

Numerical Solution of Ordinary differential equation - I

M	T	W	T	F	S
			1	2	3
5	6	7	8	9	10
12	13	14	15	16	17
19	20	21	22	23	24
26	27	28	29	30	31

* Picard's method of successive approximation:

$$y_{n+1} = y_0 + \int_{x_0}^x f(x, y_n) \cdot dx$$

* Solve by Picard method up to third approximation
 $\frac{dy}{dx} = x + y^2$, ~~initial value problem~~
 of initial value problem (IVP) $y(0) = 0$ and also
 find $y(0.1)$.

Sol: Here $y(0) = 0$, so $x = 0, y = 0$.

now, use per picard method.

$$y_{n+1} = y_0 + \int_{x_0}^x f(x, y_n) \cdot dx \Rightarrow$$

$$y_{n+1} = y_0 + \int_{x_0}^x (x + y_n^2) \cdot dx$$

now, put $n = 0$,

$$y_1 = y_0 + \int_{x_0}^x (x + y_0^2) \cdot dx$$

$$= (0) + \int_0^x (x + (0)^2) \cdot dx$$

$$= \int_0^x x \cdot dx$$

$$y_1 = \frac{x^2}{2}$$

now, put $n = 1$,

$$y_2 = y_0 + \int_{x_0}^x (x + y_1^2) \cdot dx$$

$$= 0 + \int_0^x (x + (\frac{x^2}{2})^2) \cdot dx$$

$$= \int_0^x x \cdot dx + \int_0^x \frac{x^4}{4} \cdot dx$$

$$y_2 = \frac{x^2}{2} + \frac{x^5}{20}$$

M	T	W	T	F	S	S	J
30					1	8	U
2	3	4	5	6	7	15	N
9	10	11	12	13	14	22	
16	17	18	19	20	21	28	2014
23	24	25	26	27	28	29	

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Tuesday

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now put $n=2$,

$$y_3 = y_0 + \int_{x_0}^x (x + y_2^2) \cdot dx.$$

$$= 0 + \int_0^x \left[x + \left(\frac{x^2}{2} + \frac{x^5}{20} \right)^2 \right] \cdot dx.$$

$$= \int_0^x \left[x + \frac{x^4}{4} + \frac{x^7}{20} + \frac{x^{10}}{400} \right] \cdot dx.$$

$$y_3 = \frac{x^2}{2} + \frac{x^5}{20} + \frac{x^8}{160} + \frac{x^{11}}{4400}$$

now to find $y(0.1)$, from above equation.

$$y(0.1) = \frac{(0.1)^2}{2} + \frac{(0.1)^5}{20} + \frac{(0.1)^8}{160} + \frac{(0.1)^{11}}{4400}$$

$$y(0.1) = 0.005 + 0.0000 + 0.0000 + 0.0000$$

$$y(0.1) = 0.0050$$

Happiness comes from within, it can never come from outside.

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Wednesday

M	T	W	T	F	S	S
			1	2	3	4
A	5	6	7	8	9	10
Y	12	13	14	15	16	17
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2014	26	27	28	29	30	31

* Euler's method or

7 am

Runge Kutta method for first order.

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$$y_{n+1} = y_n + h f(x_n, y_n)$$

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* Solve $\frac{dy}{dx} = x+y$ with boundary condition

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$y=1$ at $x=0$. find approximate value of

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y for $x=0.1$.

1 pm

Sol: - Here initial value of $x_0 = 0$ and $x_n = 0.1$
Now we will divide into 5 interval,

$$\text{So, } h = \frac{x_n - x_0}{n} = \frac{(0.1) - (0)}{5} = 0.02$$

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now $x_0 = 0$ then $x_1 = x_0 + h$, $x_2 = x_1 + h$, ...

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$$x_0 = 0, y_0 = 1$$

$$x_1 = 0.02$$

$$x_2 = 0.04$$

$$x_3 = 0.06$$

$$x_4 = 0.08$$

$$x_5 = 0.1$$

we have $\frac{dy}{dx} = x+y$ equation so,
Now Euler's formula,

$$y_{n+1} = y_n + h(f(x_n, y_n)) \Rightarrow y_{n+1} = y_n + h(x_n + y_n)$$

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Thursday

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now put $n=0$,

$$y_1 = y_0 + h(x_0 + y_0)$$

$$y_1 = 1 + 0.02(0 + 1) = 1.02$$

now put $n=1$,

$$y_2 = y_1 + h(x_1 + y_1) = 1.02 + 0.02(0.02 + 1.02)$$

$$y_2 = 1.0408$$

now put $n=2$,

$$y_3 = y_2 + h(x_2 + y_2) = 1.0408 + 0.02(0.04 + 1.0408)$$

$$y_3 = 1.0624$$

now put $n=3$,

$$y_4 = y_3 + h(x_3 + y_3) = 1.0624 + 0.02(0.06 + 1.0624)$$

$$y_4 = 1.0848$$

now put $n=4$,

$$y_5 = y_4 + h(x_4 + y_4) = 1.0848 + 0.02(0.08 + 1.0848)$$

$$y_5 = 1.1081$$

so, $y_0 = 1$, $y_1 = 1.02$, $y_2 = 1.0408$, $y_3 = 1.0624$, $y_4 = 1.0848$ and $y_5 = 1.1081$.

Two heads are better than one. But not if both are stupid.

* Euler's modified method

7 am

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$$y_{n+1}^{(*)} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_{n+1}, y_{n+1}^{(*)})]$$

* Given $dy/dx = x^2 + y$ with $y(0) = 1$, find $y(0.02)$ and $y(0.04)$ by Euler's modified method.

1 pm

Sol:- Here $y(0) = 1$, so $x_0 = 0$, $y_0 = 1$.
We have to find $y(0.02)$ and $y(0.04)$.

So let $x_1 = 0.02$ and $x_2 = 0.04$.

$$x_0 = 0$$

$$x_1 = 0.02$$

$$x_2 = 0.04$$

Here difference between x_1 and x_0 is 0.02
So, $h = 0.02$

We have $dy/dx = x^2 + y$

Now as Euler's modified method,

$$(i) \quad y_{n+1}^{(*)} = y_n + h(x_n^2 + y_n)$$

$$(ii) \quad y_{n+1} = y_n + \frac{h}{2} [(x_n^2 + y_n) + (x_{n+1}^2 + y_{n+1}^{(*)})]$$

Knowledge is power but action gets things done.

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Saturday

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now put $n=0$,

$$y_1^* = y_0 + h(x_0^2 + y_0) = 1 + 0.02(0^2 + 1) = 1.02$$

$$y_1 = y_0 + \frac{h}{2} [(x_0^2 + y_0) + (x_1^2 + y_1^*)]$$

$$= 1 + \frac{0.02}{2} [(0)^2 + 1 + (0.02)^2 + 1.02]$$

$$y_1 = 1.0202$$

now put $n=1$,

$$y_2^* = y_1 + h(x_1^2 + y_1) = 1.0202 + 0.02((0.02)^2 + 1.0202)$$

$$y_2^* = 1.0406$$

$$y_2 = y_1 + \frac{h}{2} [(x_1^2 + y_1) + (x_2^2 + y_2^*)]$$

$$= 1.0202 + \frac{0.02}{2} [(0.02)^2 + 1.0202 + (0.04)^2 + 1.0406]$$

$$y_2 = 1.0408$$

$$\text{so, } x_0 = 0$$

$$, y_0 = 1$$

$$x_1 = 0.02$$

$$, y_1 = 1.0202$$

$$x_2 = 0.04$$

$$, y_2 = 1.0408$$

$$\text{so, } y(0.02) = 1.0202$$

$$y(0.04) = 1.0408$$