Unit-12

Numerical Solution of ordinary differential equation – I

* Picard's method of successive approximation:-

$$y_{n+1} = y_0 + \int_{x_0}^{x} f(x, y_n)\cdot dx$$

* Solve by Picard method up to third approximation $\frac{dy}{dx} = x + y^2$, ~~of initial value problem~~ of initial value problem (IVP) $y(0)=0$ and also find $y(0.1)$.

Sol:- Here $y(0)=0$, so $x=0, y=0$.

now, as per picard method.

$$y_{n+1} = y_0 + \int_{x_0}^{x} (x, y_n)\cdot dx \rightarrow$$

$$y_{n+1} = y_0 + \int_{x_0}^{x} (x + y_n^2)\cdot dx$$

now, put $n=0$,

$$y_1 = y_0 + \int_{x_0}^{x} (x + y_0^2)\cdot dx$$

$$= (0) + \int^{x} (x + (0)^2)\cdot dx$$

$$= \int_0^{x} x \cdot dx$$

$$\boxed{y_1 = \frac{x^2}{2}}$$

now, put $n=1$,

$$y_2 = y_0 + \int_{x_0}^{x} (x + y_1^2)\cdot dx$$

$$= 0 + \int^{x} \left(x + \left(\frac{x^2}{2}\right)^2\right)\cdot dx$$

$$= \int_0^{x} x + \int_0^{x} \frac{x^4}{4}$$

$$\boxed{y_2 = \frac{x^2}{2} + \frac{x^5}{20}}$$

now put $n=2$,

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$$y_3 = y_0 + \int_{x_0}^{x} (x + y_2^2) \cdot dx.$$

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$$= 0 + \int_0^x \left[x + \left(\frac{x^2}{2} + \frac{x^5}{20}\right)^2\right] \cdot dx.$$

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$$= \int_0^x \left[x + \frac{x^4}{4} + \frac{x^7}{20} + \frac{x^{10}}{400}\right] \cdot dx.$$

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$$\boxed{y_3 = \frac{x^2}{2} + \frac{x^5}{20} + \frac{x^8}{160} + \frac{x^{11}}{4400}}$$

1 pm now to find $y(0.1)$, from above equation.

2

$$y(0.1) = \frac{(0.1)^2}{2} + \frac{(0.1)^5}{20} + \frac{(0.1)^8}{160} + \frac{(0.1)^{11}}{4400}$$

3

$$y(0.1) = 0.005 + 0.0000 + 0.000 + 0.0000$$

4

$$\boxed{y(0.1) = 0.0050}$$

5

6

7

M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

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* Euler's method or

Runge Kutta method for first order.

$$y_{n+1} = y_n + h f(x_n, y_n)$$

* Solve $\frac{dy}{dx} = x+y$ with boundary condition

$y=1$ at $x=0$. find approximate value of

y for $x=0.1$.

Sol:- Here initial value of $x_0 = 0$ and $x_n = 0.1$

Now we will divide into 5 interval,

so, $h = \frac{x_n - x_0}{n} = \frac{(0.1)-(0)}{5} \boxed{= 0.02}$

now $x_0 = 0$ then $x_1 = x_0 + h$, $x_2 = x_1 + h, \ldots$

$x_0 = 0$, $y_0 = 1$

$x_1 = 0.02$

$x_2 = 0.04$

$x_3 = 0.06$

$x_4 = 0.08$

$x_5 = 0.1$

We have $dy/dx = x+y$ equation so,

Now Euler's formula,

$y_{n+1} = y_n + h(f(x_n, y_n) \Rightarrow y_{n+1} = y_n + h(x_n + y_n)$

now put $n=0$,

$$y_1 = y_0 + h(x_0 + y_0)$$

$$y_1 = 1 + 0.02(0+1) = 1.02.$$

now put $n=1$,

$$y_2 = y_1 + h(x_1 + y_1) = 1.02 + 0.02(0.02 + 1.02)$$

$$y_2 = 1.0408.$$

now put $n=2$,

$$y_3 = y_2 + h(x_2 + y_2) = 1.0408 + 0.02(0.04 + 1.0408)$$

$$y_3 = 1.0624$$

now put $n=3$,

$$y_4 = y_3 + h(x_3 + y_3) = 1.0624 + 0.02(0.06 + 1.0624)$$

$$y_4 = 1.0848$$

now put $n=4$,

$$y_5 = y_4 + h(x_4 + y_4) = 1.0848 + 0.02(0.08 + 1.0848)$$

$$y_5 = 1.1081$$

so, $y_0 = 1$, $y_1 = 1.02$, $y_2 = 1.0408$, $y_3 = 1.0624$,

$y_4 = 1.0848$ and $y_5 = 1.1081$.

* Euler's modified method ~~oo~~ OE

$$\boxed{y^{(*)}_{n+1} = y_n + hf(x_n, y_n)}$$

$$\boxed{y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^{(*)}_{n+1})\right]}$$

* Given $dy/dx = x^2 + y$ with $y(0) = 1$, find $y(0.02)$ and $y(0.04)$ by Euler's modified method.

Sol:- Here $y(0) = 1$, so $x_0 = 0$, $y_0 = 1$.
we have to find $y(0.02)$ and $y(0.04)$.
so let $x_1 = 0.02$ and $x_2 = 0.04$.

$x_0 = 0$
$x_1 = 0.02$
$x_2 = 0.04$

Here difference between x_1 and x_0 is 0.02
so, $h = 0.02$
we have $dy/dx = x^2 + y$
Now as Euler's modified method,

(i) $y^{(*)}_{n+1} = y_n + h(x_n^2 + y_n)$

(ii) $y_{n+1} = y_n + \frac{h}{2}\left[(x_n^2 + y_n) + (x_{n+1}^2 + y^{(*)}_{n+1})\right]$

now put n=0,

$$y_1^* = y_0 + h(x_0^2 + y_0) = 1 + 0.02(0^2+1) = 1.02$$

$$y_1 = y_0 + \frac{h}{2}\left[(x_0^2 + y_0) + (x_1^2 + y_1^{(*)})\right]$$

$$= 1 + \frac{0.02}{2}\left[((0)^2 + 1) + ((0.02)^2 + 1.02)\right]$$

$$y_1 = 1.0202$$

now put n=1,

$$y_2^* = y_1 + h(x_1^2 + y_1) = 1.0202 + 0.02((0.02)^2 + 1.0202$$

$$y_2^* = 1.0406.$$

$$y_2 = y_1 + \frac{h}{2}\left[(x_1^2 + y_1) + (x_2^2 + y_2^*)\right]$$

$$= 1.0202 + \frac{0.02}{2}\left[(0.02)^2 + 1.0202 + (0.04)^2 + 1.0406\right]$$

$$y_2 = 1.0408$$

so, $x_0 = 0$, $y_0 = 1$

$x_1 = 0.02$, $y_1 = 1.0202$

$x_2 = 0.04$, $y_2 = 1.0408$

so, $y(0.02) = 1.0202$

$y(0.04) = 1.0408.$