SM5083
Assignment Number 02

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SM21MTECH12005

1. Chapter III miscellaneous example IV Q.1

1.1. show that the equation of line joining $(r_1, \theta_1), (r_2, \theta_2)$ is

$$\frac{1}{r}\sin(\theta_1 - \theta_2) = \frac{1}{r_1}\sin(\theta - \theta_2) + \frac{1}{r_2}\sin(\theta - \theta_1)$$

Solution: The python code is available at

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https://github.com/jaydeep−singh−chouhan/
    line−/blob/main/%20line.ipynb
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let

$$\begin{vmatrix} r\cos\theta & r_1\cos\theta_1 & r_2\cos\theta_2 \\ r\sin\theta & r_1\sin\theta_1 & r_2\sin\theta_2 \\ 1 & 1 & 1 \end{vmatrix} = 0 \tag{1.1.1}$$

$$\begin{aligned} r_1 r_2(\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2) \\ -rr_2(\cos\theta\sin\theta_2 - \sin\theta\cos\theta_2) \\ +rr_1(\cos\theta\sin\theta_1 - \sin\theta\cos\theta_1 = 0 \end{aligned} \tag{1.1.2}$$

$$\begin{aligned} -r_1 r_2\sin(\theta_1 - \theta_2) + rr_2\sin(\theta - \theta_2) \\ -rr_1\sin(\theta - \theta_1) = 0 \end{aligned} \tag{1.1.3}$$

now rearranging the equation 1.1.3 we get

$$\frac{1}{r}\sin(\theta_1 - \theta_2) = \frac{1}{r_1}\sin(\theta - \theta_2) + \frac{1}{r_2}\sin(\theta - \theta_1) \tag{1.1.4}$$

let us assume any r_1, r_2 , θ_1 , θ_2

P and Q are two end points of line

$$r_1 = 3, \theta_1 = \frac{\pi}{3} \tag{1.1.5}$$

$$r_2 = 4, \theta_2 = \frac{\pi}{6} \tag{1.1.6}$$

$$\mathbf{P} = \begin{pmatrix} r_1\cos\theta_1 \\ r_1\sin\theta_1 \end{pmatrix} = \begin{pmatrix} 1.5 \\ 2.59 \end{pmatrix} \tag{1.1.7}$$

$$\mathbf{Q} = \begin{pmatrix} r_2\cos\theta_2 \\ r_2\sin\theta_2 \end{pmatrix} = \begin{pmatrix} 3.46 \\ 2 \end{pmatrix} \tag{1.1.8}$$

vector equation of line PQ

$$\boxed{\mathbf{r} = t(\mathbf{Q} - \mathbf{P}) + \mathbf{P}} \tag{1.1.9}$$

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 3.46 \\ 2 \end{pmatrix} - \begin{pmatrix} 1.5 \\ 2.59 \end{pmatrix} = \begin{pmatrix} 1.96 \\ -0.59 \end{pmatrix} \tag{1.1.10}$$

$$\mathbf{r} = t\begin{pmatrix} 1.96 \\ -0.59 \end{pmatrix} + \begin{pmatrix} 1.5 \\ 2.59 \end{pmatrix} \tag{1.1.11}$$

Take any point of the line
let it be A

$$\mathbf{A} = \begin{pmatrix} 3.464 \\ 2 \end{pmatrix} \tag{1.1.12}$$

$$r = \|\mathbf{A}\| = 3.999 \tag{1.1.13}$$

$$\theta = \arctan\frac{2}{3.464} = 0.5236 \tag{1.1.14}$$

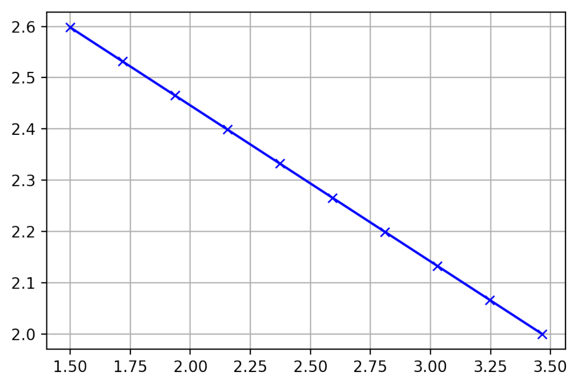

Fig. 1.1. line generated