Unit-3
Finite Difference Operators.

***Finite Difference:-**

Consider $y = f(x)$.
Let, x_0, $x_1 = x_0 + h$, $x_2 = x_1 + h$ .... be a set of points at a common interval h.
Let the corresponding values of $y = f(x)$ be
$y_0 = f(x_0)$, $y_1 = f(x_1)$; $y_2 = f(x_2)$ .....

*** Forward difference:-**

$$\Delta f(x) = f(x+h) - f(x).$$

$$\Delta(af + bg)(x) = a\Delta f(x) + b\Delta g(x).$$

$$\Delta(f(x) \cdot g(x)) = f(x+h)\Delta g(x) + g(x)\Delta f(x).$$

$$\Delta\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x)\, g(x+h)}.$$

$$\Delta C = 0.$$

$$\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x).$$

*** Backward Difference:-**

$$\nabla f(x) = f(x) - f(x-h)$$

* Given, $\log 100 = 2$, $\log 101 = 2.0043$, $\log 103 = 2.0128$, $\log 104 = 2.0170$ then find $\log 102$.

Sol:- Let the ~~aa~~ missing value is p.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
100	2				
		0.0043			
101	2.0043		$p-2.0086$		
		$p-2.0043$		$6.0257-3p$	
102	p		$4.0171-2p$		$6p-$ ~~[illegible]~~ 12.0514
		$2.0128-p$		$3p-6.0257$	
103	2.0128		$p-2.0086$		
		0.0042			
104	2.0170				

since four values are given,
so $\Delta^4 f(x) = 0$.

$6p - 12.0514 = 0$.

$\boxed{p = 2.0086.}$

* ~~express~~ Estimate the missing term from following table:-

x	0	1	2	3	4
$f(x)$	4	3	4	a	12

Sol:- Since we are given four values, so third differences are constant and fourth differences are zero.
So, $\Delta^4 f(x) = 0$, for all $x \Rightarrow (E-I)^4 f(x) = 0$.

$$(E^4 - 4E^3 + 6E^2 - 4E + 1) f(x) = 0$$
$$E^4 f(x) - 4E^3 f(x) + 6E^2 f(x) + 4E f(x) + f(x) = 0$$

$$f(x+4) - 4f(x+3) + 6f(x+2) - 4f(x+1) + f(x) = 0$$
where the interval of difference is 1.
Now, substituting $x=0$,

$$f(4) - 4f(3) + 6f(2) - 4f(1) + f(0) = 0$$
$$12 + 4(a) + 6(4) - 4(3) + 4 = 0$$
$$a = 7$$

am

* Express the given function in factorial notation. $f(x) = x^4 + 5x^3 - 3x^2 + 7x + 5$ and also find 3rd degree forward difference.

Sol:

$f(x) = x^4 + 5x^3 - 3x^2 + 7x + 5$

coefficient of $f(x)$ are $(1, 5, -3, 7, 5)$

Now By Synthetic division method,

1	1	5	-3	7	5	$x^{(0)}$
		1	6	3		
2	1	6	3	10	$x^{(1)}$	
		2	16			
3	1	8	19	$x^{(2)}$		
		3				
	1	11	$x^{(3)}$			
	1	$x^{(4)}$				

pm

so, $f(x) = x^{(4)} + 11x^{(3)} + 19x^{(2)} + 10x^{(1)} + 5$.
This is required factorial notation.

$\Delta f(x) = 4x^{(3)} + 33x^{(2)} + 38x^{(1)} + 10.$

$\Delta^2 f(x) = 12x^{(2)} + 66x^{(1)} + 38$

$\boxed{\Delta^3 f(x) = 24x^{(1)} + 66}$ 3rd degree forward difference

Relationship Between Operators:

1. Relationship between E and Δ:

$$\Delta f(x) = f(x+h) - f(x)$$
$$\Delta f(x) = Ef(x) - f(x)$$
$$\Delta f(x) = (E-1)f(x)$$
$$\Delta = (\boldsymbol{E}-\mathbf{1})$$

2. Relationship between E and ∇:

$$\nabla f(x) = f(x) - f(x-h)$$
$$\nabla f(x) = (1 - E^{-1})f(x)$$
$$\nabla f(x) = (1 - E^{-1})f(x)$$
$$\boldsymbol{\nabla} = (\mathbf{1} - \boldsymbol{E}^{-\mathbf{1}})$$

3. Relationship between E & μ:

$$\mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$$
$$\mu f(x) = \frac{1}{2}\left[E^{1/2}f(x) + E^{-1/2}f(x)\right]$$
$$\mu f(x) = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]f(x)$$
$$\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$$

4. Relationship between E & $\boldsymbol{\delta}$:

$$\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$$
$$\delta f(x) = E^{1/2}f(x) - E^{-1/2}f(x)$$
$$\delta f(x) = (E^{1/2} - E^{-\frac{1}{2}})f(x)$$
$$\delta = (E^{1/2} - E^{-\frac{1}{2}})$$