Unit-5
Interpolation with Equal Intervals.

7 am
* Newton's forward difference interpolation
* Newton's Backward difference interpolation

8
* central difference.
  1. Gauss forward
  2. Gauss Backward
  3. Stirling's formula
  4. Bessel's formula.

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1 pm
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Newton formula

* Newton forward (Δ Delta)

$$f(a+hu) = f(a) + \Delta f(a)\left(\frac{u}{1!}\right) + \Delta^2 f(a)\left(\frac{u(u-1)}{2!}\right) + \Delta^3 f(a)\left(\frac{u(u-1)(u-2)}{3!}\right) + \ldots$$

$a+hu$ = Value of x to find value of y.
a = first value of x in table
h = difference of x values in tables.
u = can be calculate from above ~~three~~ two value of a and h.

* Newton Backward (∇ Nebla)

$$f(a+hu) = f(a) + \nabla f(a)\left(\frac{u}{1!}\right) + \nabla^2 f(a)\left(\frac{u(u+1)}{2!}\right) + \nabla^3 (f(a))\left(\frac{u(u+1)(u+2)}{3!}\right) + \ldots$$

$a+hu$ = value of x to find value of y.
a = last value of x in table
h = difference of x values in tables.
u = can be calculate from above two parameters a and h.

Newtor forward

* If value of x is p then find $\boxed{f(p)}$ using Newton forward

x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
$\boxed{J_1}$	$\boxed{K_1}$				
		$K_2 - K_1 = \boxed{K_6}$			
J_2	K_2		$\boxed{K_{10}}$		
		K_7		$\boxed{K_{13}}$	
J_3	K_3		K_{11}		$\boxed{K_{15}}$
		K_8		K_{14}	
J_4	K_4		K_{12}		
		K_9			
J_5	K_5				

To find $f(p)$ in Newton formula,

$a + hu = p$, $a = J_1$, $h = |J_2 - J_1|$, p = value of x.

u = Calculate from above parameters.

In Newton formula, forward.

$f(a) = K_1$, $\Delta f(a) = K_6$, $\Delta^2 f(a) = K_{10}$

$\Delta^3 f(a) = K_{13}$, $\Delta^4 f(a) = K_{15}$.

Newton Backward

* If value of x is p then find $\boxed{f(p)}$.

x	y	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
J_1	K_1				
		K_6			
J_2	K_2		K_{10}		
		K_7		K_{13}	
J_3	K_3		K_{11}		$\boxed{K_{15}}$
		K_8		$\boxed{K_{14}}$	
J_4	K_4		$\boxed{K_{12}}$		
		$\boxed{K_9}$			
$\boxed{J_5}$	$\boxed{K_5}$				

$a + hu = p$., $a = J_5$, $h = |J_2 - J_1|$, P = value of x
u = calculate from above parameters.

In Newton backward formula,

$f(a) = K_5$, $\nabla f(a) = K_9$, $\nabla^2 f(a) = K_{12}$

$\nabla^3 f(a) = K_{14}$, $\nabla^4 f(a) = K_{15}$.

Central difference Gauss formula

* Gauss forward:

$$f(a+hu) = y_0 + \Delta y_0\left(\frac{u}{1!}\right) + \Delta^2 y_{-1}\left(\frac{u(u-1)}{2!}\right) +$$

$$\Delta^3 y_{-1}\left(\frac{u(u+1)(u-1)}{3!}\right) + \Delta^4 y_{-2}\left(\frac{u(u+1)(u-1)(u-2)}{4!}\right) + \ldots$$

$a+hu$ = value of x to find value of y.
a = Central value of x in table.
h = difference of values x in table.
u = calculated from above two Parameters.

* Gauss Backward:

$$f(a+hu) = y_0 + \Delta y_{-1}\left(\frac{u}{1!}\right) + \Delta^2 y_{-1}\left(\frac{u(u+1)}{2!}\right) +$$

$$\Delta^3 y_{-2}\left(\frac{u(u+1)(u-1)}{3!}\right) + \Delta^4 y_{-2}\left(\frac{u(u+1)(u-1)(u+2)}{4!}\right) + \ldots$$

$a+hu$ = value of x to find value of y.
a = ~~vcdo~~ Central value of x in table
h = difference of values x in table
u = calculate from above two Parameters.

* If value of x is P then find $f(x)$ usin Gauss forward and backward formula.

	x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
	J_1	$K_1(y_{-2})$				
			~~$K_2 - K_1 = K_6(\Delta y_{-2})$~~ $K_2 - K_1 = K_6(\Delta y_{-2})$			
Gauss backward	J_2	$K_2(y_{-1})$		$K_{10}(\Delta^2 y_{-2})$		
			$K_7(\Delta y_{-1})$		$K_{13}(\Delta^3 y_{-2})$	
	$\boxed{J_3}$	$\boxed{K_3(y_0)}$		$K_{11}(\Delta^2 y_{-1})$		$K_{15}(\Delta^4 y_{-2})$
			$K_8(\Delta y_0)$		$K_{14}(\Delta^3 y_{-1})$	
Gauss forward	J_4	$K_4(y_1)$		$K_{12}(\Delta^2 y_0)$		
			$K_9(\Delta y_1)$			
	J_5	$K_5(y_2)$				

- for both Gauss forward & backward:
  $a+hu = P$, $a = J_3$, $h = |J_2 - J_1|$, P = value of x.
  y = calculate using above parameters.

Central difference
stirling formula

* **Stirling formula:**

$$f(a+hu) = y_0 + \left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right)\left(\frac{u}{1!}\right) + \Delta^2 y_{-1}\left(\frac{u^2}{2!}\right) + \left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right)\left(\frac{u(u^2-1)}{3!}\right) + \Delta^4 y_{-2}\left(\frac{u^2(u^2-1)}{4!}\right) + \ldots$$

* If value of x is p then find y using stirling formula.

x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
J_1	$K_1(y_{-2})$				
		$K_2 - K_1 = K_6$ (Δy_{-2})			
J_2	$K_2(y_{-1})$		$K_{10}(\Delta^2 y_{-2})$		
		$K_7(\Delta y_{-1})$		$K_{13}(\Delta^3 y_{-2})$	
$\boxed{J_3}$	$K_3(y_0)$ →	Average →	$K_{11}(\Delta^2 y_{-1})$ →	Average →	$K_{15}(\Delta^4 y_{-2})$
		$K_8(\Delta y_0)$		$K_{14}(\Delta^3 y_{-1})$	
J_4	$K_4(y_1)$		$K_{12}(\Delta^2 y_0)$		
		$K_9(\Delta y_{-1})$			
J_5	$K_5(y_2)$				

$a + hu = p$

$a = J_3$, $h = |J_2 - J_1|$, p = value of x.

Central difference
Bessel's formula

*** Bessel's formula:**

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$$f(a+hu) = \frac{1}{2}(y_0+y_1) + \Delta y_0\left(\frac{u-1/2}{1!}\right) + \left(\frac{\Delta^2 y_0 + \Delta^2 y_{-1}}{2}\right)\left(\frac{u(u-1)}{2!}\right)$$

8

9

10

$$+ \Delta^3 y_{-1}\left(\frac{u(u-1/2)(u-1)}{3!}\right) +$$

11

$$+\left(\frac{\Delta^4 y_{-1} \;\; \Delta^4 y_{-2}}{2}\right)\left(\frac{u(u+1)(u-1)(u-2)}{4!}\right)$$

12

1 pm

2

3

4

5

6

7

Interpolation for Equal interval

Q Estimate the population in 1895 & 1925 from following statistics

Year	1891	1901	1911	1921	1931
Population	46	66	81	93	101

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

1895, 1925

$-3-(-5) = 2$, $-4-(-3) = -1$

Newton forward

$$f(a+hu) = f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a) + \cdots$$

$a+hu = 1895$, $a = 1891$, $h = 10$

$1891 + 10u = 1895$

$10u = 4$

$u = 0.4$

$46 + 0.4 \times 20 + 0.4(0.4-1) \times -5 \div 6 + 0.4(0.4-1)(0.4-2) \times 2 \div 6$

$= \text{Ans} + 0.4(0.4-1)(0.4-2)(0.4-3) \times -3 \div 24$

$$f(1895) = 46 + \frac{0.4}{1}(20) + \frac{0.4(0.4-1)}{2}(-5) + \frac{0.4(0.4-1)(0.4-2)}{6}(2) + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{24}(-3)$$

$= 54.85528$

Newton Backward

$$f(a+hu) = f(a) + \frac{u}{1!}\nabla f(a) + \frac{u(u+1)}{2!}\nabla^2 f(a) + \frac{u(u+1)(u+2)}{3!}\nabla^3 f(a) + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 f(a)$$

$a+hu = 1925$, $a = 1931$, $h = 10$

$1931 + 10u = 1925 \Rightarrow 10u = -6 \Rightarrow u = -0.6$

$$f(1925) = 101 + \frac{(-0.6)}{1}(8) + \frac{(-0.6)(-0.6+1)}{2}(-4) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{6}(-1) + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{24}(-3)$$

$= 96.8368$

Q. find the lowest degree Polynomial $y(x)$ that fit the data, find $y(5)$

x	0	2	4	6	8
y	5	9	61	209	501

x	$f(x)=y$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	5				
2	9	4			
4	61	52	48		
6	209	148	96	48	
8	501	292	144	48	0

$$f(a+hu) = f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \cdots$$

$a+hu = x$, $a=0$, $h=2$

$0+2u = x$

$\boxed{u = x/2}$

$$f(x) = 5 + \frac{x}{2}(4) + \frac{x/2\,(x/2-1)}{2}(48) + \frac{x/2\,(x/2-1)(x/2-2)}{6}(48)$$

$$f(x) = x^3 - 2x + 5$$

$$y(5) = f(5) = (5)^3 - 2(5) + 5$$

$$= 120$$

Q find Number of men getting wages between Rs. 10 & Rs. 15 from following Data

wages	0-10	10-20	20-30	30-40
Frequency	9	30	35	42

x wages	f(x) cf	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
(15) below 10	9			
		30		
below 20	30+9 = 39		5	
		35		2
below 30	39+35 = 74		7	
		42		
below 40	74+42 = 116			

Newton formula

$$f(a+hu) = f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \cdots$$

$a+hu = 15$, $a = 10$, $h = 10$

$10 + 10u = 15 \Rightarrow 10u = 5 \Rightarrow u = 0.5$

$$f(15) = 9 + \frac{0.5}{1}(30) + \frac{0.5(0.5-1)}{2}(5) + \frac{(0.5)(0.5-1)(0.5-2)}{6}(2)$$

$$= 23.5 \simeq 24$$

Hence Number of men getting wages between 10 & 15 Rs

$= 24 - 9 = 15$

Central Difference Interpolation

Q find u_9 if $u_0 = 14$, $u_4 = 24$, $u_8 = 32$, $u_{12} = 35$, $u_{16} = 40$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	14 y_{-2}				
		10 Δy_{-2}			
4	24 y_{-1}		-2 $\Delta^2 y_{-2}$		
		8 Δy_{-1}		-3 $\Delta^3 y_{-2}$	
8	32 y_0		-5 $\Delta^2 y_{-1}$		10 $\Delta^4 y_{-2}$
		3 Δy_0		7 $\Delta^3 y_{-1}$	
12	35 y_1		2 $\Delta^2 y_0$		
		5 Δy_1			
16	40 y_2				

Gauss Forward

$$f(a+hu) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{4!}\Delta^4 y_{-2}$$

$a+hu = 9$, $a = 8$, $h = 4 \Rightarrow 8 + 4u = 9 \Rightarrow u = \frac{1}{4} = 0.25$

$$f(9) = 32 + \frac{0.25}{1}(3) + \frac{(0.25-1)}{2}(-5) + \frac{(0.25+1)(0.25)(0.25-1)}{6}(7) + \frac{(0.25+1)(0.25)(0.25-1)(0.25-2)}{24}(10)$$

$= 33.1162$

Gauss Backward

$$f(a+hu) = y_0 + \frac{u}{1!}\Delta y_{-1} + \frac{u(u+1)}{2!}\Delta^2 y_{-1} + \frac{(u-1)u(u+1)}{3!}\Delta^3 y_{-2} + \frac{(u-1)u(u+1)(u+2)}{4!}\Delta^4 y_{-2}$$

$$f(9) = 32 + \frac{0.25}{1}(8) + \frac{(0.25)(0.25+1)}{2}(-5) + \frac{(0.25-1)(0.25)(0.25+1)}{6}(-3) + \frac{(0.25-1)(0.25)(0.25+1)(0.25+2)}{24}(10)$$

$= 33.1162$

Stirling formula

$$f(a+hu) = y_0 + \frac{u}{1!}\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{u^2}{2!}\Delta^2 y_{-1} + \frac{u(u^2-1)}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{u^2(u^2-1)}{4!}\Delta^4 y_{-2}$$

$$f(9) = 32 + \frac{0.25}{1}\left(\frac{8+3}{2}\right) + \frac{(0.25)^2}{2}(-5) + \frac{(0.25)((0.25)^2-1)}{6}\left(\frac{-3+7}{2}\right) + \frac{(0.25)^2((0.25)^2-1)}{24}(10)$$

$= 33.1162$

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Central Difference Interpolation

Q find u_9 if $u_0 = 14$, $u_4 = 24$, $u_8 = 32$, $u_{12} = 35$, $u_{16} = 40$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	14 y_{-2}				
		10 Δy_{-2}			
4	24 y_{-1}		-2 $\Delta^2 y_{-2}$		
		8 Δy_{-1}		-3 $\Delta^3 y_{-2}$	
8	32 y_0		-5 $\Delta^2 y_{-1}$		10 $\Delta^4 y_{-2}$
		3 Δy_0		7 $\Delta^3 y_{-1}$	
12	35 y_1		2 $\Delta^2 y_0$		$\Delta^4 y_{-1}$
		5 Δy_1			
16	40 y_2				

Bessel's formula

$$f(a+hu) = \left(\frac{y_0 + y_1}{2}\right) + \frac{(u-\frac{1}{2})}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\left(\frac{\Delta^2 y_0 + \Delta^2 y_{-1}}{2}\right) + \frac{(u-\frac{1}{2})u(u-1)}{3!}\Delta^3 y_{-1}$$
$$+ \frac{(u+1)u(u-1)(u-2)}{4!}\left(\frac{\Delta^4 y_{-1} + \Delta^4 y_{-2}}{2}\right)$$

$a + hu = 9 \quad | \quad a = 8,\ h = 4$

$8 + 4u = 9 \Rightarrow u = \frac{1}{4} = 0.25$

$$f(9) = \left(\frac{32+35}{2}\right) + \frac{(0.25-\frac{1}{2})}{1}(3) + \frac{(0.25)(0.25-1)}{2}\left(\frac{-5+2}{2}\right)$$
$$+ \frac{(0.25-\frac{1}{2})(0.25)(0.25-1)}{6}(7) + \frac{(0.25+1)(0.25)(0.25-1)(0.25-2)}{24}\left(\frac{10+0}{2}\right)$$

$= \underline{33.1162}$