Onit - 9

Interpolation with Unequal Intervals:

* Lagrange's Interpolation

If $y = f(x)$ takes the values $y_0, y_1, y_2, \ldots y_n$ corresponding to $x = x_0, x_1, x_2 \ldots x_n$ then.

$$y = f(x) =$$

$$\frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} y_1 + \ldots +$$

$$\frac{(x-x_0)(x-x_1)(x-2)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-1)\ldots(x_n-x_{n-1})} y_n.$$

* Newton's divided difference formula:-

$$y = f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0)$$

$$+ (x-x_0)(x-x_1)(x+x_2)\Delta^3 f(x_0) + \ldots$$

Many receive advice, only the wise profit by it.

Interpolation for unequal interval

Lagrange's Interpolation for unequal interval

Q find Value of y when $x=10$ by Lagrange's Interpolation formula

x	5	6	9	11
$f(x)=y$	12	13	14	16

$$f(x)=\frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)}(12)+\frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-11)}(13)+\frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)}(14)+\frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)}(16)$$

$$f(10)=\frac{(4)(1)(-2)}{(-1)(-4)(-6)}(12)+\frac{(5)(1)(-1)}{(1)(-3)(-5)}(13)+\frac{(5)(4)(-1)}{(4)(3)(-2)}(14)+\frac{(5)(4)(1)}{(6)(3)(2)}(16)$$

$$=+4-\frac{13}{3}+\frac{35}{3}+\frac{16}{3}=4+\frac{38}{3}=\frac{50}{3}=16.66$$

Interpolation for unequal interval

1 Newton Divided difference for unequal interval

Q find Value of y when $x = 10$ by Newton Divided Difference formula

x	5	6	9	11
$f(x) = y$	12	13	14	16

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_0 5	12			
		$\frac{13-12}{6-5} = 1$		
x_1 6	13		$\frac{\frac{1}{3}-1}{9-5} = -\frac{1}{6}$	
		$\frac{14-13}{9-6} = \frac{1}{3}$		$\frac{\frac{2}{15}+\frac{1}{5}}{11-5} = \frac{1}{20}$
x_2 9	14		$\frac{1-\frac{1}{3}}{11-6} = \frac{2}{15}$	
		$\frac{16-14}{11-9} = 1$		
x_3 11	16			

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0) + \cdots$$

$$f(x) = 12 + (x-5)(1) + (x-5)(x-6)\left(-\frac{1}{6}\right) + (x-5)(x-6)(x-9)\frac{1}{20}$$

$$f(10) = 12 + 5 + 5\times 4\times -\frac{1}{6} + 5\times 4\times 1\times \frac{1}{20}$$

$$= 12 + 5 - \frac{20}{6} + 1$$

$$= 18 - \frac{10}{3}$$

$$= 18 - 3.33 = 14.66$$

Q. Use Lagrange's formula to fit Polynomial to the following data Hence find $y(-2)$, $y(1)$, $y(4)$

x	-1	0	2	3
y	-8	3	1	2

$$f(x) = \frac{7x^3 - 31x^2 + 28x + 18}{6}$$

$y(-2) = f(-2) = -36.33$

$y(1) = f(1) = 3.666$

$y(4) = f(4) = 13.166$

$$f(x) = \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}(-8) + \frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)}(3) + \frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)}(1) + \frac{(x+1)(x-0)(x-2)}{(3+1)(3-0)(3-2)}(2)$$

$$f(x) = (x-2)(x-3)\left(\frac{-8x}{(-1)(-3)(-4)} + \frac{3(x+1)}{(1)(-2)(-4)}\right) + x(x+1)\left(\frac{x-3}{(3)(2)(-1)} + \frac{2(x-2)}{(4)(3)(1)}\right)$$

$$f(x) = (x^2-5x+6)\left(\frac{2x}{3} + \frac{x+1}{2}\right) + (x^2+x)\left(-\frac{(x-3)}{6} + \frac{x-2}{6}\right)$$

$$f(x) = (x^2-5x+6)\left(\frac{4x+3x+3}{6}\right) + (x^2+x)\left(\frac{-x+1+x-3}{6}\right)$$

$$f(x) = (x^2-5x+6)\left(\frac{7x+3}{6}\right) + x^2+x$$

Inverse Lagrang's formula to find value of x

Q Apply Lagrange's formula inversely to find value of x when $y = 19$ Given following

x	0	1	2
y	0	1	20

$$x = \frac{(y-1)(y-20)}{(0-1)(0-20)}(0) + \frac{(y-0)(y-20)}{(1-0)(1-20)}(1) + \frac{(y-0)(y-1)}{(20-0)(20-1)}(2)$$

$$x_{at\ y=19} = 0 + \frac{(19)(-1)}{(1)(-19)}(1) + \frac{(19)(18)}{(20)(19)}(2)$$

$$= +1 + \frac{18}{10}$$

$$= 1 + 1.8$$

$$= 2.8$$