Unit 6 Directed Graphs

Structure:

6.1 Introduction

The graphs so far studied are undirected graphs (no direction was assigned to the edge in a graph). In this unit, we shall consider directed graphs (graphs in which edges have directions). Many physical situations require directed graphs. The applications of directed graphs are in the street map of city with one-way streets, flow networks with valves in the pipes, and electrical networks. Directed graphs in the form of signal flow graphs are used for system analysis in control theory. Most of the concepts and terminology of undirected graphs are also applicable to directed graphs. In this unit we will discuss some of the properties of directed graphs which are not shared by undirected graphs.

Objectives:

After studying this unit, you should be able to:

- write the indegree and outdegree of a vertex
- explain the tournaments and Euler's digraphs
- give the different matrix representations of digraphs
- apply the properties to flows, network and traffic problem.

6.2 Types of Directed Graphs

The different types of directed graphs are discussed below:

Definition: A *directed graph* (or) a *digraph* D consists of a non-empty set V (the elements of V are normally denoted by v_1, v_2, …) and a set E (the elements of E are normally denoted by $e_1, e_2, \ldots.$) and a mapping Ψ that maps every element of E onto an ordered pair (v_i, v_j) of elements from V. The elements of V are called as *vertices or nodes or points*. The elements of E are called as *edges or arcs or lines*. If $e \in E$ and $v_i, v_j \in V$ such that $\Psi(e) = (v_i, v_j)$, then we write $e = \overrightarrow{v_i v_j}$. In this case, we say that e is an *edge between* v_i and v_j. (We also say that e is an edge from v_i to v_j). (We also say that e originates at v_i and terminates at v_j). (An edge from v_i to v_j is denoted by a line segment with an arrow directed from v_i to v_j).

Note:

i) A directed graph is also referred to as an *oriented graph*.

ii) Let D be a directed graph and $e = \overrightarrow{vu}$.

Then, we say that e is *incident out* of the vertex v, and *incident into* the vertex u. In this case, we say that the vertex v is called the *initial vertex* and the vertex u is called the *terminal vertex* of e.

Example: The graph given in Figure-6.1 is a digraph with 5 vertices and ten edges.

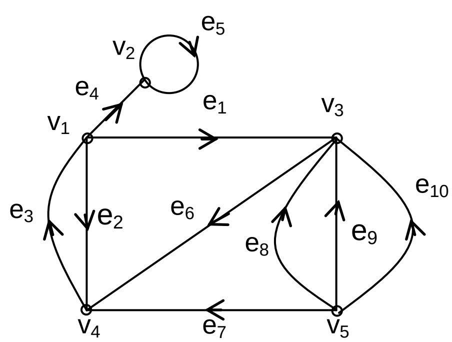


Figure 6.1: Directed Graph

Here, v_5 is the initial vertex and v_4 is the terminal vertex for the edge e_7. The edge e_5 is a self-loop.

Definition:

i) The number of edges incident out of a vertex v is called the *out degree* (or) *out-valence* of v. The out degree of a vertex v is denoted by $d^+(v)$.

ii) The number of edges incident into v is called the *indegree* (or) *in-valance* of v. The indegree of a vertex v is denoted by $d^-(v)$. Note that the degree of v is equal to the sum of indegree and out degree of v, for any vertex v in a graph. (In symbols, we can write as $d(v) = d^+(v) + d^-(v)$ for all vertices v).

Example: Consider the graph given in Fig. 6.1

i) Here, $d^+(v_1) = 3$, $d^+(v_2) = 1$, $d^+(v_3) = 1$, $d^+(v_4) = 1$, $d^+(v_5) = 4$. $d^-(v_1) = 1$,
$d^-(v_2) = 2$, $d^-(v_3) = 4$, $d^-(v_4) = 3$, $d^-(v_5) = 0$.

ii) $d(v_1) = 4 = 3 + 1 = d^+(v_1) + d^-(v_1)$; $d(v_2) = 3 = 1 + 2 = d^+(v_2) + d^-(v_2)$, and so on.

Problem: Let D be a directed graph. Show that the sum of all in-degrees of the vertices of D, is equal to the sum of all out-degrees of the vertices of D; and each sum is equal to the number of edges in D.

[In other words, $\sum_{i=1}^{n} d^+(v_i) = \sum_{i=1}^{n} d^-(v_i)$ = q where q is the number of edges in D, and $\{v_1, v_2, \ldots, v_n\}$ is the set of all vertices in D].

Solution: Let D be any digraph with n vertices and q edges. When the indegrees of vertices are counted, each edge is counted exactly once.

[This is because every edge goes to exactly one vertex].

So $\sum_{i=1}^{n} d^-(v_i)$ = q ... (i)

Similarly, when the out-degrees of vertices are counted, each edge is counted exactly once. [This is, because each edge goes out of exactly one vertex].

So $\sum_{i=1}^{n} d^+(v_i)$ = q …. (ii).

From (i) and (ii), we get that $\sum_{i=1}^{n} d^+(v_i) = \sum_{i=1}^{n} d^-(v_i)$ = q.

Example: Consider the digraph in Fig. 5.1 Here, the number of vertices $n = 5$, and the number of edges q = 10.

Now, $\sum_{i=1}^{n} d^{+}(v_i)$ = $d^+(v_1) + d^+(v_2) + d^+(v_3) + d^+(v_4) + d^+(v_5)$ = 3 + 1 + 1 + 1 + 4 = 10 = q

Also, $\sum_{i=1}^{n} d^{-}(v_i)$ = $d^-(v_1) + d^-(v_2) + d^-(v_3) + d^-(v_4) + d^-(v_5)$ = 1 + 2 + 4 + 3 + 0 = 10 = q.

Therefore, $\sum_{i=1}^{n} d^{+}(v_i) = \sum_{i=1}^{n} d^{-}(v_i)$ = q = the number of edges.

Definitions:

i) A vertex v is said to be an *isolated vertex* if the out degree of v and the indegree of v are equal to zero. (In symbols, $d^+(v) = 0 = d^-(v)$).

ii) A vertex v in a digraph D is said to be a *pendent vertex* if it is of degree 1. (In other words, a vertex is said to be a *pendent vertex* if the degree of $v = d^+(v) + d^-(v) = 1$).

iii) Two directed edges are said to be *parallel edges* if they are mapped onto the same ordered pair of vertices.

iv) In other words, two directed edges e and f are said to be *parallel edges* if both e and f originates from the same vertex, and also terminates at the same vertex. (In the graph given in Fig. 6.1, the edges e_8, e_9, and e10 are the parallel edges. The edges e_2 and e_3 are not parallel).

Note:

i) Let D be directed graph. If we disregard the orientation (that is, direction) of every edge in D, then we get an undirected graph. The undirected graph obtained (in this way) from D is called the *undirected graph corresponding* to D.

ii) Suppose, H is an undirected graph. To each edge of H we can assign a direction. Then the digraph obtained is called an *orientation* of H. (or a *directed graph associated* with H).

Definition: Let D_1 and D_2 be two digraphs. These two graphs are said to be *isomorphic* if

i) the corresponding undirected graphs are isomorphic, and

ii) the directions of corresponding edges must be same.

Example:

i) Consider the directed graphs D_1 and D_2 depicted in the figure 6.2. The two digraphs D_1 and D_2 are not isomorphic (because the direction of the edge e_4 in D_1 is different from the direction of the edge e4 in D2).

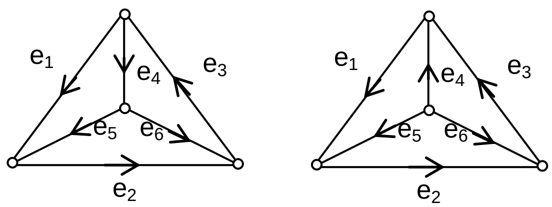


Figure 6.2: Graph D_1 Graph D_2

ii) Note that the undirected graphs corresponding to D_1 and D_2 are isomorphic.

Definition: a digraph that has does not self-loop nor parallel edges are called a *simple digraph*.

In other words, a directed graph *D* is said to be a *simple digraph* if the corresponding undirected graph is a simple graph. The two digraphs given in Fig. 6.2 are simple directed graphs.

Definitions:

i) Digraphs that have at most one directed edge between any pair of vertices, but are allowed to have self-loops are called the *asymmetric (or) anti-symmetric* digraphs.
ii) A digraph *D* is said to be a *symmetric digraph* if for every edge (*a*, *b*) in *D* there is also an edge (*b*, *a*) in *D*.
iii) A digraph that is both simple and symmetric is called a *simple symmetric digraph*.
iv) A digraph that is both simple and asymmetric is called *simple asymmetric digraph*.
v) A simple digraph is said to be a *complete symmetric digraph* if it satisfies the following condition: "Given any two vertices v_1 and v_2, there corresponds exactly one edge directed from v_1 to v_2".
vi) A *complete asymmetric digraph (or) tournament (or) a complete tournament*. is an asymmetric digraph in which there is exactly one edge between every pair of vertices.

Example: In the following figure 6.3,

i) The graph G_1 is not asymmetric since there exist two directed edges between v_1 and v_2.
ii) The graph G_2 is asymmetric.
iii) Any null graph is asymmetric.

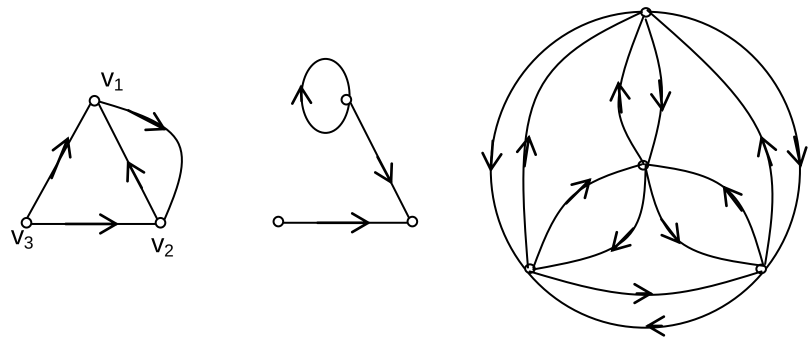


Figure 6.3 G_1 G_2 G_3

iv) The digraph G_3 is a complete symmetric digraph.

Observation:

i) A tournament (means, complete asymmetric digraph) of n vertices contains $\frac{n(n-1)}{2}$ edges.

The directed graph given in fig. 6.2 is a tournament. In this graph, the number of vertices is $n = 4$.

The number edges is 6 = $\frac{4(4-1)}{2} = \frac{n(n-1)}{2}$.

ii) A complete symmetric graph of n vertices contains $n(n - 1)$ edges. The digraph given in fig. 6.3, G_3 is a complete symmetric graph on $n = 4$ vertices.

In this graph, the number of edges is 12 = 4(4 -1) = $n(n - 1)$.

Definition: A digraph D is said to be *balanced* if the in-degree of v is equal to the out-degree of v for every vertex v in D. In other words, a digraph D is said to be *balanced* if $d^+(v) = d^-(v)$ for all vertices v in the digraph D. A *balanced digraph* is also known as *pseudo symmetric digraph (or) an isograph.*

A balanced digraph is said to be a *regular digraph* if it satisfies the following two conditions:

i) The in-degrees of all the vertices of D are equal; and

ii) The out-degrees of all the vertices of D are equal.

Self Assessment Questions

1. A directed graph is also refered to as an ———————— .
2. The number of edges incident out of a vertex v is called the————————.
3. A vertex v is said to be an isolated vertex if the out degree of v and the indegree of v are equal to————————.

6.3 Binary Relation as a Digraph

Let X be a set. Represent the elements of X by the symbols x_1, x_2, … Suppose that R is a relation on X. We represent the elements of X as vertices and draw a directed edge from x_i to x_j if $(x_i, x_j) \in R$. Then, we get a directed graph which represents the given relation R on X represented in figure 6.4.

Example: Consider the set $X = \{3, 4, 5, 7, 8\}$ and the relation $(R, >)$ on X.

Then, $R = \{(8, 3), (8, 4), (8, 5), (8, 7), (7, 3), (7, 4), (7, 5), (5, 4), (4, 3)\}$

$= \{(x, y) / x > y, x, y \in X\}$.

The graph D given below represents this relation R on X.

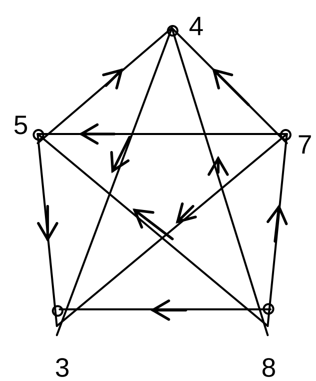


Figure 6.4: Graph represents relation R on X

It is clear that, every binary relation on a finite set can be represented by a directed graph without parallel edges. Conversely, if a directed graph D without parallel edges was given, then there corresponds a binary relation on the set of vertices.

Definition:

i) Let X be a set, and R a relation on X (that is, $R \subseteq X \times X$). If $(x, y) \in R$, then we also write xRy. The relation R is said to be *reflexive relation* if xRx for all $x \in X$.

(Note that the digraphs of a reflexive relation have a self-loop at every vertex).

We call a directed graph representing a reflexive binary relation on its vertex set as *reflexive digraph*".

A digraph in which no vertex has a self-loop is called an *irreflexive digraph*.

ii) Let X be a set, and R a relation on X. We say that R is *symmetric* if a, b $\in X$, aRb $\Rightarrow$ bRa. Note that a directed graph representing a symmetric relation is a symmetric digraph.

Example: The figure 6.5 given graph G_1 is the digraph of a relation which is symmetric but not reflexive. This relation is on the set $\{x_1, x_2, x_3, x_4\}$. Some authors represent this relation by the undirected graph given in G_2. Note that in the graph G_2, only one undirected edge is taken between a pair of vertices (that are related under the relation R).

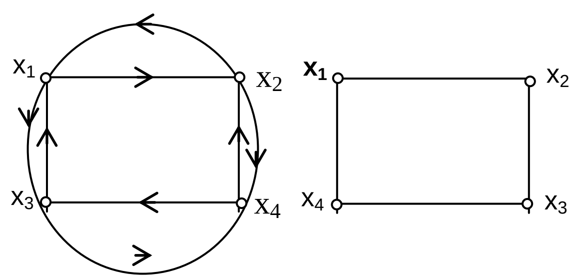


Figure 6.5: Graph G_1 Graph G_2

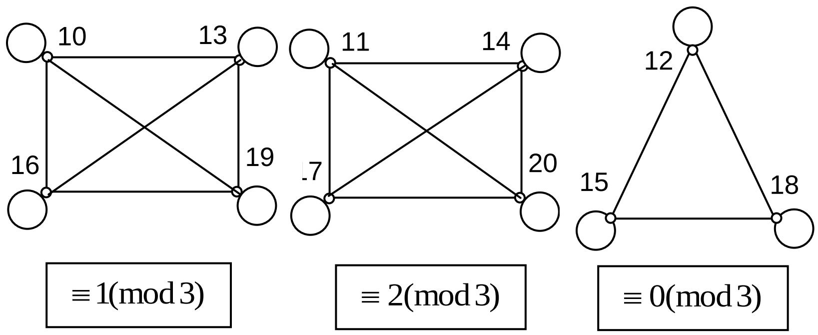
Definition: Let X be a set, and R a relation on X. The relation R is said to be *transitive* if a, b, $c \in R$, aRb, bRc $\Rightarrow$ aRc. A digraph representing a transitive relation is called a *transitive digraph.* [Observe that the graph given in 6.4 is a transitive digraph].

Let X be a set, and R a relation on X.

i) The relation R is said to be an *equivalence relation* if it is reflexive, symmetric and transitive.

ii) A digraph representing an equivalence relation is called an *equivalence digraph*.

Example: Consider the binary relation *R* (= “Congruent modulo 3”) defined on the set *X* = {10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}.



We can observe that this relation *R* on *X* is an equivalence relation. The related equivalence graph was given. We are using undirected graph here. Observe that the set of vertices is divided into three disjoint equivalence classes. Each of these sets form separate components. Each component is an undirected subgraph (due to symmetry, we are using undirected graph with a self-loop at each vertex). Also, note that (since any two elements in an equivalence class are related) any two vertices inside the component were joined by an edge.

Definition: Let *X* be a set, and *R* is a binary relation on *X*. This relation *R* on *X* may be represented by a matrix. This matrix is called as *relation matrix*. It is a (0,1)-$n \times n$-matrix, where n is the number of elements in the set $X = \{x_1, x_2, \ldots, x_n\}$.

The relation matrix (a_{ij}) is defined as follows:

$a_{ij} = 1$ if $x_i R x_j$,
$= 0$ otherwise.

Example: The relation matrix of the relation *R* (" usual grater than") on the set
X = {3, 4, 5, 7, 8}, is the Matrix

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} 3 & 4 & 7 & 5 & 8 \end{array} \\
\begin{array}{c} 3 \\ 4 \\ 7 \\ 5 \\ 8 \end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

Definitions:

i) A path $v_0e_1v_1e_2v_2 \ldots e_nv_n$ is said to be a *directed path* if e_k is oriented from v_{k-1} to v_k for each $1 \leq k \leq n$. A path which is not a directed path is called a *semi-path*. In a directed graph, the word "path" refers to either a directed path or a semi-path.

ii) Let D be a directed graph. A *directed walk* (in D) from a vertex v to u is an alternating sequence of vertices and edges beginning with v and ending with u such that each one of the edges is oriented (or directed) from the vertex preceding it to the vertex following it. [In other words, a walk $v_0e_1v_1e_2v_2 \ldots e_nv_n$ in the undirected graph D is said to be a *directed walk* if e_k is oriented from v_{k-1} to v_k for all $1 \leq k \leq n$].

Example: Consider the graph given in the Fig-6.1

i) The path $v_5\ e_8\ v_3\ e_6\ v_4\ e_3\ v_1$ is a directed path from v_5 to v_1.

ii) If we disregard the orientation in D, then $v_5\ e_7\ v_4\ e_6\ v_3\ e_1v_1$ is a path. Observe that it is not a directed path. So it is a "*semi-path*".

Since a directed walk is a walk, no edge appears more than once (but a vertex may appear more than once).

Definitions: Let D be a digraph.

i) A walk which is not a directed walk is called as *semi-walk*. We use the term "*walk*" to mean either a directed walk (or) a semi-walk.

ii) A circuit (in the corresponding undirected graph) $v_0e_1v_1e_2v_2 \ldots e_nv_n$ is said to be a *directed circuit* if e_k is oriented from v_{k-1} to v_k for all $1 \leq k \leq n$.

iii) A circuit which is not a directed circuit is a *semi-circuit*.

Example: Consider the digraph G given 6.1

i) The circuit $v_1e_1v_3e_6v_4e_3v_1$ is a directed circuit.

ii) The circuit $v_1e_1v_3e_6v_4e_2v_1$ is not a directed circuit. So it is a semi-circuit.

In undirected graphs, we define the connectedness by using the notion "path". As there are two different types of paths (namely directed path, semi-path) in digraphs, we have two different types of connectedness in the digraphs.

Definitions: Let *D* be a digraph.

i) *D* is said to be *strongly connected* if there exists at least one directed path from every vertex to every other vertex.
ii) *D* is said to be *weakly connected* if its corresponding undirected graph is connected, but *D* is not strongly connected.
iii) We say that *D* is connected if the undirected graph corresponding to *D* is connected. So the word "connected digraph" refers to both strongly connected digraph and weakly connected digraph.
iv) If *D* is not connected, then we say that *D* is a *disconnected graph*.

Example: Consider the digraphs D_1 and D_2 given in the figure 6.6.

i) The graph-D_1 is a strongly connected graph.
ii) In the graph-D_2, there is no directed path from v_1 to v_4 so it is a weakly connected digraph.

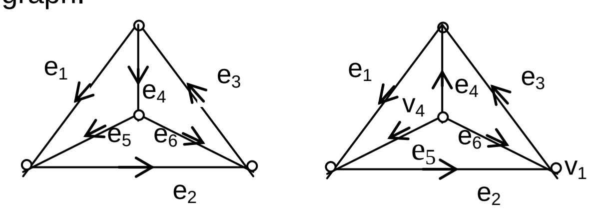


Figure 6.6: Graph-D_1 Graph-D_2

Since there are two types of connectedness in a digraph, we define two types of components.

Definitions: Let *D* be a directed graph.

i) Each maximal connected (weakly or strongly) subgraph of a digraph *D*, is said to be a *component* of *D*.
ii) Within each component of *D*, the maximal strongly connected subgraphs are said to be the *fragments* (or) *strongly connected fragments* of *D*.

Example: Consider the digraph given in the Fig-6.7.
This graph consists of two components H_1 and H_2.

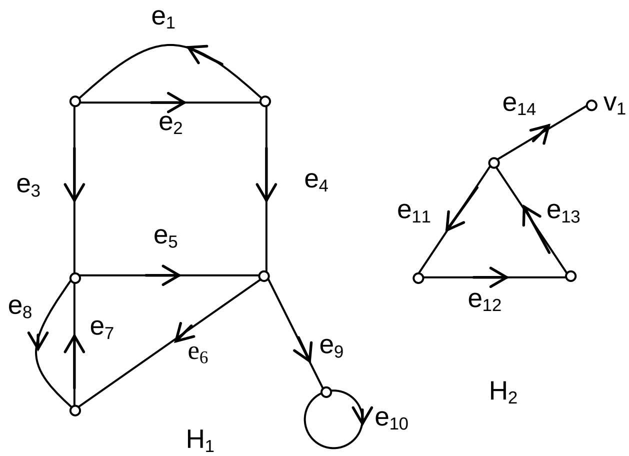


Figure 6.7: Graph with two components H_1 and H_2

i) The component H_1 contains three fragments: $\{e_1, e_2\}$; $\{e_5, e_6, e_7, e_8\}$; and $\{e_{10}\}$.
ii) The component H_2 contains a fragment $\{e_{11}, e_{12}, e_{13}\}$.
iii) It can be observed that e_3, e_4 and e_9 do not appear in any fragment of H_1.

Definitions: Let D be a digraph. The *condensation* D_c of D is a digraph obtained by using the following method:

i) Each strongly connected fragment is to be replaced by a new vertex, and
ii) For any two strongly connected components c_1 and c_2, the set of all the directed edges from c_1 to c_2 is to be replaced by a single directed edge (from c_1 to c_2).
iii) Let D be a directed graph and v, u are vertices in D. Then v said to be *accessible (or) reachable* from the vertex u, if there is a directed path from u to v.

Example: Consider the graph D given in the Fig-6.8

i) The components of D are H_1, H_2.
ii) The fragments of D are $C_1 = \{e_1, e_2\}$;
$C_2 = \{e_5, e_6, e_7, e_8\}$; $C_3 = \{e_{10}\}$; and $C_4 = \{e_{11}, e_{12}, e_{13}\}$.

iii) The fragments C_1, C_2, C_3 and C_4 are to be considered as vertices.

iv) There are two edges e_3 and e_4 from C_1 to C_2.
These two edges e_3 and e_4 are to be replaced by an edge (say f_1) from the vertices C_1 to C_2.

v) There is only one edge e_9 from the fragment C_2 to C_3. So this edge e_9 is to be drawn from the vertex C_2 to C_3.

vi) There is only one edge (e_{14}) from the fragment C_4 to the vertex v_1. So, this edge is to be drawn (in the condensation) from the vertex C_4 to v_1.

vii) Finally, we get the condensation D_c that is given in the Fig-6.8

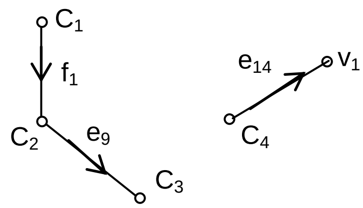


Figure 6.8

Observations:

i) The condensation D_c of a strongly connected digraph D is a vertex.

ii) The condensation D_c of a digraph D contains no directed circuits.

It is clear that a digraph D is strongly connected $\Leftrightarrow$ "v is accessible from u" for all vertices v and u in D.

Self Assessment Questions

4. Let X be a set, and R a relation on X. The relation R is said to be ——— if a, b, $c \in R$, aRb, bRc $\Rightarrow$ aRc.
5. Each maximal connected subgraph of a digraph D, is said to be a ——— of D

6.4 Euler's Digraphs

Definitions: Let D be a directed graph.

i) A directed walk that starts and ends at the same vertex is called a closed directed walk.

ii) A closed directed walk which traverses every edge of D exactly once, is called a *directed Euler line*.

iii) *D* is said to be a *Euler digraph* if it contains a directed Euler line.

Example: Consider the graph given in figure 6.9.

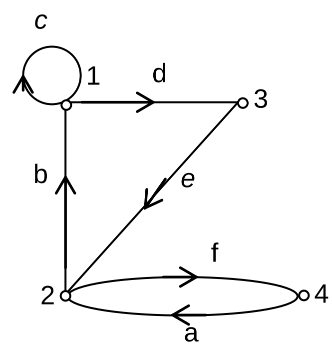


Figure 6.9

The edge sequence *f, a, b, c, d, e* forms a directed Euler line. Hence, this graph is a Euler digraph.

Theorem: Let *D* be a digraph. Then *D* is an Euler digraph if and only if *D* is connected and balanced.

[that is, $d^+(v) = d^-(v)$ for every vertex v in D].

Definition: A connected digraph containing no circuit (neither a directed circuit nor a semi-circuit) is said to be a *tree*.

Note:

i) A tree of *n* vertices contain *n* - 1 edges (directed).

ii) Trees in digraphs have additional properties (than those in undirected graphs) and variations resulting from the relative orientations of the edges.

Definitions: A digraph *D* is said to be an *arborescence* if it satisfies the following two conditions:

i) *D* contains no circuit (neither a directed circuit nor a semi-circuit).

ii) There exists exactly one vertex *v* of zero in-degree (this vertex *v* is called the *root* of the arborescence).

Example: The graph given in figure 6.10 is arborescence.

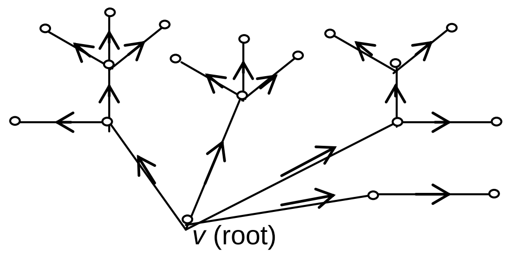


Figure 6.10

Self Assessment Questions

6. A closed directed walk which traverses every edge of *D* exactly once, is called a ————————.
7. A tree of *n* vertices contains —————— edges.

6.5 Matrix Representation of Digraphs

Definition: Let D be a digraph with p vertices. The adjacency matrix of D is a p×p matrix (a_{ij}) with

$$a_{ij} = \begin{cases} 1 \text{ if } v_i v_j \text{ is an arc of D} \\ 0 \text{ otherwise} \end{cases}.$$

It is denoted by A(D) which referred in figure 6.11.

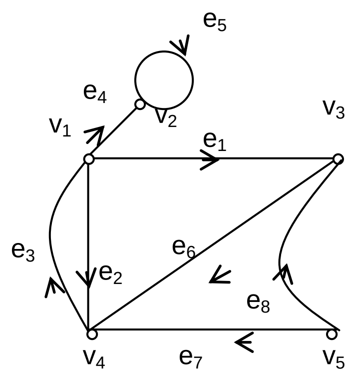


Figure 6.11

The adjacency matrix for the digraph given in figure 6.11 is $\begin{pmatrix} 01110 \\ 01000 \\ 00010 \\ 10000 \\ 00110 \end{pmatrix}$.

The sum of the i^{th} row entries of the adjacency matrix gives $d^+(v_i)$ and the sum of the i^{th} column entries gives $d^-(v_i)$ for every i.

The powers of A(D) give the number of walks from one point to another.

Definitions:

i) Let D be digraph with p vertices. The *reachability matrix* R = (r_{ij}) is the p×p matrix with

$$r_{ij} = \begin{cases} 1 \text{ if } v_j \text{ is reachable from } v_i \\ 0 \text{ otherwise, we assume that each vertex is reachable from itself.} \end{cases}$$

ii) The *distance matrix* is the p×p matrix whose $(i,j)^{th}$ entry gives the distance from the point v_i to the point v_j and is infinity if there is no path from v_i to v_j.

iii) The *detour matrix* is the p×p matrix whose $(i,j)^{th}$ entry is the length of any longest v_i-v_j path and infinity if there is no such path.

Example:

The reachable matrix, distance matrix and the detour matrix respectively for the matrix of given of 6.11 is given below.

$$\begin{pmatrix} 11110 \\ 01000 \\ 10110 \\ 11110 \\ 11110 \end{pmatrix}, \begin{pmatrix} \infty & 1 & 1 & 1 & \infty \\ \infty & 1 & \infty & \infty & \infty \\ 2 & 3 & 3 & 1 & \infty \\ 1 & 2 & 2 & 2 & \infty \\ 2 & 3 & 3 & 1 & \infty \end{pmatrix} \text{ and } \begin{pmatrix} \infty & 1 & 1 & 2 & \infty \\ \infty & 1 & \infty & \infty & \infty \\ 2 & 3 & 3 & 1 & \infty \\ 1 & 2 & 2 & 3 & \infty \\ 2 & 3 & 3 & 2 & \infty \end{pmatrix}.$$

Self Assessment Question

8. Let D be a digraph with p vertices. The adjacency matrix of D is a —— matrix

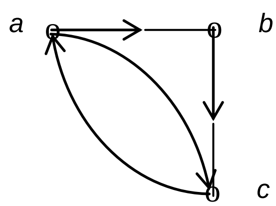
6.6 Summary

In this unit, we have investigated the fundamental features of the directed graphs. We have observed that the properties of digraphs are similar to some types of undirected graphs. The close relation ship between binary relation and digraphs was given in the unit. Directed graphs have interesting applications in telecommunications, flows and networks routing problems.

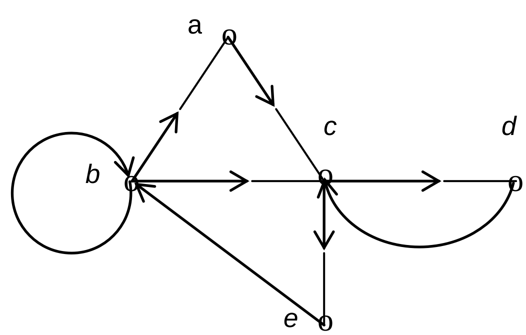
6.7 Terminal Questions

1. Find the in-degree and out-degree of the following digraphs

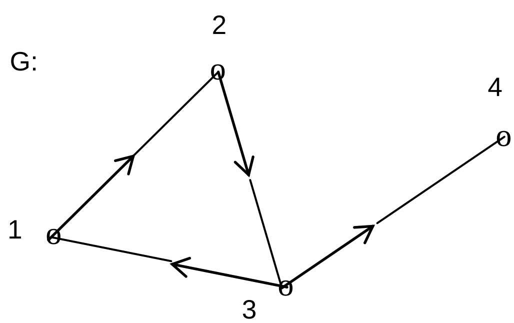
(i)

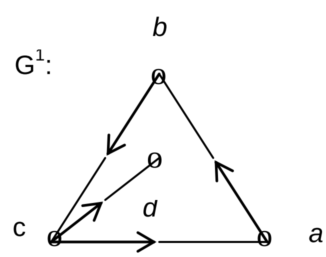


(ii)



2. Find the spanning tree of the digraph given in 1 (ii).
3. Verify whether or not the following digraphs are isomorphic? If so, write the isomorphisms.



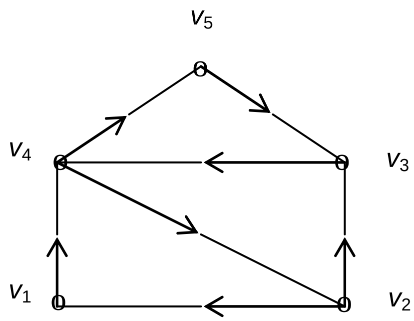


4. Draw the digraphs for the digraph D = {a, b, c, d, e} where the arcs represented by {(a, c), (a, d), (b, e), (e, c), (d, c)}. Write the in-degree and out-degree of D.

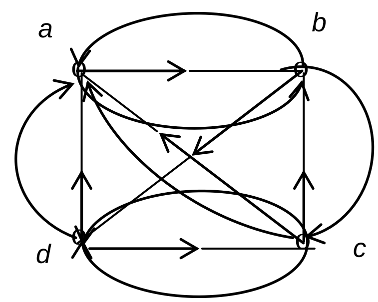
   Find also the converse D^1 (i.e., reversing the each arc direction in D) and find the indegree and outdegree of each point in D^1.

5. Give an example of a directed tree with five vertices.
6. Find in-degree and out-degree of each vertex in the following directed graphs.

(i)



(ii)



6.8 Answers

Self Assessment Questions

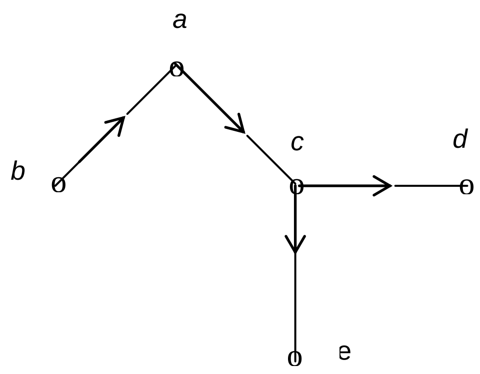
1. Oriented Graph
2. Out-Degree
3. Zero
4. Transitive
5. Component
6. Directed Euler line
7. n - 1
8. p×p

Terminal Questions

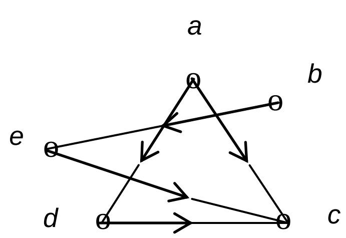
1. i) In-degree a = 1
   out-degree a = 2
   Similarly id (b) = 1, Od (b) = 1
   id (c) = 2, Od (c) = 1

   ii) id (a) = 1 id (b) = 2 id (c) = 3 id (d) = 1 id (e) = 1
   Od (a) = 1 Od (b) = 2 Od (c) = 2 Od (d) = 1 Od (e) = 1

2.

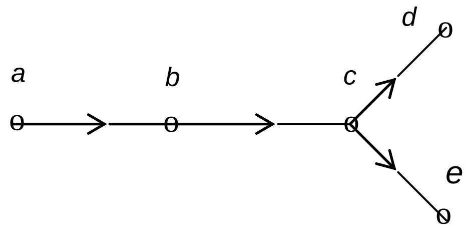


3. D and D^1 are isomorphic under the isomorphism $f(1) = a$, $f(2) = b$, $f(3) = c$, $f(4) = d$.
4. The digraph representing the given arcs is



The converse digraph also can be found in a similar way.

5.



6. i) in-degree v_1 =2, out-degree v_1 = 1
in-degree v_2 = 2, out-degree v_2 = 2
Similar for other vertices. (ii). Similar to (i).