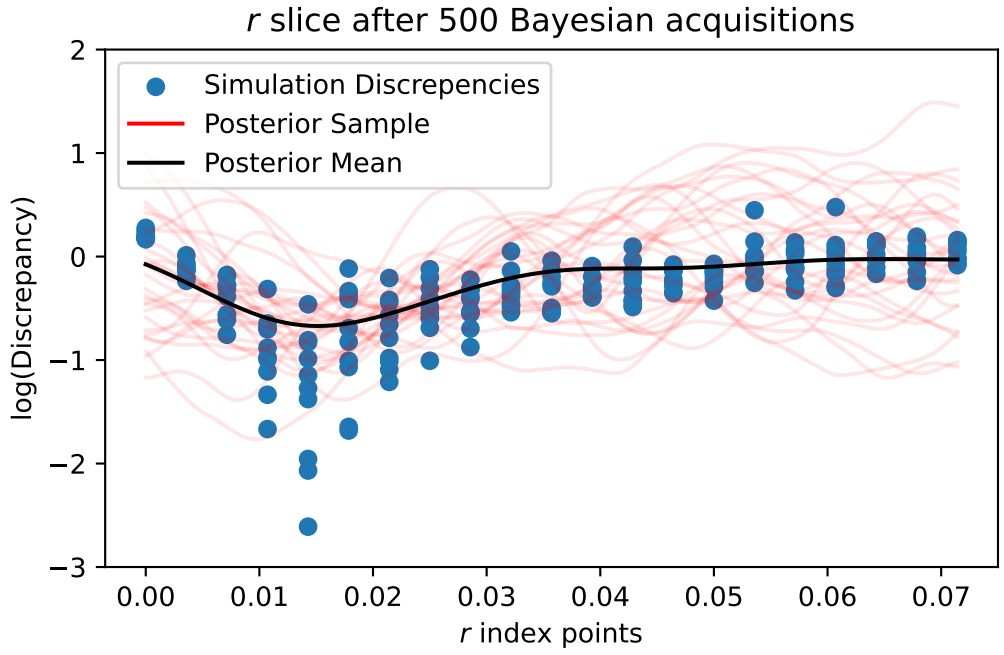

α slice after 500 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
α index points


β slice after 500 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
β index points


γ_L slice after 500 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
γ_L index points
2
1
0
−1
−2
−3
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035

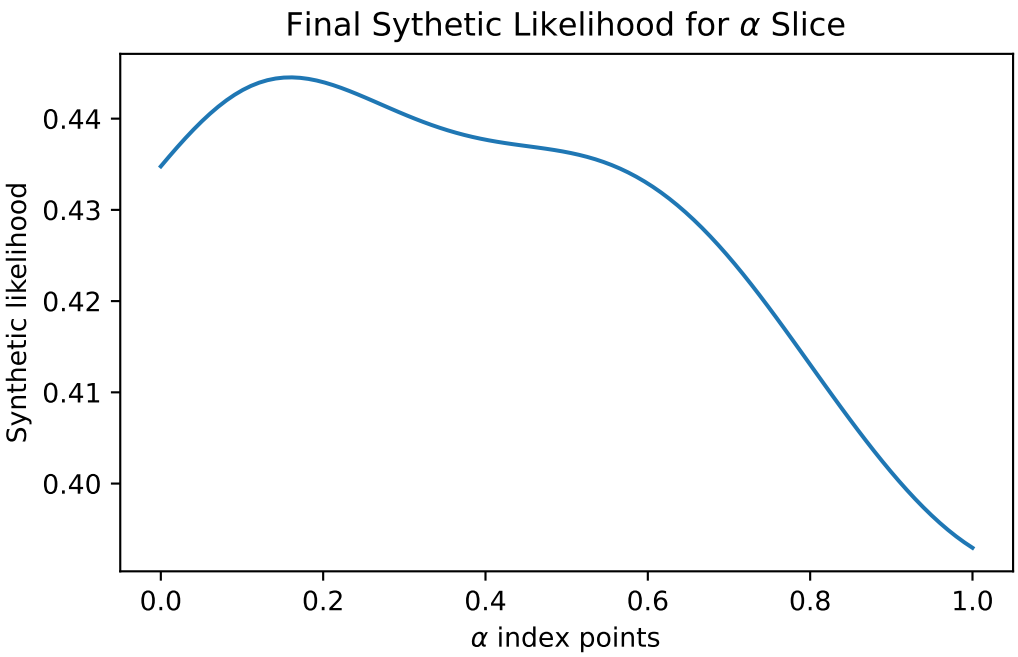

λ slice after 500 Bayesian acquisitions
Simulation Discrepencies
Posterior Sample
Posterior Mean
log(Discrepancy)
λ index points
2
1
0
−1
−2
−3
0.00
0.02
0.04
0.06
0.08
0.10

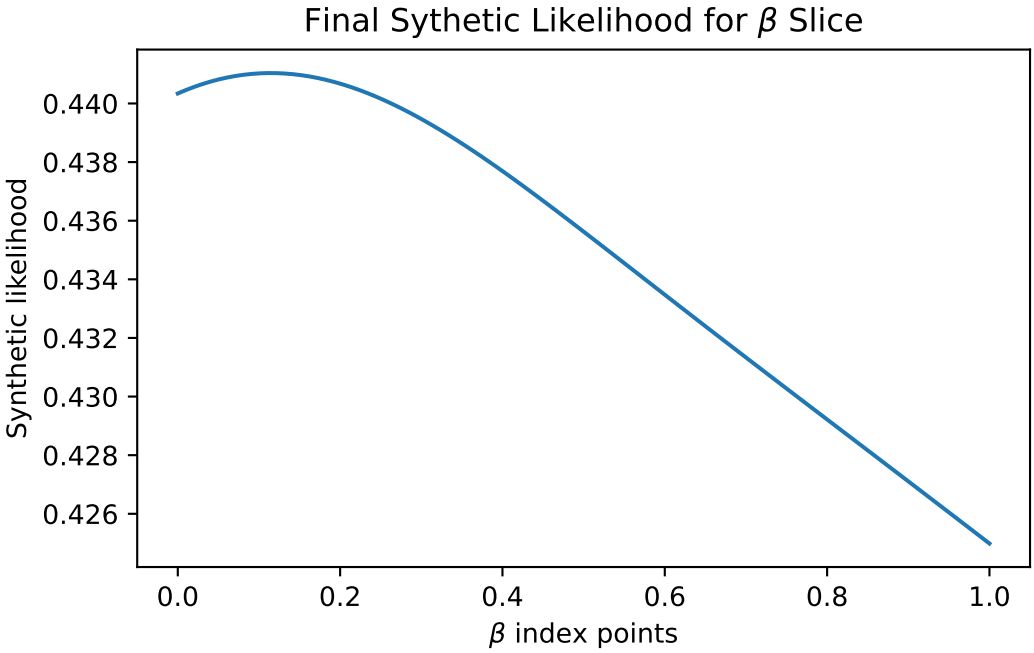


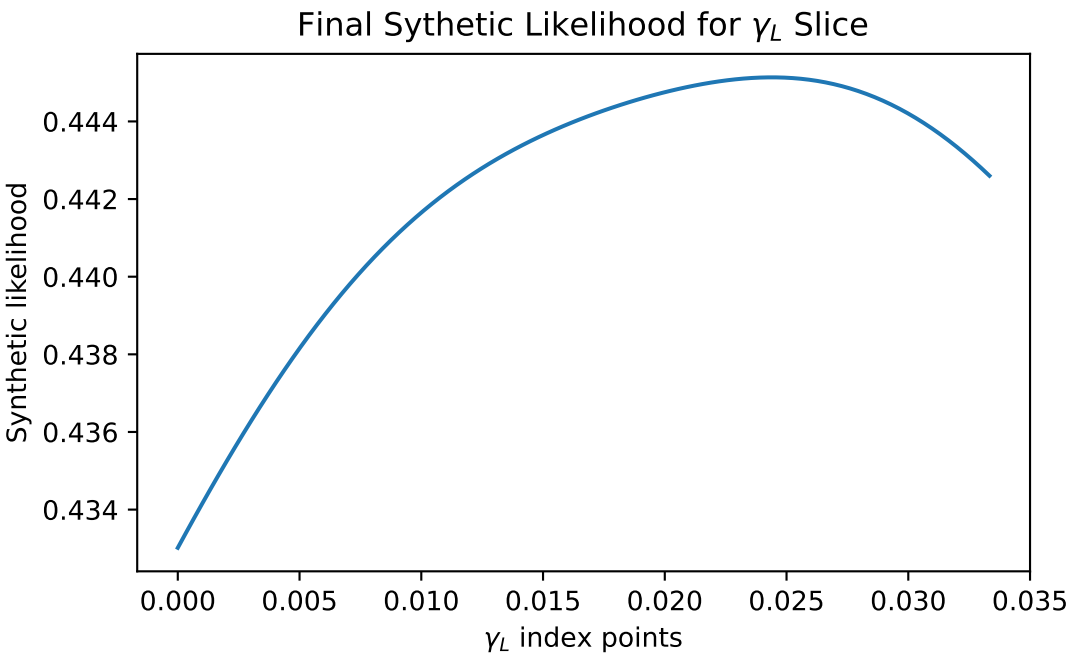


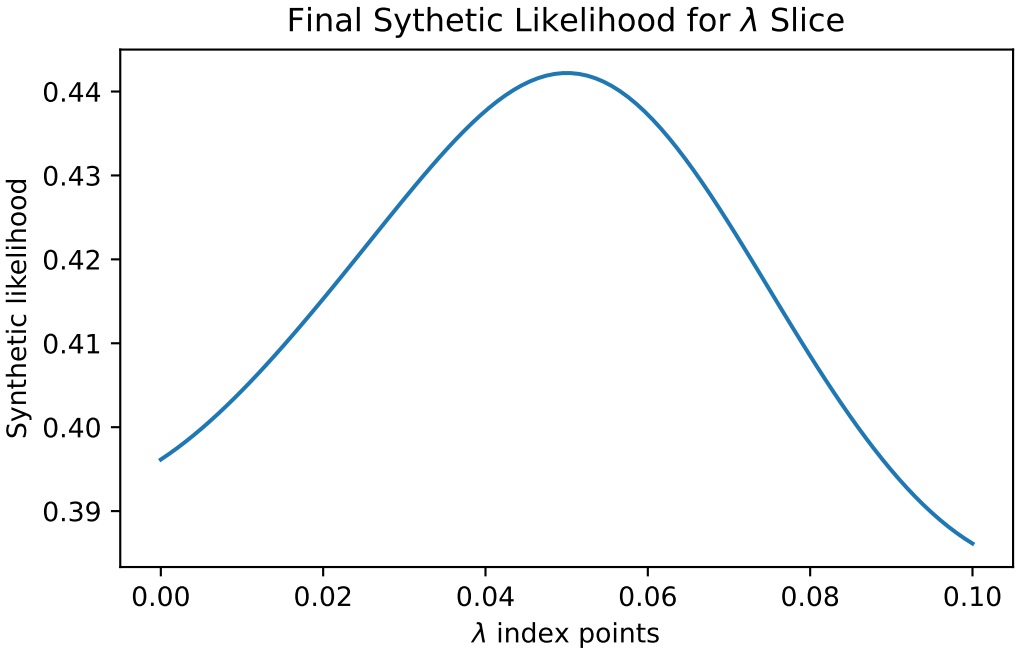

```
epsilon = -2

likelihood_dict = {}
```

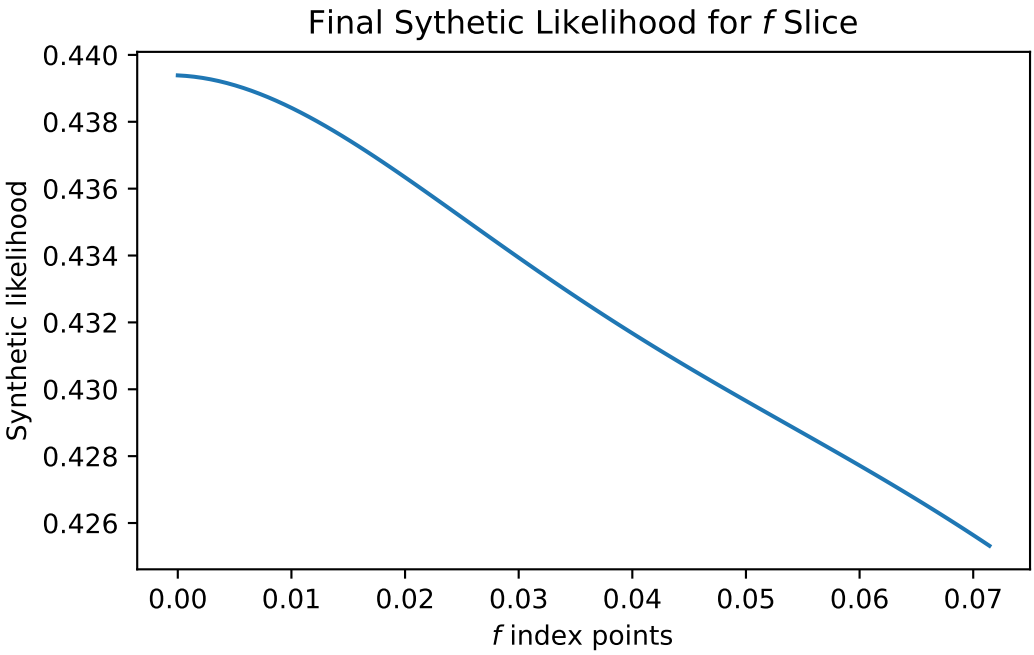
```
for var in vars:
    champ_GP_reg = tfd.GaussianProcessRegressionModel(
        kernel=kernel_champ,
        index_points=slice_indices_dfs_dict[var + "_gp_indices_df"].values,
        observation_index_points=index_vals,
        observations=obs_vals,
        observation_noise_variance=observation_noise_variance_champ,
        predictive_noise_variance=0.0,
        mean_fn=const_mean_fn(),
    )

    indices_for_lik = slice_indices_dfs_dict[var + "_gp_indices_df"].values

    mean = champ_GP_reg.mean_fn(indices_for_lik)
    variance = 30**2 * observation_noise_variance_champ.numpy()
    post_std = np.sqrt(variance)
    cdf_vals = tfd.Normal(mean, post_std).log_cdf(epsilon)
    likelihood_dict[var + "_synth_lik"] = cdf_vals

    plt.figure(figsize=(6, 3.5))
    plt.plot(
        slice_indices_dfs_dict[var + "_gp_indices_df"][var].values,
        np.exp(cdf_vals),
    )
    if var in ["f", "r"]:
        plt.xlabel("$" + var + "$ index points")
        plt.title("Final Sythetic Likelihood for $" + var + "$ Slice")
    else:
        plt.xlabel("$\\" + var + "$ index points")
        plt.title("Final Sythetic Likelihood for $\\" + var + "$ Slice")
    plt.ylabel("Synthetic likelihood")
    plt.savefig(
        "champagne_GP_images/" + var + "_slice_" + str(t) + "_synth_likelihood.pdf"
    )
    plt.show()
```

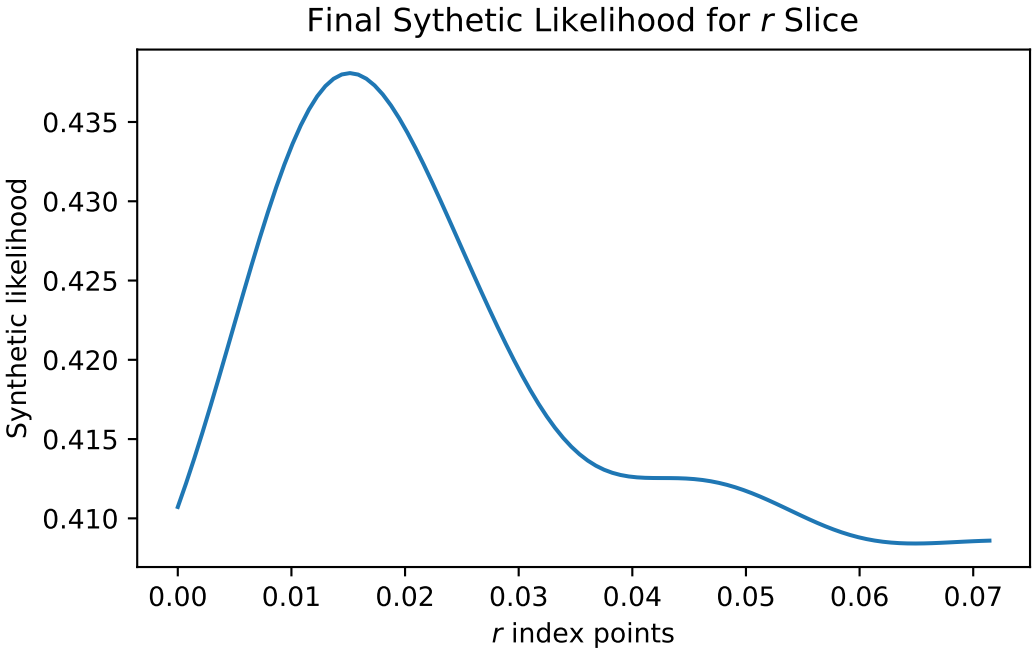

Final Sythetic Likelihood for α Slice
Synthetic likelihood
0.44
0.43
0.42
0.41
0.40
0.0
0.2
0.4
0.6
0.8
1.0
α index points


Final Sythetic Likelihood for β Slice
Synthetic likelihood
0.440
0.438
0.436
0.434
0.432
0.430
0.428
0.426
0.0
0.2
0.4
0.6
0.8
1.0
β index points


Final Sythetic Likelihood for γ_L Slice
Synthetic likelihood
0.444
0.442
0.440
0.438
0.436
0.434
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
γ_L index points


Final Sythetic Likelihood for λ Slice
Synthetic likelihood
0.44
0.43
0.42
0.41
0.40
0.39
0.00
0.02
0.04
0.06
0.08
0.10
λ index points






```
# print(index_vals[-600,].round(3))
print(index_vals[-400,].round(3))
print(index_vals[-200,].round(3))
```

```
print(index_vals[-80,].round(3))
print(index_vals[-40,].round(3))
print(index_vals[-20,].round(3))
print(index_vals[-8,].round(3))
print(index_vals[-4,].round(3))
print(index_vals[-2,].round(3))
print(index_vals[-1,].round(3))
```

```
[0.567 0.993 0.029 0.06  0.006 0.018]
[0.846 0.144 0.006 0.088 0.015 0.01 ]
[0.602 0.913 0.033 0.045 0.    0.021]
[0.554 0.003 0.033 0.064 0.009 0.033]
[0.501 0.3   0.007 0.063 0.031 0.021]
[0.967 0.699 0.019 0.062 0.006 0.   ]
[0.591 0.747 0.024 0.093 0.01  0.029]
[0.124 0.154 0.016 0.059 0.002 0.026]
[0.124 0.154 0.017 0.059 0.002 0.026]
```

```
objects_to_preserve = [
    index_vals,
    discreps,
    champ_samp,
    initial_losses_LOOCV,
    slice_samples_dict,
    slice_discrepencies_dict,
    LHC_indices_df,
    gp_samples_dict,
    likelihood_dict,
]

with open("gp_objs.pkl", "wb") as fp:
    pickle.dump(objects_to_preserve, fp)
    print("dictionary saved successfully to file")
```

```
dictionary saved successfully to file
```