

SM5083

Assignment Number 02

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SM21MTECH12005

1. CHAPTER III MISCELLANEOUS EXAMPLE IV Q.1

P and Q are two end points of line

$$r_1 = 3, \theta_1 = \frac{\pi}{3} \quad (1.1.5)$$

$$r_2 = 4, \theta_2 = \frac{\pi}{6} \quad (1.1.6)$$

$$P = (r_1 \cos \theta_1, r_1 \sin \theta_1) = (1.5, 2.59) \quad (1.1.7)$$

$$Q = (r_2 \cos \theta_2, r_2 \sin \theta_2) = (3.46, 2) \quad (1.1.8)$$

1.1. show that the equation of line joining $(r_1, \theta_1), (r_2, \theta_2)$ is

$$\frac{1}{r} \sin(\theta_1 - \theta_2) = \frac{1}{r_1} \sin(\theta - \theta_2) + \frac{1}{r_2} \sin(\theta - \theta_1)$$

Solution: The python code is available at

<https://github.com/jaydeep-singh-chouhan/line/blob/main/%20line.ipynb>

vector equation of line PQ

$$\mathbf{m} = \begin{pmatrix} 3.46 \\ 2 \end{pmatrix} - \begin{pmatrix} 1.5 \\ 2.59 \end{pmatrix} = \begin{pmatrix} 1.96 \\ -0.59 \end{pmatrix} \quad (1.1.9)$$

$$\mathbf{r} = t \begin{pmatrix} 1.96 \\ -0.59 \end{pmatrix} + \begin{pmatrix} 1.5 \\ 2.59 \end{pmatrix} \quad (1.1.10)$$

Take any point of the line
let it be A

let

$$\begin{vmatrix} r \cos \theta & r_1 \cos \theta_1 & r_2 \cos \theta_2 \\ r \sin \theta & r_1 \sin \theta_1 & r_2 \sin \theta_2 \\ 1 & 1 & 1 \end{vmatrix} = 0 \quad (1.1.1)$$

$$\begin{aligned} & r_1 r_2 (\cos \theta_1 \sin \theta_2 - \sin \theta_1 \cos \theta_2) \\ & - r r_2 (\cos \theta \sin \theta_2 - \sin \theta \cos \theta_2) \\ & + r r_1 (\cos \theta \sin \theta_1 - \sin \theta \cos \theta_1) = 0 \end{aligned} \quad (1.1.2)$$

$$\begin{aligned} & -r_1 r_2 \sin(\theta_1 - \theta_2) + r r_2 \sin(\theta - \theta_2) \\ & - r r_1 \sin(\theta - \theta_1) = 0 \end{aligned} \quad (1.1.3)$$

now rearranging the equation 1.1.3 we get

$$\frac{1}{r} \sin(\theta_1 - \theta_2) = \frac{1}{r_1} \sin(\theta - \theta_2) + \frac{1}{r_2} \sin(\theta - \theta_1) \quad (1.1.4)$$

let us assume any $r_1, r_2, \theta_1, \theta_2$

$$A = (3.464, 2) \quad (1.1.11)$$

$$\mathbf{A} = \begin{pmatrix} 3.464 \\ 2 \end{pmatrix} \quad (1.1.12)$$

$$r = \|\mathbf{A}\| = 3.999 \quad (1.1.13)$$

$$\theta = \arctan \frac{2}{3.464} = 0.5236 \quad (1.1.14)$$

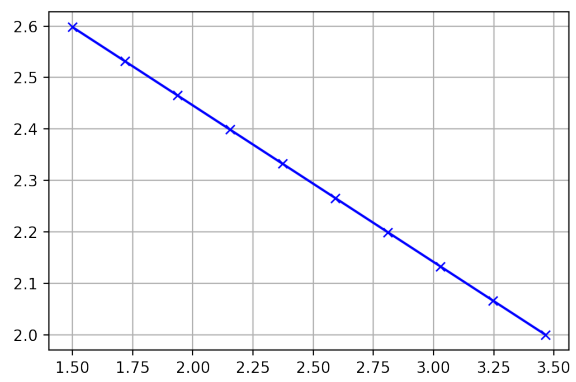


Fig. 1.1. line generated