Unit 8 Differential Equations

Structure:

8.1 Introduction

Differential equation is a branch of Mathematics which finds its application in variety of fields. First of all a given problem is converted to differential equations, which is then solved and the solution to the problem is found out.

Objectives:

At the end of the unit you would be able to

- find the solution of Differential Equations
- apply differential equations in practical situations

8.2 First order Differential Equations

Definitions

A differential equation is an equation which involves differential coefficients or differentials.

Thus,

i) $e^x dx + e^y dy = 0,$ ii) $\frac{d^2x}{dy^2} + n^2 x = 0$

iii) $y = x.\frac{dy}{dx} + \frac{x}{dy/dx}$ iv) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} \Big/ \frac{d^2y}{dx^2} = c$

v) $x.\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u$

are all examples of differential equations.

An **ordinary differential equation** is that in which all the differential coefficients have reference to a single independent variable. Thus the equation (i) to (iv) are all ordinary differential equations.

A **partial differential equation** is that in which there are two or more independent variables and partial coefficients with respect to any of them. Thus equation (v) is an example for the partial differential equation.

The **order** of a differential equation is the order of the highest derivative appearing in it.

The **degree** of a differential equation is the degree of the highest derivative occurring in it, after the equation has been expressed in a form free from radicals and fractions as far as the derivatives are concerned.

Thus from the examples above,

i) is of first order and first degree;
ii) is the second order and first degree;
iii) can be written as $y\frac{dy}{dx}=x\{.\left(\frac{dy}{dx}\right)^2+1\},$ and is clearly a first order but second degree equation.

8.3 Practical Approach to Differential Equations

Differential equations arise from many problems in oscillations of mechanical, electrical systems, bending of beams, conduction of heat, velocity of chemical reactions etc., and plays a very important role in all modern scientific and engineering studies.

The approach of an engineering student to the study of differential equations has got to be practical unlike that a student of mathematics, who is only interested in solving the differential equations without knowing as to how the differential equations are formed and how their solutions are physically interpreted.

Thus for the applied mathematics, the study of the differential equation consists of 3 phases:

i) Formation of the differential equation from the given problem
ii) Solution of this differential equation, evaluating the arbitrary constants from the given conditions, and
iii) Physical interpretation of the solution

Formation of a differential equation

An ordinary differential equation is formed in an attempt to eliminate certain arbitrary constants from a relation. In applied mathematics, every geometrical or physical problem when translated into mathematical symbols gives rise to a differential equation.

Examples

Form the differential equation of simple harmonic motion given by, $x = A\cos(nt+\alpha)$

Solution: To eliminate the constant A and α , differentiating it twice, we have,

$$\frac{dx}{dt} = -nA\sin(nt+\alpha)$$

and $\frac{d^2x}{dt^2} = -n^2A\cos(nt+\alpha) = -n^2x$

thus, $\frac{d^2x}{dt^2} + n^2x = 0,$ is the desired differential equation.

Example: Form the differential equation of all circles of radius a.

Solution: The general equation of a circle with centre at (h, k) is given by

$$(x-h)^2 + (y-k)^2 = a^2 \qquad \text{........ (1)}$$

Where h and k, the co-ordinates of the centre, and a are the constants. Differentiate twice w.r.t. x, we have,

$$2(x-h)+2(y-k).\frac{dy}{dx} = 0 \Rightarrow (x-h)+(y-k).\frac{dy}{dx} = 0$$

and $1+(y-k)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 0$

Then, $(y-k) = -\left(1+\left(\frac{dy}{dx}\right)^2\right)\Big/\frac{d^2y}{dx^2}$

And $x - h = -(y - k)\ \frac{dy}{dx}$

$$= \frac{dy}{dx}\left(1 + \left(\frac{dy}{dx}\right)^2\right) \Big/ \frac{d^2y}{dx^2}$$

Substituting these in (1), and simplifying, we get,

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = a,$$

It states that the radius of curvature of a circle at any point is constant

Solution of a differential equation

A solution (or integral) of a differential equation is a relation between the variables which satisfies the given differential equation.

For example, $x = A\cos(nt + \alpha)$ ------------ (1)

is a solution of $.\frac{d^2x}{dt^2} + n^2x = 0$ ------------ (2)

The **general** or **complete solution** of a differential equation is that in which the number of arbitrary constants is equal to the order of the differential equation. Thus, (1) is a general solution of (2) as the number of arbitrary constants A and α is the same as the order of (2).

A **particular solution** is that which can be obtained from the general solution by giving particular values to the arbitrary constants

For example, $x = A\cos\left(nt + \frac{\pi}{4}\right)$,

is the particular solution of the equation (2) as it can be derived from the general solution by putting $\alpha = \frac{\pi}{4}$.

8.4 First Order and First Degree Differential Equations

One can represent the general and particular solution of a differential equation geometrically. But it is not possible to solve a family of parabola

$y = x^2 + c$, in this approach. We shall, however, discuss some special methods of solution which are applied to the following types of equations:

(i) Equations where variables are separable
(ii) Homogeneous equations
(iii) Linear equations
(iv) Exact equations

Variables separable

If in any equation it is possible to collect all functions of x and dx on one side and all the functions of y and dy on the other side, then the variables are said to be separable. Thus the general form of such equation is $f(y)dy = \varphi(x)\,dx$.

Integrating both the sides, we get,

$\int f(y)dy = \int \phi(x)dx + c$ as its solution.

Example:

Solve $\dfrac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$

Solution: Given equation is $\dfrac{dy}{dx} = e^{-2y}\left(e^{3x} + x^2\right)$

$$e^{2y}dy = \left(e^{3x} + x^2\right)dx$$

Integrating on both the sides,

$$\int e^{2y}dy = \int \left(e^{3x} + x^2\right)dx + c$$

$$\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + c$$

$$3e^{2y} = 2\left(e^{3x} + x^3\right) = c$$

8.5 Homogeneous Equations

Homogeneous Equations are of the form $\dfrac{dy}{dx} = \dfrac{f(x, y)}{\phi(x, y)}$

Where $f(x, y)$ and $\varphi(x, y)$ are homogeneous functions of the same degree in x and y.

To solve a homogeneous equation,

i) Put $y = vx$, then $\frac{dy}{dx} = v + x.\frac{dv}{dx}$,

ii) Separate the variables v and x and then integrate.

Example:

Solve $(x^2 - y^2)dx = 2xydy$

Solution: The given equation is $\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$, which is homogeneous in x and y.

Put $y = vx$ then $\frac{dy}{dx} = v + x.\frac{dv}{dx}$

Then the given problem becomes, $v + x.\frac{dv}{dx} = \frac{1 - v^2}{2v}$

or $x.\frac{dv}{dx} = \frac{1 - v^2}{2v} - v = \frac{1 - 3v^2}{2v}$

Separating the variables, we get, $\frac{2v}{1 - 3v^2}dv = \frac{dx}{x}$

Integrating both the sides

$$\int \frac{2v}{1 - 3v^2}dv = \int \frac{dx}{x} + c$$

or $\quad -\frac{1}{3}\int \frac{-6v}{1 - 3v^2}dv = \int \frac{dx}{x} - c$

or $\quad \frac{-1}{3}\log(1 - 3v^2) = \log x + c$

or $\quad 3\log x + \log(1 - 3v^2) = -3c$

or $\quad \log x^3(1 - 3v^2) = -3c$

or $\quad x^3(1 - 3y^2/x^2) = e^{-3c} = c$

Hence the required solution is $x(x^2 - 3y^2) = c$

Equations reducible to homogeneous form

The equations of the form $\frac{dy}{dx} = \frac{ax + by + c}{a'x + b'y + c'}$ ---------- (1)

can be reduced to the homogeneous form as follows:

Case 1: when $\frac{a}{a'} \neq \frac{b}{b'}$

Putting $x = X + h, y = Y + k$ (h, k being constants)

So that, $dx = dX \& dY = dy,$ becomes,

$$\frac{dY}{dX} = \frac{aX + bY + (ah + bk + c)}{a'X + b'Y + (a'h + b'k + c')} \qquad \text{----------- (2)}$$

Choose h, k so that (2) may becomes homogeneous.

Put $\quad ah + bk + c = 0,$

and $a'h + b'k + c' = 0$

So that,

$$\frac{h}{bc' - b'c} = \frac{k}{ca' - c'a} = \frac{1}{ab' - ba'}$$

or

$$h = \frac{bc' - b'c}{ab' - ba'} \qquad \text{---------- (3)}$$

$$k = \frac{ca' - c'a}{ab' - ba'}$$

Thus, when $ab' - ba' \neq ,0$ then (2), becomes

$$\frac{dY}{dX} = \frac{aX + bY}{a'X + b'Y} \qquad \text{--------- (4)}$$

Which is a homogeneous in X, Y and can be solved by putting $Y = vX.$

Case II: When $\frac{a}{a'} = \frac{b}{b'}$

i.e. $ab' - ba' = 0,$ the above method fails as h and k become infinite or indeterminant

Now, $\quad \frac{a}{a'} = \frac{b}{b'} = \frac{1}{m}\,(say)$

$a' = am, b' = bm$ and (1) becomes

$$\frac{dy}{dx} = \frac{(ax + by) + c}{m(ax + by) + c'}$$

Put $ax + by = t,$ so that $a + b\frac{dy}{dx} = \frac{dt}{dx}$

or $\frac{dy}{dx} = \frac{1}{b}\left(\frac{dt}{dx} - a\right)$

therefore we have, $\frac{1}{b}\left(\frac{dt}{dx} - a\right) = \frac{t+c}{mt+c'}$

$$\frac{dt}{dx} = a + \frac{bt+bc}{mt+c'} = \frac{(am+b)t + ac' + bc}{mt+c'}$$

so that the variables are separable. In this solution, putting $t = ax + by$, we get the required solution of (1).

Examples

1. Solve $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$

Solution: The given equation is $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$ [this is where $\frac{a}{a'} \neq \frac{b}{b'}$] --------- (1)

Putting $x = X + h$, $y = Y + k$ (h, k being constants)

So that $dx = dX$, $dy = dY$, (1) becomes

$$\frac{dY}{dX} = \frac{Y + X + (k+h-2)}{Y - X + (k-h-4)} \qquad \text{--------- (2)}$$

Put $k + h - 2 = 0$ and $k - h - 4 = 0$,

So that, $h = -1$, $k = 3$.

Therefore (2) becomes, $\frac{dY}{dX} = \frac{Y+X}{Y-X}$ ………………………………….. (3)

Which is homogeneous in X and Y.

Put $Y = vX$, then $\frac{dY}{dX} = v + X\frac{dv}{dx}$

Therefore (3) becomes, $v + X\frac{dv}{dX} = \frac{v+1}{v-1}$

Or $X\frac{dv}{dX} = \frac{v+1}{v-1} - v = \frac{1+2v-v^2}{v-1}$

$$\frac{v-1}{1+2v-v^2}\,dv=\frac{dX}{X}$$

Integrating both the sides

$$-\frac{1}{2}\int\frac{(2-2v)}{1+2v-v^2}\;dv=\int\frac{dX}{X}+c$$

Or

$$-\frac{1}{2}\log\left(1+2v-v^2\right)=\log X+c$$

Or

$$\log\left(1+\frac{2Y}{X}-\frac{Y^2}{X^2}\right)+\log X^2=-2c$$

Or log (X^2 + 2XY – Y^2) = –2c

Or X^2 + 2XY – y^2 = e^{-2c} = c'

Putting $X = x - h = x + 1$, and $Y = y - k = y - 3$, equation (4) becomes,

$$(x+1)^2+2(x+1)(y-3)-(y-3)^2=c'$$

Which is the required solution.

8.6 Linear Equations

A differential equation is said to be linear if the different variables and its differential coefficients occurs only in the first degree and are not multiplied together.

Thus the standard form of a linear equation of the first order, commonly known as **Leibnitz's linear equation,** is

$$\frac{dy}{dx}+py=Q \qquad \text{---------- (1)}$$

Where P, Q are any functions of x.

To solve the equation, multiply both the sides by $e^{\int Pdx}$ so that we get,

$$\frac{dy}{dx}e^{\int Pdx}+y\left(e^{\int Pdx}P\right)=Qe^{\int Pdx}$$

$$i.e,.\ \frac{d}{dx}\left(ye^{\int Pdx}\right)=Qe^{\int Pdx}$$

Integrating both the sides

$$ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$$

is the required solution

Example

1. Solve $(x+1)\dfrac{dy}{dx} - y = e^{3x}\ (x+1)^2$

Solution: The given equation is, $(x+1)\dfrac{dy}{dx} - y = e^{3x}\ (x+1)^2$

Dividing both the sides by *(x + 1),* given equation becomes,

$$\frac{dy}{dx} - \frac{y}{(x+1)} = e^{3x}\ (x+1) \qquad \text{---------- (1)}$$

Which is Leibnitz's equation.

Here $P = -\dfrac{1}{x+1}$

Therefore, $\int Pdx = -\int \dfrac{dx}{x+1} = -\log(x+1) = \log(x+1)^{-1}$

And $e^{\int Pdx} = e^{\log(x+1)^{-1}} = \dfrac{1}{x+1}$

Thus the solution of (1) is,

$$ye^{\int Pdx} = \int e^{3x}(x+1)\left(\frac{1}{x+1}\right)dx + c$$

$$\frac{y}{x+1} = \int e^{3x}dx + c = \frac{1}{3}e^{3x} + c$$

$$\therefore y = \left(\frac{1}{3}e^{3x} + c\right)(x+1)$$

8.7 Bernoulli's Equation

The equation $\dfrac{dy}{dx} + Py = Qy^n,$ -------- (1)

Where P, Q are functions of x, is reducible to the Leibnitz's linear equation and is usually called the Bernoulli's equation.

To solve (1), divide both the sides of (1) by y^n, we get

$$y^{-n}\frac{dy}{dx} + Py^{1-n} = Q \qquad \text{-------- (2)}$$

Put $y^{1-n} = z,$

$\therefore$ (2) Becomes

$$(1-n)y^{-n}\frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{1}{(1-n)}\frac{dz}{dx} + Pz = Q$$

$$\frac{dz}{dx} + P(1-n)z = Q(1-n)$$

Which is Leibnitz's linear in z and can be solved easily.

Examples

1. Solve $x\frac{dy}{dx} + y = x^3y^6$

Solution: The given equation is, $x\frac{dy}{dx} + y = x^3y^6$ -------- (1)

Dividing throughout by xy^6,

$$y^{-6}\frac{dy}{dx} + \frac{y^{-5}}{x} = x^2 \quad \text{-------- (2)}$$

Put $y^{-5} = z$, so that $\left(-5y^{-6}\right)dy/dx = dz/dx$

Therefore (2) becomes, $-\frac{1}{5}\frac{dz}{dx} + \frac{z}{x} = x^2$

Or $\frac{dz}{dx} - \frac{5}{x}z = -5x^2$ --------- (3)

Which is Leibnitz's linear in z, and the intermediate form I.F. is,

I.F. = $e^{\int Pdx} = e^{-\int\left(\frac{5}{x}\right)dx} = e^{\log x^{-5}} = x^{-5}$

Therefore the solution of (3) is,

$$z(I.F) = \int\left(-5x^2\right)(I.F)\,dx + c$$

$$zx^{-5} = \int\left(-5x^2\right)x^{-5}dx + c$$

Or $y^{-5}x^{-5} = -5.\frac{x^{-2}}{-2} + c$ [since $z = y^{-5}$]

Dividing both sides by $y^{-5}x^{-5}$, we get,

$1 = (2.5 + cx^2)\, x^3 y^5$

8.8 Exact Differential Equations

1. **Definition:** A differential equation of the form

   $M(x,y)dx + N(x,y)dy = 0,$

   is said to be exact if its left hand member is the exact differential of some function *u(x, y).*

   i.e., $du \equiv Mdx + Ndy = 0$

   Its solution therefore, is u(x, y) = c.

2. **Theorem:** The necessary and sufficient condition for the differential equation *Mdx* + *Ndy* = *0* to be exact is

   $$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

   Condition is necessary:

   The equation *M(x, y) dx* + *N(x, y) dy* = *0* will be exact, if *Mdx* + *Ndy* $\equiv du$ ----- (1)

   Where u is some function of x and y.

   But $$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy \qquad \text{------- (2)}$$

   Equating coefficients of dx and dy in (1) and (2), we get

   $$M = \frac{\partial u}{\partial x} \quad \text{and}\, N = \frac{\partial u}{\partial y}$$

   $$\therefore \frac{\partial M}{\partial y} = \frac{\partial^2 u}{\partial y \partial x} \quad \text{and} \quad \frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial x \partial y}$$

   But, $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$ (Assumption)

   $$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

   Which is the necessary condition of exactness.

Condition is sufficient:

i.e., if $\frac{\partial \text{M}}{\partial \text{y}} = \frac{\partial \text{N}}{\partial \text{x}}$, then *Mdx* + *Ndy* = *0* is exact.

Let $\int Mdx = u,$ where *u* is supposed constant while performing integration. Then

$$\frac{\partial}{\partial y}\left(\int Mdx\right)=\frac{\partial u}{\partial x}\ i.e., M=\frac{\partial u}{\partial x} \qquad \text{------- (3)}$$

$$\therefore \frac{\partial M}{\partial y}=\frac{\partial^2 u}{\partial y \partial x} \quad \text{since} \ \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x} \ \text{(given)}$$

Or $\frac{\partial N}{\partial x}=\frac{\partial^2 u}{\partial x \partial y}=\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)$ and $\frac{\partial^2 u}{\partial y \partial x}=\frac{\partial^2 u}{\partial x \partial y}$

Integrating both the sides, w.r.t x (taking y as constant)

$N=\frac{\partial u}{\partial y}+f(y)$, where f(y) is a function of y alone. -------- (4)

$$\therefore Mdx+Ndy=\frac{\partial u}{\partial x}dx+\left\{\frac{\partial u}{\partial y}+f(y)\right\}dy \qquad \text{[by (3) and (4).]}$$

$$=\left\{\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy\right\}+f(y)dy \qquad \text{-------- (5)}$$

$$=du+f(y)dy=d[u+\int f(y)\,dy]$$

Which shows that Mdx + Ndy = 0 is exact.

Method of solution: By equation (5), equation *Mdx + Ndy = 0* becomes

$$d\left[u+\int f(y)dy\right]=0$$

Integrating $u+\int f(y)dy=c$

But $u=\int_{(y\,const)} Mdx=c \ and \ f(y)$=terms of N not containing x.

$\therefore$ The solution of Mdx + Ndy = 0 is

$$\int_{(y\,const)} Mdx+\int \text{(terms of N not containing x)}\, dy=c$$

Provided $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

Examples

1. Solve $\frac{dy}{dx}-\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$

   Solution: Given equation can be written as,
   (ycos x + sin y + y) dx + (sin x + x cos y + x) dy = 0

Here,

$M = y \cos x + \sin y + y$

$N = \sin x + x \cos y + x$

$$\therefore \frac{\partial M}{\partial y} = \cos x + \cos y + 1 = \frac{\partial N}{\partial x}$$

Thus the equation is exact and its solution is

$$\int_{(y\,const)} M dx + \int (terms\ of\ N\ not\ containing\ x) dy = c$$

i.e., $\int_{(y\,const)} (y \cos x + \sin y + y) dx + \int (0) dy = c$

Or $y \sin x + (\sin y + y)x = c$

Self Assessment Questions

1. Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$
2. Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$
3. Solve $(3y + 2x + 4)dx - (4x + 6y + 5)dy = 0$

8.9 Summary

In this unit, we study the first order differential equations. The practical approach to differential equations is clearly explained with suitable examples. Solving first order first degree differential equation by the variable separable method is discussed here. Equations reducible to homogeneous form is discussed here with example. The linear equations, Bernoulli's equation and exact differential equation is solved here in a simple manner with proper examples.

8.10 Terminal Questions

1. Derive the necessary and sufficient condition for the differential equation *Mdx + Ndy* to be exact
2. Briefly describe Bernoulli's equation

8.11 Answers

Self Assessment Questions

1. Dividing throughout by $\cos^2 y$,

$$\sec^2 y\frac{dy}{dx}+2x\frac{\sin y\cos y}{\cos^2 y}=x^3$$

$$\sec^2 y\frac{dy}{dx}+2x\tan y=x^3 \qquad \text{-------- (1)}$$

Put tan $y = z$, so that $\sec^2 y\frac{dy}{dx}=\frac{dz}{dx}$

Therefore equation (1) becomes,

$$\frac{dz}{dx}+2xz=x^3,$$

Which is Leibnitz's linear equation in z.

$$I.F.=e^{\int Pdx}=e^{\int 2xdx}=e^{x^2}$$

Therefore the solution is,

$$ze^{x^2}=\int e^{x^2}x^3dx+c=\frac{1}{2}\left(x^2-1\right)e^{x^2}+c$$

Replacing z by tan y, we get

$$\tan y=\frac{1}{2}\left(x^2-1\right)+c.e^{-x^2}$$

Which is the required form

2. This equation contains y^2 and $\tan^{-1}$ y and is, therefore not a linear in y, it can be written as,

$$\left(1+y^2\right)\frac{dx}{dy}=\tan^{-1}y-x$$

$$\Longrightarrow \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$$

Which is leibnitz's equation in x.

Therefore, we have intermediately form I.F. as,

$$I.F.=e^{\int Pdx}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

Thus the solution is,

$$x(I.F) = \int \frac{tan^{-1} y}{1+y^2}(I.F)dy + c$$

Or $x.e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2}.e^{\tan^{-1} y}.dy + c$ $\left[\begin{array}{l} Put\ t = tan^{-1} y \\ dy/1+y^2 = dt \end{array}\right.$

Therefore

$$x.e^t = \int t.e^t.dt + c$$

$$= t.e^t - \int 1.e^t.dt + c$$

$$= t.e^t - e^t + c$$

$$= \left(\tan^{-1} y - 1\right).e^{\tan^{-1} y} + c$$

Or $x = \tan^{-1} y - 1 + c.e^{-\tan^{-1} y}$

3. The given equation is, $\frac{dy}{dx} = \frac{(2x+3y)+4}{2(2x+3y)+5}$ -------- (1)

Putting $2x + 3y = t$, so that $2 + 3\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{1}{3}\left(\frac{dt}{dx} - 2\right) = \frac{t+4}{2t+5}$$

$\therefore$ (1) becomes, $\frac{dt}{dx} = 2 + \frac{3t+12}{2t+5} = \frac{7t+22}{2t+5}$

$$\frac{2t+5}{7t+22}dt = dx$$

Integrating both the sides, $\int \frac{2t+5}{7t+22}dt = \int dx + c$

Or $\int\left(\frac{2}{7} - \frac{9}{7}.\frac{1}{7t+22}\right)dt = x + c$

Or $\frac{2}{7}t - \frac{9}{49}log(7t+22) = x + c$

Putting $t = 2x + 3y$, we have

14(2x + 3y) – 9 log (14x + 21y + 22) = 49x + 49 c

Or $-21x + 42y - 9\log(14x + 21y + 22) = c'$

Which is the required solution

Terminal Questions

1. Refer to Section 16.8
2. Refer to Section 16.7