Unit 12 Probability

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12.1 Introductions

Even in day – to – day life uncertainty plays an important role. When we are unable to forecast the future with certainty, we make statements like "probably it will rain in the evening", "Ram has a better chance of winning the elections" etc. Although we are not sure of the happening of some event we make statements like those mentioned above.

It is interesting to note that the seed of probability theory was thrown when a French nobleman Anokine Gombould (1607 – 1684) sought an explanation from the mathematician Blaise Pascal (1623 – 1662) regarding the frequent occurrence of some combinations of number in the roll of dice. Pierre de Fermat (1601 – 1655) and Blaise Pascal were working on this problem.

Another problem was posed to Blaise Pascal. If a game of change is stopped in the middle, how should the two players divide the stake ? This was another problem leading to the concept of probability, J. Bernoullis (1654 – 1705) first treatise on probability was published in 1718. Other mathematicians who were instrumental in the development of probability were chebyshev (1821 – 1894), A. Markov (1856 – 1920), Liapounoff (who enunciated central limit theorem), De Moivre, T. Bayes and P.S. Laplace.

Initially ideas of probability and statistics were used to explain natural phenomena. Now it is an indispensable tool in many decisions regarding business also.

Objectives:

At the end of the unit you would be able to

- understand the idea of Probability
- apply Baye's theorem in problems

12.2 Concept of Probability

Consider the following statements.

1. A particular medicine is effective except for one out of 1000 patients.
2. There is likely to be moderate to heavy rains in most part of Karnataka
3. Getting a head or a tail in the toss of a coin are equally likely
4. When a single die is rolled, any number from 1 to 6 is equally likely
5. Only 3 out of 2 million parts is likely to be defective

In all the above statements the outcome is not certain. But we are able to list all possible outcomes. For example, we are not certain about the number likely to be seen in a roll of a single die but we know that only one of the six numbers 1, 2, 3, 4, 5, 6 will definitely be seen. Statement 4 is about the likelihood of something to happen. From statement 1 we are not able to say to whom the medicine is likely to be ineffective but we can say that it is ineffective only for one patient when the medicine is administered to 1000 patients. Let us consider statement 3. Although we cannot say whether we get a tail or head, we can say that we will be getting head half the number of times when a coin is tossed several times.

Before proceeding to study the rigorous definition of probability let us understand that Probability is a numerical measure of the likelihood of an event to happen.

It is a number between 0 and 1, 0 and 1 representing the impossibility and certainty.

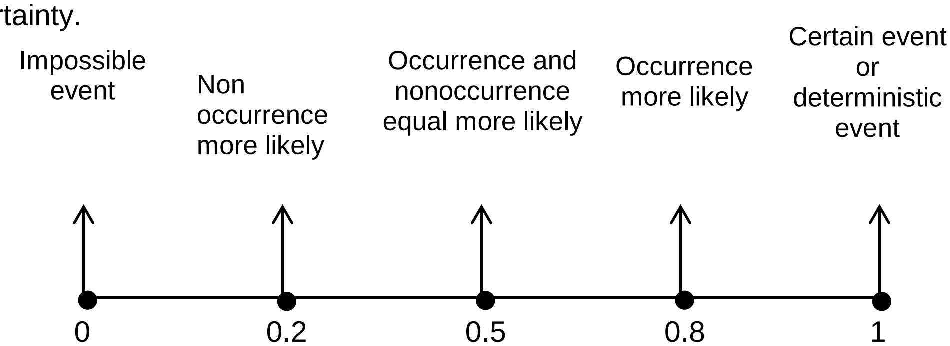

Fig. 12.1: Probability of an event

For example, non occurrence of rain is more likely in summer and occurrence of rain is more likely in rainy season. Head or tails are equally likely in the toss of a coin.

12.3 Sample Space and Events

For defining probability we need the definition of an experiment. (to be more precise random experiment).

Definition: An experiment is a process that generates well – defined outcomes.

When we perform an experiment we call it a trial.
For each trial there is one and only one outcome among the several well – defined outcomes.

Example: Find all the possible outcomes of the following experiments

a) Tossing a single coin
b) Tossing two coins
c) Tossing three coins
d) Roll of a single die
e) Roll of two dice
f) Play a one day cricket match

Solution: The possible outcomes are

a) H, T (H and T denotes head and tail respectively)
b) HH, HT, TH, TT
c) HHH, HHT, THH, HTH, HTT, TTH, THT, TTT
d) 1, 2, 3, 4, 5, 6
e) 11, 12, 13, 14, 15, 16
21, 22, 23, 25, 25, 26
31, 32, 33, 34, 35, 36
41, 42, 43, 44, 45, 46
51, 52, 53, 54, 55, 56
61, 62, 63, 64, 65, 66
f) win, defeat, tie

Definition: The sample space for an experiment is the set of all possible outcomes.

Example: Find the sample space for the following experiments.
a) Number of heads in a toss of two coins
b) Sum on the roll of two dice
c) Sum on the roll of three dice
d) Play a cricket game

Solution: the sample space is
a) {2, 1, 0}
b) {2, 3, 4, 5, 6, 7, 8, 10, 11, 12}
c) {3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18}
d) {win, defeat, tie}

Definition: An event in an experiment is a subset of the sample space.

Definition: A single outcome is called an elementary event.

Note: Any event consists of elementary events.

Example: Write down the following events as a subset of the respective sample space
a) Getting at least one head in a toss of two coins
b) Getting a sum of 6 or more in a roll of two dice
c) Getting two defective parts when five parts are inspected
d) India winning at least one in 3 matches played against Australia.

Solution:
a) $\{HT, TH, HH\} \subset \{HT, TH, HH, TT\}$
b) $\{6, 7, 8, 9, 10, 11, 12\} \subset \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
c) $\{2\} \subset \{0, 1, 2, 3, 4, 5\}$
d) $\{1, 2, 3\} \subset \{0, 1, 2, 3\}$

S.A.Q. 1: Write the following events as a subset of the respective sample space.
a) Getting even number of heads in a toss of 3 coins
b) Winning in alternate matches when 5 matches are played
c) Getting a sum of 4 or 10 in a roll of two dice
d) Winning at least one match when three matches are played

12.4 Three approaches to Probability

As the concept of probability was developed for more than two centuries, the statisticians defined it in several ways. In 1933 the Russian mathematician A. N. Kolmogorov developed probability theory using the axiomatic approach which was used to define various mathematical objects in the last century. The axiomatic approach unified all the three approaches of probability. We study three approaches to probability which were developed before Kolmogorov's unified approach in this section. Kolmogorov's axiomatic approach is developed in the next section.

Classical Probability

The classical probability is also called mathematical, or a priori probability. We need a few preliminary definitions before defining the classical probability of an event.

Definition: The outcomes of an experiment are equally likely if there is no reason to expect one outcome in preference to other outcomes.

For example, Head or Tail are equally likely in tossing a single coin. Any number from 1 to 6 is equally likely in the roll of a die.

Definition: Two or more events are mutually exclusive. If Head falls then Tail cannot fall and vice versa. The appearance of 1, 2, 3, 4, 5, 6 are mutually exclusive. For when outcome of the appearance of 1 occurs, the remaining cannot occur.

Definition: A collection of events is collectively exhaustive if they, when taken together constitute the entire sample space.

For example, the events 1, 2, 3, 4, 5, 6 are collectively exhaustive. So also Head and Tail in the toss of a single coin.

Definition: If the outcomes of an experiment are, equally likely, collectively exhaustive and mutually exclusive then the probability of an event E is defined by

$$\text{Probability of E} = \frac{\text{Number of outcomes favourable to the happening of E}}{\text{Total number of outcomes of the experiment}}$$

Note: The classical probability is called a priori probability since we are able to calculate the probability of an event in advance (that is, without repeating the experiment). Of course this definition is applicable only when the outcomes of an experiment are mutually exclusive, collectively exhaustive and equally likely. In the case of the toss of a coin, if we can assume the validity of these three conditions then we say that the coin is unbiased. Similarly we defined an unbiased die.

Example: Find the classical probability of the following events.

a) Getting at least one Head in a toss of two coins

b) Getting a sum of 10 in a roll of two dice.

Solution: Let E denote the given event.

a) HT, TH, HH are the 3 outcomes favorable to the event E and the total number of outcomes is 4. So $P(E) = \frac{3}{4} = 0.75$

b) (4,6), (5,5) and (6,4) are the 3 outcomes favorable to the event and the total number of outcomes is 36. Hence $P(E) = \frac{3}{36} = \frac{1}{12}$

Note A classical way of monitoring possibility is as follows: If odds in favour of E are x : y or x to y, then $P(E) = \frac{x}{x+y}$

Statistical or empirical Probability

In this approach we repeat the experiment a large number of times and define the probability of an event.

Definition: If n trials are performed and m trials are favourable to the occurrence of an event E, then the probability of the event E is defined by Probability of $E = \underset{n\to\infty}{lt} \frac{m}{n}$

Note: We assume that such a limit exists.

Example: A bag contains 3 red balls, 4 green balls and 5 blue balls. Find the probability of choosing 2 red balls, 1 green ball and one blue ball.

Solution: Denote the required event by E. As there are 12 balls in all, the total number of choosing 4 balls is C (12, 4). We can choose 2 red balls from 3 red balls in C (3, 2) ways. One green ball can be chosen in C(4, 1) ways and one blue ball can be chosen in C(5, 1) ways. By multiplication principle, the number of outcomes favorable to E is C(3, 2) , C(4, 1) C(5, 1).

So $P(E) = \frac{C(3,2)\,C(4,1)\,C(5,1)}{C(12,4)}$

$$= \frac{3.4.5}{12.11.10.9}\,1.2.3.4$$

$$= \frac{4}{33}$$

Example: There are 25 cards having the numbers 1, 2, …….., 25, written in them. If one card is chosen what is the probability that the number in the card is divisible by 3 or 7.

Solution: The numbers divisible by 3 are 3, 6, 9, 12, 15, 18, 21, 24 and those divisible by 7 are 7, 14 (and 21 which is already considered). So the number of favourable outcomes is 10.

$$P(E) = \frac{10}{25} = 0.4$$

Bayesian or subjective probability

In the last two approaches either we make certain assumptions about the outcomes or we assume that we can have a large number of trials. When we have an experiment that can be performed only once or only a few times, the earlier methods fail. In such cases we resort to subjective approach.

Definition: The subjective probability of an event is the probability assigned to an event by an individual based on the evidence available to him, if there is any.

Many of the social and managerial decisions are concerned with specific unique situations. In such cases the decision makes has to frame subjective probability for these events.

When a new product is developed, the marketing manager makes prediction based on subjective probability framed by him.

S.A.Q. 2: A bag contains 3 red, 6 yellow and 7 blue balls. What is the probability that the two balls drawn are yellow and blue ?

S.A.Q. 3: A ball is drawn from a bag containing 10 black and 7 white balls. What I the probability that it is white ?

S.A.Q. 4: One number is chosen from each of the two sets {1, 2, 3, 4, 5, 6, 7, 8, 9} and {2, 4, 6, 8, 10}. What is the probability that the sum of these two numbers is 13 ?

12.5 Kolmogorov's Axiomatic Approach to Probability

As we saw earlier, Kolmogorov proposed axiomatic theory of probability in 1933. In axiomatic approach to any theory, a minimal set of properties are taken as axioms (by an axiom we mean an assumption). They are taken as a basis and all other properties are deduced from the axioms. This is how many of modern mathematical objects are defined.

Before proceeding further, recall the definition of a sample space and events discussed in earlier sections.

Definition: Let S be a sample spacer and β be a collection of events defined in the sample space S.

Then the probability of an event A is defined by a function P: B → R (R is the set of all real numbers) Satisfying the following axioms.

(A_1) for each $A \in \beta, P(A)$ satisfies $0 \le P(A) \le 1$.

(A_2) P(S) = 1

(A_3) If A_1, A_2, ….., A_n, ……. Is a sequence of mutually exclusive (disjoint events in β then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Note: In most of the applications we take only a finite number of disjoint events A_1, ………., A_n. In this case, (A_3) reduces to

$$P(A_1 \cup A_2 .... \cup A_n) = \sum_{i=1}^{n} P(A_i) = P(A_1) + P(A_2) + ... + P(A_n) \quad (12.1)$$

In particular if A and B are mutually exclusive then

$$P(A \cup B) = P(A) + P(B) \quad (12.2)$$

Using the axiomatic approach, we can deduce all properties of probability which hold good for classical and statistical probabilities.

We derive a few important properties of probability using (A_1) (A_2) (A_3) or (12.1). In most of the proofs (12.1) or (12.2) is used.

Property 12.1: The probability of an impossible event is zero.

Proof: The impossible event is the empty set ϕ. We know that $S = S \cup \phi$ and the union $S \cup \phi$ is a disjoint union. Then

$$P(S) = P(S \cup \phi)$$

$$= P(S) + P(\phi) \quad \text{(by 12.2)}$$

Canceling P(S) on both sides. We get $P(\phi) = 0.$

Property 12.2: If $\overline{A}$ is the complement of the event A, that is $\overline{A} = S - A,$ then $P(\overline{A}) = 1 - P(A)$

Proof: We know that $s = A \cup \overline{A}$ (see figure 12.2)

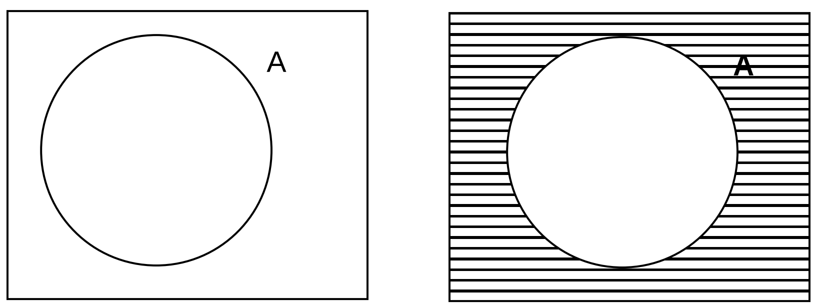

Fig. 12.2: The complementary event

So $P(S) = P(A) + P(\overline{A})$

As $P(S) = 1\ (by\ (A_2)),\ P(A) + P(\overline{A}) = 1$ or

$P(\overline{A}) = 1 - P(A)$

Theorem: (Addition theorem) If A and B are any two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \text{.........................} (12.3)$$

Proof: We write $A \cup B$ as a disjoint union.

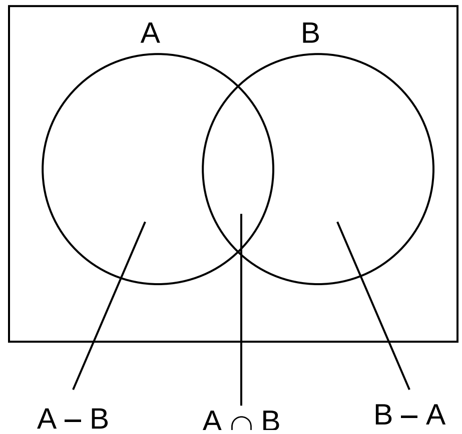

Figure 12.3 $A \cup B$ as disjoint union

From Fig 12.3, we see that $A = (A - B) \cup (A \cap B)$
and the union is disjoint.
By (12.2), $P(A) = P(A - B) + P(A \cap B)$ ……… (12.4)
Similarly $B = (B - A) \cup (A \cap B)$
And the union is disjoint. By (12.2), $P(B) = P(B - A) + P(A \cap B)$ ……. (12.5)
As $A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$ and the union on RHS is disjoint.

$$P(A \cup B) = P(A - B) + P(A \cap B) + P(B - A)$$
$$= \lfloor P(A - B) + P(A \cap B) \rfloor + \lfloor P(B - A) + P(A \cap B) \rfloor - P(A \cap B)$$
$$= P(A) + P(B) - P(A \cap B) \text{ by (12.4) and (12.5)}$$

Note: (12.4) and (12.5) are useful for doing problems.
Thus (12.3) is proved

Corollary (Extended Addition theorem)
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
…. (12.6)

Note: Fig. 12.5 will help you to prove (12.6)

Property 12.3 If $B \subseteq A,$ then $P(B) \leq P(A)$

Proof: From Fig. 12.4, we see that

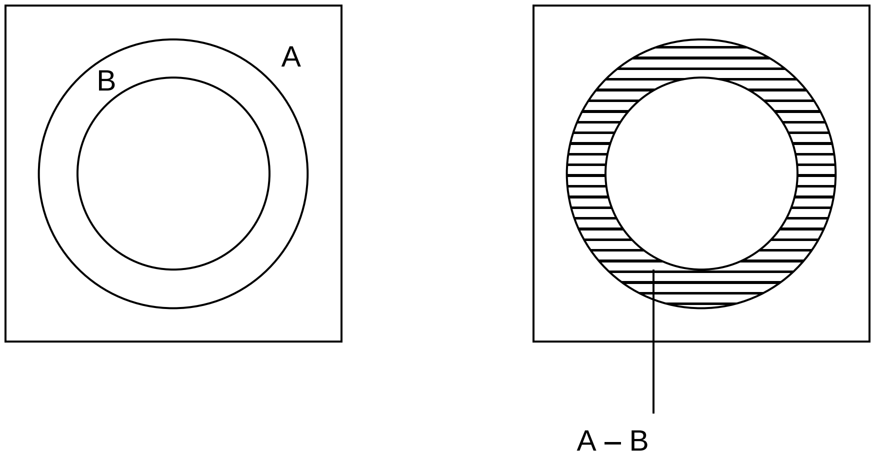

Figure 12.4, Venn Diagram for property 12.3

$A = B \cup (A - B)$ and the union is disjoint. Hence

$P(A) = P(B \cup (A - B))$

$= P(B) + P(A - B)$ by (12.2)

As $P(A - B) \geq 0$ $(by\ (A_1))$, $P(A) \geq P(B)$ *or* $P(B) \leq P(A)$

Example: If A, B, C are any three events, write the following events using $A, B, C, \overline{A}, \overline{B}, \overline{C}$ and set operations.

a) only A occurs
b) A and B occur but not C
c) All the three events occur
d) None of them occur
e) At least one of them occur
f) At least two of them occur
g) Exactly one of them occurs
h) Exactly two of them occur

Solution: We represent A, B, C and their complements in Fig. 12.5

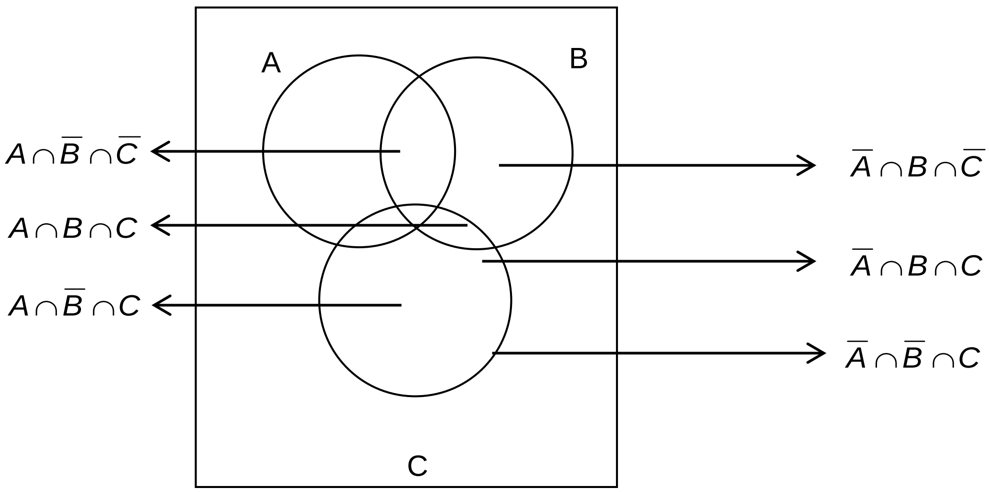

Figure 12.5: Three events

Now using the Venn diagram given in Fig 12.5, we can represent the events as follows.

a) $A \cap \overline{B} \cap \overline{C}$ b) $A \cap B \cap \overline{C}$

c) $A \cap B \cap C$ d) $\overline{(A \cap B \cap C)}$

e) $A \cup B \cup C$ f) $(A \cap B) \cup (B \cap C) \cup (C \cap A)$

g) $\left(A \cap \overline{B} \cap \overline{C}\right) \cup \left(\overline{A} \cap B \cap \overline{C}\right) \cup \left(\overline{A} \cap \overline{B} \cap C\right)$

h) $\left(A \cap B \cap \overline{C}\right) \cup \left(A \cap \overline{B} \cap C\right) \cup \left(\overline{A} \cap B \cap C\right)$

Example: If two dice are thrown what is the probability that the sum is a) greater than 9 b) neither 3 or 9 c) less than 4

Solution:

a) The favourable outcomes are (4,6), (5,5), (6,4), (5,6), (6,5), (6,6). So $P(E) = \frac{6}{36} = \frac{1}{6}$.

b) Let A and B denote the events that the sum is 3 and 9 respectively. Then $P(A) = \frac{2}{36}$ and $P(B) = \frac{4}{36}$. As A and B are mutually exclusive $P(A \cap B) = 0.$

So $P(A \cup B) = \frac{2}{36} + \frac{4}{36} = \frac{6}{36} = \frac{1}{6}$.

P(Sum is neither 3 or 9).

$= P(\overline{A} \cap \overline{B})$

$= P(\overline{A \cup B})$ (By De Morgan's law $\overline{A \cup B} = \overline{A} \cap \overline{B}$

$= 1 - P(A \cup B)$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

c) The favorable outcomes are 11, 12, 21. So probability is $\frac{3}{36} = \frac{1}{12}$.

Example: A bag contains two red balls, three blue balls and five green balls. Three balls are drawn at random. Find the probability that

a) the three balls are of different colours
b) two balls are of the same colour
c) all the three are of the same colour

Solution Let E denote the given event..

a) We can choose one red ball in *C (2, 1)* ways, etc. So

$$P(E) = \frac{C(2,1)C(3,1)C(5,1)}{C(10,3)}$$

$$= \frac{2.3.5}{10.9.8}.1.2.3 = \frac{1}{4}$$

b) Let E_1, E_2, E_3 denote the events that two balls among the three are red, blue and green respectively.

$$P(E_1) = \frac{C(2,2)C(8,1)}{C(10,3)} = \frac{1.8}{10.9.8}.1.2.3 = \frac{1}{15}$$

Similarly

$$P(E_2) = \frac{C(3,2)C(7,1)}{C(10,3)} = \frac{3.7}{10.9.8}.1.2.3 = \frac{7}{40}$$

$$P(E_3) = \frac{C(5,2)C(5,1)}{C(10,3)} = \frac{5.4}{1.2} \cdot \frac{5}{10.9.8}.1.2.3 = \frac{5}{12}$$

So $P(E) = P(E_1) + P(E_2) + P(E_3)$

$$= \frac{1}{15} + \frac{7}{40} + \frac{5}{12}$$

$$= \frac{8+21+50}{120}$$

$$= \frac{79}{120}$$

c) As there are only two red balls, the chosen three balls are of the same colour only if they are all blue or green.

Let E_1, E_2 denote the events that the three balls are blue and green respectively.

$$P(E_1) = \frac{C(3,3)}{C(10,3)} = \frac{1}{10.9.8}.1.2.3 = \frac{1}{120}$$

$$P(E_2) = \frac{C(5,3)}{C(10,3)} = \frac{5.4}{1.2} \cdot \frac{1.2.3}{10.9.8}.1.2.3 = \frac{1}{12}$$

So $P(E) = P(E_1) + P(E_2)$

$$= \frac{1}{120} + \frac{1}{12}$$

$$= \frac{1+10}{120}$$

$$= \frac{11}{120}$$

Example: Two dice are rolled. If A is the event that the number in the first die is odd and B is the event that the number in the second die is at least 3, find $P(A \cup B), P(A \cap B), P(A-B), P(B-A)$.

Solution: $P(A) = \frac{1}{2},\ P(B) = \frac{2}{3}$

The outcomes favourable to $A \cap B$ are 13, 14, 15, 16, 33, 34, 35, 36, 53, 54, 55, 56. So $P(A \cap B) = \frac{12}{36} = \frac{1}{3}$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{2}{3} - \frac{1}{3}$$

$$= \frac{5}{6}$$

$P(A-B) = P(A) - P(A \cap B)$ (By (12.4))

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6}$$

$$P(B-A) = P(B) - P(A \cap B) \qquad \text{(By (12.5))}$$

$$= \frac{2}{3} - \frac{1}{3}$$

$$= \frac{1}{3}$$

S.A.Q.5: A bag contains 6 white and 10 black balls. Three balls are drawn at random. Find the probability that

a) all the three are black
b) none of them is black
c) two of them are black
d) two of them are white
e) all the three are of the same colour

S.A.Q.6: The following table gives a distribution of wages of 1,000 workers:

Wages (in Rs.)	120 – 140	140 – 160	160 – 180	180 – 200	200 – 220	220 – 240	240 – 260
No. of workers	9	118	478	200	142	35	18

An individual is selected at random from the above group. What is the probability that his wages are (i) under Rs. 160 (ii) above Rs. 200, (iii) between Rs. 169 to 200 ?

S.A.Q. 7: Let a sample space be $S = \{a_1, a_2, a_3\}$. Which of the following defines probability space on S?

i) $P(a_1) = \frac{1}{4}$, $P(a_2) = \frac{1}{3}$, $P(a_3) = \frac{1}{2}$.

ii) $P(a_1) = \frac{2}{3}$, $P(a_2) = -\frac{1}{3}$, $P(a_3) = \frac{2}{3}$.

iii) $P(a_1) = 0$, $P(a_2) = \frac{1}{3}$, $P(a_3) = \frac{2}{3}$

S.A.Q. 8: Out of numbers 1 to 100, one is selected at random. What is the probability that it is divisible by 4 or 5?

S.A.Q. 9: The chance of an accident in a factory in a year is 1 in 5 in Bomay, 2 in 20 in Poona, 10 in 120 in Nagpur. Find the chances that a accident may happen in (i) at least one of them (ii) all of them.

S.A.Q. 10: A person is known to hit a target in 3 out of 5 shots, whereas another person is known to hit in 2 out of 3 shots. Find the probability that the target being hit in all when they both try.

S.A.Q. 11: A six faced dice is biased that is twice as likely to show an even number as an odd number when it is thrown. What is the probability that the sum of the two numbers is even ?

S.A.Q. 12: The odd in favour of one student passing a test are 3 : 6. The odds against another student passing at are 3:5. What are odds that (i) both pass, (ii) both fail ?

S.A.Q. 13: If two dice are thrown, what is the probability that the sum is (a) greater than 8, and (b) neither 7 or 11 ?

S.A.Q. 14: Let A and B two events such that $P(A) = \frac{3}{4}$ *and* $P(B) = \frac{5}{8}$.

Show that i) $P(A \cup B) \geq \frac{3}{4}$

ii) $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$

S.A.Q. 15: From a group of children, 5 boys and 3 girls, three children are selected at random. Calculate the probabilities that the selected group contain i) no girl, ii) only one girl, iii) one particular girl, (iv) at least one girl, and v) more girls than boys.

S.A.Q. 16: According to the census Bureau, deaths in the United States occur at a rate of 2, 425,000 per year. The National Centre for Health statistics reported that the three leading causes of death during 1997 were heart disease (725, 790), cancer (537, 390) and stroke (159, 877). Let H, c and δ represent the events that a person dies of heart disease, cancer and stroke, respectively.

a) Use the data to estimate *P(H), P(C)* and *P(S).*

b) Are the events H and C mutually exclusive. Find $P(H \cap C)$.

c) What is the probability that a person dies from heart disease or cancer?

d) What is the probability that a person dies from cancer or a stroke?
e) Find the probability that someone dies from a cause other than one of these three.

12.6 Conditional Probability and Independence of Events

Let us consider a class having both boys and girls. Suppose a girl is selected. What is the probability that the selected girl gets a first class? The required probability is the probability of B (getting a first class) given that A(The selected student is a girl) has already happened. Such a probability is called conditional probability.

Definition 12.12: Let A and B be two events in the same sample space and $P(B) > 0$. Then the conditional probability $P(A/B)$ is the probability for A to happen given that B has already happened.

The following theorem gives us a method of finding the conditional probability.

Theorem 12.2: (Multiplicative law for probability and conditional probability) For any two events A and B,

$P(A \cap B) = P(B)\,P(A/B)$, provided $P(B) > 0$.

$\qquad = P(A)\,P(B/A)$, provided $P(A) > 0$.

Proof: Let the total number of outcomes be N. Let n_A, n_B, n_{AB} denote the number of outcomes favourable to the events A, B and $A \cap B$ respectively. Then $P(A) = \frac{n_A}{N}$, $P(B) = \frac{n_B}{N}$ and $P(A \cap B) = \frac{n_{AB}}{N}$. Let us calculate $P(A/B)$ using the classical approach. As B has already happened, the total number of outcomes is n_B. The number of outcomes favourable to A given B is the number of outcomes favourable to $A \cap B$. Hence

$$P(A/B) = \frac{n_{AB}}{n_B}$$

$$= \frac{\frac{n_{AB}}{N}}{\frac{n_B}{N}}$$

$$= \frac{P(A \cap B)}{P(B)}$$

So $P(A \cap B) = P(B)\,P(A/B)$ proving the first identity. The second identity can be proved similarly.

Example 12.11: The following table shows the distribution of blood types in a state in India

	A	**B**	**AB**	**O**
Rh+	33%	11%	4%	42%
Rh-	3%	2%	2%	3%

Find the following probabilities

a) The probability that a person has type O blood.

b) The probability that a person is Rh-

c) The probability that a married couple are both Rh+

d) The probability that a married couple have type AB blood

e) The probability that a person has type B blood given that the person is Rh-

f) The probability that a person has type B blood given that the person is Rh+

Solution: Let A, B, AB, O, Rh+ and Rh- denote the events that a person has type A blood etc.

a) $P(O) = P(O \cap Rh+) + P(O \cap Rh-) = 0.33 + 0.33 = 0.36$

b) $\mathrm{P(Rh-) = P(A \cap Rh) + P(B \cap Rh-) + P(AB \cap Rh-) + (PO \cap Rh-)}$

$= 0.03 + 0.02 + 0.02 + 0.03 = 0.10$

c) *P (Married couple are both Rh+)*

= P[(husband is Rh+) $\cap$ *(wife is Rh+)]*

= [P (husband is Rh+)][P(wife is Rh+)]assuming independent.

= [1 – P (Rh –)] [1 – P(Rh –)]

= (0.9) (0.9) = 0.81

d) *P(a* couple have *AB)*

= P(husband has AB) P(wife has AB)

= (0.04 + 0.02) (0.04 + 0.02)

= 0.0036

e) $P(O / Rh-) = \dfrac{P(O \cap Rh-)}{P(Rh-)} = \dfrac{0.03}{0.10} = 0.3$

f) $P(B / Rh+) = \dfrac{P(B \cap Rh+)}{P(Rh+)} = \dfrac{0.11}{0.90} = 0.122$

Let A be the event that it rains heavily in Sikkim and B be the event that you will score a first class in Bio informatics. Obviously the events A and B have no dependence among themselves. We formulate this in the following definition.

Definition: Two events A and B are independent if the occurrence or non – occurrence of one does not affect the occurrence of the other. This happen when

$P(A/B) = P(A)$ and $P(B/A) = P(B)$ ……………….. (12.7)

Note: We know that $P(A/B) = \frac{P(A \cap B)}{P(B)}$

P(A/B) = P(A), (as P(A $\cap$ B) = P(A) P(B))

Also, $P(B/A) = \frac{P(A \cap B)}{P(A)} = P(B)$ when $P(A \cap B) = P(A).P(B)$

So *P(A/B) = P(A)* implies *P(B/A) = P(B)*

So we note the following

A and B are independent if $P(A \cap B) = P(A)\,P(B)$ ……………………… (12.8)

So (12.8) can be taken as the working definition of independence of two events.

Example: A bag has 20 blue balls and 10 green balls. Two balls are taken from the bag one after the other. Find the probability that both are blue if.

i) The first ball is not replaced before taking out the second ball
ii) The first ball is replaced before taking out the second

Solution: Let A denote the event that the first ball is blue and B be the event that the second ball is blue. So we have to find $P(A \cap B)$

As the bag has 20 blue balls and the total number of balls is 30, $P(A) = \frac{C(20,1)}{C(30,1)} = \frac{20}{30} = \frac{2}{3}$

i) When the first ball is not replaced and the second ball is taken out there are 19 balls. So

$$P(A) = \frac{C(20,1)}{C(29,1)} = \frac{19}{29}$$

So $P(A \cap B) = P(B/A)\,P(A)$ (By theorem 12.2)

$$= \frac{19}{29} \cdot \frac{2}{3}$$

$= \frac{38}{87}$

ii) When the first ball is replaced before the second ball is taken out, there are 20 blue balls and 30 balls in all before the second choice

So $P(B/A) = \frac{C(20,1)}{C(30,1)} = \frac{20}{30}$

So $P(A \cap B) = P(B/A)\,P(A) = \frac{20}{30} \cdot \frac{20}{30} = \frac{4}{9}$

Note: in the above example, A and B are not independent in (i) but independent in (ii)

Example 12.13: If A and B are independent show that

a) $A \text{ and } \overline{B}$ are independent

b) $\overline{A} \text{ and } \overline{B}$ are independent

Solution: As A and B are independent

$P(A \cap B) = P(A)\,P(B)$

We know that $A = (A \cap B) \cup (A \cap \overline{B})$ and the union is disjoint. Hence

$P(A) = P(A \cap \overline{B}) + P(A \cap B)$

$P(A \cap \overline{B}) = P(A) - P(A \cap B)$

$= P(A) - P(A)\,P(B)$ ($\because$ A and B are independent)

$= P(A)\,[1 - P(B)]$

$= P(A)\,P(\overline{B})$ (by property 12.2)

Hence A and $\overline{B}$ are independent

b) $P(\overline{A} \cap \overline{B}) = \overline{P(A \cup B)}\left(\because \overline{A \cap B} = \overline{A} \cap \overline{B} \text{ by De Morgan's law}\right)$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$ (by Addition theorem)

$= 1 - [P(A) + P(B) - P(A)\,P(B)]$ (since A and B are independent)

$= 1 - P(A) - P(B) + P(A)\,P(B)$

$= 1 - P(A) - P(B)\,[1 - P(A)]$

$= [1 - P(A)]\,[1 - P(B)]$

$= P(\overline{A})\,P(\overline{B})$

Hence $\overline{A} \text{ and } \overline{B}$ are independent.

Example: One third of the students in a class are girls and the rest are boys. The probability that a girl gets a first class is 0.4 and that of a boy is 0.3. If a student having first class is selected, find the probability that the student is a girl.

Solution: Let A, B and C denote the event that a student is a boy, a girl and a student having first class. We are given the following

$P(A) = \frac{2}{3}, \qquad P(B) = \frac{1}{3}, \qquad P(C/A) = \frac{3}{10}$ and $P(C/B) = \frac{4}{10}$

So $P(A \cap C) = P(C/A)\,P(A) = \frac{3}{10}.\frac{2}{3} = \frac{1}{5}$. Similarly $P(C \cap B) = \frac{4}{30}$

$P(C) = P(C \cap (A \cup B))$ since $A \cup B = 5$

$= P((C \cap A) \cup (C \cap B))$ by Demorgan's law

$= P(C \cap A) + P(C \cap B)$ by Addition theorem

$= \frac{3}{10} + \frac{4}{30} = \frac{13}{30}$

We are required to find $P(B/C)$

$$P(B/C) = \frac{P(B \cap C))}{P(C)} = \frac{\frac{4}{30}}{\frac{13}{30}} = \frac{4}{13}$$

Example: The probability that a 60 – year old man to be alive for 5 years is 0.80 and the same probability for a 55 – year old woman is 0.85. Find the probability that a couple of ages 60 and 50 respectively will be alive for the next 5 years.

Solution: We assume that the age expectation of the couple are independent. Let A, B denote the probability that the husband and wife will be alive for the next 5 years.

P(both will be alive for next 5 years)

$= P(A \cap B)$

$= P(A)\ P(B)$

$= (0.80)\ (0.85)$

$= 0.68$

We can extend the concept of independence to more than two events.

Definition: Three events A, B and C are mutually independent if the occurrence or non occurrence of any one of the events does not affect the occurrence of other events.

Note: When A, B, C are mutually independent then A and B are independent etc. So the working definition of 3 mutually independent events A, B, C can be given as follows.

A, B, C are mutually independent if

$P(A \cap B) = P(A)\, P(B),\ P(B \cap C) = P(B)\, P(C),$

$P(C \cap A) = P(C)\, P(A)$ *and*

$P(A \cap B \cap C) = P(A)\, P(B)\, P(C)$ ….. (12.9)

Example: A difficult problem is given to the students of 1^{st}, 2^{nd} and 3^{rd} rank by a professor. The probability that these students solve the problem are $\frac{3}{4}, \frac{1}{2}, \frac{2}{5}$ respectively. Find the probability that the problem is solved.

Solution: Let A denote the event that A solves the problem etc. Let us find the probability that the problem is not solved by any of them (Assume independence of A, B, C and hence $\overline{A}, \overline{B}$ *and* $\overline{C}$)

Then $P\left(\overline{A} \cap \overline{B} \cap \overline{C}\right) = P\left(\overline{A}\right) P\left(\overline{B}\right) P\left(\overline{C}\right)$

$$= \left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{2}\right)\left(1 - \frac{2}{5}\right)$$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)$$

$$= \frac{3}{40}$$

Probability that the problem is solved = $P(A \cup B \cup C) = 1 - P\left(\overline{A \cup B \cup C}\right)$

$$= 1 - P\left(\overline{A} \cap \overline{B} \cap \overline{C}\right)$$

$$\because \overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$$

by De Morgan's law

$$= 1 - \frac{3}{40}$$

$$= \frac{37}{40}$$

S.A.Q. 17: If $P(A) = 0.25$, $P(B/A) = 0.5$ and $P(A/B) = 0.25$ find $P(A \cup B), P(A \cap B), P(\overline{A} \cup B), P(A \cup \overline{B}), P(\overline{A} \cup \overline{B})$ *and* $P(\overline{A} \cap \overline{B})$

S.A.Q. 18: An article consists of two parts A and B. The probabilities of defect in A and B are 0.08 and 0.04. What is the probability that the assembled part will not have any defect?

S.A.Q. 19: From a bag containing 3 red and 4 black balls two balls are drawn in succession without replacement. Find the probability that both the balls are (i) red (ii) black (iii) of the same colour.

12.7 Baye's theorem

We have seen that subjective probability is used when some event may happen only once or a few times. But after assuming subjective probability we may get some new information. This information can be used to revise the subjective probability. Baye's theorem is used for revising probability on the basis of new information.

Reverend Thomas Bayes (1702 – 1761), a Christian Priest, enunciated Baye's theorem which has significant applications in many areas of business administration especially marketing.

Theorem (Baye's theorem)

If $E_1, E_2, \ldots\ldots, E_n$ are mutually exclusive events with $P(E_i) > 0, i = 1, 2, \ldots.. n$ then for any arbitrary event A which is a subset of $\bigcup_{i-1}^{n} E_i$ such that $P(A) > 0$, we have

$$P(E_i / A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

Note: $P(E_1). \ldots\ldots, P(E_n)$ are called a priori or prior probabilities. A denotes some new information. Then we revise the probabilities $P(E_i)$ as $P(E_i/A)$. The revised probabilities are called posteriori probabilities.

This process is illustrated in Fig. 12.6

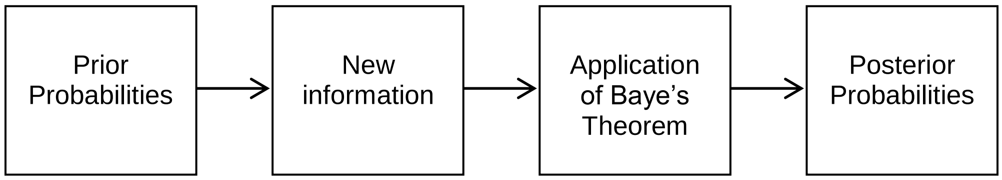

Figure 12.6 Prior and posterior probabilities

Example: A company has three plants A, B and C manufacturing the same spare part in the ratio 30:45:25. The percentage of defective parts in the plants are 3%, 2% and 5% respectively. A part is chosen at random and found to be defective. What is the probability that it is manufactured by plants A, B or C ?

Solution: Let A, B, C denote the event that it is manufactured in plants A, B and C respectively.

Let P(D) be the probability that a spare part is defective. Given

$P(A) = 0.3$	$P(B) = 0.45,$	$P(C) = 0.25$
$P(D/A) = 0.03$	$P(D/B) = 0.02$	$P(D/C) = 0.05$

Probability that the chosen defective part is manufactured by plant

$$A = P(A/D)$$

$$= \frac{P(A)\,P(D/A)}{P(A)\,P(D/A) + P(B)P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{(0.3)\,(0.03)}{(0.3)\,(0.03) + (0.45)\,(0.02) + (0.25)\,(0.05)}$$

$$= 0.295$$

Similarly,

$P(B/D) = 0.295$

$P(C/D) = 0.410$

Example: A physician believes that the people in a particular region are prone to diseases A and B. He estimates that P(A) = 0.6 and P(B) = 0.4. The diseases have symptoms S_1, S_2 and S_3. Given that the patient has the diseases, the probabilities for him to have symptoms S_1, S_2 and S_3 are given in the following table.

Disease	Symptoms		
	S_1	S_2	S_3
A	0.15	0.10	0.15
B	0.80	0.15	0.03

If a patient has symptom S_1, find the probability that he has disease A.

Solution We are given that

$P(A) = 0.6$ $P(B) = 0.4$

$P(S_1/A) = 0.15$ $P(S_2/A) = 0.10$ $P(S_3/A) = 0.15$

$P(S_1/B) = 0.80$ $P(S_2/B) = 0.15$ $P(S_3/B) = 0.03$

The required probability

$= P(A/S_1)$

$$= \frac{P(A)\,P(S_1 / A)}{P(A)\,P(S_1 / A) + P(B)\,P(S_1 / B)}$$

$$= \frac{(0.6)(0.15)}{(0.6)(0.15) + (0.4)(0.80)}$$

$= 0.738$

S.A.Q.20: The contents of vessels I, II and III are as follows:

1. white, 2 black and 1 red balls
2. white, 1 black and 1 red balls, and
4. white, 5 black and 3 red balls.

One vessel is chosen at random and two balls are drawn. They happen to be white and red. What is the probability that they come from vessels I, II or III ?

S.A.Q. 21: A company has three machines M_1, M_2, M_3 which produces 20%, 30% and 50% of the products respectively. Their respective defective percentages are 7, 3 and 5. From these products one is chosen and inspected. It is defective. What is the probability that it has been made by machine M_2 ?

12.8 Summary

In this unit we discussed about the concept of probability. The different basic term of probability is well defined with examples. Different types of probability, Baye's theorem and its application is discussed with clear cut examples.

12.9 Terminal Questions

1. Five persons are selected from a group of 8 men, 6 women and 6 children. Find the probability that 3 of the 5 persons selected are children.
2. A bag contains 5 white, 6 black and 3 green balls. Find the probability that a ball drawn at random is white or green.
3. Find the probability of getting (i) only one head and (ii) at least 4 heads in six tosses of an unbiased coin.
4. Find the chance of getting more than 15 in rolling three dice
5. A five-digit number is formed from the digits 2, 3, 5, 6, 8, no repetition being allowed. Find the probability that the number is (i) odd (ii) even.
6. A sample space has 5 elementary events E_1, E_2, E_3, E_4, E_5. *If* $P(E_3) = 0.4$, $P(E_4) = 2$, $P(E_5)$ and $P(E_1) = P(E_2) = 0.15$, determine $P(E_4)$, $P(E_5)$, $P(\{E_4, E_5\})$, $P(\{E_1, E_2, E_3\})$
7. A problem is probability is given to three students A, B and C. The probabilities that they solve the problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. Find the probabilities that
   i) A alone solves the problem
   ii) Just two of them solve the problem
   iii) The problem is solved

   (Mention the assumptions you make in solving the problem).
8. A large company dumps its chemical waste in a local river. The probability that either a fish or an animal dies on drinking the water is $\frac{11}{21}$. The probability that only a fish dies is $\frac{1}{3}$ and the probability that only an animal dies in $\frac{2}{7}$. What is the probability, (i) that both will dies ? (ii) none of them will die ?
9. In a college, the percentage of students reading the new Indian Express, Deccan Herald and both are 20%, 30% and 15%. Find the probability that a student of the college
   i) reads at least one newspaper

ii) reads none of them

iii) reads only Deccan Herald

10. A team of 6 students is to be selected from a class consisting of 7 boys and 4 girls. Find the probability that the team consists of a) exactly two girls b) at least 2 girls.

11. 40 candidates appeared for examination in Papers A and B. 16 students passed in paper A, 14 passed in Paper B and 16 failed in both. If one student is selected at random what is the probability that he

    a) passed in both papers

    b) failed only in A

    c) failed in A or B

12. In a town, only 80% of the children born reach the age of 15 and only 85% of them reach the age of 30. 2.5% of persons aged 30 die in one year. What is the probability that a person will reach the age of 31.

13. If A, B, C are mutually exclusive and collectively exhaustive and $2\,P(C) = 3P\,(A) = 6P(B)$ find $P(A), P(B), P(C).$

14. Two vessels contain 20 white, 12 red and 18 black balls; 6 white, 14 red and 30 black balls respectively. One ball is taken out from each vessel. Find the probability that a) both are red b) both are of the same colour

15. The probability that a candidate passes in Biostatistics is 0.6 and that the probability that he passes in Genetics 50.5. What is the probability that he passes in only one of the papers ? (Mention the assumption you make).

16. The following table gives the frequency distribution of 50 professors according to age and their salaries.

Age in years	**Salary**			
	10000 – 15000	**15000 – 20000**	**20000 – 25000**	**25000 – 30000**
20 – 30	16	6	–	–
30 – 40	4	10	4	4
40 – 50	–	4	18	12
50 – 60	–	–	12	10

If a professor is chosen at random find the probability that

a) he is in the age group 30 – 40 years and earns more than 20,000

b) he earns in the range 15000 – 20000 and less than 40 years old.

17. In a class of 20 boys and 40 girls, half the boys and half the girls have two – wheelers. Find the probability that a randomly selected student is a boy or has a two wheeler.

18. Three fair (unbiased) coins are tossed. If the first coin shows a head find the probability of getting all heads.

19. If $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$ *and* $P(A \cup B)=\frac{1}{2}$ find *P B/A and P(A/B).*

20. If $P(A)=\frac{1}{3}, P(B)=\frac{1}{2}, P(A / B = \frac{1}{6},$ find *P(B/A) and P(B/A).*

21. The records of 400 examinees are given below.

Score	**Educational Qualification**			
	B.A.	**B.Sc**	**B.Com**	**Total**
Below 50	90	30	60	**180**
Between 50 & 60	20	70	70	160
Above 60	10	30	20	60
Total	120	130	150	400

If an examinee is selected from this group find

i) The probability that he is a commerce graduate

ii) The probability that he is a science graduate given that his score is above 60

iii) The probability that his score is below 50 given that he is a B.A graduate.

22. There are 3 boxes containing respectively 1 white, 2 red, 3 black balls; 2 white, 3 red, 1 black ball; 3 white, 1 red and 2 black b alls. A box is chosen at random and from it two balls are drawn at random. The two balls are 1 red and 1 white. What is the probability that they come from (i) the first box (ii) second box (iii) third box?

23. An item is manufactured by three machines M_1, M_2 and M_3. Out of the total manufactured during a specified production period, 50% are manufactured on M_1, 30% on M_2 and 20% on M_3.
It is also known that 2% of the item produced by M_1 and M_2 are defective, white 3% of those manufactured by M_3 are defective. All the items are put into one bin. From the bin, one item is drawn at random and is found to be defective. What is the probability that it was made on M_1, M_2 or M_3.

12.10 Answers

Self Assessment Questions

1. a) {TTT, HHT, HTH, THH}
   b) {WLWLW, LWLWL}
   c) {13, 22, 31, 46, 55, 64}
   d) {WLL, LWL, LLW, WWL, WLW, LWW, WWW}

2. $\frac{C(6,1)\,C(7,1)}{C(16,2)} = \frac{7}{20}$

3. $\frac{7}{17}$

4. The sum 13 can be obtained from the pairs (3, 10), (5, 8), (7, 6), (9, 4). The total number of pairs that can be chosen is 9(5) = 45.

5. Answer: $\frac{4}{45}$

   a) $\frac{C(10,3)}{C(16,3)} = \frac{3}{14}$

   b) $\frac{C(6,3)}{C(16,3)} = \frac{1}{28}$

   c) $\frac{C(10,2)\,C(6,1)}{C(16,3)} = \frac{27}{56}$

   d) $\frac{15}{56}$

e) $\frac{3}{14}+\frac{1}{28}=\frac{1}{4}$

6. Total number of workers = 1000. (i) 0.127 (ii) 0.195 (iii) 0.596

7. i) As $P(S) = P(a1) + P(a2) + P(a3) \neq 1$, answer is No.

   ii) As $P(a_3)=\frac{-1}{3}, No$

   iii) $P(S) = 1;$ Yes

8. Let A and B denote divisibility by 4, 5. Outcomes favourable to A are 4, 8, …… , 100. So $P(A) = 0.25$. Similarly $P(B) = 0.2$ and $P(A \cap B) = 0.05.$ Answer: 0.4

9. i) Prob (accident occurs in none of them)

   $=P\left(\overline{A}\cap\overline{B}\cap\overline{C}\right)=\frac{4}{5},\frac{9}{10},\frac{11}{12}=\frac{33}{50}.$

   Answer: $\frac{17}{50}$

   ii) $P(A\cap B\cap C)=\frac{1}{600}$

10. Assume independence. (In case of independence, $P(A \cap B) = P(A) P(B)$. See later sections).

    Answer: $\frac{13}{15}$

11. Let P (odd number) = a. Treat the die as a coin showing 'even' or 'odd'. Then $2a + a = 1$ or $a=\frac{1}{3}.$ Prob (sum is even = Prob (both are even) + Prob (both are odd) = $\frac{2}{3}\cdot\frac{2}{3}+\frac{1}{3}\cdot\frac{1}{3}=\frac{5}{9}.$

12. $P(A)=\frac{3}{3+6}=\frac{1}{3}.$ For the second, odds for passing are 5 : 3. So $P(B)=\frac{5}{8}.$ Assume independence i) $\frac{1}{3}\cdot\frac{5}{8}=\frac{5}{24}$ ii) $\frac{2}{3}\cdot\frac{3}{8}=\frac{1}{4}.$

13. a) Favourable outcomes are: 36, 45, 54, 63, 46, 55, 64, 56, 65, 66.

    Answer: $\frac{5}{18}.$

b) $P(7 \text{ or } 11) = P(7) + P(11) = \frac{1}{6} + \frac{1}{18}$. Answer: $\frac{7}{9}$.

14. As $P(A) \le P(A \cup B)$, (i) follows. $P(A \cap B) \le P(B) = \frac{5}{8}$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) \ge \frac{3}{4} + \frac{5}{8} - 1$$

(since P$(A \cup B) \le 1$ and so $- P(A \cup B) \ge -1)$. So (ii) follows.

15. (i) $\frac{C(5, 2)}{C(8, 3)} = \frac{5}{28}$

(ii) $\frac{C(3,1)(5, 2)}{C(8, 3)} = \frac{15}{28}$

(iii) $\frac{C(7, 2)}{(8, 3)} = \frac{3}{8}$

(iv) $1 - \frac{5}{28} = \frac{23}{28}$

(v) $\frac{1}{56} + \frac{15}{56} = \frac{2}{7}$

16. a) $P(H) = \frac{725790}{2{,}425{,}000}$ $P(C) = \frac{537390}{2425000}$ $P(S) = \frac{159877}{2425000}$

b) Yes. They are also independence. Answer: $P(H)\ P(C)$

c) $P(H) + P(C)$

d) $P(C) + P(S)$

e) $P(H \cup C \cup S) = P(H) + P(C) + P(S) - P(H)\,P(C) - P(C)\,P(S) - P(H)\,P(S) + P(H)\,P(C)\,P(S)$. (We assume independence of H, C, S)

17. $P(A \cap B) = 0.125$, $P(B) = 0.5$,

So $P(A \cup B) = 0.25 + 0.5 - 0.125 = 0.625$.

$P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) = 0.375$. $P(\overline{A} \cup \overline{B}) = 1 - P(A \cap B) = 0.875$

$P(\overline{A} \cup B) = P(B) - P(B) - P(A \cap B) = 0.375$.

$P(A \cup \overline{B}) = P(A) - P(A \cap B) = 0.125$

18. Let x, y be the events that A and B do not have any defect respectively. $P(x) = 0.92$ and $P(y) = 0.96$. Assuming independence, the answer is (0.92) (0.96) = 0.8832.

19. Let A and B denote the events that the ball is red in first and second attempt (i) $P(A \cap B) = P(A)\,P(B/A) = \frac{3}{7} \cdot \frac{2}{6} = \frac{1}{7}$ (ii) $\frac{2}{7}$ (iii) $\frac{1}{7} + \frac{2}{7} = \frac{3}{7}$

20. *P(E1) = P(E2) = P(E3) = 1*

$P(A/E_1) = \frac{1}{5}$ $P(A/E_2) = \frac{1}{3}$ $P(A/E_3) = \frac{2}{11}$ $P(E_1/A) = \frac{33}{118}$

$P(E_2/A) = \frac{55}{118}$ So $P(E_3/A) = 1 - \frac{33}{118} - \frac{55}{118} = \frac{30}{118.}$

21. Answer: $\frac{(0.3)(0.3)}{(0.2)(0.07) + (0.3)(0.03) + (0.5)(0.05)} = 0.1875$

Terminal Questions

1. $\frac{C(6,3)\,C(14,2)}{C(20,5)} = \frac{455}{3876}$

2. $\frac{4}{7}$

3. One head can appear in 6 ways.

   (i) $\frac{3}{32}$ (ii) $\frac{C(6,4) + C(6,5) + C(6,6)}{64} = \frac{11}{32}$

4. We should get a sum of 16, 17, 18. Answer: $\frac{5}{108}$.

5. An odd number ends in 3 or 5. Number of favourable outcomes is 2(4!).

   Answer: $\frac{2(4\,!)}{5\,!} = \frac{2}{5};\ \frac{3}{5}$

6. Let $P(E_5) = x$. Then $0.15 + 0.15 + 0.4 + 2x + x = 1$. Hence $x = 0.1$

   Answers: *0.2, 0.1, 0.3, 0.7*

7. a) $P(A \cap \overline{B} \cap \overline{C}) = \frac{1}{4}$

   b) $P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C) = \frac{5}{12}$

   c) $P(\overline{A} \cap \overline{B} \cap \overline{C}) = \frac{1}{4}$. $P(A \cup B \cup C) = \frac{3}{4}$

8. Given that $P(F \cup A) = \frac{11}{21}, P(F) = \frac{1}{3}$ *and* $P(A) = \frac{2}{7}$, use addition theorem,

i) $P(F \cap A) = \frac{2}{21}$ ii) $P(\overline{F} \cap \overline{A}) = 1 - \frac{2}{21} = \frac{19}{21}$

9. Given that $P(A) = 0.2, P(B) = 0.3$ and $P(A \cap B) = 0.15$

(i) $P(A \cup B) = 0.35$

(ii) $P(\overline{A} \cap \overline{B}) = 0.65$

(iii) $P(\overline{A} \cap B) = 0.15$

10. a) $\frac{5}{11}$

b) $\frac{C(4,2)C(7,4) + c(4,3)C(7,3) + c(4,4)C(7,2)}{C(11,6)} = \frac{53}{66}$

11. Given that $P(A) = \frac{16}{40},\ P(B) = \frac{14}{40}$ *and So,* $P(A \cup B) = \frac{24}{40}$.

a) $\frac{3}{20}$ b) $P(\overline{A} \cap B) = \frac{1}{5}$ c) $P(\overline{A} \cup \overline{B}) = \frac{17}{20}$

12. P (Fifteen) = 0.8, P (thirty/ Fifteen) = 0.85 and P (thirty one/ thirty) = 0.975.

Answer: (0.8) (0.85) (0.975) = 0.663

13. $1 = P(A) + P(B) + P(C) = P(A) + \frac{1}{2}P(A) + \frac{3}{2}P(A)$

So, $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{6}$, $P(C) = \frac{1}{2}$.

14. a) $\frac{12}{50} \cdot \frac{14}{50} = \frac{42}{625}$

b) $\frac{12}{50} \cdot \frac{14}{50} + \frac{20}{50} \cdot \frac{6}{50} + \frac{18}{50} \cdot \frac{30}{50} = \frac{207}{625}$

15. Assume independence of B and G. Answer:

$P(B \cap \overline{G}) + P(\overline{B} \cap G) = 0.5$

16. Total number of professors = 100

a) $4 + \frac{4}{100} = (0.08)$ b) 0.16

17. Given that $P(B)=\frac{1}{3}$, $P(B\cap W)=\frac{10}{60}=1/6$, $P(W)=\frac{1}{2}$

    Answer: $P(B\cup W)=2/3$

18. Let A denote getting H is the first toss and B denote getting H in second and third tosses.

    Answer: $P(B/A)=P(A\cap B)/P(A)=\frac{1}{8}\div\frac{1}{2}=\frac{1}{4}$

19. $P(A\cap B)=\frac{1}{12}$ $\quad P(B/A)=\frac{1}{4}$ and $P(A/B)=\frac{1}{3}$

20. $P(A\cap B)=\frac{1}{12}$

    $P(B/A)=P(A\cap B)/P(A)=\frac{1}{12}\div\frac{1}{3}=\frac{1}{4}$

    $P(B\cap\overline{A})=P(B)-P(B\cap A)=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$

    $\therefore P(B/\overline{A})=\frac{5}{12}\div P(\overline{A})=\frac{5}{12}\div\frac{2}{3}=\frac{5}{8}$

21. (i) $\frac{3}{8}$ (ii) $P(S/Score>60)=\frac{1}{2}$ (iii) $P(Score<50/A)=\frac{3}{4}$

22. $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$

    $P(A/E_1)=\frac{2}{15}$ $\quad P(A/E_2)=\frac{6}{15}$

    $P(A/E_3)=\frac{3}{15}$

    $P(E_1/A)=\frac{2}{11}$, $\quad P(E_2/A)=\frac{6}{11}$ Hence $P(E_3/A)=\frac{3}{11}$

23. $P(E_1) = 0.5$ $P(E_2) = 0.3$ $P(E_3) = 0.2$ $P(A/E_1) = 0.02$ etc

    $P(E_1/A) = 0.454$, $P(E_2/A) = 0.273$, $P(E_3/A) = 0.273$