# Inference on the Champagne Model using a Gaussian Process

#### TODO

• Change outputs

# Setting up the Champagne Model

## **Imports**

```
import pandas as pd
import numpy as np
from typing import Any
import matplotlib.pyplot as plt
import multiprocessing as mp
import pickle
import random
from scipy.stats import qmc
from scipy.stats import norm
import tensorflow as tf
import tensorflow_probability as tfp
from tensorflow_probability.python.distributions import normal
tfb = tfp.bijectors
tfd = tfp.distributions
tfk = tfp.math.psd_kernels
tfp_acq = tfp.experimental.bayesopt.acquisition
```

```
gpu_devices = tf.config.experimental.list_physical_devices("GPU")
for device in gpu_devices:
   tf.config.experimental.set_memory_growth(device, True)
```

2024-06-07 11:31:57.506397: I tensorflow/core/util/port.cc:113] oneDNN custom operations are 2024-06-07 11:31:58.026668: I tensorflow/core/platform/cpu\_feature\_guard.cc:210] This Tensor To enable the following instructions: AVX2 AVX512F AVX512\_VNNI FMA, in other operations, rebraced and tensorflow/compiler/tf2tensorrt/utils/py\_utils.cc:38] TF-TRT Was 2024-06-07 11:32:03.473568: W tensorflow/core/common\_runtime/gpu/gpu\_device.cc:2251] Cannot Skipping registering GPU devices...

#### Model itself

```
np.random.seed(590154)
population = 10000
initial_infecteds = 100
epidemic_length = 1000 # not used
number_of_events = 200000
pv_champ_alpha = 0.95 * 0.13 # prop of effective care
pv_champ_beta = 0.429 # prop of radical cure
pv_champ_gamma_L = 1 / 383 # liver stage clearance rate
pv_champ_delta = 0.05 # prop of imported cases
pv_champ_lambda = 0.01 # transmission rate
pv_champ_f = 1 / 69 # relapse frequency
pv_champ_r = 1 / 60 # blood stage clearance rate
gamma_L_max = 1 / 30
lambda_max = 0.05
f_max = 1 / 20
r_max = 1 / 15
upper_bounds = np.array([1, 1, gamma_L_max, lambda_max, f_max, r_max])
num_lhc_samples = 50
initial_repeats = 1
dis_mean_n = 30
```

```
def champagne_stochastic(
    alpha_,
    beta_,
    gamma_L,
    lambda_,
    f,
    r,
    N=population,
    I_L=initial_infecteds,
    I_0=0,
    S_L=0,
    delta_=0,
    # end_time=epidemic_length,
    num_events=number_of_events,
    seed=12,
):
    np.random.seed(seed)
    if (0 > (alpha_ or beta_)) or (1 < (alpha_ or beta_)):</pre>
        return "Alpha or Beta out of bounds"
    if 0 > (gamma_L or lambda_ or f or r):
        return "Gamma, lambda, f or r out of bounds"
    t = 0
    S_0 = N - I_L - I_0 - S_L
    inc_counter = 0
    list_of_outcomes = [
        {"t": 0, "S_0": S_0, "S_L": S_L, "I_0": I_0, "I_L": I_L, "inc_counter": 0}
    ]
    prop_new = alpha_ * beta_ * f / (alpha_ * beta_ * f + gamma_L)
    i = 0
    while (i < num_events) or (t < 30):
        i += 1
        if S_0 == N:
            while t < 31:
                t += 1
                new_stages = {
                    "t": t,
                    "S_0": N,
```

```
"S L": 0,
            "I_0": 0,
            "I L": 0,
            "inc_counter": inc_counter,
        }
        list_of_outcomes.append(new_stages)
   break
S_0_{t_0} = (1 - alpha) * lambda * (I_L + I_0) / N * S_0
S_0_{t_0} = alpha_* (1 - beta_) * lambda_* (I_0 + I_L) / N * S_0
I_0_{to}S_0 = r * I_0 / N
I_0_{to}I_L = lambda_* (I_L + I_0) / N * I_0
I_L_{to}I_0 = gamma_L * I_L
I_L_{to}S_L = r * I_L
S_L_{0} = (gamma_L + (f + lambda_ * (I_0 + I_L) / N) * alpha_ * beta_) * S_L
S_L_{to}I_L = (f + lambda_* (I_0 + I_L) / N) * (1 - alpha_) * S_L
total_rate = (
   S_0_to_I_L
   + S_0_to_S_L
   + I_0_to_S_0
   + I_0_to_I_L
   + I_L_to_I_0
   + I_L_to_S_L
   + S_L_to_S_0
   + S_L_to_I_L
)
delta_t = np.random.exponential(1 / total_rate)
new_stages_prob = [
   S_0_to_I_L / total_rate,
   S_0_to_S_L / total_rate,
   I_0_to_S_0 / total_rate,
   I_0_to_I_L / total_rate,
   I_L_to_I_0 / total_rate,
   I_L_to_S_L / total_rate,
   S_L_to_S_0 / total_rate,
   S_L_to_I_L / total_rate,
t += delta t
silent_incidences = np.random.poisson(
```

```
delta_t * alpha_* * beta_* lambda_* (I_L + I_0) * S_0 / N
    + delta_t * alpha_* * (1 - beta_) * (f + lambda_* * (I_L + I_0) / N) * S_L
)
new_stages = np.random.choice(
    {
            "t": t,
            "S_0": S_0 - 1,
            "S_L": S_L,
            "I_0": I_0,
            "I_L": I_L + 1,
            "inc_counter": inc_counter + silent_incidences + 1,
       },
            "t": t,
            "S_0": S_0 - 1,
            "S_L": S_L + 1,
            "I_0": I_0,
            "I_L": I_L,
            "inc_counter": inc_counter + silent_incidences + 1,
        },
        {
            "t": t,
            "S_0": S_0 + 1,
            "S_L": S_L,
            "I_0": I_0 - 1,
            "I_L": I_L,
            "inc_counter": inc_counter + silent_incidences,
        },
            "t": t,
            "S_0": S_0,
            "S_L": S_L,
            "I_0": I_0 - 1,
            "I_L": I_L + 1,
            "inc_counter": inc_counter + silent_incidences,
        },
            "t": t,
            "S_0": S_0,
```

```
"S_L": S_L,
            "I_0": I_0 + 1,
            "I_L": I_L - 1,
            "inc_counter": inc_counter + silent_incidences,
        },
        {
            "t": t,
            "S_0": S_0,
            "S_L": S_L + 1,
            "I_0": I_0,
            "I_L": I_L - 1,
            "inc_counter": inc_counter + silent_incidences,
        },
        {
            "t": t,
            "S_0": S_0 + 1,
            "S_L": S_L - 1,
            "I_0": I_0,
            "I_L": I_L,
            "inc_counter": inc_counter
            + silent_incidences
            + np.random.binomial(1, prop_new),
        },
            "t": t,
            "S_0": S_0,
            "S_L": S_L - 1,
            "I_0": I_0,
            "I_L": I_L + 1,
            "inc_counter": inc_counter + silent_incidences + 1,
        },
   ],
   p=new_stages_prob,
)
list_of_outcomes.append(new_stages)
S_0 = new_stages["S_0"]
I_0 = new_stages["I_0"]
I_L = new_stages["I_L"]
S_L = new_stages["S_L"]
```

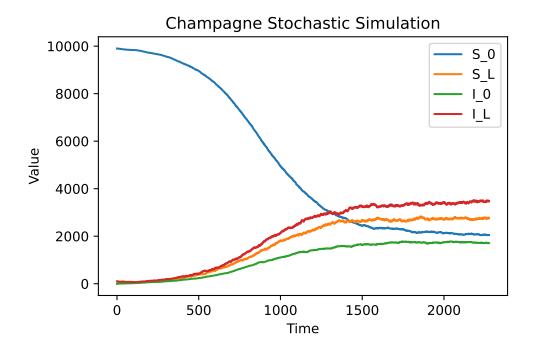
```
inc_counter = new_stages["inc_counter"]

outcome_df = pd.DataFrame(list_of_outcomes)
    return outcome_df

champ_samp = champagne_stochastic(
    pv_champ_alpha,
    pv_champ_beta,
    pv_champ_gamma_L,
    pv_champ_lambda,
    pv_champ_f,
    pv_champ_r,
)
```

# Plotting outcome

```
champ_samp.drop("inc_counter", axis=1).plot(x="t", legend=True)
plt.xlabel("Time")
plt.ylabel("Value")
plt.title("Champagne Stochastic Simulation")
plt.savefig("champagne_GP_images/champagne_simulation.pdf")
plt.show()
```



## **Function that Outputs Final Prevalence**

```
def incidence(df, start, days):
    start_ind = df[df["t"].le(start)].index[-1]
    end_ind = df[df["t"].le(start + days)].index[-1]
    incidence_week = df.iloc[end_ind]["inc_counter"] - df.iloc[start_ind]["inc_counter"]
    return incidence_week

def champ_sum_stats(alpha_, beta_, gamma_L, lambda_, f, r, seed=12301923):
    champ_df_ = champagne_stochastic(alpha_, beta_, gamma_L, lambda_, f, r, seed=seed)
    first_month_inc = incidence(champ_df_, 0, 30)
    fin_t = champ_df_.iloc[-1]["t"]
    fin_week_inc = incidence(champ_df_, fin_t - 7, 7)

first_month_ind = champ_df_[champ_df_["t"].le(30)].index[-1]
    first_month_prev = (
        champ_df_.iloc[first_month_ind]["I_0"]
        + champ_df_.iloc[first_month_ind]["I_U"]
    )
    fin_prev = champ_df_.iloc[-1]["I_0"] + champ_df_.iloc[-1]["I_L"]
```

```
return np.array([fin_week_inc, fin_prev, first_month_inc, first_month_prev])
  observed_sum_stats = champ_sum_stats(
      pv_champ_alpha,
      pv_champ_beta,
      pv_champ_gamma_L,
      pv_champ_lambda,
      pv_champ_f,
      pv_champ_r,
  )
  print(observed_sum_stats)
  def single_discrepency(alpha_, beta_, gamma_L, lambda_, f, r, seed=12301923):
      x = champ_sum_stats(alpha_, beta_, gamma_L, lambda_, f, r, seed=seed)
      return np.log(np.linalg.norm((x - observed_sum_stats) / observed_sum_stats))
  def discrepency_fn(alpha_, beta_, gamma_L, lambda_, f, r, mean_of=dis_mean_n):
      seed = int(np.random.uniform() * 1000000)
      with mp.Pool(processes=mp.cpu_count()) as pool:
          args = [
              (alpha_, beta_, gamma_L, lambda_, f, r, seed * i) for i in range(mean_of)
          results = pool.starmap(single_discrepency, args)
      mean_obs = np.mean(results)
      return mean_obs
[ 461. 5205. 42. 87.]
```

# Gaussian Process Regression on Final Prevalence Discrepency

```
my_seed = np.random.default_rng(seed=1795) # For replicability
variables_names = ["alpha", "beta", "gamma_L", "lambda", "f", "r"]
LHC_sampler = qmc.LatinHypercube(d=6, seed=my_seed)
```

```
LHC_samples = LHC_sampler.random(n=num_lhc_samples)
  # Using Champagne Initialisation table 2
  LHC_samples[:, 2] = gamma_L_max * LHC_samples[:, 2]
  LHC_samples[:, 3] = lambda_max * LHC_samples[:, 3]
  LHC_samples[:, 4] = f_max * LHC_samples[:, 4]
  LHC samples[:, 5] = r max * LHC samples[:, 5]
  LHC_samples = np.repeat(LHC_samples, initial_repeats, axis = 0)
  LHC_indices_df = pd.DataFrame(LHC_samples, columns=variables_names)
  print(LHC_indices_df.head())
      alpha
                                   lambda
                                                  f
                 beta gamma_L
0 0.100008 0.122349 0.009668 0.015376 0.016920 0.015954
1 \quad 0.659225 \quad 0.590955 \quad 0.001070 \quad 0.038947 \quad 0.007433 \quad 0.003318
2 0.503558 0.005003 0.031832 0.027053 0.002028 0.019736
3 0.011840 0.630562 0.023631 0.033488 0.035622 0.035127
4 0.271011 0.942434 0.014052 0.030138 0.031133 0.051736
```

#### **Generate Discrepencies**

 $\begin{bmatrix} -0.72112073 & 0.85173495 & -0.10445358 & 1.04268897 & 0.56652755 & 0.46286618 \end{bmatrix}$ 

### **Differing Methods to Iterate Function**

```
# import timeit
# def function1():
     np.vectorize(champ sum stats)(random indices df['alpha'],
     random indices df['beta'], random indices df['gamma L'],
     random_indices_df['lambda'], random_indices_df['f'], random_indices_df['r'])
     pass
# def function2():
     random_indices_df.apply(
          lambda x: champ_sum_stats(
              x['alpha'], x['beta'], x['gamma_L'], x['lambda'], x['f'], x['r']),
              axis = 1)
#
     pass
# # Time function1
# time_taken_function1 = timeit.timeit(
      "function1()", globals=globals(), number=100)
# # Time function2
# time taken function2 = timeit.timeit(
      "function2()", globals=globals(), number=100)
# print("Time taken for function1:", time_taken_function1)
# print("Time taken for function2:", time_taken_function2)
```

Time taken for function1: 187.48960775700016 Time taken for function2: 204.06618941299985

#### Constrain Variables to be Positive

```
constrain_positive = tfb.Shift(np.finfo(np.float64).tiny)(tfb.Exp())
```

#### **Custom Quadratic Mean Function**

```
class quad_mean_fn(tf.Module):
    def __init__(self):
        super(quad_mean_fn, self).__init__()
        # self.amp_alpha_mean = tfp.util.TransformedVariable(
              bijector=constrain_positive,
              initial value=1.0,
              dtype=np.float64,
        #
              name="amp alpha mean",
        # )
        # self.alpha_tp = tf.Variable(pv_champ_alpha, dtype=np.float64, name="alpha_tp")
        # self.amp_beta_mean = tfp.util.TransformedVariable(
              bijector=constrain_positive,
              initial_value=0.5,
              dtype=np.float64,
        #
              name="amp_beta_mean",
        # )
        # self.beta_tp = tf.Variable(pv_champ_beta, dtype=np.float64, name="beta_tp")
        self.amp_gamma_L_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_gamma_L_mean",
        )
        # self.gamma_L_tp = tfp.util.TransformedVariable(
              bijector=constrain_positive,
              initial_value=1.0,
              dtype=np.float64,
              name="gamma_L_tp",
        self.amp_lambda_mean = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_lambda_mean",
```

```
# self.lambda_tp = tfp.util.TransformedVariable(
          bijector=constrain_positive,
          initial_value=1.0,
    #
          dtype=np.float64,
    #
          name="lambda_tp",
    # )
    self.amp_f_mean = tfp.util.TransformedVariable(
        bijector=constrain_positive,
        initial_value=1.0,
        dtype=np.float64,
        name="amp_f_mean",
    # self.f_tp = tfp.util.TransformedVariable(
          bijector=constrain_positive,
          initial_value=1.0,
          dtype=np.float64,
    #
         name="f_tp",
    # )
    self.amp_r_mean = tfp.util.TransformedVariable(
        bijector=constrain_positive,
        initial value=1.0,
        dtype=np.float64,
        name="amp_r_mean",
    )
    # self.r_tp = tfp.util.TransformedVariable(
          bijector=constrain_positive,
    #
          initial_value=1.0,
          dtype=np.float64,
          name="r_tp",
    # )
    # self.bias_mean = tfp.util.TransformedVariable(
          bijector=constrain_positive,
    #
          initial_value=1.0,
    #
          dtype=np.float64,
          name="bias_mean",
    # )
    self.bias_mean = tf.Variable(-1.5, dtype=np.float64, name="bias_mean")
def __call__(self, x):
    return (
```

```
self.bias_mean
# + self.amp_alpha_mean * (x[..., 0] - self.alpha_tp) ** 2
# + self.amp_beta_mean * (x[..., 1] - self.beta_tp) ** 2
# + self.amp_gamma_L_mean * (x[..., 2] - self.gamma_L_tp) ** 2
# + self.amp_lambda_mean * (x[..., 3] - self.lambda_tp) ** 2
# + self.amp_f_mean * (x[..., 4] - self.f_tp) ** 2
# + self.amp_r_mean * (x[..., 5] - self.r_tp) ** 2
+ self.amp_gamma_L_mean * (x[..., 2]) ** 2
+ self.amp_lambda_mean * (x[..., 3]) ** 2
+ self.amp_f_mean * (x[..., 4]) ** 2
+ self.amp_r_mean * (x[..., 5]) ** 2
)

quad_mean_fn().__call__(x=np.array([[1.0, 1.0, 1.0, 1.0, 1.0, 1.0]])) # should return 1
```

<tf.Tensor: shape=(1,), dtype=float64, numpy=array([2.5])>

#### **Custom Linear Mean Function**

```
class lin_mean_fn(tf.Module):
    def __init__(self):
        super(lin_mean_fn, self).__init__()
        # self.amp_alpha_lin = tfp.util.TransformedVariable(
              bijector=constrain_positive,
        #
              initial_value=1.0,
              dtype=np.float64,
        #
              name="amp_alpha_lin",
        # )
        # self.amp_beta_lin = tfp.util.TransformedVariable(
              bijector=constrain_positive,
        #
              initial_value=0.5,
              dtype=np.float64,
        #
              name="amp_beta_lin",
        # )
        self.amp_gamma_L_lin = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_gamma_L_lin",
```

```
self.amp_lambda_lin = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_lambda_lin",
        self.amp_f_lin = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_f_lin",
        self.amp_r_lin = tfp.util.TransformedVariable(
            bijector=constrain_positive,
            initial_value=1.0,
            dtype=np.float64,
            name="amp_r_lin",
        )
        # self.bias_lin = tfp.util.TransformedVariable(
              bijector=constrain_positive,
              initial value=1.0,
              dtype=np.float64,
              name="bias_lin",
        # )
        self.bias_lin = tf.Variable(0.0, dtype=np.float64, name="bias_mean")
    def __call__(self, x):
        return (
            self.bias_lin
            \# + self.amp_alpha_lin * (x[..., 0])
            # + self.amp_beta_lin * (x[..., 1])
            + self.amp_gamma_L_lin * (x[..., 2])
            + self.amp_lambda_lin * (x[..., 3])
            + self.amp_f_lin * (x[..., 4])
            + self.amp_r_lin * (x[..., 5])
        )
class const_mean_fn(tf.Module):
    def __init__(self):
        super(const_mean_fn, self).__init__()
```

```
self.bias_lin = tf.Variable(0.0, dtype=np.float64, name="bias_mean")
def __call__(self, x):
    return self.bias_lin
```

#### Making the ARD Kernel

```
index_vals = LHC_indices_df.values
obs_vals = random_discrepencies
amplitude_champ = tfp.util.TransformedVariable(
    bijector=constrain_positive,
    initial_value=4.0,
    dtype=np.float64,
    name="amplitude_champ",
)
observation_noise_variance_champ = tfp.util.TransformedVariable(
    bijector=constrain_positive,
    initial_value=1.,
    dtype=np.float64,
    name="observation_noise_variance_champ",
)
length_scales_champ = tfp.util.TransformedVariable(
    bijector=tfb.Sigmoid(
        np.float64(0.0),
        [1.0 / 2, 1.0 / 2, gamma_L_max / 2, lambda_max / 2, f_max / 2, r_max / 2],
    ),
    initial_value=[1 / 8, 1 / 8, gamma_L_max / 8, lambda_max / 8, f_max / 8, r_max / 8],
    dtype=np.float64,
    name="length_scales_champ",
)
kernel_champ = tfk.FeatureScaled(
    tfk.MaternFiveHalves(amplitude=amplitude_champ),
    scale_diag=length_scales_champ,
)
```

#### Define the Gaussian Process with Quadratic Mean Function and ARD Kernel

#### Train the Hyperparameters

#### Leave One Out Predictive Log-likelihood

```
# predictive log stuff
@tf.function(autograph=False, jit_compile=False)
def optimize():
    with tf.GradientTape() as tape:
        K = (
            champ_GP.kernel.matrix(index_vals, index_vals)
            + tf.eye(index_vals.shape[0], dtype=np.float64)
            * observation_noise_variance_champ
        )
        means = champ_GP.mean_fn(index_vals)
        K_inv = tf.linalg.inv(K)
        K_inv_y = K_inv @ tf.reshape(obs_vals - means, shape=[obs_vals.shape[0], 1])
        K_inv_diag = tf.linalg.diag_part(K_inv)
        log_var = tf.math.log(K_inv_diag)
        log_mu = tf.reshape(K_inv_y, shape=[-1]) ** 2
        loss = -tf.math.reduce_sum(log_var - log_mu)
    grads = tape.gradient(loss, champ_GP.trainable_variables)
    Adam_optim.apply_gradients(zip(grads, champ_GP.trainable_variables))
```

```
return loss

num_iters = 10000

lls_ = np.zeros(num_iters, np.float64)
tolerance = 1e-6  # Set your desired tolerance level
previous_loss = float("inf")

for i in range(num_iters):
    loss = optimize()
    lls_[i] = loss

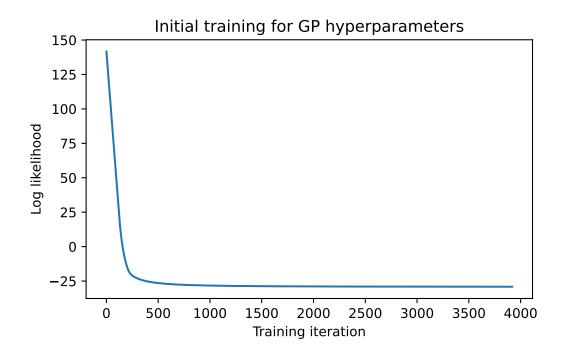
# Check if change in loss is less than tolerance
if abs(loss - previous_loss) < tolerance:
    print(f"Hyperparameter convergence reached at iteration {i+1}.")
    lls_ = lls_[range(i + 1)]
    break

previous_loss = loss</pre>
```

Hyperparameter convergence reached at iteration 3920.

```
print("Trained parameters:")
for var in champ_GP.trainable_variables:
    if "bias" in var.name:
        print("{} is {}\n".format(var.name, var.numpy().round(3)))
    else:
        if "length" in var.name:
            print(
                "{} is {}\n".format(
                    var.name,
                    tfb.Sigmoid(
                        np.float64(0.0),
                             1.0 / 2,
                             1.0 / 2,
                             gamma_L_max / 2,
                             lambda_max / 2,
                             f_max / 2,
                             r_max / 2,
```

```
],
                    .forward(var)
                    .numpy()
                    .round(3),
                )
             )
         else:
             print(
                "{} is {}\n".format(
                    var.name, constrain_positive.forward(var).numpy().round(3)
                )
             )
  initial_losses_LOOCV = lls_
Trained parameters:
amplitude_champ:0 is 0.729
observation_noise_variance_champ:0 is 0.001
bias_mean:0 is 0.595
  plt.figure(figsize=(6, 3.5))
  plt.plot(lls_)
  plt.title("Initial training for GP hyperparameters")
  plt.xlabel("Training iteration")
  plt.ylabel("Log likelihood")
  plt.savefig("champagne_GP_images/hyperparam_loss_log_discrep.pdf")
  plt.show()
```



# Creating slices across one variable dimension

```
plot_samp_no = 21
plot_samp_times = 10
plot_gp_no = 100
gp_samp_no = 30
slice_samples_dict = {
    "alpha_slice_samples": np.repeat(np.concatenate(
            np.linspace(0, 1, plot_samp_no, dtype=np.float64).reshape(-1, 1), # alpha
            np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1), # beta
            np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1),
            np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1), # lambda
            np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1), # f
            np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1),
        ),
        axis=1,
    ), plot_samp_times, axis = 0),
    "alpha_gp_samples": np.concatenate(
```

```
(
       np.linspace(0, 1, plot_gp_no, dtype=np.float64).reshape(-1, 1), # alpha
       np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1), # beta
       np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1), # gamma_L
       np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1), # lambda
       np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1), # f
       np.repeat(pv champ r, plot gp no).reshape(-1, 1), # r
   ),
   axis=1,
),
"beta_slice_samples": np.repeat(np.concatenate(
    (
       np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1), # alpha
       np.linspace(0, 1, plot_samp_no, dtype=np.float64).reshape(-1, 1), # beta
       np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1), # gamma_L
       np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1), # lambda
       np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1), # f
       np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1), # r
   ),
   axis=1,
), plot_samp_times, axis = 0),
"beta gp samples": np.concatenate(
    (
       np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1), # alpha
       np.linspace(0, 1, plot_gp_no, dtype=np.float64).reshape(-1, 1), # beta
       np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1), # gamma_L
       np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1), # lambda
       np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1), # f
       np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1), # r
   ),
   axis=1,
"gamma_L_slice_samples": np.repeat(np.concatenate(
    (
       np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1), # alpha
       np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1), # beta
       np.linspace(0, gamma_L_max, plot_samp_no, dtype=np.float64).reshape(-1, 1), #
       np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1), # lambda
       np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1), # f
       np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1), # r
   ),
```

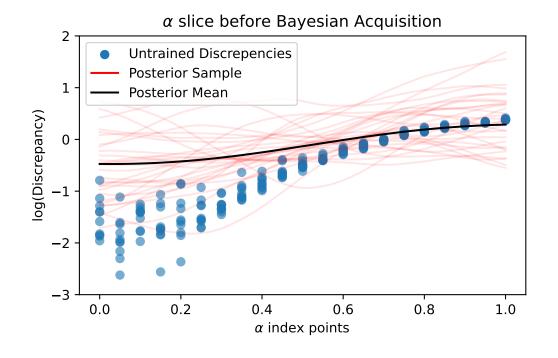
```
axis=1,
), plot_samp_times, axis = 0),
"gamma_L_gp_samples": np.concatenate(
       np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1), # alpha
       np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1), # beta
       np.linspace(0, gamma_L_max, plot_gp_no, dtype=np.float64).reshape(-1, 1), # g
       np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1),
                                                               # lambda
       np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1), # f
       np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1), # r
   ),
   axis=1.
),
"lambda slice samples": np.repeat(np.concatenate(
       np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1), # alpha
       np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1), # beta
       np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1), # gamma_L
       np.linspace(0, lambda_max, plot_samp_no, dtype=np.float64).reshape(-1, 1), # 1
       np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1), # f
       np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1), # r
   ),
   axis=1,
), plot_samp_times, axis = 0),
"lambda_gp_samples": np.concatenate(
    (
       np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1), # alpha
       np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1), # beta
       np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1), # gamma_L
       np.linspace(0, lambda_max, plot_gp_no, dtype=np.float64).reshape(-1, 1), # lam
       np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1), # f
       np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1), # r
   ),
   axis=1,
"f_slice_samples": np.repeat(np.concatenate(
    (
       np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1), # alpha
       np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1), # beta
       np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1), # gamma_L
       np.repeat(pv_champ_lambda, plot_samp_no).reshape(-1, 1), # lambda
```

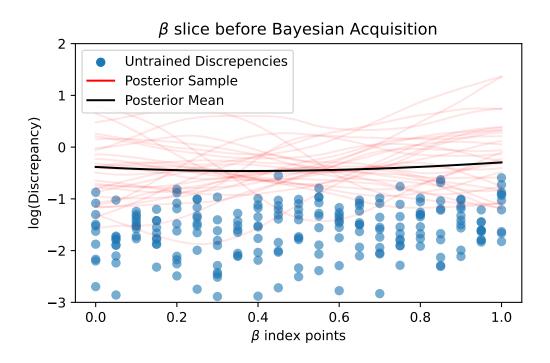
```
np.linspace(0, f_max, plot_samp_no, dtype=np.float64).reshape(-1, 1), # f
           np.repeat(pv_champ_r, plot_samp_no).reshape(-1, 1), # r
       ),
       axis=1,
    ), plot_samp_times, axis = 0),
    "f gp samples": np.concatenate(
           np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1), # alpha
           np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1), # beta
           np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1), # gamma_L
           np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1), # lambda
           np.linspace(0, f_max, plot_gp_no, dtype=np.float64).reshape(-1, 1), # f
           np.repeat(pv_champ_r, plot_gp_no).reshape(-1, 1), # r
       ),
       axis=1,
    "r_slice_samples": np.repeat(np.concatenate(
           np.repeat(pv_champ_alpha, plot_samp_no).reshape(-1, 1), # alpha
           np.repeat(pv_champ_beta, plot_samp_no).reshape(-1, 1), # beta
           np.repeat(pv_champ_gamma_L, plot_samp_no).reshape(-1, 1), # gamma_L
           np.repeat(pv champ lambda, plot samp no).reshape(-1, 1), # lambda
           np.repeat(pv_champ_f, plot_samp_no).reshape(-1, 1), # f
           np.linspace(0, r_max, plot_samp_no, dtype=np.float64).reshape(-1, 1), # r
       ),
       axis=1,
    ), plot_samp_times, axis = 0),
    "r_gp_samples": np.concatenate(
        (
           np.repeat(pv_champ_alpha, plot_gp_no).reshape(-1, 1), # alpha
           np.repeat(pv_champ_beta, plot_gp_no).reshape(-1, 1), # beta
           np.repeat(pv_champ_gamma_L, plot_gp_no).reshape(-1, 1), # gamma_L
           np.repeat(pv_champ_lambda, plot_gp_no).reshape(-1, 1), # lambda
           np.repeat(pv_champ_f, plot_gp_no).reshape(-1, 1), # f
           np.linspace(0, r_max, plot_gp_no, dtype=np.float64).reshape(-1, 1), # r
       ),
       axis=1,
    ),
}
```

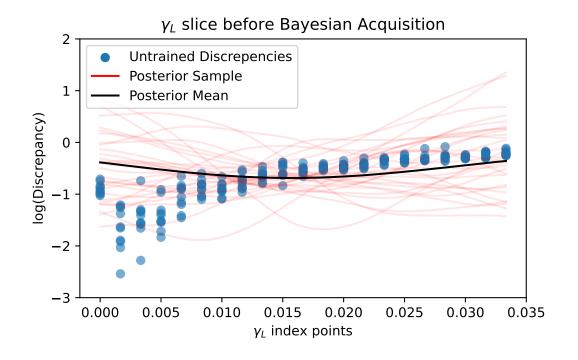
# Plotting the GPs across different slices

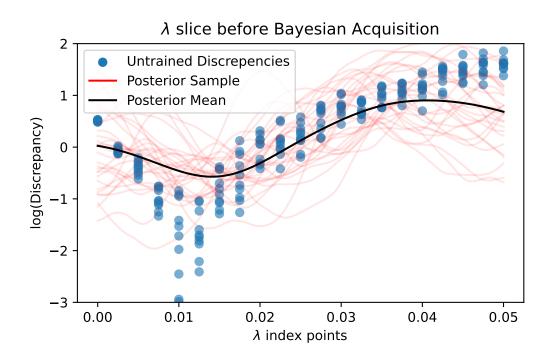
```
GP seed = tfp.random.sanitize seed(4362)
vars = ["alpha", "beta", "gamma_L", "lambda", "f", "r"]
slice_indices_dfs_dict = {}
slice_index_vals_dict = {}
slice_discrepencies_dict = {}
gp_samples_dict = {}
for var in vars:
    val df = pd.DataFrame(
        slice_samples_dict[var + "_slice_samples"], columns=variables_names
    slice_indices_dfs_dict[var + "_slice_indices_df"] = val_df
    slice_index_vals_dict[var + "_slice_index_vals"] = val_df.values
    df_temp = val_df.assign(seed=range(val_df.shape[0]))
    seed = int(np.random.uniform() * 1000000)
    with mp.Pool(processes=mp.cpu_count()) as pool:
        args = list(df_temp.itertuples(index=False, name=None))
        results = pool.starmap(single_discrepency, args)
    discreps = results
    slice_discrepencies_dict[var + "_slice_discrepencies"] = discreps
    gp_samples_df = pd.DataFrame(
        slice_samples_dict[var + "_gp_samples"], columns=variables_names
    slice_indices_dfs_dict[var + "_gp_indices_df"] = gp_samples_df
    slice_index_vals_dict[var + "_gp_index_vals"] = gp_samples_df.values
    champ_GP_reg_plot = tfd.GaussianProcessRegressionModel(
        kernel=kernel champ,
        index_points=gp_samples_df.values,
        observation_index_points=index_vals,
        observations=obs_vals,
        observation_noise_variance=observation_noise_variance_champ,
        predictive_noise_variance=0.0,
        mean_fn=const_mean_fn(),
    GP_samples = champ_GP_reg_plot.sample(gp_samp_no, seed=GP_seed)
```

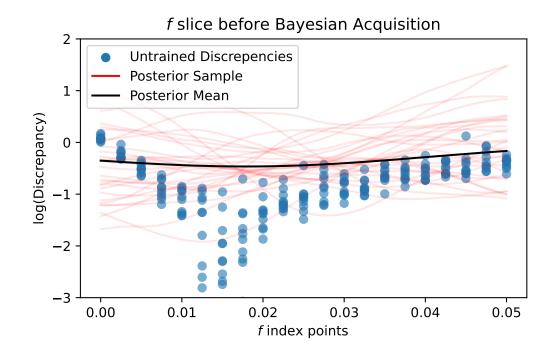
```
gp_samples_dict[var + "initial_gp_samps"] = GP_samples
plt.figure(figsize=(6, 3.5))
plt.scatter(
    val_df[var].values,
    discreps,
    label="Untrained Discrepencies",
    alpha=0.6,
)
for i in range(gp_samp_no):
    plt.plot(
        gp_samples_df[var].values,
        GP_samples[i, :],
        c="r",
        alpha=0.1,
        label="Posterior Sample" if i == 0 else None,
plt.plot(
    slice_indices_dfs_dict[var + "_gp_indices_df"][var].values,
    champ_GP_reg_plot.mean_fn(
        slice_indices_dfs_dict[var + "_gp_indices_df"].values
    ),
    c="black",
    alpha=1,
    label="Posterior Mean",
)
leg = plt.legend(loc="upper left")
for lh in leg.legend_handles:
    lh.set_alpha(1)
if var in ["f", "r"]:
    plt.xlabel("$" + var + "$ index points")
    plt.title("$" + var + "$ slice before Bayesian Acquisition")
else:
    plt.xlabel("$\\" + var + "$ index points")
    plt.title("$\\" + var + "$ slice before Bayesian Acquisition")
# if var not in ["alpha", "beta"]:
     plt.xscale("log", base=np.e)
plt.ylabel("log(Discrepancy)")
plt.ylim((-3, 2))
plt.savefig("champagne_GP_images/initial_" + var + "_slice_log_discrep.pdf")
plt.show()
```

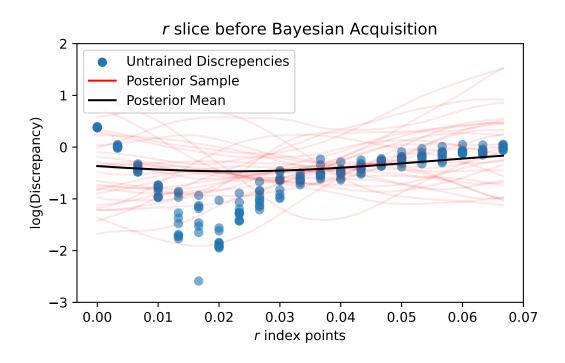












# Acquiring the next datapoint to test

#### Proof that .variance returns what we need in acquisition function

```
champ_GP_reg = tfd.GaussianProcessRegressionModel(
      kernel=kernel_champ,
      observation_index_points=index_vals,
      observations=obs_vals,
      observation_noise_variance=observation_noise_variance_champ,
      mean_fn=const_mean_fn(),
  )
  new guess = np.array([0.4, 0.4, 0.004, 0.04, 0.01, 0.17])
  mean_t = champ_GP_reg.mean_fn(new_guess)
  variance_t = champ_GP_reg.variance(index_points=[new_guess])
  kernel_self = kernel_champ.apply(new_guess, new_guess)
  kernel others = kernel champ.apply(new guess, index vals)
  K = kernel_champ.matrix(
      index_vals, index_vals
  ) + observation_noise_variance_champ * np.identity(index_vals.shape[0])
  inv_K = np.linalg.inv(K)
  print("Self Kernel is {}".format(kernel_self.numpy().round(3)))
  print("Others Kernel is {}".format(kernel_others.numpy().round(3)))
  print(inv_K)
  my_var_t = kernel_self - kernel_others.numpy() @ inv K @ kernel_others.numpy()
  print("Variance function is {}".format(variance_t.numpy().round(3)))
  print("Variance function is {}".format(my_var_t.numpy().round(3)))
Self Kernel is 0.532
Others Kernel is [0.
                            0.
                                  0.001 0.002 0.005 0.
                                                         0.001 0.
                                                                          0.002 0.
                      0.
                                                                    0.
                                        0.001 0.001 0.001 0.006 0.001
0.
     0.001 0.004 0.004 0.
                            0.007 0.
0.
      0.001 0.003 0.
                                              0.
                       0.
                             0.
                                   0.
                                         0.
                                                    0.
                                                          0.
                                                                0.
0.
      0.
           Ο.
                0.006 0.006 0.
                                   0.
                                         0.
                                              0.
                                                    0.003 0.
                                                                0.001
0.
      0.
           ]
[[ 2.43907699e+00 9.04941335e-03 -1.95938089e-02 ... -6.70431227e-02
  1.05179552e-03 -5.63875095e-01]
 -5.55692566e-01 2.17876770e-03]
 [-1.95938089e-02 \ 1.72781117e-02 \ 2.24292081e+00 \ \dots \ -3.87518498e-01
```

```
1.19500590e-02 -3.92962545e-01]
...
[-6.70431227e-02 1.04767362e-03 -3.87518498e-01 ... 2.33092109e+00
-5.42814807e-04 1.22896549e-02]
[ 1.05179552e-03 -5.55692566e-01 1.19500590e-02 ... -5.42814807e-04
    2.15512928e+00 5.89726568e-03]
[-5.63875095e-01 2.17876770e-03 -3.92962545e-01 ... 1.22896549e-02
    5.89726568e-03 2.66090472e+00]]
Variance function is [0.532]
Variance function is 0.532
```

#### Loss function

```
next_alpha = tfp.util.TransformedVariable(
    initial_value=0.5,
    bijector=tfb.Sigmoid(),
    dtype=np.float64,
    name="next_alpha",
)
next_beta = tfp.util.TransformedVariable(
    initial_value=0.5,
    bijector=tfb.Sigmoid(),
    dtype=np.float64,
    name="next_beta",
)
next_gamma_L = tfp.util.TransformedVariable(
    initial_value=gamma_L_max/2,
    bijector=tfb.Sigmoid(np.float64(0.), gamma_L_max),
    dtype=np.float64,
    name="next_gamma_L",
)
next_lambda = tfp.util.TransformedVariable(
    initial_value=lambda_max/2,
    bijector=tfb.Sigmoid(np.float64(0.), lambda_max),
    dtype=np.float64,
    name="next_lambda",
)
```

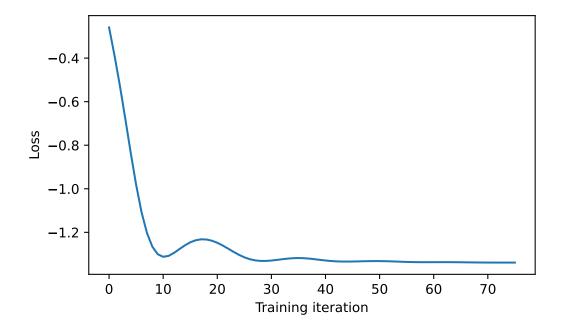
```
next_f = tfp.util.TransformedVariable(
      initial_value=f_max/2,
      bijector=tfb.Sigmoid(np.float64(0.), f_max),
      dtype=np.float64,
      name="next_f",
  )
  next_r = tfp.util.TransformedVariable(
      initial_value=r_max/2,
      bijector=tfb.Sigmoid(np.float64(0.), r_max),
      dtype=np.float64,
      name="next_r",
  )
  next_vars = (
      (next_alpha.trainable_variables[0],
      next_beta.trainable_variables[0],
      next_gamma_L.trainable_variables[0],
      next_lambda.trainable_variables[0],
      next_f.trainable_variables[0],
      next_r.trainable_variables[0],)
  )
  next_vars
(<tf.Variable 'next_alpha:0' shape=() dtype=float64, numpy=0.0>,
<tf.Variable 'next_beta:0' shape=() dtype=float64, numpy=0.0>,
<tf.Variable 'next_gamma_L:0' shape=() dtype=float64, numpy=0.0>,
<tf.Variable 'next_lambda:0' shape=() dtype=float64, numpy=0.0>,
<tf.Variable 'next_f:0' shape=() dtype=float64, numpy=0.0>,
<tf.Variable 'next_r:0' shape=() dtype=float64, numpy=0.0>)
  eta_t = tf.constant(1.0, dtype=np.float64)
  def UCB_loss(champ_GP_reg):
      next_guess = tf.reshape(
          tf.stack([next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]),
          [1, 6],
      mean_t = champ_GP_reg.mean_fn(next_guess)
      std_t = tf.math.sqrt(
```

```
champ_GP_reg.variance(index_points=next_guess)
          - observation_noise_variance_champ
      return tf.squeeze(mean_t - std_t)
  optimizer fast = tf.keras.optimizers.Adam(learning rate=0.1)
  @tf.function(autograph=False, jit_compile=False)
  def opt_var():
      with tf.GradientTape() as tape:
          loss = UCB_loss(champ_GP_reg)
      grads = tape.gradient(loss, next_vars)
      optimizer_fast.apply_gradients(zip(grads, next_vars))
      return loss
  num_iters = 10000
  lls_ = np.zeros(num_iters, np.float64)
  tolerance = 1e-6 # Set your desired tolerance level
  previous_loss = float("inf")
  for i in range(num_iters):
      loss = opt_var()
      lls_[i] = loss
      # Check if change in loss is less than tolerance
      if abs(loss - previous_loss) < tolerance:</pre>
          print(f"Acquisition function convergence reached at iteration {i+1}.")
          lls_= lls_[range(i + 1)]
          break
      previous_loss = loss
  print("Trained parameters:")
  for var in [next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]:
      print("{} is {}".format(var.name, (var.bijector.forward(var).numpy().round(3))))
Acquisition function convergence reached at iteration 76.
Trained parameters:
next_alpha is 0.541
next_beta is 0.632
```

```
next_lambda is 0.025
next_f is 0.025
next_r is 0.034

plt.figure(figsize=(6, 3.5))
plt.plot(lls_)
plt.xlabel("Training iteration")
plt.ylabel("Loss")
plt.savefig("champagne_GP_images/bolfi_optim_loss_log_discrep.pdf")
plt.show()
```

next\_gamma\_L is 0.017



```
K_inv_y = K_inv @ tf.reshape(obs_vals - means, shape=[obs_vals.shape[0], 1])
        K_inv_diag = tf.linalg.diag_part(K_inv)
        log_var = tf.math.log(K_inv_diag)
        log_mu = tf.reshape(K_inv_y, shape=[-1]) ** 2
        return -tf.math.reduce_sum(log_var - log_mu)
    @tf.function(autograph=False, jit compile=False)
    def opt_GP():
        with tf.GradientTape() as tape:
            loss = L00 loss(
                champ_GP, index_vals, obs_vals, observation_noise_variance_champ
            )
        grads = tape.gradient(loss, champ_GP.trainable_variables)
        optimizer slow.apply gradients(zip(grads, champ GP.trainable_variables))
        return loss
    num_iters = 10000
    lls_ = np.zeros(num_iters, np.float64)
    tolerance = 1e-6  # Set your desired tolerance level
    previous_loss = float("inf")
    for i in range(num_iters):
        loss = opt_GP()
        # Check if change in loss is less than tolerance
        if abs(loss - previous_loss) < tolerance:</pre>
            print(f"Hyperparameter convergence reached at iteration {i+1}.")
            break
        previous_loss = loss
    for var in optimizer_slow.variables:
        var.assign(tf.zeros_like(var))
def update_GP_MLE(champ_GP):
    @tf.function(autograph=False, jit_compile=False)
    def train_model():
        with tf.GradientTape() as tape:
            loss = -champ_GP.log_prob(obs_vals)
        grads = tape.gradient(loss, champ_GP.trainable_variables)
```

```
optimizer_slow.apply_gradients(zip(grads, champ_GP.trainable_variables))
        return loss
    num_iters = 10000
    lls_ = np.zeros(num_iters, np.float64)
    tolerance = 1e-6 # Set your desired tolerance level
    previous_loss = float("inf")
    for i in range(num_iters):
        loss = train_model()
        # Check if change in loss is less than tolerance
        if abs(loss - previous_loss) < tolerance:</pre>
            print(f"Hyperparameter convergence reached at iteration {i+1}.")
            break
        previous_loss = loss
    for var in optimizer_slow.variables:
        var.assign(tf.zeros_like(var))
# def UCB_loss(eta_t, champ_GP_reg):
     next_guess = tf.reshape(
          tf.stack([next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]),
#
          [1, 6],
#
     mean_t = champ_GP_reg.mean_fn(next_guess)
      std_t = champ_GP_reg.stddev(index_points=next_guess)
     return tf.squeeze(mean_t - eta_t * std_t)
def update_var_UCB(eta_t, champ_GP_reg, next_vars):
    optimizer_fast = tf.keras.optimizers.Adam(learning_rate=0.1)
    @tf.function(autograph=False, jit_compile=False)
    def opt_var():
        with tf.GradientTape() as tape:
            loss = UCB_loss(eta_t, champ_GP_reg)
        grads = tape.gradient(loss, next_vars)
        optimizer_fast.apply_gradients(zip(grads, next_vars))
```

```
return loss
    num_iters = 10000
    lls_ = np.zeros(num_iters, np.float64)
    tolerance = 1e-3 # Set your desired tolerance level
    previous_loss = float("inf")
    for i in range(num_iters):
        loss = opt_var()
        lls_[i] = loss
        # Check if change in loss is less than tolerance
        if abs(loss - previous_loss) < tolerance:</pre>
            print(f"Acquisition function convergence reached at iteration {i+1}.")
            break
        previous_loss = loss
    next_guess = tf.reshape(
        tf.stack([next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]),
    )
    print(
        "The final UCB loss was {}".format(loss.numpy().round(3))
        + " with predicted mean of {}".format(
            champ_GP_reg.mean_fn(next_guess).numpy().round(3)
        )
    for var in optimizer_fast.variables:
        var.assign(tf.zeros_like(var))
def update_var_EI(GP_reg, alpha, beta, gamma_L, lambda_, f, r, min_obs):
    def EI_loss(alpha, beta, gamma_L, lambda_, f, r, min_obs):
        next_guess = tf.reshape(
            tf.stack([alpha, beta, gamma_L, lambda_, f, r]),
            [1, 6],
        )
        mean_t = GP_reg.mean_fn(next_guess)
        std_t = GP_reg.stddev(index_points=next_guess)
```

```
delt = min_obs - mean_t
    return -tf.squeeze(
        delt * tfd.Normal(0, np.float64(1)).cdf(delt / std_t)
        + std_t * tfd.Normal(0, np.float64(1)).prob(delt / std_t)
optimizer_fast = tf.keras.optimizers.Adam(learning_rate=0.1)
@tf.function(autograph=False, jit_compile=False)
def opt_var():
    with tf.GradientTape() as tape:
        loss = EI_loss(alpha, beta, gamma_L, lambda_, f, r, min_obs)
    grads = tape.gradient(loss, next_vars)
    optimizer_fast.apply_gradients(zip(grads, next_vars))
    return loss
num_iters = 10000
lls_ = np.zeros(num_iters, np.float64)
tolerance = 1e-6  # Set your desired tolerance level
previous_loss = np.float64("inf")
for i in range(num_iters):
    loss = opt_var()
    lls [i] = loss
    # Check if change in loss is less than tolerance
    if abs(loss - previous_loss) < tolerance:</pre>
        print(f"Acquisition function convergence reached at iteration {i+1}.")
        lls_ = lls_ [range(i + 1)]
        break
    previous_loss = loss
next_guess = tf.reshape(
    tf.stack([next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]),
    [1, 6],
print(
    "The final EI loss was {}".format(loss.numpy().round(3))
    + " with predicted mean of {}".format(
```

```
champ_GP_reg.mean_fn(next_guess).numpy().round(3)
        )
    )
def new_eta_t(t, d, exploration_rate):
    # return np.log((t + 1) ** (d * 2 + 2) * np.pi**2 / (3 * exploration_rate))
    return np.sqrt(np.log((t + \frac{1}{2}) ** (d * \frac{2}{2} + \frac{2}{2}) * np.pi**2 / (\frac{3}{2} * exploration_rate)))
# optimizer_fast = tf.keras.optimizers.Adam(learning_rate=1.)
# update var EI()
# plt.figure(figsize=(6, 3.5))
# plt.plot(lls )
# plt.xlabel("Training iteration")
# plt.ylabel("Loss")
# plt.show()
num_slice_updates = 15
all_slices = [
    np.linspace(0, 1, num_slice_updates + 2, dtype=np.float64)[1:-1], # alpha
    np.linspace(0, 1, num_slice updates + 2, dtype=np.float64)[1:-1], # beta
    np.linspace(0, gamma_L_max, num_slice_updates + 2, dtype=np.float64)[
        1:-1
    ], # gamma_L
    np.linspace(0, lambda_max, num_slice_updates + 2, dtype=np.float64)[1:-1], # lambda
    np.linspace(0, f_max, num_slice_updates + 2, dtype=np.float64)[1:-1], # f
    np.linspace(0, r_max, num_slice_updates + 2, dtype=np.float64)[1:-1], # r
]
exploration_rate = 1
update_GP_hp_freq = 20  # how many iterations before updating GP hyperparams
eta_t = tf.Variable(0, dtype=np.float64, name="eta_t")
min_obs = tf.Variable(100, dtype=np.float64, name="min_obs", shape=())
min_index = index_vals[
    champ GP reg.mean fn(index vals) == min(champ GP reg.mean fn(index vals))
] [0]
simulation_reps = 20
```

```
for t in range (501):
   min_index = index_vals[
       champ GP reg.mean fn(index vals) == min(champ GP reg.mean fn(index vals))
   ][
       0,
   optimizer slow = tf.keras.optimizers.Adam()
   # eta_t.assign(new_eta_t(t, d, exploration_rate))
   min_obs.assign(min(champ_GP_reg.mean_fn(index_vals)))
   print("Iteration " + str(t))
   # print(eta_t)
   # for var in [next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]:
         var.assign(
   #
             var.bijector.forward(np.float64(100000000.0))
             * np.float64(np.random.uniform())
   #
   index_update = 0
   for var in [next_alpha, next_beta, next_gamma_L, next_lambda, next_f, next_r]:
       if np.random.uniform() > 0.5:
           var.assign(min_index[index_update])
       else:
           var.assign(
               var.bijector.forward(np.float64(100000000.0))
               * np.float64(np.random.uniform())
           )
       index_update += 1
   # update_var_UCB(eta_t, champ_GP_reg)
   update_var_EI(
       champ_GP_reg,
       next_alpha,
       next_beta,
       next_gamma_L,
       next_lambda,
       next_f,
       next_r,
       min_obs,
```

```
)
bayes_params = np.array(
        next_alpha.numpy(),
       next_beta.numpy(),
       next_gamma_L.numpy(),
        next_lambda.numpy(),
       next_f.numpy(),
       next_r.numpy(),
    ]
).reshape(1, -1)
print("The next parameters to simulate from are {}".format(bayes_params.round(3)))
if t < 6:
    new_params = np.repeat(
       np.array(
            next_alpha.numpy(),
                    next_beta.numpy(),
                    next_gamma_L.numpy(),
                    next_lambda.numpy(),
                    next_f.numpy(),
                    next_r.numpy(),
                ]
            ]
        ),
        num_slice_updates,
        axis=0,
    new_params[:, t % 6] = all_slices[t % 6]
else:
    new_params = np.repeat(
        np.array(
            next_alpha.numpy(),
                    next_beta.numpy(),
                    next_gamma_L.numpy(),
                    next_lambda.numpy(),
```

```
next_f.numpy(),
                    next_r.numpy(),
                ]
            ]
        ),
        4,
        axis=0,
    new_params[:, t % 6] = np.random.uniform(0, upper_bounds[t % 6], 4)
new_params = np.append(
    new_params,
    np.array(
        next_alpha.numpy(),
                next_beta.numpy(),
                next_gamma_L.numpy(),
                next_lambda.numpy(),
                next_f.numpy(),
                next_r.numpy(),
            ]
        ]
    ),
    axis=0,
)
new_params_reps = np.repeat(new_params, dis_mean_n, axis=0)
index_vals = np.append(index_vals, new_params, axis=0)
with mp.Pool(processes=mp.cpu_count()) as pool:
    args = [
        (a, b, c, d, e, f, int(g * np.random.uniform()))
        for (a, b, c, d, e, f), g in zip(
            list(map(tuple, new_params_reps)), range(new_params_reps.shape[0])
        )
    results = pool.starmap(single_discrepency, args)
new_discrepencies = np.mean(np.array(results).reshape(-1, dis_mean_n), axis=1)
```

```
print("The mean of the samples was {}".format(new_discrepencies[-1].round(3)))
obs_vals = np.append(obs_vals, new_discrepencies)
champ_GP_reg = tfd.GaussianProcessRegressionModel(
   kernel=kernel_champ,
   observation_index_points=index_vals,
   observations=obs vals,
   observation_noise_variance=observation_noise_variance_champ,
   predictive_noise_variance=0.0,
   mean_fn=const_mean_fn(),
)
if t % update_GP_hp_freq == 0:
   champ_GP = tfd.GaussianProcess(
       kernel=kernel_champ,
       observation_noise_variance=observation_noise_variance_champ,
       index_points=index_vals,
       mean_fn=const_mean_fn(),
   update GP LOO(champ GP, index vals, obs vals, observation noise variance champ)
   # update_GP_MLE(champ_GP)
   min_value = min(champ_GP_reg.mean_fn(index_vals))
   min_index = index_vals[champ_GP_reg.mean_fn(index_vals) == min_value][0,]
   print(
       "The minimum predicted mean of the observed indices is {}".format(
           min_value.numpy().round(3)
       + " at the point \n{}".format(min_index.round(3))
if (t > 0) & (t \% 50 == 0):
   print("Trained parameters:")
   for train_var in champ_GP.trainable_variables:
       if "bias" in train_var.name:
           print("{} is {}\n".format(train_var.name, train_var.numpy().round(3)))
       else:
           if "length" in train_var.name:
               print(
                   "{} is {}\n".format(
```

```
train var.name,
                    tfb.Sigmoid(
                        np.float64(0.0),
                            1.0 / 2,
                            1.0 / 2,
                            gamma_L_max / 2,
                            lambda_max / 2,
                            f_max / 2,
                            r_{max} / 2,
                        ],
                    .forward(train_var)
                    .numpy()
                    .round(3),
                )
            )
        else:
            print(
                "{} is {}\n".format(
                    train_var.name,
                    constrain_positive.forward(train_var).numpy().round(3),
                )
            )
for var in vars:
    champ_GP_reg_plot = tfd.GaussianProcessRegressionModel(
        kernel=kernel_champ,
        index_points=slice_indices_dfs_dict[var + "_gp_indices_df"].values,
        observation_index_points=index_vals,
        observations=obs_vals,
        observation_noise_variance=observation_noise_variance_champ,
        predictive_noise_variance=0.0,
        mean_fn=const_mean_fn(),
    GP_samples = champ_GP_reg_plot.sample(gp_samp_no, seed=GP_seed)
    gp_samples_dict[var + "_gp_samps" + str(t) + "iters"] = GP_samples
    plt.figure(figsize=(6, 3.5))
    plt.scatter(
        slice_indices_dfs_dict[var + "_slice_indices_df"][var].values,
```

```
slice_discrepencies_dict[var + "_slice_discrepencies"],
    label="Untrained Discrepencies",
    alpha=0.6,
)
for i in range(gp_samp_no):
    plt.plot(
        slice_indices_dfs_dict[var + "_gp_indices_df"][var].values,
        GP_samples[i, :],
        c="r",
        alpha=0.1,
        label="Posterior Sample" if i == 0 else None,
    )
plt.plot(
    slice_indices_dfs_dict[var + "_gp_indices_df"][var].values,
    champ_GP_reg_plot.mean_fn(
        slice_indices_dfs_dict[var + "_gp_indices_df"].values
    ),
    c="black",
    alpha=1,
    label="Posterior Mean",
leg = plt.legend(loc="upper left")
for lh in leg.legend_handles:
    lh.set_alpha(1)
if var in ["f", "r"]:
    plt.xlabel("$" + var + "$ index points")
    plt.title(
        "$" + var + "$ slice after " + str(t) + " Bayesian acquisitions"
    )
else:
    plt.xlabel("$\\" + var + "$ index points")
    plt.title(
        "\" + var + "\" slice after " + str(t) + " Bayesian acquisitions"
    )
plt.ylabel("log(Discrepancy)")
plt.ylim((-3, 2))
plt.savefig(
    "champagne_GP_images/"
   + var
    + "_slice_"
    + str(t)
```

```
)
              plt.show()
Iteration 0
Acquisition function convergence reached at iteration 169.
The final EI loss was -0.014 with predicted mean of [0.246]
                                                             0.05 0.05 0.
                                                                               11
The next parameters to simulate from are [[0.001 1.
                                                        0.
The mean of the samples was 2.031
The minimum predicted mean of the observed indices is -0.958 at the point
[0.169 0.759 0.018 0.012 0.026 0.04 ]
Iteration 1
Acquisition function convergence reached at iteration 113.
The final EI loss was -0.151 with predicted mean of [-0.81]
The next parameters to simulate from are [[0.126 0.559 0.012 0.011 0.018 0.038]]
The mean of the samples was -0.831
Iteration 2
Acquisition function convergence reached at iteration 50.
The final EI loss was -0.093 with predicted mean of [-0.471]
The next parameters to simulate from are [[0.736 0.102 0.033 0.012 0.022 0.005]]
The mean of the samples was -0.348
Iteration 3
Acquisition function convergence reached at iteration 79.
The final EI loss was -0.094 with predicted mean of [-0.686]
The next parameters to simulate from are [[0.144 0.042 0.001 0.009 0.014 0.031]]
The mean of the samples was -0.373
Iteration 4
Acquisition function convergence reached at iteration 79.
The final EI loss was -0.116 with predicted mean of [-0.886]
The next parameters to simulate from are [[0.694 0.002 0.007 0.011 0.031 0.
                                                                               11
The mean of the samples was 0.329
Iteration 5
Acquisition function convergence reached at iteration 150.
The final EI loss was -0.427 with predicted mean of [-1.481]
The next parameters to simulate from are [[0.776 0.183 0.007 0.014 0.023 0.007]]
The mean of the samples was -1.266
Iteration 6
Acquisition function convergence reached at iteration 96.
The final EI loss was -0.107 with predicted mean of [-1.212]
The next parameters to simulate from are [[0.762\ 0.208\ 0.007\ 0.012\ 0.022\ 0.002]]
The mean of the samples was -0.33
Iteration 7
```

+ "\_bolfi\_updates\_log\_discrep.pdf"

Acquisition function convergence reached at iteration 78.

The final EI loss was -0.11 with predicted mean of [-0.858]

The next parameters to simulate from are [[0.049 0.061 0.015 0.012 0.01 0.035]]

The mean of the samples was -0.811

Iteration 8

Acquisition function convergence reached at iteration 117.

The final EI loss was -0.101 with predicted mean of [-0.79]

The next parameters to simulate from are [[0.154 0.671 0.022 0.01 0.011 0.049]]

The mean of the samples was -0.276

Iteration 9

Acquisition function convergence reached at iteration 122.

The final EI loss was -0.009 with predicted mean of [0.443]

The next parameters to simulate from are  $[[0.638 \ 0.001 \ 0.05 \ 0.05 \ 0.05]]$ 

The mean of the samples was 1.825

Iteration 10

Acquisition function convergence reached at iteration 193.

The final EI loss was -0.02 with predicted mean of [0.191]

The next parameters to simulate from are [[0.396 0.999 0.033 0.05 0. 0. ]]

The mean of the samples was 1.691

Iteration 11

Acquisition function convergence reached at iteration 192.

The final EI loss was -0.104 with predicted mean of [-0.888]

The next parameters to simulate from are [[0.096 0.747 0.016 0.01 0.033 0.033]]

The mean of the samples was -1.158

Iteration 12

Acquisition function convergence reached at iteration 75.

The final EI loss was -0.09 with predicted mean of [-1.03]

The next parameters to simulate from are [[0.083 0.422 0.015 0.011 0.031 0.036]]

The mean of the samples was -1.261

Iteration 13

Acquisition function convergence reached at iteration 123.

The final EI loss was -0.11 with predicted mean of [-1.239]

The next parameters to simulate from are [[0.145 0.489 0.015 0.012 0.037 0.037]]

The mean of the samples was -0.969

Iteration 14

Acquisition function convergence reached at iteration 72.

The final EI loss was -0.108 with predicted mean of [-1.285]

The next parameters to simulate from are [[0.074 0.609 0.018 0.011 0.027 0.035]]

The mean of the samples was -1.25

Iteration 15

Acquisition function convergence reached at iteration 55.

The final EI loss was -0.072 with predicted mean of [-0.991]

The next parameters to simulate from are [[0.062 0.055 0.014 0.011 0.03 0.035]]

The mean of the samples was -0.982

Iteration 16

Acquisition function convergence reached at iteration 198.

The final EI loss was -0.022 with predicted mean of [0.02]

The next parameters to simulate from are [[0.999 0.992 0.033 0. 0.05 0.066]]

The mean of the samples was 0.4

Iteration 17

Acquisition function convergence reached at iteration 37.

The final EI loss was -0.092 with predicted mean of [-1.26]

The next parameters to simulate from are [[0.086 0.312 0.017 0.009 0.027 0.034]]

The mean of the samples was -1.061

Iteration 18

Acquisition function convergence reached at iteration 88.

The final EI loss was -0.067 with predicted mean of [-1.07]

The next parameters to simulate from are [[0.029 0.642 0.017 0.01 0.035 0.037]]

The mean of the samples was -1.165

Iteration 19

Acquisition function convergence reached at iteration 56.

The final EI loss was -0.027 with predicted mean of [-0.204]

The next parameters to simulate from are [[0.002 0.999 0.025 0.002 0.003 0.037]]

The mean of the samples was 0.256

Iteration 20

Acquisition function convergence reached at iteration 65.

The final EI loss was -0.071 with predicted mean of [-0.887]

The next parameters to simulate from are [[0.272 0.378 0.024 0.013 0.031 0.04 ]]

The mean of the samples was -0.993

The minimum predicted mean of the observed indices is -1.189 at the point

[0.776 0.183 0.007 0.014 0.023 0.007]

Iteration 21

Acquisition function convergence reached at iteration 110.

The final EI loss was -0.096 with predicted mean of [-0.981]

The next parameters to simulate from are [[0.937 0.05 0.007 0.014 0.021 0.007]]

The mean of the samples was -0.369

Iteration 22

Acquisition function convergence reached at iteration 75.

The final EI loss was -0.124 with predicted mean of [-1.07]

The next parameters to simulate from are [[0.003 0.575 0.019 0.012 0.035 0.028]]

The mean of the samples was -0.68

Iteration 23

Acquisition function convergence reached at iteration 79.

The final EI loss was -0.11 with predicted mean of [-1.019]

The next parameters to simulate from are [[0.042 0.988 0.018 0.011 0.028 0.031]]

The mean of the samples was -1.101

Iteration 24

Acquisition function convergence reached at iteration 95.

The final EI loss was -0.157 with predicted mean of [-1.216]

The next parameters to simulate from are [[0.075 0.284 0.02 0.01 0.029 0.038]]

The mean of the samples was -1.061

Iteration 25

Acquisition function convergence reached at iteration 68.

The final EI loss was -0.083 with predicted mean of [-0.684]

The next parameters to simulate from are [[0.314 0.337 0.022 0.014 0.019 0.022]]

The mean of the samples was -1.156

Iteration 26

Acquisition function convergence reached at iteration 55.

The final EI loss was -0.115 with predicted mean of [-1.008]

The next parameters to simulate from are [[0.251 0.824 0.022 0.013 0.024 0.028]]

The mean of the samples was -1.086

Iteration 27

Acquisition function convergence reached at iteration 83.

The final EI loss was -0.102 with predicted mean of [-0.907]

The next parameters to simulate from are [[0.435 0.226 0.021 0.015 0.019 0.016]]

The mean of the samples was -0.865

Iteration 28

Acquisition function convergence reached at iteration 83.

The final EI loss was -0.104 with predicted mean of [-1.081]

The next parameters to simulate from are [[0.116 0.966 0.023 0.01 0.033 0.037]]

The mean of the samples was -0.886

Iteration 29

Acquisition function convergence reached at iteration 22.

The final EI loss was -0.116 with predicted mean of [-1.142]

The next parameters to simulate from are [[0.256 0.651 0.014 0.013 0.02 0.025]]

The mean of the samples was -1.23

Iteration 30

Acquisition function convergence reached at iteration 90.

The final EI loss was -0.108 with predicted mean of [-1.174]

The next parameters to simulate from are [[0.16 0.871 0.015 0.013 0.018 0.027]]

The mean of the samples was -1.208

Iteration 31

Acquisition function convergence reached at iteration 90.

The final EI loss was -0.1 with predicted mean of [-1.185]

The next parameters to simulate from are [[0.175 0.768 0.015 0.014 0.025 0.025]]

The mean of the samples was -1.203

Iteration 32

Acquisition function convergence reached at iteration 30.

The final EI loss was -0.054 with predicted mean of [-0.556]

The next parameters to simulate from are [[0.032 0.013 0.025 0.012 0.037 0.044]]

The mean of the samples was -0.83

Iteration 33

Acquisition function convergence reached at iteration 77.

The final EI loss was -0.089 with predicted mean of [-0.989]

The next parameters to simulate from are [[0.147 0.608 0.015 0.013 0.009 0.022]]

The mean of the samples was -0.938

Iteration 34

Acquisition function convergence reached at iteration 56.

The final EI loss was -0.077 with predicted mean of [-1.041]

The next parameters to simulate from are [[0.236 0.612 0.021 0.015 0.012 0.026]]

The mean of the samples was -0.98

Iteration 35

Acquisition function convergence reached at iteration 157.

The final EI loss was -0.013 with predicted mean of [0.237]

The next parameters to simulate from are [[0.001 0.998 0.033 0.05 0.05 0.024]]

The mean of the samples was 1.888

Iteration 36

Acquisition function convergence reached at iteration 65.

The final EI loss was -0.102 with predicted mean of [-1.294]

The next parameters to simulate from are [[0.134 0.836 0.014 0.012 0.024 0.029]]

The mean of the samples was -1.344

Iteration 37

Acquisition function convergence reached at iteration 59.

The final EI loss was -0.08 with predicted mean of [-0.833]

The next parameters to simulate from are  $[[0.01 \ 0.639 \ 0.025 \ 0.011 \ 0.039 \ 0.045]]$ 

The mean of the samples was -0.801

Iteration 38

Acquisition function convergence reached at iteration 170.

The final EI loss was -0.066 with predicted mean of [-0.868]

The next parameters to simulate from are [[0.005 0.998 0.014 0.008 0.037 0.043]]

The mean of the samples was -1.048

Iteration 39

Acquisition function convergence reached at iteration 128.

The final EI loss was -0.067 with predicted mean of [-1.073]

The next parameters to simulate from are [[0.123 0.999 0.016 0.014 0.025 0.03 ]]

The mean of the samples was -1.053

Iteration 40

Acquisition function convergence reached at iteration 112.

The final EI loss was -0.063 with predicted mean of [-0.785]

The next parameters to simulate from are [[0.26 0.197 0.033 0.014 0.026 0.021]]

The mean of the samples was -0.456

Hyperparameter convergence reached at iteration 6306.

The minimum predicted mean of the observed indices is -1.297 at the point  $[0.134\ 0.836\ 0.014\ 0.012\ 0.024\ 0.029]$ 

Iteration 41

Acquisition function convergence reached at iteration 74.

The final EI loss was -0.067 with predicted mean of [-0.933]

The next parameters to simulate from are [[0.217 0.009 0.023 0.011 0.029 0.034]]

The mean of the samples was -1.086

Iteration 42

Acquisition function convergence reached at iteration 74.

The final EI loss was -0.068 with predicted mean of [-1.292]

The next parameters to simulate from are [[0.152 0.758 0.016 0.012 0.023 0.03 ]]

The mean of the samples was -1.272

Iteration 43

Acquisition function convergence reached at iteration 150.

The final EI loss was -0.014 with predicted mean of [0.053]

The next parameters to simulate from are [[0.998 0.011 0. 0.05 0. 0.066]]

The mean of the samples was 0.577

Iteration 44

Acquisition function convergence reached at iteration 77.

The final EI loss was -0.055 with predicted mean of [-1.296]

The next parameters to simulate from are [[0.156 0.793 0.016 0.012 0.023 0.028]]

The mean of the samples was -1.247

Iteration 45

Acquisition function convergence reached at iteration 135.

The final EI loss was -0.074 with predicted mean of [-1.26]

The next parameters to simulate from are [[0.146 0.825 0.009 0.012 0.024 0.025]]

The mean of the samples was -1.472

Iteration 46

Acquisition function convergence reached at iteration 78.

The final EI loss was -0.065 with predicted mean of [-1.33]

The next parameters to simulate from are [[0.133 0.908 0.008 0.013 0.022 0.026]]

The mean of the samples was -1.484

Iteration 47

Acquisition function convergence reached at iteration 166.

The final EI loss was -0.085 with predicted mean of [-1.401]

The next parameters to simulate from are [[0.134 0.789 0.008 0.012 0.017 0.026]]

The mean of the samples was -1.453

Iteration 48

Acquisition function convergence reached at iteration 109.

The final EI loss was -0.005 with predicted mean of [0.177]

The next parameters to simulate from are [[0.994 0.997 0. 0.05 0.05 0.03 ]]

The mean of the samples was 0.912

Iteration 49

Acquisition function convergence reached at iteration 136.

The final EI loss was -0.031 with predicted mean of [-0.776]

The next parameters to simulate from are [[0.204 0.008 0.028 0.014 0.016 0.04 ]]

The mean of the samples was -1.062

Iteration 50

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.85]

The next parameters to simulate from are [[0.131 0.012 0.008 0.046 0.016 0.061]]

The mean of the samples was 1.164

Trained parameters:

amplitude\_champ:0 is 0.791

length\_scales\_champ:0 is [0.322 0.5 0.014 0.006 0.025 0.009]

observation\_noise\_variance\_champ:0 is 0.051

bias\_mean:0 is 0.65

Iteration 51

Acquisition function convergence reached at iteration 108.

The final EI loss was -0.009 with predicted mean of [0.034]

The next parameters to simulate from are [[0.986 0.994 0. 0.028 0. 0.066]]

The mean of the samples was 0.5

Iteration 52

Acquisition function convergence reached at iteration 86.

The final EI loss was -0.064 with predicted mean of [-1.368]

The next parameters to simulate from are [[0.177 0.849 0.007 0.012 0.017 0.023]]

The mean of the samples was -1.364

Iteration 53

Acquisition function convergence reached at iteration 133.

The final EI loss was -0.074 with predicted mean of [-1.471]

The next parameters to simulate from are [[0.159 0.721 0.008 0.012 0.022 0.024]]

The mean of the samples was -1.549

Iteration 54

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.738]

The next parameters to simulate from are [[0.014 0.721 0.008 0.031 0.022 0.024]]

The mean of the samples was 0.883

Iteration 55

Acquisition function convergence reached at iteration 177.

The final EI loss was -0.027 with predicted mean of [-0.77]

The next parameters to simulate from are [[0.297 0.009 0.029 0.013 0.027 0.047]]

The mean of the samples was -0.776

Iteration 56

Acquisition function convergence reached at iteration 81.

The final EI loss was -0.021 with predicted mean of [-0.596]

The next parameters to simulate from are [[0.001 0.009 0.031 0.014 0.012 0.047]]

The mean of the samples was -0.953

Iteration 57

Acquisition function convergence reached at iteration 126.

The final EI loss was -0.032 with predicted mean of [-0.881]

The next parameters to simulate from are [[0.006 0.418 0.025 0.016 0.022 0.044]]

The mean of the samples was -0.873

Iteration 58

Acquisition function convergence reached at iteration 75.

The final EI loss was -0.053 with predicted mean of [-1.414]

The next parameters to simulate from are [[0.177 0.698 0.006 0.013 0.021 0.023]]

The mean of the samples was -1.461

Iteration 59

Acquisition function convergence reached at iteration 40.

The final EI loss was -0.01 with predicted mean of [-0.291]

The next parameters to simulate from are [[0.956 0.026 0.025 0.014 0.047 0.012]]

The mean of the samples was -0.063

Iteration 60

Acquisition function convergence reached at iteration 117.

The final EI loss was -0.006 with predicted mean of [0.073]

The next parameters to simulate from are [[0.998 0.976 0. 0. 0.05 0.066]]

The mean of the samples was 0.512

Hyperparameter convergence reached at iteration 7718.

The minimum predicted mean of the observed indices is -1.523 at the point

[0.159 0.721 0.008 0.012 0.022 0.024]

Iteration 61

Acquisition function convergence reached at iteration 146.

The final EI loss was -0.024 with predicted mean of [-0.886]

The next parameters to simulate from are [[0.003 0.998 0.015 0.013 0.031 0.046]]

The mean of the samples was -0.602

Iteration 62

Acquisition function convergence reached at iteration 108.

The final EI loss was -0.026 with predicted mean of [-0.863]

The next parameters to simulate from are [[0.417 0.103 0.024 0.015 0.016 0.031]]

The mean of the samples was -1.038

Iteration 63

Acquisition function convergence reached at iteration 66.

The final EI loss was -0.008 with predicted mean of [0.005]

The next parameters to simulate from are [[0.009 0.994 0.033 0.017 0. 0.001]]

The mean of the samples was 0.494

Iteration 64

Acquisition function convergence reached at iteration 128.

The final EI loss was -0.061 with predicted mean of [-1.523]

The next parameters to simulate from are [[0.133 0.795 0.009 0.013 0.021 0.023]]

The mean of the samples was -1.575

Iteration 65

Acquisition function convergence reached at iteration 41.

The final EI loss was -0.05 with predicted mean of [-1.55]

The next parameters to simulate from are [[0.163 0.789 0.009 0.012 0.021 0.024]]

The mean of the samples was -1.772

Iteration 66

Acquisition function convergence reached at iteration 73.

The final EI loss was -0.015 with predicted mean of [-0.515]

The next parameters to simulate from are [[0.374 0.998 0.032 0.017 0.012 0.031]]

The mean of the samples was -0.966

Iteration 67

Acquisition function convergence reached at iteration 67.

The final EI loss was -0.006 with predicted mean of [0.044]

The next parameters to simulate from are  $[[0.014\ 0.012\ 0.033\ 0.016\ 0.05\ 0.\ ]]$ 

The mean of the samples was 0.541

Iteration 68

Acquisition function convergence reached at iteration 55.

The final EI loss was -0.014 with predicted mean of [-0.607]

The next parameters to simulate from are [[0.014 0.684 0.017 0.005 0.047 0.044]]

The mean of the samples was -0.738

Iteration 69

Acquisition function convergence reached at iteration 174.

The final EI loss was -0.018 with predicted mean of [-0.618]

The next parameters to simulate from are [[0.002 0.008 0.032 0.015 0.028 0.055]]

The mean of the samples was -0.874

Iteration 70

Acquisition function convergence reached at iteration 72.

The final EI loss was -0.024 with predicted mean of [-0.885]

The next parameters to simulate from are [[0.378 0.528 0.033 0.016 0.02 0.034]]

The mean of the samples was -0.945

Iteration 71

Acquisition function convergence reached at iteration 74.

The final EI loss was -0.035 with predicted mean of [-1.433]

The next parameters to simulate from are [[0.119 0.785 0.007 0.011 0.022 0.024]]

The mean of the samples was -1.907

Iteration 72

Acquisition function convergence reached at iteration 72.

The final EI loss was -0.006 with predicted mean of [-0.142]

The next parameters to simulate from are [[0.97 0.987 0. 0.015 0.013 0.005]]

The mean of the samples was 0.344

Iteration 73

Acquisition function convergence reached at iteration 66.

The final EI loss was -0.071 with predicted mean of [-1.616]

The next parameters to simulate from are [[0.092 0.786 0.005 0.01 0.022 0.023]]

The mean of the samples was -1.808

Iteration 74

Acquisition function convergence reached at iteration 129.

The final EI loss was -0.083 with predicted mean of [-1.738]

The next parameters to simulate from are [[0.074 0.62 0.004 0.01 0.023 0.023]]

The mean of the samples was -1.596

Iteration 75

Acquisition function convergence reached at iteration 42.

The final EI loss was -0.007 with predicted mean of [-0.602]

The next parameters to simulate from are [[0.56 0.639 0.033 0.015 0.024 0.02 ]]

The mean of the samples was -0.834

Iteration 76

Acquisition function convergence reached at iteration 39.

The final EI loss was -0.009 with predicted mean of [-0.697]

The next parameters to simulate from are [[0.672 0.181 0.03 0.015 0.014 0.023]]

The mean of the samples was -0.85

Iteration 77

Acquisition function convergence reached at iteration 88.

The final EI loss was -0.005 with predicted mean of [-0.507]

The next parameters to simulate from are [[0.002 0.994 0.014 0.003 0.035 0.05 ]]

The mean of the samples was -0.352

Iteration 78

Acquisition function convergence reached at iteration 60.

The final EI loss was -0.057 with predicted mean of [-1.735]

The next parameters to simulate from are [[0.1 0.821 0.004 0.01 0.022 0.023]]

The mean of the samples was -1.424

Iteration 79

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.009 with predicted mean of [-0.345]

The next parameters to simulate from are [[0.038 0.963 0.031 0.012 0.048 0.058]]

The mean of the samples was -0.591

Iteration 80

Acquisition function convergence reached at iteration 73.

The final EI loss was -0.002 with predicted mean of [0.28]

The next parameters to simulate from are [[0.001 0.996 0. 0.05 0. 0.]]

The mean of the samples was 2.085

Hyperparameter convergence reached at iteration 2324.

The minimum predicted mean of the observed indices is -1.761 at the point  $[0.119\ 0.785\ 0.007\ 0.011\ 0.022\ 0.024]$ 

Iteration 81

Acquisition function convergence reached at iteration 130.

The final EI loss was -0.086 with predicted mean of [-1.788]

The next parameters to simulate from are [[0.083 0.662 0.007 0.01 0.023 0.024]]

The mean of the samples was -1.611

Iteration 82

Acquisition function convergence reached at iteration 143.

The final EI loss was -0.011 with predicted mean of [-0.958]

The next parameters to simulate from are [[0.002 0.001 0.024 0.014 0.018 0.037]]

The mean of the samples was -1.125

Iteration 83

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.578]

The next parameters to simulate from are [[0.119 0.916 0.007 0.03 0.017 0.024]]

The mean of the samples was 0.591

Iteration 84

Acquisition function convergence reached at iteration 79.

The final EI loss was -0.016 with predicted mean of [-1.099]

The next parameters to simulate from are [[0.495 0.488 0.028 0.015 0.023 0.026]]

The mean of the samples was -0.963

Iteration 85

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.749]

The next parameters to simulate from are [[0.121 0.667 0.024 0.011 0.022 0.024]]

The mean of the samples was -0.709

Iteration 86

Acquisition function convergence reached at iteration 83.

The final EI loss was -0.01 with predicted mean of [-0.771]

The next parameters to simulate from are [[0.461 0.729 0.028 0.015 0.005 0.022]]

The mean of the samples was -0.847

Iteration 87

Acquisition function convergence reached at iteration 107.

The final EI loss was -0.055 with predicted mean of [-1.752]

The next parameters to simulate from are [[0.095 0.76 0.007 0.01 0.022 0.024]]

The mean of the samples was -1.59

Iteration 88

Acquisition function convergence reached at iteration 88.

The final EI loss was -0.054 with predicted mean of [-1.708]

The next parameters to simulate from are [[0.086 0.654 0.006 0.011 0.023 0.023]]

The mean of the samples was -1.556

Iteration 89

Acquisition function convergence reached at iteration 76.

The final EI loss was -0.002 with predicted mean of [0.151]

The next parameters to simulate from are [[0.004 0.006 0.033 0.036 0. 0. ]]

The mean of the samples was 1.462

Iteration 90

Acquisition function convergence reached at iteration 67.

The final EI loss was -0.008 with predicted mean of [-0.47]

The next parameters to simulate from are [[0.003 0.127 0.018 0.008 0.047 0.055]]

The mean of the samples was -0.727

Iteration 91

Acquisition function convergence reached at iteration 143.

The final EI loss was -0.043 with predicted mean of [-1.474]

The next parameters to simulate from are [[0.19 0.325 0.007 0.01 0.023 0.022]]

The mean of the samples was -1.708

Iteration 92

Acquisition function convergence reached at iteration 50.

The final EI loss was -0.003 with predicted mean of [0.007]

The next parameters to simulate from are [[0.981 0.985 0.032 0.022 0.049 0.063]]

The mean of the samples was 0.459

Iteration 93

Acquisition function convergence reached at iteration 83.

The final EI loss was -0.063 with predicted mean of [-1.75]

The next parameters to simulate from are [[0.16 0.451 0.006 0.01 0.023 0.021]]

The mean of the samples was -1.86

Iteration 94

Acquisition function convergence reached at iteration 162.

The final EI loss was -0.012 with predicted mean of [-0.951]

The next parameters to simulate from are [[0.002 0.002 0.024 0.011 0.019 0.048]]

The mean of the samples was -0.805

Iteration 95

Acquisition function convergence reached at iteration 30.

The final EI loss was -0.008 with predicted mean of [-0.868]

The next parameters to simulate from are [[0.374 0.73 0.025 0.017 0.016 0.034]]

The mean of the samples was -0.881

Iteration 96

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.279]

The next parameters to simulate from are [[0.16 0.6 0.014 0.041 0.023 0.022]]

The mean of the samples was 1.273

Iteration 97

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.381]

The next parameters to simulate from are [[0.341 0.451 0.006 0.044 0.023 0.021]]

The mean of the samples was 1.285

Iteration 98

Acquisition function convergence reached at iteration 184.

The final EI loss was -0.084 with predicted mean of [-1.791]

The next parameters to simulate from are [[0.169 0.422 0.007 0.009 0.023 0.023]]

The mean of the samples was -1.778

Iteration 99

Acquisition function convergence reached at iteration 43.

The final EI loss was -0.003 with predicted mean of [-0.191]

The next parameters to simulate from are [[0.14 0.998 0.001 0.007 0.046 0.014]]

The mean of the samples was -1.349

Iteration 100

Acquisition function convergence reached at iteration 127.

The final EI loss was -0.003 with predicted mean of [0.047]

The next parameters to simulate from are [[0.998 0.005 0.002 0. 0. 0.067]]

The mean of the samples was 0.646

Hyperparameter convergence reached at iteration 2485.

The minimum predicted mean of the observed indices is -1.77 at the point

[0.16 0.451 0.006 0.01 0.023 0.021]

Trained parameters:

amplitude\_champ:0 is 0.782

length\_scales\_champ:0 is [0.309 0.5 0.011 0.006 0.02 0.012]

observation\_noise\_variance\_champ:0 is 0.064

bias\_mean:0 is 0.707

Iteration 101

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.218]

The next parameters to simulate from are [[0.161 0.45 0.026 0.041 0.04 0.042]]

The mean of the samples was 1.248

Iteration 102

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.841]

The next parameters to simulate from are [[0.16 0.66 0.006 0.031 0.024 0.021]]

The mean of the samples was 0.804

Iteration 103

Acquisition function convergence reached at iteration 58.

The final EI loss was -0.061 with predicted mean of [-1.757]

The next parameters to simulate from are [[0.171 0.41 0.006 0.009 0.024 0.021]]

The mean of the samples was -1.933

Iteration 104

Acquisition function convergence reached at iteration 43.

The final EI loss was -0.006 with predicted mean of [-0.611]

The next parameters to simulate from are [[0.329 0.014 0.014 0.012 0. 0.021]]

The mean of the samples was -0.392

Iteration 105

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.617]

The next parameters to simulate from are [[0.459 0.761 0.005 0.036 0.007 0.021]]

The mean of the samples was 0.627

Iteration 106

Acquisition function convergence reached at iteration 117.

The final EI loss was -0.055 with predicted mean of [-1.786]

The next parameters to simulate from are [[0.18 0.351 0.005 0.009 0.027 0.021]]

The mean of the samples was -1.69

Iteration 107

Acquisition function convergence reached at iteration 40.

The final EI loss was -0.006 with predicted mean of [-0.662]

The next parameters to simulate from are [[0.176 0.037 0.023 0.017 0.003 0.037]]

The mean of the samples was -0.758

Iteration 108

Acquisition function convergence reached at iteration 86.

The final EI loss was -0.06 with predicted mean of [-1.835]

The next parameters to simulate from are [[0.169 0.409 0.006 0.01 0.026 0.023]]

The mean of the samples was -1.593

Iteration 109

Acquisition function convergence reached at iteration 37.

The final EI loss was -0.048 with predicted mean of [-1.807]

The next parameters to simulate from are [[0.163 0.432 0.006 0.01 0.025 0.021]]

The mean of the samples was -1.714

Iteration 110

Acquisition function convergence reached at iteration 123.

The final EI loss was -0.04 with predicted mean of [-1.804]

The next parameters to simulate from are [[0.161 0.487 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.793

Iteration 111

Acquisition function convergence reached at iteration 120.

The final EI loss was -0.035 with predicted mean of [-1.721]

The next parameters to simulate from are [[0.183 0.296 0.007 0.009 0.026 0.023]]

The mean of the samples was -1.707

Iteration 112

Acquisition function convergence reached at iteration 12.

The final EI loss was -0.003 with predicted mean of [-0.572]

The next parameters to simulate from are [[0.091 0.267 0.013 0.005 0.036 0.049]]

The mean of the samples was -0.545

Iteration 113

Acquisition function convergence reached at iteration 95.

The final EI loss was -0.006 with predicted mean of [-0.612]

The next parameters to simulate from are [[0.507 0.004 0.033 0.018 0.026 0.031]]

The mean of the samples was -0.577

Iteration 114

Acquisition function convergence reached at iteration 68.

The final EI loss was -0.005 with predicted mean of [-0.647]

The next parameters to simulate from are [[0.47 0.985 0.033 0.012 0.021 0.035]]

The mean of the samples was -0.572

Iteration 115

Acquisition function convergence reached at iteration 58.

The final EI loss was -0.005 with predicted mean of [-0.684]

The next parameters to simulate from are [[0.468 0.256 0.023 0.019 0.003 0.026]]

The mean of the samples was -0.807

Iteration 116

Acquisition function convergence reached at iteration 85.

The final EI loss was -0.002 with predicted mean of [0.123]

The next parameters to simulate from are [[0.987 0.002 0. 0. 0. 0.035]]

The mean of the samples was 0.599

Iteration 117

Acquisition function convergence reached at iteration 73.

The final EI loss was -0.002 with predicted mean of [0.093]

The next parameters to simulate from are [[0.003 0.01 0. 0.022 0.001 0. ]]

The mean of the samples was 0.774

Iteration 118

Acquisition function convergence reached at iteration 145.

The final EI loss was -0.037 with predicted mean of [-1.768]

The next parameters to simulate from are [[0.145 0.417 0.006 0.009 0.025 0.021]]

The mean of the samples was -1.813

Iteration 119

Acquisition function convergence reached at iteration 112.

The final EI loss was -0.037 with predicted mean of [-1.796]

The next parameters to simulate from are [[0.148 0.424 0.006 0.009 0.025 0.021]]

The mean of the samples was -1.763

Iteration 120

Acquisition function convergence reached at iteration 285.

The final EI loss was -0.031 with predicted mean of [-1.783]

The next parameters to simulate from are [[0.175 0.415 0.006 0.009 0.025 0.02 ]]

The mean of the samples was -1.738

Hyperparameter convergence reached at iteration 1824.

The minimum predicted mean of the observed indices is -1.799 at the point  $[0.148\ 0.424\ 0.006\ 0.009\ 0.025\ 0.021]$ 

Iteration 121

Acquisition function convergence reached at iteration 54.

The final EI loss was -0.004 with predicted mean of [-0.476]

The next parameters to simulate from are [[0.005 0.481 0.028 0.012 0.029 0.066]]

The mean of the samples was -0.812

Iteration 122

Acquisition function convergence reached at iteration 72.

The final EI loss was -0.034 with predicted mean of [-1.798]

The next parameters to simulate from are [[0.142 0.489 0.006 0.009 0.024 0.022]]

The mean of the samples was -1.736

Iteration 123

Acquisition function convergence reached at iteration 35.

The final EI loss was -0.004 with predicted mean of [-0.48]

The next parameters to simulate from are [[0.267 0.304 0.033 0.015 0.036 0.066]]

The mean of the samples was -0.443

Iteration 124

Acquisition function convergence reached at iteration 50.

The final EI loss was -0.035 with predicted mean of [-1.79]

The next parameters to simulate from are [[0.167 0.355 0.007 0.009 0.024 0.021]]

The mean of the samples was -1.473

Iteration 125

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.823]

The next parameters to simulate from are [[0.162 0.439 0. 0.01 0.025 0.022]]

The mean of the samples was -0.364

Iteration 126

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.027]

The next parameters to simulate from are [[0.666 0.469 0.007 0.048 0.039 0.023]]

The mean of the samples was 1.364

Iteration 127

Acquisition function convergence reached at iteration 58.

The final EI loss was -0.005 with predicted mean of [-0.479]

The next parameters to simulate from are [[0.008 0.584 0.02 0.012 0.048 0.067]]

The mean of the samples was -0.221

Iteration 128

Acquisition function convergence reached at iteration 37.

The final EI loss was -0.001 with predicted mean of [0.243]

The next parameters to simulate from are [[0.991 0.826 0. 0.012 0.048 0.066]]

The mean of the samples was 0.578

Iteration 129

Acquisition function convergence reached at iteration 44.

The final EI loss was -0.035 with predicted mean of [-1.788]

The next parameters to simulate from are [[0.147 0.552 0.007 0.01 0.023 0.022]]

The mean of the samples was -1.63

Iteration 130

Acquisition function convergence reached at iteration 76.

The final EI loss was -0.011 with predicted mean of [-0.927]

The next parameters to simulate from are [[0.046 0.893 0.005 0.005 0.05 0.016]]

The mean of the samples was -0.68

Iteration 131

Acquisition function convergence reached at iteration 115.

The final EI loss was -0.001 with predicted mean of [0.113]

The next parameters to simulate from are [[0.001 0.003 0.033 0. 0. 0.001]]

The mean of the samples was 0.499

Iteration 132

Acquisition function convergence reached at iteration 103.

The final EI loss was -0.005 with predicted mean of [-0.654]

The next parameters to simulate from are [[0.013 0.01 0.033 0.017 0.01 0.035]]

The mean of the samples was -0.801

Iteration 133

Acquisition function convergence reached at iteration 75.

The final EI loss was -0.025 with predicted mean of [-1.688]

The next parameters to simulate from are [[0.163 0.264 0.007 0.009 0.024 0.023]]

The mean of the samples was -1.61

Iteration 134

Acquisition function convergence reached at iteration 81.

The final EI loss was -0.002 with predicted mean of [-0.135]

The next parameters to simulate from are [[0.019 0.007 0.012 0.017 0. 0.066]]

The mean of the samples was 0.083

Iteration 135

Acquisition function convergence reached at iteration 34.

The final EI loss was -0.014 with predicted mean of [-1.298]

The next parameters to simulate from are [[0.253 0.002 0.01 0.009 0.025 0.025]]

The mean of the samples was -1.557

Iteration 136

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.273]

The next parameters to simulate from are [[0.146 0.488 0.007 0.043 0.022 0.01 ]]

The mean of the samples was 1.491

Iteration 137

Acquisition function convergence reached at iteration 55.

The final EI loss was -0.033 with predicted mean of [-1.786]

The next parameters to simulate from are [[0.147 0.544 0.007 0.01 0.023 0.022]]

The mean of the samples was -1.823

Iteration 138

Acquisition function convergence reached at iteration 68.

The final EI loss was -0.032 with predicted mean of [-1.789]

The next parameters to simulate from are [[0.152 0.511 0.007 0.01 0.023 0.022]]

The mean of the samples was -1.919

Iteration 139

Acquisition function convergence reached at iteration 33.

The final EI loss was -0.004 with predicted mean of [-0.631]

The next parameters to simulate from are [[0.21 0.987 0.013 0.009 0.049 0.045]]

The mean of the samples was -0.65

Iteration 140

Acquisition function convergence reached at iteration 58.

The final EI loss was -0.001 with predicted mean of [0.18]

The next parameters to simulate from are [[0.99 0.989 0. 0.05 0. 0.038]]

The mean of the samples was 0.643

Hyperparameter convergence reached at iteration 1792.

The minimum predicted mean of the observed indices is -1.802 at the point

[0.152 0.511 0.007 0.01 0.023 0.022]

Iteration 141

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.636]

The next parameters to simulate from are [[0.152 0.514 0.007 0.009 0.023 0.042]]

The mean of the samples was -0.679

Iteration 142

Acquisition function convergence reached at iteration 150.

The final EI loss was -0.007 with predicted mean of [-0.791]

The next parameters to simulate from are [[0.005 0.617 0.033 0.015 0.03 0.045]]

The mean of the samples was -0.942

Iteration 143

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.768]

The next parameters to simulate from are [[0.152 0.511 0.007 0.03 0.023 0.022]]

The mean of the samples was 0.738

Iteration 144

Acquisition function convergence reached at iteration 11.

The final EI loss was -0.004 with predicted mean of [-0.957]

The next parameters to simulate from are [[0.119 0.26 0.01 0.008 0.037 0.032]]

The mean of the samples was -1.374

Iteration 145

Acquisition function convergence reached at iteration 81.

The final EI loss was -0.018 with predicted mean of [-1.552]

The next parameters to simulate from are [[0.134 0.18 0.008 0.009 0.026 0.027]]

The mean of the samples was -1.61

Iteration 146

Acquisition function convergence reached at iteration 82.

The final EI loss was -0.036 with predicted mean of [-1.806]

The next parameters to simulate from are [[0.121 0.505 0.006 0.01 0.024 0.023]]

The mean of the samples was -1.886

Iteration 147

Acquisition function convergence reached at iteration 29.

The final EI loss was -0.006 with predicted mean of [-0.673]

The next parameters to simulate from are [[0.401 0.938 0. 0.007 0.048 0.009]]

The mean of the samples was -0.459

Iteration 148

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.02]

The next parameters to simulate from are [[0.824 0.182 0.006 0.01 0.024 0.023]]

The mean of the samples was -0.047

Iteration 149

Acquisition function convergence reached at iteration 78.

The final EI loss was -0.033 with predicted mean of [-1.815]

The next parameters to simulate from are [[0.131 0.512 0.006 0.01 0.024 0.023]]

The mean of the samples was -1.816

Iteration 150

Acquisition function convergence reached at iteration 14.

The final EI loss was -0.001 with predicted mean of [-0.283]

The next parameters to simulate from are [[0.041 0.025 0.005 0.012 0.001 0.044]]

The mean of the samples was 0.203

Trained parameters:

amplitude\_champ:0 is 0.738

length\_scales\_champ:0 is [0.347 0.5 0.008 0.006 0.02 0.012]

observation\_noise\_variance\_champ:0 is 0.073

bias\_mean:0 is 0.709

Iteration 151

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.516]

The next parameters to simulate from are [[0.131 0.512 0.006 0.045 0.049 0.011]]

The mean of the samples was 1.652

Iteration 152

Acquisition function convergence reached at iteration 76.

The final EI loss was -0.002 with predicted mean of [-0.34]

The next parameters to simulate from are [[0.007 0.004 0.019 0.007 0.031 0.067]]

The mean of the samples was -0.464

Iteration 153

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.415]

The next parameters to simulate from are [[0.43 0.512 0.004 0.046 0.024 0.015]]

The mean of the samples was 1.336

Iteration 154

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.194]

The next parameters to simulate from are [[0.131 0.858 0.006 0.009 0.024 0.064]]

The mean of the samples was -0.195

Iteration 155

Acquisition function convergence reached at iteration 80.

The final EI loss was -0.004 with predicted mean of [-0.703]

The next parameters to simulate from are [[0.011 0.33 0.007 0.004 0.049 0.032]]

The mean of the samples was -1.086

Iteration 156

Acquisition function convergence reached at iteration 38.

The final EI loss was -0.005 with predicted mean of [-0.672]

The next parameters to simulate from are [[0.065 0.949 0.033 0.019 0.015 0.038]]

The mean of the samples was -0.681

Iteration 157

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.787]

The next parameters to simulate from are [[0.039 0.071 0.006 0.032 0.016 0.023]]

The mean of the samples was 0.933

Iteration 158

Acquisition function convergence reached at iteration 66.

The final EI loss was -0.008 with predicted mean of [-0.93]

The next parameters to simulate from are [[0.506 0.993 0.001 0.013 0.038 0.006]]

The mean of the samples was -1.035

Iteration 159

Acquisition function convergence reached at iteration 99.

The final EI loss was -0.012 with predicted mean of [-1.373]

The next parameters to simulate from are [[0.254 0.005 0.007 0.009 0.031 0.026]]

The mean of the samples was -1.068

Iteration 160

Acquisition function convergence reached at iteration 46.

The final EI loss was -0.001 with predicted mean of [0.08]

The next parameters to simulate from are [[0.988 0.023 0.033 0.03 0.05 0.052]]

The mean of the samples was 0.893

Hyperparameter convergence reached at iteration 1863.

The minimum predicted mean of the observed indices is -1.816 at the point  $[0.131\ 0.512\ 0.006\ 0.01\ 0.024\ 0.023]$ 

Iteration 161

Acquisition function convergence reached at iteration 62.

The final EI loss was -0.032 with predicted mean of [-1.817]

The next parameters to simulate from are [[0.133 0.492 0.006 0.01 0.023 0.023]]

The mean of the samples was -1.715

Iteration 162

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.46]

The next parameters to simulate from are [[0.885 0.511 0.006 0.031 0.024 0.023]]

The mean of the samples was 0.542

Iteration 163

Acquisition function convergence reached at iteration 105.

The final EI loss was -0.003 with predicted mean of [-0.338]

The next parameters to simulate from are [[0.418 0.991 0.033 0.013 0.049 0.029]]

The mean of the samples was -0.857

Iteration 164

Acquisition function convergence reached at iteration 21.

The final EI loss was -0.001 with predicted mean of [-0.244]

The next parameters to simulate from are [[0.58 0.758 0.001 0.019 0.012 0.001]]

The mean of the samples was 0.293

Iteration 165

Acquisition function convergence reached at iteration 52.

The final EI loss was -0.004 with predicted mean of [-0.501]

The next parameters to simulate from are [[0.346 0.673 0.033 0.009 0.049 0.038]]

The mean of the samples was -0.706

Iteration 166

Acquisition function convergence reached at iteration 50.

The final EI loss was -0.001 with predicted mean of [0.165]

The next parameters to simulate from are [[0.004 0.983 0. 0.039 0. 0.066]]

The mean of the samples was 0.565

Iteration 167

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.663]

The next parameters to simulate from are [[0.727 0.508 0.008 0.037 0.024 0.024]]

The mean of the samples was 0.723

Iteration 168

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.004]

The next parameters to simulate from are [[0.127 0.811 0.007 0.047 0.036 0.058]]

The mean of the samples was 1.461

Iteration 169

Acquisition function convergence reached at iteration 68.

The final EI loss was -0.003 with predicted mean of [-0.764]

The next parameters to simulate from are [[0.024 0.378 0.017 0.017 0.01 0.034]]

The mean of the samples was -1.142

Iteration 170

Acquisition function convergence reached at iteration 41.

The final EI loss was -0.005 with predicted mean of [-0.71]

The next parameters to simulate from are [[0.006 0.939 0.025 0.017 0.002 0.034]]

The mean of the samples was -0.9

Iteration 171

Acquisition function convergence reached at iteration 26.

The final EI loss was -0.005 with predicted mean of [-0.761]

The next parameters to simulate from are [[0.455 0.697 0.026 0.011 0.042 0.026]]

The mean of the samples was -0.825

Iteration 172

Acquisition function convergence reached at iteration 118.

The final EI loss was -0.03 with predicted mean of [-1.811]

The next parameters to simulate from are [[0.131 0.494 0.006 0.01 0.023 0.023]]

The mean of the samples was -1.82

Iteration 173

Acquisition function convergence reached at iteration 40.

The final EI loss was -0.004 with predicted mean of [-1.073]

The next parameters to simulate from are [[0.13 0. 0.02 0.015 0.013 0.029]]

The mean of the samples was -0.992

Iteration 174

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.54]

The next parameters to simulate from are [[0.132 0.495 0.023 0.01 0.016 0.023]]

The mean of the samples was -0.537

Iteration 175

Acquisition function convergence reached at iteration 51.

The final EI loss was -0.012 with predicted mean of [-1.342]

The next parameters to simulate from are [[0.028 0.023 0.01 0.01 0.023 0.024]]

The mean of the samples was -1.227

Iteration 176

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.012]

The next parameters to simulate from are [[0.728 0.446 0.018 0.01 0.024 0.035]]

The mean of the samples was -0.019

Iteration 177

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.163]

The next parameters to simulate from are [[0.658 0.488 0.006 0.05 0.024 0.006]]

The mean of the samples was 1.39

Iteration 178

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.003 with predicted mean of [-0.844]

The next parameters to simulate from are [[0.004 0. 0.011 0.014 0.014 0.027]]

The mean of the samples was -1.395

Iteration 179

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.238]

The next parameters to simulate from are [[0.733 0.429 0.006 0.009 0.026 0.05 ]]

The mean of the samples was 0.294

Iteration 180

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.001 with predicted mean of [0.191]

The next parameters to simulate from are [[0.004 0.01 0.033 0.029 0.05 0. ]]

The mean of the samples was 1.165

Hyperparameter convergence reached at iteration 1968.

The minimum predicted mean of the observed indices is -1.8 at the point

[0.161 0.487 0.006 0.01 0.024 0.022]

Iteration 181

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.664]

The next parameters to simulate from are [[0.161 0.313 0.022 0.01 0.024 0.022]]

The mean of the samples was -0.642

Iteration 182

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.094]

The next parameters to simulate from are [[0.667 0.145 0.006 0.009 0.025 0.044]]

The mean of the samples was 0.0

Iteration 183

Acquisition function convergence reached at iteration 74.

The final EI loss was -0.005 with predicted mean of [-0.812]

The next parameters to simulate from are [[0.007 0.042 0.014 0.017 0.001 0.027]]

The mean of the samples was -0.806

Iteration 184

Acquisition function convergence reached at iteration 77.

The final EI loss was -0.027 with predicted mean of [-1.805]

The next parameters to simulate from are [[0.142 0.517 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.706

Iteration 185

Acquisition function convergence reached at iteration 48.

The final EI loss was -0.003 with predicted mean of [-0.853]

The next parameters to simulate from are [[0.273 0.46 0.015 0.018 0.009 0.033]]

The mean of the samples was -0.804

Iteration 186

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.747]

The next parameters to simulate from are [[0.142 0.517 0.006 0.01 0.045 0.022]]

The mean of the samples was -0.787

Iteration 187

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.973]

The next parameters to simulate from are [[0.96 0.46 0.009 0.047 0.024 0.006]]

The mean of the samples was 1.035

Iteration 188

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.074]

The next parameters to simulate from are [[0.142 0.299 0.006 0.01 0. 0.022]]

The mean of the samples was -0.026

Iteration 189

WARNING:tensorflow:5 out of the last 55 calls to <function update\_var\_EI.<locals>.opt\_var at Acquisition function convergence reached at iteration 65.

The final EI loss was -0.001 with predicted mean of [0.071]

The next parameters to simulate from are [[0.011 0.987 0. 0. 0.001 0.067]]

The mean of the samples was 0.65

Iteration 190

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.489]

The next parameters to simulate from are [[0.101 0.413 0.006 0.046 0.014 0.022]]

The mean of the samples was 1.534

Iteration 191

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.001 with predicted mean of [0.011]

The next parameters to simulate from are [[0.009 0.987 0.002 0.015 0.049 0.066]]

The mean of the samples was 0.97

Iteration 192

Acquisition function convergence reached at iteration 91.

The final EI loss was -0.007 with predicted mean of [-1.133]

The next parameters to simulate from are [[0.366 0.077 0.012 0.011 0.023 0.02 ]]

The mean of the samples was -1.632

Iteration 193

Acquisition function convergence reached at iteration 24.

The final EI loss was -0.013 with predicted mean of [-1.311]

The next parameters to simulate from are [[0.403 0.196 0.011 0.011 0.031 0.017]]

The mean of the samples was -1.409

Iteration 194

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.602]

The next parameters to simulate from are [[0.142 0.344 0.023 0.01 0.018 0.022]]

The mean of the samples was -0.592

Iteration 195

Acquisition function convergence reached at iteration 66.

The final EI loss was -0.017 with predicted mean of [-1.332]

The next parameters to simulate from are [[0.485 0.272 0.013 0.013 0.025 0.016]]

The mean of the samples was -1.626

Iteration 196

Acquisition function convergence reached at iteration 72.

The final EI loss was -0.019 with predicted mean of [-1.42]

The next parameters to simulate from are [[0.45 0.229 0.015 0.012 0.029 0.021]]

The mean of the samples was -1.389

Iteration 197

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.242]

The next parameters to simulate from are [[0.142 0.517 0.006 0.022 0.024 0.022]]

The mean of the samples was 0.152

Iteration 198

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.849]

The next parameters to simulate from are [[0.142 0.518 0.015 0.01 0.019 0.022]]

The mean of the samples was -0.827

Iteration 199

Acquisition function convergence reached at iteration 46.

The final EI loss was -0.001 with predicted mean of [0.079]

The next parameters to simulate from are [[0.46 0.985 0. 0.023 0.001 0.041]]

The mean of the samples was 0.41

Iteration 200

Acquisition function convergence reached at iteration 52.

The final EI loss was -0.015 with predicted mean of [-1.401]

The next parameters to simulate from are [[0.495 0.044 0.012 0.013 0.026 0.015]]

The mean of the samples was -1.245

Hyperparameter convergence reached at iteration 1909.

The minimum predicted mean of the observed indices is -1.801 at the point

[0.142 0.517 0.006 0.01 0.024 0.022]

Trained parameters:

amplitude\_champ:0 is 0.71

length\_scales\_champ:0 is [0.305 0.5 0.008 0.005 0.018 0.013]

observation\_noise\_variance\_champ:0 is 0.07

## bias\_mean:0 is 0.751

Iteration 201

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.026 with predicted mean of [-1.799]

The next parameters to simulate from are [[0.147 0.437 0.006 0.01 0.023 0.023]]

The mean of the samples was -1.895

Iteration 202

Acquisition function convergence reached at iteration 114.

The final EI loss was -0.025 with predicted mean of [-1.805]

The next parameters to simulate from are [[0.147 0.435 0.006 0.01 0.023 0.023]]

The mean of the samples was -1.752

Iteration 203

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.008 with predicted mean of [-1.093]

The next parameters to simulate from are [[0.171 0.631 0.009 0.007 0.041 0.031]]

The mean of the samples was -1.09

Iteration 204

Acquisition function convergence reached at iteration 38.

The final EI loss was -0.001 with predicted mean of [-0.073]

The next parameters to simulate from are [[0.999 0.905 0.006 0.016 0.046 0.001]]

The mean of the samples was 0.336

Iteration 205

Acquisition function convergence reached at iteration 41.

The final EI loss was -0.003 with predicted mean of [-0.561]

The next parameters to simulate from are [[0.059 0.018 0.03 0.009 0.049 0.053]]

The mean of the samples was -0.762

Iteration 206

Acquisition function convergence reached at iteration 77.

The final EI loss was -0.023 with predicted mean of [-1.798]

The next parameters to simulate from are [[0.145 0.527 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.679

Iteration 207

Acquisition function convergence reached at iteration 96.

The final EI loss was -0.014 with predicted mean of [-1.533]

The next parameters to simulate from are [[0.31 0.221 0.011 0.01 0.027 0.024]]

The mean of the samples was -1.421

Iteration 208

Acquisition function convergence reached at iteration 56.

The final EI loss was -0.024 with predicted mean of [-1.796]

The next parameters to simulate from are [[0.143 0.438 0.006 0.01 0.024 0.023]]

The mean of the samples was -1.55

Iteration 209

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.608]

The next parameters to simulate from are [[0.143 0.438 0.006 0.027 0.024 0.023]]

The mean of the samples was 0.585

Iteration 210

Acquisition function convergence reached at iteration 72.

The final EI loss was -0.003 with predicted mean of [-0.611]

The next parameters to simulate from are [[0.008 0.692 0.002 0.003 0.05 0.033]]

The mean of the samples was -1.178

Iteration 211

Acquisition function convergence reached at iteration 51.

The final EI loss was -0.015 with predicted mean of [-1.244]

The next parameters to simulate from are [[0.302 0.997 0.003 0.01 0.042 0.015]]

The mean of the samples was -1.602

Iteration 212

Acquisition function convergence reached at iteration 35.

The final EI loss was -0.016 with predicted mean of [-1.402]

The next parameters to simulate from are [[0.31 0.981 0.005 0.01 0.046 0.015]]

The mean of the samples was -1.618

Iteration 213

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.056]

The next parameters to simulate from are [[0.143 0.524 0.006 0.01 0.001 0.015]]

The mean of the samples was -0.291

Iteration 214

Acquisition function convergence reached at iteration 67.

The final EI loss was -0.013 with predicted mean of [-1.327]

The next parameters to simulate from are [[0.506 0.444 0.011 0.012 0.031 0.014]]

The mean of the samples was -1.489

Iteration 215

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.326]

The next parameters to simulate from are [[0.844 0.413 0.006 0.01 0.024 0.048]]

The mean of the samples was 0.355

Iteration 216

Acquisition function convergence reached at iteration 44.

The final EI loss was -0.002 with predicted mean of [-0.147]

The next parameters to simulate from are [[0.013 0.008 0.015 0.001 0.05 0.066]]

The mean of the samples was 0.011

Iteration 217

Acquisition function convergence reached at iteration 74.

The final EI loss was -0.022 with predicted mean of [-1.769]

The next parameters to simulate from are [[0.141 0.611 0.006 0.01 0.023 0.022]]

The mean of the samples was -1.781

Iteration 218

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.456]

The next parameters to simulate from are [[0.034 0.98 0.006 0.029 0.013 0.023]]

The mean of the samples was 0.598

Iteration 219

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.573]

The next parameters to simulate from are [[0.145 0.196 0.017 0.01 0.023 0.016]]

The mean of the samples was -0.51

Iteration 220

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.858]

The next parameters to simulate from are [[0.148 0.772 0.017 0.01 0.024 0.023]]

The mean of the samples was -0.841

Hyperparameter convergence reached at iteration 1915.

The minimum predicted mean of the observed indices is -1.79 at the point

[0.143 0.438 0.006 0.01 0.024 0.023]

Iteration 221

WARNING:tensorflow:5 out of the last 81 calls to <function update\_var\_EI.<locals>.opt\_var at Acquisition function convergence reached at iteration 88.

The final EI loss was -0.004 with predicted mean of [-0.579]

The next parameters to simulate from are [[0.001 0.985 0.033 0.016 0.02 0.056]]

The mean of the samples was -1.009

Iteration 222

Acquisition function convergence reached at iteration 58.

The final EI loss was -0.001 with predicted mean of [0.093]

The next parameters to simulate from are [[0.998 0.016 0. 0.033 0.049 0.066]]

The mean of the samples was 1.171

Iteration 223

Acquisition function convergence reached at iteration 44.

The final EI loss was -0.006 with predicted mean of [-0.92]

The next parameters to simulate from are [[0.23 0.26 0.016 0.008 0.045 0.029]]

The mean of the samples was -1.084

Iteration 224

Acquisition function convergence reached at iteration 56.

The final EI loss was -0.001 with predicted mean of [0.14]

The next parameters to simulate from are [[0.181 0.008 0. 0. 0.001 0.066]]

The mean of the samples was 0.647

Iteration 225

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.612]

The next parameters to simulate from are [[0.143 0.868 0.006 0.049 0.023 0.008]]

The mean of the samples was 1.769

Iteration 226

Acquisition function convergence reached at iteration 74.

The final EI loss was -0.023 with predicted mean of [-1.766]

The next parameters to simulate from are [[0.134 0.669 0.007 0.01 0.023 0.022]]

The mean of the samples was -1.732

Iteration 227

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.322]

The next parameters to simulate from are [[0.143 0.548 0.004 0.01 0.024 0.052]]

The mean of the samples was -0.356

Iteration 228

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.224]

The next parameters to simulate from are [[0.196 0.993 0.015 0.046 0.031 0.023]]

The mean of the samples was 1.488

Iteration 229

Acquisition function convergence reached at iteration 67.

The final EI loss was -0.021 with predicted mean of [-1.793]

The next parameters to simulate from are [[0.159 0.445 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.673

Iteration 230

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.148]

The next parameters to simulate from are [[0.879 0.038 0.006 0.009 0.019 0.034]]

The mean of the samples was 0.182

Iteration 231

Acquisition function convergence reached at iteration 58.

The final EI loss was -0.009 with predicted mean of [-1.327]

The next parameters to simulate from are [[0.61 0.336 0.014 0.013 0.027 0.012]]

The mean of the samples was -1.31

Iteration 232

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.007]

The next parameters to simulate from are [[0.176 0.129 0.006 0.019 0.023 0.022]]

The mean of the samples was 0.011

Iteration 233

Acquisition function convergence reached at iteration 17.

The final EI loss was -0.002 with predicted mean of [-0.434]

The next parameters to simulate from are [[0.029 0.035 0.016 0.021 0.002 0.039]]

The mean of the samples was -0.636

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.485]

The next parameters to simulate from are [[0.142 0.517 0.006 0.01 0.007 0.022]]

The mean of the samples was -0.488

Iteration 235

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.274]

The next parameters to simulate from are [[0.224 0.409 0.015 0.041 0.024 0.015]]

The mean of the samples was 1.321

Iteration 236

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.002 with predicted mean of [-0.509]

The next parameters to simulate from are [[0.022 0.007 0.026 0.014 0.05 0.058]]

The mean of the samples was -0.149

Iteration 237

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.016 with predicted mean of [-1.444]

The next parameters to simulate from are [[0.289 0.985 0.002 0.01 0.047 0.017]]

The mean of the samples was -1.176

Iteration 238

Acquisition function convergence reached at iteration 56.

The final EI loss was -0.004 with predicted mean of [-0.553]

The next parameters to simulate from are [[0.081 0.878 0.033 0.019 0.003 0.054]]

The mean of the samples was -0.868

Iteration 239

Acquisition function convergence reached at iteration 108.

The final EI loss was -0.01 with predicted mean of [-1.53]

The next parameters to simulate from are [[0.398 0.32 0.011 0.012 0.025 0.019]]

The mean of the samples was -1.431

Iteration 240

Acquisition function convergence reached at iteration 40.

The final EI loss was -0.005 with predicted mean of [-1.459]

The next parameters to simulate from are [[0.531 0.249 0.013 0.011 0.028 0.015]]

The mean of the samples was -1.455

Hyperparameter convergence reached at iteration 1899.

The minimum predicted mean of the observed indices is -1.796 at the point

[0.142 0.517 0.006 0.01 0.024 0.022]

Iteration 241

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.247]

The next parameters to simulate from are [[0.77 0.519 0.006 0.006 0.024 0.022]]

The mean of the samples was 0.191

Acquisition function convergence reached at iteration 61.

The final EI loss was -0.001 with predicted mean of [0.155]

The next parameters to simulate from are [[0.997 0.994 0.012 0. 0. 0.065]]

The mean of the samples was 0.597

Iteration 243

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.421]

The next parameters to simulate from are [[0.142 0.816 0.016 0.045 0.008 0.008]]

The mean of the samples was 1.566

Iteration 244

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.426]

The next parameters to simulate from are [[0.142 0.726 0.006 0.049 0.024 0.026]]

The mean of the samples was 1.577

Iteration 245

Acquisition function convergence reached at iteration 30.

The final EI loss was -0.001 with predicted mean of [-0.329]

The next parameters to simulate from are [[0.334 0.019 0.019 0.011 0.035 0.067]]

The mean of the samples was -0.344

Iteration 246

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.019 with predicted mean of [-1.525]

The next parameters to simulate from are [[0.345 0.824 0.006 0.01 0.04 0.015]]

The mean of the samples was -1.448

Iteration 247

Acquisition function convergence reached at iteration 91.

The final EI loss was -0.011 with predicted mean of [-1.346]

The next parameters to simulate from are [[0.394 0.003 0.014 0.011 0.028 0.025]]

The mean of the samples was -1.373

Iteration 248

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.142]

The next parameters to simulate from are [[0.142 0.517 0.006 0.021 0.024 0.033]]

The mean of the samples was 0.225

Iteration 249

Acquisition function convergence reached at iteration 16.

The final EI loss was -0.005 with predicted mean of [-0.961]

The next parameters to simulate from are [[0.489 0.075 0.022 0.012 0.036 0.026]]

The mean of the samples was -0.998

Iteration 250

Acquisition function convergence reached at iteration 59.

The final EI loss was -0.006 with predicted mean of [-0.897]

The next parameters to simulate from are [[0.002 0.495 0.033 0.015 0.016 0.057]]

The mean of the samples was -1.018

Trained parameters:

amplitude\_champ:0 is 0.703

observation\_noise\_variance\_champ:0 is 0.072

bias\_mean:0 is 0.742

Iteration 251

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.357]

The next parameters to simulate from are [[0.996 0.517 0.013 0.01 0.024 0.022]]

The mean of the samples was 0.374

Iteration 252

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.39]

The next parameters to simulate from are [[0.967 0.605 0.006 0.032 0.009 0.022]]

The mean of the samples was 0.501

Iteration 253

Acquisition function convergence reached at iteration 89.

The final EI loss was -0.015 with predicted mean of [-1.442]

The next parameters to simulate from are [[0.252 0.99 0.003 0.01 0.04 0.012]]

The mean of the samples was -1.418

Iteration 254

Acquisition function convergence reached at iteration 23.

The final EI loss was -0.004 with predicted mean of [-0.805]

The next parameters to simulate from are [[0.432 0.965 0.029 0.016 0.034 0.029]]

The mean of the samples was -0.916

Iteration 255

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.61]

The next parameters to simulate from are [[0.686 0.517 0.006 0.037 0.024 0.034]]

The mean of the samples was 0.657

Iteration 256

Acquisition function convergence reached at iteration 81.

The final EI loss was -0.022 with predicted mean of [-1.789]

The next parameters to simulate from are [[0.15 0.575 0.006 0.01 0.024 0.023]]

The mean of the samples was -1.845

Iteration 257

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.673]

The next parameters to simulate from are [[0.142 0.231 0.006 0.028 0.024 0.022]]

The mean of the samples was 0.681

Iteration 258

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.32]

The next parameters to simulate from are [[0.869 0.517 0.015 0.01 0.024 0.041]]

The mean of the samples was 0.274

Iteration 259

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.546]

The next parameters to simulate from are [[0.142 0.186 0.026 0.01 0.024 0.022]]

The mean of the samples was -0.551

Iteration 260

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.242]

The next parameters to simulate from are [[0.142 0.06 0.006 0.022 0.024 0.022]]

The mean of the samples was 0.23

Hyperparameter convergence reached at iteration 1911.

The minimum predicted mean of the observed indices is -1.795 at the point

[0.142 0.517 0.006 0.01 0.024 0.022]

Iteration 261

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.325]

The next parameters to simulate from are [[0.724 0.506 0.006 0.03 0.004 0.023]]

The mean of the samples was 0.207

Iteration 262

Acquisition function convergence reached at iteration 75.

The final EI loss was -0.02 with predicted mean of [-1.795]

The next parameters to simulate from are [[0.156 0.515 0.006 0.01 0.024 0.023]]

The mean of the samples was -1.746

Iteration 263

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.812]

The next parameters to simulate from are [[0.86 0.515 0.006 0.042 0.024 0.023]]

The mean of the samples was 0.839

Iteration 264

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.248]

The next parameters to simulate from are [[0.157 0.274 0.006 0.01 0.024 0.061]]

The mean of the samples was -0.321

Iteration 265

Acquisition function convergence reached at iteration 126.

The final EI loss was -0.02 with predicted mean of [-1.794]

The next parameters to simulate from are [[0.156 0.515 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.734

Iteration 266

Acquisition function convergence reached at iteration 85.

The final EI loss was -0.02 with predicted mean of [-1.786]

The next parameters to simulate from are [[0.145 0.604 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.873

Iteration 267

Acquisition function convergence reached at iteration 62.

The final EI loss was -0.02 with predicted mean of [-1.789]

The next parameters to simulate from are [[0.148 0.607 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.959

Iteration 268

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.323]

The next parameters to simulate from are [[0.146 0.515 0.006 0.01 0.024 0.06 ]]

The mean of the samples was -0.305

Iteration 269

Acquisition function convergence reached at iteration 55.

The final EI loss was -0.021 with predicted mean of [-1.797]

The next parameters to simulate from are [[0.147 0.617 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.92

Iteration 270

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.072]

The next parameters to simulate from are [[0.338 0.577 0.001 0.01 0.024 0.031]]

The mean of the samples was -0.104

Iteration 271

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.644]

The next parameters to simulate from are [[0.15 0.575 0.006 0.01 0.024 0.044]]

The mean of the samples was -0.641

Iteration 272

Acquisition function convergence reached at iteration 30.

The final EI loss was -0.003 with predicted mean of [-0.56]

The next parameters to simulate from are [[0.013 0.917 0.03 0.014 0.004 0.065]]

The mean of the samples was -0.377

Iteration 273

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.022 with predicted mean of [-1.802]

The next parameters to simulate from are [[0.145 0.62 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.725

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.317]

The next parameters to simulate from are [[0.691 0.536 0.001 0.01 0.024 0.059]]

The mean of the samples was 0.426

Iteration 275

Acquisition function convergence reached at iteration 70.

The final EI loss was -0.021 with predicted mean of [-1.798]

The next parameters to simulate from are [[0.144 0.621 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.74

Iteration 276

Acquisition function convergence reached at iteration 44.

The final EI loss was -0.004 with predicted mean of [-0.975]

The next parameters to simulate from are [[0.514 0.52 0.02 0.014 0.032 0.018]]

The mean of the samples was -1.07

Iteration 277

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.343]

The next parameters to simulate from are [[0.702 0.207 0.006 0.01 0.024 0.022]]

The mean of the samples was -0.366

Iteration 278

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.289]

The next parameters to simulate from are [[0.894 0.387 0.006 0.01 0.011 0.022]]

The mean of the samples was 0.28

Iteration 279

Acquisition function convergence reached at iteration 63.

The final EI loss was -0.019 with predicted mean of [-1.799]

The next parameters to simulate from are [[0.149 0.595 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.765

Iteration 280

Acquisition function convergence reached at iteration 77.

The final EI loss was -0.006 with predicted mean of [-1.363]

The next parameters to simulate from are [[0.385 0.919 0.002 0.011 0.038 0.013]]

The mean of the samples was -1.154

Hyperparameter convergence reached at iteration 1948.

The minimum predicted mean of the observed indices is -1.803 at the point

[0.156 0.515 0.006 0.01 0.024 0.022]

Iteration 281

Acquisition function convergence reached at iteration 54.

The final EI loss was -0.0 with predicted mean of [0.106]

The next parameters to simulate from are  $[[0.005 \ 0.013 \ 0.$  0. 0.04 0.001]]

The mean of the samples was 0.348

Acquisition function convergence reached at iteration 70.

The final EI loss was -0.004 with predicted mean of [-0.841]

The next parameters to simulate from are [[0.19 0.288 0.033 0.018 0.012 0.047]]

The mean of the samples was -0.873

Iteration 283

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-1.389]

The next parameters to simulate from are [[0.159 0.521 0.007 0.009 0.034 0.023]]

The mean of the samples was -1.495

Iteration 284

Acquisition function convergence reached at iteration 58.

The final EI loss was -0.003 with predicted mean of [-0.801]

The next parameters to simulate from are [[0.028 0.975 0.033 0.011 0.028 0.055]]

The mean of the samples was -0.816

Iteration 285

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.027]

The next parameters to simulate from are [[0.167 0.533 0.01 0.009 0.003 0.037]]

The mean of the samples was 0.048

Iteration 286

Acquisition function convergence reached at iteration 67.

The final EI loss was -0.002 with predicted mean of [-0.379]

The next parameters to simulate from are [[0.006 0.971 0.033 0.006 0.037 0.067]]

The mean of the samples was -0.474

Iteration 287

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.009 with predicted mean of [-1.556]

The next parameters to simulate from are [[0.177 0.132 0.009 0.011 0.02 0.025]]

The mean of the samples was -1.788

Iteration 288

Acquisition function convergence reached at iteration 38.

The final EI loss was -0.011 with predicted mean of [-1.433]

The next parameters to simulate from are [[0.223 0.958 0.005 0.009 0.039 0.018]]

The mean of the samples was -1.675

Iteration 289

Acquisition function convergence reached at iteration 67.

The final EI loss was -0.003 with predicted mean of [-1.173]

The next parameters to simulate from are [[0.529 0.525 0.006 0.013 0.029 0.012]]

The mean of the samples was -1.571

Iteration 290

Acquisition function convergence reached at iteration 42.

The final EI loss was -0.0 with predicted mean of [0.343]

The next parameters to simulate from are [[0.007 0.992 0.019 0.025 0.049 0. ]]

The mean of the samples was 0.902

Iteration 291

Acquisition function convergence reached at iteration 61.

The final EI loss was -0.001 with predicted mean of [-0.076]

The next parameters to simulate from are [[0.971 0.969 0.032 0.013 0. 0.002]]

The mean of the samples was -0.303

Iteration 292

Acquisition function convergence reached at iteration 88.

The final EI loss was -0.004 with predicted mean of [-0.722]

The next parameters to simulate from are [[0.013 0.619 0.024 0.019 0. 0.044]]

The mean of the samples was -0.777

Iteration 293

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.52]

The next parameters to simulate from are [[0.143 0.55 0.027 0.01 0.024 0.022]]

The mean of the samples was -0.505

Iteration 294

Acquisition function convergence reached at iteration 89.

The final EI loss was -0.01 with predicted mean of [-1.45]

The next parameters to simulate from are [[0.198 0.999 0.006 0.009 0.044 0.015]]

The mean of the samples was -1.134

Iteration 295

Acquisition function convergence reached at iteration 55.

The final EI loss was -0.02 with predicted mean of [-1.795]

The next parameters to simulate from are [[0.147 0.591 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.701

Iteration 296

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.792]

The next parameters to simulate from are [[0.488 0.364 0.006 0.01 0.023 0.023]]

The mean of the samples was -0.775

Iteration 297

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.099]

The next parameters to simulate from are [[0.15 0.244 0.006 0.04 0.034 0.039]]

The mean of the samples was 1.397

Iteration 298

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.909]

The next parameters to simulate from are [[0.771 0.575 0.003 0.042 0.029 0.023]]

The mean of the samples was 0.943

Acquisition function convergence reached at iteration 46.

The final EI loss was -0.002 with predicted mean of [-0.947]

The next parameters to simulate from are [[0.318 0.989 0.011 0.015 0.021 0.019]]

The mean of the samples was -1.317

Iteration 300

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.05]

The next parameters to simulate from are [[0.724 0.575 0.009 0.01 0.05 0.023]]

The mean of the samples was -0.13

Hyperparameter convergence reached at iteration 1978.

The minimum predicted mean of the observed indices is -1.809 at the point

[0.15 0.575 0.006 0.01 0.024 0.023]

Trained parameters:

amplitude\_champ:0 is 0.694

length\_scales\_champ:0 is [0.279 0.5 0.009 0.006 0.019 0.013]

observation\_noise\_variance\_champ:0 is 0.073

bias\_mean:0 is 0.755

Iteration 301

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.13]

The next parameters to simulate from are [[0.995 0.151 0.006 0.01 0.024 0.018]]

The mean of the samples was 0.358

Iteration 302

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.777]

The next parameters to simulate from are [[0.363 0.575 0.006 0.035 0.018 0.027]]

The mean of the samples was 0.765

Iteration 303

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.617]

The next parameters to simulate from are [[0.15 0.553 0.024 0.01 0.024 0.023]]

The mean of the samples was -0.63

Iteration 304

Acquisition function convergence reached at iteration 68.

The final EI loss was -0.009 with predicted mean of [-1.457]

The next parameters to simulate from are [[0.298 0.64 0.009 0.01 0.038 0.02 ]]

The mean of the samples was -1.511

Iteration 305

Acquisition function convergence reached at iteration 74.

The final EI loss was -0.003 with predicted mean of [-0.922]

The next parameters to simulate from are [[0.003 0.387 0.025 0.016 0.005 0.033]]

The mean of the samples was -0.992

Iteration 306

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.671]

The next parameters to simulate from are [[0.15 0.167 0.022 0.01 0.024 0.023]]

The mean of the samples was -0.677

Iteration 307

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.436]

The next parameters to simulate from are [[0.805 0.587 0.007 0.01 0.011 0.065]]

The mean of the samples was 0.438

Iteration 308

Acquisition function convergence reached at iteration 29.

The final EI loss was -0.001 with predicted mean of [-0.069]

The next parameters to simulate from are [[0.979 0.83 0.033 0.015 0.038 0.004]]

The mean of the samples was -0.009

Iteration 309

Acquisition function convergence reached at iteration 38.

The final EI loss was -0.006 with predicted mean of [-1.391]

The next parameters to simulate from are [[0.36 0.607 0.008 0.012 0.032 0.016]]

The mean of the samples was -1.619

Iteration 310

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.414]

The next parameters to simulate from are [[0.15 0.133 0.006 0.019 0.038 0.023]]

The mean of the samples was 0.287

Iteration 311

Acquisition function convergence reached at iteration 39.

The final EI loss was -0.003 with predicted mean of [-0.737]

The next parameters to simulate from are [[0.377 0.123 0.033 0.011 0.031 0.035]]

The mean of the samples was -0.967

Iteration 312

Acquisition function convergence reached at iteration 62.

The final EI loss was -0.022 with predicted mean of [-1.804]

The next parameters to simulate from are [[0.149 0.625 0.007 0.01 0.025 0.023]]

The mean of the samples was -1.617

Iteration 313

Acquisition function convergence reached at iteration 44.

The final EI loss was -0.002 with predicted mean of [-0.717]

The next parameters to simulate from are [[0.227 0.999 0.033 0.016 0.001 0.044]]

The mean of the samples was -0.585

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.565]

The next parameters to simulate from are [[0.15 0.545 0.006 0.01 0.009 0.015]]

The mean of the samples was -0.753

Iteration 315

Acquisition function convergence reached at iteration 38.

The final EI loss was -0.005 with predicted mean of [-1.165]

The next parameters to simulate from are [[0. 0.562 0.008 0.006 0.035 0.031]]

The mean of the samples was -1.343

Iteration 316

Acquisition function convergence reached at iteration 68.

The final EI loss was -0.02 with predicted mean of [-1.797]

The next parameters to simulate from are [[0.137 0.631 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.746

Iteration 317

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.29]

The next parameters to simulate from are [[0.15 0.575 0.006 0.043 0.024 0.039]]

The mean of the samples was 1.257

Iteration 318

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.116]

The next parameters to simulate from are [[0.15 0.576 0.006 0.01 0.002 0.042]]

The mean of the samples was 0.192

Iteration 319

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.235]

The next parameters to simulate from are [[0.512 0.575 0.006 0.044 0.024 0.005]]

The mean of the samples was 1.3

Iteration 320

Acquisition function convergence reached at iteration 72.

The final EI loss was -0.019 with predicted mean of [-1.796]

The next parameters to simulate from are [[0.141 0.616 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.895

Hyperparameter convergence reached at iteration 1886.

The minimum predicted mean of the observed indices is -1.805 at the point

[0.15 0.575 0.006 0.01 0.024 0.023]

Iteration 321

Acquisition function convergence reached at iteration 8.

The final EI loss was -0.003 with predicted mean of [-0.873]

The next parameters to simulate from are [[0.086 0.288 0.032 0.009 0.03 0.042]]

The mean of the samples was -0.771

Acquisition function convergence reached at iteration 18.

The final EI loss was -0.001 with predicted mean of [-0.702]

The next parameters to simulate from are [[0.054 0.32 0.001 0.006 0.05 0.032]]

The mean of the samples was -0.068

Iteration 323

Acquisition function convergence reached at iteration 46.

The final EI loss was -0.004 with predicted mean of [-1.057]

The next parameters to simulate from are [[0.006 0.369 0.014 0.006 0.048 0.034]]

The mean of the samples was -0.911

Iteration 324

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.466]

The next parameters to simulate from are [[0.15 0.575 0.006 0.042 0.044 0.031]]

The mean of the samples was 1.481

Iteration 325

Acquisition function convergence reached at iteration 40.

The final EI loss was -0.001 with predicted mean of [-0.142]

The next parameters to simulate from are [[0.989 0.023 0.033 0.018 0.002 0.01 ]]

The mean of the samples was 0.278

Iteration 326

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.308]

The next parameters to simulate from are [[0.985 0.578 0.006 0.01 0.049 0.032]]

The mean of the samples was 0.432

Iteration 327

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.521]

The next parameters to simulate from are [[0.15 0.575 0.029 0.011 0.024 0.023]]

The mean of the samples was -0.52

Iteration 328

Acquisition function convergence reached at iteration 83.

The final EI loss was -0.002 with predicted mean of [-0.445]

The next parameters to simulate from are [[0.031 0.005 0.033 0.017 0.01 0.065]]

The mean of the samples was -0.768

Iteration 329

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.321]

The next parameters to simulate from are [[0.89 0.575 0.006 0.01 0.024 0.023]]

The mean of the samples was 0.288

Iteration 330

Acquisition function convergence reached at iteration 65.

The final EI loss was -0.002 with predicted mean of [-0.598]

The next parameters to simulate from are [[0.295 0.975 0.025 0.009 0.043 0.051]]

The mean of the samples was -0.516

Iteration 331

Acquisition function convergence reached at iteration 36.

The final EI loss was -0.004 with predicted mean of [-1.414]

The next parameters to simulate from are [[0.602 0.36 0.009 0.013 0.027 0.01 ]]

The mean of the samples was -1.481

Iteration 332

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.397]

The next parameters to simulate from are [[0.15 0.821 0.006 0.01 0.024 0.054]]

The mean of the samples was -0.354

Iteration 333

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.417]

The next parameters to simulate from are [[0.15 0.575 0.006 0.01 0.023 0.053]]

The mean of the samples was -0.371

Iteration 334

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.055]

The next parameters to simulate from are [[0.315 0.625 0.006 0.04 0.024 0.039]]

The mean of the samples was 1.034

Iteration 335

Acquisition function convergence reached at iteration 84.

The final EI loss was -0.001 with predicted mean of [0.06]

The next parameters to simulate from are [[0.607 0.999 0.033 0. 0. 0.001]]

The mean of the samples was 0.471

Iteration 336

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.213]

The next parameters to simulate from are [[0.15 0.575 0.002 0.044 0.024 0.037]]

The mean of the samples was 1.309

Iteration 337

Acquisition function convergence reached at iteration 40.

The final EI loss was -0.004 with predicted mean of [-0.779]

The next parameters to simulate from are [[0.265 0.076 0.026 0.009 0.046 0.038]]

The mean of the samples was -0.928

Iteration 338

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.357]

The next parameters to simulate from are [[0.15 0.575 0.006 0.043 0.023 0.023]]

The mean of the samples was 1.355

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.995]

The next parameters to simulate from are [[0.148 0.576 0.017 0.038 0.035 0.001]]

The mean of the samples was 1.444

Iteration 340

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.002 with predicted mean of [-0.795]

The next parameters to simulate from are [[0.133 0.663 0.02 0.02 0. 0.03 ]]

The mean of the samples was -0.71

Hyperparameter convergence reached at iteration 1951.

The minimum predicted mean of the observed indices is -1.805 at the point

[0.15 0.575 0.006 0.01 0.024 0.023]

Iteration 341

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.483]

The next parameters to simulate from are [[0.15 0.303 0.009 0.01 0.024 0.056]]

The mean of the samples was -0.468

Iteration 342

Acquisition function convergence reached at iteration 59.

The final EI loss was -0.019 with predicted mean of [-1.802]

The next parameters to simulate from are [[0.139 0.585 0.006 0.01 0.024 0.023]]

The mean of the samples was -1.844

Iteration 343

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.365]

The next parameters to simulate from are  $[[0.874\ 0.626\ 0.006\ 0.01\ 0.024\ 0.039]]$ 

The mean of the samples was 0.362

Iteration 344

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.044]

The next parameters to simulate from are [[0.743 0.576 0.019 0.01 0.01 0.023]]

The mean of the samples was -0.045

Iteration 345

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.311]

The next parameters to simulate from are [[0.901 0.431 0.006 0.01 0.015 0.023]]

The mean of the samples was 0.288

Iteration 346

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.644]

The next parameters to simulate from are [[0.15 0.575 0.006 0.01 0.024 0.044]]

The mean of the samples was -0.614

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.231]

The next parameters to simulate from are [[0.15 0.496 0.006 0.039 0.024 0.023]]

The mean of the samples was 1.254

Iteration 348

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.366]

The next parameters to simulate from are [[0.259 0.599 0.013 0.009 0.024 0.053]]

The mean of the samples was -0.318

Iteration 349

Acquisition function convergence reached at iteration 66.

The final EI loss was -0.004 with predicted mean of [-1.22]

The next parameters to simulate from are [[0.45 0.625 0.013 0.014 0.024 0.017]]

The mean of the samples was -1.128

Iteration 350

Acquisition function convergence reached at iteration 42.

The final EI loss was -0.002 with predicted mean of [-0.399]

The next parameters to simulate from are [[0.346 0.904 0.031 0.023 0.001 0.044]]

The mean of the samples was -0.654

Trained parameters:

amplitude\_champ:0 is 0.684

length\_scales\_champ:0 is [0.28 0.5 0.009 0.005 0.018 0.013]

observation\_noise\_variance\_champ:0 is 0.071

bias\_mean:0 is 0.755

Iteration 351

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.018 with predicted mean of [-1.804]

The next parameters to simulate from are [[0.137 0.621 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.831

Iteration 352

Acquisition function convergence reached at iteration 42.

The final EI loss was -0.001 with predicted mean of [-0.15]

The next parameters to simulate from are [[0.342 0.04 0.031 0.022 0. 0.067]]

The mean of the samples was -0.302

Iteration 353

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.826]

The next parameters to simulate from are [[0.15 0.575 0.001 0.01 0.024 0.023]]

The mean of the samples was -0.727

Acquisition function convergence reached at iteration 51.

The final EI loss was -0.003 with predicted mean of [-1.265]

The next parameters to simulate from are [[0.559 0.458 0.006 0.013 0.021 0.011]]

The mean of the samples was -1.54

Iteration 355

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.06]

The next parameters to simulate from are [[0.286 0.221 0.032 0.016 0.022 0.008]]

The mean of the samples was 0.083

Iteration 356

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.288]

The next parameters to simulate from are [[0.15 0.575 0.005 0.04 0.024 0.016]]

The mean of the samples was 1.302

Iteration 357

Acquisition function convergence reached at iteration 54.

The final EI loss was -0.003 with predicted mean of [-0.975]

The next parameters to simulate from are [[0.216 0.999 0.015 0.017 0.012 0.024]]

The mean of the samples was -1.094

Iteration 358

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.926]

The next parameters to simulate from are [[0.479 0.575 0.025 0.04 0.019 0.023]]

The mean of the samples was 0.892

Iteration 359

Acquisition function convergence reached at iteration 35.

The final EI loss was -0.001 with predicted mean of [-0.5]

The next parameters to simulate from are [[0.158 0.632 0.024 0.008 0.035 0.066]]

The mean of the samples was -0.443

Iteration 360

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.27]

The next parameters to simulate from are [[0.15 0.245 0.001 0.044 0.01 0.023]]

The mean of the samples was 1.328

Hyperparameter convergence reached at iteration 1959.

The minimum predicted mean of the observed indices is -1.805 at the point

[0.15 0.575 0.006 0.01 0.024 0.023]

Iteration 361

Acquisition function convergence reached at iteration 47.

The final EI loss was -0.001 with predicted mean of [-0.277]

The next parameters to simulate from are [[0.079 0.984 0.033 0.023 0.001 0.067]]

The mean of the samples was -0.557

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.894]

The next parameters to simulate from are [[0.307 0.66 0.014 0.034 0.043 0.045]]

The mean of the samples was 0.987

Iteration 363

Acquisition function convergence reached at iteration 69.

The final EI loss was -0.01 with predicted mean of [-1.503]

The next parameters to simulate from are [[0.207 0.008 0.011 0.011 0.02 0.025]]

The mean of the samples was -1.625

Iteration 364

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.26]

The next parameters to simulate from are [[0.007 0.575 0.028 0.045 0.025 0.048]]

The mean of the samples was 1.423

Iteration 365

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.448]

The next parameters to simulate from are [[0.587 0.42 0.006 0.01 0.024 0.023]]

The mean of the samples was -0.429

Iteration 366

Acquisition function convergence reached at iteration 64.

The final EI loss was -0.002 with predicted mean of [-0.611]

The next parameters to simulate from are [[0.27 0.352 0.031 0.021 0.002 0.034]]

The mean of the samples was -0.951

Iteration 367

Acquisition function convergence reached at iteration 100.

The final EI loss was -0.018 with predicted mean of [-1.796]

The next parameters to simulate from are [[0.134 0.659 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.814

Iteration 368

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.808]

The next parameters to simulate from are [[0.811 0.621 0.006 0.041 0.024 0.022]]

The mean of the samples was 0.778

Iteration 369

Acquisition function convergence reached at iteration 36.

The final EI loss was -0.001 with predicted mean of [-0.07]

The next parameters to simulate from are [[0.849 0.011 0.033 0.008 0.049 0. ]]

The mean of the samples was 0.24

Iteration 370

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.034]

The next parameters to simulate from are [[0.156 0.602 0.006 0.017 0.012 0.061]]

The mean of the samples was -0.164

Iteration 371

Acquisition function convergence reached at iteration 43.

The final EI loss was -0.002 with predicted mean of [-0.684]

The next parameters to simulate from are [[0.304 0.842 0.032 0.02 0. 0.029]]

The mean of the samples was -0.91

Iteration 372

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.547]

The next parameters to simulate from are [[0.137 0.129 0.006 0.023 0.038 0.022]]

The mean of the samples was 0.63

Iteration 373

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.492]

The next parameters to simulate from are [[0.017 0.624 0.006 0.022 0.034 0.034]]

The mean of the samples was 0.602

Iteration 374

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.416]

The next parameters to simulate from are [[0.373 0.613 0.006 0.027 0.024 0.022]]

The mean of the samples was 0.468

Iteration 375

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.238]

The next parameters to simulate from are [[0.137 0.621 0.006 0.01 0.024 0.002]]

The mean of the samples was 0.245

Iteration 376

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.021 with predicted mean of [-1.797]

The next parameters to simulate from are [[0.129 0.704 0.007 0.01 0.024 0.023]]

The mean of the samples was -1.736

Iteration 377

Acquisition function convergence reached at iteration 75.

The final EI loss was -0.002 with predicted mean of [-0.632]

The next parameters to simulate from are [[0.01 0.002 0.023 0.007 0.05 0.041]]

The mean of the samples was -0.713

Iteration 378

Acquisition function convergence reached at iteration 29.

The final EI loss was -0.001 with predicted mean of [-0.536]

The next parameters to simulate from are [[0.561 0.845 0.021 0.015 0.048 0.021]]

The mean of the samples was -0.724

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.222]

The next parameters to simulate from are [[0.147 0.575 0.001 0.01 0.024 0.044]]

The mean of the samples was -0.192

Iteration 380

Acquisition function convergence reached at iteration 39.

The final EI loss was -0.002 with predicted mean of [-0.913]

The next parameters to simulate from are [[0.271 0.857 0.033 0.013 0.036 0.035]]

The mean of the samples was -0.883

Hyperparameter convergence reached at iteration 1925.

The minimum predicted mean of the observed indices is -1.803 at the point

[0.15 0.575 0.006 0.01 0.024 0.023]

Iteration 381

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.191]

The next parameters to simulate from are [[0.149 0.404 0.006 0.003 0.024 0.023]]

The mean of the samples was -0.178

Iteration 382

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.1]

The next parameters to simulate from are [[0.136 0.575 0.014 0.01 0.024 0.006]]

The mean of the samples was -0.0

Iteration 383

Acquisition function convergence reached at iteration 42.

The final EI loss was -0.006 with predicted mean of [-1.319]

The next parameters to simulate from are [[0.014 0.995 0.007 0.011 0.022 0.023]]

The mean of the samples was -1.298

Iteration 384

Acquisition function convergence reached at iteration 47.

The final EI loss was -0.006 with predicted mean of [-1.384]

The next parameters to simulate from are [[0.332 0.444 0.012 0.009 0.04 0.022]]

The mean of the samples was -1.371

Iteration 385

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.017 with predicted mean of [-1.803]

The next parameters to simulate from are [[0.141 0.583 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.739

Iteration 386

Acquisition function convergence reached at iteration 76.

The final EI loss was -0.017 with predicted mean of [-1.806]

The next parameters to simulate from are [[0.148 0.569 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.834

Acquisition function convergence reached at iteration 34.

The final EI loss was -0.016 with predicted mean of [-1.801]

The next parameters to simulate from are [[0.138 0.631 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.867

Iteration 388

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.378]

The next parameters to simulate from are [[0.148 0.569 0.006 0.01 0.032 0.054]]

The mean of the samples was -0.374

Iteration 389

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.771]

The next parameters to simulate from are [[0.715 0.569 0.006 0.036 0.033 0.022]]

The mean of the samples was 0.817

Iteration 390

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.956]

The next parameters to simulate from are [[0.148 0.569 0.006 0.01 0.043 0.022]]

The mean of the samples was -1.059

Iteration 391

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.016 with predicted mean of [-1.794]

The next parameters to simulate from are [[0.134 0.693 0.007 0.01 0.025 0.023]]

The mean of the samples was -1.604

Iteration 392

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.361]

The next parameters to simulate from are [[0.67 0.051 0.006 0.049 0.039 0.012]]

The mean of the samples was 1.617

Iteration 393

Acquisition function convergence reached at iteration 82.

The final EI loss was -0.016 with predicted mean of [-1.805]

The next parameters to simulate from are [[0.152 0.539 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.714

Iteration 394

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-1.285]

The next parameters to simulate from are [[0.152 0.539 0.006 0.01 0.036 0.022]]

The mean of the samples was -1.195

Iteration 395

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.17]

The next parameters to simulate from are [[0.152 0.539 0.029 0.043 0.014 0.022]]

The mean of the samples was 1.298

Iteration 396

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.391]

The next parameters to simulate from are [[0.152 0.999 0.006 0.01 0.024 0.051]]

The mean of the samples was -0.358

Iteration 397

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.002 with predicted mean of [-0.936]

The next parameters to simulate from are [[0.496 0.017 0.028 0.012 0.025 0.026]]

The mean of the samples was -1.081

Iteration 398

Acquisition function convergence reached at iteration 91.

The final EI loss was -0.006 with predicted mean of [-1.401]

The next parameters to simulate from are [[0.474 0. 0.014 0.01 0.03 0.02]]

The mean of the samples was -1.607

Iteration 399

Acquisition function convergence reached at iteration 86.

The final EI loss was -0.014 with predicted mean of [-1.771]

The next parameters to simulate from are [[0.129 0.754 0.007 0.01 0.025 0.023]]

The mean of the samples was -1.718

Iteration 400

Acquisition function convergence reached at iteration 52.

The final EI loss was -0.002 with predicted mean of [-0.827]

The next parameters to simulate from are [[0.438 0.005 0.028 0.013 0.041 0.035]]

The mean of the samples was -0.673

Hyperparameter convergence reached at iteration 1957.

The minimum predicted mean of the observed indices is -1.803 at the point

[0.152 0.539 0.006 0.01 0.024 0.022]

Trained parameters:

amplitude\_champ:0 is 0.671

observation\_noise\_variance\_champ:0 is 0.071

bias\_mean:0 is 0.746

Iteration 401

Acquisition function convergence reached at iteration 62.

The final EI loss was -0.001 with predicted mean of [0.036]

The next parameters to simulate from are  $[[0.944\ 0.97\ 0.001\ 0.\ 0.\ ]]$ 

The mean of the samples was 0.376

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.323]

The next parameters to simulate from are [[0.152 0.539 0.006 0.041 0.024 0.021]]

The mean of the samples was 1.29

Iteration 403

Acquisition function convergence reached at iteration 58.

The final EI loss was -0.007 with predicted mean of [-1.488]

The next parameters to simulate from are [[0.268 0.002 0.013 0.011 0.022 0.027]]

The mean of the samples was -1.525

Iteration 404

Acquisition function convergence reached at iteration 57.

The final EI loss was -0.016 with predicted mean of [-1.804]

The next parameters to simulate from are [[0.152 0.534 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.777

Iteration 405

Acquisition function convergence reached at iteration 28.

The final EI loss was -0.015 with predicted mean of [-1.8]

The next parameters to simulate from are [[0.146 0.566 0.006 0.01 0.024 0.022]]

The mean of the samples was -2.03

Iteration 406

Acquisition function convergence reached at iteration 115.

The final EI loss was -0.016 with predicted mean of [-1.804]

The next parameters to simulate from are [[0.155 0.548 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.866

Iteration 407

Acquisition function convergence reached at iteration 28.

The final EI loss was -0.001 with predicted mean of [-0.65]

The next parameters to simulate from are [[0.529 0.25 0.019 0.01 0.046 0.015]]

The mean of the samples was -0.879

Iteration 408

Acquisition function convergence reached at iteration 50.

The final EI loss was -0.015 with predicted mean of [-1.804]

The next parameters to simulate from are [[0.155 0.542 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.877

Iteration 409

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.098]

The next parameters to simulate from are [[0.529 0.83 0.008 0.01 0.024 0.046]]

The mean of the samples was 0.125

Iteration 410

Acquisition function convergence reached at iteration 65.

The final EI loss was -0.017 with predicted mean of [-1.81]

The next parameters to simulate from are [[0.168 0.534 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.691

Iteration 411

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.314]

The next parameters to simulate from are [[0.155 0.542 0.006 0.023 0.024 0.056]]

The mean of the samples was 0.338

Iteration 412

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.801]

The next parameters to simulate from are [[0.693 0.542 0.006 0.045 0.026 0.062]]

The mean of the samples was 0.778

Iteration 413

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.717]

The next parameters to simulate from are [[0.462 0.542 0.006 0.01 0.024 0.022]]

The mean of the samples was -0.664

Iteration 414

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.004 with predicted mean of [-1.225]

The next parameters to simulate from are [[0.367 0.741 0.012 0.011 0.047 0.02 ]]

The mean of the samples was -1.293

Iteration 415

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.542]

The next parameters to simulate from are [[0.155 0.542 0.026 0.01 0.024 0.022]]

The mean of the samples was -0.542

Iteration 416

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.8]

The next parameters to simulate from are [[0.155 0.542 0.017 0.01 0.024 0.022]]

The mean of the samples was -0.832

Iteration 417

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.381]

The next parameters to simulate from are [[0.494 0.542 0.006 0.026 0.024 0.022]]

The mean of the samples was 0.34

Iteration 418

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.905]

The next parameters to simulate from are [[0.129 0.542 0.006 0.036 0.024 0.065]]

The mean of the samples was 0.959

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.922]

The next parameters to simulate from are [[0.155 0.743 0.006 0.035 0.024 0.065]]

The mean of the samples was 0.855

Iteration 420

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.346]

The next parameters to simulate from are [[0.965 0.542 0.006 0.004 0.024 0.022]]

The mean of the samples was 0.384

Hyperparameter convergence reached at iteration 1961.

The minimum predicted mean of the observed indices is -1.812 at the point

[0.155 0.542 0.006 0.01 0.024 0.022]

Iteration 421

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.361]

The next parameters to simulate from are [[0.162 0.203 0.006 0.038 0.003 0.064]]

The mean of the samples was 0.332

Iteration 422

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.378]

The next parameters to simulate from are [[0.155 0.542 0.005 0.01 0.024 0.053]]

The mean of the samples was -0.388

Iteration 423

Acquisition function convergence reached at iteration 59.

The final EI loss was -0.001 with predicted mean of [-0.069]

The next parameters to simulate from are [[0.419 0.999 0.032 0.028 0. 0.066]]

The mean of the samples was -0.289

Iteration 424

Acquisition function convergence reached at iteration 29.

The final EI loss was -0.016 with predicted mean of [-1.806]

The next parameters to simulate from are [[0.169 0.544 0.006 0.01 0.024 0.022]]

The mean of the samples was -1.637

Iteration 425

Acquisition function convergence reached at iteration 54.

The final EI loss was -0.0 with predicted mean of [-0.002]

The next parameters to simulate from are [[0.988 0.026 0.033 0.007 0.003 0. ]]

The mean of the samples was 0.226

Iteration 426

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.33]

The next parameters to simulate from are [[0.152 0.786 0.008 0.01 0.009 0.036]]

The mean of the samples was -0.238

Acquisition function convergence reached at iteration 48.

The final EI loss was -0.0 with predicted mean of [0.07]

The next parameters to simulate from are [[0.964 0.026 0.019 0. 0.049 0.001]]

The mean of the samples was 0.321

Iteration 428

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.019]

The next parameters to simulate from are [[0.269 0.75 0.006 0.01 0.001 0.023]]

The mean of the samples was 0.032

Iteration 429

Acquisition function convergence reached at iteration 46.

The final EI loss was -0.004 with predicted mean of [-1.076]

The next parameters to simulate from are [[0.397 0.31 0.012 0.008 0.05 0.021]]

The mean of the samples was -1.297

Iteration 430

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.402]

The next parameters to simulate from are [[0.152 0.212 0.016 0.045 0.024 0.022]]

The mean of the samples was 1.461

Iteration 431

Acquisition function convergence reached at iteration 40.

The final EI loss was -0.015 with predicted mean of [-1.808]

The next parameters to simulate from are [[0.156 0.537 0.006 0.01 0.024 0.022]]

The mean of the samples was -2.056

Iteration 432

Acquisition function convergence reached at iteration 58.

The final EI loss was -0.002 with predicted mean of [-0.851]

The next parameters to simulate from are [[0.533 0.477 0.033 0.014 0.034 0.033]]

The mean of the samples was -0.886

Iteration 433

Acquisition function convergence reached at iteration 40.

The final EI loss was -0.007 with predicted mean of [-1.4]

The next parameters to simulate from are [[0.084 0.991 0.007 0.009 0.031 0.025]]

The mean of the samples was -1.703

Iteration 434

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-1.157]

The next parameters to simulate from are [[0.152 0.415 0.003 0.01 0.024 0.022]]

The mean of the samples was -1.022

Iteration 435

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.661]

The next parameters to simulate from are [[0.152 0.058 0.014 0.028 0.024 0.022]]

The mean of the samples was 0.692

Iteration 436

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.463]

The next parameters to simulate from are [[0.152 0.534 0.03 0.01 0.024 0.022]]

The mean of the samples was -0.48

Iteration 437

Acquisition function convergence reached at iteration 49.

The final EI loss was -0.015 with predicted mean of [-1.743]

The next parameters to simulate from are [[0.112 0.817 0.007 0.01 0.026 0.025]]

The mean of the samples was -1.877

Iteration 438

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.54]

The next parameters to simulate from are [[0.15 0.199 0.028 0.01 0.024 0.023]]

The mean of the samples was -0.553

Iteration 439

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.667]

The next parameters to simulate from are [[0.612 0.575 0.006 0.042 0.024 0.066]]

The mean of the samples was 0.701

Iteration 440

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.751]

The next parameters to simulate from are [[0.15 0.165 0.02 0.01 0.024 0.023]]

The mean of the samples was -0.786

Hyperparameter convergence reached at iteration 1939.

The minimum predicted mean of the observed indices is -1.812 at the point

[0.152 0.534 0.006 0.01 0.024 0.022]

Iteration 441

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.874]

The next parameters to simulate from are [[0.634 0.545 0.006 0.01 0.024 0.007]]

The mean of the samples was -0.901

Iteration 442

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.612]

The next parameters to simulate from are [[0.152 0.534 0.006 0.048 0.024 0.022]]

The mean of the samples was 1.555

Iteration 443

Acquisition function convergence reached at iteration 29.

The final EI loss was -0.001 with predicted mean of [-0.332]

The next parameters to simulate from are [[0.683 0.063 0.02 0.017 0. 0.015]]

The mean of the samples was -0.6

Iteration 444

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.127]

The next parameters to simulate from are [[0.439 0.534 0.009 0.044 0.015 0.022]]

The mean of the samples was 1.055

Iteration 445

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.347]

The next parameters to simulate from are [[0.153 0.984 0.006 0.026 0.024 0.06 ]]

The mean of the samples was 0.385

Iteration 446

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.55]

The next parameters to simulate from are [[0.152 0.528 0.006 0.01 0.022 0.047]]

The mean of the samples was -0.504

Iteration 447

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.396]

The next parameters to simulate from are [[0.152 0.534 0.006 0.01 0.006 0.022]]

The mean of the samples was -0.397

Iteration 448

Acquisition function convergence reached at iteration 52.

The final EI loss was -0.009 with predicted mean of [-1.698]

The next parameters to simulate from are [[0.175 0.195 0.008 0.01 0.021 0.025]]

The mean of the samples was -1.649

Iteration 449

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.911]

The next parameters to simulate from are [[0.289 0.62 0.015 0.01 0.024 0.022]]

The mean of the samples was -0.924

Iteration 450

Acquisition function convergence reached at iteration 35.

The final EI loss was -0.001 with predicted mean of [-1.122]

The next parameters to simulate from are [[0.085 0.344 0.02 0.014 0.019 0.036]]

The mean of the samples was -1.296

Trained parameters:

amplitude\_champ:0 is 0.661

length\_scales\_champ:0 is [0.276 0.5 0.009 0.005 0.018 0.013]

observation\_noise\_variance\_champ:0 is 0.072

bias\_mean:0 is 0.744

Iteration 451

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.194]

The next parameters to simulate from are [[0.152 0.82 0.006 0.046 0.002 0.022]]

The mean of the samples was 1.277

Iteration 452

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.641]

The next parameters to simulate from are [[0.261 0.534 0.006 0.036 0.016 0.051]]

The mean of the samples was 0.684

Iteration 453

Acquisition function convergence reached at iteration 80.

The final EI loss was -0.018 with predicted mean of [-1.758]

The next parameters to simulate from are [[0.112 0.823 0.007 0.01 0.026 0.025]]

The mean of the samples was -1.672

Iteration 454

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.896]

The next parameters to simulate from are [[0.156 0.537 0.003 0.032 0.031 0.05 ]]

The mean of the samples was 1.026

Iteration 455

Acquisition function convergence reached at iteration 77.

The final EI loss was -0.015 with predicted mean of [-1.813]

The next parameters to simulate from are [[0.153 0.555 0.006 0.01 0.024 0.023]]

The mean of the samples was -1.943

Iteration 456

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.062]

The next parameters to simulate from are [[0.828 0.555 0.014 0.01 0.024 0.023]]

The mean of the samples was 0.044

Iteration 457

Acquisition function convergence reached at iteration 55.

The final EI loss was -0.004 with predicted mean of [-1.526]

The next parameters to simulate from are [[0.152 0. 0.01 0.011 0.02 0.025]]

The mean of the samples was -1.824

Iteration 458

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.383]

The next parameters to simulate from are [[0.153 0.555 0.006 0.01 0.021 0.052]]

The mean of the samples was -0.397

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.173]

The next parameters to simulate from are [[0.153 0.126 0.006 0.01 0.015 0.059]]

The mean of the samples was -0.179

Iteration 460

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.162]

The next parameters to simulate from are [[0.447 0.555 0.006 0.01 0.007 0.037]]

The mean of the samples was 0.13

Hyperparameter convergence reached at iteration 1964.

The minimum predicted mean of the observed indices is -1.816 at the point

[0.153 0.555 0.006 0.01 0.024 0.023]

Iteration 461

Acquisition function convergence reached at iteration 71.

The final EI loss was -0.007 with predicted mean of [-1.66]

The next parameters to simulate from are [[0.168 0.013 0.01 0.011 0.021 0.025]]

The mean of the samples was -1.534

Iteration 462

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-1.016]

The next parameters to simulate from are [[0.153 0.667 0.002 0.01 0.024 0.023]]

The mean of the samples was -0.993

Iteration 463

Acquisition function convergence reached at iteration 36.

The final EI loss was -0.001 with predicted mean of [-0.83]

The next parameters to simulate from are [[0.199 0.03 0.025 0.008 0.035 0.046]]

The mean of the samples was -0.762

Iteration 464

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.332]

The next parameters to simulate from are [[0.085 0.555 0.006 0.01 0.024 0.001]]

The mean of the samples was 0.339

Iteration 465

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.055]

The next parameters to simulate from are [[0.036 0.555 0.006 0.035 0.024 0.023]]

The mean of the samples was 1.114

Iteration 466

Acquisition function convergence reached at iteration 40.

The final EI loss was -0.004 with predicted mean of [-1.321]

The next parameters to simulate from are [[0.343 0.739 0.006 0.01 0.049 0.014]]

The mean of the samples was -1.31

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.486]

The next parameters to simulate from are [[0.622 0.274 0.006 0.01 0.024 0.023]]

The mean of the samples was -0.49

Iteration 468

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.381]

The next parameters to simulate from are [[0.94 0.697 0.006 0.01 0.033 0.023]]

The mean of the samples was 0.339

Iteration 469

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-1.075]

The next parameters to simulate from are [[0.237 0.622 0.006 0.007 0.025 0.024]]

The mean of the samples was -0.878

Iteration 470

Acquisition function convergence reached at iteration 42.

The final EI loss was -0.004 with predicted mean of [-0.946]

The next parameters to simulate from are [[0.011 0.998 0.006 0.006 0.05 0.031]]

The mean of the samples was -1.078

Iteration 471

Acquisition function convergence reached at iteration 48.

The final EI loss was -0.0 with predicted mean of [0.073]

The next parameters to simulate from are [[0.999 0.016 0.033 0.05 0.001 0.064]]

The mean of the samples was 0.618

Iteration 472

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.461]

The next parameters to simulate from are [[0.156 0.537 0.006 0.01 0.02 0.048]]

The mean of the samples was -0.42

Iteration 473

Acquisition function convergence reached at iteration 63.

The final EI loss was -0.004 with predicted mean of [-1.324]

The next parameters to simulate from are [[0.382 0.013 0.013 0.009 0.036 0.023]]

The mean of the samples was -1.319

Iteration 474

Acquisition function convergence reached at iteration 68.

The final EI loss was -0.002 with predicted mean of [-0.885]

The next parameters to simulate from are [[0.003 0.996 0.024 0.015 0.015 0.043]]

The mean of the samples was -1.047

Iteration 475

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.506]

The next parameters to simulate from are [[0.156 0.657 0.003 0.026 0.024 0.022]]

The mean of the samples was 0.547

Iteration 476

Acquisition function convergence reached at iteration 62.

The final EI loss was -0.0 with predicted mean of [-0.107]

The next parameters to simulate from are [[0.003 0.992 0.025 0. 0.049 0.066]]

The mean of the samples was 0.073

Iteration 477

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.876]

The next parameters to simulate from are [[0.156 0.537 0.009 0.01 0.026 0.014]]

The mean of the samples was -0.757

Iteration 478

Acquisition function convergence reached at iteration 50.

The final EI loss was -0.002 with predicted mean of [-1.013]

The next parameters to simulate from are [[0.005 0.991 0.021 0.015 0.014 0.03 ]]

The mean of the samples was -0.87

Iteration 479

Acquisition function convergence reached at iteration 62.

The final EI loss was -0.015 with predicted mean of [-1.815]

The next parameters to simulate from are [[0.159 0.548 0.006 0.01 0.024 0.023]]

The mean of the samples was -1.674

Iteration 480

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.481]

The next parameters to simulate from are [[0.153 0.555 0.006 0.01 0.01 0.009]]

The mean of the samples was -0.454

Hyperparameter convergence reached at iteration 1949.

The minimum predicted mean of the observed indices is -1.814 at the point

[0.153 0.555 0.006 0.01 0.024 0.023]

Iteration 481

Acquisition function convergence reached at iteration 8.

The final EI loss was -0.0 with predicted mean of [-0.603]

The next parameters to simulate from are [[0.113 0.493 0.024 0.014 0.035 0.056]]

The mean of the samples was -0.53

Iteration 482

Acquisition function convergence reached at iteration 60.

The final EI loss was -0.015 with predicted mean of [-1.813]

The next parameters to simulate from are [[0.157 0.546 0.006 0.01 0.024 0.023]]

The mean of the samples was -1.718

Iteration 483

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [1.095]

The next parameters to simulate from are [[0.153 0.555 0.015 0.036 0.024 0.023]]

The mean of the samples was 1.021

Iteration 484

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.719]

The next parameters to simulate from are [[0.153 0.555 0.006 0.01 0.05 0.023]]

The mean of the samples was -0.77

Iteration 485

Acquisition function convergence reached at iteration 74.

The final EI loss was -0.013 with predicted mean of [-1.78]

The next parameters to simulate from are [[0.138 0.735 0.007 0.01 0.025 0.024]]

The mean of the samples was -1.699

Iteration 486

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.426]

The next parameters to simulate from are [[0.961 0.555 0.001 0.01 0.032 0.044]]

The mean of the samples was 0.468

Iteration 487

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.409]

The next parameters to simulate from are [[0.413 0.555 0.027 0.026 0.04 0.023]]

The mean of the samples was 0.358

Iteration 488

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.78]

The next parameters to simulate from are [[0.153 0.555 0.019 0.01 0.024 0.023]]

The mean of the samples was -0.806

Iteration 489

Acquisition function convergence reached at iteration 42.

The final EI loss was -0.0 with predicted mean of [0.052]

The next parameters to simulate from are [[0.008 0.977 0. 0. 0.001 0.002]]

The mean of the samples was 0.387

Iteration 490

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.838]

The next parameters to simulate from are [[0.153 0.796 0.006 0.015 0.024 0.023]]

The mean of the samples was -0.712

Iteration 491

Acquisition function convergence reached at iteration 43.

The final EI loss was -0.0 with predicted mean of [0.039]

The next parameters to simulate from are [[0.956 0.003 0. 0.019 0.001 0.002]]

The mean of the samples was 0.252

Iteration 492

Acquisition function convergence reached at iteration 44.

The final EI loss was -0.0 with predicted mean of [0.071]

The next parameters to simulate from are [[0.988 0.981 0.033 0. 0.048 0.023]]

The mean of the samples was 0.434

Iteration 493

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.277]

The next parameters to simulate from are [[0.153 0.223 0.007 0.016 0.024 0.014]]

The mean of the samples was -0.266

Iteration 494

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.734]

The next parameters to simulate from are [[0.69 0.555 0.006 0.039 0.024 0.032]]

The mean of the samples was 0.707

Iteration 495

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.434]

The next parameters to simulate from are [[0.565 0.282 0.007 0.002 0.007 0.058]]

The mean of the samples was 0.49

Iteration 496

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.44]

The next parameters to simulate from are [[0.154 0.027 0.032 0.01 0.024 0.024]]

The mean of the samples was -0.457

Iteration 497

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.995]

The next parameters to simulate from are [[0.153 0.556 0.022 0.036 0.048 0.023]]

The mean of the samples was 1.139

Iteration 498

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [-0.005]

The next parameters to simulate from are [[0.696 0.555 0.017 0.01 0.004 0.023]]

The mean of the samples was 0.024

Iteration 499

Acquisition function convergence reached at iteration 2.

The final EI loss was -0.0 with predicted mean of [0.034]

The next parameters to simulate from are [[0.781 0.419 0.006 0.01 0.024 0.023]]

The mean of the samples was 0.043

Iteration 500

Acquisition function convergence reached at iteration 10.

The final EI loss was -0.0 with predicted mean of [-0.232]

The next parameters to simulate from are [[0.183 0.96 0.024 0.014 0.047 0.014]]

The mean of the samples was -0.202

Hyperparameter convergence reached at iteration 1968.

The minimum predicted mean of the observed indices is -1.812 at the point  $[0.153\ 0.555\ 0.006\ 0.01\ 0.024\ 0.023]$ 

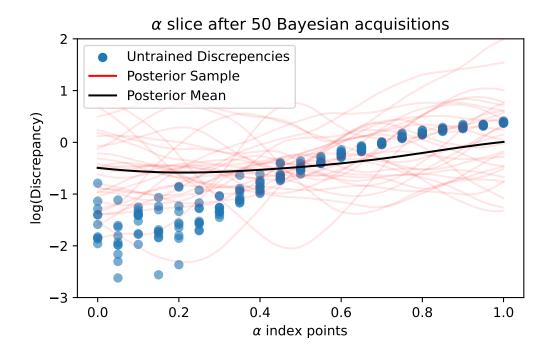
Trained parameters:

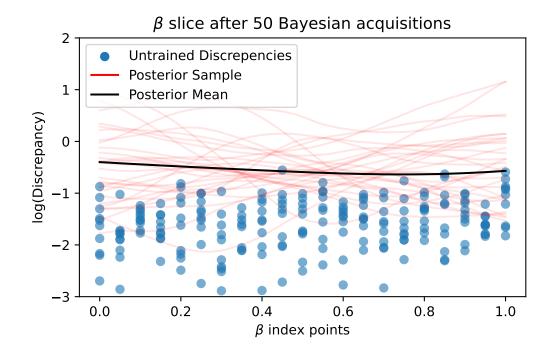
amplitude\_champ:0 is 0.65

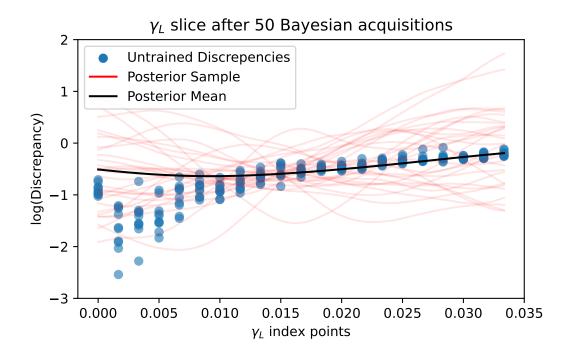
length\_scales\_champ:0 is [0.271 0.5 0.009 0.005 0.018 0.014]

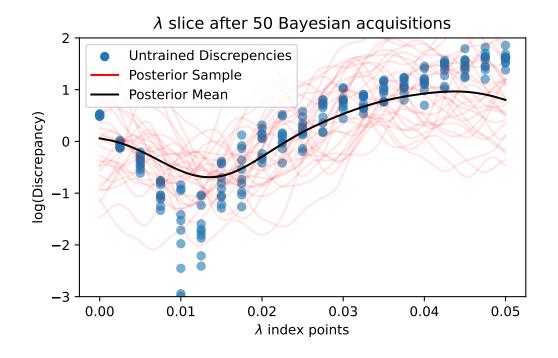
observation\_noise\_variance\_champ:0 is 0.071

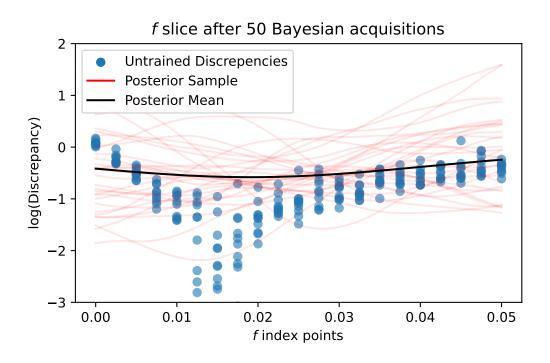
bias\_mean:0 is 0.75

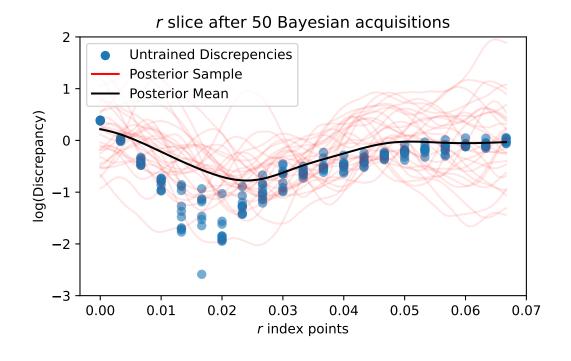


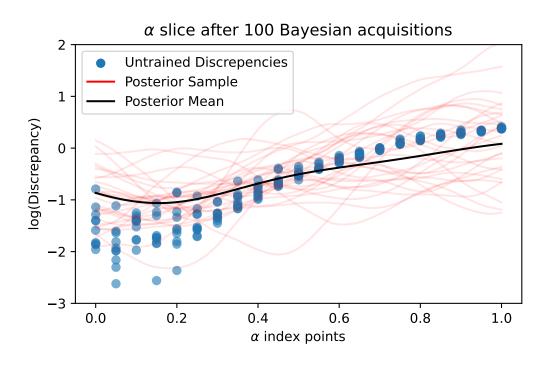


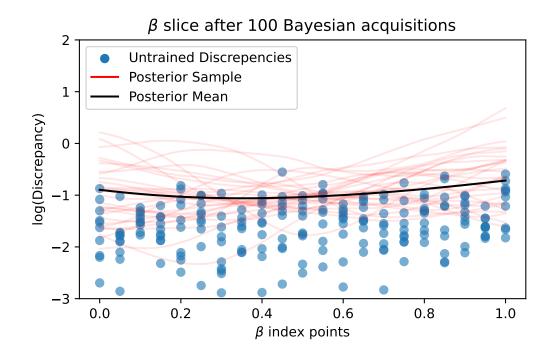


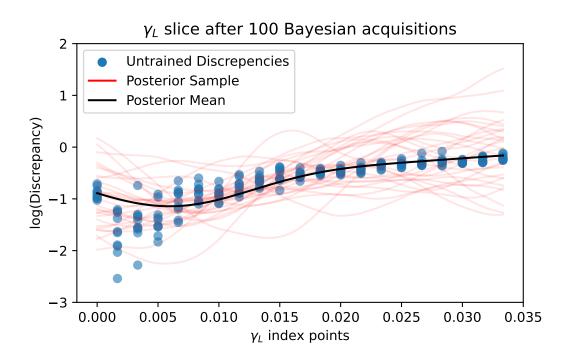


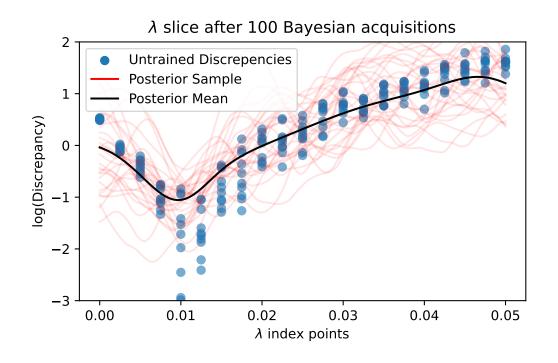


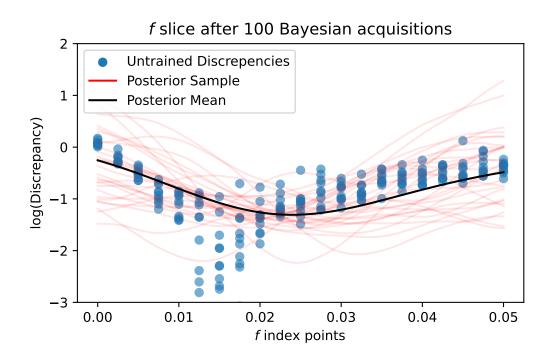


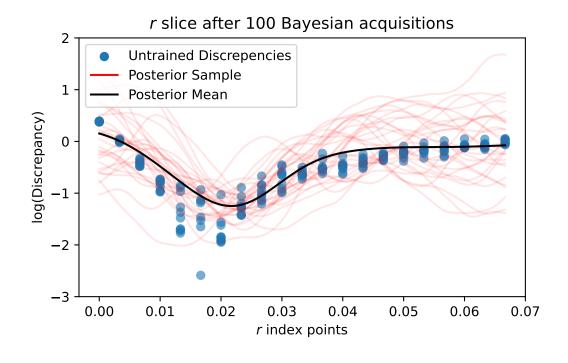


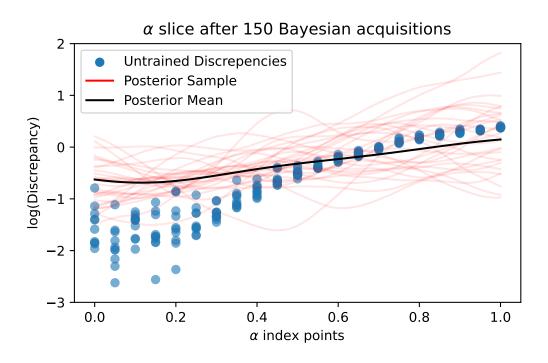


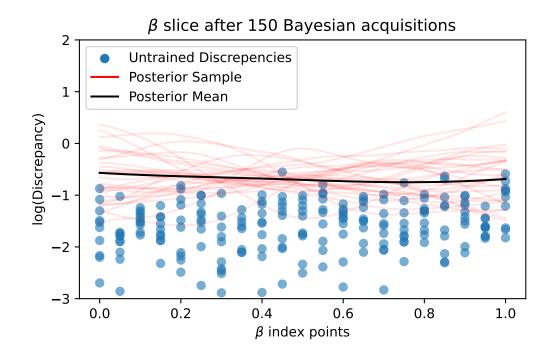


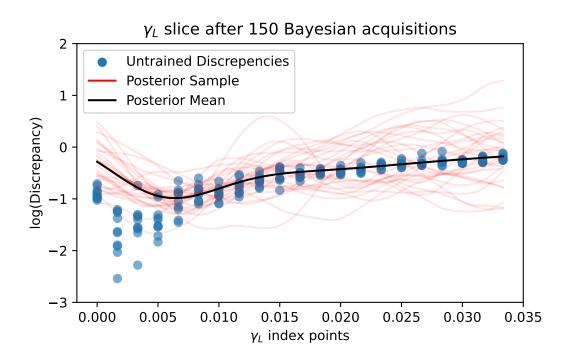


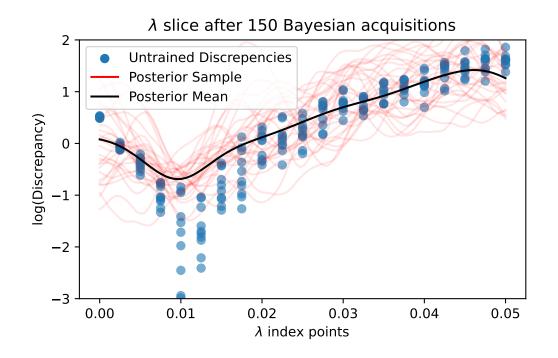


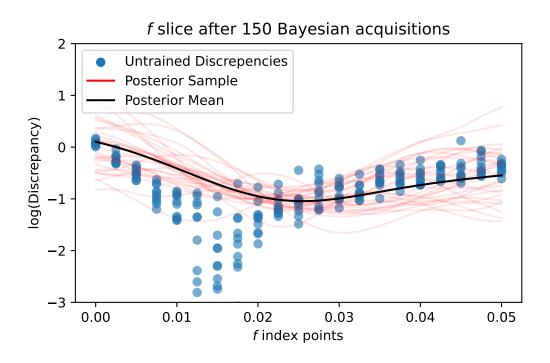


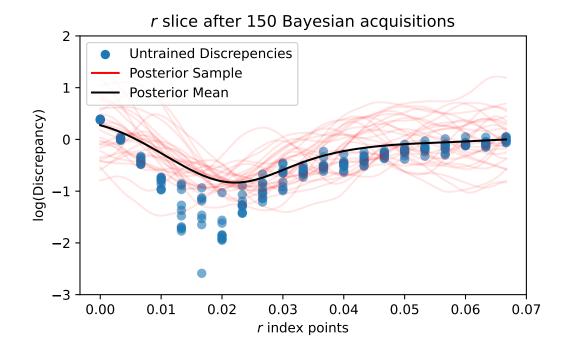


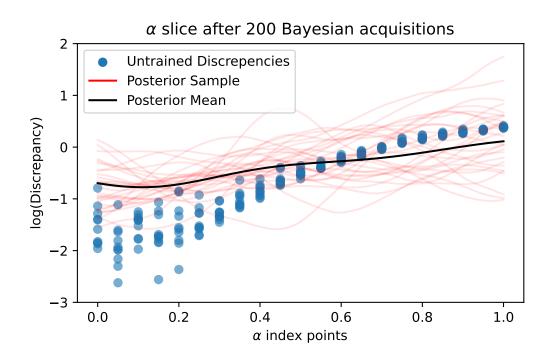


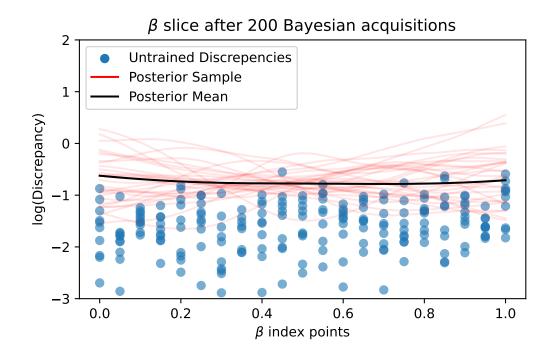


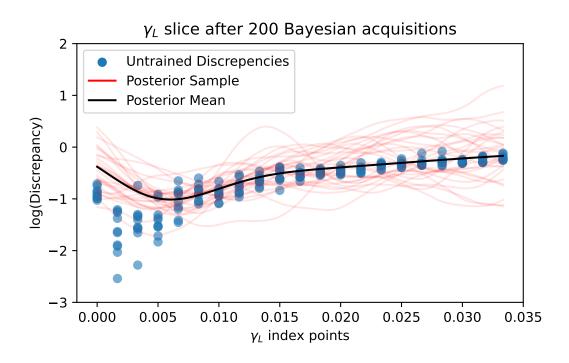


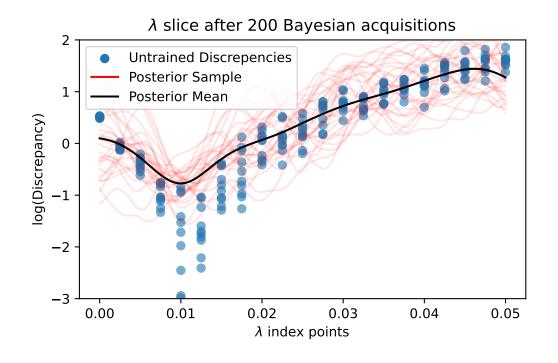


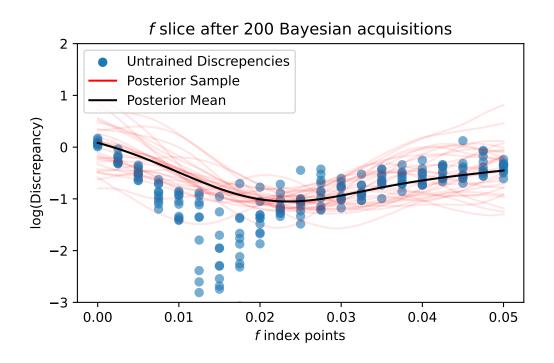


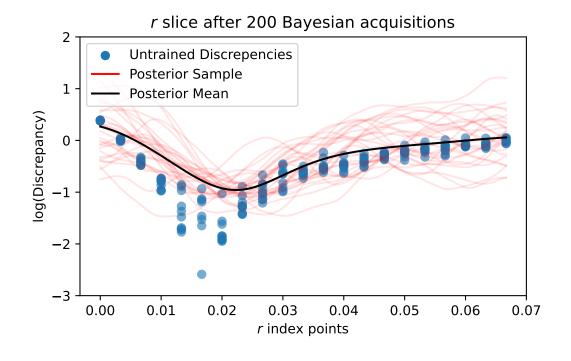


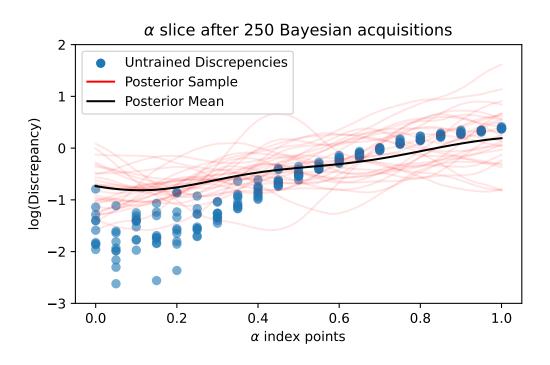


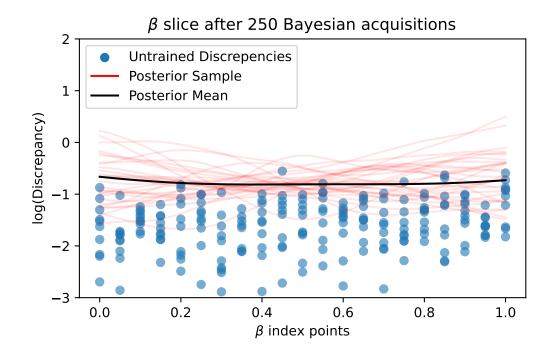


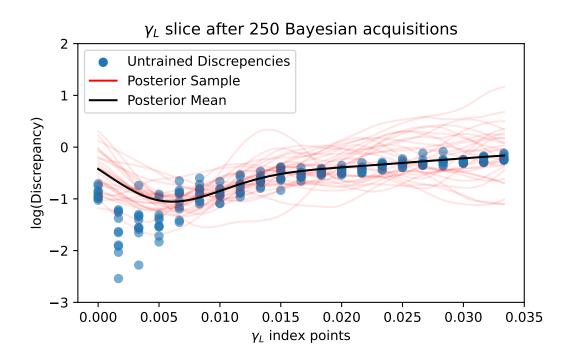


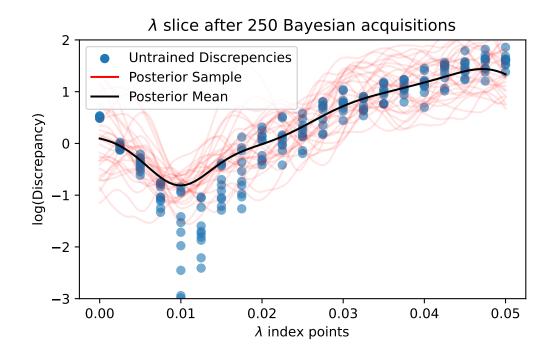


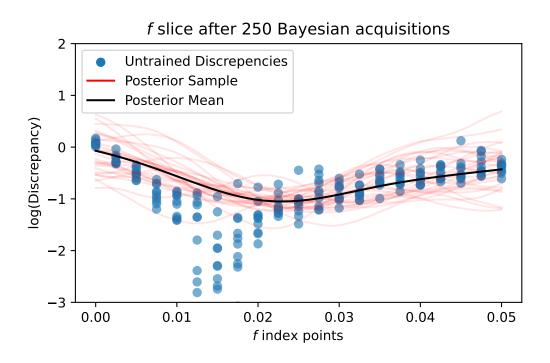


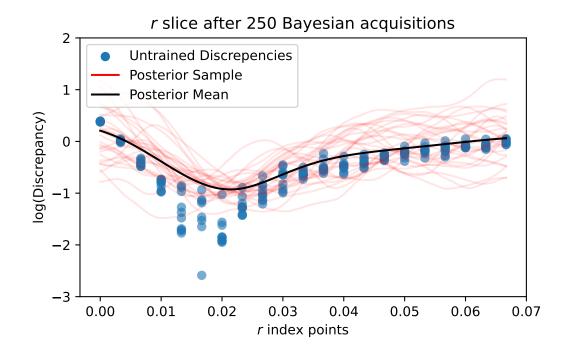


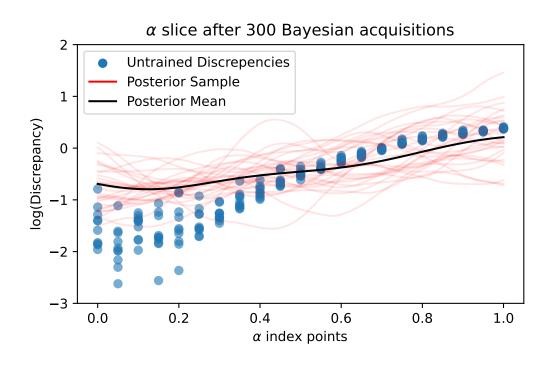


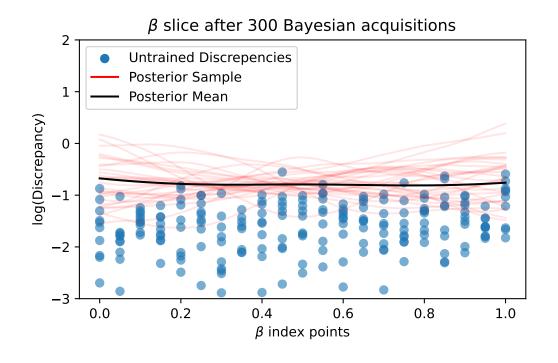


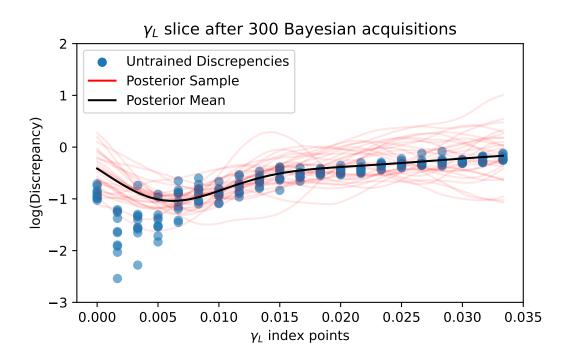


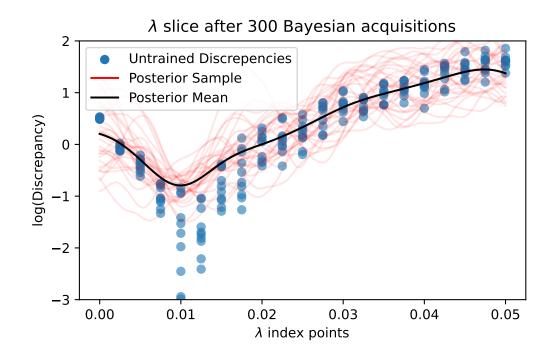


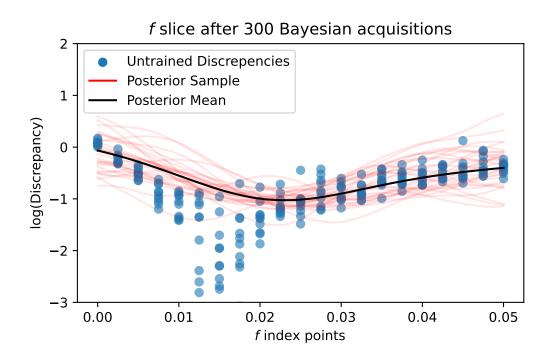


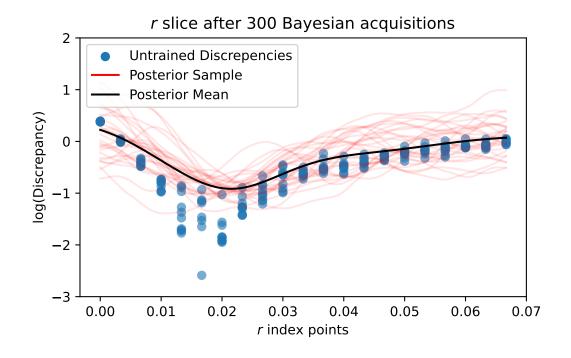


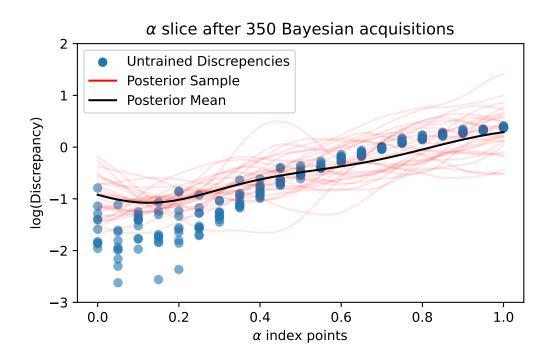


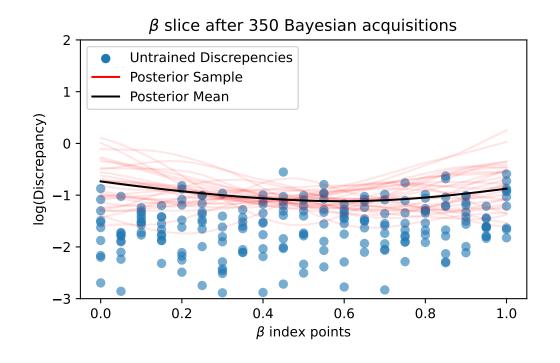


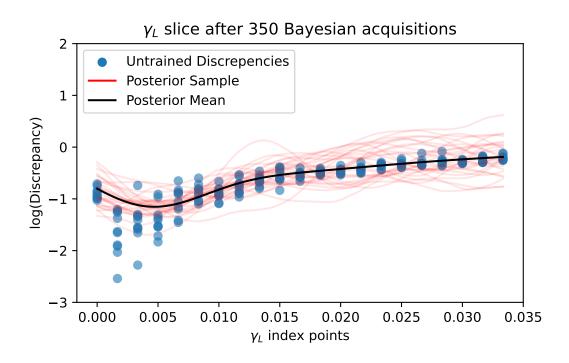


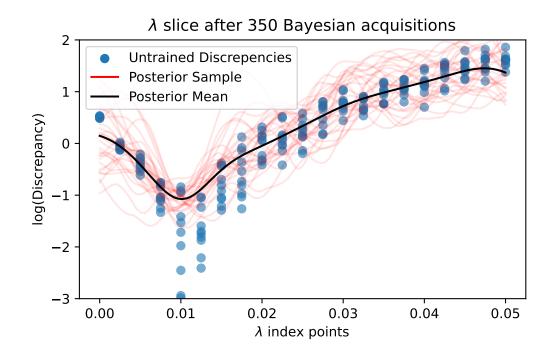


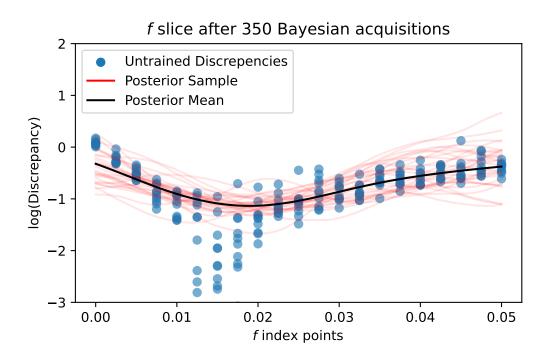


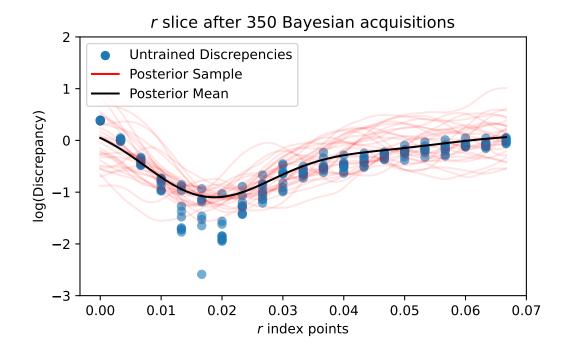


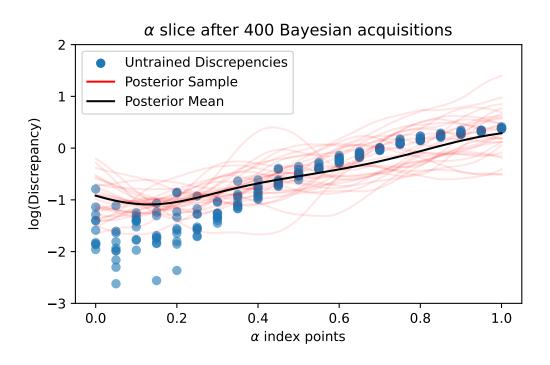


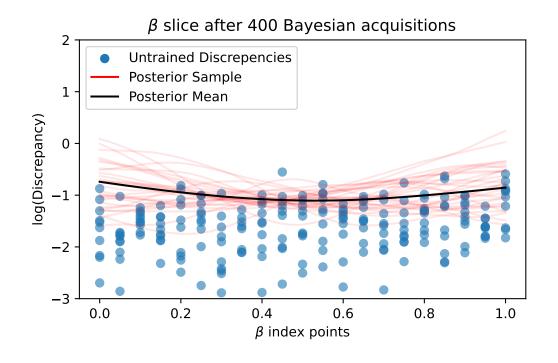


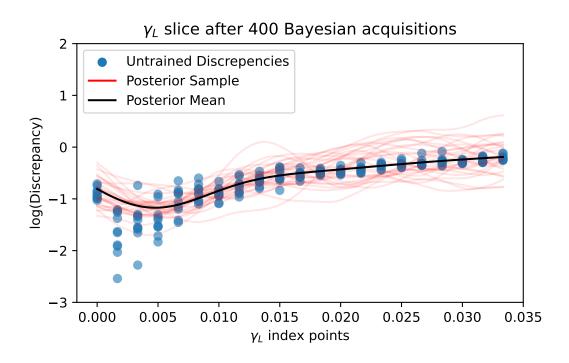


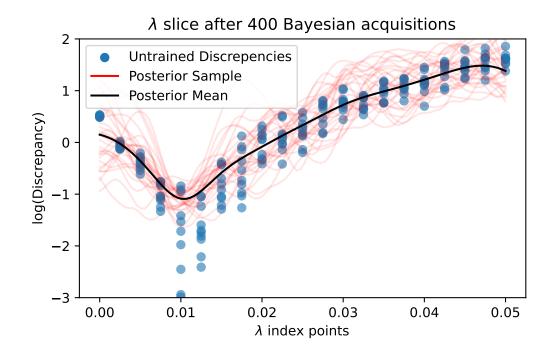


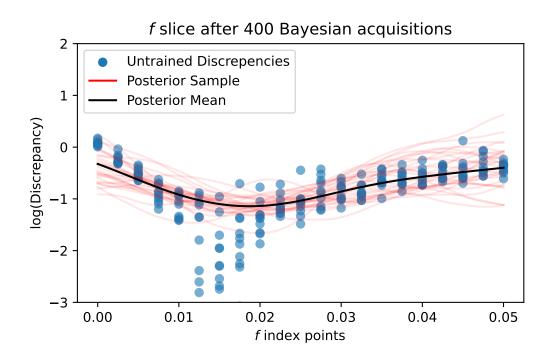


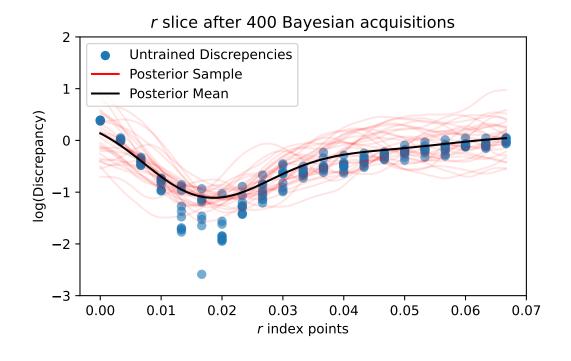


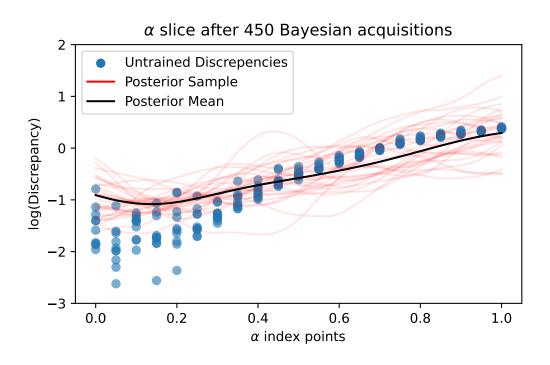


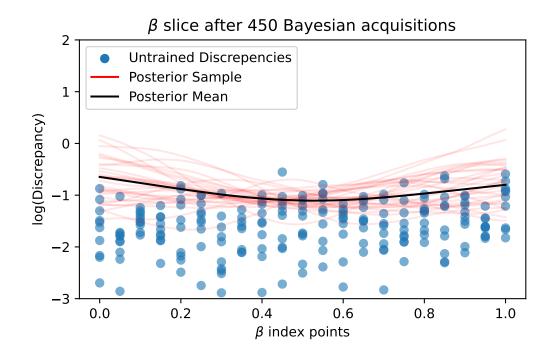


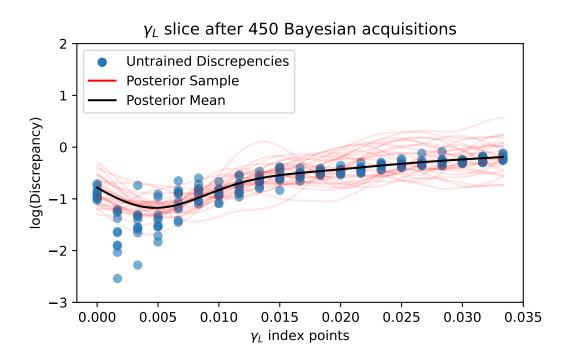


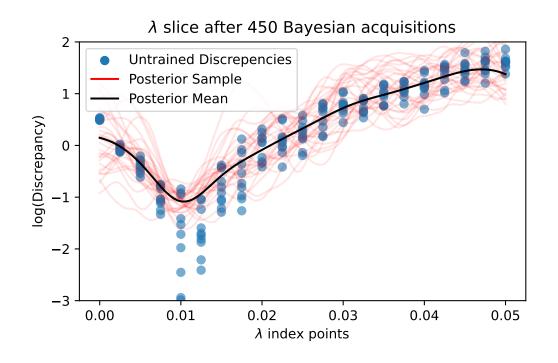


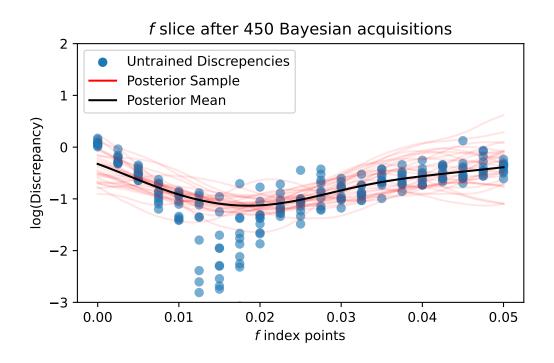


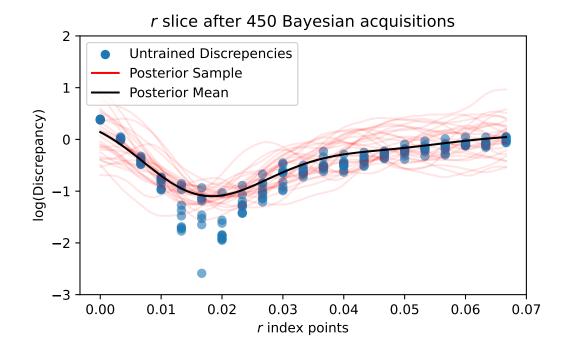


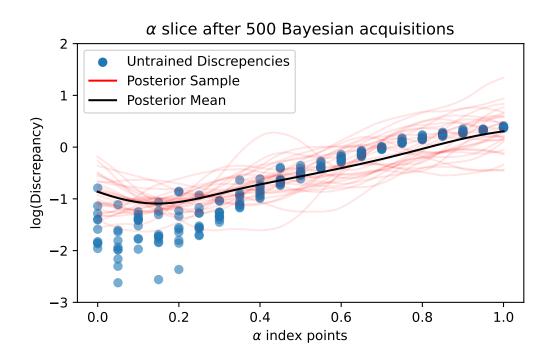


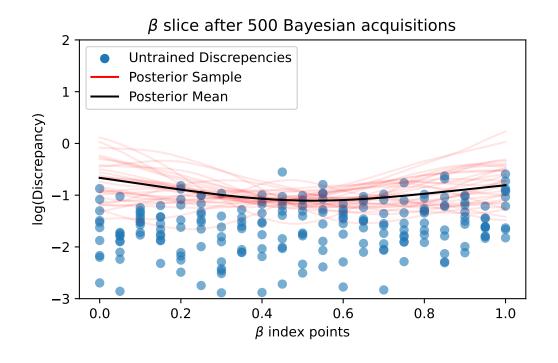


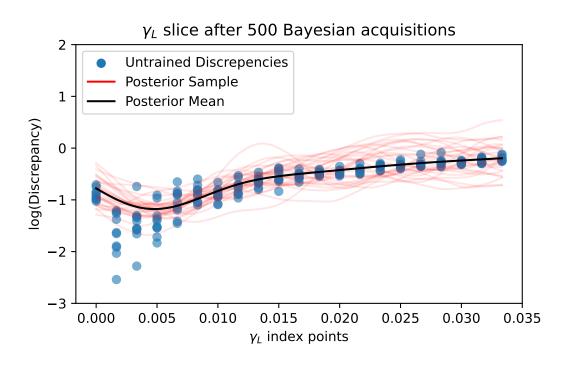


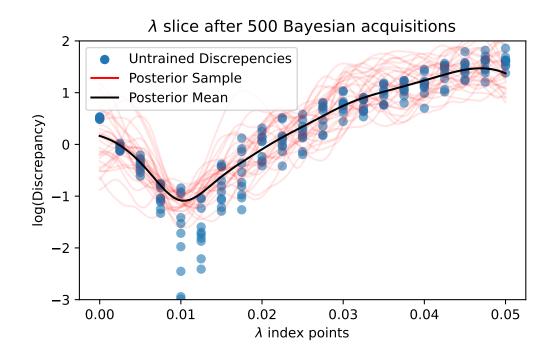


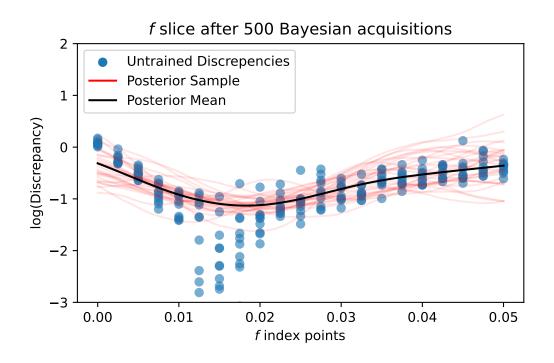


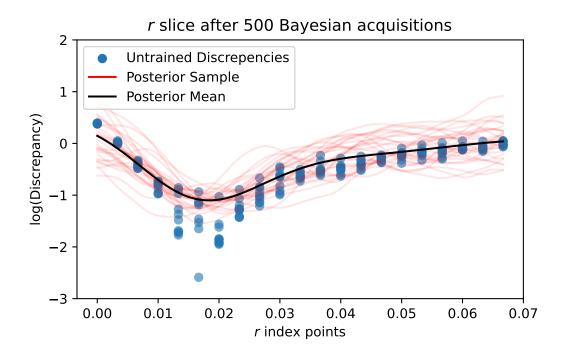








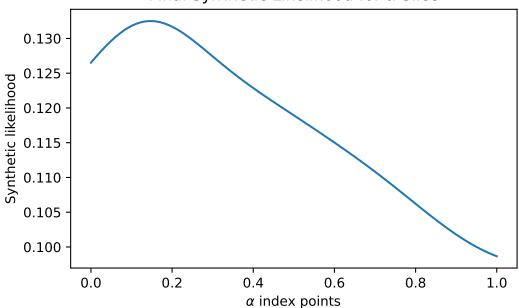


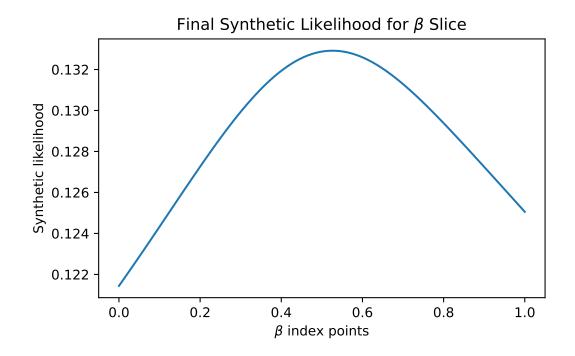


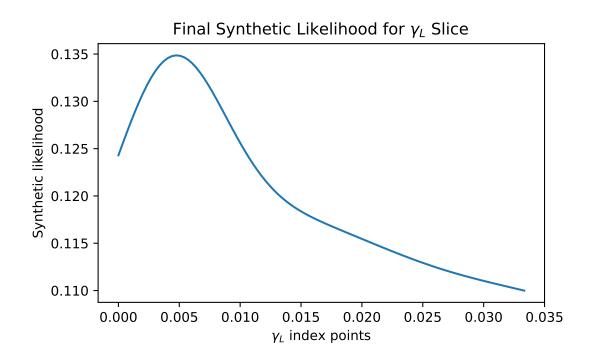
```
epsilon = -10
likelihood_dict = {}
for var in vars:
    champ_GP_reg = tfd.GaussianProcessRegressionModel(
        kernel=kernel_champ,
        index_points=slice_indices_dfs_dict[var + "_gp_indices_df"].values,
        observation_index_points=index_vals,
        observations=obs_vals,
        observation_noise_variance=observation_noise_variance_champ,
        predictive_noise_variance=0.0,
        mean_fn=const_mean_fn(),
    )
    indices_for_lik = slice_indices_dfs_dict[var + "_gp_indices_df"].values
    mean = champ_GP_reg.mean_fn(indices_for_lik)
    likelihood_dict[var + "_slice_means"] = mean
    variance = dis_mean_n**2 * observation_noise_variance_champ.numpy()
    post_std = np.sqrt(variance)
    log_cdf_vals = tfd.Normal(mean, post_std).log_cdf(epsilon)
```

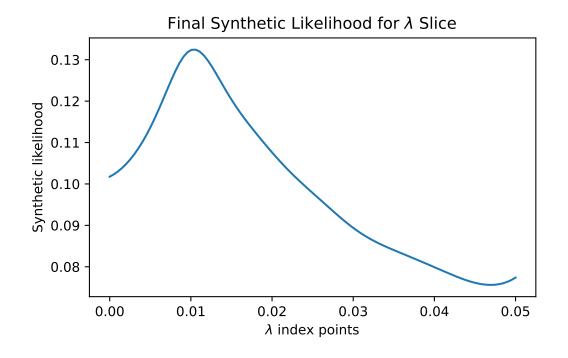
```
likelihood_dict[var + "_synth_log_lik"] = log_cdf_vals
plt.figure(figsize=(6, 3.5))
plt.plot(
    slice_indices_dfs_dict[var + "_gp_indices_df"][var].values,
    np.exp(log_cdf_vals),
)
if var in ["f", "r"]:
    plt.xlabel("$" + var + "$ index points")
    plt.title("Final Synthetic Likelihood for $" + var + "$ Slice")
else:
    plt.xlabel("$\\" + var + "$ index points")
    plt.title("Final Synthetic Likelihood for $\\" + var + "$ Slice")
plt.ylabel("Synthetic likelihood")
plt.savefig(
    "champagne_GP_images/" + var + "_slice_" + str(t) + "_synth_likelihood.pdf"
plt.show()
```

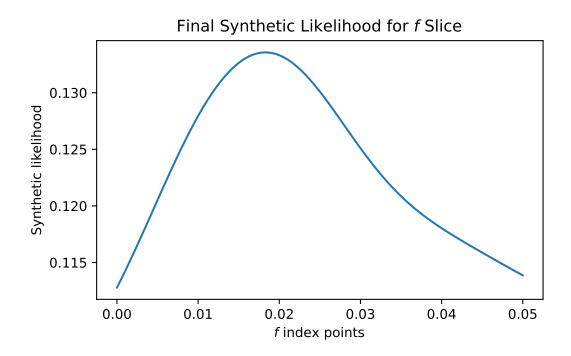


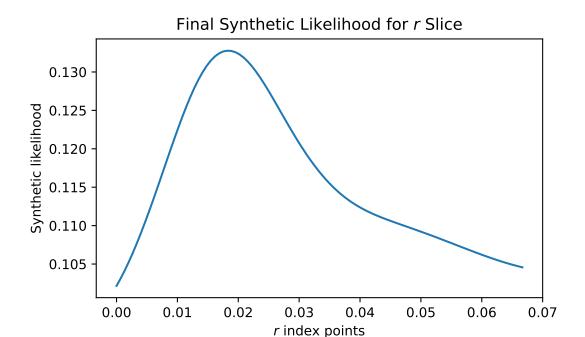












```
# print(index_vals[-600,].round(3))
  print(index_vals[-400,].round(3))
  print(index_vals[-200,].round(3))
  print(index_vals[-80,].round(3))
  print(index vals[-40,].round(3))
  print(index_vals[-20,].round(3))
  print(index_vals[-8,].round(3))
  print(index_vals[-4,].round(3))
  print(index_vals[-2,].round(3))
  print(index_vals[-1,].round(3))
[0.162 0.689 0.006 0.038 0.003 0.064]
[0.168 0.013 0.01 0.011 0.021 0.038]
[0.138 0.735 0.007 0.01 0.025 0.028]
[0.153 0.411 0.007 0.016 0.024 0.014]
[0.153 0.556 0.022 0.036 0.048 0.028]
[0.781 0.24 0.006 0.01 0.024 0.023]
[0.183 0.96 0.012 0.014 0.047 0.014]
[0.183 0.96 0.023 0.014 0.047 0.014]
[0.183 0.96 0.024 0.014 0.047 0.014]
```

```
objects_to_preserve = [
    index_vals,
    discreps,
    champ_samp,
    initial_losses_LOOCV,
    slice_samples_dict,
    slice_discrepencies_dict,
    LHC_indices_df,
    gp_samples_dict,
    likelihood_dict,
]

with open("gp_objs.pkl", "wb") as fp:
    pickle.dump(objects_to_preserve, fp)
    print("dictionary saved successfully to file")
```

dictionary saved successfully to file