Unit - II. Numerical Integration.

*** Trapezoidal Rule formulae :-** (for even, odd).

It is applied for even or odd ~~intervals~~. number of total interval.

$$\int_{x_0}^{x_n} y \cdot dx = h\left(\frac{y_0 + y_n}{2} + y_1 + y_2 + \overset{\text{All other}}{\cdots\cdots} + y_{n-1}\right)$$

*** Simpson 1/3 Rule :-** (for even).

It is applicable for only even number of interval.

$$\int_{x_0}^{x_n} y \cdot dx = \frac{h}{3}\Big[(y_0 + y_n) + 2(y_2 + y_4 + y_6 + \overset{\text{even}}{\cdots\cdots} + y_{n-2}) + 4(y_1 + y_3 + y_5 + \overset{\text{odd}}{\cdots\cdots} + y_{n-1})\Big]$$

*** Simpson 3/8 Rule :-** (for 3 multiple).

It is applicable ~~for~~ if total no. of interval is multiple of 3.

$$\int_{x_0}^{x_n} y \cdot dx = \frac{3h}{8}\Big[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + \cdots\cdots + y_{n-1}) + 2(y_3 + y_6 + \cdots\cdots + y_{n-3})\Big]$$

$n = \frac{x_n - x_0}{h}$

and

$h = \frac{x_n - x_0}{n}$

h is difference betn x.

n is total interval

* Evalute $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule, simpson 1/3 Rule and simpson 3/8 rule. and also find find value of π in each case.

Sol:-

Now we will take 6 interval. so;

$$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}.$$

Now take $x_0 = 0$ so, and $x_1 = x_0 + h$, $x_2 = x_1 + h \ldots$

$x_0 = 0$, $y_0 = \frac{1}{1+x_0^2} = 1$

$x_1 = 1/6$, $y_1 = 1/1+x_1^2 = .36/37$

$x_2 = 2/6$, $y_2 = 1/1+x_2^2 = 0.9$

$x_3 = 3/6$, $y_3 = 1/1+x_3^2 = 0.8$

$x_4 = 4/6$, $y_4 = 1/1+x_4^2 = 9/13$

$x_5 = 5/6$, $y_5 = 1/1+x_5^2 = 36/61$

$x_6 = 1$, $y_6 = 1/1+x_6^2 = 0.5$

Now integration of equation,

$$\int_0^1 \frac{1}{1+x^2}dx = (\tan^{-1}x)_0^1 = (\tan^{-1}1 - \tan^{-1}0) = \pi/4 - 0 = \boxed{\frac{\pi}{4}} \quad (1)$$

(i) By Trapezoidal formula:-

$$\int_0^1 \frac{1}{1+x^2} \cdot dx = h\left[\frac{y_0+y_6}{2} + y_1 + y_2 + y_3 + y_4 + y_5\right]$$

$$= \frac{1}{6}\left[\frac{1+0.5}{2} + \frac{36}{37} + 0.9 + 0.8 + \frac{9}{13} + \frac{36}{61}\right]$$

$$\boxed{= 0.78423} \quad (2)$$

(ii) By simpson 1/3 rule:-

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{h}{3}\left[(y_0+y_6)+2(y_2+y_4)+4(y_1+y_3+y_5)\right]$$

$$=\frac{1}{6}\times\frac{1}{3}\left[(1+0.5)+2\left(0.9+\frac{9}{13}\right)+4\left(\frac{36}{37}+0.8+\frac{36}{61}\right)\right]$$

$$\boxed{=0.785396} \quad \text{———— (3)}$$

(iii) By simpson 3/8 rule:-

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$=\frac{3}{8}\times\frac{1}{6}\left[(1+0.5)+3\left(\frac{36}{37}+0.9+\frac{9}{13}+\frac{36}{61}\right)+2(0.8)\right]$$

$$\boxed{=0.785394} \quad \text{———— (4)}$$

Now comparing equation (1) and (2), for Trapezoidal.

$\pi/4 = 0.78423$

$\pi = 4(0.78423) \boxed{= 3.1369.}$

Now comparing equation (1) and (3), for simpson 1/3

$\pi/4 = 0.785396$

$\pi = 4 \times 0.785396 \boxed{= 3.14158}$

Now comparing equation (1) and (4), for simpson 3/8.

$\pi/4 = 0.785394$

$\pi = 4(0.785394) \boxed{= 3.14157}$

* Given that,

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$y = \log x$	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

then Evaluate $\int_4^{5.2} \log x . dx$. by Simpson 3/8 rule.

Sol: Here total 6 interval so $n = 6$,

$$h = \frac{b-a}{n} \text{ or } \frac{x_n - x_0}{n}$$

$$= \frac{5.2 - 4}{6}$$

$$h = 0.2$$

$$\int_4^{5.2} \log x . dx = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3\right]$$

$$= \frac{3(0.2)}{8}\left[(1.3863 + 1.6487) + 3(1.4351 + 1.4816 + 1.5686 + 1.6094) + 2(1.5\right.$$

$$= 1.82784$$