Unit 2 Mathematical Logic

Structure:

2.1 Introduction

Logic is the study of general patterns of reasoning, without reference to particular meaning or contexts. If an object is either black or white, and if it is not black, then logic leads us to the conclusion that it must be white. Observe that logical reasoning from the given hypotheses cannot reveal what ‘black’ or ‘white’ mean, or why an object can not be both.

Logic can find applications in many branches of sciences and social sciences. Logic, in fact, is the theoretical basis for many areas of computer science such as digital logic circuit design, automata theory and artificial intelligence.

In this chapter, we shall learn about statements, truth values of a statement, compound statements, basic logical connectives, truth tables, tautologies, logical equivalence, duality, algebra of statements, use of Venn diagrams in logic and finally, some simple applications of logic in switching circuits.

Objectives:

At the end of the unit you would be able to

- understand the ideas in Mathematical Logic
- apply the concept of Mathematical Logic in circuits

2.2 Statement

A *statement* is a sentence which is either true or false, but not both simultaneously.

Note: A sentence which is both *true* and *false* simultaneously is not a statement, rather, it is a *paradox*.

Example:

(a) Each of the following sentences:

i) New Delhi is in India.
ii) Two plus two is four.
iii) Roses are red.
iv) The sun is a star.
v) Every square is a rectangle.

is true and so each of them is a statement.

(b) Each of the following sentences:

i) The earth is a star.
ii) Two plus two is five.
iii) Every rectangle is a square.
iv) 8 is less than 6.
v) Every set is a finite set.

is false and so each of them is a statement.

Example:

a) Each of the sentences:

i) Open the door.
ii) Switch on the fan.
iii) Do your homework.

can not be assigned true or false and so none of them is a statement. Infact, each of them is a *command*.

b) Each of the sentences:

i) Did you meet Rahman?
ii) Where are you going?

iii) Have you ever seen TajMahal?

can not be assigned true or false and so none of them is a statement. Infact, each of them is a question.

c) Each of the sentences:
   i) May you live long!
   ii) May God bless you!

   can not be assigned true or false and so none of them is a statement. Infact, each of them is *optative*.

d) Each of the sentences:
   i) Hurrah! We have won the match.
   ii) Alas! I have failed.

   can not be assigned true or false and so none of them is a statement. In fact, each of them is *exclamatory*.

e) Each of the sentences:
   i) Good morning to all.
   ii) Wish you best of luck.

   can not be assigned true or false and so none of them is a statement. In fact, each them is a *wish*.

f) Each of the sentences:
   i) Please do me a favour.
   ii) Give me a glass of water.

   can not be assigned true or false and so none of them is a statement. In fact, each them is a request.

g) Each of the following sentences:
   i) x is a natural number
   ii) He is a college student.

   is an open sentence because the truth or falsity of *(i)* depends on *x* and that of *(ii)* depends on the reference for the pronoun *he*. We may observe that for some values of x like $x = 1, 2, \ldots$ etc, *(i)* may be true and for some other values like $x = \frac{1}{2}, \frac{1}{3}, \ldots$ etc, *(i)* is false. . However, at a particular point of time or situation they are either true or false. Since, we are interested only in the fact that it is true or false, sentences *(i)* and *(ii)* can be considered as statements.

It is useful to have some notation to represent statements. Let us represent the statements by lower case letter like *p, q, r, s, …..* Thus, a statement 'New Delhi is city may be represented or denoted by *p* and we write

*p :*New Delhi is a city.

similarly, we may denote a statement '$2 + 3 = 6$' by *q* and write

$q : 2 + 3 = 6.$

Truth value of a statement: The truth or falsity of a statement is called its truth value. Every statement must be either true or false. No statement can be both true and false at the same time. If a statement is true, we say that its truth value is TRUE or *T* and if it is false we say that its truth value is FALSE or *F.*

Example: The statements in earlier Example *(a)* have the truth value T while the statements in earlier Example *(b)* have the truth value F.

Simple statement: A statement is said to be *simple*, if it cannot be broken down into two or more sentences. The statements that we considered in earlier Example *(a)* and *(b)* are all simple statements.

Compound statement: New statements that can be formed by combining two or more simple statements are called *compound statements*. Thus, a compound statement is the one which is made up of two or more simple statements.

Example:

a) The statement "Roses are red and Violets are blue" is a compound statement which is a combination of two simple statements "Roses are red" and "Violets are blue".
b) The statement "Gita is sick or Rehana is well" is a compound statement made up of two simple statements "Gita is sick" and "Rehana is well".
c) The statement "It is raining today and $2 + 2 = 4$" is a compound statement composed of two simple statements "It is raining today" and "$2 + 2 = 4$".

Simple statements, which on combining, form compound statements, are called *sub-statements* or component statements of the compound statements. The compound statements *S* consisting of sub-statements *p, q, r,...* is denoted by *S (p, q, r,...).*

A fundamental property of a compound statements is that its truth value is completely determined by the truth value of each of its sub-statements together with the way in which they are connected to form the compound statement.

2.3 Basic Logical Connectives

There are many ways of combining simple statements to form compound statements. The words which combine simple statements to form compound statements are called *connectives*. In the English language, we combine two or more statements to form a new statement by using the connectives 'and', 'or', etc. with a variety of meanings. Because the use of these connectives in English language is not always precise and unambiguous, it is necessary to define a set of connectives with definite meanings in the language of logic, called *object language*. We now define connectives for object language which *corresponds* to the connectives discussed above. Three basic connectives (logical) are *conjunction* which corresponds to the English word 'and' ; disjunction which corresponds to the word 'or' ; and *negation* which corresponds to the word 'not'.

Throughout we use the symbol '$\wedge$' to denote conjunction ; '$\vee$' to denote disjunction and the symbol '~' to denote negation.

Note:. Negation is called a connective although it does not combine two or more statements. In fact, it only *modifies* a statement.

2.4 Conjunction

If two simple statements p and q are connected by the word 'and', then the resulting compound statement *"p* and *q"* is called a conjunction of p and q and is written in symbolic form as "$p \wedge q$".

Example: Form the conjunction of the following simple statements:

p : Dinesh is a boy.

q : Nagma is a girl.

Solution: The conjunction of the statement p and q is given by

$p \wedge q$: Dinesh is a boy and Nagma is a girl.

Example: Translate the following statement into symbolic form

"Jack and Jill went up the hill."

Solution: The given statement can be rewritten as

"Jack went up the hill and Jill went up the hill"

Let *p :* Jack went up the hill and *q :* Jill went up the hill.

Then the given statement in symbolic form is $p \wedge q$.

Regarding truth value of conjuction of statements, We have

(D_1) : $T \wedge T = T$

(D_2): $T \wedge F = F \wedge T = F \wedge F = F$

Example: Write the truth value of each of the following four statements:

i) Delhi is in India and $2 + 3 = 6$.
ii) Delhi is in India and $2 + 3 = 5$.
iii) Delhi is in Nepal and $2 + 3 = 5$.
iv) Delhi is in Nepal and $2 + 3 = 6$.

Solution: In view of (D_1) and (D_2) above, we observe that statement *(i)* has the truth value F as the truth value of the statement "$2 + 3 = 6$" is *F.* Also, statement *(ii)* has the truth value *T* as both the statement "Delhi is in India" and "$2 + 3 = 5$" has the truth value *T*. Similarly, the truth value of both the statements *(iii)* and *(iv)* is *F*.

2.5 Disjunction

If two simple statements p and q are connected by the word 'or', then the resulting compound statement *"p or q"* is called disjunction of *statements p* and *q* and is written in symbolic form as "$p \vee q$".

Example: Form the disjunction of the following simple statements:

p : The sun shines.

q : It rains.

Solution: The disjunction of the statements *p* and *q* is given by

$p \vee q$: The sun shines or it rains.

Regarding truth value of disjuction of statements, We have

(D_3): $T \vee T = T \vee F = F \vee T = T$

(D_4): $F \vee F = F$

Example: Write the truth value of each of the following statements:

i) India is in Asia or $2 + 2 = 4$.

ii) India is in Asia or $2 + 2 = 5$.

iii) India is in Europe or $2 + 2 = 4$.

iv) India is in Europe or $2 + 2 = 5$.

Solution: In view of (D_3) and (D_4) above, we observe that only the last statement has truth value F as both the sub-statements "India is in Europe" and "$2 + 2 = 5$" have the truth value F. The remaining statements *(i)* to *(iii)* have the truth value T as at least one of the sub-statements of these statements has the truth value T.

2.6 Negation

An assertion that a statement fails or denial of a statement is called the negation of the statement. The negation of a statement is generally formed by introducing the word "not" at some proper place in the statement or by prefixing the statement with "It is not the case that" or "It is false that".

The negation of a statement p in symbolic form is written as "$\sim p$".

Example: Write the negation of the statement

p : New Delhi is a city.

Solution: The negation of p is given by

$\sim p$:New Delhi is not a city

or $\sim p$: It is not the case that New Delhi is a city.

or $\sim p$: It is false that New Delhi is a city

Example: Write the negation of the following statements:

p : I went to my class yesterday.

q : $2 + 3 = 6$

r : All natural numbers are integers.

Solution: Negation of the statement p is given by

$\sim p$: I did not go to my class yesterday.

or

It is not the case that I went to my class yesterday.

or

It is false that I went to my class yesterday.

or

I was absent from my class yesterday.

The negation of the statement q is given by

$\sim q : 2 + 3 \neq 6$

or

It is not the case that $2 + 3 = 6$

or

It is false that $2 + 3 = 6$

The negation of the statement r is given by

~ r : Not all natural numbers are integers.

or

There exists a natural number which is not an integer.

or

it is not the case that all natural numbers are integers.

or

It is false that all natural numbers are integers.

Regarding the truth value of the negation $\sim p$ of a statement *p.* we have

(D_5) : ~ p has truth value *T* whenever p has truth value *F.*

(D_6) : ~ p has truth value *F* whenever p has truth value *T.*

Example: Write the truth value of the negation of each of the following statements::

i) *p :* Every square is a rectangle.
ii) *q :* The earth is a star.
iii) $r : 2 + 3 < 4$

Solution: In view of (D_5) and (D_6), we observe that the truth value of $\sim p$ is *F* as the truth value of p is *T*. Similarly, the truth value of both $\sim q$ and $\sim r$ is *T* as the truth value of both statements *q* and r is *F*

2.7 Negation of Compound Statements

I) **Negation of conjunction:** Recall that a conjunction $p \wedge q$ consists of two sub-statements *p* and *q* both of which exist simultaneously. Therefore, the negation of the conjunction would mean the negation of at least one of the two sub-statements. Thus, we have

(D_7): The negation of a conjunction $p \wedge q$ is the disjunction of the negation of *p* and the negation of *q.* Equivalently, we write

$$\sim (p \wedge q) = \sim p \vee \sim q$$

Example: Write the negation of each of the following conjunctions:

a) Paris is in France and London is in England.

b) $2 + 3 = 5$ *and* $8 < 10$.

Solution:

(a) Write p *:*Paris is in France and q *:*London is in England.

Then, the conjunction in *(a)* is given by $p \wedge q$.

Now $\sim p$ *:*Paris is not in France, and

$\sim q$ *:*London is not in England.

Therefore, using *(D_7),* negation of $p \wedge q$ is given by

$\sim(p \wedge q)$ = Paris is not in France or London is not in England.

(b) Write $p : 2+3 = 5$ and $q : 8 < 10$.

Then the conjunction in *(b)* is given by $p \wedge q$.

Now $\sim p : 2 + 3 \neq 5$ and $\sim q : 8 \nless 10$

Then, using *(D_7),* negation of $p \wedge q$ is given by

$\sim (p \wedge q) = 2 + 3 \neq 5$ or $(8 \nless 10)$.

(II) **Negation of disjunction:** Recall that a disjunction $p \vee q$ is consisting of two sub-statements p and q which are such that either p or q or both exist. Therefore, the negation of the disjunction would mean the negation of both p and q simultaneously. Thus, in symbolic form, we have

(D_8): The negation of a disjunction $p \vee q$ is the conjunction of the negation of p and the negation of q. Equivalently, we write

$\sim (p \vee q) = \sim p \wedge \sim q$

Example: Writ the negation of each of the following disjunction:

a) Ram is in class X or Rahim is in Class XII

b) 7 is greater than 4 or 6 is less than 7

Solution:

a) Let p *:* Ram is in class X and q *:* Rahim is in Class XII.

Then, the disjunction in *(a)* is given by $p \vee q$.

Now $\sim p$ *:* Ram is not in Class X.

$\sim q$ *:* Rahim is not in Class XII.

Then, using *(D_8),* negation of $p \vee q$ is given by

$\sim (p \vee q)$ *:* Ram is not in Class X and Rahim is not in Class XII.

b) Write p : 7 is greater than 4, and q : 6 is less than 7.
Then, using (D_8), negation of $p \vee q$ is given by

$\sim (p \vee q)$: 7 is not greater than 4 and 6 is not less than 7.

(III) **Negation of a negation:** As already remarked the negation is not a connective but a modifier. It only modifies a given statement and applies only to a single simple statement. Therefore, in view of (D_5) and (D_6), for a statement p, we have

(D_9): Negation of negation of a statement is the statement itself Equivalently, we write

$\sim (\sim p) = p$

Example: Verify (D_9) for the statement

p : Roses are red.

Solution: The negation of p is given by

$\sim p$: Roses are not red.

Therefore, the negation of negation of p is

$\sim (\sim p)$: It is not the case that Roses are not red.

or

It is false that Roses are not red.

or

Roses are red.

Conditional Statement: Statement of the type "If p then q" are called *conditional* statements and are denoted by $p \rightarrow q$ read as 'p implies q'.

Another common statement is of the form "p if and only if q". Such statements are called ***bi-conditional* statements** and are denoted by $p \leftrightarrow q$.

Regarding the truth values of $p \rightarrow q$ and $p \leftrightarrow q$, we have

a) the conditional $p \rightarrow q$ is false only if p is true and q is false *and in all other cases it is true*
b) the bi-conditional $p \leftrightarrow q$ is true whenever p and q have the same truth values otherwise it is false.

One may verify that $p \rightarrow q = (\sim p) \vee q$

2.8 Truth Tables

A truth table consists of rows and columns. The initial columns are filled with the possible truth values of the sub-statements and the last column is filled with the truth values of the compound statement *S* (the truth values of *S* depends on the truth values of the sub-statements entered in the initial columns).

Note: If there are n statements, then the number of rows in the truth table is 2^n

Example: Construct the truth table for *~p.*

Solution: Note that one simple statement *~p* is consisting of only one simple statement p. Therefore, there must be 2^1 $(= 2)$ rows in the truth table. It is necessary to consider all possible truth values of *p.*

In view of (D_5) above, recall that p has the truth value *T* if and only if *~p* has the truth value *F.* Therefore, the truth table for *~p* is given by

Table 21 Truth table for ~ *p*

P	*~ p*
T	*F*
F	*T*

Example: Construct the truth table for $p \wedge (\sim p)$

Solution: Note that the compound statement $p \wedge (\sim p)$ is consisting of only one simple statement *p.* Therefore, there must be 2^1 $(= 2)$ rows in the truth table. It is necessary to consider all possible truth values of *p.*

Step 1: Enter all possible truth values of *p.* namely, *T* and *F* in the first column of the truth table (Table 2.2).

Table 2.2

P	*~ p*	$p \wedge (\sim p)$
T		
F		

Step 2: Using (D_5) and (D_6), enter the truth values of ~ *p* in the second column of the truth table (Table 2.3).

Table 2.3

p	$\sim p$	$p \wedge (\sim p)$
T	F	
F	T	

Step 3: Finally, using (D_2) enter the truth values of $p \wedge (\sim p)$ in the last column of the truth table (Table 2.4)

Table 2.4

p	$\sim p$	$p \wedge (\sim p)$
T	F	F
F	T	F

Example: Construct the truth table for $p \wedge q$.

Solution: The compound statement $p \wedge q$ is consisting of two simple statements p and q. Therefore, there must be $2^2 (= 4)$ rows in the truth table of $p \wedge q$. Now enter all possible truth values of statements p and q namely *TT, TF, FT* and *FF* in first two columns of Table 2.5.

Table 2.5

P	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

Then, in view of (D_1) and (D_2) above, enter the truth values of the compound statement $p \wedge q$ in the truth table (Table 18.6) to complete the truth table.

Table 2.6: Truth table for $p \wedge q$

P	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example: Construct the truth table for $p \vee q$.

Solution:

Table 2.7: Truth table for $p \vee q$.

P	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

In view of (D_3) and (D_4) above, recall that the compound statement $p \vee q$ has the truth value F if and only if both p and q have the truth value F; otherwise $p \vee q$ has truth value T. Thus, the truth table for $p \vee q$ is as given in Table 2.7.

Example: Construct the truth table of the following

a) $\sim [p \wedge (\sim q)]$

b) $(p \wedge q) \wedge (\sim p)$

c) $\sim[(\sim p) \vee (\sim q)]$

Solution:

a) Truth table for $\sim [p \wedge (\sim q)]$ is given by

Table 2.8: Truth table for $\sim [p \wedge (\sim q)]$

p	q	$\sim q$	$p \wedge (\sim q)$	$\sim[p \wedge (\sim q)]$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

b) Truth table for $(p \wedge q) \wedge (\sim p)$ is given by

Table 2.9: Truth table for $(p \wedge q) \wedge (\sim p)$

p	q	$p \wedge q$	$\sim p$	$(p \wedge q) \wedge (\sim p)$
T	T	T	F	F
T	F	F	F	F
F	T	F	T	F
F	F	F	T	F

c) Truth table for ~ *[(~p) v (~q)]* is given by

Table 2.10 : Truth table for ~ *[(~p) ∨ (~q)]*

p	*q*	*~p*	*~q*	*(~ p) ∨ (~ q)*	*~ [(~ p)] ∧ [(~q)]*
T	*T*	*F*	*F*	*F*	*T*
T	*F*	*F*	*T*	*T*	*F*
F	*T*	*T*	*F*	*T*	*F*
F	*F*	*T*	*T*	*T*	*F*

2.9 Tautologies

A statement is said to be a tautology if it is true for all logical possibilities. In other words, a statement is called tautology if its truth value is *T* and only *T* in the last column of its truth table. Analogously, a statement is said to be a *contradiction* if it is false for all logical possibilities. In other words, a statement is called contradiction if its truth value is *F* and only *F* in the last column of its truth table. A straight forward method to determine whether a given statement is tautology (or contradiction) is to construct its truth table.

Example: The statement *p ∨ (~p)* is a tautology since it contains *T* in the last column of its truth table (Table 2.11)

Table 2.11: Truth table for *p ∨ (~p)*

p	*~p*	*p ∨ (~p)*
T	*F*	*T*
F	*T*	*T*

Example: The statement *p ∧ (~p)* is a contradiction since it contains *F* in the last column of its truth table (Table 2.12)

Table 2.12: Truth table for *p ∧ (~ p)*

p	*~p*	*p ∧ (~p)*
T	*F*	*F*
F	*T*	*F*

Remark: The negation of a tautology is a contradiction since it is always false, and the negation of a contradiction is a tautology since it is always true.

SAQ 1: Show that

a) $\sim [p \vee (\sim p)]$ is a contradiction.

b) $\sim [p \wedge (\sim p)]$ is a tautology.

Example: Show that

a) $(p \vee q) \vee (\sim p)$ is a tautology.

b) $(p \wedge q) \wedge (\sim p)$ is a contradiction.

Solution:

a) The truth table for $(p \vee q) \vee (\sim p)$ is given by

Table 2.15: Truth table for $(p \vee q) \vee (\sim p)$

P	*q*	$p \vee q$	$\sim p$	$(p \vee q) \vee (\sim p)$
T	*T*	*T*	*F*	*T*
T	*F*	*T*	*F*	*T*
F	*T*	*T*	*T*	*T*
F	*F*	*F*	*T*	*T*

Since the truth table for $(p \vee q) \vee (\sim p)$ contains only *T* in the last column, it follows that $(p \vee q) \vee (\sim p)$ is a tautology.

b) Recall Table 2.9 which is the truth table for $(p \wedge q) \wedge (\sim p)$ and observe that it contains only F in the last column. Therefore, $(p \wedge q) \wedge (\sim p)$ is a contradiction.

2.10 Logical Equivalence

Two statements S_1 $(p, q, r, ...)$ and S_2 $(p, q, r, ...)$ are said to be logically equivalent, or simply equivalent if they have the same truth values for all logical possibilities and is denoted by

$S_1 (p, q, r,...) \equiv S_2 (p, q, r,...)$.

In other words, S_1 and S_2 are logically equivalent if they have identical truth tables (by identical truth tables we mean the entries in the last column of the truth tables are same).

Example: Show that $\sim (p \wedge q)$ is logically equivalent to $(\sim p) \vee (\sim q)$.

Solution: The truth tables for both the statements are

Table 2.16: Truth table for $\sim (p \wedge q)$

p	q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Table 2.17: Truth table for $(\sim p) \vee (\sim q)$

p	q	$\sim p$	$\sim q$	$(\sim p) \vee (\sim q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Now, observe that the entries (truth values) in the last column of both the tables are same. Hence, the statement $\sim(p \wedge q)$ is equivalent to the statement $(\sim p) \vee (\sim q)$.

Example: Let

p : The South-West monsoon is very good this year and

q : Rivers are rising.

Give verbal translation of $\sim [(\sim p) \vee (\sim q)]$.

Solution: we have

$$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$$

Therefore, the statement $\sim [(\sim p) \vee (\sim q)]$ is the same as the negation of the statement $\sim(p \wedge q)$ which is the same as the conjunction $p \wedge q$. Thus, the verbal translation for $\sim [(\sim p) \vee (\sim q)]$ is

"The South-West monsoon is very good this year and rivers are rising"

Example: Prove the following:

a) $\sim [p \vee (\sim q)] \equiv (\sim p) \wedge q$

b) $\sim [(\sim p) \wedge q] \equiv p \vee (\sim q)$

c) $\sim (\sim p) \equiv p$

Solution:

a) The truth tables for $\sim[p \vee (\sim q)]$ and $(\sim p) \wedge q$ are given by

Table 2.18: Truth table for $\sim[p \vee (\sim q)]$

p	q	$\sim q$	$p \vee (\sim q)$	$\sim[p \vee (\sim q)]$
T	T	F	T	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

Table 2.19: Truth table for $(\sim p) \wedge q$

p	q	$\sim p$	$(\sim p) \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

The last column of the two tables are the same.

b) It follows in view of the truth Table 2.20

Table 2.20: Truth table for $p \vee (\sim q)$ and $\sim[(\sim p) \wedge q]$

p	q	$\sim p$	$\sim q$	$(\sim p) \wedge q$	$p \vee (\sim q)$	$\sim[(\sim p) \wedge q]$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	F	T	T

c) The assertion follows in view of Table 2.21

Table 2.21: Truth table for ~(~p)

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

2.11 Applications

The logic that we have discussed so far is called two-value logic because we have considered only those statements which are having truth values

True or False. A similar situation exists in various electrical and mechanical devices. Claude Shannon, in late 1930's, was first to notice an analogy between the operations of switching devices and the operations of logical connectives. He used this analogy with great success to solve problems of circuit design.

Observe that an electric switch which is used for turning 'on' and 'off' an electric light is a two-state device. We shall now explain various electric networks with the help of logical connectives. For this, first we discuss how an electric switch works. Observe that, in Fig. 2.1, we have shown two positions of a simple switch.

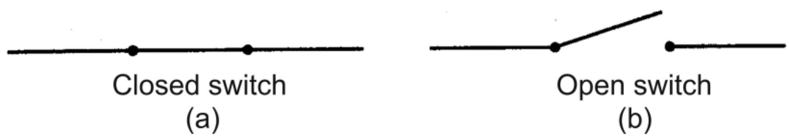


Fig. 2.1

In *(a)* when switch is closed (i.e. on), current can flow from one terminal to the other. In *(b),* when the switch is open (i.e. off), current can not flow.

Let us now consider the example of an electric lamp controlled by switch. Such a circuit is given in Fig. 2.2.

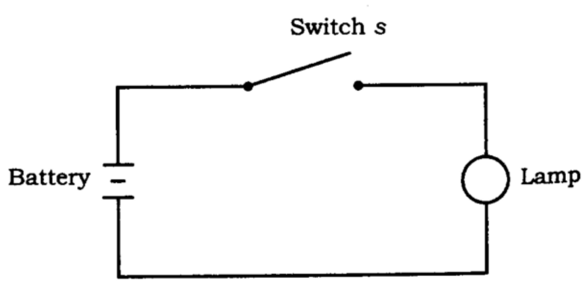


Fig. 2.2

Observe that when the switch s is open, no current flows in the circuit and therefore, the lamp is 'off'. But when switch s is closed, the lamp is 'on'. Thus the lamp is on if and only if the switch s is closed.

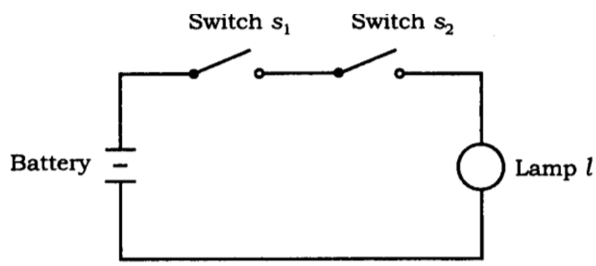
If we denote the statements as

p : The switch s is closed

l : The lamp l is 'on'

then, by using logic, the above circuit can be expressed as $p \equiv l$.

Next, consider an extension of the above circuit in which we have taken two switches $s_1$and s_2 in series as shown in Fig. 2.3.



Switches s_1 and s_2 are in series

Fig. 2.3

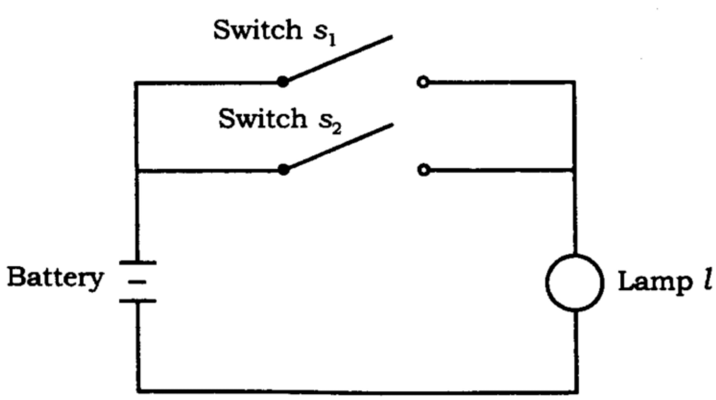
Here, observe that the lamp is 'on' if and only if both the switches s_1 and $s_2$are closed.

If we denote the statements as:

p : the switch s_1 is closed.
q : the switch s_2 is closed.
l : the lamp l is 'on'.

then the above circuit can be expressed as $p \wedge q \equiv l$.

Now, we consider a circuit in which two switches s_1 and s_2 are connected in parallel (Fig. 2.4).



Switches s_1 and s_2 are in parallel

Fig. 2.4

If we denote the statements as:

p : the switch s_1 is closed.

q : the switch s_2 is closed.

l : the lamp *l* is 'on'.

then the above circuit can be expressed as $p \vee q \equiv l$.

SAQ 2: Express the following circuit in Fig. 2.5 in symbolic form of logic.

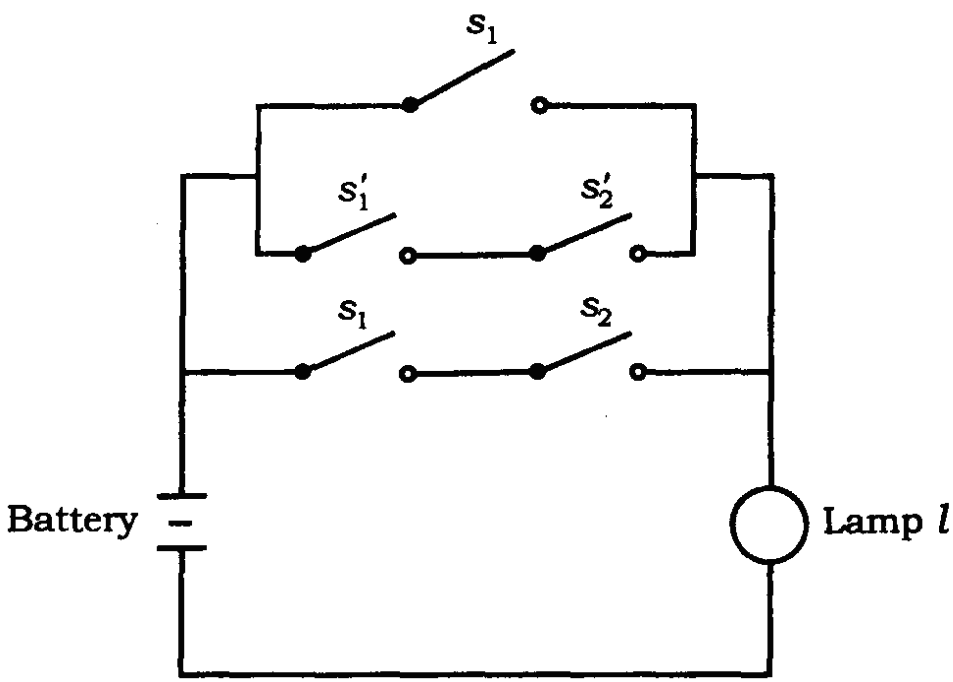


Fig. 2.5

2.12 Summary

In this unit we study the truth values of statements. The different basic logical connectives are discussed in detail with some standard examples. Compound statements and the negation are clearly explained. The concept of Tautology, Contradiction and Logical Equivalence is discussed in detail with example wherever necessary. The applications of mathematical logic to switching circuits is dealt with standard examples.

2.13 Terminal Questions

1. Define Tautology and Contradiction
2. Draw the truth tables of Conjunction, disjunction and Biconditional statements.

2.14 Answers

Self Assessment Questions

1. a) The truth table of $\sim [p \vee (\sim p)]$ is given by

Table 2.13: Truth table for $\sim [p \vee (\sim p)]$

P	$\sim p$	$p \vee (\sim p)$	$\sim [p \vee (\sim p)]$
T	F	T	F
F	T	T	F

Since it contains only F in the last column of its truth table, it follows that
$\sim [p \vee (\sim p)]$ is a contradiction.

b) The truth table of $\sim [p \wedge (\sim p)]$ is given by

Table 2.14: Truth table for $\sim [p \wedge (\sim p)]$

P	$\sim p$	$p \wedge (\sim p)$	$\sim [p \wedge (\sim p)]$
T	F	F	T
F	T	F	T

Since it contains only T in the last column of its truth table, it follows that $\sim [p \wedge (\sim p)]$ is a tautology.

2. Observe that the lamp is 'on' if and only if either s_1 and s_2 both are closed or s_1 and s_2 both are open or only s_1 is closed.
If we denote the statements as
p : The switch s_1 is closed
q : The switch s_2 is closed
l : The lamp l is 'on'
then

~p: The switch s_1 is open.
or
The switch s_1 is not closed.
~ q: The switch s_2 is open.
or
The switch s_2 is not closed.

Therefore, the circuit in Fig. 2.5 in symbolic form of logic may be expressed as
$p \vee [(\sim p) \wedge (\sim q)] \vee (p \wedge q) \equiv l.$