

2014

MAY

06

Wk - 19 • 126-239

Tuesday

Unit-10

# Numerical Differentiation

M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

7 am \* Derivatives using Newton's formulae.

① Forward diff -

$$f(u+hu) = f(u) + h\Delta f(u) + \frac{h^2}{2!} \Delta^2 f(u) + \frac{h^3}{3!} \Delta^3 f(u) + \dots$$

$$+ \frac{h^3}{3!} \Delta^3 f(u) + \dots$$

$$\frac{dy}{dx} = \frac{1}{h} \left[ \Delta f(u) + \frac{(2u-1)}{2} \Delta^2 f(u) + \right.$$

$$\left. \frac{(3u^2-6u+2)}{6} \Delta^3 f(u) + \frac{(4u^3-18u^2+22u+6)}{24} \Delta^4 f(u) + \dots \right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2} \left[ \Delta^2 f(u) + (4-1) \Delta^3 f(u) + \right.$$

$$\left. \frac{(12u^2-36u+22)}{24} \Delta^4 f(u) + \dots \right]$$

Also formula can be use for find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for Newton's forward differentiation,

$$\left( \frac{dy}{dx} \right)_{x=x_0} = \frac{1}{h} \left[ \Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \frac{1}{5} \Delta^5 y_0 - \frac{1}{6} \Delta^6 y_0 + \dots \right]$$

$$\left( \frac{d^2y}{dx^2} \right)_{x=x_0} = \frac{1}{h^2} \left[ \Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 - \frac{5}{6} \Delta^5 y_0 + \frac{137}{180} \Delta^6 y_0 + \dots \right]$$



M	T	W	T	F	S	S	J
30	3	4	5	6	7	8	U
2	10	11	12	13	14	15	N
9	17	18	19	20	21	22	
16	24	25	26	27	28	29	2014

MAY

2014

Wk - 19 • 127-238

07

② Backward :-

$$f(a+hy) = f(a) + \nabla f(a) \left( \frac{y}{1!} \right) + \nabla^2 f(a) \left( \frac{y(y+1)}{2!} \right) + \nabla^3 f(a) \left( \frac{y(y+1)(y+2)}{3!} \right) + \dots$$

$$\frac{dy}{dx} = \frac{1}{h} \left[ \nabla f(a) + \frac{(2y+1)}{2} \nabla^2 f(a) + \frac{(3y^2+6y+2)}{6} \nabla^3 f(a) + \frac{(4y^3+18y^2+22y+6)}{24} \nabla^4 f(a) + \dots \right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2} \left[ \nabla^2 f(a) + (y+1) \nabla^3 f(a) + \frac{(12y^2+36y+22)}{24} \nabla^4 f(a) + \dots \right]$$

Also, to find  $(dy/dx)$  and  $(d^2y/dx^2)$  for Newton's backward difference formula :-

$$\left( \frac{dy}{dx} \right)_{x=x_n} = \frac{1}{h} \left[ \nabla y_n + \frac{1}{2} \nabla^2 y_n + \frac{1}{3} \nabla^3 y_n + \frac{1}{4} \nabla^4 y_n + \frac{1}{5} \nabla^5 y_n + \frac{1}{6} \nabla^6 y_n + \dots \right]$$

$$\left( \frac{d^2y}{dx^2} \right)_{x=x_n} = \frac{1}{h^2} \left[ \nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \frac{5}{6} \nabla^5 y_n + \frac{137}{180} \nabla^6 y_n + \dots \right]$$