Unit - 2
Approximation and Round off

*** Absolute Error :-**

Absolute Error is the numerical difference between its true value of a quantity and its approximate value. If X is true quantity & X_a is its approximate value then absolute Error E_a is

$$E_a = |\text{True value} - \text{approximate val.}| = |X - X_a|$$

*** Relative Error :-**

Relative Error is the absolute error divided by the true value of the quantity and this is denoted by E_r.,

$$E_r = \frac{\text{Absolute Error}}{\text{True value}} = \frac{E_a}{X} = \frac{|X - X_a|}{X}$$

*** Percentage Error :-**

The percentage Error $E_p = \frac{E_a}{X} \times 100 = E_r \times 100$.

*** Error - Propogation :-**

If we perform any operation like addition, multiplication, power and division, we have errors then result will get also the error.

(i) **Addition :-** Let $z = x + y$ be the addition of two numbers x and y and $z_A = x_A + y_A$ be sum of approximated values of x and y. so error in z and z_A is given as.

$$E_z = E_x + E_y$$
$$E_z = z - z_A$$

7 am

Example:- find equivalent Resistance of $R_1 = 2 \pm 0.2\,\Omega$ and $R_2 = 4 \pm 0.4\,\Omega$

8 **Sol:-**

$$R = R_1 + R_2$$

$$= 2 + 4$$

9 $$\boxed{R = 6}$$

Now adding absolute errors,

10 $$E_R = E_{R_1} + E_{R_2}$$

$$= 0.2 + 0.4$$

11 $$\boxed{E_R = 0.6}$$

12 So equivalent Resistence is 6 ± 0.6 with error.

1 pm

(ii) **Multiplications:-**

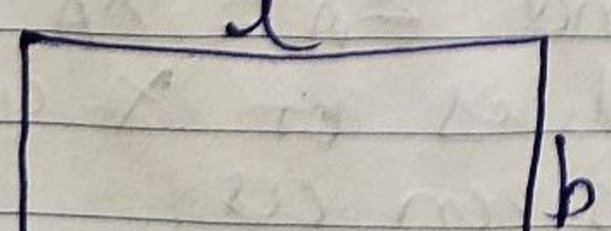
2 Let $Z = X \cdot Y$ be the product of two number X and Y. consider $Z_A = X_A \cdot Y_A$ be the product of approximates, so.

3

4 $$\frac{e_Z}{Z} \simeq \frac{e_Y}{Y} + \frac{e_X}{X}$$

5 $$e_Z \simeq \left(\frac{e_Y}{Y} + \frac{e_X}{X}\right) Z.$$

6 **Example**

7 

$l = 4 \pm 0.4\ m$

$b = 2 \pm 0.2\ m$

Area = ?

Sol:-

$$\text{Area} = l \times b = 4 \times 2 = 8\ m^2$$

Only way to know if a person is trustworthy is to trust him.

$$E_z = \left(\frac{E_x}{x} + \frac{E_y}{y}\right) z$$

$$= \left(\frac{0.4}{4} + \frac{0.2}{2}\right) 8$$

$$E_z = 1.6\ m^2$$

So Area of rectangle with Error is $(8 \pm 1.6)m^2$

(iii) Division :-

Let $Z = \frac{X}{Y}$ be the division. $\therefore Z_A = \frac{X_A}{Y_A}$ be the approximates, so,

$$E_z = \frac{E_x}{Y} - \frac{X E_y}{Y^2}$$

(iv) Power :-

Let $Z = \frac{A^P B^Q}{C^R}$

$$E_z = \left[P\left(\frac{E_A}{A}\right) + Q\left(\frac{E_B}{B}\right) + R\left(\frac{E_C}{C}\right)\right] . Z.$$