

2014

MAY

Wk - 19 • 128-237

Thursday

08

# Unit - II Numerical Integration.

|      |    |    |    |    |    |
|------|----|----|----|----|----|
| M    | M  | T  | W  | T  | F  |
| A    | 5  | 6  | 7  | 8  | 9  |
| Y    | 12 | 13 | 14 | 15 | 16 |
|      | 19 | 20 | 21 | 22 | 23 |
| 2014 | 26 | 27 | 28 | 29 | 30 |

7 am \* Trapezoidal Rule formula:- (even, odd) <sup>for</sup>

8 It is applied for even or odd ~~intervals~~ number of total interval.

9 
$$\int_{x_0}^{x_n} y \cdot dx = h \left( \frac{y_0 + y_n}{2} + y_1 + y_2 + \dots + y_{n-1} \right)$$

10 <sup>All other</sup>

11 \* Simpson 1/3 Rule :- (even <sup>for</sup> ~~intervals~~).

12 It is applicable for only even number of interval.

1 pm 
$$\int_{x_0}^{x_n} y \cdot dx = \frac{h}{3} \left[ (y_0 + y_n) + 2(y_2 + y_4 + y_6 + \dots + y_{n-2}) + 4(y_1 + y_3 + y_5 + \dots + y_{n-1}) \right]$$

2 
$$h = \frac{x_n - x_0}{n}$$

3 
$$h = \frac{x_n - x_0}{n}$$

4 
$$h = \frac{x_n - x_0}{n}$$

5 
$$h = \frac{x_n - x_0}{n}$$

6 \* Simpson 3/8 Rule :- (for 3 multiple).

7 It is applicable ~~for~~ if total no. of interval is multiple of 3.

8 
$$\int_{x_0}^{x_n} y \cdot dx = \frac{3h}{8} \left[ (y_0 + y_n) + 3(y_1 + y_2 + y_4 + \dots + y_{n-1}) + 2(y_3 + y_6 + \dots + y_{n-3}) \right]$$

9 
$$h = \frac{x_n - x_0}{n}$$

10 
$$h = \frac{x_n - x_0}{n}$$

11 
$$h = \frac{x_n - x_0}{n}$$

12 
$$h = \frac{x_n - x_0}{n}$$



| M  | T  | W  | T  | F  | S  | S  |
|----|----|----|----|----|----|----|
| 30 | 3  | 4  | 5  | 6  | 7  | 8  |
| 2  | 10 | 11 | 12 | 13 | 14 | 15 |
| 9  | 17 | 18 | 19 | 20 | 21 | 22 |
| 16 | 24 | 25 | 26 | 27 | 28 | 29 |

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Friday

09

\* Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  using Trapezoidal rule,

Simpson  $1/3$  Rule and Simpson  $3/8$  rule. and also find value of  $\pi$  in each case.

Soln:-

Now we will take 6 interval. so,

$$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

Now take  $x_0=0$  so, and  $x_1 = x_0 + h$ ,  $x_2 = x_1 + h$ ...

|             |                               |
|-------------|-------------------------------|
| $x_0 = 0$   | $y_0 = \frac{1}{1+x_0^2} = 1$ |
| $x_1 = 1/6$ | $y_1 = 1/(1+x_1^2) = .36/37$  |
| $x_2 = 2/6$ | $y_2 = 1/(1+x_2^2) = 0.9$     |
| $x_3 = 3/6$ | $y_3 = 1/(1+x_3^2) = 0.8$     |
| $x_4 = 4/6$ | $y_4 = 1/(1+x_4^2) = 9/13$    |
| $x_5 = 5/6$ | $y_5 = 1/(1+x_5^2) = 36/61$   |
| $x_6 = 1$   | $y_6 = 1/(1+x_6^2) = 0.5$     |

Now integration of equation,

$$\int_0^1 \frac{1}{1+x^2} dx = (\tan^{-1} x)_0^1 = (\tan^{-1} 1 - \tan^{-1} 0) = \pi/4 - 0 = \frac{\pi}{4} \quad \text{--- (1)}$$

(i) By Trapezoidal formula:-

$$\begin{aligned} \int_0^1 \frac{1}{1+x^2} dx &= h \left[ \frac{y_0 + y_6}{2} + y_1 + y_2 + y_3 + y_4 + y_5 \right] \\ &= \frac{1}{6} \left[ \frac{1 + 0.5}{2} + \frac{36}{37} + 0.9 + 0.8 + \frac{9}{13} + \frac{36}{61} \right] \\ &= 0.78423 \quad \text{--- (2)} \end{aligned}$$



| M    | T  | W  | T  | F  | S  |
|------|----|----|----|----|----|
| A    | 5  | 6  | 7  | 1  | 2  |
| Y    | 12 | 13 | 14 | 8  | 9  |
|      | 19 | 20 | 21 | 15 | 16 |
| 2014 | 26 | 27 | 28 | 22 | 23 |
|      |    |    | 29 | 24 | 25 |
|      |    |    |    | 30 | 31 |

(ii) By Simpson  $1/3$  rule: -

7 am

$$\int_0^1 \frac{1}{1+x} dx = \frac{h}{3} [(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

9

10

$$= \frac{1}{6} \times \frac{1}{3} [(1 + 0.5) + 2(0.9 + \frac{9}{13}) + 4(\frac{36}{37} + 0.8 + \frac{36}{61})]$$

$$= 0.785396$$

③

(iii) By Simpson  $3/8$  rule: -

12

1 pm

$$\int_0^1 \frac{1}{1+x^2} dx = \frac{3h}{8} [(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

2

$$= \frac{3}{8} \times \frac{1}{6} [(1 + 0.5) + 3(\frac{36}{37} + 0.9 + \frac{9}{13} + \frac{36}{61}) + 2(0.8)]$$

3

$$= 0.785394$$

④

Now comparing equation ① and ②, for Trapezoidal.

$$\pi/4 = 0.78423$$

$$\pi = 4(0.78423) = 3.1369$$

Now comparing equation ① and ③, for Simpson  $1/3$

$$\pi/4 = 0.785396$$

$$\pi = 4 \times 0.785396 = 3.14158$$

Now comparing equation ① and ④, for Simpson  $3/8$ .

$$\pi/4 = 0.785394$$

$$\pi = 4(0.785394) = 3.14157$$



| M  | T  | W  | T  | F  | S  | S  | J    |
|----|----|----|----|----|----|----|------|
| 30 |    |    |    |    |    | 1  | U    |
| 2  | 3  | 4  | 5  | 6  | 7  | 8  | N    |
| 9  | 10 | 11 | 12 | 13 | 14 | 15 |      |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 |      |
| 23 | 24 | 25 | 26 | 27 | 28 | 29 | 2014 |

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Sunday

11

\* Given that

|      |              |        |        |        |        |        |        |        |
|------|--------------|--------|--------|--------|--------|--------|--------|--------|
| 7 am | $x$          | 4.0    | 4.2    | 4.4    | 4.6    | 4.8    | 5.0    | 5.2    |
|      | $y = \log x$ | 1.3863 | 1.4351 | 1.4816 | 1.5281 | 1.5688 | 1.6094 | 1.6487 |

then Evaluate  $\int_4^{5.2} \log x \cdot dx$  by Simpson  $3/8$  rule.

Sol: Here total 6 interval so  $n = 6$ ,

$$h = \frac{b-a}{n} \text{ or } \frac{x_n - x_0}{n}$$

$$= \frac{5.2 - 4}{6}$$

$$\boxed{h = 0.2}$$

$$\int_4^{5.2} \log x \cdot dx = \frac{3h}{8} [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3(0.2)}{8} [(1.3863 + 1.6487) + 3(1.4351 + 1.4816 + 1.5688 + 1.6094) + 2(1.5281)]$$

$$\boxed{= 1.82714}$$

Its not the size of a weapon that matter, but fury of the attack.