Inference on the Champagne Model using a Gaussian Process

TODO

- Set seed for LHC and stuff
- Change to log discrepency with custom observation variance
- Change from MLE to cross validation

Setting up the Champagne Model

Imports

```
import pandas as pd
import numpy as np
from typing import Any
import matplotlib.pyplot as plt

from scipy.stats import qmc

import tensorflow as tf
import tensorflow_probability as tfp

tfb = tfp.bijectors
tfd = tfp.distributions
tfk = tfp.math.psd_kernels
# tfp_acq = tfp.experimental.bayesopt.acquisition
```

```
2024-04-19 04:49:24.512285: I tensorflow/core/platform/cpu_feature_guard.cc:210] This Tensorl
To enable the following instructions: AVX2 FMA, in other operations, rebuild TensorFlow with
2024-04-19 04:49:25.148310: W tensorflow/compiler/tf2tensorrt/utils/py_utils.cc:38] TF-TRT Wa
```

Model itself

```
np.random.seed(590154)

population = 1000
initial_infecteds = 10
epidemic_length = 1000
number_of_events = 15000

pv_champ_alpha = 0.4  # prop of effective care
pv_champ_beta = 0.4  # prop of radical cure
pv_champ_gamma_L = 1 / 223  # liver stage clearance rate
pv_champ_delta = 0.05  # prop of imported cases
pv_champ_lambda = 0.04  # transmission rate
pv_champ_f = 1 / 72  # relapse frequency
pv_champ_r = 1 / 60  # blood stage clearance rate


def champagne_stochastic(
    alpha_,
    beta_,
    gamma_L,
    lambda_,
    f,
    r,
    N=population,
    I_L=initial_infecteds,
    I_0=0,
    S_L=0,
    delta_=0,
    end_time=epidemic_length,
    num_events=number_of_events,
):
    if (0 > (alpha_ or beta_)) or (1 < (alpha_ or beta_)):
        return "Alpha or Beta out of bounds"
    if 0 > (gamma_L or lambda_ or f or r):
        return "Gamma, lambda, f or r out of bounds"
```

```
t = 0
S_0 = N - I_L - I_0 - S_L
inc_counter = 0

list_of_outcomes = [
    {"t": 0, "S_0": S_0, "S_L": S_L, "I_0": I_0, "I_L": I_L, "inc_counter": 0}
]

prop_new = alpha_*beta_*f/(alpha_*beta_*f + gamma_L)

for i in range(num_events):
    if S_0 == N:
        while t < 31:
            t += 1
            new_stages = {
                "t": t,
                "S_0": N,
                "S_L": 0,
                "I_0": 0,
                "I_L": 0,
                "inc_counter": inc_counter,
            }
            list_of_outcomes.append(new_stages)
        break

    S_0_to_I_L = (1 - alpha_) * lambda_ * (I_L + I_0) / N * S_0
    S_0_to_S_L = alpha_ * (1 - beta_) * lambda_ * (I_0 + I_L) / N * S_0
    I_0_to_S_0 = r * I_0 / N
    I_0_to_I_L = lambda_ * (I_L + I_0) / N * I_0
    I_L_to_I_0 = gamma_L * I_L
    I_L_to_S_L = r * I_L
    S_L_to_S_0 = (gamma_L + (f + lambda_ * (I_0 + I_L) / N) * alpha_ * beta_) * S_L
    S_L_to_I_L = (f + lambda_ * (I_0 + I_L) / N) * (1 - alpha_) * S_L

    total_rate = (
        S_0_to_I_L
        + S_0_to_S_L
        + I_0_to_S_0
        + I_0_to_I_L
        + I_L_to_I_0
        + I_L_to_S_L
        + S_L_to_S_0
```

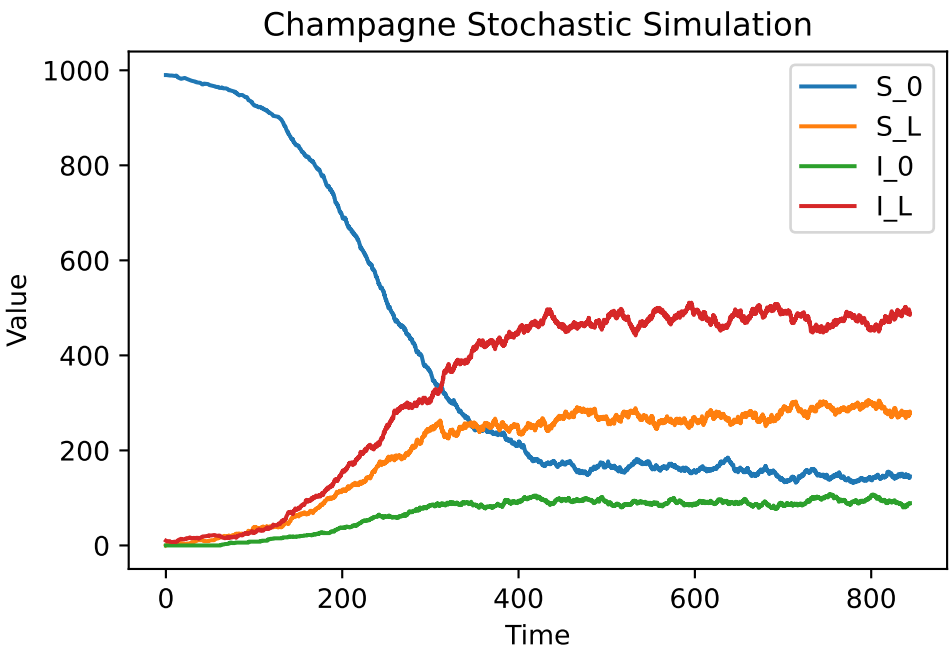
```
            + S_L_to_I_L
        )

        delta_t = np.random.exponential(1 / total_rate)
        new_stages_prob = [
            S_0_to_I_L / total_rate,
            S_0_to_S_L / total_rate,
            I_0_to_S_0 / total_rate,
            I_0_to_I_L / total_rate,
            I_L_to_I_0 / total_rate,
            I_L_to_S_L / total_rate,
            S_L_to_S_0 / total_rate,
            S_L_to_I_L / total_rate,
        ]
        t += delta_t
        silent_incidences = np.random.poisson(
            delta_t * alpha_ * beta_ * lambda_ * (I_L + I_0) * S_0 / N
        )

        new_stages = np.random.choice(
            [
                {
                    "t": t,
                    "S_0": S_0 - 1,
                    "S_L": S_L,
                    "I_0": I_0,
                    "I_L": I_L + 1,
                    "inc_counter": inc_counter + silent_incidences + 1,
                },
                {
                    "t": t,
                    "S_0": S_0 - 1,
                    "S_L": S_L + 1,
                    "I_0": I_0,
                    "I_L": I_L,
                    "inc_counter": inc_counter + silent_incidences + 1,
                },
                {
                    "t": t,
                    "S_0": S_0 + 1,
                    "S_L": S_L,
                    "I_0": I_0 - 1,
```

```
                "I_L": I_L,
                "inc_counter": inc_counter + silent_incidences,
            },
            {
                "t": t,
                "S_0": S_0,
                "S_L": S_L,
                "I_0": I_0 - 1,
                "I_L": I_L + 1,
                "inc_counter": inc_counter + silent_incidences,
            },
            {
                "t": t,
                "S_0": S_0,
                "S_L": S_L,
                "I_0": I_0 + 1,
                "I_L": I_L - 1,
                "inc_counter": inc_counter + silent_incidences,
            },
            {
                "t": t,
                "S_0": S_0,
                "S_L": S_L + 1,
                "I_0": I_0,
                "I_L": I_L - 1,
                "inc_counter": inc_counter + silent_incidences,
            },
            {
                "t": t,
                "S_0": S_0 + 1,
                "S_L": S_L - 1,
                "I_0": I_0,
                "I_L": I_L,
                "inc_counter": inc_counter
                + silent_incidences
                + np.random.binomial(1, prop_new),
            },
            {
                "t": t,
                "S_0": S_0,
                "S_L": S_L - 1,
                "I_0": I_0,
```

```
                    "I_L": I_L + 1,
                    "inc_counter": inc_counter + silent_incidences + 1,
                },
            ],
            p=new_stages_prob,
        )

        list_of_outcomes.append(new_stages)

        S_0 = new_stages["S_0"]
        I_0 = new_stages["I_0"]
        I_L = new_stages["I_L"]
        S_L = new_stages["S_L"]
        inc_counter = new_stages["inc_counter"]

    outcome_df = pd.DataFrame(list_of_outcomes)
    return outcome_df


champ_samp = champagne_stochastic(
    pv_champ_alpha,
    pv_champ_beta,
    pv_champ_gamma_L,
    pv_champ_lambda,
    pv_champ_f,
    pv_champ_r,
)  # .melt(id_vars='t')
```

Plotting outcome

```
champ_samp.drop("inc_counter", axis=1).plot(x="t", legend=True)
plt.xlabel("Time")
plt.ylabel("Value")
plt.title("Champagne Stochastic Simulation")
plt.savefig("champagne_GP_images/champagne_simulation.pdf")
plt.show()
```



Function that Outputs Final Prevalence

```
def incidence(df, start, days):
    start_ind = df[df["t"].le(start)].index[-1]
    end_ind = df[df["t"].le(start + days)].index[-1]
    incidence_week = df.iloc[end_ind]["inc_counter"] - df.iloc[start_ind]["inc_counter"]
    return incidence_week


def champ_sum_stats(alpha_, beta_, gamma_L, lambda_, f, r):
    champ_df_ = champagne_stochastic(alpha_, beta_, gamma_L, lambda_, f, r)
    fin_t = champ_df_.iloc[-1]["t"]
    first_month_inc = incidence(champ_df_, 0, 30)
    fin_t = champ_df_.iloc[-1]["t"]
    fin_week_inc = incidence(champ_df_, fin_t - 7, 7)
    fin_prev = champ_df_.iloc[-1]["I_0"] + champ_df_.iloc[-1]["I_L"]

    return np.array([fin_prev, first_month_inc, fin_week_inc])


observed_sum_stats = champ_sum_stats(
```

```
    pv_champ_alpha,
    pv_champ_beta,
    pv_champ_gamma_L,
    pv_champ_lambda,
    pv_champ_f,
    pv_champ_r,
)


def discrepency_fn(alpha_, beta_, gamma_L, lambda_, f, r): # best is L1 norm
    x = champ_sum_stats(alpha_, beta_, gamma_L, lambda_, f, r)
    return np.log(np.sum(np.abs((x - observed_sum_stats) / observed_sum_stats)))
```

Testing the variances across different values of params etc.

```
# samples = 30
# cor_sums = np.zeros(samples)
# for i in range(samples):
#     cor_sums[i] = discrepency_fn(
#         pv_champ_alpha,
#         pv_champ_beta,
#         pv_champ_gamma_L,
#         pv_champ_lambda,
#         pv_champ_f,
#         pv_champ_r,
#     )

# cor_mean = np.mean(cor_sums)
# cor_s_2 = sum((cor_sums - cor_mean) ** 2) / (samples - 1)
# print(cor_mean, cor_s_2)

# doub_sums = np.zeros(samples)
# for i in range(samples):
#     doub_sums[i] = discrepency_fn(
#         2 * pv_champ_alpha,
#         2 * pv_champ_beta,
#         2 * pv_champ_gamma_L,
#         2 * pv_champ_lambda,
#         2 * pv_champ_f,
#         2 * pv_champ_r,
#     )
```

```
# doub_mean = np.mean(doub_sums)
# doub_s_2 = sum((doub_sums - doub_mean) ** 2) / (samples - 1)
# print(doub_mean, doub_s_2)

# half_sums = np.zeros(samples)
# for i in range(samples):
#     half_sums[i] = discrepency_fn(
#         pv_champ_alpha / 2,
#         pv_champ_beta / 2,
#         pv_champ_gamma_L / 2,
#         pv_champ_lambda / 2,
#         pv_champ_f / 2,
#         pv_champ_r / 2,
#     )

# half_mean = np.mean(half_sums)
# half_s_2 = sum((half_sums - half_mean) ** 2) / (samples - 1)
# print(half_mean, half_s_2)

# rogue_sums = np.zeros(samples)
# for i in range(samples):
#     rogue_sums[i] = discrepency_fn(
#         pv_champ_alpha / 2,
#         pv_champ_beta / 2,
#         pv_champ_gamma_L / 2,
#         pv_champ_lambda / 2,
#         pv_champ_f / 2,
#         pv_champ_r / 2,
#     )

# rogue_mean = np.mean(rogue_sums)
# rogue_s_2 = sum((rogue_sums - rogue_mean) ** 2) / (samples - 1)
# print(rogue_mean, rogue_s_2)

# plt.figure(figsize=(7, 4))
# plt.scatter(
#     np.array([half_mean, cor_mean, doub_mean, rogue_mean]),
#     np.array([half_s_2, cor_s_2, doub_s_2, rogue_s_2]),
# )
# plt.title("variance and mean")
# plt.xlabel("mean")
# plt.ylabel("variance")
```

```
# plt.show()
```

Gaussian Process Regression on Final Prevalence Discrepency

```
my_seed = np.random.default_rng(seed=1795)  # For replicability

num_samples = 30

variables_names = ["alpha", "beta", "gamma_L", "lambda", "f", "r"]

pv_champ_alpha = 0.4  # prop of effective care
pv_champ_beta = 0.4  # prop of radical cure
pv_champ_gamma_L = 1 / 223  # liver stage clearance rate
pv_champ_lambda = 0.04  # transmission rate
pv_champ_f = 1 / 72  # relapse frequency
pv_champ_r = 1 / 60  # blood stage clearance rate

samples = np.concatenate(
    (
        my_seed.uniform(low=0, high=1, size=(num_samples, 1)),  # alpha
        my_seed.uniform(low=0, high=1, size=(num_samples, 1)),  # beta
        my_seed.exponential(scale=pv_champ_gamma_L, size=(num_samples, 1)),  # gamma_L
        my_seed.exponential(scale=pv_champ_lambda, size=(num_samples, 1)),  # lambda
        my_seed.exponential(scale=pv_champ_f, size=(num_samples, 1)),  # f
        my_seed.exponential(scale=pv_champ_r, size=(num_samples, 1)),  # r
    ),
    axis=1,
)

LHC_sampler = qmc.LatinHypercube(d=6, seed=my_seed)
LHC_samples = LHC_sampler.random(n=num_samples)
LHC_samples[:, 2] = -pv_champ_gamma_L * np.log(LHC_samples[:, 2])
LHC_samples[:, 3] = -pv_champ_lambda * np.log(LHC_samples[:, 3])
LHC_samples[:, 4] = -pv_champ_f * np.log(LHC_samples[:, 4])
LHC_samples[:, 5] = -pv_champ_r * np.log(LHC_samples[:, 5])

LHC_samples = np.repeat(LHC_samples, 3, axis = 0)

random_indices_df = pd.DataFrame(samples, columns=variables_names)
```

```
LHC_indices_df = pd.DataFrame(LHC_samples, columns=variables_names)

print(random_indices_df.head())
print(LHC_indices_df.head())
```

```
      alpha      beta   gamma_L    lambda         f         r
0  0.201552  0.081511  0.004695  0.017172  0.007355  0.021370
1  0.332324  0.374497  0.003022  0.020210  0.001350  0.002604
2  0.836050  0.570164  0.002141  0.043572  0.001212  0.008367
3  0.566773  0.347186  0.001925  0.016830  0.000064  0.003145
4  0.880603  0.316884  0.000425  0.012374  0.000358  0.003491
      alpha      beta   gamma_L    lambda         f         r
0  0.066680  0.570582  0.001707  0.002226  0.004358  0.003743
1  0.066680  0.570582  0.001707  0.002226  0.004358  0.003743
2  0.066680  0.570582  0.001707  0.002226  0.004358  0.003743
3  0.132042  0.551592  0.013131  0.036829  0.002851  0.002075
4  0.132042  0.551592  0.013131  0.036829  0.002851  0.002075
```

Generate Discrepencies

```
random_discrepencies = LHC_indices_df.apply(
    lambda x: discrepency_fn(
        x["alpha"], x["beta"], x["gamma_L"], x["lambda"], x["f"], x["r"]
    ),
    axis=1,
)

print(random_discrepencies.head())
```

```
0    0.542551
1    0.627749
2    0.650314
3    0.644435
4    0.667979
dtype: float64
```

Differing Methods to Iterate Function

```
# import timeit

# def function1():
#     np.vectorize(champ_sum_stats)(random_indices_df['alpha'],
#     random_indices_df['beta'], random_indices_df['gamma_L'],
#     random_indices_df['lambda'], random_indices_df['f'], random_indices_df['r'])
#     pass

# def function2():
#     random_indices_df.apply(
#         lambda x: champ_sum_stats(
#             x['alpha'], x['beta'], x['gamma_L'], x['lambda'], x['f'], x['r']),
#             axis = 1)
#     pass

# # Time function1
# time_taken_function1 = timeit.timeit(
#     "function1()", globals=globals(), number=100)

# # Time function2
# time_taken_function2 = timeit.timeit(
#     "function2()", globals=globals(), number=100)

# print("Time taken for function1:", time_taken_function1)
# print("Time taken for function2:", time_taken_function2)
```

Time taken for function1: 187.48960775700016 Time taken for function2: 204.06618941299985

Constrain Variables to be Positive

```
constrain_positive = tfb.Shift(np.finfo(np.float64).tiny)(tfb.Exp())
```

```
2024-04-19 04:50:03.131297: I external/local_xla/xla/stream_executor/cuda/cuda_executor.cc:99
2024-04-19 04:50:03.169039: W tensorflow/core/common_runtime/gpu/gpu_device.cc:2251] Cannot d
Skipping registering GPU devices...
```

Custom Quadratic Mean Function

```
class quad_mean_fn(tf.Module):
    def __init__(self):
        super(quad_mean_fn, self).__init__()
        self.amp_alpha_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_alpha_mean",
        )
        self.alpha_tp = tf.Variable(pv_champ_alpha, dtype=np.float64, name="alpha_tp")
        self.amp_beta_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_beta_mean",
        )
        self.beta_tp = tf.Variable(pv_champ_beta, dtype=np.float64, name="beta_tp")
        self.amp_gamma_L_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_gamma_L_mean",
        )
        self.gamma_L_tp = tf.Variable(
            pv_champ_gamma_L, dtype=np.float64, name="gamma_L_tp"
        )
        self.amp_lambda_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_lambda_mean",
        )
        self.lambda_tp = tf.Variable(
            pv_champ_lambda, dtype=np.float64, name="lambda_tp"
        )
        self.amp_f_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_f_mean",
```

```
        )
        self.f_tp = tf.Variable(pv_champ_f, dtype=np.float64, name="f_tp")
        self.amp_r_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_r_mean",
        )
        self.r_tp = tf.Variable(pv_champ_r, dtype=np.float64, name="r_tp")
        # self.bias_mean = tfp.util.TransformedVariable(
        #     bijector=constrain_positive,
        #     initial_value=50.0,
        #     dtype=np.float64,
        #     name="bias_mean",
        # )
        self.bias_mean = tf.Variable(0.0, dtype=np.float64, name="bias_mean")

    def __call__(self, x):
        return (
            self.amp_alpha_mean * (x[..., 0] - self.alpha_tp) ** 2
            + self.amp_beta_mean * (x[..., 1] - self.beta_tp) ** 2
            + self.amp_gamma_L_mean * (x[..., 2] - self.gamma_L_tp) ** 2
            + self.amp_lambda_mean * (x[..., 3] - self.lambda_tp) ** 2
            + self.amp_f_mean * (x[..., 4] - self.f_tp) ** 2
            + self.amp_r_mean * (x[..., 5] - self.r_tp) ** 2
            + self.bias_mean
        )
```

Making the ARD Kernel

```
index_vals = LHC_indices_df.values
obs_vals = random_discrepencies.values

amplitude_champ = tfp.util.TransformedVariable(
    bijector=constrain_positive,
    initial_value=1.0,
    dtype=np.float64,
    name="amplitude_champ",
)
```

```
observation_noise_variance_champ = tfp.util.TransformedVariable(
    bijector=constrain_positive,
    initial_value=0.03,
    dtype=np.float64,
    name="observation_noise_variance_champ",
)
```

```
length_scales_champ = tfp.util.TransformedVariable(
    bijector=constrain_positive,
    initial_value=[0.05, 0.05, 0.05, 0.05, 0.05, 0.05],
    dtype=np.float64,
    name="length_scales_champ",
)
```

```
kernel_champ = tfk.FeatureScaled(
    tfk.MaternFiveHalves(amplitude=amplitude_champ),
    scale_diag=length_scales_champ,
)
```

Define the Gaussian Process with Quadratic Mean Function and ARD Kernel

```
# Define Gaussian Process with the custom kernel
champ_GP = tfd.GaussianProcess(
    kernel=kernel_champ,
    observation_noise_variance=observation_noise_variance_champ,
    index_points=index_vals,
    mean_fn=quad_mean_fn(),
)

print(champ_GP.trainable_variables)

Adam_optim = tf.optimizers.Adam(learning_rate=0.01)
```

```
(<tf.Variable 'amplitude_champ:0' shape=() dtype=float64, numpy=0.0>, <tf.Variable 'length_s
array([-2.99573227, -2.99573227, -2.99573227, -2.99573227, -2.99573227,
       -2.99573227])>, <tf.Variable 'observation_noise_variance_champ:0' shape=() dtype=float
```

Train the Hyperparameters

```
# predictive log stuff
@tf.function(autograph=False, jit_compile=False)
def optimize():
    with tf.GradientTape() as tape:
        K = (
            champ_GP.kernel.matrix(index_vals, index_vals)
            + tf.eye(index_vals.shape[0], dtype=np.float64)
            * observation_noise_variance_champ
        )
        means = champ_GP.mean_fn(index_vals)
        K_inv = tf.linalg.inv(K)
        K_inv_y = K_inv @ tf.reshape(obs_vals - means, shape=[obs_vals.shape[0], 1])
        K_inv_diag = tf.linalg.diag_part(K_inv)
        log_var = tf.math.log(K_inv_diag)
        log_mu = tf.reshape(K_inv_y, shape=[-1]) ** 2
        loss = -tf.math.reduce_sum(log_var - log_mu)
    grads = tape.gradient(loss, champ_GP.trainable_variables)
    Adam_optim.apply_gradients(zip(grads, champ_GP.trainable_variables))
    return loss


num_iters = 10000

lls_ = np.zeros(num_iters, np.float64)
tolerance = 1e-6  # Set your desired tolerance level
previous_loss = float("inf")

for i in range(num_iters):
    loss = optimize()
    lls_[i] = loss

    # Check if change in loss is less than tolerance
    if abs(loss - previous_loss) < tolerance:
        print(f"Hyperparameter convergence reached at iteration {i+1}.")
        lls_ = lls_[range(i + 1)]
        break

    previous_loss = loss
```

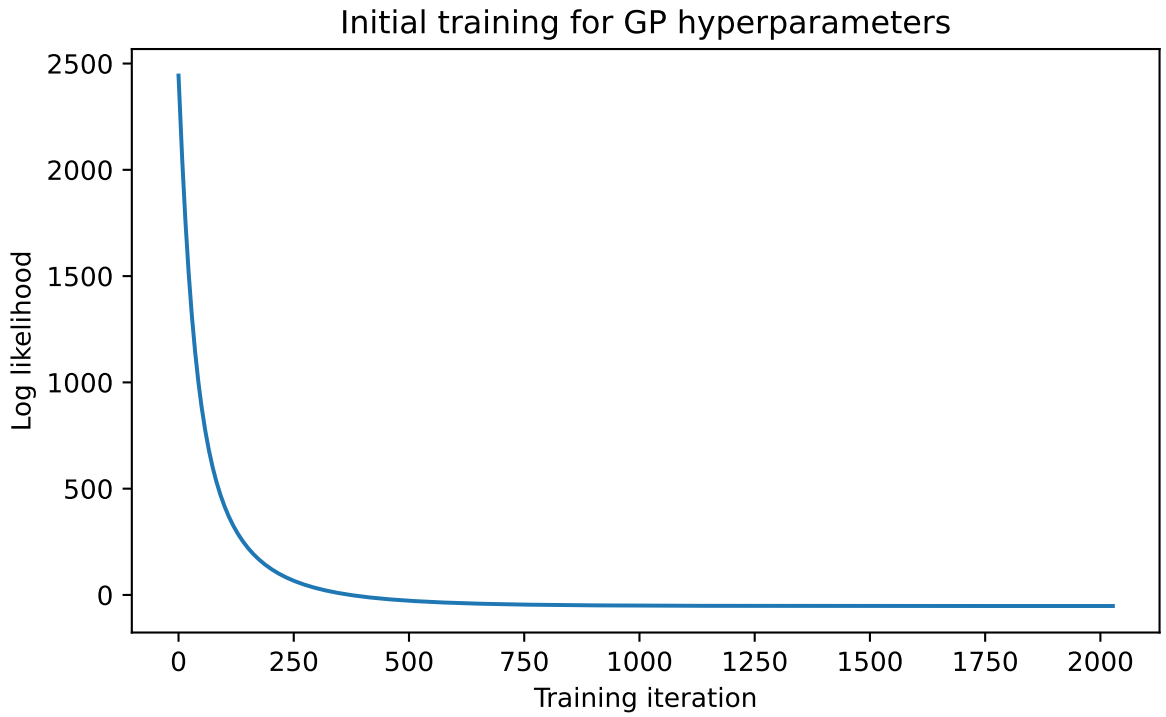
```
Hyperparameter convergence reached at iteration 2028.
```

```
print("Trained parameters:")
for var in champ_GP.trainable_variables:
    if "tp" in var.name or "bias" in var.name:
        print("{} is {}\n".format(var.name, var.numpy().round(3)))
    else:
        print(
            "{} is {}\n".format(
                var.name, constrain_positive.forward(var).numpy().round(3)
            )
        )
```

```
Trained parameters:
amplitude_champ:0 is 0.812

length_scales_champ:0 is [0.004 0.014 0.032 0.017 0.022 0.018]

observation_noise_variance_champ:0 is 0.238

alpha_tp:0 is -0.575

amp_alpha_mean:0 is 0.252

amp_beta_mean:0 is 1.231

amp_f_mean:0 is 1128.319

amp_gamma_L_mean:0 is 4.401

amp_lambda_mean:0 is 96.25

amp_r_mean:0 is 48.536

beta_tp:0 is 0.522

bias_mean:0 is -1.332

f_tp:0 is 0.015

gamma_L_tp:0 is -0.103

lambda_tp:0 is 0.041
```

```
r_tp:0 is 0.173
```

```
plt.figure(figsize=(7, 4))
plt.plot(lls_)
plt.title("Initial training for GP hyperparameters")
plt.xlabel("Training iteration")
plt.ylabel("Log likelihood")
plt.savefig("champagne_GP_images/hyperparam_loss.pdf")
plt.show()
```



Creating slices across one variable dimension

```
plot_samp_no = 21
plot_gp_no = 200
gp_samp_no = 50
```

```
slice_samples_dict = {
    "alpha_slice_samples": np.repeat(np.concatenate(
```

```
        (
            np.linspace(0, 1, plot_samp_no, dtype=np.float64).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), 3, axis = 0),
    "alpha_gp_samples": np.concatenate(
        (
            np.linspace(0, 1, plot_gp_no, dtype=np.float64).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ),
    "beta_slice_samples": np.repeat(np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.linspace(0, 1, plot_samp_no, dtype=np.float64).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), 3, axis = 0),
    "beta_gp_samples": np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.linspace(0, 1, plot_gp_no, dtype=np.float64).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
```

```
    ),
    "gamma_L_slice_samples": np.repeat(np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
            -10*pv_champ_gamma_L
            * np.log(
                np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
            ).reshape(
                -1, 1
            ),  # gamma_L
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), 3, axis = 0),
    "gamma_L_gp_samples": np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.linspace(
                -10*pv_champ_gamma_L
                * np.log(
                    np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
                ).reshape(-1, 1)[0],
                -10*pv_champ_gamma_L
                * np.log(
                    np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
                ).reshape(-1, 1)[-1], plot_gp_no, dtype=np.float64
            ),  # gamma_L
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ),
    "lambda_slice_samples": np.repeat(np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
```

```
            -pv_champ_lambda
            * np.log(
                np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
            ).reshape(
                -1, 1
            ),  # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), 3, axis = 0),
    "lambda_gp_samples": np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.linspace(
                -pv_champ_lambda
                * np.log(
                    np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
                ).reshape(-1, 1)[0],
                -pv_champ_lambda
                * np.log(
                    np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
                ).reshape(-1, 1)[-1], plot_gp_no, dtype=np.float64
            ),  # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ),
    "f_slice_samples": np.repeat(np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            -10*pv_champ_f
            * np.log(
                np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
            ).reshape(
                -1, 1
```

```
            ),  # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ), 3, axis = 0),
    "f_gp_samples": np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.linspace(
                -10*pv_champ_f
                * np.log(
                    np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
                ).reshape(-1, 1)[0],
                -10*pv_champ_f
                * np.log(
                    np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
                ).reshape(-1, 1)[-1], plot_gp_no, dtype=np.float64
            ),  # f
            np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1),  # r
        ),
        axis=1,
    ),
    "r_slice_samples": np.repeat(np.concatenate(
        (
            np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1),  # f
            -2*pv_champ_r
            * np.log(
                np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
            ).reshape(
                -1, 1
            ),  # r
        ),
        axis=1,
    ), 3, axis = 0),
    "r_gp_samples": np.concatenate(
```

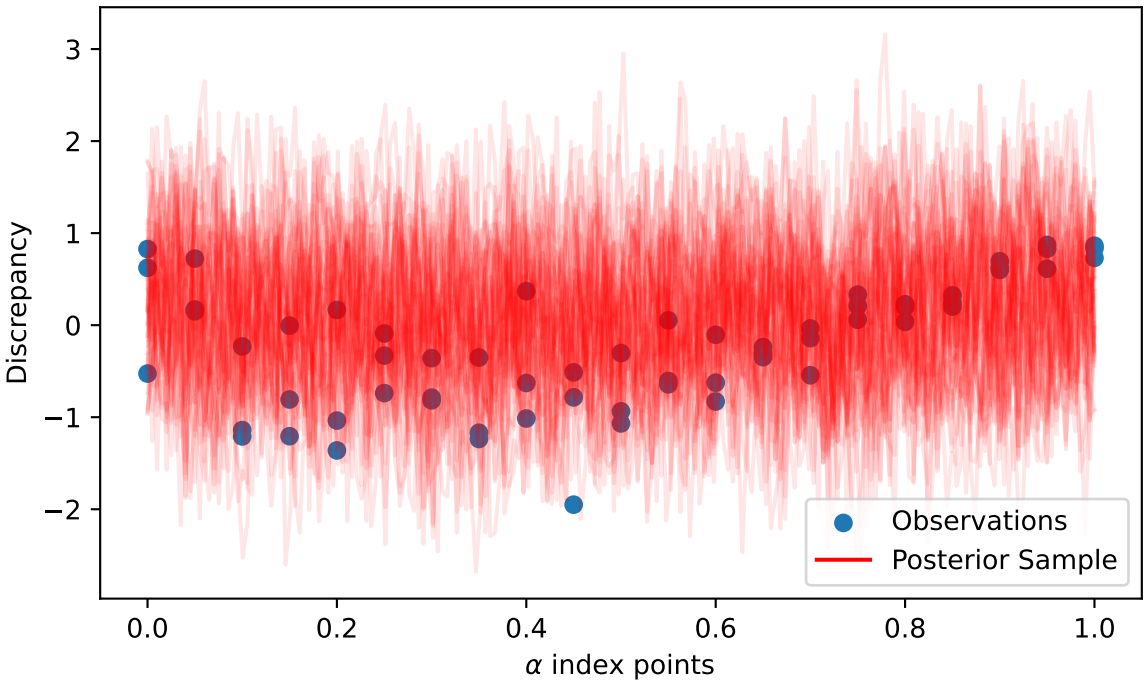
```
        (
            np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1),  # alpha
            np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1),  # beta
            np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1),  # gamma_L
            np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),  # lambda
            np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1),  # f
            np.linspace(
                -2*pv_champ_r
                * np.log(
                    np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
                ).reshape(-1, 1)[0],
                -2*pv_champ_r
                * np.log(
                    np.linspace(0, 1, plot_samp_no + 2, dtype=np.float64)[1:-1]
                ).reshape(-1, 1)[-1], plot_gp_no, dtype=np.float64
            ),  # r
        ),
        axis=1,
    ),
}
```

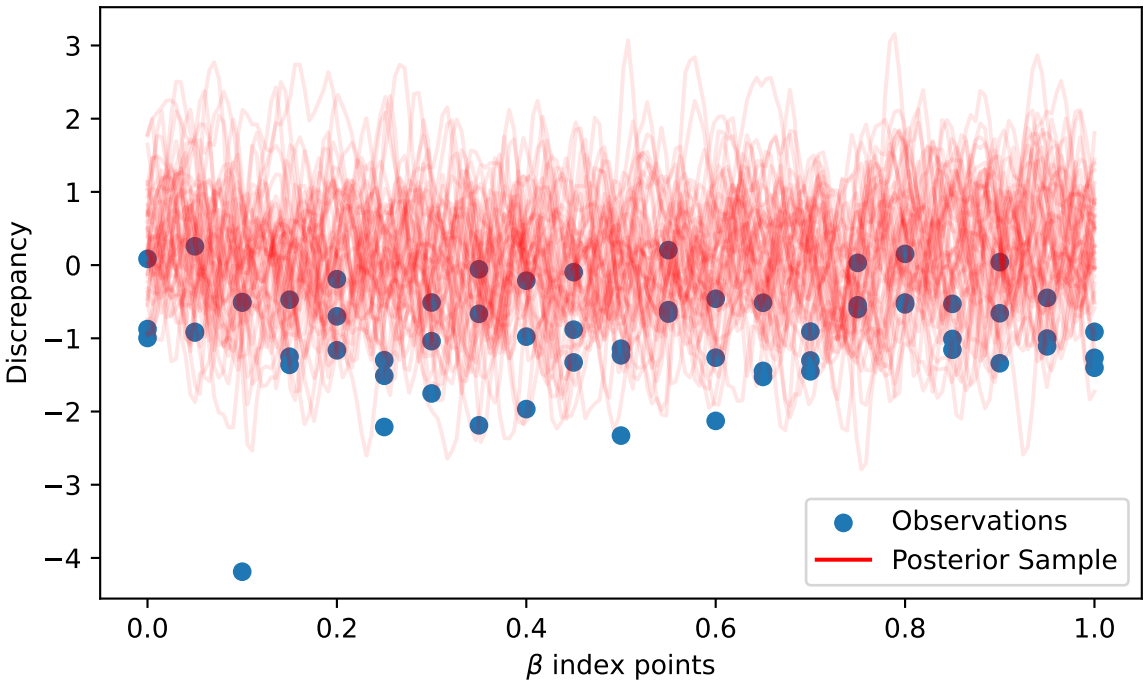
Plotting the GPs across different slices

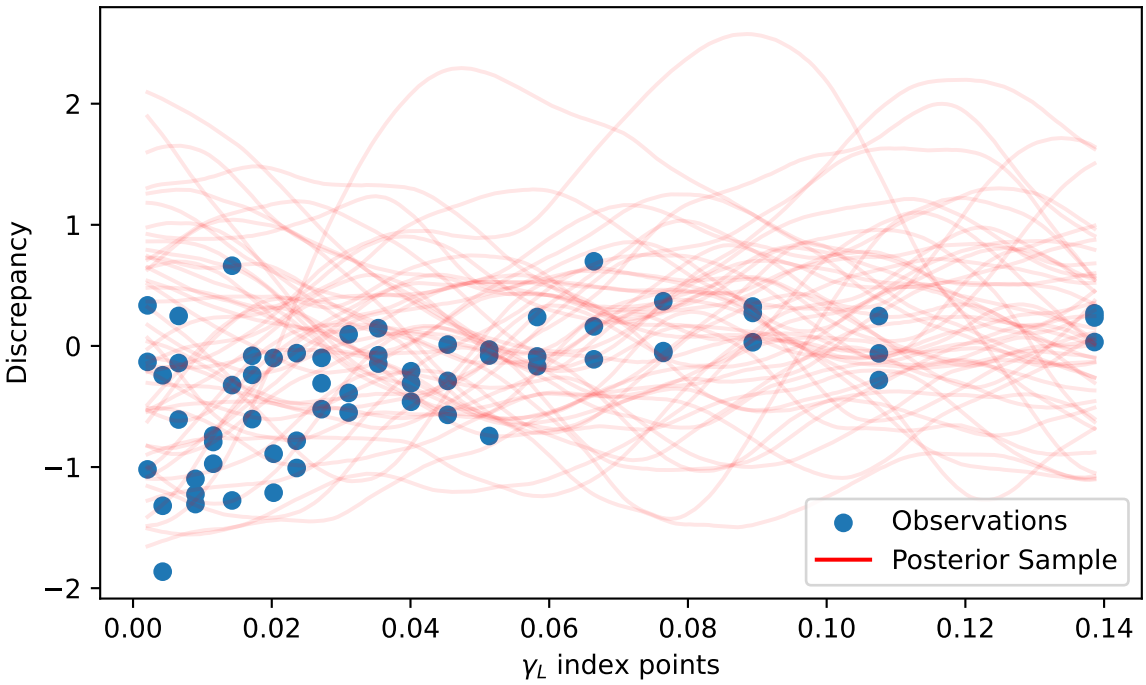
```
GP_seed = tfp.random.sanitize_seed(4362)
vars = ["alpha", "beta", "gamma_L", "lambda", "f", "r"]
slice_indices_dfs_dict = {}
slice_index_vals_dict = {}
slice_discrepencies_dict = {}

for var in vars:
    val_df = pd.DataFrame(
        slice_samples_dict[var + "_slice_samples"], columns=variables_names
    )
    slice_indices_dfs_dict[var + "_slice_indices_df"] = val_df
    slice_index_vals_dict[var + "_slice_index_vals"] = val_df.values
    discreps = val_df.apply(
        lambda x: discrepency_fn(
            x["alpha"], x["beta"], x["gamma_L"], x["lambda"], x["f"], x["r"]
        ),
        axis=1,
```

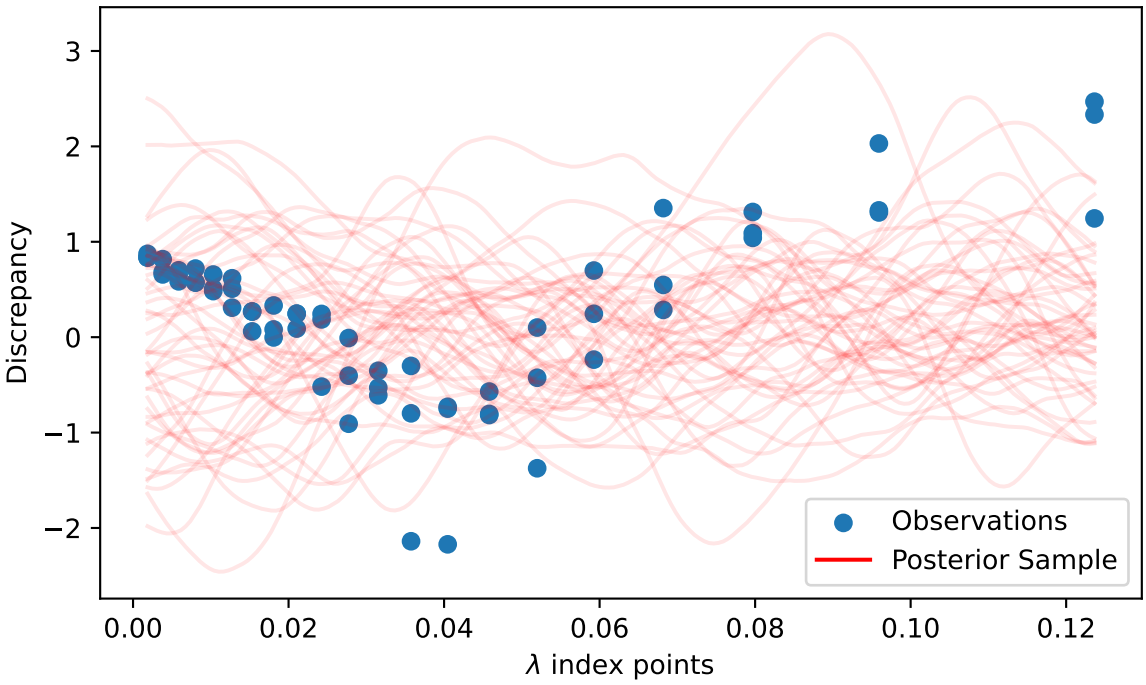
```
    )
    slice_discrepencies_dict[var + "_slice_discrepencies"] = discreps


    gp_samples_df = pd.DataFrame(
        slice_samples_dict[var + "_gp_samples"], columns=variables_names
    )
    slice_indices_dfs_dict[var + "_gp_indices_df"] = gp_samples_df
    slice_index_vals_dict[var + "_gp_index_vals"] = gp_samples_df.values

    champ_GP_reg = tfd.GaussianProcessRegressionModel(
        kernel=kernel_champ,
        index_points=gp_samples_df.values,
        observation_index_points=index_vals,
        observations=obs_vals,
        observation_noise_variance=observation_noise_variance_champ,
        predictive_noise_variance=0.0,
        mean_fn=quad_mean_fn(),
    )
    GP_samples = champ_GP_reg.sample(gp_samp_no, seed=GP_seed)

    plt.figure(figsize=(7, 4))
    plt.scatter(
        val_df[var].values,
        discreps,
        label="Observations",
    )
    for i in range(gp_samp_no):
        plt.plot(
            gp_samples_df[var].values,
            GP_samples[i, :],
            c="r",
            alpha=0.1,
            label="Posterior Sample" if i == 0 else None,
        )
    leg = plt.legend(loc="lower right")
    for lh in leg.legend_handles:
        lh.set_alpha(1)
    if var in ["f", "r"]:
        plt.xlabel("$" + var + "$ index points")
    else:
        plt.xlabel("$\\" + var + "$ index points")
```

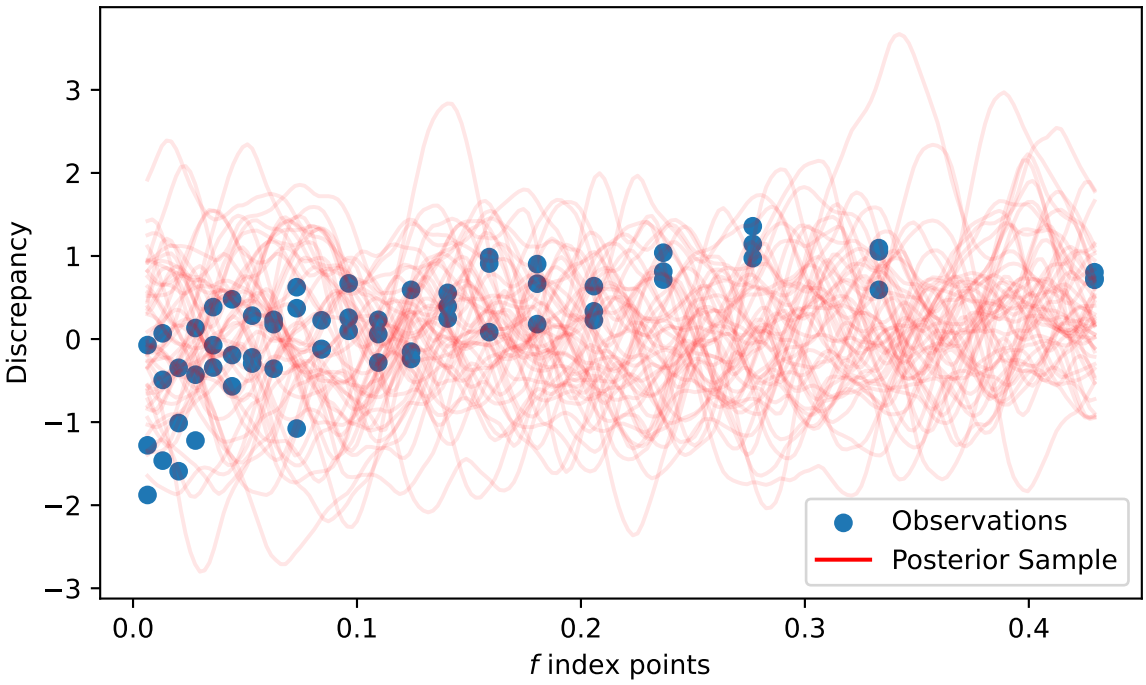
```
    # if var not in ["alpha", "beta"]:
    #     plt.xscale("log", base=np.e)
    plt.ylabel("Discrepancy")
    plt.savefig("champagne_GP_images/initial_" + var + "_slice.pdf")
    plt.show()
```

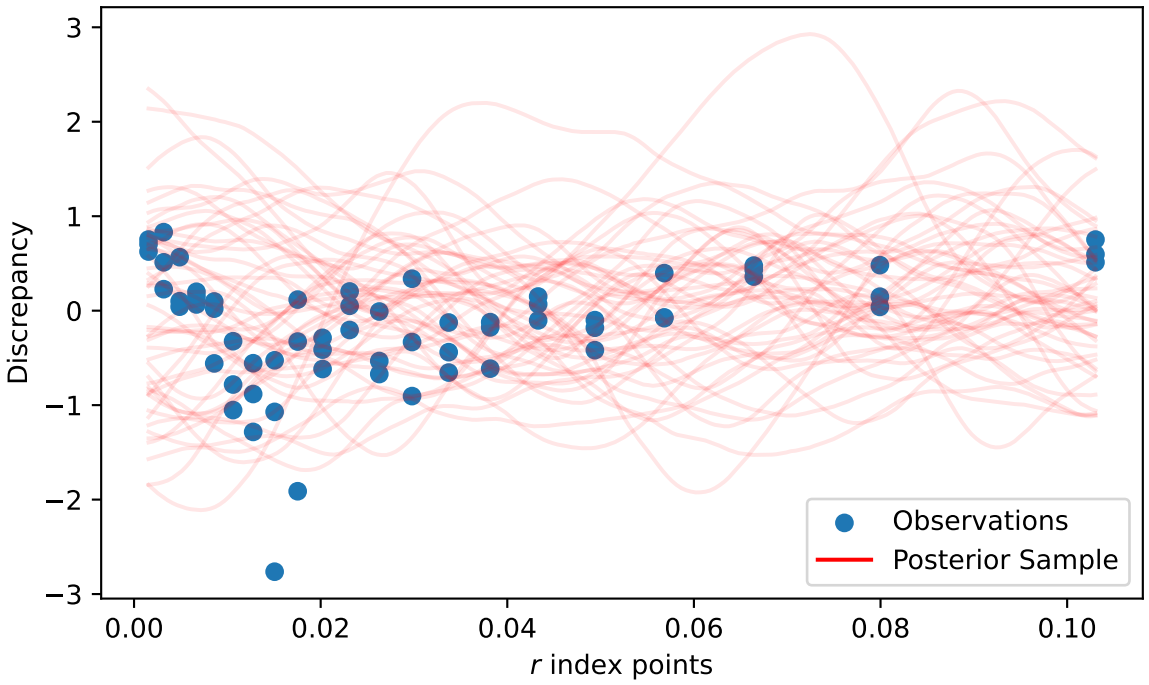



Discrepancy
3
2
1
0
−1
−2
−3
−4
0.0
0.2
0.4
0.6
0.8
1.0
β index points
Observations
Posterior Sample


Discrepancy
2
1
0
−1
−2
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
γ_L index points
Observations
Posterior Sample


Discrepancy
3
2
1
0
−1
−2
0.00
0.02
0.04
0.06
0.08
0.10
0.12
λ index points
Observations
Posterior Sample


Discrepancy
3
2
1
0
−1
−2
−3
0.0
0.1
0.2
0.3
0.4
f index points
Observations
Posterior Sample



Acquiring the next datapoint to test

Proof that .variance returns what we need in acquisition function

```
new_guess = np.array([0.4, 0.4, 0.004, 0.04, 0.01, 0.17])
mean_t = champ_GP_reg.mean_fn(new_guess)
variance_t = champ_GP_reg.variance(index_points=[new_guess])

kernel_self = kernel_champ.apply(new_guess, new_guess)
kernel_others = kernel_champ.apply(new_guess, index_vals)
K = kernel_champ.matrix(
    index_vals, index_vals
) + observation_noise_variance_champ * np.identity(index_vals.shape[0])
inv_K = np.linalg.inv(K)
print("Self Kernel is {}".format(kernel_self.numpy().round(3)))
print("Others Kernel is {}".format(kernel_others.numpy().round(3)))
print(inv_K)
my_var_t = kernel_self - kernel_others.numpy() @ inv_K @ kernel_others.numpy()
```

```
print("Variance function is {}".format(variance_t.numpy().round(3)))
print("Variance function is {}".format(my_var_t.numpy().round(3)))
```

```
Self Kernel is 0.659
Others Kernel is [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
[[ 2.94799119e+000 -1.24829656e+000 -1.24829656e+000 ... -6.13992477e-146
  -6.13992477e-146 -6.13992477e-146]
 [-1.24829656e+000  2.94799119e+000 -1.24829656e+000 ... -6.13992477e-146
  -6.13992477e-146 -6.13992477e-146]
 [-1.24829656e+000 -1.24829656e+000  2.94799119e+000 ... -6.13992477e-146
  -6.13992477e-146 -6.13992477e-146]
 ...
 [-6.13992477e-146 -6.13992477e-146 -6.13992477e-146 ...  2.94799119e+000
  -1.24829656e+000 -1.24829656e+000]
 [-6.13992477e-146 -6.13992477e-146 -6.13992477e-146 ... -1.24829656e+000
   2.94799119e+000 -1.24829656e+000]
 [-6.13992477e-146 -6.13992477e-146 -6.13992477e-146 ... -1.24829656e+000
  -1.24829656e+000  2.94799119e+000]]
Variance function is [0.659]
Variance function is 0.659
```

Loss function

```
next_alpha = tfp.util.TransformedVariable(
    initial_value=0.5,
    bijector=tfb.Sigmoid(),
    dtype=np.float64,
    name="next_alpha",
)

next_beta = tfp.util.TransformedVariable(
    initial_value=0.5,
    bijector=tfb.Sigmoid(),
    dtype=np.float64,
    name="next_beta",
)
```

```
next_gamma_L = tfp.util.TransformedVariable(
    initial_value=0.1,
    bijector=constrain_positive,
    dtype=np.float64,
    name="next_gamma_L",
)

next_lambda = tfp.util.TransformedVariable(
    initial_value=0.1,
    bijector=constrain_positive,
    dtype=np.float64,
    name="next_lambda",
)

next_f = tfp.util.TransformedVariable(
    initial_value=0.1,
    bijector=constrain_positive,
    dtype=np.float64,
    name="next_f",
)

next_r = tfp.util.TransformedVariable(
    initial_value=0.1,
    bijector=constrain_positive,
    dtype=np.float64,
    name="next_r",
)

next_vars = [
    v.trainable_variables[0]
    for v in [next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]
]
```

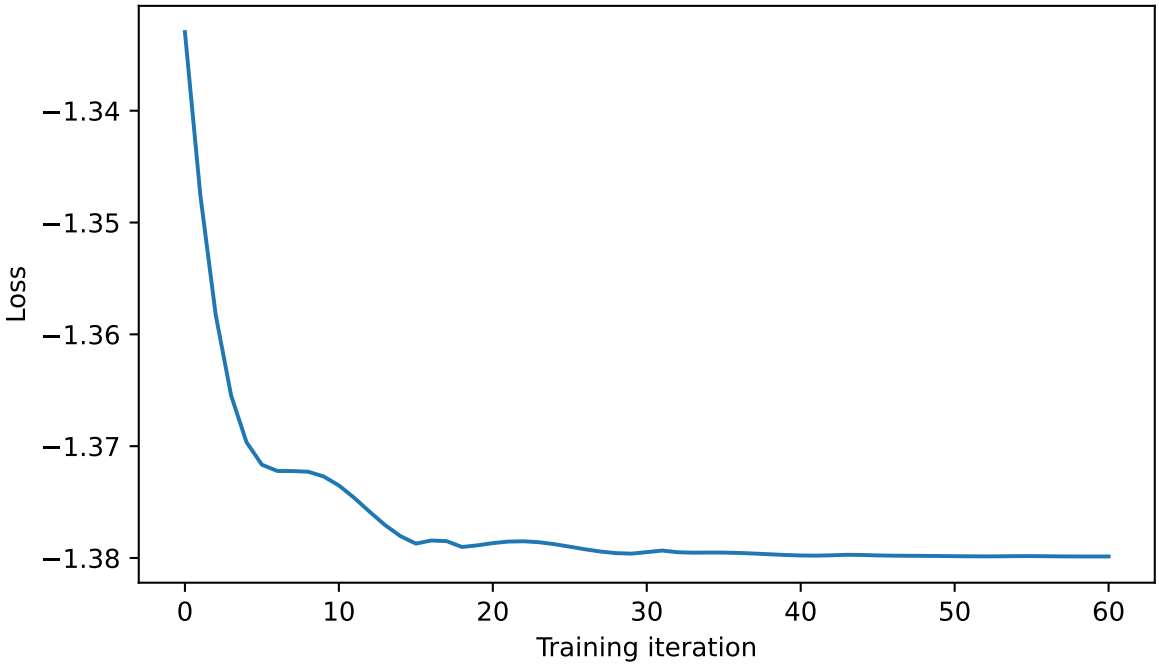
```
Adam_optim = tf.optimizers.Adam(learning_rate=0.1)


@tf.function(autograph=False, jit_compile=False)
def optimize():
    with tf.GradientTape() as tape:
        next_guess = tf.reshape(
            [
                tfb.Sigmoid().forward(next_vars[0]),
```

```
                tfb.Sigmoid().forward(next_vars[1]),
                constrain_positive.forward(next_vars[2]),
                constrain_positive.forward(next_vars[3]),
                constrain_positive.forward(next_vars[4]),
                constrain_positive.forward(next_vars[5]),
            ],
            [1, 6],
        )
        mean_t = champ_GP_reg.mean_fn(next_guess)
        std_t = champ_GP_reg.stddev(index_points=next_guess)
        loss = tf.squeeze(mean_t - 1.7 * std_t)
    grads = tape.gradient(loss, next_vars)
    Adam_optim.apply_gradients(zip(grads, next_vars))
    return loss


num_iters = 10000

lls_ = np.zeros(num_iters, np.float64)
tolerance = 1e-6  # Set your desired tolerance level
previous_loss = float("inf")

for i in range(num_iters):
    loss = optimize()
    lls_[i] = loss

    # Check if change in loss is less than tolerance
    if abs(loss - previous_loss) < tolerance:
        print(f"Acquisition function convergence reached at iteration {i+1}.")
        lls_ = lls_[range(i + 1)]
        break

    previous_loss = loss

print("Trained parameters:")
for var in next_vars:
    if ("alpha" in var.name) | ("beta" in var.name):
        print(
            "{} is {}".format(var.name, (tfb.Sigmoid().forward(var).numpy().round(3)))
        )
    else:
        print(
```

```
            "{} is {}".format(
                var.name, constrain_positive.forward(var).numpy().round(3)
            )
        )
```

```
Acquisition function convergence reached at iteration 61.
Trained parameters:
next_alpha:0 is 0.39
next_beta:0 is 0.4
next_gamma_L:0 is 0.012
next_lambda:0 is 0.042
next_f:0 is 0.015
next_r:0 is 0.017
```

```
plt.figure(figsize=(7, 4))
plt.plot(lls_)
plt.xlabel("Training iteration")
plt.ylabel("Loss")
plt.savefig("champagne_GP_images/bolfi_optim_loss.pdf")
plt.show()
```



```
def update_GP():
    @tf.function
    def opt_GP():
        with tf.GradientTape() as tape:
            K = (
                champ_GP.kernel.matrix(index_vals, index_vals)
                + tf.eye(index_vals.shape[0], dtype=np.float64)
                * observation_noise_variance_champ
            )
            means = champ_GP.mean_fn(index_vals)
            K_inv = tf.linalg.inv(K)
            K_inv_y = K_inv @ tf.reshape(obs_vals - means, shape=[obs_vals.shape[0], 1])
            K_inv_diag = tf.linalg.diag_part(K_inv)
            log_var = tf.math.log(K_inv_diag)
            log_mu = tf.reshape(K_inv_y, shape=[-1]) ** 2
            loss = -tf.math.reduce_sum(log_var - log_mu)
        grads = tape.gradient(loss, champ_GP.trainable_variables)
        optimizer_slow.apply_gradients(zip(grads, champ_GP.trainable_variables))
        return loss

    num_iters = 10000

    lls_ = np.zeros(num_iters, np.float64)
    tolerance = 1e-6  # Set your desired tolerance level
    previous_loss = float("inf")

    for i in range(num_iters):
        loss = opt_GP()
        lls_[i] = loss.numpy()

        # Check if change in loss is less than tolerance
        if abs(loss - previous_loss) < tolerance:
            print(f"Hyperparameter convergence reached at iteration {i+1}.")
            lls_ = lls_[range(i + 1)]
            break

        previous_loss = loss
    for var in optimizer_slow.variables:
        var.assign(tf.zeros_like(var))


def update_var():
```

```
    @tf.function
    def opt_var():
        with tf.GradientTape() as tape:
            next_guess = tf.reshape(
                [
                    tfb.Sigmoid().forward(next_vars[0]),
                    tfb.Sigmoid().forward(next_vars[1]),
                    tfb.Sigmoid().forward(next_vars[2]),
                    tfb.Sigmoid().forward(next_vars[3]),
                    tfb.Sigmoid().forward(next_vars[4]),
                    tfb.Sigmoid().forward(next_vars[5]),
                ],
                [1, 6],
            )
            mean_t = champ_GP_reg.mean_fn(next_guess)
            std_t = champ_GP_reg.stddev(index_points=next_guess)
            loss = tf.squeeze(mean_t - eta_t * std_t)
        grads = tape.gradient(loss, next_vars)
        optimizer_fast.apply_gradients(zip(grads, next_vars))
        return loss

    num_iters = 10000

    lls_ = np.zeros(num_iters, np.float64)
    tolerance = 1e-6  # Set your desired tolerance level
    previous_loss = float("inf")

    for i in range(num_iters):
        loss = opt_var()
        lls_[i] = loss

        # Check if change in loss is less than tolerance
        if abs(loss - previous_loss) < tolerance:
            print(f"Acquisition function convergence reached at iteration {i+1}.")
            lls_ = lls_[range(i + 1)]
            break

        previous_loss = loss
    print(loss)
    for var in optimizer_fast.variables:
        var.assign(tf.zeros_like(var))
```

```
# EI = tfp_acq.GaussianProcessExpectedImprovement(champ_GP_reg, obs_vals)

def new_eta_t(t, d, exploration_rate):
    return np.log((t + 1) ** (d / 2 + 2) * np.pi**2 / (3 * exploration_rate))
```

```
exploration_rate = 0.000000001
d = 6
update_freq = 20  # how many iterations before updating GP hyperparams
eta_t = tf.Variable(0,dtype=np.float64, name = "eta_t")

for t in range(401):
    print(index_vals[-1,])
    optimizer_fast = tf.optimizers.Adam(learning_rate=0.01)
    optimizer_slow = tf.optimizers.Adam()
    eta_t.assign(new_eta_t(t, d, exploration_rate))
    print(eta_t)
    print(t)

    for var in next_vars:
        var.assign(my_seed.uniform(0,1))
    update_var()

    new_discrepency = discrepency_fn(
        next_alpha.numpy(),
        next_beta.numpy(),
        next_gamma_L.numpy(),
        next_lambda.numpy(),
        next_f.numpy(),
        next_r.numpy(),
    )

    index_vals = np.append(
        index_vals,
        np.array(
            [
                next_alpha.numpy(),
                next_beta.numpy(),
                next_gamma_L.numpy(),
                next_lambda.numpy(),
                next_f.numpy(),
                next_r.numpy(),
            ]
```

```
        ).reshape(1, -1),
        axis=0,
    )
    obs_vals = np.append(obs_vals, new_discrepency)

    if t % update_freq == 0:
        champ_GP = tfd.GaussianProcess(
            kernel=kernel_champ,
            observation_noise_variance=observation_noise_variance_champ,
            index_points=index_vals,
            mean_fn=quad_mean_fn(),
        )
        update_GP()

    champ_GP_reg = tfd.GaussianProcessRegressionModel(
        kernel=kernel_champ,
        observation_index_points=index_vals,
        observations=obs_vals,
        observation_noise_variance=observation_noise_variance_champ,
        predictive_noise_variance=0.0,
        mean_fn=quad_mean_fn(),
    )

    if (t > 0) & (t % 50 == 0):
        print("Trained parameters:")
        for var in champ_GP.trainable_variables:
            if "tp" in var.name or "bias" in var.name:
                print("{} is {}\n".format(var.name, var.numpy().round(3)))
            else:
                print(
                    "{} is {}\n".format(
                        var.name, constrain_positive.forward(var).numpy().round(3)
                    )
                )
        for var in vars:
            champ_GP_reg = tfd.GaussianProcessRegressionModel(
                kernel=kernel_champ,
                index_points=slice_indices_dfs_dict[var + "_gp_indices_df"].values,
                observation_index_points=index_vals,
                observations=obs_vals,
                observation_noise_variance=observation_noise_variance_champ,
                predictive_noise_variance=0.0,
```

```
                mean_fn=quad_mean_fn(),
            )
            GP_samples = champ_GP_reg.sample(gp_samp_no, seed=GP_seed)

            plt.figure(figsize=(7, 4))
            plt.scatter(
                slice_indices_dfs_dict[var + "_slice_indices_df"][var].values,
                slice_discrepencies_dict[var + "_slice_discrepencies"],
                label="Observations",
            )
            for i in range(gp_samp_no):
                plt.plot(
                    slice_indices_dfs_dict[var + "_gp_indices_df"][var].values,
                    GP_samples[i, :],
                    c="r",
                    alpha=0.1,
                    label="Posterior Sample" if i == 0 else None,
                )
            leg = plt.legend(loc="lower right")
            for lh in leg.legend_handles:
                lh.set_alpha(1)
            if var in ["f", "r"]:
                plt.xlabel("$" + var + "$ index points")
            else:
                plt.xlabel("$\\" + var + "$ index points")
            plt.ylabel("Discrepancy")
            plt.savefig(
                "champagne_GP_images/" + var + "_slice_" + str(t) + "_bolfi_updates.pdf"
            )
            plt.show()

# print(index_vals[-600,])
print(index_vals[-400,])
print(index_vals[-200,])
print(index_vals[-80,])
print(index_vals[-40,])
print(index_vals[-20,])
print(index_vals[-8,])
print(index_vals[-4,])
print(index_vals[-2,])
print(index_vals[-1,])
```

```
[0.73704431 0.30911903 0.00761646 0.06693362 0.00576551 0.00166694]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=21.9141133199771>
0
Acquisition function convergence reached at iteration 2666.
tf.Tensor(-17.78874687060007, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 7800.
[0.39221966 0.39970067 0.02899882 0.0479134  0.02846772 0.02929494]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=25.379849222776826>
1
Acquisition function convergence reached at iteration 1112.
tf.Tensor(-22.730142691981158, shape=(), dtype=float64)
[0.39283988 0.39974583 0.05218201 0.06311031 0.0626069  0.05049378]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=27.40717476331765>
2
Acquisition function convergence reached at iteration 1847.
tf.Tensor(-24.533776952547804, shape=(), dtype=float64)
[0.42350174 0.40040677 0.0402677  0.05459336 0.03367625 0.03600863]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=28.845585125576555>
3
Acquisition function convergence reached at iteration 1276.
tf.Tensor(-25.820394899334243, shape=(), dtype=float64)
[0.41763582 0.40000224 0.05277395 0.06897497 0.06060338 0.06171969]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=29.961302882147603>
4
Acquisition function convergence reached at iteration 1862.
tf.Tensor(-26.81930270465397, shape=(), dtype=float64)
[0.36034139 0.39415216 0.03710994 0.03870981 0.04029362 0.0430834 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=30.872910666117377>
5
Acquisition function convergence reached at iteration 2591.
tf.Tensor(-27.638611436731995, shape=(), dtype=float64)
[0.36870129 0.3983159  0.02836085 0.03866608 0.03434788 0.02850899]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=31.643664065253667>
6
Acquisition function convergence reached at iteration 1991.
tf.Tensor(-28.32914165887645, shape=(), dtype=float64)
[0.43195719 0.39956583 0.03425216 0.05068014 0.03824232 0.03221141]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=32.311321028376284>
7
Acquisition function convergence reached at iteration 2212.
tf.Tensor(-28.937435571563405, shape=(), dtype=float64)
[0.43933387 0.4004751  0.03201407 0.04560929 0.03239232 0.03532994]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=32.9002362066582>
```

```
8
Acquisition function convergence reached at iteration 1812.
tf.Tensor(-29.44922703079004, shape=(), dtype=float64)
[0.45050417 0.39394791 0.04180651 0.04159937 0.0430031  0.03929272]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=33.42703878494733>
9
Acquisition function convergence reached at iteration 1844.
tf.Tensor(-29.9155911186239, shape=(), dtype=float64)
[0.45483834 0.32816092 0.04216976 0.05046356 0.0342986  0.0367868 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=33.90358968396895>
10
Acquisition function convergence reached at iteration 2627.
tf.Tensor(-30.36236540837716, shape=(), dtype=float64)
[0.38524183 0.39973838 0.02393813 0.03694681 0.02108335 0.02629369]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=34.338646568917106>
11
Acquisition function convergence reached at iteration 1859.
tf.Tensor(-30.739440196664027, shape=(), dtype=float64)
[0.4443185  0.37394141 0.03107991 0.04009049 0.04063852 0.04412184]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=34.738860107284786>
12
Acquisition function convergence reached at iteration 2130.
tf.Tensor(-31.107536289971556, shape=(), dtype=float64)
[0.45156721 0.36729014 0.03138008 0.04320764 0.03691667 0.03661629]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=35.109399968053395>
13
Acquisition function convergence reached at iteration 2804.
tf.Tensor(-31.44519757625343, shape=(), dtype=float64)
[0.39060621 0.37992654 0.02561778 0.03556688 0.01966719 0.02201585]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=35.45436432548815>
14
Acquisition function convergence reached at iteration 2039.
tf.Tensor(-31.75211491811652, shape=(), dtype=float64)
[0.43524164 0.37675622 0.03505846 0.05014093 0.03933715 0.03930604]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=35.777056931176006>
15
Acquisition function convergence reached at iteration 3213.
tf.Tensor(-32.039020706382495, shape=(), dtype=float64)
[0.44564213 0.38214477 0.02244043 0.03498445 0.02357376 0.01547231]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=36.080180040258185>
16
Acquisition function convergence reached at iteration 1788.
tf.Tensor(-32.292425297471006, shape=(), dtype=float64)
```

```
[0.35400502 0.33410572 0.03576265 0.03312959 0.04127181 0.04267237]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=36.36597210945793>
17
Acquisition function convergence reached at iteration 1808.
tf.Tensor(-32.61320319895427, shape=(), dtype=float64)
[0.35507771 0.34996857 0.03974691 0.04895156 0.03790217 0.04015779]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=36.6363082158093>
18
Acquisition function convergence reached at iteration 1998.
tf.Tensor(-32.85174366209961, shape=(), dtype=float64)
[0.35259631 0.31765883 0.03295236 0.03821793 0.02826982 0.03369905]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=36.89277468774706>
19
Acquisition function convergence reached at iteration 2124.
tf.Tensor(-33.04353757913332, shape=(), dtype=float64)
[0.44278579 0.36300244 0.03494325 0.05174521 0.02810994 0.01777726]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=37.13672550859422>
20
Acquisition function convergence reached at iteration 1753.
tf.Tensor(-33.244046021828744, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 7276.
[0.44510822 0.34452681 0.03850225 0.04086614 0.04039291 0.04031763]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=37.36932558676868>
21
Acquisition function convergence reached at iteration 2796.
tf.Tensor(-57.035113822170295, shape=(), dtype=float64)
[0.39999615 0.40179478 0.02711408 0.11494929 0.03135914 0.03170133]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=37.59158439962285>
22
Acquisition function convergence reached at iteration 2584.
tf.Tensor(-57.36518795534702, shape=(), dtype=float64)
[0.4        0.39952965 0.02713847 0.18062168 0.03241767 0.02865027]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=37.80438247171683>
23
Acquisition function convergence reached at iteration 2661.
tf.Tensor(-57.66593799682296, shape=(), dtype=float64)
[0.40000021 0.46829243 0.02610189 0.26964181 0.0316131  0.0316526 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=38.00849244431811>
24
Acquisition function convergence reached at iteration 2563.
tf.Tensor(-57.95130334146802, shape=(), dtype=float64)
[0.40000033 0.32501183 0.02933459 0.37698712 0.02784387 0.03058598]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=38.204596010084515>
```

```
25
Acquisition function convergence reached at iteration 2641.
tf.Tensor(-58.25063698894303, shape=(), dtype=float64)
[0.40000033 0.32501157 0.02743124 0.37698844 0.02649867 0.03464409]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=38.39329764999875>
26
Acquisition function convergence reached at iteration 2609.
tf.Tensor(-58.53867224424658, shape=(), dtype=float64)
[0.40000033 0.32501116 0.02707541 0.37698875 0.03214992 0.02936156]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=38.575135870853124>
27
Acquisition function convergence reached at iteration 2446.
tf.Tensor(-58.81629084771838, shape=(), dtype=float64)
[0.40000033 0.3250111  0.0246869  0.37699025 0.02802404 0.03495182]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=38.75059246990947>
28
Acquisition function convergence reached at iteration 2551.
tf.Tensor(-59.08406982944294, shape=(), dtype=float64)
[0.40000033 0.32501044 0.02620631 0.37698976 0.03103394 0.03131354]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=38.92010022828788>
29
Acquisition function convergence reached at iteration 2679.
tf.Tensor(-59.34277715077198, shape=(), dtype=float64)
[0.40000033 0.32501007 0.02606826 0.37699005 0.03267397 0.03060543]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=39.084049342402835>
30
Acquisition function convergence reached at iteration 2555.
tf.Tensor(-59.59305942424514, shape=(), dtype=float64)
[0.40000033 0.32500992 0.02484472 0.37699077 0.03428405 0.0289071 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=39.242792833975734>
31
Acquisition function convergence reached at iteration 2485.
tf.Tensor(-59.83538542592146, shape=(), dtype=float64)
[0.40000033 0.32500942 0.02558987 0.37699081 0.02963009 0.03291808]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=39.396651127309504>
32
Acquisition function convergence reached at iteration 2592.
tf.Tensor(-60.07022765948244, shape=(), dtype=float64)
[0.40000033 0.32500953 0.0230445  0.37699217 0.03142877 0.03448497]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=39.545915943057906>
33
Acquisition function convergence reached at iteration 2574.
tf.Tensor(-60.298040984942716, shape=(), dtype=float64)
```

```
[0.40000033 0.32500872 0.02591523 0.37699097 0.03347024 0.02882915]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=39.69085362742417>
34
Acquisition function convergence reached at iteration 2739.
tf.Tensor(-60.519242770922105, shape=(), dtype=float64)
[0.40000033 0.32500832 0.02611873 0.3769912  0.03209781 0.03169829]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=39.83170801225765>
35
Acquisition function convergence reached at iteration 2670.
tf.Tensor(-60.7342633250138, shape=(), dtype=float64)
[0.40000033 0.32500814 0.02518537 0.37699183 0.03269831 0.03154487]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=39.968702883198226>
36
Acquisition function convergence reached at iteration 2725.
tf.Tensor(-60.94335837292397, shape=(), dtype=float64)
[0.40000033 0.32500773 0.02596405 0.37699172 0.03062535 0.03331191]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=40.10204411860903>
37
Acquisition function convergence reached at iteration 2655.
tf.Tensor(-61.14687789366604, shape=(), dtype=float64)
[0.40000033 0.32500716 0.02877683 0.37699044 0.03082717 0.02892077]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=40.23192155062534>
38
Acquisition function convergence reached at iteration 2552.
tf.Tensor(-61.34513897183365, shape=(), dtype=float64)
[0.40000033 0.32500679 0.02994189 0.37699009 0.02698347 0.03040348]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=40.358510590546786>
39
Acquisition function convergence reached at iteration 2603.
tf.Tensor(-61.53837913904654, shape=(), dtype=float64)
[0.40000033 0.32500675 0.02709534 0.37699179 0.02652602 0.03472375]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=40.48197365349864>
40
Acquisition function convergence reached at iteration 2744.
tf.Tensor(-61.726814964215755, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 6666.
[0.40000033 0.32500667 0.02523443 0.37699281 0.03171648 0.03309773]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=40.602461411393946>
41
Acquisition function convergence reached at iteration 2443.
tf.Tensor(-49.54692871533463, shape=(), dtype=float64)
[0.39999988 0.33212752 0.02893151 0.10225044 0.03324863 0.09774256]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=40.720113898444914>
```

```
42
Acquisition function convergence reached at iteration 2319.
tf.Tensor(-49.68564689650818, shape=(), dtype=float64)
[0.4        0.4245553  0.02587831 0.16621659 0.03654049 0.10896737]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=40.83506148956841>
43
Acquisition function convergence reached at iteration 3116.
tf.Tensor(-49.82754968890441, shape=(), dtype=float64)
[0.4        0.32165201 0.02476353 0.1643589  0.028292   0.01929277]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=40.9474257688287>
44
Acquisition function convergence reached at iteration 2450.
tf.Tensor(-49.95528923155126, shape=(), dtype=float64)
[0.4        0.33999481 0.02986381 0.1638256  0.03344655 0.11359785]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.057320302422575>
45
Acquisition function convergence reached at iteration 2786.
tf.Tensor(-50.07957702141229, shape=(), dtype=float64)
[0.4        0.47693052 0.02309943 0.26678461 0.03050303 0.04350592]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.1648513285274>
46
Acquisition function convergence reached at iteration 2649.
tf.Tensor(-50.210557623620495, shape=(), dtype=float64)
[0.4        0.47695737 0.02291671 0.26470434 0.03157633 0.05514125]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.27011837451656>
47
Acquisition function convergence reached at iteration 2313.
tf.Tensor(-50.35659484735212, shape=(), dtype=float64)
[0.4        0.50000519 0.03022099 0.14469459 0.02957089 0.0521528 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.373214810530236>
48
Acquisition function convergence reached at iteration 4496.
tf.Tensor(-50.46765594888723, shape=(), dtype=float64)
[0.4        0.5069243  0.0131696  0.22793901 0.02004211 0.01360628]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.47422834711783>
49
Acquisition function convergence reached at iteration 4744.
tf.Tensor(-50.62309287633811, shape=(), dtype=float64)
[0.39999593 0.46325498 0.01282067 0.0681624  0.01831846 0.00233292]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.57324148359873>
50
Acquisition function convergence reached at iteration 2286.
tf.Tensor(-50.73143887673563, shape=(), dtype=float64)
```

```
Trained parameters:
amplitude_champ:0 is 1.221

length_scales_champ:0 is [8.4450e+00 2.2000e-02 1.0980e+00 9.0000e-03 1.5797e+01 2.0000e-02]

observation_noise_variance_champ:0 is 0.218

alpha_tp:0 is 0.266

amp_alpha_mean:0 is 0.184

amp_beta_mean:0 is 0.085

amp_f_mean:0 is 163.524

amp_gamma_L_mean:0 is 221.147

amp_lambda_mean:0 is 19.848

amp_r_mean:0 is 38.281

beta_tp:0 is 1.064

bias_mean:0 is -3.31

f_tp:0 is 0.086

gamma_L_tp:0 is 0.058

lambda_tp:0 is -0.28

r_tp:0 is -0.085

[0.39999998 0.46613933 0.02791102 0.10068829 0.03444146 0.10461499]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.67033191288424>
51
Acquisition function convergence reached at iteration 2507.
tf.Tensor(-50.83632967331244, shape=(), dtype=float64)
[0.4        0.39868873 0.0289854  0.14337429 0.03477284 0.16894655]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.76557288773771>
52
Acquisition function convergence reached at iteration 2579.
tf.Tensor(-50.94151832006703, shape=(), dtype=float64)
```

```
[0.4        0.42728158 0.030184   0.2674713  0.03391423 0.10977548]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.85903355279847>
53
Acquisition function convergence reached at iteration 2145.
tf.Tensor(-51.05563203959497, shape=(), dtype=float64)
[0.4        0.42750764 0.02749713 0.2670784  0.03384678 0.10982441]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=41.950779246139454>
54
Acquisition function convergence reached at iteration 2339.
tf.Tensor(-51.16753568742614, shape=(), dtype=float64)
[0.4        0.42762105 0.02613942 0.26689211 0.03645289 0.1098476 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.040871773652846>
55
Acquisition function convergence reached at iteration 2435.
tf.Tensor(-51.27747347278128, shape=(), dtype=float64)
[0.4        0.4276279  0.02802268 0.26688105 0.03533065 0.10984908]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.12936965914985>
56
Acquisition function convergence reached at iteration 2505.
tf.Tensor(-51.38547852014271, shape=(), dtype=float64)
[0.4        0.4276256  0.02953551 0.26688497 0.03417998 0.10984868]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.2163283727092>
57
Acquisition function convergence reached at iteration 2381.
tf.Tensor(-51.49165271094059, shape=(), dtype=float64)
[0.4        0.42761988 0.03057121 0.26689455 0.03212982 0.1098476 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.3018005395057>
58
Acquisition function convergence reached at iteration 2368.
tf.Tensor(-51.59599967930548, shape=(), dtype=float64)
[0.4        0.42763166 0.02856765 0.26687569 0.03432226 0.10985019]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.38583613108761>
59
Acquisition function convergence reached at iteration 2225.
tf.Tensor(-51.6986189512012, shape=(), dtype=float64)
[0.4        0.42764305 0.02739012 0.26685741 0.03455107 0.10985265]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.46848264084366>
60
Acquisition function convergence reached at iteration 2234.
tf.Tensor(-51.79948110713925, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 7981.
[0.4        0.42765614 0.02985382 0.26683621 0.03191099 0.10985533]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.54978524520256>
```

```
61
Acquisition function convergence reached at iteration 2418.
tf.Tensor(-41.79476212309139, shape=(), dtype=float64)
[0.4        0.41503362 0.02875072 0.26068509 0.03448197 0.08986597]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.62978695193477>
62
Acquisition function convergence reached at iteration 2282.
tf.Tensor(-41.873440534011614, shape=(), dtype=float64)
[0.4        0.41504106 0.02896745 0.26067143 0.03330066 0.08987139]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.70852873677546>
63
Acquisition function convergence reached at iteration 2332.
tf.Tensor(-41.95084254359311, shape=(), dtype=float64)
[0.4        0.41501229 0.02848453 0.2606631  0.03416134 0.0899134 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.78604966945529>
64
Acquisition function convergence reached at iteration 2252.
tf.Tensor(-42.027085646039076, shape=(), dtype=float64)
[0.4        0.41496859 0.02604563 0.2606433  0.03599186 0.08998194]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.86238703010923>
65
Acquisition function convergence reached at iteration 2422.
tf.Tensor(-42.10207308170517, shape=(), dtype=float64)
[0.4        0.41497006 0.02903158 0.26064346 0.03420152 0.0899834 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=42.93757641693193>
66
Acquisition function convergence reached at iteration 2242.
tf.Tensor(-42.17604535152612, shape=(), dtype=float64)
[0.4        0.4149648  0.02752767 0.26064344 0.03451704 0.08998636]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.011651845857635>
67
Acquisition function convergence reached at iteration 2569.
tf.Tensor(-42.24877395353778, shape=(), dtype=float64)
[0.4        0.41503919 0.02998546 0.26066884 0.03410419 0.08987765]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.0846458429634>
68
Acquisition function convergence reached at iteration 2404.
tf.Tensor(-42.32056151100876, shape=(), dtype=float64)
[0.4        0.41498623 0.0315451  0.26064851 0.03101087 0.08995878]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.1565895302239>
69
Acquisition function convergence reached at iteration 2413.
tf.Tensor(-42.39130459541977, shape=(), dtype=float64)
```

```
[0.4        0.41501774 0.02881339 0.26065987 0.0343689  0.08991196]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.22751270518368>
70
Acquisition function convergence reached at iteration 2347.
tf.Tensor(-42.461034175857314, shape=(), dtype=float64)
[0.4        0.4149832  0.03044881 0.26064657 0.03200933 0.0899655 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.29744391505738>
71
Acquisition function convergence reached at iteration 2330.
tf.Tensor(-42.52977253059661, shape=(), dtype=float64)
[0.4        0.41494638 0.03240259 0.26063187 0.02914726 0.09002483]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.366410525719054>
72
Acquisition function convergence reached at iteration 2287.
tf.Tensor(-42.59762438095901, shape=(), dtype=float64)
[0.4        0.41502542 0.02607862 0.26066269 0.03619127 0.08990051]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.43443878599795>
73
Acquisition function convergence reached at iteration 2342.
tf.Tensor(-42.66448235203446, shape=(), dtype=float64)
[0.4        0.41503865 0.02730948 0.26066781 0.03540934 0.08988072]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.50155388765865>
74
Acquisition function convergence reached at iteration 2478.
tf.Tensor(-42.730412661511004, shape=(), dtype=float64)
[0.4        0.41503792 0.03016604 0.26066737 0.03328756 0.08988211]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.56778002140876>
75
Acquisition function convergence reached at iteration 2512.
tf.Tensor(-42.79551727000197, shape=(), dtype=float64)
[0.4        0.41503715 0.02909845 0.26066696 0.03471395 0.08988374]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.63314042924552>
76
Acquisition function convergence reached at iteration 2323.
tf.Tensor(-42.85981440260102, shape=(), dtype=float64)
[0.4        0.41502559 0.02954853 0.260663   0.03294842 0.08990065]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.69765745342506>
77
Acquisition function convergence reached at iteration 2313.
tf.Tensor(-42.923237292799584, shape=(), dtype=float64)
[0.4        0.41492966 0.03054562 0.26062614 0.03162426 0.09005044]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.761352582312206>
78
```

```
Acquisition function convergence reached at iteration 2458.
tf.Tensor(-42.98583712246017, shape=(), dtype=float64)
[0.4        0.41502637 0.02791103 0.26066323 0.03556759 0.08990073]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.82424649334651>
79
Acquisition function convergence reached at iteration 2418.
tf.Tensor(-43.04764777967137, shape=(), dtype=float64)
[0.4        0.41502229 0.03225201 0.26066184 0.03013633 0.08990675]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.886359093339294>
80
Acquisition function convergence reached at iteration 2450.
tf.Tensor(-43.10871098812502, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 6990.
[0.4        0.41503008 0.03112634 0.26066457 0.03192807 0.08989524]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=43.94770955629837>
81
Acquisition function convergence reached at iteration 2186.
tf.Tensor(-39.33508117205074, shape=(), dtype=float64)
[0.4        0.42167062 0.02678944 0.26234613 0.03485739 0.11872276]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.008316358960094>
82
Acquisition function convergence reached at iteration 2286.
tf.Tensor(-39.3893368765644, shape=(), dtype=float64)
[0.4        0.42168073 0.02944057 0.26232843 0.03280487 0.11873195]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.06819731419367>
83
Acquisition function convergence reached at iteration 2435.
tf.Tensor(-39.442948736751454, shape=(), dtype=float64)
[0.4        0.42168088 0.0303229  0.26232832 0.03282474 0.11873236]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.127369602428686>
84
Acquisition function convergence reached at iteration 2319.
tf.Tensor(-39.49598689868739, shape=(), dtype=float64)
[0.4        0.42168045 0.03152462 0.26232924 0.03037255 0.11873224]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.18584980124464>
85
Acquisition function convergence reached at iteration 2403.
tf.Tensor(-39.548355708286955, shape=(), dtype=float64)
[0.4        0.42168146 0.03266142 0.2623276  0.02939661 0.11873342]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.24365391325002>
86
Acquisition function convergence reached at iteration 2293.
tf.Tensor(-39.60019505626122, shape=(), dtype=float64)
```

```
[0.4        0.42168077 0.02996222 0.26232897 0.03221799 0.11873306]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.30079739236813>
87
Acquisition function convergence reached at iteration 2292.
tf.Tensor(-39.65140329515509, shape=(), dtype=float64)
[0.4        0.4216808  0.02884802 0.26232905 0.0334983  0.11873335]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.3572951686378>
88
Acquisition function convergence reached at iteration 2464.
tf.Tensor(-39.70197897294381, shape=(), dtype=float64)
[0.4        0.42168091 0.02967476 0.262329   0.03375547 0.11873371]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.41316167162843>
89
Acquisition function convergence reached at iteration 2412.
tf.Tensor(-39.75203423476153, shape=(), dtype=float64)
[0.4        0.42168131 0.03156304 0.26232843 0.03105364 0.11873433]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.468410852561355>
90
Acquisition function convergence reached at iteration 2179.
tf.Tensor(-39.8016033884315, shape=(), dtype=float64)
[0.4        0.42168092 0.03040016 0.26232926 0.03073495 0.11873423]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.523056205222304>
91
Acquisition function convergence reached at iteration 2464.
tf.Tensor(-39.850500408695126, shape=(), dtype=float64)
[0.4        0.42168071 0.02964256 0.26232978 0.03382052 0.11873429]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.57711078574338>
92
Acquisition function convergence reached at iteration 2346.
tf.Tensor(-39.89898682237473, shape=(), dtype=float64)
[0.4        0.42168073 0.02541573 0.26232988 0.03711514 0.11873455]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.63058723132712>
93
Acquisition function convergence reached at iteration 2470.
tf.Tensor(-39.94683928052617, shape=(), dtype=float64)
[0.4        0.42168057 0.03090828 0.2623303  0.03234256 0.11873466]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.6834977779798>
94
Acquisition function convergence reached at iteration 2384.
tf.Tensor(-39.994263994885415, shape=(), dtype=float64)
[0.4        0.42168092 0.03141791 0.2623298  0.03102455 0.11873522]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.735854277316285>
95
```

```
Acquisition function convergence reached at iteration 2369.
tf.Tensor(-40.04116996569641, shape=(), dtype=float64)
[0.4        0.42168095 0.03248751 0.26232989 0.02938481 0.11873548]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.78766821249402>
96
Acquisition function convergence reached at iteration 510.
tf.Tensor(-40.054846445227795, shape=(), dtype=float64)
[0.39999999 0.37659485 0.14765064 0.24769536 0.15434564 0.15582404]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.838950713329965>
97
Acquisition function convergence reached at iteration 2904.
tf.Tensor(-40.11079511812075, shape=(), dtype=float64)
[0.4        0.32344596 0.02020302 0.37877079 0.02442825 0.01073071]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.889712570650055>
98
Acquisition function convergence reached at iteration 3068.
tf.Tensor(-40.15624848144134, shape=(), dtype=float64)
[0.4        0.32359137 0.02125457 0.37873491 0.02564233 0.01044926]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.93996424991756>
99
Acquisition function convergence reached at iteration 2370.
tf.Tensor(-40.22383039726846, shape=(), dtype=float64)
[0.4        0.42447548 0.02767386 0.26165409 0.0352365  0.11811508]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=44.9897159041834>
100
Acquisition function convergence reached at iteration 2139.
tf.Tensor(-40.26846737857364, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 9462.
Trained parameters:
amplitude_champ:0 is 0.88

length_scales_champ:0 is [4.27873e+03 2.60000e-02 3.58685e+02 7.00000e-03 5.53223e+02 5.4500

observation_noise_variance_champ:0 is 0.365

alpha_tp:0 is 0.45

amp_alpha_mean:0 is 0.826

amp_beta_mean:0 is 0.196

amp_f_mean:0 is 0.861
```

```
amp_gamma_L_mean:0 is 24.325

amp_lambda_mean:0 is 16.909

amp_r_mean:0 is 0.647

beta_tp:0 is 0.4

bias_mean:0 is -2.069

f_tp:0 is -0.082

gamma_L_tp:0 is 0.128

lambda_tp:0 is -0.311

r_tp:0 is 0.499

[0.4        0.42447559 0.02743326 0.2616537  0.03388131 0.11811591]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.03897738639846>
101
Acquisition function convergence reached at iteration 2544.
tf.Tensor(-39.57644728534682, shape=(), dtype=float64)
[0.4        0.35820317 0.0246595  0.41861325 0.02816426 0.03545684]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.08775826112528>
102
Acquisition function convergence reached at iteration 2524.
tf.Tensor(-39.619378047062064, shape=(), dtype=float64)
[0.4        0.35820002 0.02634275 0.41861063 0.02912486 0.03281034]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.136067815683965>
103
Acquisition function convergence reached at iteration 2616.
tf.Tensor(-39.66186357560802, shape=(), dtype=float64)
[0.4        0.3581955  0.02845801 0.41860694 0.03045662 0.0294046 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.18391507076472>
104
Acquisition function convergence reached at iteration 2782.
tf.Tensor(-39.703961541414785, shape=(), dtype=float64)
[0.4        0.35820019 0.02723549 0.41861098 0.02990495 0.03287014]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.23130879053744>
105
Acquisition function convergence reached at iteration 2402.
tf.Tensor(-39.74576690439068, shape=(), dtype=float64)
```

```
[0.4        0.35819902 0.02638556 0.41861001 0.02913232 0.03178887]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.278257492286635>
106
Acquisition function convergence reached at iteration 2502.
tf.Tensor(-39.787077775204395, shape=(), dtype=float64)
[0.4        0.35819821 0.02531924 0.41860939 0.03173301 0.03109237]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.3247694555982>
107
Acquisition function convergence reached at iteration 2551.
tf.Tensor(-39.82800216385163, shape=(), dtype=float64)
[0.4        0.35819881 0.02650216 0.41860998 0.03048254 0.03148316]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.37085273112282>
108
Acquisition function convergence reached at iteration 2718.
tf.Tensor(-39.86852151277273, shape=(), dtype=float64)
[0.4        0.35820006 0.02824841 0.41861109 0.02870838 0.03232234]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.41651514893918>
109
Acquisition function convergence reached at iteration 2461.
tf.Tensor(-39.908798087065236, shape=(), dtype=float64)
[0.4        0.35820355 0.02476462 0.41861404 0.028061   0.03488494]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.46176432653877>
110
Acquisition function convergence reached at iteration 2492.
tf.Tensor(-39.94862742669121, shape=(), dtype=float64)
[0.4        0.35820339 0.02411408 0.41861403 0.02919831 0.03475942]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.506607676452575>
111
Acquisition function convergence reached at iteration 2541.
tf.Tensor(-39.988062770614704, shape=(), dtype=float64)
[0.4        0.35820127 0.02777865 0.41861237 0.02699262 0.03311064]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.551052413538805>
112
Acquisition function convergence reached at iteration 2526.
tf.Tensor(-40.0271901021595, shape=(), dtype=float64)
[0.4        0.35819579 0.02662951 0.41860778 0.03236642 0.0288634 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.59510556194958>
113
Acquisition function convergence reached at iteration 2598.
tf.Tensor(-40.06594346106279, shape=(), dtype=float64)
[0.4        0.35819704 0.02831401 0.41860893 0.03024065 0.02975121]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.638773961793355>
114
```

```
Acquisition function convergence reached at iteration 2556.
tf.Tensor(-40.104402515149864, shape=(), dtype=float64)
[0.4        0.35820076 0.02757709 0.41861205 0.02831374 0.03232243]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.68206427550893>
115
Acquisition function convergence reached at iteration 2616.
tf.Tensor(-40.142491921595926, shape=(), dtype=float64)
[0.4        0.35819528 0.0269684  0.41860749 0.03307626 0.02819198]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.72498299396588>
116
Acquisition function convergence reached at iteration 2661.
tf.Tensor(-40.18026879867483, shape=(), dtype=float64)
[0.4        0.35819683 0.02640699 0.41860892 0.03312057 0.02932813]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.76753644230543>
117
Acquisition function convergence reached at iteration 2650.
tf.Tensor(-40.21773920185358, shape=(), dtype=float64)
[0.4        0.35819986 0.02549642 0.41861156 0.03222256 0.03153663]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.80973078553475>
118
Acquisition function convergence reached at iteration 2605.
tf.Tensor(-40.25489369199908, shape=(), dtype=float64)
[0.4        0.3582007  0.0252157  0.41861227 0.03178778 0.03200649]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.85157203388733>
119
Acquisition function convergence reached at iteration 2589.
tf.Tensor(-40.291709118975696, shape=(), dtype=float64)
[0.4        0.35819682 0.02745639 0.41860906 0.03174607 0.0290402 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.89306604796081>
120
Acquisition function convergence reached at iteration 2570.
tf.Tensor(-40.32822201223798, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 8253.
[0.4        0.35819688 0.02979553 0.41860916 0.02867151 0.0289973 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.93421854364338>
121
Acquisition function convergence reached at iteration 2661.
tf.Tensor(-42.27015302006576, shape=(), dtype=float64)
[0.4        0.34996079 0.0285002  0.41289658 0.02873278 0.03162467]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=45.975035096839186>
122
Acquisition function convergence reached at iteration 2643.
tf.Tensor(-42.30778620942449, shape=(), dtype=float64)
```

```
[0.4        0.34996084 0.02699603 0.4128967  0.03183795 0.03012252]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.015521148002286>
123
Acquisition function convergence reached at iteration 2527.
tf.Tensor(-42.34513765016151, shape=(), dtype=float64)
[0.4        0.349961   0.02652228 0.4128969  0.02836458 0.03336533]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.05568200648861>
124
Acquisition function convergence reached at iteration 2757.
tf.Tensor(-42.382089903441944, shape=(), dtype=float64)
[0.4        0.34996099 0.02799733 0.41289698 0.03201009 0.02942875]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.09552285473449>
125
Acquisition function convergence reached at iteration 2654.
tf.Tensor(-42.418825720124296, shape=(), dtype=float64)
[0.4        0.34996102 0.02837651 0.41289709 0.03236239 0.02746481]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.13504875227006>
126
Acquisition function convergence reached at iteration 2804.
tf.Tensor(-42.45524544442323, shape=(), dtype=float64)
[0.4        0.34996122 0.02734076 0.41289732 0.03043567 0.03246739]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.174264639575185>
127
Acquisition function convergence reached at iteration 2315.
tf.Tensor(-42.49150979076144, shape=(), dtype=float64)
[0.4        0.3499613  0.02583562 0.41289745 0.02915617 0.03172767]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.213175341785465>
128
Acquisition function convergence reached at iteration 2465.
tf.Tensor(-42.527341936068616, shape=(), dtype=float64)
[0.4        0.34996133 0.02485409 0.41289757 0.032808   0.03013918]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.25178557225502>
129
Acquisition function convergence reached at iteration 2690.
tf.Tensor(-42.562857005507155, shape=(), dtype=float64)
[0.4        0.34996143 0.02879407 0.41289772 0.02950841 0.03068162]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.29009993598286>
130
Acquisition function convergence reached at iteration 2602.
tf.Tensor(-42.598208562771646, shape=(), dtype=float64)
[0.4        0.34996151 0.02633681 0.41289786 0.03176956 0.03071612]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.328122932908954>
131
```

```
Acquisition function convergence reached at iteration 2518.
tf.Tensor(-42.63326814994526, shape=(), dtype=float64)
[0.4        0.34996164 0.02767947 0.41289803 0.02750264 0.03268562]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.36585896108587>
132
Acquisition function convergence reached at iteration 2684.
tf.Tensor(-42.66799651896033, shape=(), dtype=float64)
[0.4        0.3499616  0.02892921 0.41289808 0.03165424 0.02782588]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.40331231973166>
133
Acquisition function convergence reached at iteration 2545.
tf.Tensor(-42.70255578370327, shape=(), dtype=float64)
[0.4        0.34996175 0.0288731  0.41289827 0.02826736 0.03064685]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.44048721216925>
134
Acquisition function convergence reached at iteration 2689.
tf.Tensor(-42.73678768357254, shape=(), dtype=float64)
[0.4        0.34996178 0.02874941 0.41289837 0.03116785 0.02878875]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.477387748657364>
135
Acquisition function convergence reached at iteration 2664.
tf.Tensor(-42.77081723608485, shape=(), dtype=float64)
[0.4        0.34996196 0.0280274  0.41289858 0.02775121 0.03314743]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.51401794911773>
136
Acquisition function convergence reached at iteration 2615.
tf.Tensor(-42.80458903617369, shape=(), dtype=float64)
[0.4        0.34996199 0.02801476 0.41289868 0.02930982 0.03137338]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.550381745763126>
137
Acquisition function convergence reached at iteration 2691.
tf.Tensor(-42.83809087183666, shape=(), dtype=float64)
[0.4        0.34996204 0.02771938 0.41289879 0.03145417 0.02998506]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.58648298563056>
138
Acquisition function convergence reached at iteration 2586.
tf.Tensor(-42.87139061965415, shape=(), dtype=float64)
[0.4        0.34996217 0.02796915 0.41289896 0.02825804 0.03221101]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.622325433023626>
139
Acquisition function convergence reached at iteration 2726.
tf.Tensor(-42.9043909386148, shape=(), dtype=float64)
[0.4        0.34996216 0.02823448 0.41289903 0.03195552 0.02888502]
```

```
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.65791277186794>
140
Acquisition function convergence reached at iteration 2638.
tf.Tensor(-42.93721716114503, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 9392.
[0.4        0.34996231 0.02825599 0.41289921 0.02861163 0.03189647]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.693248607983406>
141
Acquisition function convergence reached at iteration 2640.
tf.Tensor(-43.44857411578866, shape=(), dtype=float64)
[0.4        0.34392204 0.02407823 0.40732339 0.032677   0.03253343]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.72833647127664>
142
Acquisition function convergence reached at iteration 2635.
tf.Tensor(-43.481258108967296, shape=(), dtype=float64)
[0.4        0.34392209 0.02605266 0.4073235  0.03316788 0.02958994]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.76317981785711>
143
Acquisition function convergence reached at iteration 2678.
tf.Tensor(-43.51372948499878, shape=(), dtype=float64)
[0.4        0.34392215 0.02587852 0.40732362 0.0305471  0.03315986]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.79778203207997>
144
Acquisition function convergence reached at iteration 2537.
tf.Tensor(-43.54601243896091, shape=(), dtype=float64)
[0.4        0.34392221 0.02520233 0.40732374 0.03082594 0.03257599]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.83214642851878>
145
Acquisition function convergence reached at iteration 2682.
tf.Tensor(-43.577991213663964, shape=(), dtype=float64)
[0.4        0.34392227 0.02785355 0.40732386 0.02722422 0.03388326]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.86627625387079>
146
Acquisition function convergence reached at iteration 2579.
tf.Tensor(-43.60981665190283, shape=(), dtype=float64)
[0.4        0.34392232 0.02753815 0.40732397 0.03133731 0.02944896]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.90017468879768>
147
Acquisition function convergence reached at iteration 2554.
tf.Tensor(-43.641433753183605, shape=(), dtype=float64)
[0.4        0.34392238 0.02445984 0.40732409 0.03423714 0.02946306]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.9338448497044>
148
```

```
Acquisition function convergence reached at iteration 2725.
tf.Tensor(-43.6727584293347, shape=(), dtype=float64)
[0.4        0.34392243 0.02840589 0.4073242  0.02781641 0.03303123]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=46.96728979045838>
149
Acquisition function convergence reached at iteration 2530.
tf.Tensor(-43.70398127829021, shape=(), dtype=float64)
[0.4        0.34392249 0.02693214 0.40732432 0.02710688 0.03394515]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.00051250405172>
150
Acquisition function convergence reached at iteration 2465.
tf.Tensor(-43.73497517683253, shape=(), dtype=float64)
Trained parameters:
amplitude_champ:0 is 0.932

length_scales_champ:0 is [5.769044e+03 2.400000e-02 7.477870e+02 6.000000e-03 1.263811e+03
 6.323700e+01]

observation_noise_variance_champ:0 is 0.325

alpha_tp:0 is 0.481

amp_alpha_mean:0 is 1.108

amp_beta_mean:0 is 0.2

amp_f_mean:0 is 0.824

amp_gamma_L_mean:0 is 21.675

amp_lambda_mean:0 is 13.618

amp_r_mean:0 is 0.331

beta_tp:0 is 0.268

bias_mean:0 is -2.516

f_tp:0 is -0.08

gamma_L_tp:0 is 0.121

lambda_tp:0 is -0.402
```

```
r_tp:0 is 0.468

[0.4        0.34392254 0.02458337 0.40732443 0.02834174 0.03492267]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.03351592420849>
151
Acquisition function convergence reached at iteration 2507.
tf.Tensor(-43.765683017124566, shape=(), dtype=float64)
[0.4        0.34392259 0.02939421 0.40732454 0.02809611 0.02976323]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.06630292693928>
152
Acquisition function convergence reached at iteration 2596.
tf.Tensor(-43.79622933008898, shape=(), dtype=float64)
[0.4        0.34392265 0.02862907 0.40732465 0.02797769 0.03168963]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.09887633204525>
153
Acquisition function convergence reached at iteration 2714.
tf.Tensor(-43.82658667402182, shape=(), dtype=float64)
[0.4        0.3439227  0.02546071 0.40732476 0.03130071 0.03307332]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.13123890457334>
154
Acquisition function convergence reached at iteration 2547.
tf.Tensor(-43.856781979123994, shape=(), dtype=float64)
[0.4        0.34392276 0.02582714 0.40732487 0.02788865 0.03464076]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.16339335622479>
155
Acquisition function convergence reached at iteration 2512.
tf.Tensor(-43.886764991852374, shape=(), dtype=float64)
[0.4        0.34392281 0.02367255 0.40732498 0.03353542 0.03095922]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.19534234671864>
156
Acquisition function convergence reached at iteration 2751.
tf.Tensor(-43.916471298184014, shape=(), dtype=float64)
[0.4        0.34392286 0.02711882 0.40732509 0.02971752 0.03306462]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.227088485111935>
157
Acquisition function convergence reached at iteration 2565.
tf.Tensor(-43.9460894683923, shape=(), dtype=float64)
[0.4        0.34392291 0.02723354 0.4073252  0.03179174 0.02918811]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.25863433107826>
158
Acquisition function convergence reached at iteration 2663.
tf.Tensor(-43.97549231119324, shape=(), dtype=float64)
```

```
[0.4        0.34392296 0.02484329 0.4073253  0.03116921 0.03350782]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.289982396146236>
159
Acquisition function convergence reached at iteration 2445.
tf.Tensor(-44.0047444366178, shape=(), dtype=float64)
[0.4        0.34392303 0.02589716 0.40732542 0.03136853 0.03040821]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.321135144899415>
160
Acquisition function convergence reached at iteration 2661.
tf.Tensor(-44.03374507101983, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 9572.
[0.4        0.34392307 0.02450757 0.40732552 0.03100086 0.03398283]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.35209499613902>
161
Acquisition function convergence reached at iteration 2771.
tf.Tensor(-44.83685779759875, shape=(), dtype=float64)
[0.4        0.30708395 0.02734674 0.37893799 0.02998638 0.03264977]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.38286432401091>
162
Acquisition function convergence reached at iteration 2681.
tf.Tensor(-44.86604344189629, shape=(), dtype=float64)
[0.4        0.30708389 0.02844896 0.378938   0.03019018 0.03034923]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.41344545909809>
163
Acquisition function convergence reached at iteration 2550.
tf.Tensor(-44.89508961551824, shape=(), dtype=float64)
[0.4        0.30708382 0.02645819 0.378938   0.02935564 0.03271072]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.443840689480005>
164
Acquisition function convergence reached at iteration 2630.
tf.Tensor(-44.92388079378257, shape=(), dtype=float64)
[0.4        0.30708375 0.02809404 0.37893801 0.03024357 0.03038904]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.474052261759816>
165
Acquisition function convergence reached at iteration 2581.
tf.Tensor(-44.95255641951991, shape=(), dtype=float64)
[0.4        0.30708368 0.02642901 0.37893802 0.02968983 0.03267545]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.50408238206088>
166
Acquisition function convergence reached at iteration 2514.
tf.Tensor(-44.981013890270894, shape=(), dtype=float64)
[0.4        0.30708362 0.03036815 0.37893802 0.02753562 0.0288343 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.5339332169934>
```

```
167
Acquisition function convergence reached at iteration 2536.
tf.Tensor(-45.00936413587423, shape=(), dtype=float64)
[0.4        0.30708355 0.02385056 0.37893803 0.03376177 0.03067606]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.56360689459247>
168
Acquisition function convergence reached at iteration 2668.
tf.Tensor(-45.03745139189252, shape=(), dtype=float64)
[0.4        0.30708348 0.02824406 0.37893803 0.02620626 0.03411014]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.59310550522841>
169
Acquisition function convergence reached at iteration 2667.
tf.Tensor(-45.06544493752126, shape=(), dtype=float64)
[0.4        0.30708342 0.02541865 0.37893804 0.03088997 0.03321988]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.6224311024904>
170
Acquisition function convergence reached at iteration 2793.
tf.Tensor(-45.093211457478304, shape=(), dtype=float64)
[0.4        0.30708335 0.02758661 0.37893805 0.03086326 0.03163732]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.651585704044365>
171
Acquisition function convergence reached at iteration 2626.
tf.Tensor(-45.12087940460605, shape=(), dtype=float64)
[0.4        0.30708329 0.02894046 0.37893805 0.03038825 0.02891971]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.680571292466>
172
Acquisition function convergence reached at iteration 2661.
tf.Tensor(-45.14837664727406, shape=(), dtype=float64)
[0.4        0.30708322 0.02658485 0.37893806 0.03272912 0.02968416]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.70938981604975>
173
Acquisition function convergence reached at iteration 2711.
tf.Tensor(-45.175683877511695, shape=(), dtype=float64)
[0.4        0.30708316 0.02783448 0.37893806 0.03238298 0.02882637]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.73804318959467>
174
Acquisition function convergence reached at iteration 2701.
tf.Tensor(-45.2028625562682, shape=(), dtype=float64)
[0.4        0.3070831  0.02759743 0.37893807 0.03199852 0.0295573 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.76653329516786>
175
Acquisition function convergence reached at iteration 2672.
tf.Tensor(-45.229909712309535, shape=(), dtype=float64)
```

```
[0.4        0.30708303 0.02487326 0.37893808 0.03156136 0.03316523]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.79486198284625>
176
Acquisition function convergence reached at iteration 2569.
tf.Tensor(-45.25676857134883, shape=(), dtype=float64)
[0.4        0.30708297 0.02869119 0.37893808 0.027603   0.03171664]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.823031071437526>
177
Acquisition function convergence reached at iteration 2659.
tf.Tensor(-45.283467371661835, shape=(), dtype=float64)
[0.4        0.30708291 0.02724523 0.37893809 0.03239046 0.02920128]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.85104234918088>
178
Acquisition function convergence reached at iteration 2586.
tf.Tensor(-45.31002457680654, shape=(), dtype=float64)
[0.4        0.30708285 0.03017627 0.37893809 0.02865326 0.02860187]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.87889757442815>
179
Acquisition function convergence reached at iteration 2520.
tf.Tensor(-45.33650449525428, shape=(), dtype=float64)
[0.4        0.30708279 0.02393581 0.3789381  0.03152555 0.03305828]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.90659847630623>
180
Acquisition function convergence reached at iteration 2361.
tf.Tensor(-45.36278899376913, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 9857.
[0.4        0.30708272 0.02674703 0.3789381  0.02948713 0.03068649]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.93414675536108>
181
Acquisition function convergence reached at iteration 2748.
tf.Tensor(-45.78218534883362, shape=(), dtype=float64)
[0.4        0.35509011 0.02763802 0.40806386 0.03017086 0.03196917]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.961544084184204>
182
Acquisition function convergence reached at iteration 2783.
tf.Tensor(-45.80838824008161, shape=(), dtype=float64)
[0.4        0.35509018 0.02701719 0.40806395 0.03125875 0.03183154]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=47.98879210802203>
183
Acquisition function convergence reached at iteration 2773.
tf.Tensor(-45.83444090215917, shape=(), dtype=float64)
[0.4        0.35509025 0.02811457 0.40806404 0.03180722 0.02968027]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.01589244536873>
```

```
184
Acquisition function convergence reached at iteration 2688.
tf.Tensor(-45.86039928239696, shape=(), dtype=float64)
[0.4        0.35509033 0.0257908  0.40806413 0.03237224 0.03135986]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.04284668854311>
185
Acquisition function convergence reached at iteration 2641.
tf.Tensor(-45.88619262427464, shape=(), dtype=float64)
[0.4        0.3550904  0.02597759 0.40806422 0.03019802 0.03313287]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.06965640425003>
186
Acquisition function convergence reached at iteration 2678.
tf.Tensor(-45.91182572247459, shape=(), dtype=float64)
[0.4        0.35509047 0.02651592 0.40806431 0.02972878 0.03321952]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.09632313412685>
187
Acquisition function convergence reached at iteration 2504.
tf.Tensor(-45.9373737756437, shape=(), dtype=float64)
[0.4        0.35509054 0.02534002 0.40806439 0.03212059 0.03072117]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.12284839527531>
188
Acquisition function convergence reached at iteration 2617.
tf.Tensor(-45.96272229390063, shape=(), dtype=float64)
[0.4        0.35509061 0.02506144 0.40806448 0.03298445 0.03092939]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.14923368077953>
189
Acquisition function convergence reached at iteration 2757.
tf.Tensor(-45.98793238717396, shape=(), dtype=float64)
[0.4        0.35509068 0.02533945 0.40806457 0.0320309  0.03274886]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.17548046021025>
190
Acquisition function convergence reached at iteration 2571.
tf.Tensor(-46.01307985134728, shape=(), dtype=float64)
[0.4        0.35509075 0.0248436  0.40806465 0.03161203 0.03239071]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.20159018011601>
191
Acquisition function convergence reached at iteration 2524.
tf.Tensor(-46.03806978490953, shape=(), dtype=float64)
[0.4        0.35509082 0.0240259  0.40806474 0.03175741 0.03274257]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.22756426450153>
192
Acquisition function convergence reached at iteration 2568.
tf.Tensor(-46.062892622697134, shape=(), dtype=float64)
```

```
[0.4        0.35509089 0.02577916 0.40806483 0.03073931 0.03227044]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.25340411529374>
193
Acquisition function convergence reached at iteration 2578.
tf.Tensor(-46.08758873698556, shape=(), dtype=float64)
[0.4        0.35509096 0.02810214 0.40806491 0.02958734 0.03063628]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.27911111279584>
194
Acquisition function convergence reached at iteration 2730.
tf.Tensor(-46.11214509996523, shape=(), dtype=float64)
[0.4        0.35509103 0.02831841 0.408065   0.02959842 0.03153881]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.304686616129686>
195
Acquisition function convergence reached at iteration 2545.
tf.Tensor(-46.13664631978558, shape=(), dtype=float64)
[0.4        0.3550911  0.02789452 0.40806508 0.03062514 0.02951407]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.33013196366704>
196
Acquisition function convergence reached at iteration 2710.
tf.Tensor(-46.16095465734698, shape=(), dtype=float64)
[0.4        0.35509116 0.02717623 0.40806516 0.03242389 0.02970447]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.355448473449776>
197
Acquisition function convergence reached at iteration 2606.
tf.Tensor(-46.18517954627162, shape=(), dtype=float64)
[0.4        0.35509123 0.02886915 0.40806525 0.03027451 0.02900304]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.38063744359956>
198
Acquisition function convergence reached at iteration 2624.
tf.Tensor(-46.209297579423456, shape=(), dtype=float64)
[0.4        0.3550913  0.02568786 0.40806533 0.03077125 0.03275355]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.40570015271729>
199
Acquisition function convergence reached at iteration 2645.
tf.Tensor(-46.233254807967995, shape=(), dtype=float64)
[0.4        0.35509137 0.02739098 0.40806541 0.02861673 0.03300469]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.43063786027248>
200
Acquisition function convergence reached at iteration 2685.
tf.Tensor(-46.25708891125458, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 9728.
Trained parameters:
amplitude_champ:0 is 0.966
```

```
length_scales_champ:0 is [6.516520e+03 2.400000e-02 1.279288e+03 5.000000e-03 2.238005e+03
 1.206181e+03]

observation_noise_variance_champ:0 is 0.285

alpha_tp:0 is 0.504

amp_alpha_mean:0 is 1.491

amp_beta_mean:0 is 0.299

amp_f_mean:0 is 2.884

amp_gamma_L_mean:0 is 16.461

amp_lambda_mean:0 is 14.765

amp_r_mean:0 is 0.631

beta_tp:0 is 0.284

bias_mean:0 is -2.793

f_tp:0 is -0.144

gamma_L_tp:0 is 0.159

lambda_tp:0 is -0.379

r_tp:0 is 0.546

[0.4        0.35509143 0.02861177 0.40806549 0.02899    0.03141044]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.45545180698313>
201
Acquisition function convergence reached at iteration 2604.
tf.Tensor(-46.75186371323863, shape=(), dtype=float64)
[0.4        0.35973608 0.03005014 0.40837935 0.02910603 0.02843961]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.48014321518604>
202
Acquisition function convergence reached at iteration 2628.
tf.Tensor(-46.77574277972098, shape=(), dtype=float64)
[0.4        0.35973615 0.02673574 0.40837943 0.03155895 0.03069784]
```

```
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.504713289198186>
203
Acquisition function convergence reached at iteration 2578.
tf.Tensor(-46.79949115278465, shape=(), dtype=float64)
[0.4        0.35973621 0.02667915 0.4083795  0.03259641 0.02899983]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.52916321566914>
204
Acquisition function convergence reached at iteration 2677.
tf.Tensor(-46.823092592527786, shape=(), dtype=float64)
[0.4        0.35973628 0.02740323 0.40837958 0.03109129 0.03074386]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.55349416392501>
205
Acquisition function convergence reached at iteration 2656.
tf.Tensor(-46.846612279331396, shape=(), dtype=float64)
[0.4        0.35973635 0.02710372 0.40837965 0.02915041 0.03292244]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.57770728630395>
206
Acquisition function convergence reached at iteration 2528.
tf.Tensor(-46.8700504171045, shape=(), dtype=float64)
[0.4        0.35973641 0.02517205 0.40837972 0.02748851 0.03543565]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.601803718483694>
207
Acquisition function convergence reached at iteration 2705.
tf.Tensor(-46.89326916445002, shape=(), dtype=float64)
[0.4        0.35973648 0.02843845 0.4083798  0.02757765 0.03308218]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.625784579801156>
208
Acquisition function convergence reached at iteration 2614.
tf.Tensor(-46.9164580386301, shape=(), dtype=float64)
[0.4        0.35973654 0.02809431 0.40837987 0.03005431 0.03047706]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.64965097356445>
209
Acquisition function convergence reached at iteration 2778.
tf.Tensor(-46.93949220523049, shape=(), dtype=float64)
[0.4        0.35973661 0.02729957 0.40837994 0.03082232 0.03193234]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.67340398735743>
210
Acquisition function convergence reached at iteration 2506.
tf.Tensor(-46.96250681727318, shape=(), dtype=float64)
[0.4        0.35973667 0.0262264  0.40838001 0.03112463 0.03080208]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.69704469333716>
211
Acquisition function convergence reached at iteration 2626.
```

```
tf.Tensor(-46.98530534982813, shape=(), dtype=float64)
[0.4        0.35973673 0.02887071 0.40838008 0.02705127 0.032415  ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.72057414852423>
212
Acquisition function convergence reached at iteration 2533.
tf.Tensor(-47.00807165645318, shape=(), dtype=float64)
[0.4        0.3597368  0.02741041 0.40838016 0.02800589 0.03271033]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.743993395086356>
213
Acquisition function convergence reached at iteration 2809.
tf.Tensor(-47.030640801782695, shape=(), dtype=float64)
[0.4        0.35973686 0.02734972 0.40838023 0.03149147 0.03133719]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.767303460615416>
214
Acquisition function convergence reached at iteration 2416.
tf.Tensor(-47.05326974672322, shape=(), dtype=float64)
[0.4        0.35973692 0.02455917 0.4083803  0.02990342 0.03321962]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.79050535839793>
215
Acquisition function convergence reached at iteration 2723.
tf.Tensor(-47.075609949532144, shape=(), dtype=float64)
[0.4        0.35973699 0.02604997 0.40838037 0.03105312 0.03275483]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.8136000876794>
216
Acquisition function convergence reached at iteration 2614.
tf.Tensor(-47.09796070390157, shape=(), dtype=float64)
[0.4        0.35973705 0.0239315  0.40838044 0.0321527  0.0330875 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.836588633922545>
217
Acquisition function convergence reached at iteration 2421.
tf.Tensor(-47.12018704280942, shape=(), dtype=float64)
[0.4        0.35973711 0.02786181 0.40838051 0.02813343 0.03113578]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.859471969059605>
218
Acquisition function convergence reached at iteration 2613.
tf.Tensor(-47.142250182529494, shape=(), dtype=float64)
[0.4        0.35973717 0.02863305 0.40838058 0.0279984  0.03179321]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.88225105173891>
219
Acquisition function convergence reached at iteration 2387.
tf.Tensor(-47.16433376666702, shape=(), dtype=float64)
[0.4        0.35973723 0.02545036 0.40838065 0.02967361 0.03227846]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.904926827565866>
```

```
220
Acquisition function convergence reached at iteration 2626.
tf.Tensor(-47.18618762716159, shape=(), dtype=float64)
[0.4        0.35973729 0.02508318 0.40838071 0.03338026 0.03049611]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.9275002293385>
221
Acquisition function convergence reached at iteration 2714.
tf.Tensor(-48.06973923208916, shape=(), dtype=float64)
[0.4        0.36635518 0.02847505 0.4104757  0.03129224 0.02927366]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.949972177277694>
222
Acquisition function convergence reached at iteration 2446.
tf.Tensor(-48.09191681985814, shape=(), dtype=float64)
[0.4        0.36635523 0.0269936  0.41047576 0.03083551 0.02963424]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.9723435792523>
223
Acquisition function convergence reached at iteration 2569.
tf.Tensor(-48.11391597850003, shape=(), dtype=float64)
[0.4        0.36635529 0.02434163 0.41047582 0.03399729 0.03007557]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=48.9946153309992>
224
Acquisition function convergence reached at iteration 2355.
tf.Tensor(-48.1358581271015, shape=(), dtype=float64)
[0.4        0.36635535 0.02747216 0.41047588 0.02909226 0.03006853]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.016788316338534>
225
Acquisition function convergence reached at iteration 2590.
tf.Tensor(-48.15764652305267, shape=(), dtype=float64)
[0.4        0.36635541 0.02360069 0.41047594 0.03297837 0.03233362]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.03886340738411>
226
Acquisition function convergence reached at iteration 2471.
tf.Tensor(-48.17939248938663, shape=(), dtype=float64)
[0.4        0.36635547 0.02393793 0.41047599 0.02994638 0.03421865]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.0608414647493>
227
Acquisition function convergence reached at iteration 2638.
tf.Tensor(-48.20093504956708, shape=(), dtype=float64)
[0.4        0.36635553 0.02921799 0.41047605 0.02985786 0.02923993]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.0827233377483>
228
Acquisition function convergence reached at iteration 2551.
tf.Tensor(-48.22250377967378, shape=(), dtype=float64)
```

```
[0.4        0.36635559 0.02672547 0.41047611 0.03061595 0.03111038]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.104509864593076>
229
Acquisition function convergence reached at iteration 2715.
tf.Tensor(-48.243911969910876, shape=(), dtype=float64)
[0.4        0.36635564 0.0256778  0.41047617 0.03137466 0.03278346]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.12620187258607>
230
Acquisition function convergence reached at iteration 2385.
tf.Tensor(-48.265326744726885, shape=(), dtype=float64)
[0.4        0.3663557  0.02514659 0.41047623 0.03115819 0.03105676]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.14780017830865>
231
Acquisition function convergence reached at iteration 2382.
tf.Tensor(-48.28656454942722, shape=(), dtype=float64)
[0.4        0.36635576 0.02709076 0.41047629 0.0288617  0.03105237]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.1693055878056>
232
Acquisition function convergence reached at iteration 2602.
tf.Tensor(-48.30769111046222, shape=(), dtype=float64)
[0.4        0.36635582 0.02476653 0.41047634 0.03089358 0.03343129]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.19071889676561>
233
Acquisition function convergence reached at iteration 2504.
tf.Tensor(-48.32876205373608, shape=(), dtype=float64)
[0.4        0.36635587 0.02713859 0.4104764  0.02843679 0.03243307]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.2120408906979>
234
Acquisition function convergence reached at iteration 2677.
tf.Tensor(-48.349718350019174, shape=(), dtype=float64)
[0.4        0.36635593 0.02476175 0.41047646 0.03200835 0.03282653]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.23327234510515>
235
Acquisition function convergence reached at iteration 2670.
tf.Tensor(-48.37060870902301, shape=(), dtype=float64)
[0.4        0.36635599 0.02490973 0.41047652 0.03088511 0.03376452]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.25441402565276>
236
Acquisition function convergence reached at iteration 2614.
tf.Tensor(-48.391391450152966, shape=(), dtype=float64)
[0.4        0.36635604 0.02863261 0.41047657 0.0279785  0.03183207]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.275466688334475>
237
```

```
Acquisition function convergence reached at iteration 2768.
tf.Tensor(-48.41207812619455, shape=(), dtype=float64)
[0.4        0.3663561  0.02776976 0.41047663 0.03061087 0.03147806]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.29643107963466>
238
Acquisition function convergence reached at iteration 2679.
tf.Tensor(-48.432718445733386, shape=(), dtype=float64)
[0.4        0.36635615 0.02783113 0.41047669 0.03112874 0.03014706]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.31730793668706>
239
Acquisition function convergence reached at iteration 2609.
tf.Tensor(-48.45329173920072, shape=(), dtype=float64)
[0.4        0.36635621 0.02605566 0.41047674 0.02835055 0.03444329]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.33809798743037>
240
Acquisition function convergence reached at iteration 2793.
tf.Tensor(-48.47369947640735, shape=(), dtype=float64)
[0.4        0.36635626 0.02688601 0.4104768  0.0307071  0.03264526]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.35880195076054>
241
Acquisition function convergence reached at iteration 2590.
tf.Tensor(-48.70274439934764, shape=(), dtype=float64)
[0.4        0.36383051 0.02609543 0.40842067 0.02782975 0.03468397]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.379420536679845>
242
Acquisition function convergence reached at iteration 2526.
tf.Tensor(-48.723136359759565, shape=(), dtype=float64)
[0.4        0.36383056 0.02370362 0.40842073 0.0344961  0.02977122]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.39995444644311>
243
Acquisition function convergence reached at iteration 2582.
tf.Tensor(-48.74341329233375, shape=(), dtype=float64)
[0.4        0.36383062 0.02527553 0.40842079 0.02802471 0.03528052]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.42040437270074>
244
Acquisition function convergence reached at iteration 2561.
tf.Tensor(-48.76359081997149, shape=(), dtype=float64)
[0.4        0.36383067 0.02859223 0.40842084 0.03018931 0.02913858]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.440770999638914>
245
Acquisition function convergence reached at iteration 2603.
tf.Tensor(-48.78372163649369, shape=(), dtype=float64)
[0.4        0.36383073 0.02717236 0.4084209  0.02835913 0.03320184]
```

```
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.46105500311699>
246
Acquisition function convergence reached at iteration 2584.
tf.Tensor(-48.80376599647128, shape=(), dtype=float64)
[0.4        0.36383078 0.02690949 0.40842096 0.03080476 0.03097547]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.481257050802014>
247
Acquisition function convergence reached at iteration 2488.
tf.Tensor(-48.82375886667788, shape=(), dtype=float64)
[0.4        0.36383084 0.02595971 0.40842101 0.02872071 0.03336954]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.50137780230064>
248
Acquisition function convergence reached at iteration 2423.
tf.Tensor(-48.84363483736502, shape=(), dtype=float64)
[0.4        0.36383089 0.02787523 0.40842107 0.02926332 0.02998842]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.52141790928833>
249
Acquisition function convergence reached at iteration 2576.
tf.Tensor(-48.86342716443007, shape=(), dtype=float64)
[0.4        0.36383095 0.02444706 0.40842113 0.03245827 0.03192388]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.54137801563602>
250
Acquisition function convergence reached at iteration 2621.
tf.Tensor(-48.88311225114659, shape=(), dtype=float64)
Trained parameters:
amplitude_champ:0 is 0.988

length_scales_champ:0 is [6.375627e+03 2.400000e-02 1.646964e+03 5.000000e-03 2.840766e+03
 1.420613e+03]

observation_noise_variance_champ:0 is 0.266

alpha_tp:0 is 0.516

amp_alpha_mean:0 is 1.629

amp_beta_mean:0 is 0.241

amp_f_mean:0 is 3.557

amp_gamma_L_mean:0 is 16.77

amp_lambda_mean:0 is 12.293
```

```
amp_r_mean:0 is 0.391

beta_tp:0 is 0.183

bias_mean:0 is -3.226

f_tp:0 is -0.125

gamma_L_tp:0 is 0.149

lambda_tp:0 is -0.464

r_tp:0 is 0.569

[0.4        0.363831   0.0285789  0.40842118 0.02677779 0.03295528]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.56125875753422>
251
Acquisition function convergence reached at iteration 2557.
tf.Tensor(-48.902783097810065, shape=(), dtype=float64)
[0.4        0.36383106 0.02576685 0.40842124 0.03285111 0.02974136]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.5810607636147>
252
Acquisition function convergence reached at iteration 2495.
tf.Tensor(-48.92236750285696, shape=(), dtype=float64)
[0.4        0.36383111 0.02612505 0.40842129 0.02798716 0.03387825]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.60078465506979>
253
Acquisition function convergence reached at iteration 2424.
tf.Tensor(-48.94187271634224, shape=(), dtype=float64)
[0.4        0.36383116 0.02511118 0.40842135 0.03241314 0.02991151]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.62043104576923>
254
Acquisition function convergence reached at iteration 2696.
tf.Tensor(-48.96122422986934, shape=(), dtype=float64)
[0.4        0.36383122 0.02612887 0.4084214  0.02973567 0.03375324]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.64000054237491>
255
Acquisition function convergence reached at iteration 2508.
tf.Tensor(-48.9806163328158, shape=(), dtype=float64)
[0.4        0.36383127 0.02351174 0.40842146 0.03119163 0.03371826]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.6594937444532>
256
```

```
Acquisition function convergence reached at iteration 2517.
tf.Tensor(-48.99987061849509, shape=(), dtype=float64)
[0.4        0.36383132 0.02460529 0.40842151 0.02817906 0.03535948]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.67891124458519>
257
Acquisition function convergence reached at iteration 2592.
tf.Tensor(-49.019009665943884, shape=(), dtype=float64)
[0.4        0.36383137 0.02842555 0.40842157 0.0275839  0.03224642]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.69825362847479>
258
Acquisition function convergence reached at iteration 2748.
tf.Tensor(-49.03810306706539, shape=(), dtype=float64)
[0.4        0.36383143 0.02611724 0.40842162 0.03177607 0.03211162]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.71752147505474>
259
Acquisition function convergence reached at iteration 2687.
tf.Tensor(-49.05716770962763, shape=(), dtype=float64)
[0.4        0.36383148 0.02420051 0.40842167 0.03212175 0.03335633]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.736715356590565>
260
Acquisition function convergence reached at iteration 2718.
tf.Tensor(-49.07609666739888, shape=(), dtype=float64)
[0.4        0.36383153 0.0280864  0.40842173 0.03057624 0.03071265]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.75583583878259>
261
Acquisition function convergence reached at iteration 2578.
tf.Tensor(-49.716037685629544, shape=(), dtype=float64)
[0.4        0.36268941 0.02692198 0.40742376 0.02681313 0.03452888]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.774883480865924>
262
Acquisition function convergence reached at iteration 2691.
tf.Tensor(-49.735064789027824, shape=(), dtype=float64)
[0.4        0.36268946 0.0269278  0.40742381 0.03251185 0.02974519]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.79385883570868>
263
Acquisition function convergence reached at iteration 2607.
tf.Tensor(-49.75405759960787, shape=(), dtype=float64)
[0.4        0.36268951 0.02765455 0.40742387 0.03263473 0.02778273]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.81276244990821>
264
Acquisition function convergence reached at iteration 2609.
tf.Tensor(-49.77297327621704, shape=(), dtype=float64)
[0.4        0.36268957 0.02753784 0.40742392 0.03232728 0.02842867]
```

```
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.8315948638856>
265
Acquisition function convergence reached at iteration 2565.
tf.Tensor(-49.79182541157848, shape=(), dtype=float64)
[0.4        0.36268962 0.02740287 0.40742398 0.03235489 0.02818778]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.85035661197835>
266
Acquisition function convergence reached at iteration 2500.
tf.Tensor(-49.81063204043631, shape=(), dtype=float64)
[0.4        0.36268967 0.02505637 0.40742403 0.03285675 0.03011085]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.86904822253138>
267
Acquisition function convergence reached at iteration 2621.
tf.Tensor(-49.82930080963016, shape=(), dtype=float64)
[0.4        0.36268972 0.02651977 0.40742409 0.0308347  0.03169852]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.8876702179863>
268
Acquisition function convergence reached at iteration 2518.
tf.Tensor(-49.84793001455604, shape=(), dtype=float64)
[0.4        0.36268978 0.02927994 0.40742414 0.02930777 0.02869019]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.906223114968974>
269
Acquisition function convergence reached at iteration 2534.
tf.Tensor(-49.866509527415666, shape=(), dtype=float64)
[0.4        0.36268983 0.027248   0.40742419 0.02917235 0.03183727]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.924707424375605>
270
Acquisition function convergence reached at iteration 2641.
tf.Tensor(-49.88498970257954, shape=(), dtype=float64)
[0.4        0.36268988 0.02604228 0.40742425 0.0308248  0.03241177]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.943123651457086>
271
Acquisition function convergence reached at iteration 2588.
tf.Tensor(-49.903408996744965, shape=(), dtype=float64)
[0.4        0.36268993 0.02833723 0.4074243  0.02891911 0.03110628]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.9614722959019>
272
Acquisition function convergence reached at iteration 2678.
tf.Tensor(-49.92176460218481, shape=(), dtype=float64)
[0.4        0.36268998 0.02569039 0.40742435 0.03289786 0.03077974]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.97975385191745>
273
Acquisition function convergence reached at iteration 2630.
```

```
tf.Tensor(-49.94005185345026, shape=(), dtype=float64)
[0.4        0.36269003 0.02828605 0.40742441 0.02772333 0.0326009 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=49.997968808309956>
274
Acquisition function convergence reached at iteration 2643.
tf.Tensor(-49.958272174951446, shape=(), dtype=float64)
[0.4        0.36269008 0.02835654 0.40742446 0.0283448  0.03207314]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.016117648562854>
275
Acquisition function convergence reached at iteration 2543.
tf.Tensor(-49.976466971387524, shape=(), dtype=float64)
[0.4        0.36269013 0.02653286 0.40742451 0.02932645 0.032591  ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.03420085091379>
276
Acquisition function convergence reached at iteration 2571.
tf.Tensor(-49.99454551601768, shape=(), dtype=float64)
[0.4        0.36269018 0.02756869 0.40742456 0.02846501 0.03239037]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.05221888843029>
277
Acquisition function convergence reached at iteration 2558.
tf.Tensor(-50.01258151038314, shape=(), dtype=float64)
[0.4        0.36269023 0.0269956  0.40742462 0.02690006 0.03424919]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.07017222908393>
278
Acquisition function convergence reached at iteration 2535.
tf.Tensor(-50.03057362899666, shape=(), dtype=float64)
[0.4        0.36269028 0.02331491 0.40742467 0.03130751 0.03396856]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.08806133582335>
279
Acquisition function convergence reached at iteration 2509.
tf.Tensor(-50.04843321599992, shape=(), dtype=float64)
[0.4        0.36269033 0.02903381 0.40742472 0.02843053 0.02997753]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.10588666664583>
280
Acquisition function convergence reached at iteration 2725.
tf.Tensor(-50.06623076810098, shape=(), dtype=float64)
[0.4        0.36269038 0.02818517 0.40742477 0.02997647 0.03129427]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.12364867466767>
281
Acquisition function convergence reached at iteration 2495.
tf.Tensor(-50.767616216748245, shape=(), dtype=float64)
[0.4        0.36114165 0.02855405 0.40727381 0.02787926 0.03104196]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.14134780819329>
```

```
282
Acquisition function convergence reached at iteration 2711.
tf.Tensor(-50.785547517568595, shape=(), dtype=float64)
[0.4        0.3611417  0.02476136 0.40727386 0.03208826 0.03298758]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.158984510783135>
283
Acquisition function convergence reached at iteration 2778.
tf.Tensor(-50.803407410564034, shape=(), dtype=float64)
[0.4        0.36114175 0.02655903 0.40727392 0.03133228 0.0322909 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.17655922132035>
284
Acquisition function convergence reached at iteration 2616.
tf.Tensor(-50.821250407039784, shape=(), dtype=float64)
[0.4        0.3611418  0.02839021 0.40727397 0.02945558 0.0307077 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.19407237407636>
285
Acquisition function convergence reached at iteration 2461.
tf.Tensor(-50.83905006248126, shape=(), dtype=float64)
[0.4        0.36114185 0.0277258  0.40727402 0.02935566 0.0304259 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.211524398775204>
286
Acquisition function convergence reached at iteration 2675.
tf.Tensor(-50.856729729851224, shape=(), dtype=float64)
[0.4        0.3611419  0.02444193 0.40727408 0.03102712 0.03413612]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.22891572065683>
287
Acquisition function convergence reached at iteration 2495.
tf.Tensor(-50.87440930877072, shape=(), dtype=float64)
[0.4        0.36114195 0.02379811 0.40727413 0.03175091 0.03274337]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.246246760539265>
288
Acquisition function convergence reached at iteration 2692.
tf.Tensor(-50.89192998771412, shape=(), dtype=float64)
[0.4        0.361142   0.02581568 0.40727419 0.03099274 0.03287522]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.2635179348797>
289
Acquisition function convergence reached at iteration 2480.
tf.Tensor(-50.90950704927778, shape=(), dtype=float64)
[0.4        0.36114205 0.02377731 0.40727424 0.03072191 0.03368393]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.28072965583456>
290
Acquisition function convergence reached at iteration 2580.
tf.Tensor(-50.926910362892116, shape=(), dtype=float64)
```

```
[0.4        0.3611421  0.02694659 0.40727429 0.03252468 0.02873264]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.29788233131851>
291
Acquisition function convergence reached at iteration 2351.
tf.Tensor(-50.94436696865937, shape=(), dtype=float64)
[0.4        0.36114215 0.02639038 0.40727435 0.03001727 0.03045707]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.314976365062435>
292
Acquisition function convergence reached at iteration 2633.
tf.Tensor(-50.96165178132494, shape=(), dtype=float64)
[0.4        0.3611422  0.02483478 0.4072744  0.03212009 0.03230447]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.33201215667051>
293
Acquisition function convergence reached at iteration 2588.
tf.Tensor(-50.978940919081325, shape=(), dtype=float64)
[0.4        0.36114225 0.02500073 0.40727445 0.02931291 0.03454809]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.3489901016762>
294
Acquisition function convergence reached at iteration 2493.
tf.Tensor(-50.99618622293166, shape=(), dtype=float64)
[0.4        0.3611423  0.02411955 0.4072745  0.02853997 0.03538369]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.365910591597405>
295
Acquisition function convergence reached at iteration 2697.
tf.Tensor(-51.01328687670857, shape=(), dtype=float64)
[0.4        0.36114234 0.02609344 0.40727456 0.030787   0.03279889]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.3827740139906>
296
Acquisition function convergence reached at iteration 2730.
tf.Tensor(-51.030368362161155, shape=(), dtype=float64)
[0.4        0.36114239 0.02741644 0.40727461 0.03190492 0.03019202]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.39958075250412>
297
Acquisition function convergence reached at iteration 2651.
tf.Tensor(-51.04745618517789, shape=(), dtype=float64)
[0.4        0.36114244 0.02414489 0.40727466 0.03211264 0.03317729]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.416331186930535>
298
Acquisition function convergence reached at iteration 2611.
tf.Tensor(-51.06441376193958, shape=(), dtype=float64)
[0.4        0.36114249 0.02949479 0.40727471 0.02649981 0.03192745]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.43302569325811>
299
```

```
Acquisition function convergence reached at iteration 2644.
tf.Tensor(-51.08134457912174, shape=(), dtype=float64)
[0.4        0.36114254 0.02853577 0.40727476 0.02895876 0.03125078]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.44966464372148>
300
Acquisition function convergence reached at iteration 2671.
tf.Tensor(-51.098207152664905, shape=(), dtype=float64)
Trained parameters:
amplitude_champ:0 is 1.023

length_scales_champ:0 is [5.067021e+03 2.400000e-02 2.242284e+03 5.000000e-03 3.807867e+03
 1.580158e+03]

observation_noise_variance_champ:0 is 0.243

alpha_tp:0 is 0.526

amp_alpha_mean:0 is 1.961

amp_beta_mean:0 is 0.328

amp_f_mean:0 is 3.511

amp_gamma_L_mean:0 is 5.126

amp_lambda_mean:0 is 11.133

amp_r_mean:0 is 0.576

beta_tp:0 is 0.183

bias_mean:0 is -3.363

f_tp:0 is -0.144

gamma_L_tp:0 is 0.179

lambda_tp:0 is -0.506

r_tp:0 is 0.561

[0.4        0.36114258 0.02838398 0.40727481 0.0321179  0.02796912]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.46624840685145>
```

```
301
Acquisition function convergence reached at iteration 2734.
tf.Tensor(-51.544264310789224, shape=(), dtype=float64)
[0.4        0.3657343  0.02850488 0.40737642 0.03008403 0.03080867]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.48277734752395>
302
Acquisition function convergence reached at iteration 2685.
tf.Tensor(-51.56120351144001, shape=(), dtype=float64)
[0.4        0.36573434 0.0251335  0.40737647 0.03195929 0.03253779]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.49925182700821>
303
Acquisition function convergence reached at iteration 2766.
tf.Tensor(-51.5780141036832, shape=(), dtype=float64)
[0.4        0.36573439 0.02802904 0.40737651 0.03030799 0.0314353 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.51567220301416>
304
Acquisition function convergence reached at iteration 2520.
tf.Tensor(-51.59487307806823, shape=(), dtype=float64)
[0.4        0.36573444 0.02526106 0.40737656 0.03316547 0.02962384]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.53203882973901>
305
Acquisition function convergence reached at iteration 2543.
tf.Tensor(-51.61162624948331, shape=(), dtype=float64)
[0.4        0.36573448 0.02317798 0.4073766  0.03144602 0.03400998]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.548352057913085>
306
Acquisition function convergence reached at iteration 2512.
tf.Tensor(-51.62829645491605, shape=(), dtype=float64)
[0.4        0.36573453 0.02617591 0.40737665 0.02821243 0.03377175]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.56461223484497>
307
Acquisition function convergence reached at iteration 2821.
tf.Tensor(-51.644853522872, shape=(), dtype=float64)
[0.4        0.36573457 0.02697399 0.40737669 0.03074768 0.03269451]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.58081970446583>
308
Acquisition function convergence reached at iteration 2584.
tf.Tensor(-51.66149077597304, shape=(), dtype=float64)
[0.4        0.36573462 0.02512234 0.40737673 0.03067889 0.03315699]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.59697480737306>
309
Acquisition function convergence reached at iteration 2495.
tf.Tensor(-51.67801226862934, shape=(), dtype=float64)
```

```
[0.4        0.36573466 0.02743152 0.40737678 0.02985705 0.03058016]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.61307788087327>
310
Acquisition function convergence reached at iteration 2508.
tf.Tensor(-51.69446702753421, shape=(), dtype=float64)
[0.4        0.36573471 0.02858466 0.40737682 0.02971924 0.02918752]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.629129259024516>
311
Acquisition function convergence reached at iteration 2764.
tf.Tensor(-51.7108383102924, shape=(), dtype=float64)
[0.4        0.36573475 0.02731885 0.40737687 0.03185339 0.03065063]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.64512927267787>
312
Acquisition function convergence reached at iteration 2510.
tf.Tensor(-51.72727894702973, shape=(), dtype=float64)
[0.4        0.3657348  0.02381944 0.40737691 0.03300875 0.03144569]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.66107824951837>
313
Acquisition function convergence reached at iteration 2637.
tf.Tensor(-51.743544300852385, shape=(), dtype=float64)
[0.4        0.36573484 0.02702549 0.40737695 0.02873815 0.03327423]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.67697651410527>
314
Acquisition function convergence reached at iteration 2629.
tf.Tensor(-51.759806839961456, shape=(), dtype=float64)
[0.4        0.36573488 0.02594784 0.407377   0.03252232 0.03054507]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.692824387911664>
315
Acquisition function convergence reached at iteration 2563.
tf.Tensor(-51.77599233718762, shape=(), dtype=float64)
[0.4        0.36573493 0.03043637 0.40737704 0.02843809 0.02820412]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.7086221893635>
316
Acquisition function convergence reached at iteration 2581.
tf.Tensor(-51.79217312511463, shape=(), dtype=float64)
[0.4        0.36573497 0.02769269 0.40737708 0.02672426 0.03377377]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.724370233877984>
317
Acquisition function convergence reached at iteration 2511.
tf.Tensor(-51.808306265470065, shape=(), dtype=float64)
[0.4        0.36573502 0.02506828 0.40737712 0.03264203 0.03049305]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.740068833901326>
318
```

```
Acquisition function convergence reached at iteration 2599.
tf.Tensor(-51.82432971852811, shape=(), dtype=float64)
[0.4        0.36573506 0.0262422  0.40737717 0.03341092 0.02869918]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.755718298945965>
319
Acquisition function convergence reached at iteration 2612.
tf.Tensor(-51.840321347618286, shape=(), dtype=float64)
[0.4        0.3657351  0.02857083 0.40737721 0.02894044 0.03094524]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.77131893562718>
320
Acquisition function convergence reached at iteration 2594.
tf.Tensor(-51.85626752747388, shape=(), dtype=float64)
[0.4        0.36573515 0.02983918 0.40737725 0.0278148  0.0301279 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.786871047699144>
321
Acquisition function convergence reached at iteration 2772.
tf.Tensor(-52.18639646015569, shape=(), dtype=float64)
[0.4        0.3707827  0.02798402 0.40796218 0.0311762  0.03060216]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.80237493609039>
322
Acquisition function convergence reached at iteration 2848.
tf.Tensor(-52.202340888599224, shape=(), dtype=float64)
[0.4        0.37078273 0.02731842 0.40796222 0.03076495 0.03242285]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.81783089893875>
323
Acquisition function convergence reached at iteration 2477.
tf.Tensor(-52.21833331313406, shape=(), dtype=float64)
[0.4        0.37078277 0.02575361 0.40796225 0.02757049 0.0344662 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.83323923162579>
324
Acquisition function convergence reached at iteration 2538.
tf.Tensor(-52.234177068290194, shape=(), dtype=float64)
[0.4        0.37078281 0.02517318 0.40796229 0.02894173 0.03434749]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.84860022681064>
325
Acquisition function convergence reached at iteration 2588.
tf.Tensor(-52.24994044553974, shape=(), dtype=float64)
[0.4        0.37078285 0.02784531 0.40796232 0.03193377 0.0283062 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.86391417446337>
326
Acquisition function convergence reached at iteration 2544.
tf.Tensor(-52.2657260016199, shape=(), dtype=float64)
[0.4        0.37078289 0.02641786 0.40796236 0.0272408  0.03451923]
```

```
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.87918136189782>
327
Acquisition function convergence reached at iteration 2624.
tf.Tensor(-52.281399917095186, shape=(), dtype=float64)
[0.4        0.37078293 0.02751312 0.40796239 0.03212153 0.0288768 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.89440207380396>
328
Acquisition function convergence reached at iteration 2754.
tf.Tensor(-52.297034382004405, shape=(), dtype=float64)
[0.4        0.37078296 0.02833534 0.40796242 0.02876752 0.03248426]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.909576592279734>
329
Acquisition function convergence reached at iteration 2612.
tf.Tensor(-52.31268990839065, shape=(), dtype=float64)
[0.4        0.370783   0.02590886 0.40796246 0.03210457 0.0309437 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.92470519686242>
330
Acquisition function convergence reached at iteration 2617.
tf.Tensor(-52.32824599012347, shape=(), dtype=float64)
[0.4        0.37078304 0.02763433 0.40796249 0.02655289 0.03421392]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.939788164559545>
331
Acquisition function convergence reached at iteration 2611.
tf.Tensor(-52.343797898179346, shape=(), dtype=float64)
[0.4        0.37078308 0.02281108 0.40796253 0.03136698 0.03489181]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.95482576987932>
332
Acquisition function convergence reached at iteration 2620.
tf.Tensor(-52.359212803277416, shape=(), dtype=float64)
[0.4        0.37078312 0.02938456 0.40796256 0.03045102 0.02808584]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.9698182848606>
333
Acquisition function convergence reached at iteration 2734.
tf.Tensor(-52.374630843217616, shape=(), dtype=float64)
[0.4        0.37078315 0.02782854 0.40796259 0.03120883 0.03046026]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.984765979102434>
334
Acquisition function convergence reached at iteration 2629.
tf.Tensor(-52.39006525806738, shape=(), dtype=float64)
[0.4        0.37078319 0.02358302 0.40796263 0.03093949 0.03471408]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=50.999669119793126>
335
Acquisition function convergence reached at iteration 2409.
```

```
tf.Tensor(-52.40543689045541, shape=(), dtype=float64)
[0.4        0.37078323 0.02463477 0.40796266 0.03137256 0.03159772]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.01452797173891>
336
Acquisition function convergence reached at iteration 2579.
tf.Tensor(-52.420650980609, shape=(), dtype=float64)
[0.4        0.37078326 0.02927586 0.40796269 0.02864534 0.03000164]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.0293427973922>
337
Acquisition function convergence reached at iteration 2605.
tf.Tensor(-52.435902482362216, shape=(), dtype=float64)
[0.4        0.3707833  0.02696285 0.40796273 0.03210039 0.02954052]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.044113856879356>
338
Acquisition function convergence reached at iteration 2550.
tf.Tensor(-52.45110183601192, shape=(), dtype=float64)
[0.4        0.37078334 0.02844241 0.40796276 0.03019686 0.02924274]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.05884140802814>
339
Acquisition function convergence reached at iteration 2718.
tf.Tensor(-52.46624442241307, shape=(), dtype=float64)
[0.4        0.37078337 0.02542504 0.40796279 0.03203844 0.03237133]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.07352570639468>
340
Acquisition function convergence reached at iteration 2734.
tf.Tensor(-52.48132806966011, shape=(), dtype=float64)
Hyperparameter convergence reached at iteration 9564.
[0.4        0.37078341 0.02840572 0.40796283 0.0296456  0.03138637]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.088167005290124>
341
Acquisition function convergence reached at iteration 2588.
tf.Tensor(-52.878616428318495, shape=(), dtype=float64)
[0.4        0.37463841 0.02543591 0.407515   0.0286684  0.03466636]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.1027655558068>
342
Acquisition function convergence reached at iteration 2474.
tf.Tensor(-52.8937679721073, shape=(), dtype=float64)
[0.4        0.37463844 0.0254382  0.40751502 0.03026566 0.03238948]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.117321606844094>
343
Acquisition function convergence reached at iteration 2738.
tf.Tensor(-52.90877655808367, shape=(), dtype=float64)
[0.4        0.37463847 0.02832485 0.40751505 0.02807321 0.03300281]
```

```
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.1318354051339>
344
Acquisition function convergence reached at iteration 2480.
tf.Tensor(-52.923879856936374, shape=(), dtype=float64)
[0.4        0.3746385  0.02629767 0.40751508 0.03218298 0.02920784]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.14630719526572>
345
Acquisition function convergence reached at iteration 2735.
tf.Tensor(-52.938820307430106, shape=(), dtype=float64)
[0.4        0.37463853 0.02734701 0.4075151  0.03082543 0.03154513]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.1607372197114>
346
Acquisition function convergence reached at iteration 2737.
tf.Tensor(-52.95376291106366, shape=(), dtype=float64)
[0.4        0.37463856 0.02817517 0.40751513 0.03205828 0.02894221]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.17512571884947>
347
Acquisition function convergence reached at iteration 2429.
tf.Tensor(-52.96877382802758, shape=(), dtype=float64)
[0.4        0.37463859 0.02471399 0.40751515 0.02835218 0.03452544]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.18947293098924>
348
Acquisition function convergence reached at iteration 2621.
tf.Tensor(-52.98359797378965, shape=(), dtype=float64)
[0.4        0.37463862 0.02426685 0.40751518 0.03272606 0.03216705]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.2037790923944>
349
Acquisition function convergence reached at iteration 2636.
tf.Tensor(-52.99840514878718, shape=(), dtype=float64)
[0.4        0.37463865 0.02667695 0.4075152  0.0305955  0.03189216]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.21804443730643>
350
Acquisition function convergence reached at iteration 2745.
tf.Tensor(-53.01315794176707, shape=(), dtype=float64)
Trained parameters:
amplitude_champ:0 is 1.036

length_scales_champ:0 is [3.231844e+03 2.600000e-02 2.709781e+03 5.000000e-03 4.445590e+03
 1.574275e+03]

observation_noise_variance_champ:0 is 0.232

alpha_tp:0 is 0.533
```

```
amp_alpha_mean:0 is 2.107

amp_beta_mean:0 is 0.342

amp_f_mean:0 is 4.567

amp_gamma_L_mean:0 is 5.718

amp_lambda_mean:0 is 12.14

amp_r_mean:0 is 1.133

beta_tp:0 is 0.206

bias_mean:0 is -3.275

f_tp:0 is -0.135

gamma_L_tp:0 is 0.176

lambda_tp:0 is -0.464

r_tp:0 is 0.522

[0.4        0.37463868 0.02771195 0.40751523 0.03060653 0.03140736]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.23226919796759>
351
Acquisition function convergence reached at iteration 2532.
tf.Tensor(-53.027949196402744, shape=(), dtype=float64)
[0.4        0.37463871 0.02737538 0.40751526 0.02768567 0.03301948]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.246453604643584>
352
Acquisition function convergence reached at iteration 2682.
tf.Tensor(-53.04261033859327, shape=(), dtype=float64)
[0.4        0.37463874 0.02833884 0.40751528 0.02774686 0.03286917]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.26059788564597>
353
Acquisition function convergence reached at iteration 2670.
tf.Tensor(-53.05729139019281, shape=(), dtype=float64)
[0.4        0.37463877 0.02528783 0.40751531 0.03140637 0.03285927]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.27470226735418>
354
```

```
Acquisition function convergence reached at iteration 2520.
tf.Tensor(-53.0719414060307, shape=(), dtype=float64)
[0.4        0.3746388  0.02439197 0.40751533 0.03350599 0.03022995]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.288766974237255>
355
Acquisition function convergence reached at iteration 2608.
tf.Tensor(-53.0864987756294, shape=(), dtype=float64)
[0.4        0.37463883 0.02500143 0.40751536 0.0311262  0.03301535]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.3027922288753>
356
Acquisition function convergence reached at iteration 2628.
tf.Tensor(-53.10103211562489, shape=(), dtype=float64)
[0.4        0.37463885 0.02489216 0.40751538 0.03019721 0.03413703]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.3167782519806>
357
Acquisition function convergence reached at iteration 2426.
tf.Tensor(-53.11555259005315, shape=(), dtype=float64)
[0.4        0.37463888 0.0270755  0.40751541 0.0284675  0.03185966]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.3307252624185>
358
Acquisition function convergence reached at iteration 2574.
tf.Tensor(-53.12996910335243, shape=(), dtype=float64)
[0.4        0.37463891 0.02777651 0.40751543 0.02843345 0.03217406]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.34463347722788>
359
Acquisition function convergence reached at iteration 2675.
tf.Tensor(-53.14436459686946, shape=(), dtype=float64)
[0.4        0.37463894 0.02735563 0.40751546 0.02909839 0.03282301]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.358503111641504>
360
Acquisition function convergence reached at iteration 2527.
tf.Tensor(-53.158783445073105, shape=(), dtype=float64)
[0.4        0.37463897 0.02415128 0.40751548 0.03416533 0.02972786]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.37233437910596>
361
Acquisition function convergence reached at iteration 2478.
tf.Tensor(-53.511761058396594, shape=(), dtype=float64)
[0.4        0.37874158 0.02808962 0.4065084  0.0290989  0.03034286]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.38612749130136>
362
Acquisition function convergence reached at iteration 2551.
tf.Tensor(-53.52613537907804, shape=(), dtype=float64)
[0.4        0.3787416  0.02750875 0.40650842 0.02945133 0.03137209]
```

```
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.399882658160806>
363
Acquisition function convergence reached at iteration 2448.
tf.Tensor(-53.54050982736811, shape=(), dtype=float64)
[0.4        0.37874162 0.02667802 0.40650844 0.02962731 0.03138891]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.413600087889556>
364
Acquisition function convergence reached at iteration 2489.
tf.Tensor(-53.55481161736438, shape=(), dtype=float64)
[0.4        0.37874164 0.02615125 0.40650846 0.02986804 0.03209157]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.42727998698393>
365
Acquisition function convergence reached at iteration 2688.
tf.Tensor(-53.56902652347192, shape=(), dtype=float64)
[0.4        0.37874166 0.02714099 0.40650847 0.03121357 0.03100943]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.44092256024995>
366
Acquisition function convergence reached at iteration 2647.
tf.Tensor(-53.58327348046853, shape=(), dtype=float64)
[0.4        0.37874168 0.02534197 0.40650849 0.03278862 0.03110165]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.454528010821754>
367
Acquisition function convergence reached at iteration 2621.
tf.Tensor(-53.59745384866059, shape=(), dtype=float64)
[0.4        0.3787417  0.02700445 0.40650851 0.03256791 0.0289692 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.46809654017974>
368
Acquisition function convergence reached at iteration 2584.
tf.Tensor(-53.61162914240776, shape=(), dtype=float64)
[0.4        0.37874171 0.02505637 0.40650852 0.03290406 0.03075211]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.48162834816845>
369
Acquisition function convergence reached at iteration 2758.
tf.Tensor(-53.62570246804669, shape=(), dtype=float64)
[0.4        0.37874173 0.02577216 0.40650854 0.03203704 0.03228348]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.49512363301428>
370
Acquisition function convergence reached at iteration 2375.
tf.Tensor(-53.6398695668577, shape=(), dtype=float64)
[0.4        0.37874175 0.02488514 0.40650856 0.02890988 0.03346782]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.50858259134284>
371
Acquisition function convergence reached at iteration 2499.
```

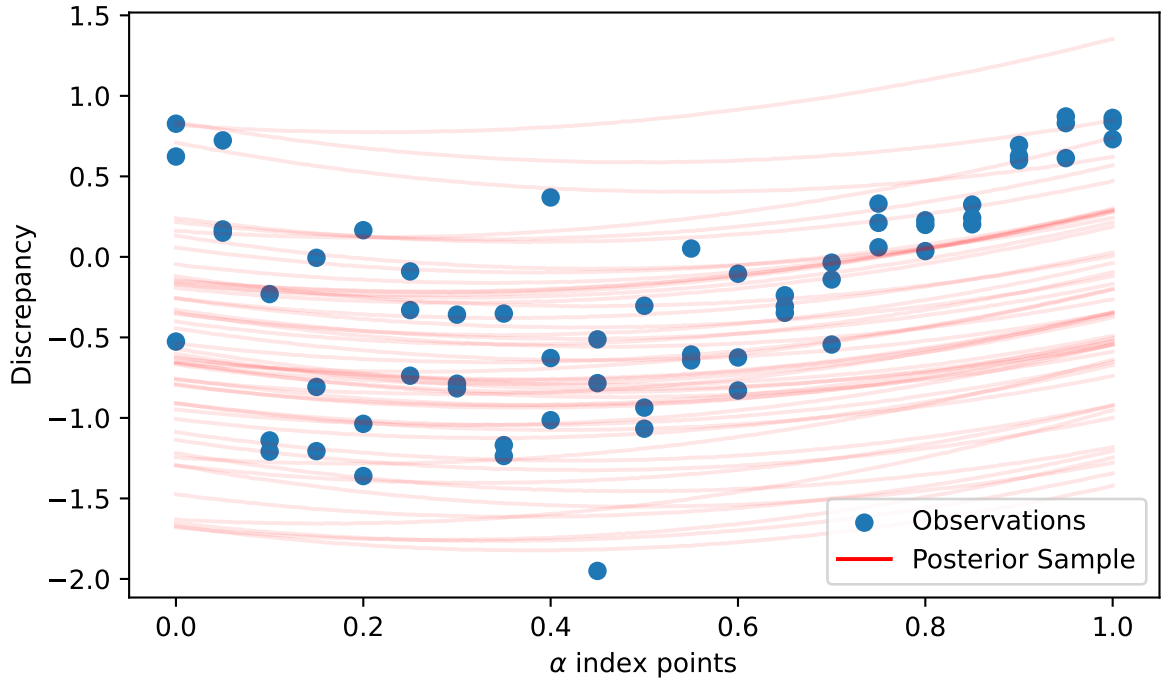
```
tf.Tensor(-53.65387925116737, shape=(), dtype=float64)
[0.4        0.37874177 0.02391659 0.40650858 0.03272076 0.03158583]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.52200541819618>
372
Acquisition function convergence reached at iteration 2796.
tf.Tensor(-53.6678018213297, shape=(), dtype=float64)
[0.4        0.37874179 0.02641108 0.40650859 0.03107335 0.0328173 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.53539230704976>
373
Acquisition function convergence reached at iteration 2638.
tf.Tensor(-53.68177551069908, shape=(), dtype=float64)
[0.4        0.37874181 0.02877067 0.40650861 0.02879885 0.03105272]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.54874344982915>
374
Acquisition function convergence reached at iteration 2701.
tf.Tensor(-53.69570982747627, shape=(), dtype=float64)
[0.4        0.37874183 0.02604787 0.40650863 0.03016094 0.03347173]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.56205903692658>
375
Acquisition function convergence reached at iteration 2771.
tf.Tensor(-53.70956849640053, shape=(), dtype=float64)
[0.4        0.37874185 0.02726589 0.40650864 0.03192466 0.03069145]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.57533925721715>
376
Acquisition function convergence reached at iteration 2659.
tf.Tensor(-53.72344756908479, shape=(), dtype=float64)
[0.4        0.37874187 0.02633346 0.40650866 0.03230507 0.03056326]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.58858429807504>
377
Acquisition function convergence reached at iteration 2664.
tf.Tensor(-53.73727247710299, shape=(), dtype=float64)
[0.4        0.37874189 0.02520784 0.40650868 0.03121165 0.0330887 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.60179434538923>
378
Acquisition function convergence reached at iteration 2695.
tf.Tensor(-53.75102331844545, shape=(), dtype=float64)
[0.4        0.37874191 0.02761329 0.40650869 0.0305988  0.03115433]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.61496958357926>
379
Acquisition function convergence reached at iteration 2512.
tf.Tensor(-53.76479591477377, shape=(), dtype=float64)
[0.4        0.37874192 0.02841189 0.40650871 0.02920437 0.0300674 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.62811019561061>
```

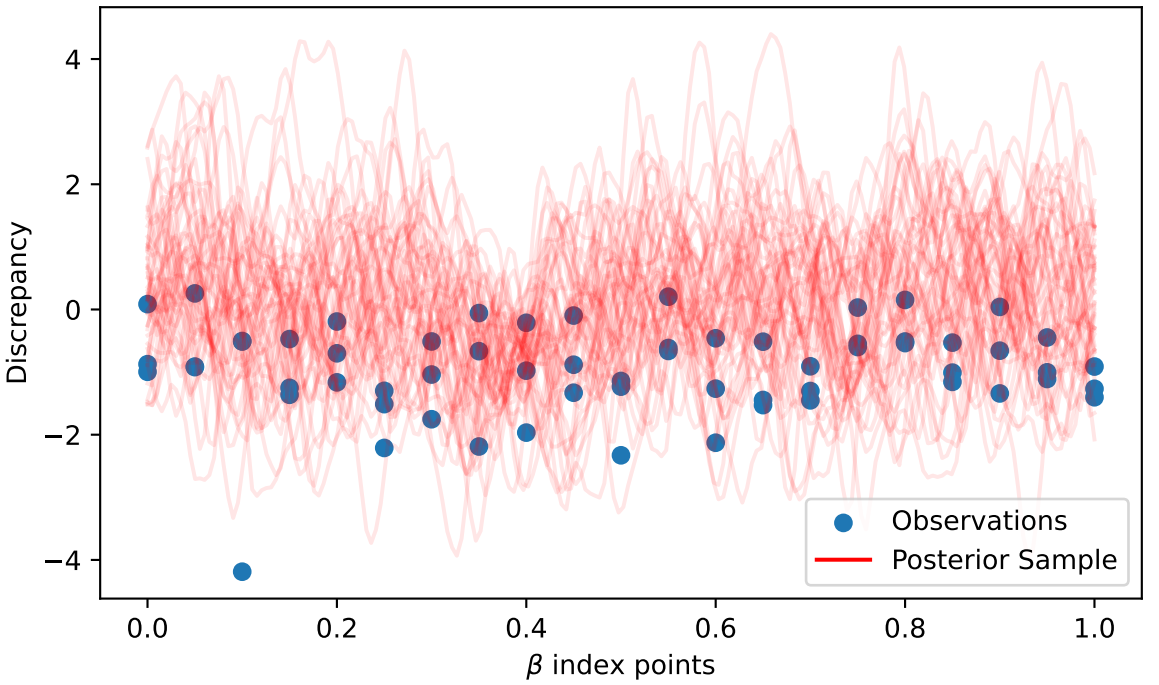
```
380
Acquisition function convergence reached at iteration 2478.
tf.Tensor(-53.778525729166994, shape=(), dtype=float64)
[0.4        0.37874194 0.0266368  0.40650873 0.02781429 0.03332149]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.641216363009974>
381
Acquisition function convergence reached at iteration 2505.
tf.Tensor(-52.86070700447297, shape=(), dtype=float64)
[0.4        0.36120098 0.02379502 0.41345058 0.03009863 0.03443406]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.65428826588033>
382
Acquisition function convergence reached at iteration 2433.
tf.Tensor(-52.874105699016596, shape=(), dtype=float64)
[0.4        0.36120102 0.02584206 0.41345062 0.02913394 0.03272093]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.667326082915736>
383
Acquisition function convergence reached at iteration 2608.
tf.Tensor(-52.88740720483023, shape=(), dtype=float64)
[0.4        0.36120107 0.0287293  0.41345067 0.02825007 0.03138749]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.68032999141602>
384
Acquisition function convergence reached at iteration 2751.
tf.Tensor(-52.900720150607995, shape=(), dtype=float64)
[0.4        0.36120111 0.02691023 0.41345072 0.03041938 0.03261705]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.69330016730125>
385
Acquisition function convergence reached at iteration 2523.
tf.Tensor(-52.91406888451523, shape=(), dtype=float64)
[0.4        0.36120115 0.02563609 0.41345077 0.03140047 0.03136045]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.70623678512601>
386
Acquisition function convergence reached at iteration 2494.
tf.Tensor(-52.92733426207121, shape=(), dtype=float64)
[0.4        0.36120119 0.02616721 0.41345082 0.02827978 0.03358104]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.71914001809347>
387
Acquisition function convergence reached at iteration 2687.
tf.Tensor(-52.94049157417491, shape=(), dtype=float64)
[0.4        0.36120124 0.02898036 0.41345087 0.03011675 0.02971836]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.73201003806933>
388
Acquisition function convergence reached at iteration 2619.
tf.Tensor(-52.95373752279627, shape=(), dtype=float64)
```

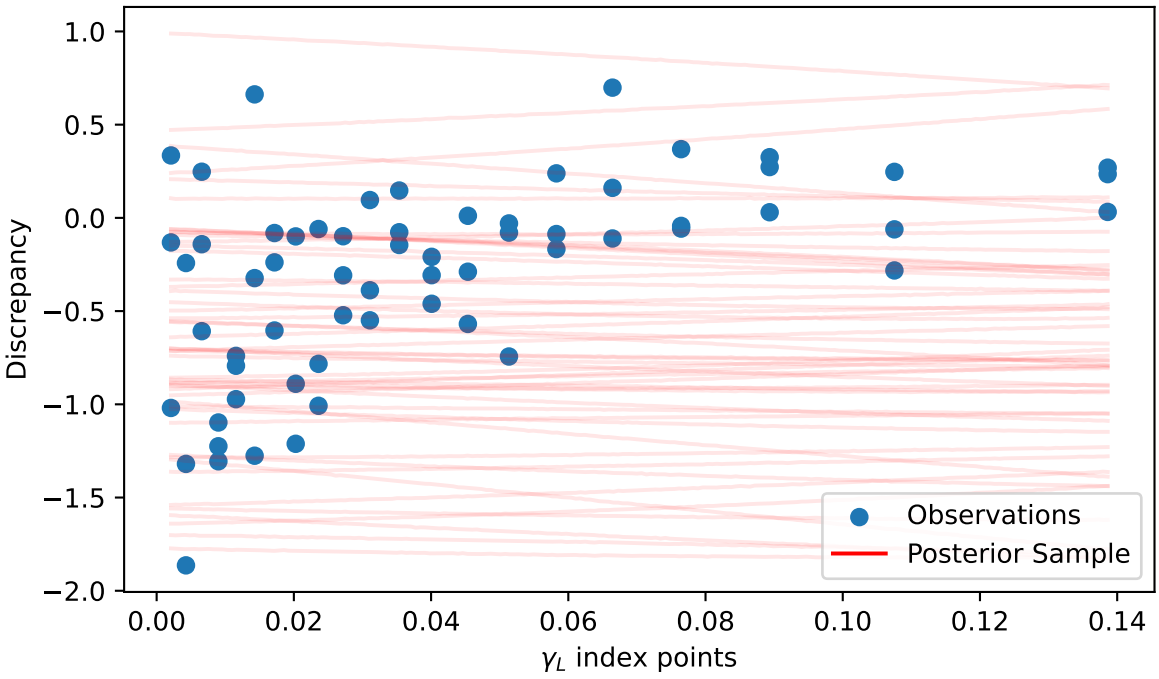
```
[0.4        0.36120128 0.0241165  0.41345092 0.03047614 0.03456727]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.74484701559556>
389
Acquisition function convergence reached at iteration 2497.
tf.Tensor(-52.96690500922203, shape=(), dtype=float64)
[0.4        0.36120132 0.02481446 0.41345096 0.03409142 0.0287017 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.75765111990393>
390
Acquisition function convergence reached at iteration 2699.
tf.Tensor(-52.979977288599635, shape=(), dtype=float64)
[0.4        0.36120136 0.02714903 0.41345101 0.03046874 0.03190034]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.770422518929415>
391
Acquisition function convergence reached at iteration 2572.
tf.Tensor(-52.99310812457336, shape=(), dtype=float64)
[0.4        0.3612014  0.02470257 0.41345106 0.03301968 0.03092478]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.78316137932341>
392
Acquisition function convergence reached at iteration 2565.
tf.Tensor(-53.00615647837687, shape=(), dtype=float64)
[0.4        0.36120145 0.02539826 0.41345111 0.03394672 0.02875086]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.79586786646677>
393
Acquisition function convergence reached at iteration 2489.
tf.Tensor(-53.01919866694353, shape=(), dtype=float64)
[0.4        0.36120149 0.02648308 0.41345116 0.02766006 0.03372372]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.80854214448271>
394
Acquisition function convergence reached at iteration 2650.
tf.Tensor(-53.03215877934904, shape=(), dtype=float64)
[0.4        0.36120153 0.02574669 0.4134512  0.0306385  0.03298178]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.821184376249505>
395
Acquisition function convergence reached at iteration 2598.
tf.Tensor(-53.045118339504384, shape=(), dtype=float64)
[0.4        0.36120157 0.02704851 0.41345125 0.02774203 0.03381841]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.83379472341306>
396
Acquisition function convergence reached at iteration 2641.
tf.Tensor(-53.05803536544464, shape=(), dtype=float64)
[0.4        0.36120161 0.02684384 0.4134513  0.02877571 0.03346797]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.84637334639929>
397
```

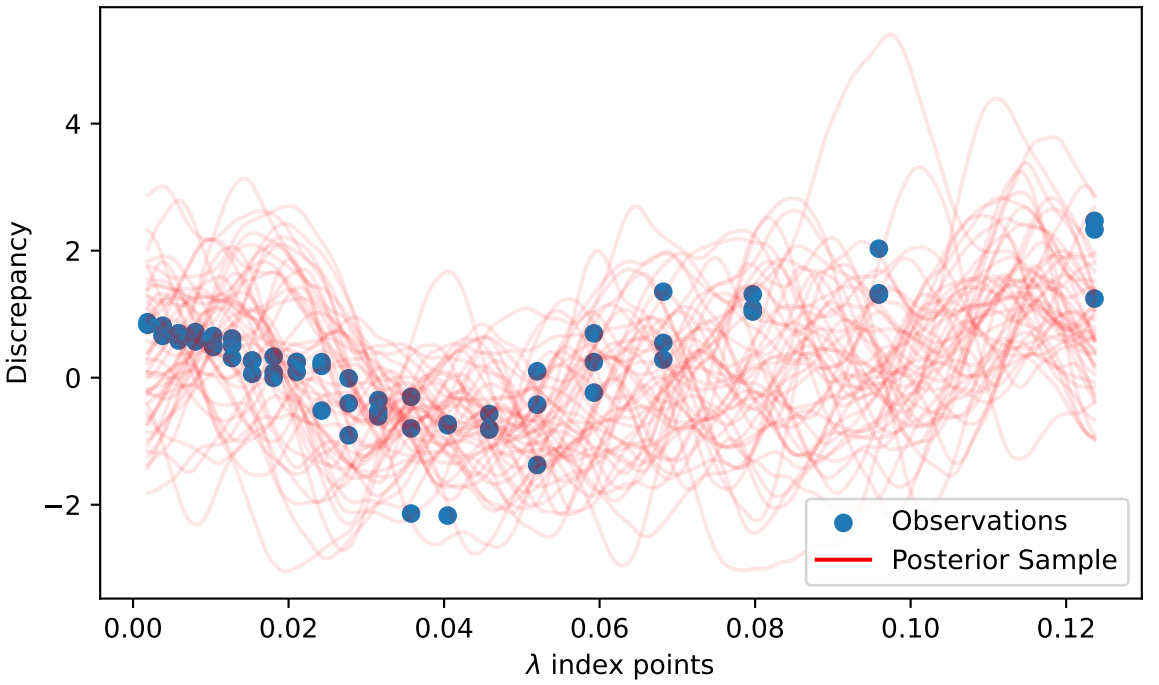
```
Acquisition function convergence reached at iteration 2537.
tf.Tensor(-53.070951336185814, shape=(), dtype=float64)
[0.4        0.36120165 0.02668583 0.41345134 0.02812191 0.03345958]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.85892040442642>
398
Acquisition function convergence reached at iteration 2475.
tf.Tensor(-53.08381524593387, shape=(), dtype=float64)
[0.4        0.36120169 0.02782177 0.41345139 0.02805717 0.03167255]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.87143605551701>
399
Acquisition function convergence reached at iteration 2395.
tf.Tensor(-53.096683924107346, shape=(), dtype=float64)
[0.4        0.36120173 0.02472463 0.41345144 0.02961447 0.0331761 ]
<tf.Variable 'eta_t:0' shape=() dtype=float64, numpy=51.88392045650995>
400
Acquisition function convergence reached at iteration 2617.
tf.Tensor(-53.10941145497907, shape=(), dtype=float64)
Trained parameters:
amplitude_champ:0 is 1.03

length_scales_champ:0 is [9.400000e-02 2.500000e-02 3.519769e+03 6.000000e-03 5.520439e+03
 2.371162e+03]

observation_noise_variance_champ:0 is 0.218

alpha_tp:0 is 0.348

amp_alpha_mean:0 is 0.72

amp_beta_mean:0 is 0.055

amp_f_mean:0 is 4.21

amp_gamma_L_mean:0 is 11.846

amp_lambda_mean:0 is 10.797

amp_r_mean:0 is 2.018

beta_tp:0 is 0.108

bias_mean:0 is -3.448
```

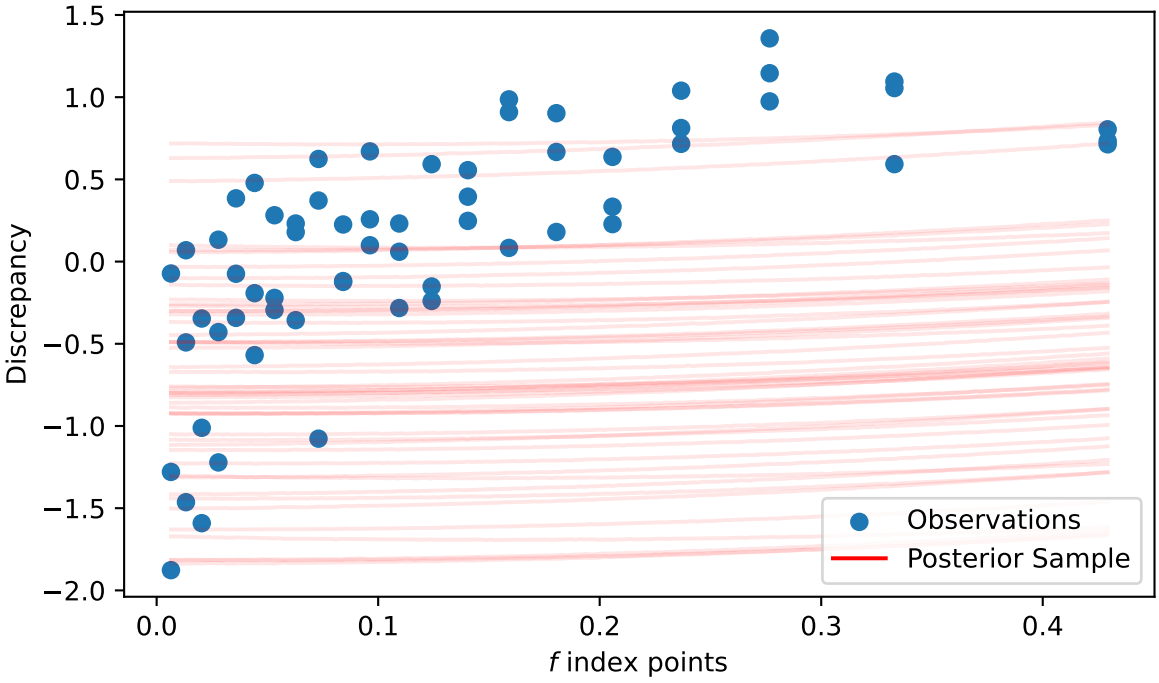
```
f_tp:0 is -0.104

gamma_L_tp:0 is 0.122

lambda_tp:0 is -0.51

r_tp:0 is 0.521

[0.39283988 0.39974583 0.05218201 0.06311031 0.0626069  0.05049378]
[0.4        0.35973608 0.03005014 0.40837935 0.02910603 0.02843961]
[0.4        0.3707827  0.02798402 0.40796218 0.0311762  0.03060216]
[0.4        0.37874158 0.02808962 0.4065084  0.0290989  0.03034286]
[0.4        0.36120098 0.02379502 0.41345058 0.03009863 0.03443406]
[0.4        0.36120149 0.02648308 0.41345116 0.02766006 0.03372372]
[0.4        0.36120165 0.02668583 0.41345134 0.02812191 0.03345958]
[0.4        0.36120173 0.02472463 0.41345144 0.02961447 0.0331761 ]
[0.4        0.36120177 0.0273468  0.41345149 0.02901419 0.03249182]
```

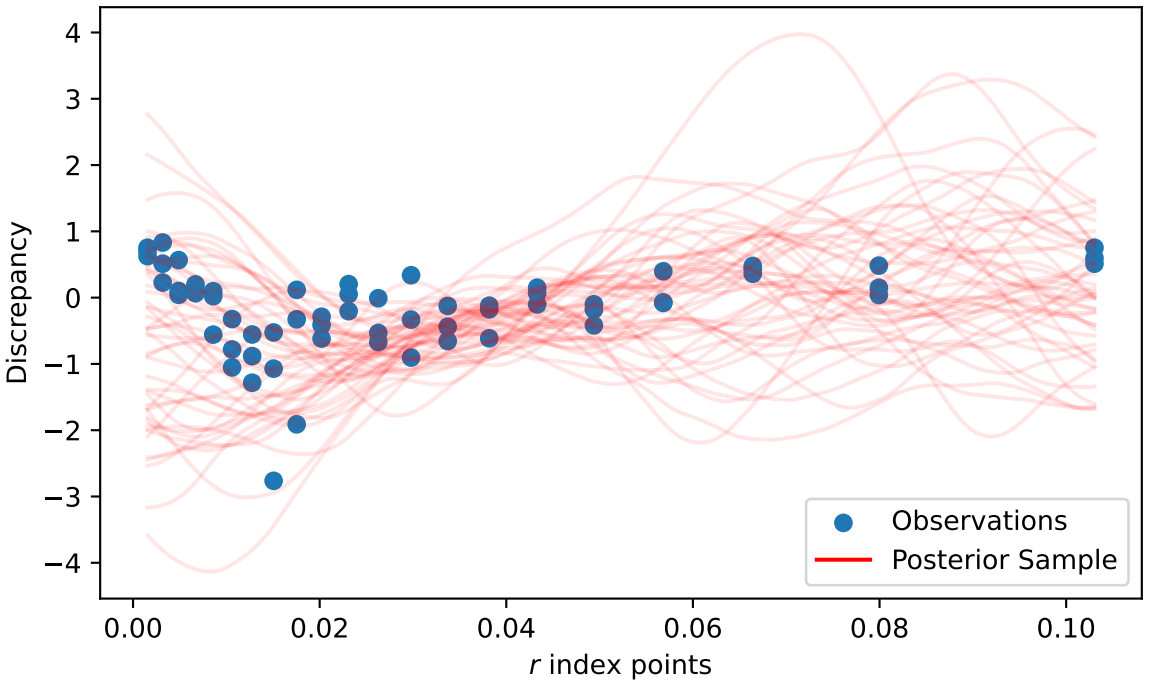


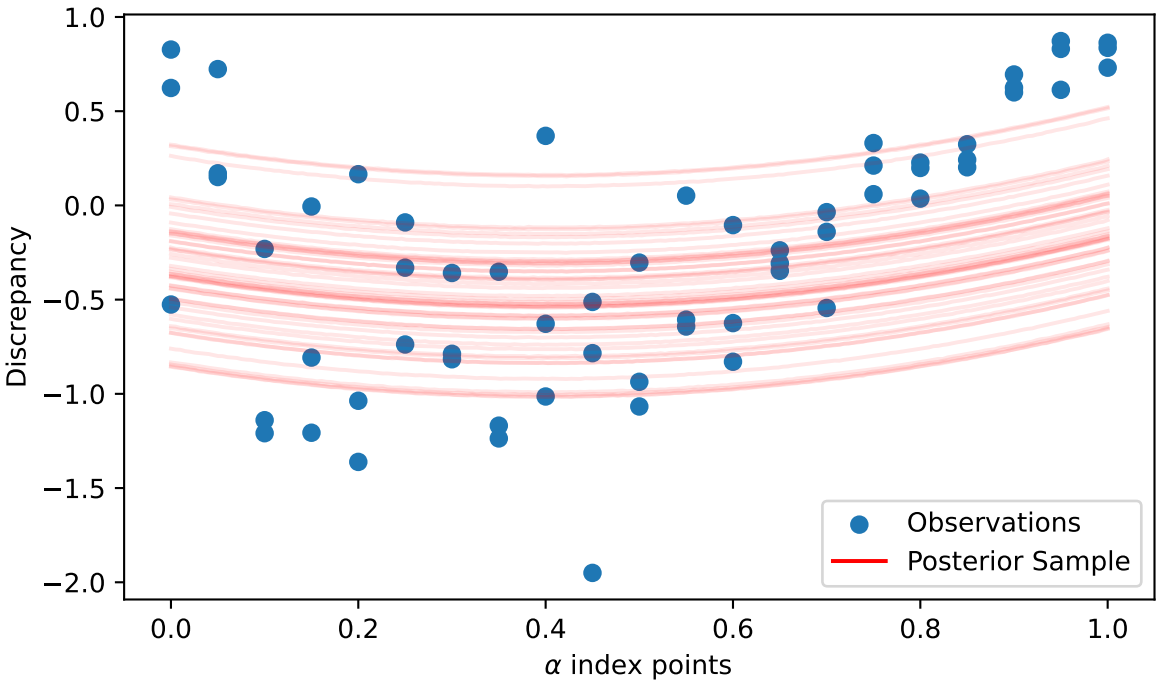

Discrepancy
4
2
0
−2
−4
0.0
0.2
0.4
0.6
0.8
1.0
β index points
Observations
Posterior Sample


Discrepancy
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
γ_L index points
Observations
Posterior Sample


Discrepancy
4
2
0
−2
0.00
0.02
0.04
0.06
0.08
0.10
0.12
λ index points
Observations
Posterior Sample


Discrepancy
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.1
0.2
0.3
0.4
f index points
Observations
Posterior Sample


Discrepancy
4
3
2
1
0
−1
−2
−3
−4
0.00
0.02
0.04
0.06
0.08
0.10
r index points
Observations
Posterior Sample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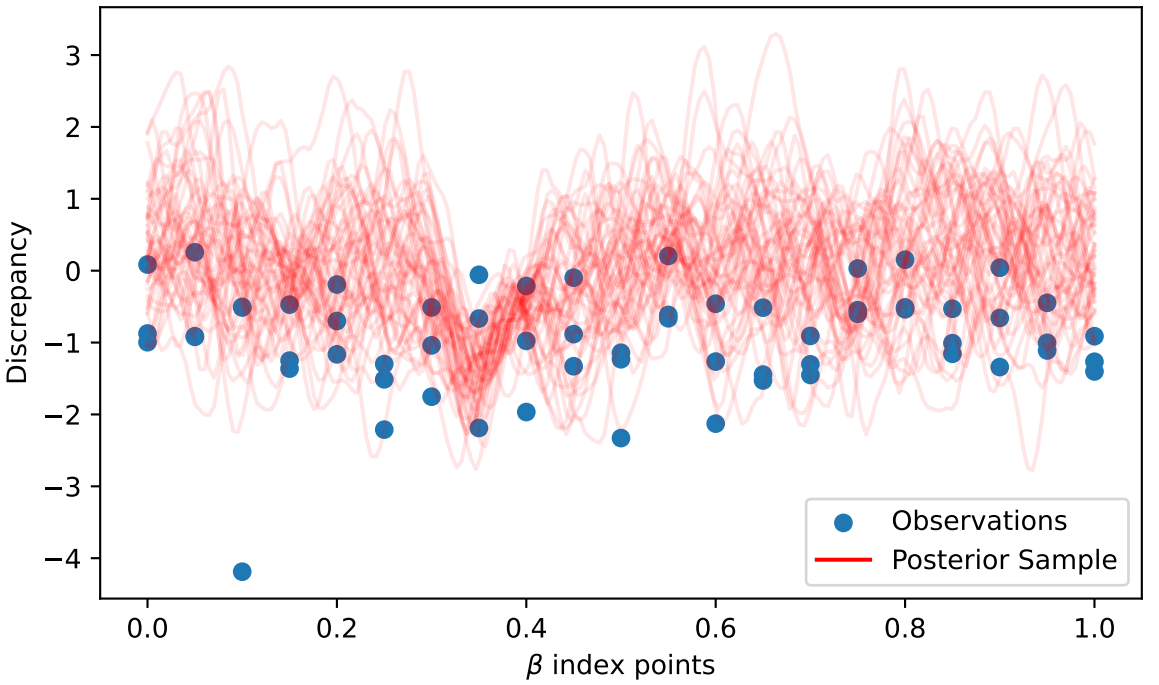
0.8
1.0
α index points
Observations
Posterior Sample


Discrepancy
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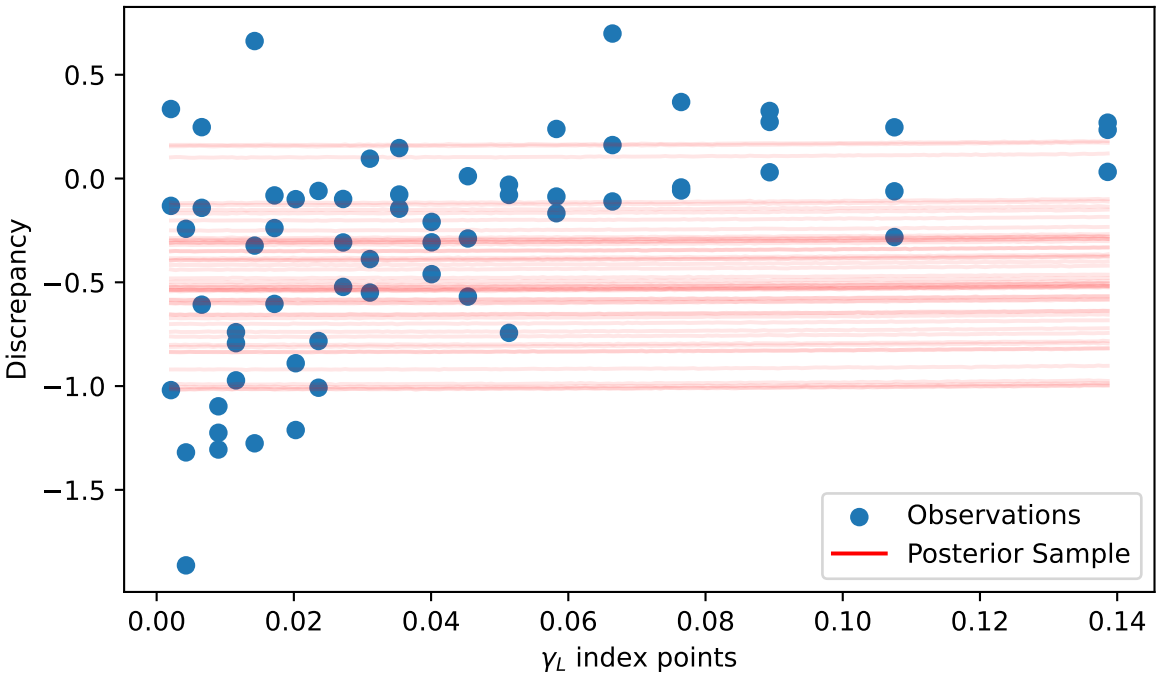
0.4
0.6
0.8
1.0
β index points
Observations
Posterior Sample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0.10
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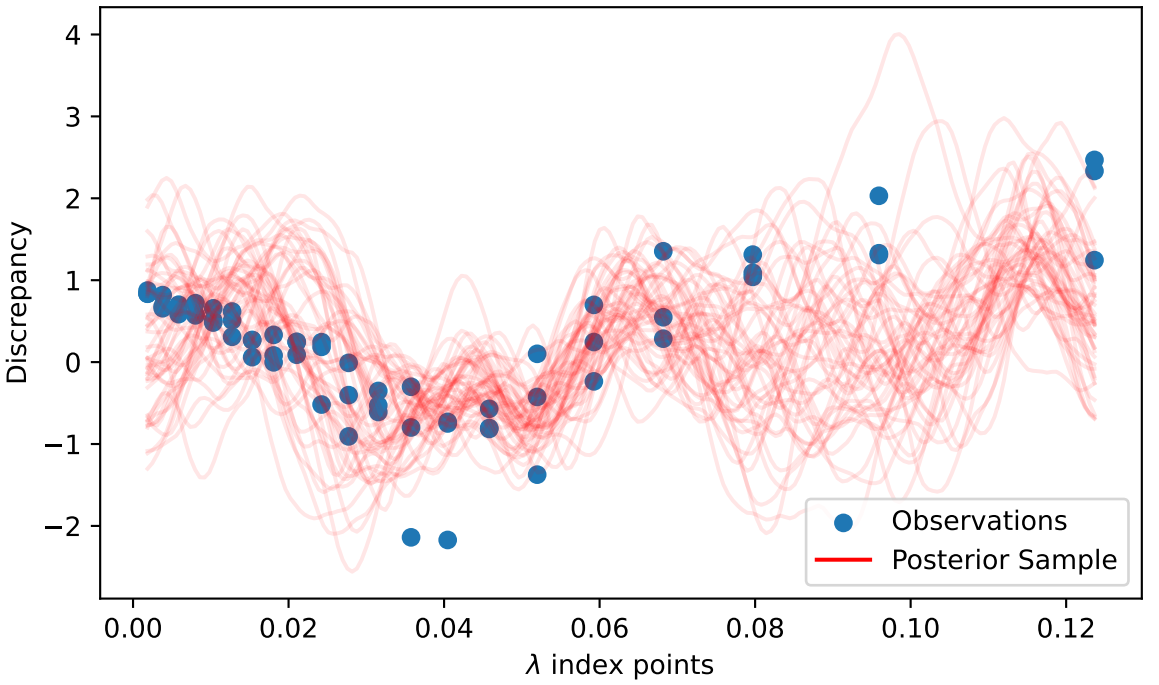
0.14
γ_L index points
Observations
Posterior Sample


Discrepancy
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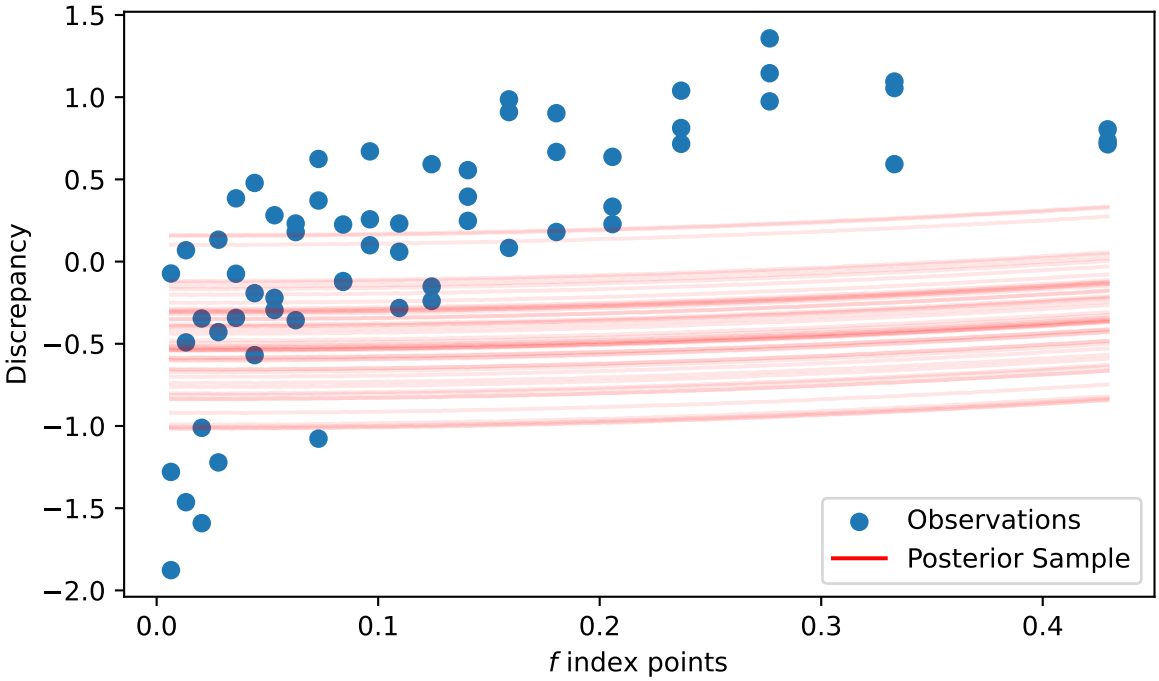
0.10
0.12
λ index points
Observations
Posterior Sample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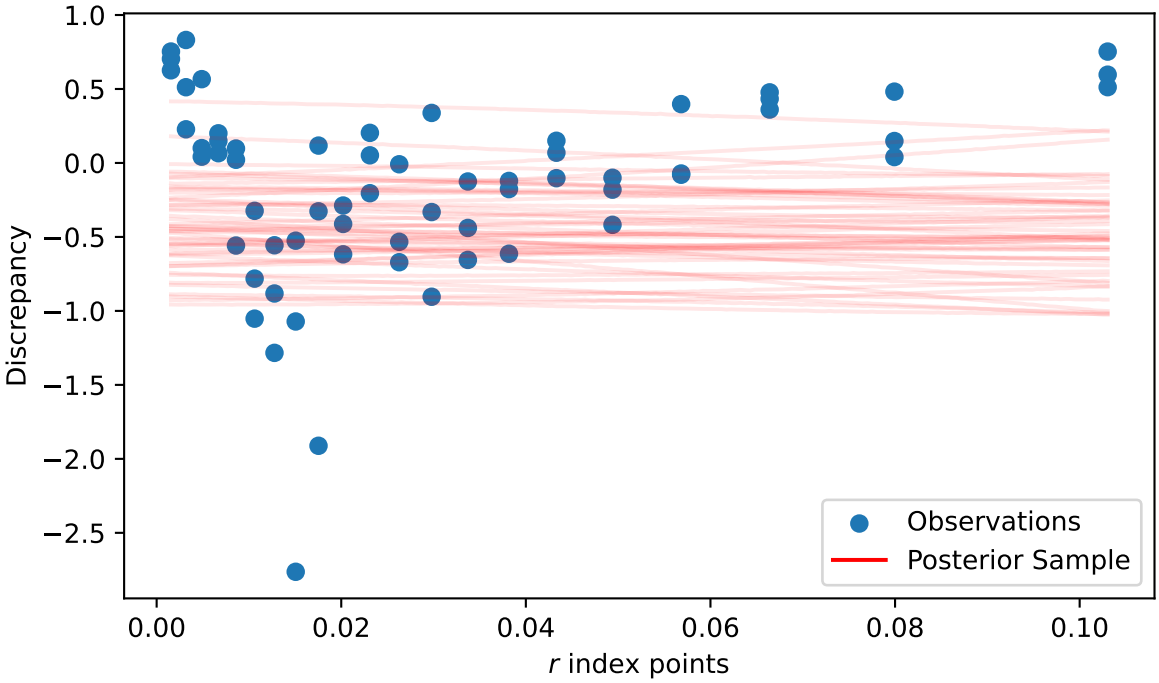
0.2
0.3
0.4
f index points
Observations
Posterior Sample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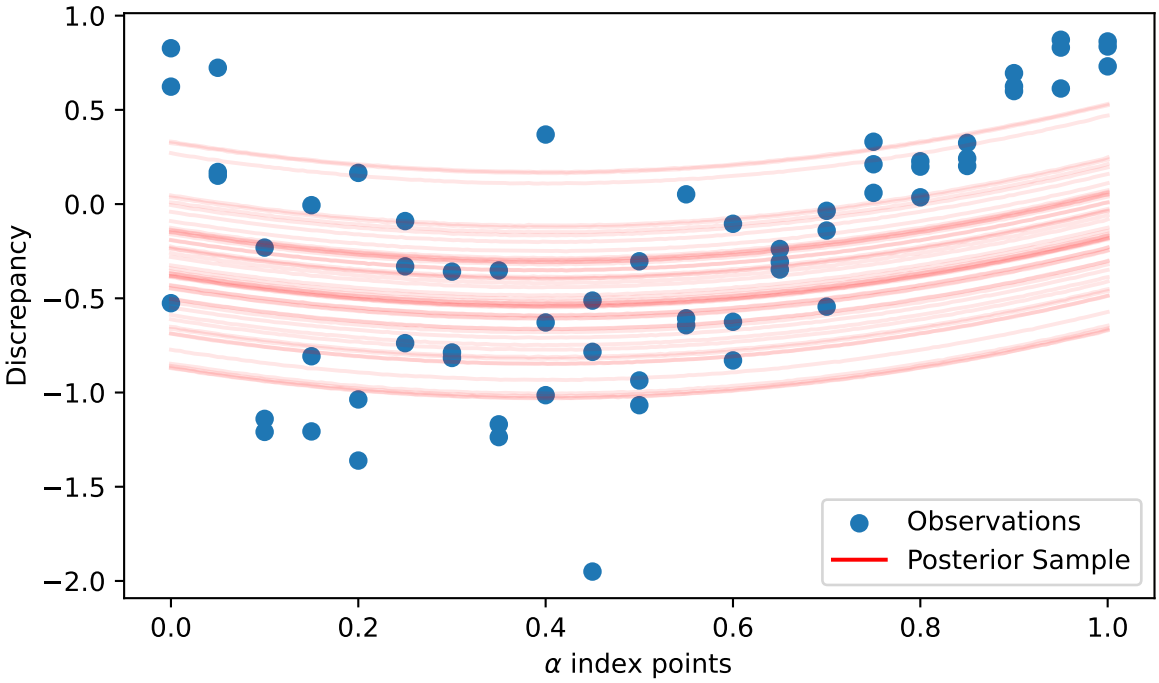
0.10
r index points
Observations
Posterior Sample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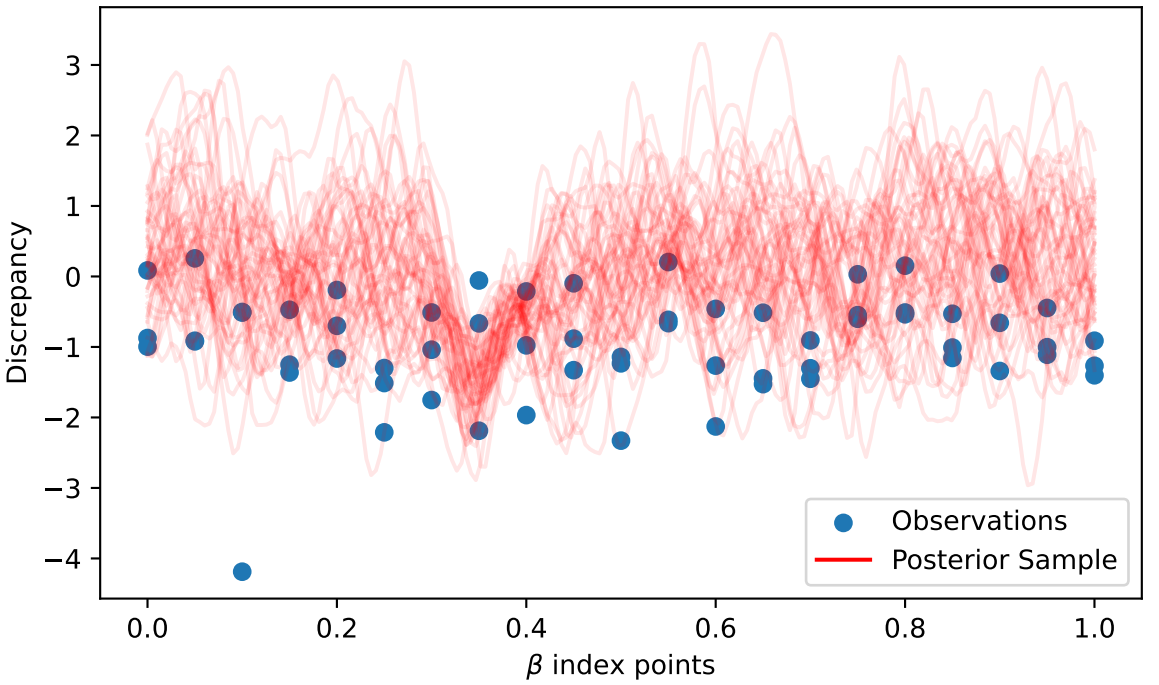
0.4
0.6
0.8
1.0
α index points
Observations
Posterior Sample


Discrepancy
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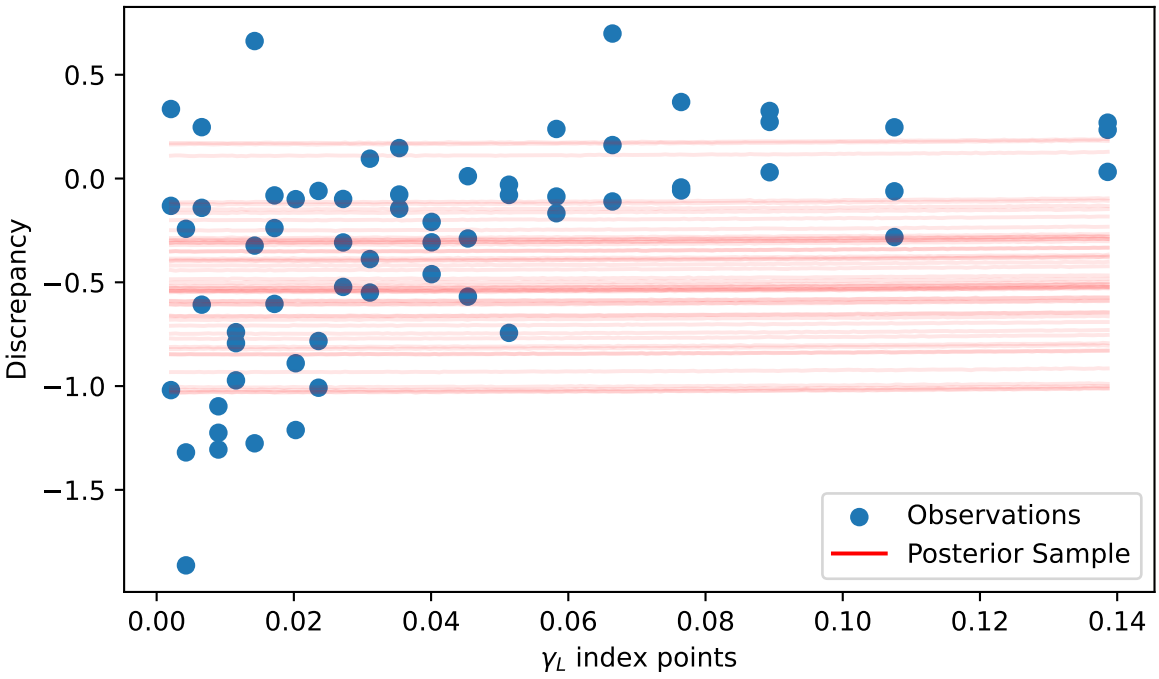
β index points
Observations
Posterior Sample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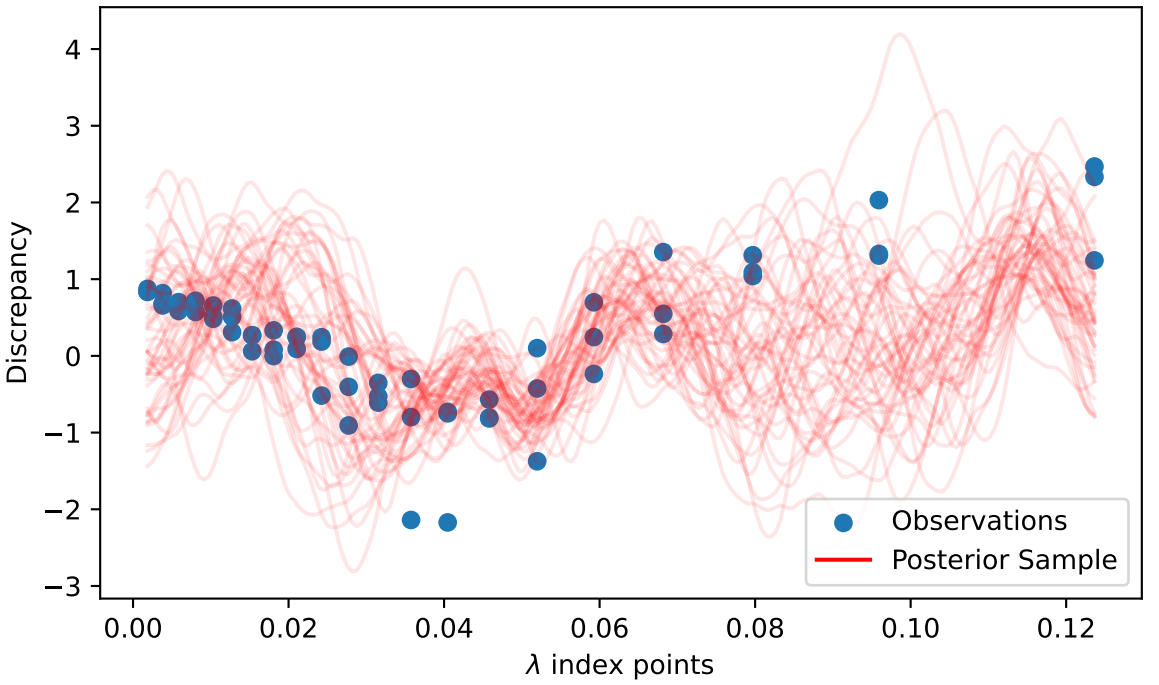
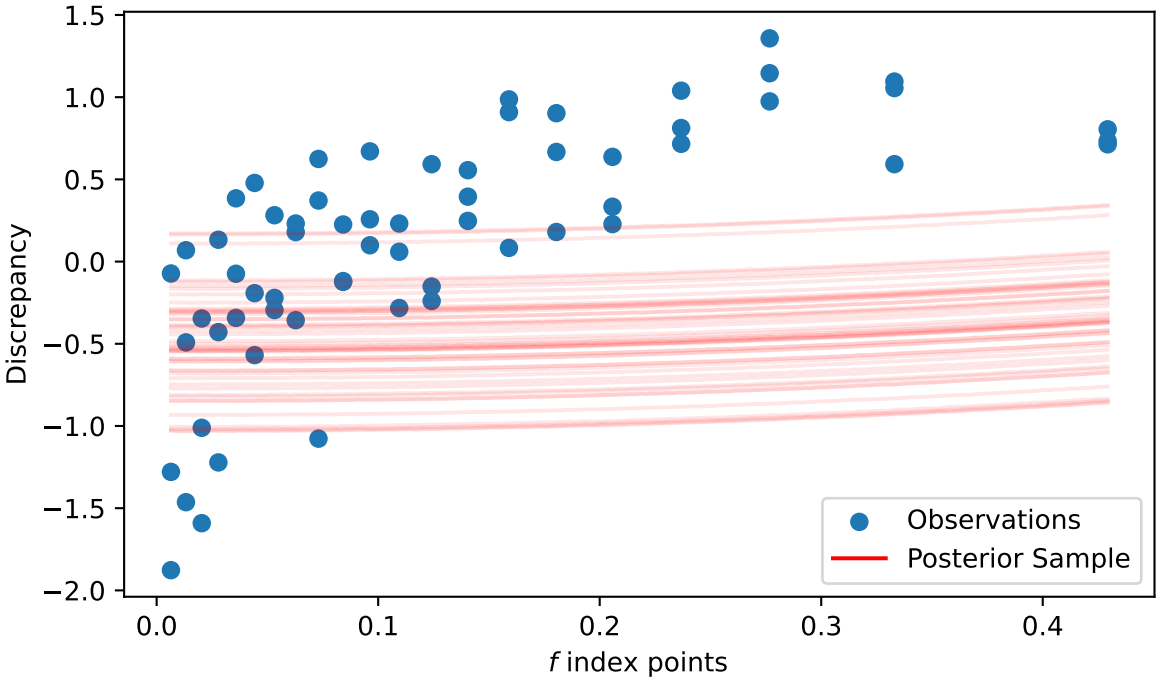
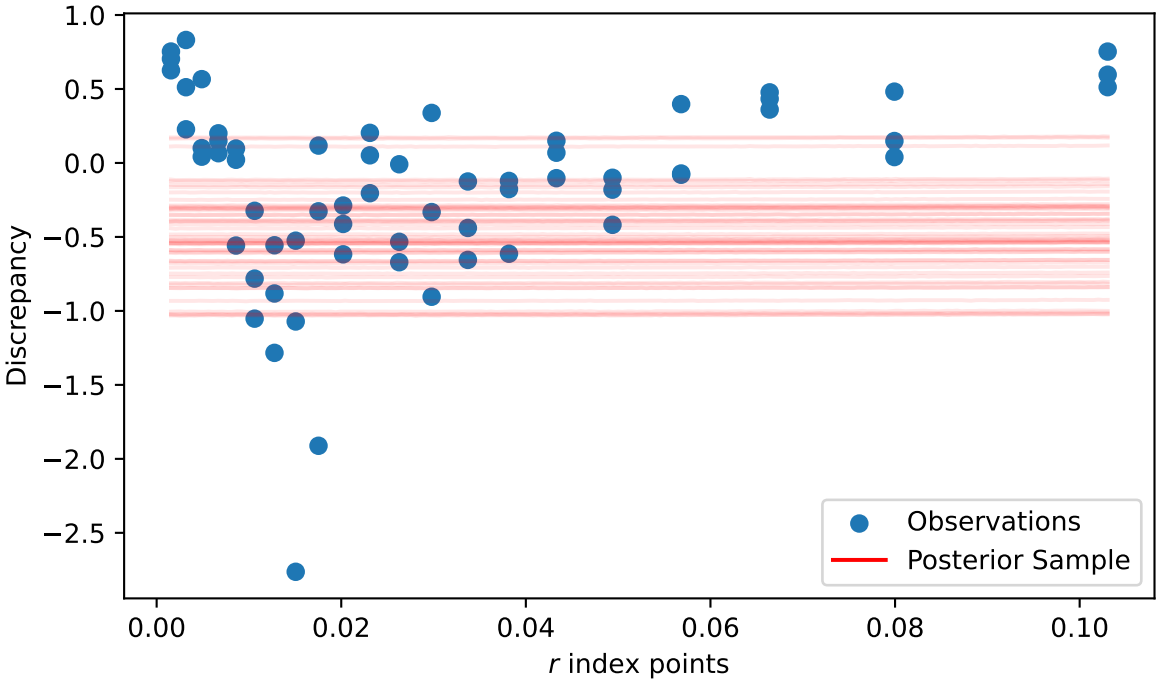
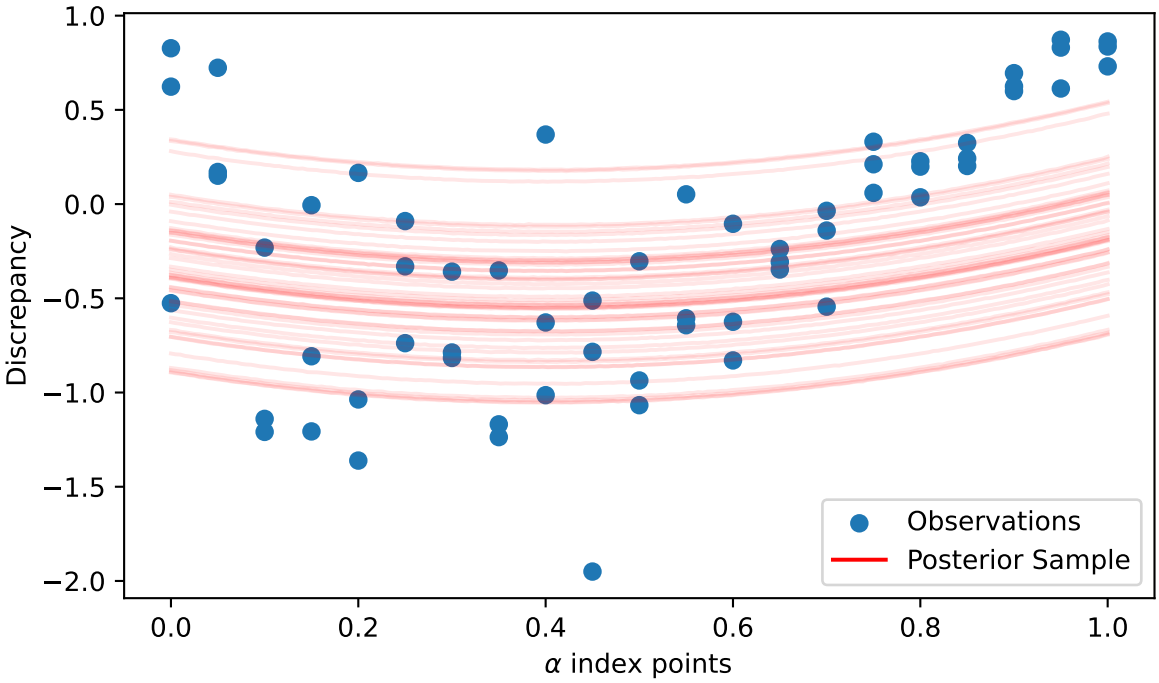
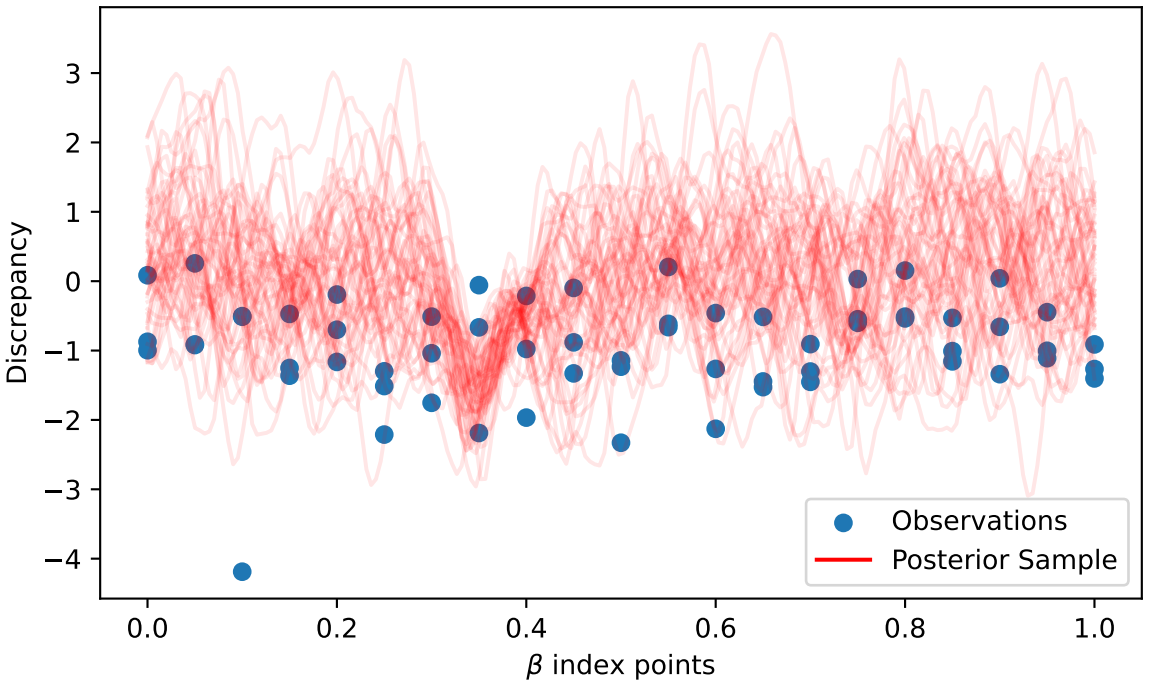
0.12
0.14
γ_L index points
Observations
Posterior Sample


Discrepancy
λ index points
Observations
Posterior Sample


Discrepancy
f index points
Observations
Posterior Sample


Discrepancy
r index points
Observations
Posterior Sample


Discrepancy
α index points
Observations
Posterior Sample


Discrepancy
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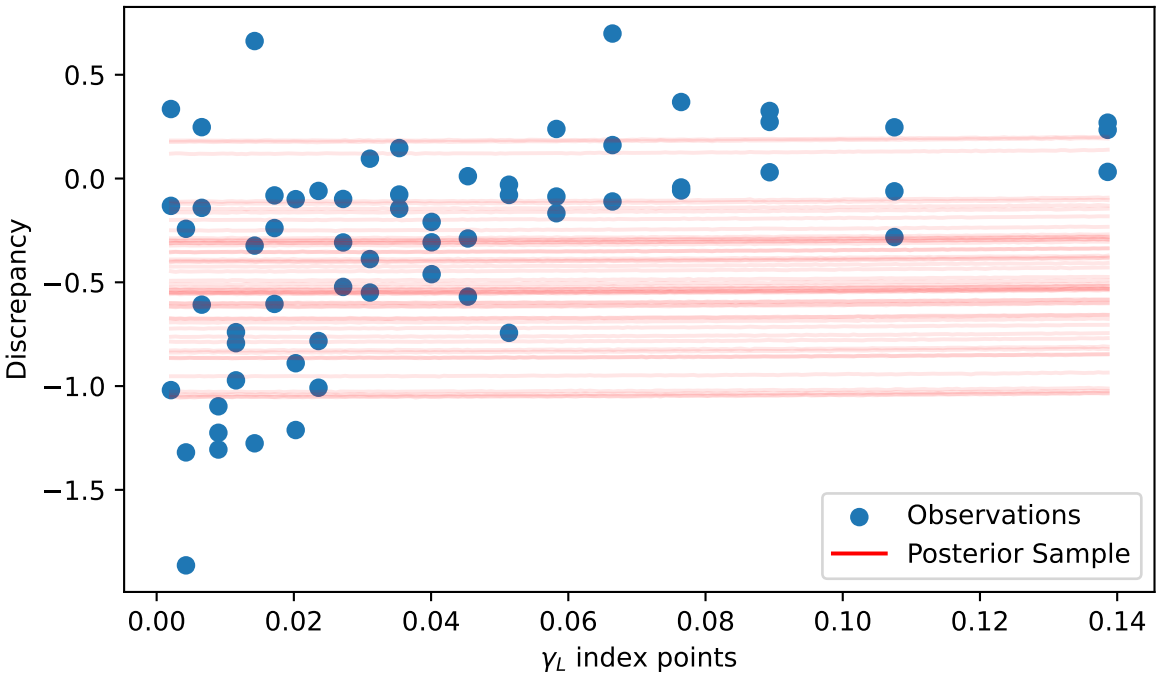
0.4
0.6
0.8
1.0
β index points
Observations
Posterior Sample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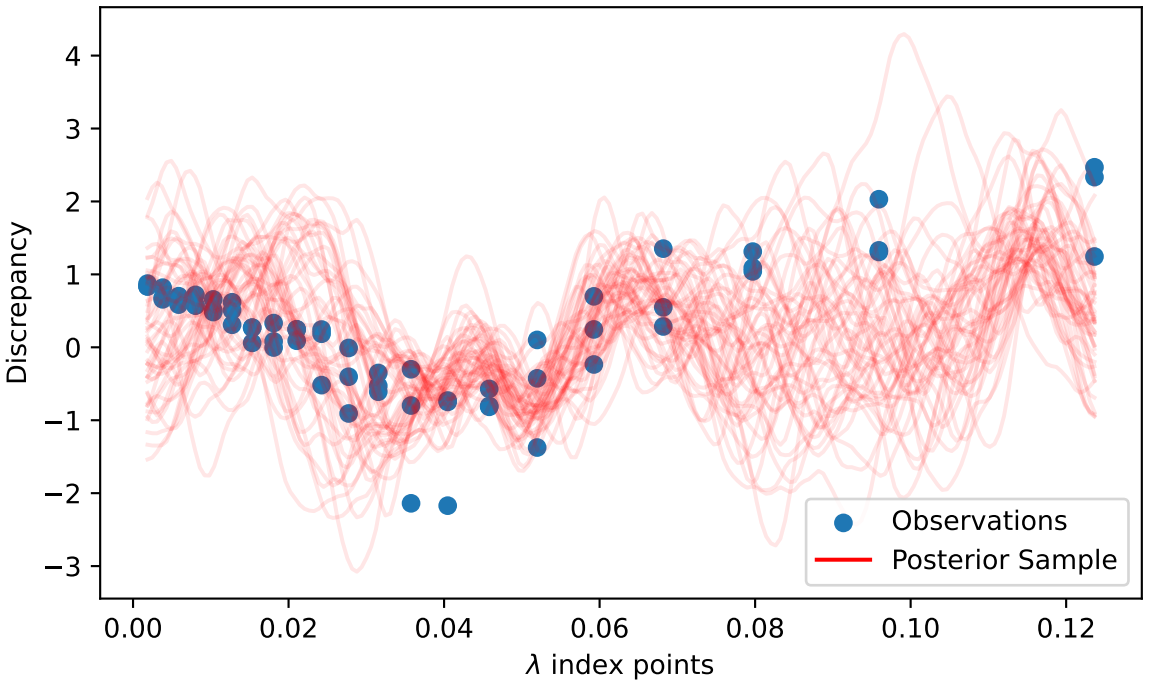
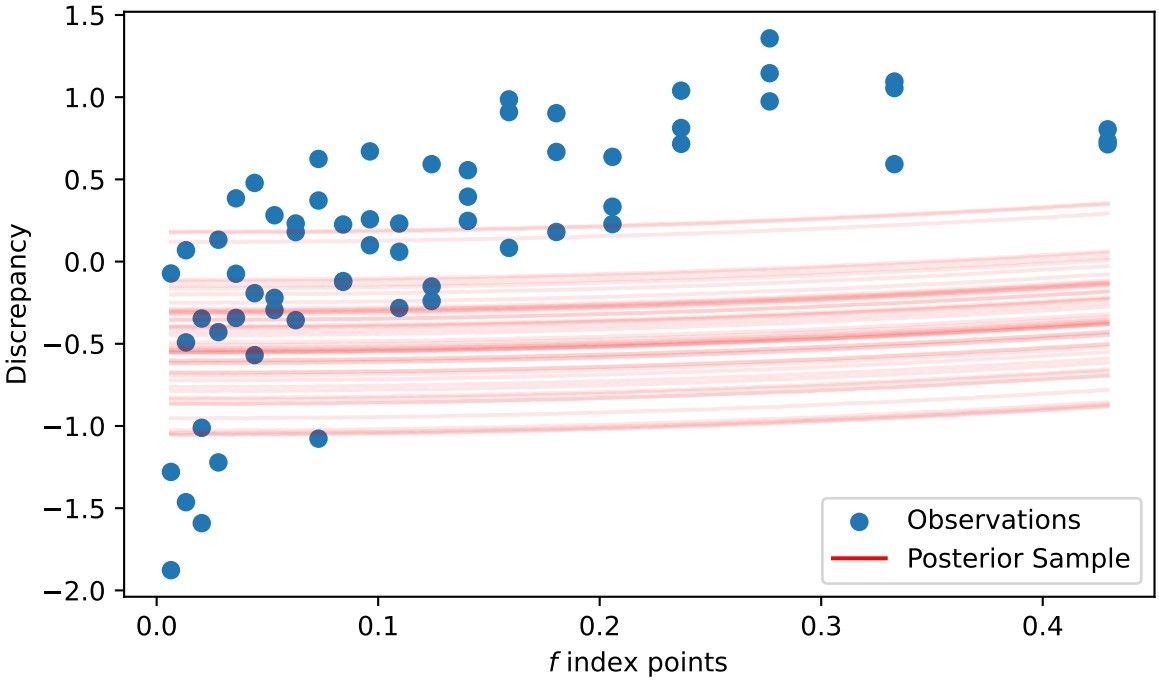
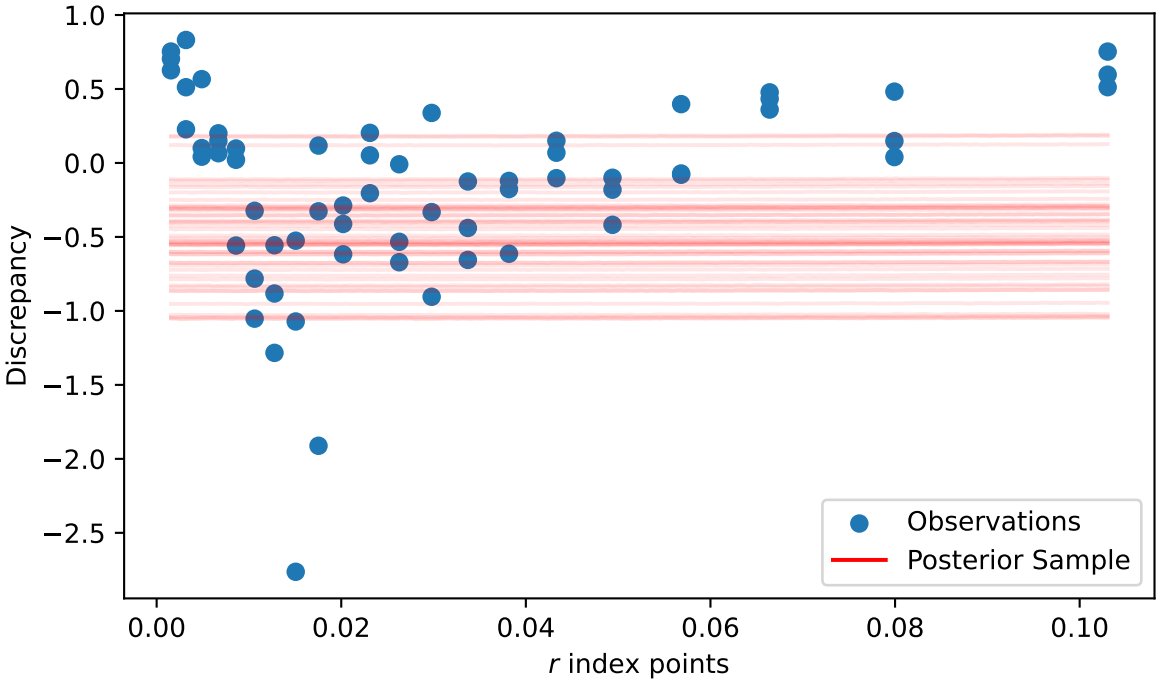
γ_L index points
Observations
Posterior Sample


Discrepancy
λ index points
Observations
Posterior Sample


Discrepancy
f index points
Observations
Posterior Sample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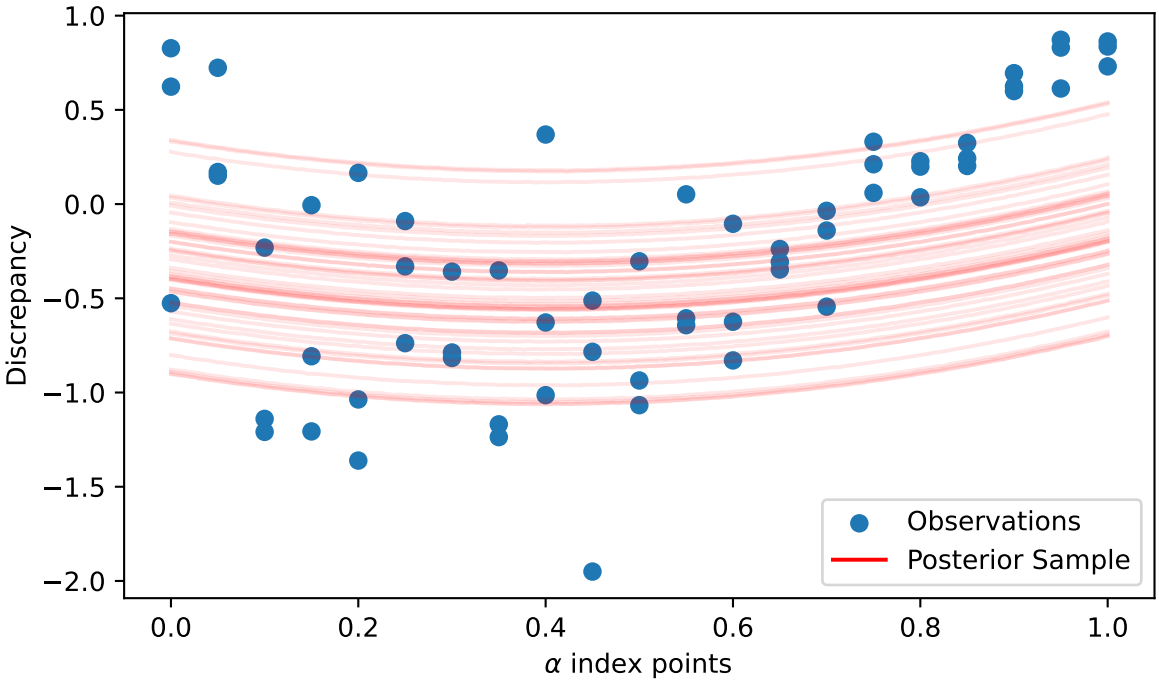
r index points
Observations
Posterior Sample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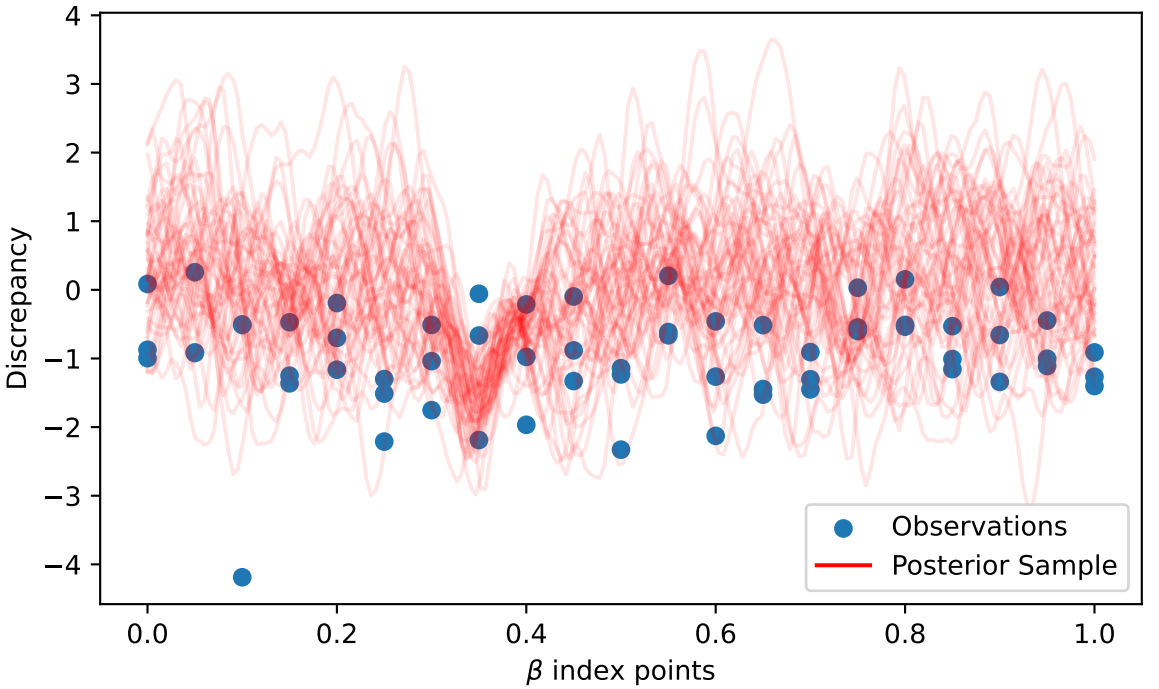
α index points
Observations
Posterior Sample


Discrepancy
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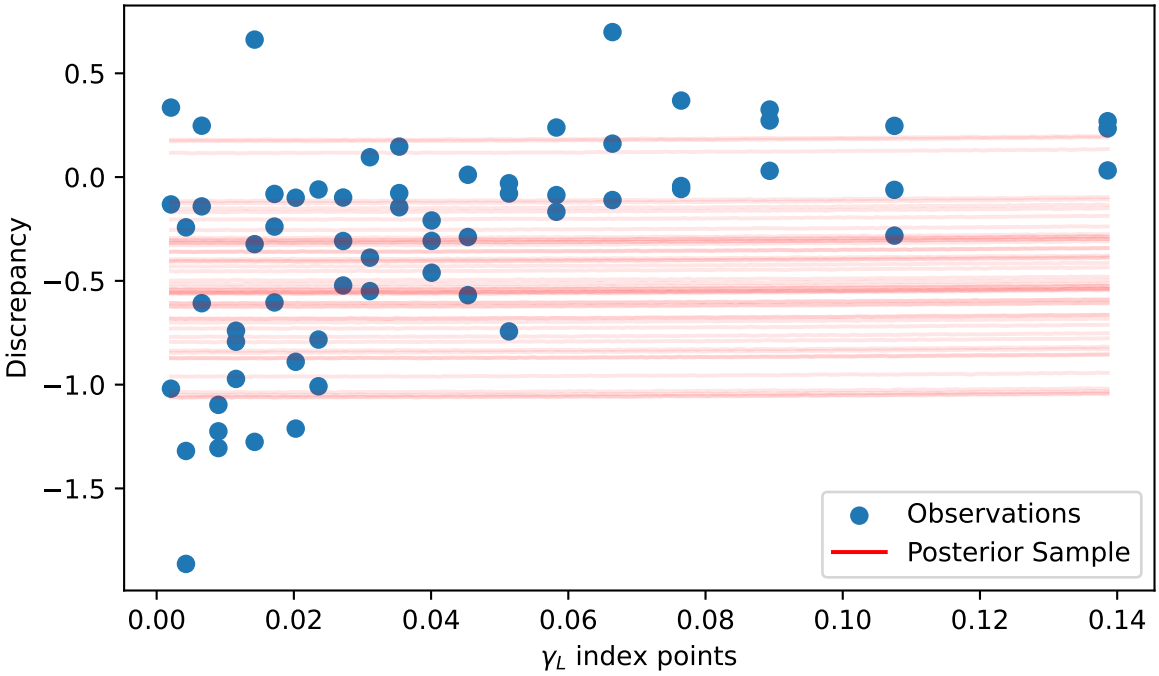
β index points
Observations
Posterior Sample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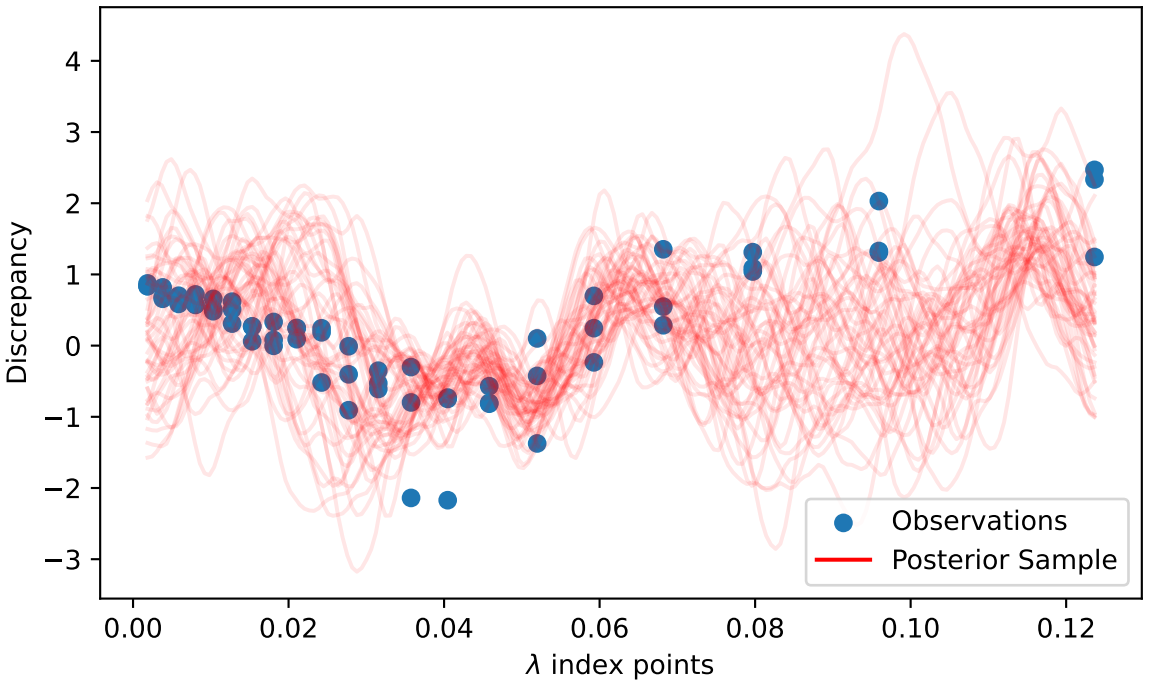
0.14
γ_L index points
Observations
Posterior Sample


Discrepancy
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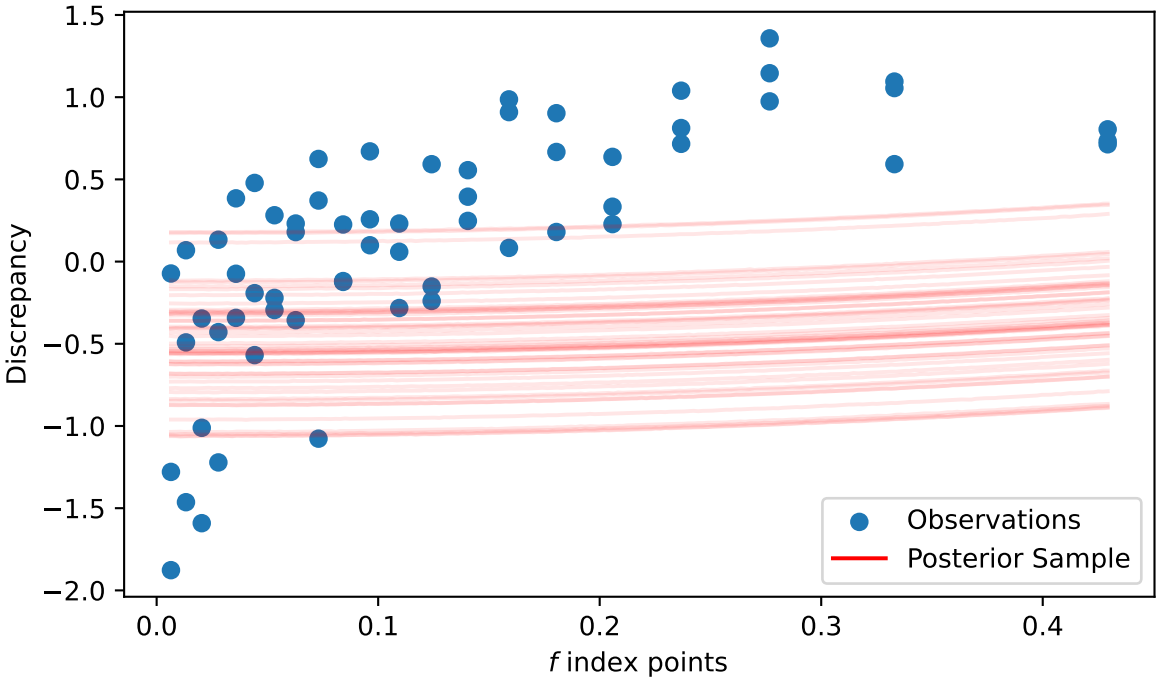
λ index points
Observations
Posterior Sample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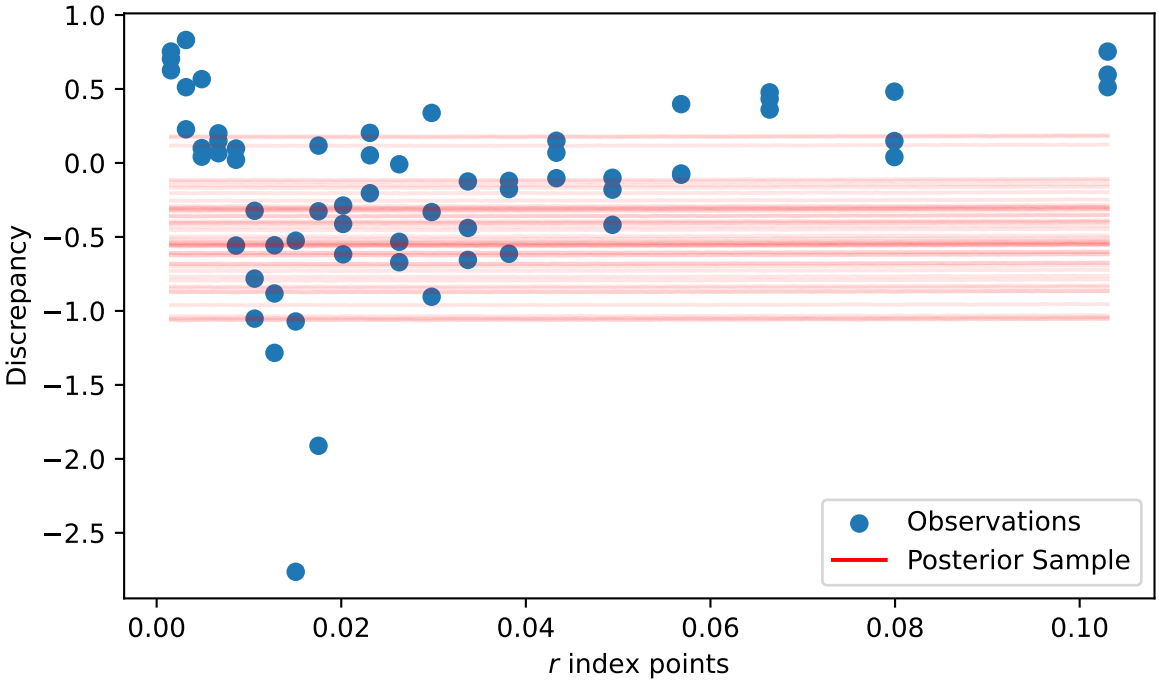
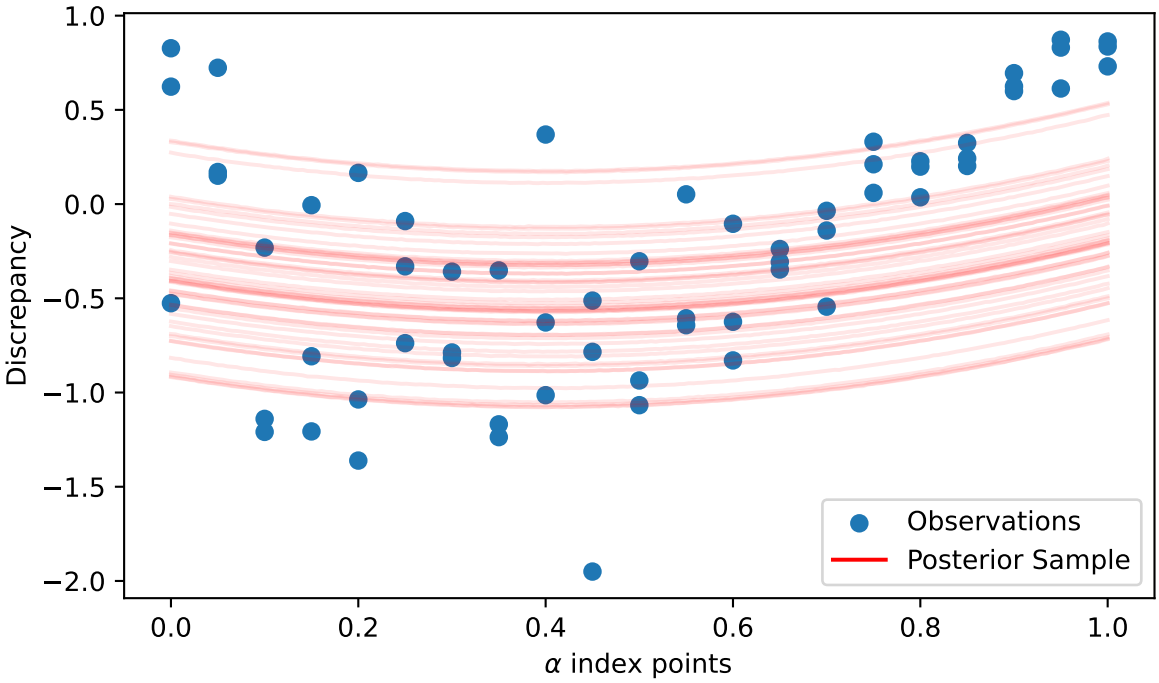
f index points
Observations
Posterior Sample


Discrepancy
r index points
Observations
Posterior Sample


Discrepancy
α index points
Observations
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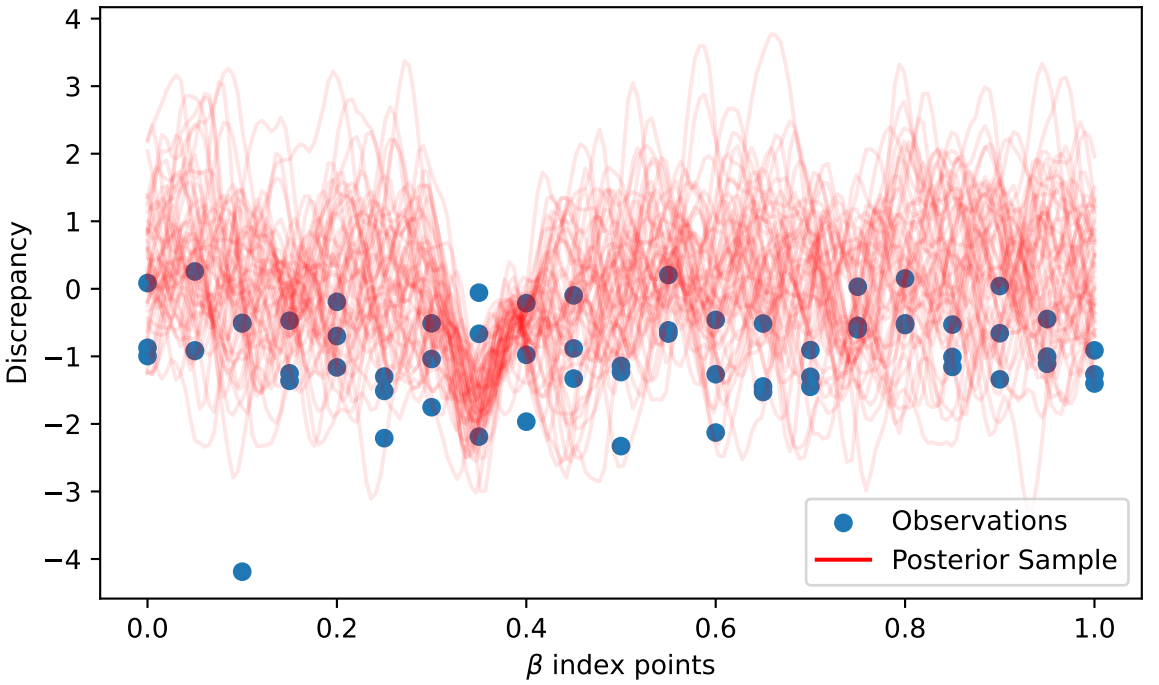
Discrepancy
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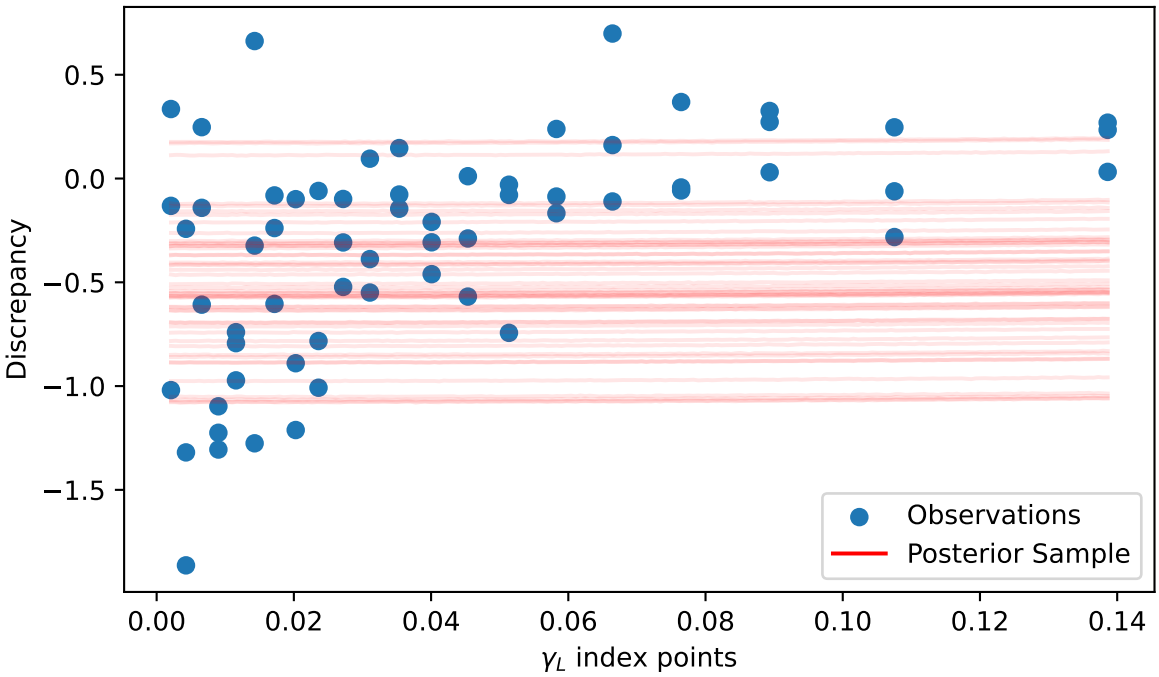
β index points
Observations
Posterior Sample


Discrepancy
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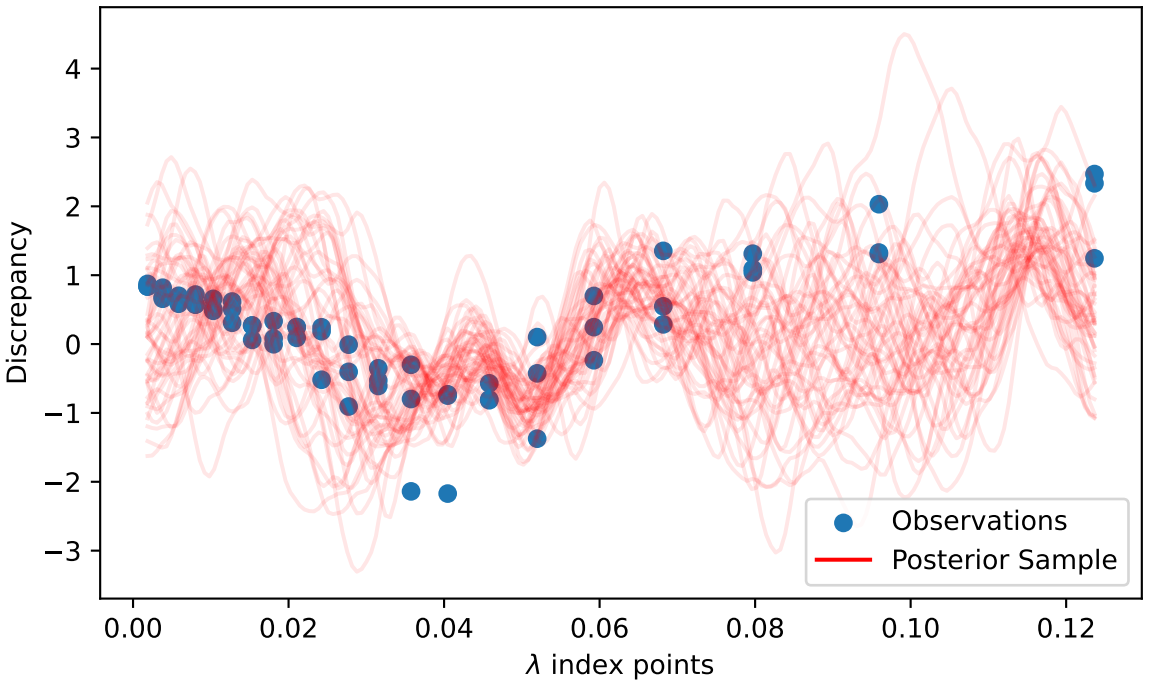
γ_L index points
Observations
Posterior Sample


Discrepancy
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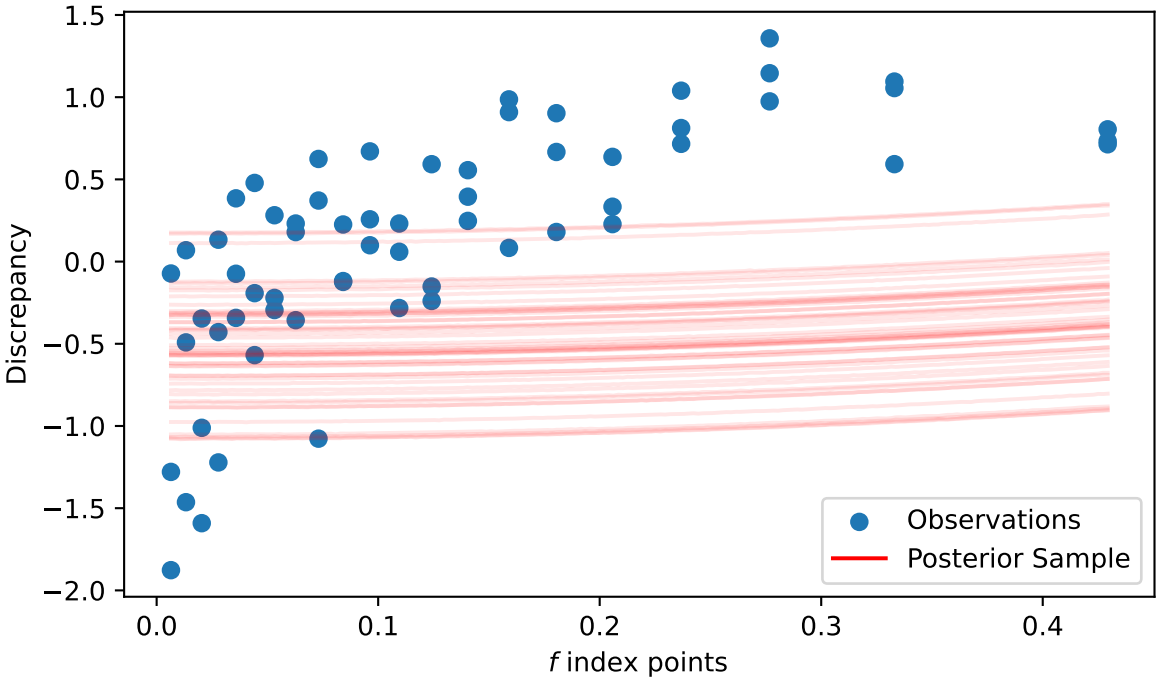
λ index points
Observations
Posterior Sample


Discrepancy
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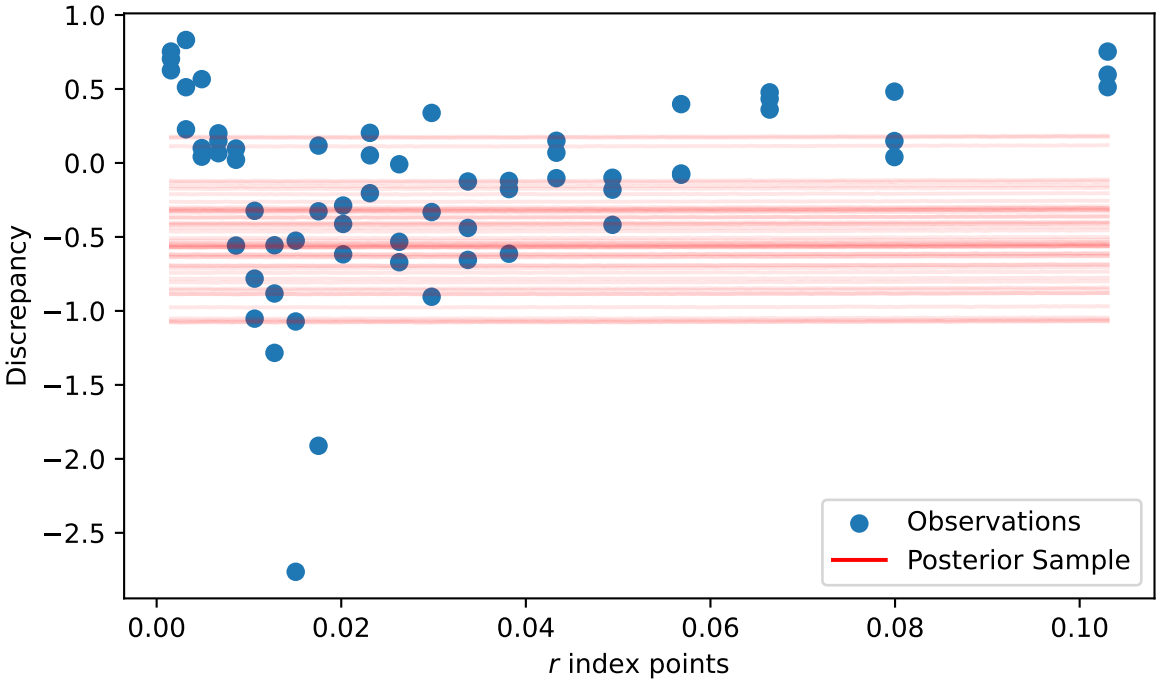
f index points
Observations
Posterior Sample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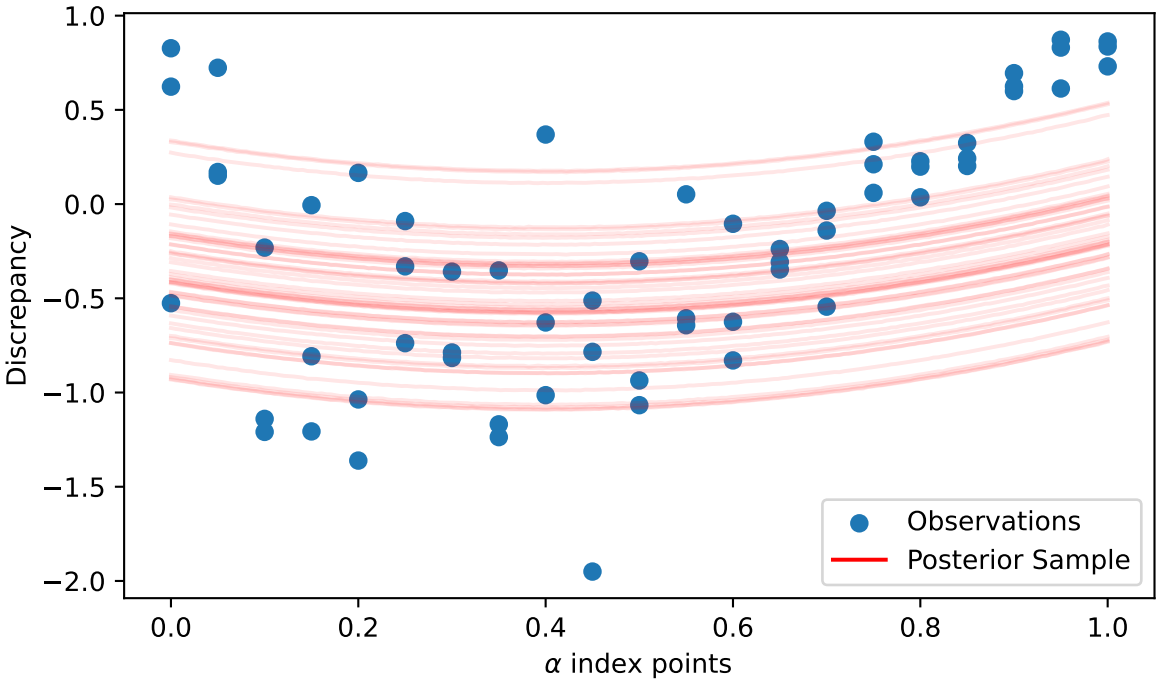
r index points
Observations
Posterior Sample


Discrepancy
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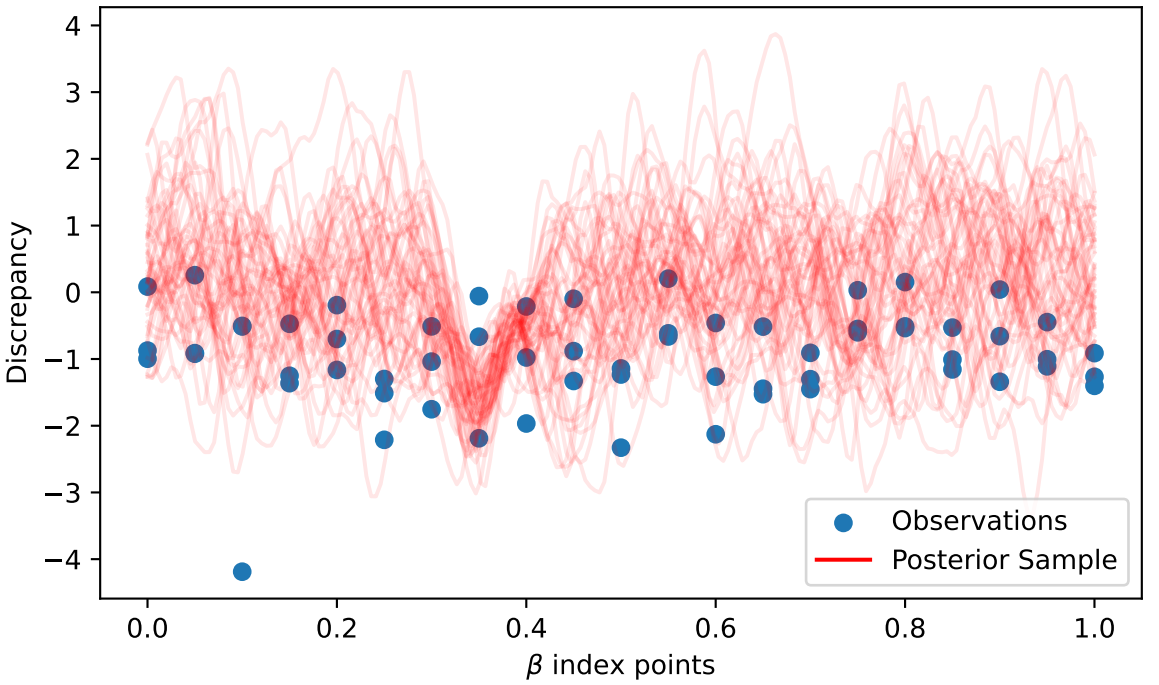
α index points
Observations
Posterior Sample


Discrepancy
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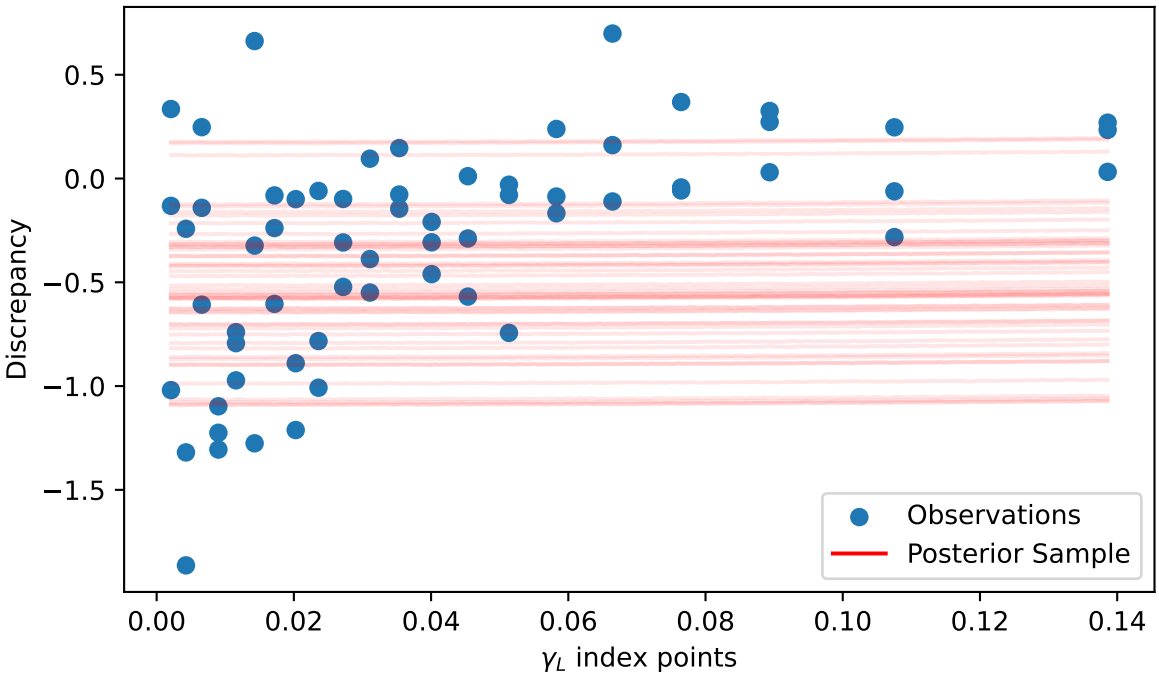
β index points
Observations
Posterior Sample


Discrepancy
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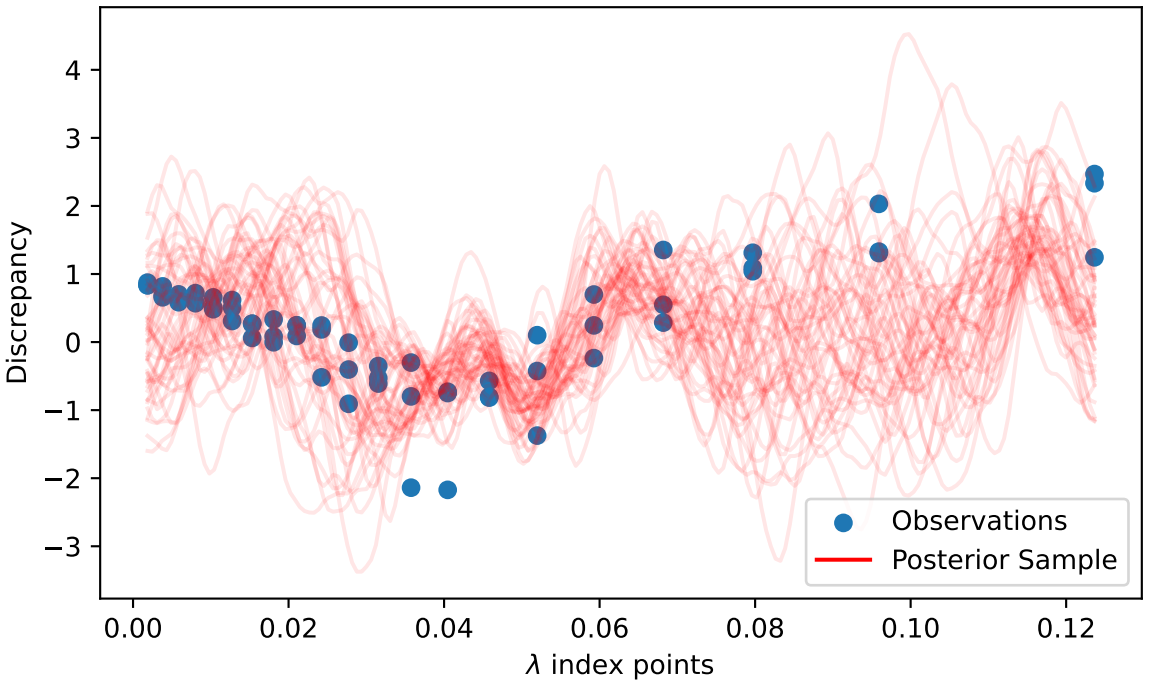
γ_L index points
Observations
Posterior Sample


Discrepancy
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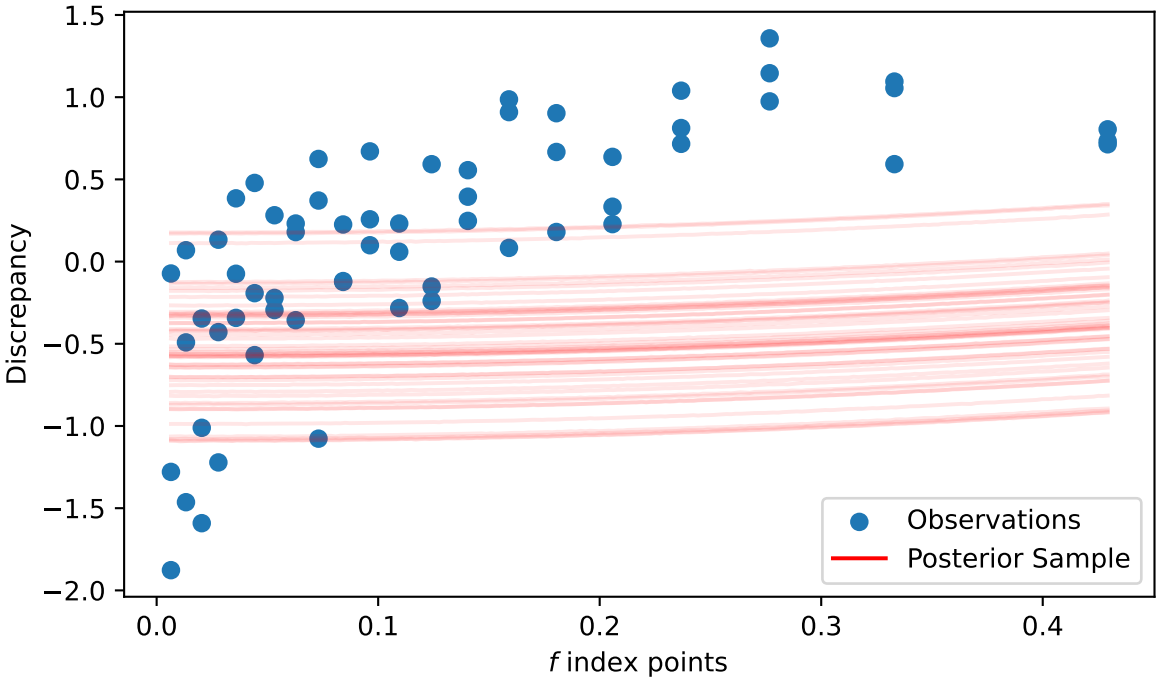
λ index points
Observations
Posterior Sample


Discrepancy
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0.2
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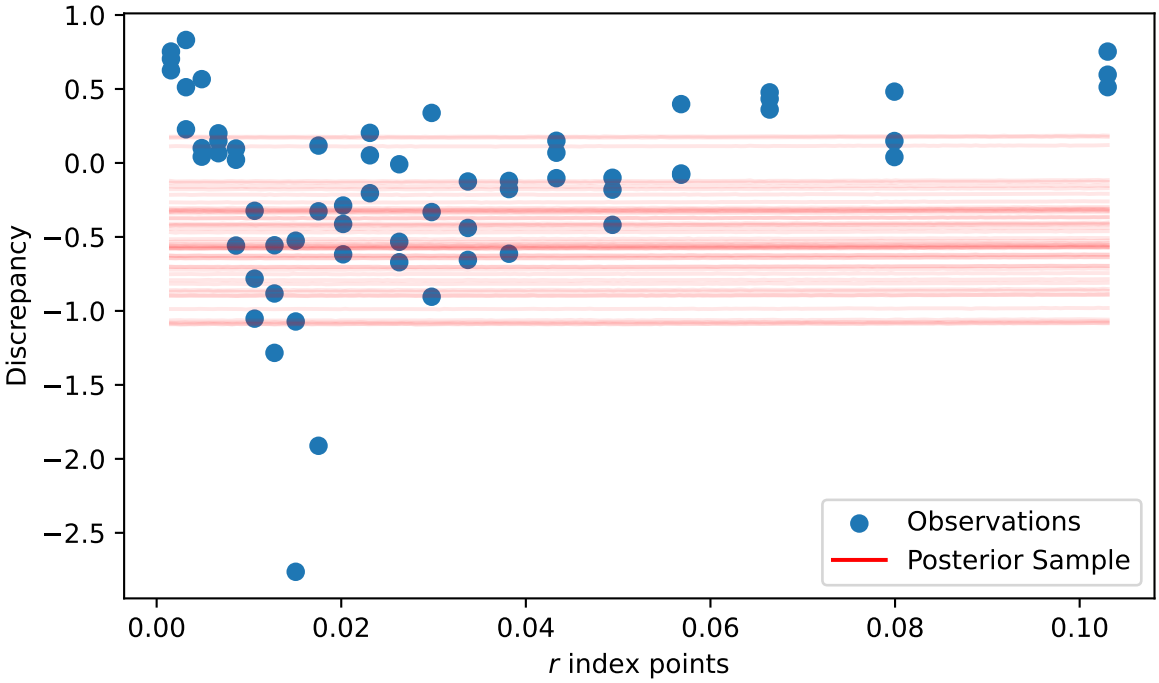
f index points
Observations
Posterior Sample


Discrepancy
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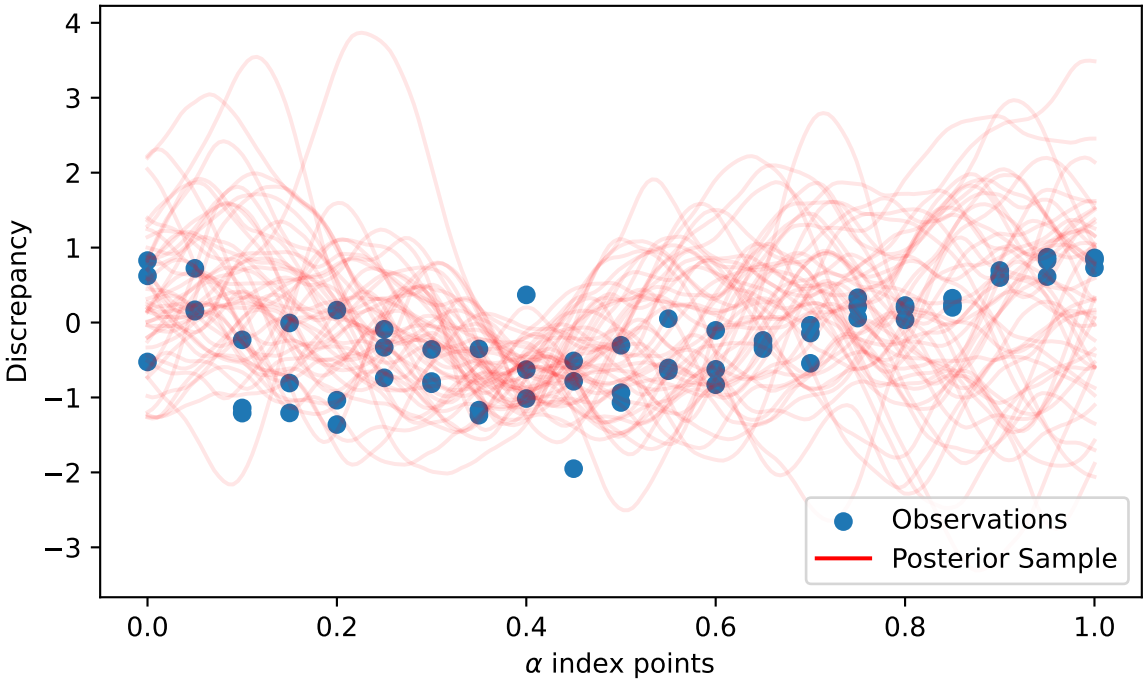
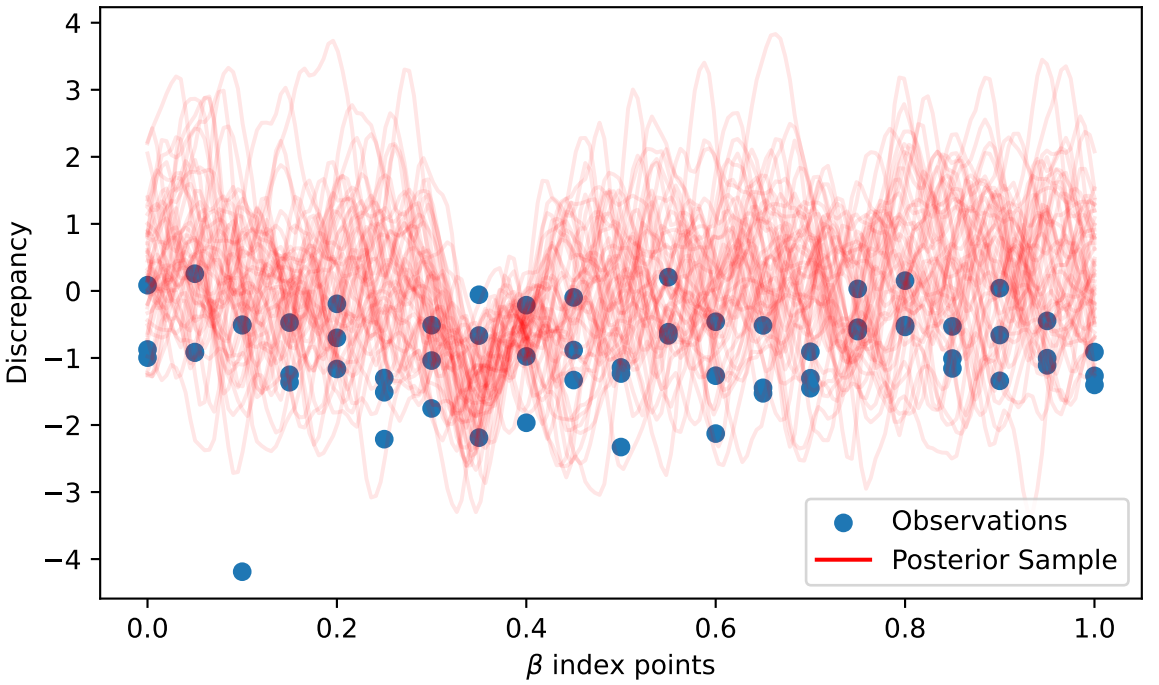
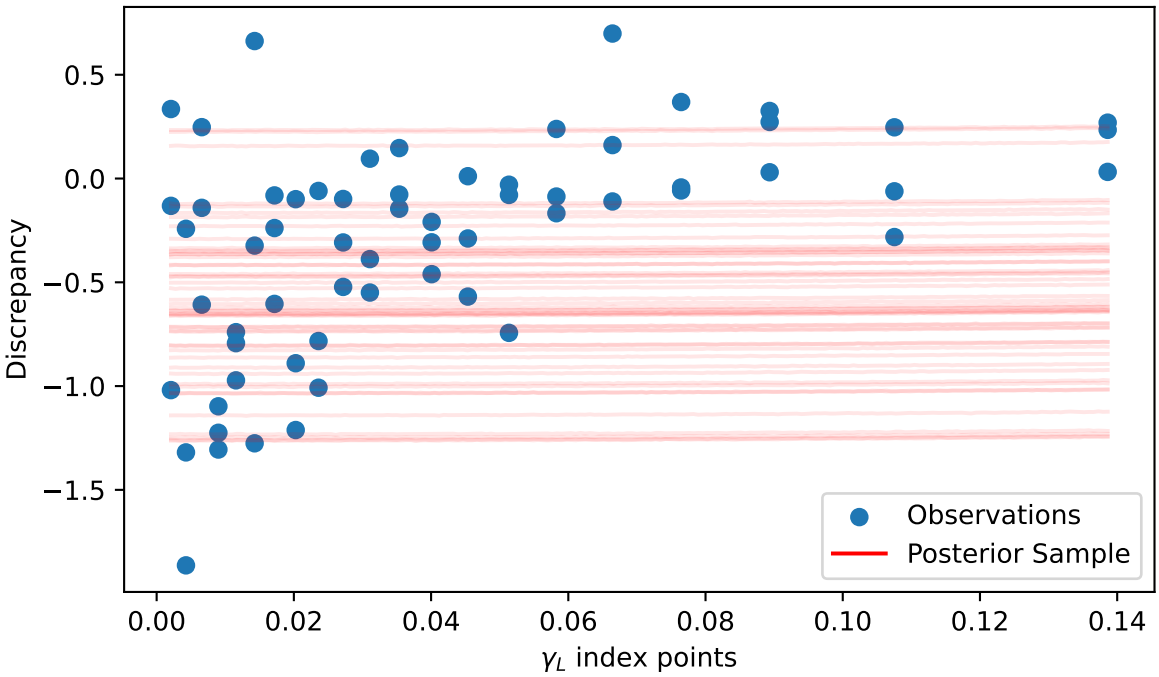
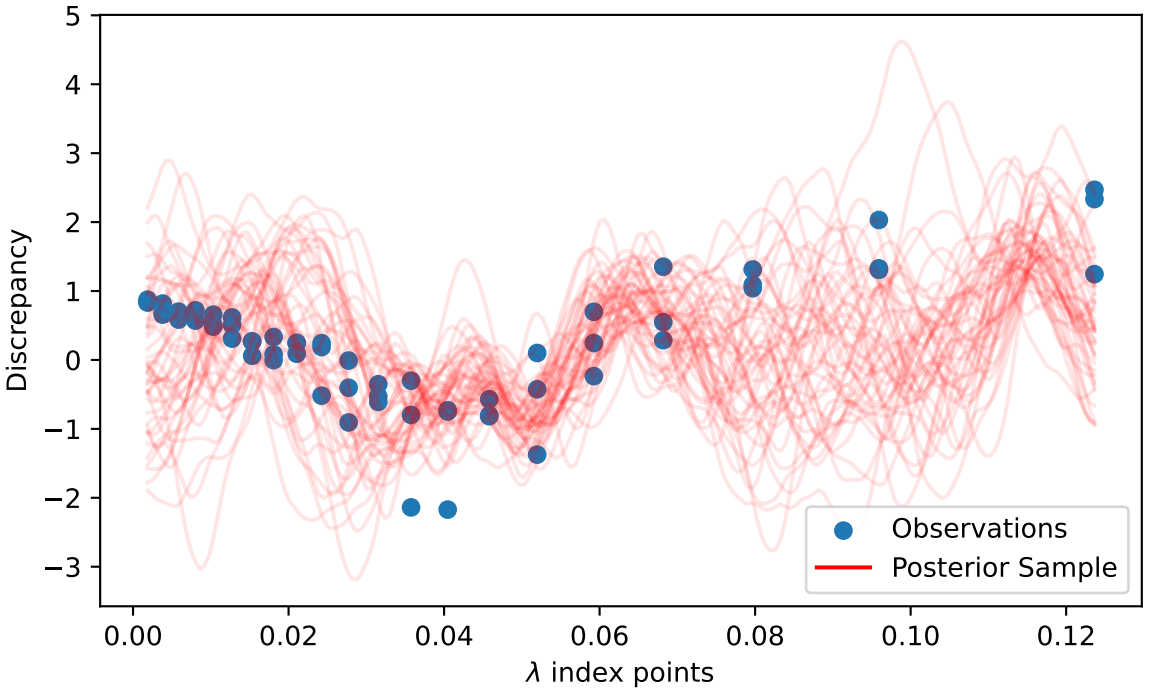
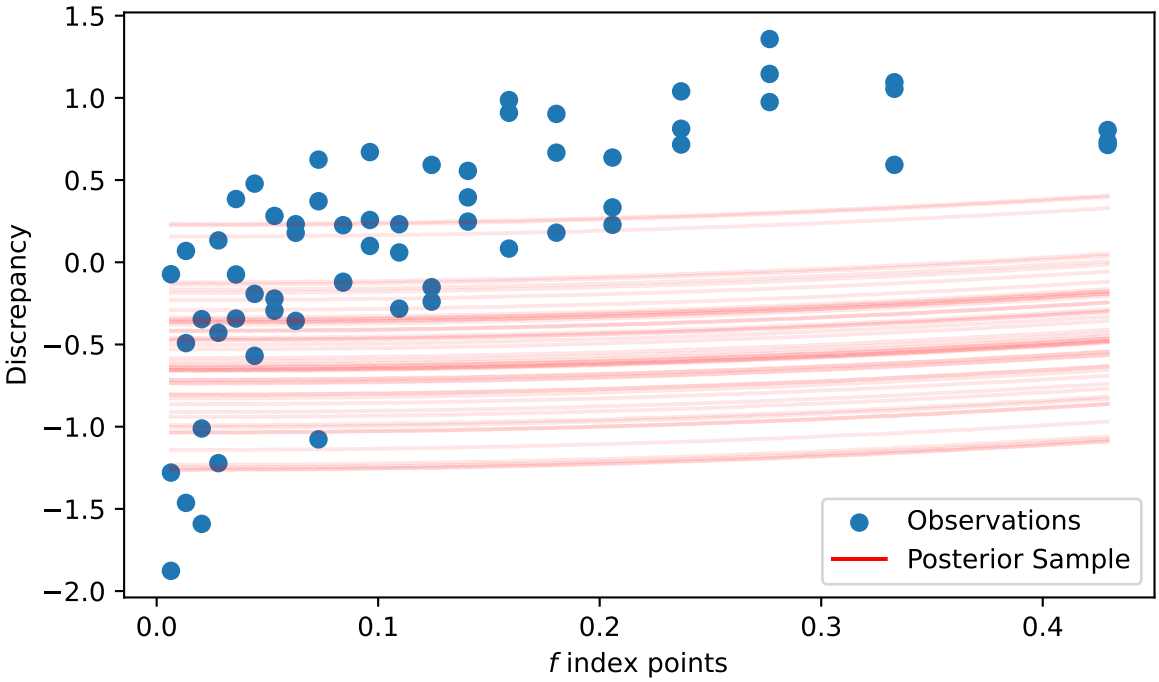
r index points
Observations
Posterior Sample

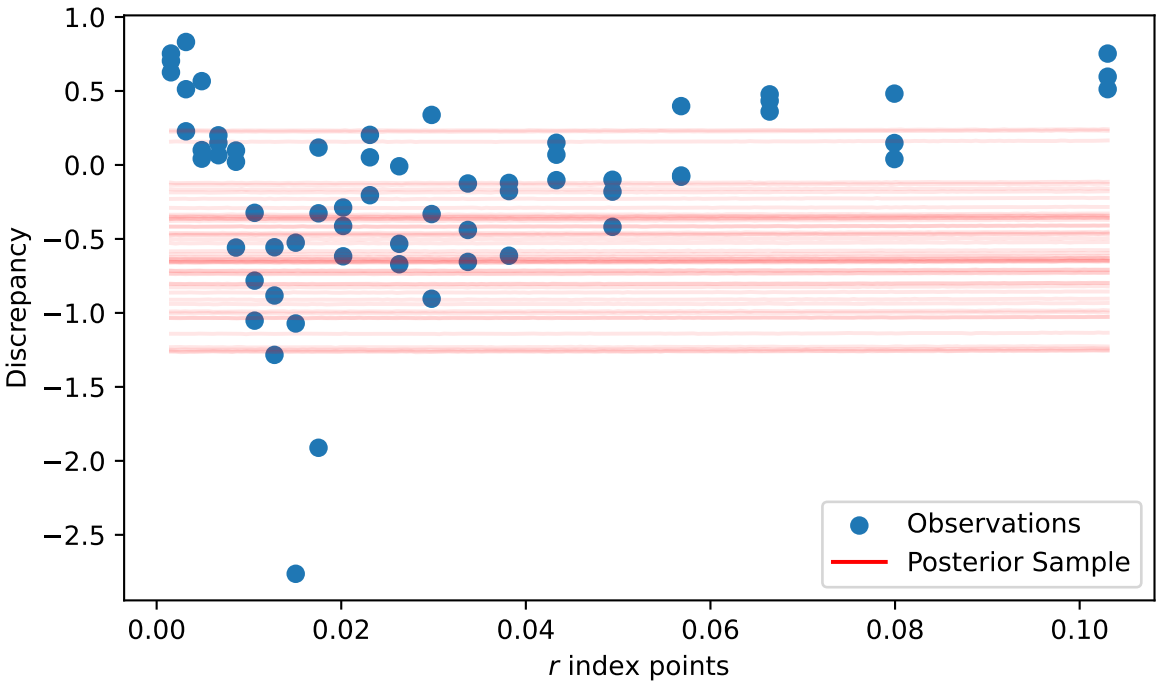

Discrepancy
4
3
2
1
0
−1
−2
−3
0.0
0.2
0.4
0.6
0.8
1.0
α index points
Observations
Posterior Sample


Discrepancy
4
3
2
1
0
−1
−2
−3
−4
0.0
0.2
0.4
0.6
0.8
1.0
β index points
Observations
Posterior Sample


Discrepancy
0.5
0.0
−0.5
−1.0
−1.5
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
γ_L index points
Observations
Posterior Sample


Discrepancy
λ index points
Observations
Posterior Sample


Discrepancy
f index points
Observations
Posterior Sample


1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
Discrepancy
0.00
0.02
0.04
0.06
0.08
0.10
r index points
Observations
Posterior Sample