1

SM5083 Assignment Number 02

Jaydeep singh chouhan SM21MTECH12005

1. Chapter III miscellaneous example IV Q.1

1.1. show that the equation of line joining $(r_1, \theta_1), (r_2, \theta_2)$ is

$$\frac{1}{r}\sin(\theta_1 - \theta_2) = \frac{1}{r_1}\sin(\theta - \theta_2) + \frac{1}{r_2}\sin(\theta - \theta_1)$$

Solution: The python code is available at

https://github.com/jaydeep-singh-chouhan/line-/blob/main/%20line.ipynb

let

$$\begin{vmatrix} r\cos\theta & r_1\cos\theta_1 & r_2\cos\theta_2 \\ r\sin\theta & r_1\sin\theta_1 & r_2\sin\theta_2 \\ 1 & 1 & 1 \end{vmatrix} = 0 \quad (1.1.1)$$

$$r_1 r_2(\cos \theta_1 \sin \theta_2 - \sin \theta_1 \cos \theta_2)$$
$$-r r_2(\cos \theta \sin \theta_2 - \sin \theta \cos \theta_2)$$
$$+r r_1(\cos \theta \sin \theta_1 - \sin \theta \cos \theta_1 = 0$$
(1.1.2)

$$-r_1 r_2 \sin(\theta_1 - \theta_2) + r r_2 \sin(\theta - \theta_2)$$
$$-r r_1 \sin(\theta - \theta_1) = 0$$
(1.1.3)

now rearranging the equation 1.1.3 we get

$$\frac{1}{r}\sin(\theta_1 - \theta_2) = \frac{1}{r_1}\sin(\theta - \theta_2) + \frac{1}{r_2}\sin(\theta - \theta_1)$$
(1.1.4)

let us assume any r_1, r_2 , θ_1 , θ_2

P and Q are two end points of line

$$r_1 = 3, \theta_1 = \frac{\pi}{3} \tag{1.1.5}$$

$$r_2 = 4, \theta_2 = \frac{\pi}{6}$$
 (1.1.6)

$$\mathbf{P} = \begin{pmatrix} r_1 \cos \theta_1 \\ r_1 \sin \theta_1 \end{pmatrix} = \begin{pmatrix} 1.5 \\ 2.59 \end{pmatrix} \tag{1.1.7}$$

$$\mathbf{Q} = \begin{pmatrix} r_2 \cos \theta_2 \\ r_2 \sin \theta_2 \end{pmatrix} = \begin{pmatrix} 3.46 \\ 2 \end{pmatrix} \tag{1.1.8}$$

vector equation of line PQ

$$\mathbf{m} = \begin{pmatrix} 3.46 \\ 2 \end{pmatrix} - \begin{pmatrix} 1.5 \\ 2.59 \end{pmatrix} = \begin{pmatrix} 1.96 \\ -0.59 \end{pmatrix} \tag{1.1.9}$$

$$\mathbf{r} = t \begin{pmatrix} 1.96 \\ -0.59 \end{pmatrix} + \begin{pmatrix} 1.5 \\ 2.59 \end{pmatrix} \tag{1.1.10}$$

Take any point of the line let it be A

$$\mathbf{A} = \begin{pmatrix} 3.464 \\ 2 \end{pmatrix} \tag{1.1.11}$$

$$r = ||\mathbf{A}|| = 3.999 \tag{1.1.12}$$

$$\theta = \arctan \frac{2}{3.464} = 0.5236 \tag{1.1.13}$$

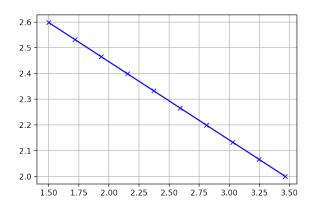


Fig. 1.1. line generated