Unit-10

Numerical Differentiation

* Derivatives using Newton's formula.

(1) forward:-

$$f(a+hu) = f(a) + \Delta f(a)\left(\frac{u}{1!}\right) + \Delta^2 f(a)\left(\frac{u(u-1)}{2!}\right) + \Delta^3 f(a)\left(\frac{u(u-1)(u-2)}{3!}\right) + \ldots\ldots$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta f(a) + \frac{(2u-1)}{2}\Delta^2 f(a) + \frac{(3u^2-6u+2)}{6}\Delta^3 f(a) + \left(\frac{4u^3-18u^2+22u+6}{24}\right)\Delta^4 f(a) + \ldots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 f(a) + (u-1)\Delta^3 f(a) + \left(\frac{12u^2-36u+22}{24}\right)\Delta^4 f(a) + \ldots\right]$$

Also formula can be use for find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for Newton's forward differeniution.

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - \frac{1}{6}\Delta^6 y_0 + \ldots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \frac{137}{180}\Delta^6 y_0 + \ldots\right]$$

② Backward :-

$$f(a+hu) = f(a) + \nabla f(a)\left(\frac{u}{1!}\right) + \nabla^2 f(a)\left(\frac{u(u+1)}{2!}\right) + \nabla^3 f(a)\left(\frac{u(u+1)(u+2)}{3!}\right) + \dots$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla f(a) + \left(\frac{2u+1}{2}\right)\nabla^2 f(a) + \left(\frac{3u^2+6u+2}{6}\right)\nabla^3 f(a) + \left(\frac{4u^3+18u^2+22u+6}{24}\right)\nabla^4 f(a) + \dots\right]$$

$$\frac{d^2y}{dx} = \frac{1}{h^2}\left[\nabla^2 f(a) + (u+1)\nabla^3 f(a) + \left(\frac{12u^2+36u+22}{24}\right)\nabla^4 f(a) + \dots\right]$$

Also, to find (dy/dx) and (d^2y/dx^2) for Newton's backward difference formula :-

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \frac{1}{5}\nabla^5 y_n + \frac{1}{6}\nabla^6 y_n + \dots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \frac{137}{180}\nabla^6 y_n + \dots\right]$$