

pausa double code :

$\Phi \in V$

## ME639 - Intro to Robotics

## Assignment 6 and 7

problem: ① A straight line movement from

A (0.4, 0.06, 0.1) to B (0.40, 0.01, 0.1)

⇒ derive desired end point trajectory,  
position, velocity and acceleration with  
time.

Answer: starting point,

$$A \equiv (x_0, y_0, z_0) = (0.4, 0.06, 0.1)$$

end point B  $\Rightarrow (x_f, y_f, z_f) = (0.4, 0.01, 0.1)$ .

⇒ y co-ordinate of end point is changing, we  
can consider that during the motion x and  
z coordinates are constant.

⇒ so, we will define trajectory for y coordinate  
only.

⇒ Using cubic polynomial for trajectory,  
we can find trajectory as,

$$y(t) = y_0 + 3(y_f - y_0)\frac{t^2}{T^2} + (-2)(y_f - y_0)\frac{t^3}{T^3}$$

T is total time to reach  
from A to B.

So, for our trajectory,

$$y(t) = 0.06 + \frac{(-0.15)t^2}{T^2} + \frac{(0.1)t^3}{T^3} \quad m$$

$$\dot{y}(t) = \frac{-0.3t}{T^2} + \frac{0.3t^2}{T^3} \quad m/sec$$

$$\ddot{y}(t) = \frac{-0.3}{T^2} + \frac{(0.6)t}{T^3} \quad m/sec^2$$

⇒ Plots and python files are submitted with PDF

problem: (2): Selected manipulator: SCARA.

⇒ Python file and desired trajectory in joint space are attached with submission.

⇒  $\theta_1, \theta_2$  and  $d$  are calculated using inverse kinematics.

⇒  $\dot{\theta}_1, \dot{\theta}_2$  and  $\dot{d}$  are calculated using following equations:

⇒ For SCARA Robot,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -l_2 \sin(\theta_1 + \theta_2) - l_1 \sin \theta_1, & -l_2 \cos(\theta_1 + \theta_2), & 0 \\ +l_2 \cos(\theta_1 + \theta_2) + l_1 \cos \theta_1, & l_2 \sin(\theta_1 + \theta_2), & 0 \\ 0, & 0, & 1 \end{bmatrix} \times \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d} \end{bmatrix}$$

so,

$$J_{3 \times 3} = \begin{bmatrix} -l_1 \sin \theta_1 - l_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) & 0 \\ l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

now, for inverse kinematics,

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d} \end{bmatrix} = J^T \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

$$\text{So, } J^T = \begin{bmatrix} \frac{\cos(\theta_1 + \theta_2)}{l_1 \sin \theta_2} & \frac{\sin(\theta_1 + \theta_2)}{l_1 \sin \theta_2} & 0 \\ -\frac{(l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2))}{l_1 l_2 \sin \theta_2} & \frac{-l_2 \sin(\theta_1 + \theta_2) - l_1 \sin \theta_1}{l_1 l_2 \sin \theta_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for our case,  $\dot{x} = 0$  and  $\dot{z} = 0$ ,

so,

$$\dot{\theta}_1 = \frac{\sin(\theta_1 + \theta_2)}{l_1 \sin \theta_2} \dot{y}$$

$$\dot{\theta}_2 = \frac{-(l_1 \sin(\theta_1 + \theta_2) + l_1 \sin \theta_1)}{l_1 l_2 \sin \theta_2} \dot{y}$$



→ Now, acceleration,

$$\ddot{\theta}_1 = \frac{d}{dt} \left( \frac{\sin(\theta_1 + \theta_2)}{l_1 \sin \theta_2} \dot{\gamma} \right)$$

$$= \ddot{\gamma} \frac{\sin(\theta_1 + \theta_2)}{l_1 \sin \theta_2} + \frac{d}{dt} \left( \frac{\sin(\theta_1 + \theta_2)}{l_1 \sin \theta_2} \right) \dot{\gamma}$$

$$\ddot{\theta}_1 = \ddot{\gamma} \frac{\sin(\theta_1 + \theta_2)}{l_1 \sin \theta_2} + \left( \frac{s_2 c_{12}(\theta_1 + \theta_2) - s_{12} c_2 \dot{\theta}_2}{l_1 \sin^2 \theta_2} \right) \dot{\gamma}$$

$$\ddot{\theta}_2 = -\ddot{\theta}_1 + \frac{d}{dt} \left( \frac{l_1 \sin \theta_1}{l_1 l_2 \sin \theta_2} \dot{\gamma} \right)$$

$$= -\ddot{\theta}_1 + \frac{d}{dt} \left( \frac{\sin \theta_1}{l_2 \sin \theta_2} \right) \dot{\gamma}$$

$$+ \ddot{\gamma} \frac{\cos \theta_1}{l_2 \sin \theta_2}$$

$$= -\ddot{\theta}_1 - \frac{\sin \theta_1}{l_2 \sin \theta_2} \ddot{\gamma} - \left( \frac{\cos \theta_1 \sin \theta_2}{l_2 \sin^2 \theta_2} - \frac{\sin \theta_1 \cos \theta_2}{l_2 \sin^2 \theta_2} \right) \dot{\gamma}$$

$$\ddot{\theta}_2 = -\ddot{\theta}_1 - \frac{\sin \theta_1}{l_2 \sin \theta_2} \ddot{\gamma} + \left( \frac{\sin(\theta_1 - \theta_2)}{l_2 \sin^2 \theta_2} \right) \dot{\gamma}$$

problem ③ Implement four controllers

for above desired trajectory.

(a) PFD control that might yield a small steady state error with damping ratio.

## Dynamics.

$$M = \begin{bmatrix} \alpha + \beta + \alpha r \cos \theta_2 & \beta + \alpha r \cos \theta_2 & 0 \\ \beta + r \cos \theta_2 & \beta & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

$$C = \begin{bmatrix} -r \sin \theta_2 \ddot{\theta}_2 & -r \sin \theta_2 (\dot{\theta}_1 + \dot{\theta}_2) & 0 \\ r \sin \theta_2 \ddot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 \\ 0 \\ m_3 g \end{bmatrix}, \quad x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ d \end{bmatrix}, \quad F = \begin{bmatrix} \tau_1 \\ \tau_2 \\ F_3 \end{bmatrix}$$

$$\ddot{x}M + C\dot{x} + Q = F$$

$$J = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$$

⇒ considering motor dynamics,

$$\ddot{x}(M+J) + (C+B)\dot{x} + Q = F$$

$$B = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix}$$

$\Rightarrow$  (control law),

$$\tau_1 = 2\Delta\dot{\theta}_1 + 1 \cdot \Delta\ddot{\theta}_1$$

$$\tau_2 = 2\Delta\dot{\theta}_2 + 1 \cdot \Delta\ddot{\theta}_2$$

$$\tau_3 = 10\Delta\dot{d} + 0.001\Delta\ddot{d}$$

for  $\tau_3$ , we can add  $m_3 g$  to compensate for gravity.

(b) PD control with feedforward control,

$$\tau_1 = 2\Delta\dot{\theta}_1 + \Delta\ddot{\theta}_1 + (\alpha + \beta + 2\tau\cos\theta_{2d} + J_1)\ddot{\theta}_1$$

$$\tau_2 = 2\Delta\dot{\theta}_2 + \Delta\ddot{\theta}_2 + (\beta + J_2)\ddot{\theta}_2$$

$$\tau_3 = 10\Delta\dot{d} + 0.001\Delta\ddot{d} + m_3 \cdot \ddot{d}$$

$$\approx 10\Delta\dot{d} + 0.001\Delta\ddot{d}$$

(c) PD along with feedforward disturbance cancellation,

$$\begin{aligned} \tau_1 = & 2\Delta\dot{\theta}_1 + \Delta\ddot{\theta}_1 + (\alpha + \beta + \tau\cos\theta_{2d} + J_1)\ddot{\theta}_1 + \\ & (\beta + \tau\cos\theta_{2d})\ddot{\theta}_2 - \tau\sin\theta_{2d}\ddot{\theta}_2\ddot{\theta}_1 + \\ & - \tau\sin\theta_{2d}(\dot{\theta}_1 + \dot{\theta}_2) \cdot \dot{\theta}_2 \end{aligned}$$

$$\begin{aligned} \tau_2 = & \cancel{2\Delta\dot{\theta}_2} + 2\Delta\dot{\theta}_2 + \Delta\ddot{\theta}_2 + (\beta + J_2)\ddot{\theta}_2 \\ & (\beta + \tau\cos\theta_{2d})\ddot{\theta}_1 + \tau\sin\theta_{2d}\dot{\theta}_1\dot{\theta}_2 \end{aligned}$$

$$\tau_3 = 10\Delta\dot{d} + 0.001\Delta\ddot{d} + m_3 g$$

(4) Multivariable control:

$$U = \begin{bmatrix} K_{p1} & 0 & 0 \\ 0 & K_{p2} & 0 \\ 0 & 0 & K_{p3} \end{bmatrix} \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \Delta d \end{bmatrix} + \begin{bmatrix} K_{d1} & 0 & 0 \\ 0 & K_{d2} & 0 \\ 0 & 0 & K_{d3} \end{bmatrix} \begin{bmatrix} \Delta \dot{\theta}_1 \\ \Delta \dot{\theta}_2 \\ \Delta \dot{d} \end{bmatrix} \\ + M \begin{bmatrix} \ddot{\theta}_1^d \\ \ddot{\theta}_2^d \\ \ddot{d}^d \end{bmatrix} + C \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d} \end{bmatrix} + G$$

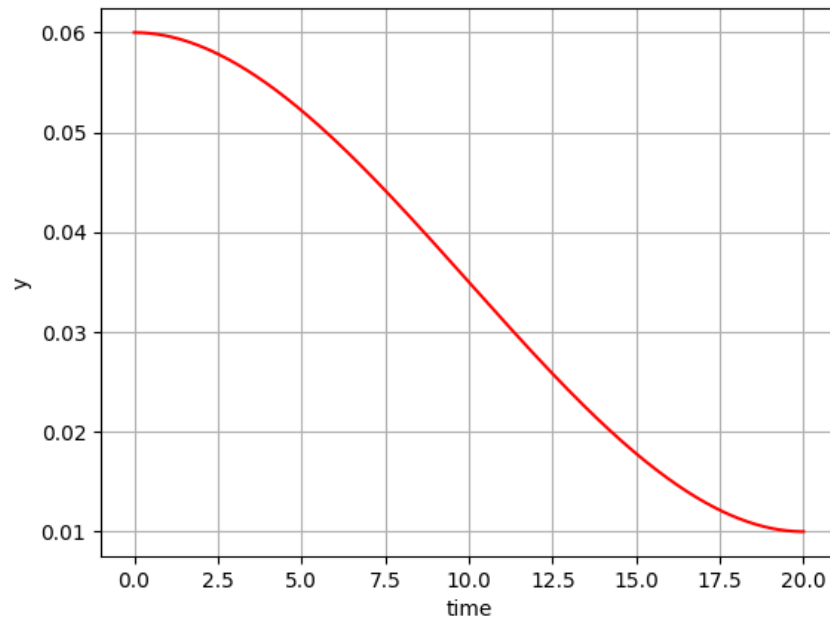
⇒ codes and plots are attached with submission.

⇒ problems (4)(5)(6): codes and plots are attached.

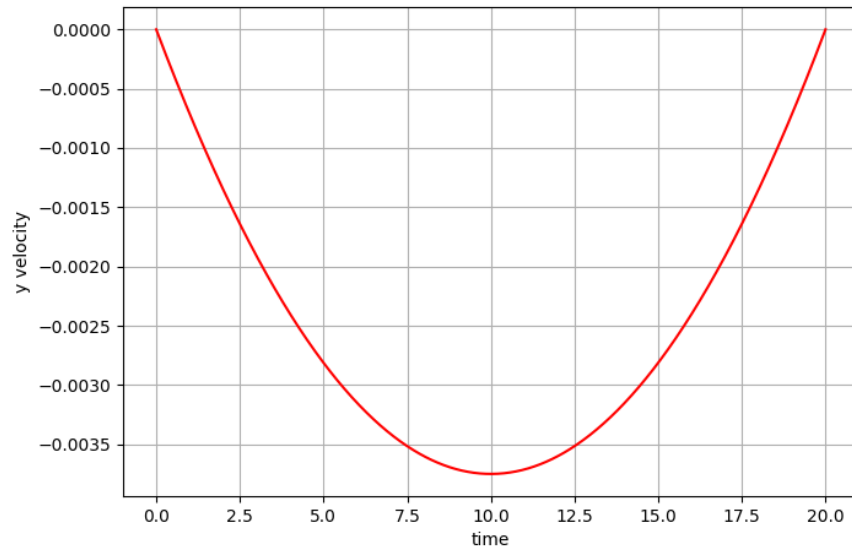


## Problem 1

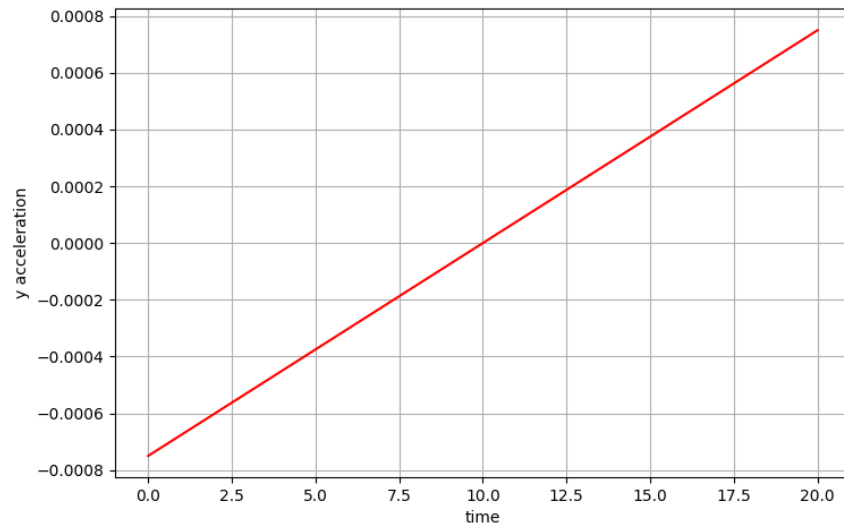
Problem1: End Effector Y axis motion from 0.06 to 0.01



Problem1: End Effector Y axis motion from 0.06 to 0.01, velocity

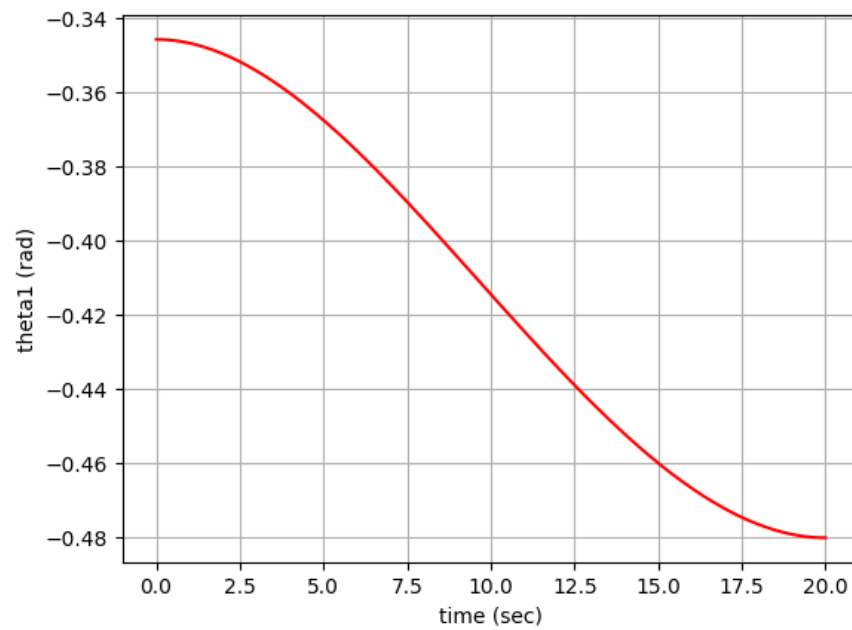


Problem1: End Effector Y axis motion from 0.06 to 0.01, acceleration

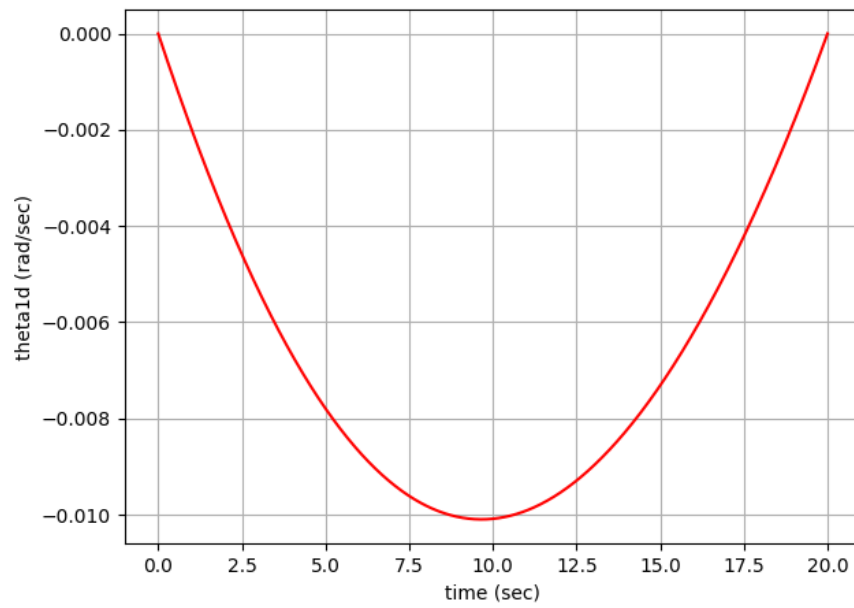


Problem 2

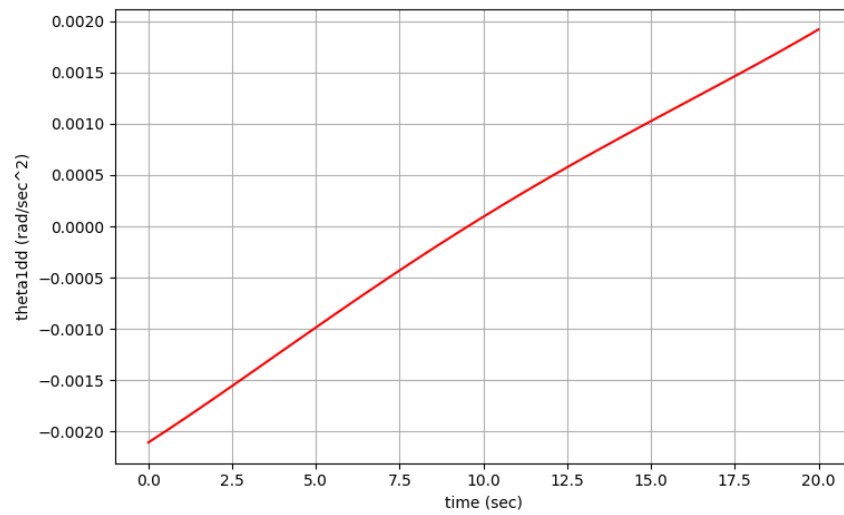
Problem2: Theta1 motion for A to B motion of End Effector



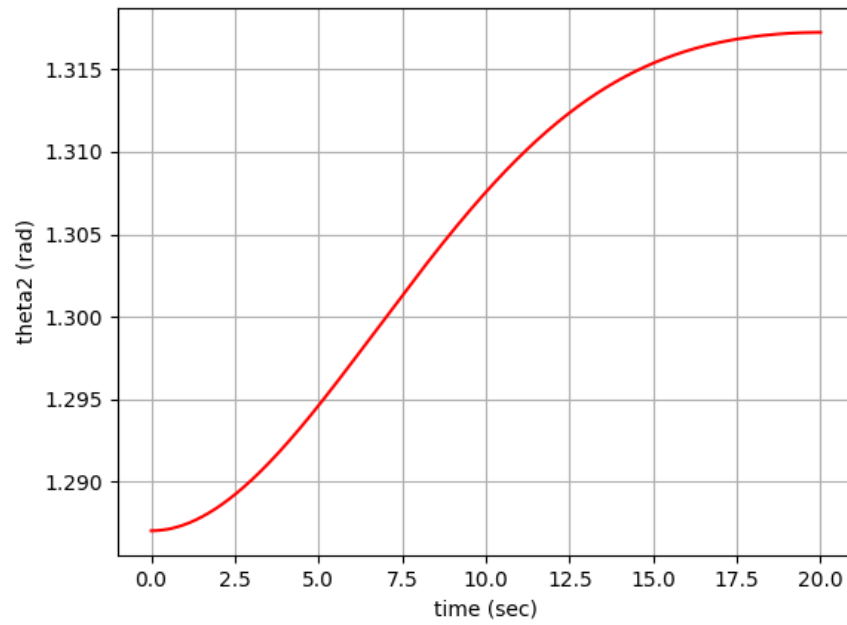
Problem2: theta1 velocity motion for A to B motion of End Effector



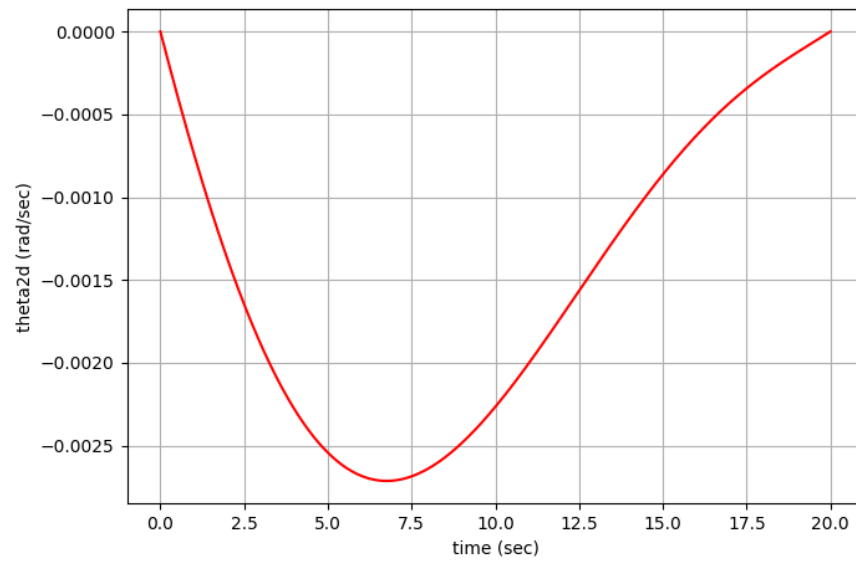
Problem2: theta1 acceleration motion for A to B motion of End Effector



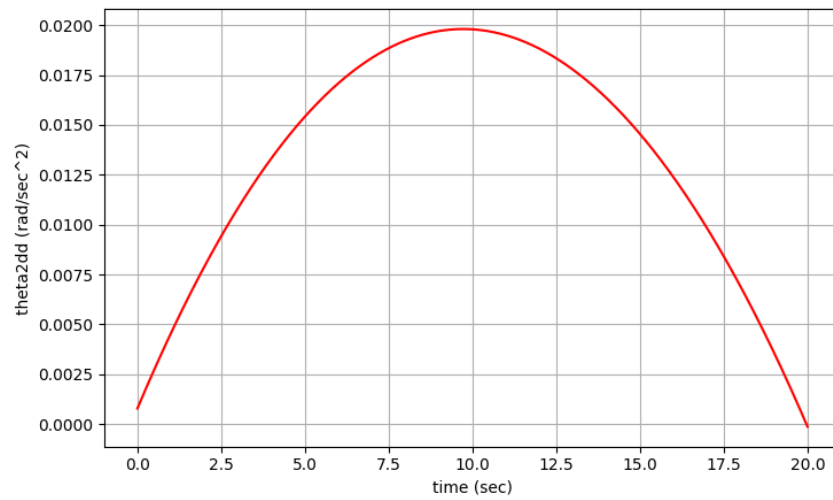
Problem2: Theta2 motion for A to B motion of End Effector



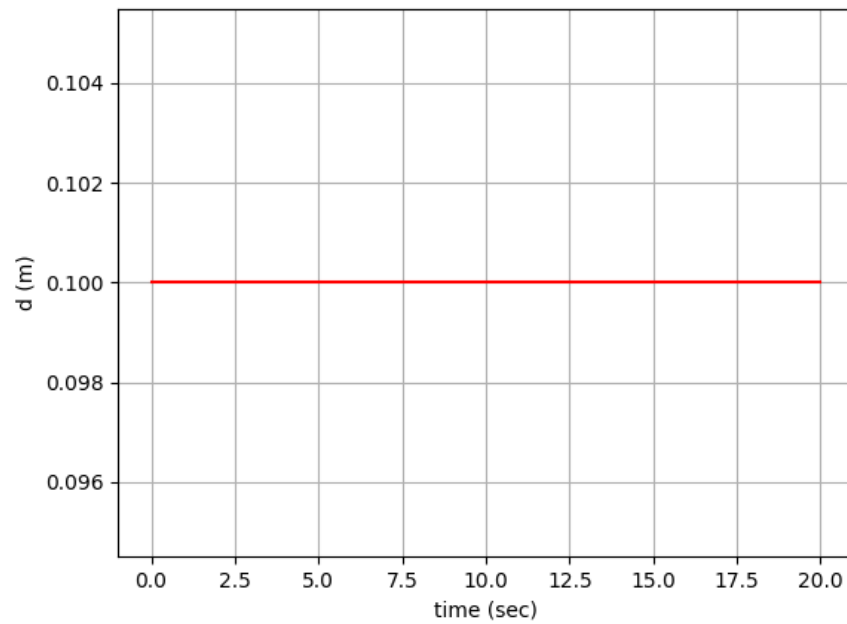
Problem2: theta2 velocity motion for A to B motion of End Effector



Problem2: theta2 acceleration motion for A to B motion of End Effector



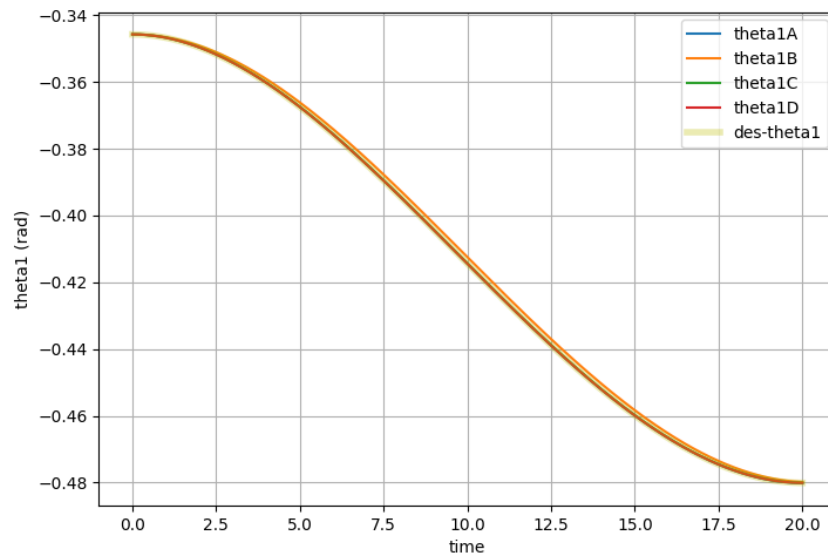
Problem2: d (Linear) motion for A to B motion of End Effector



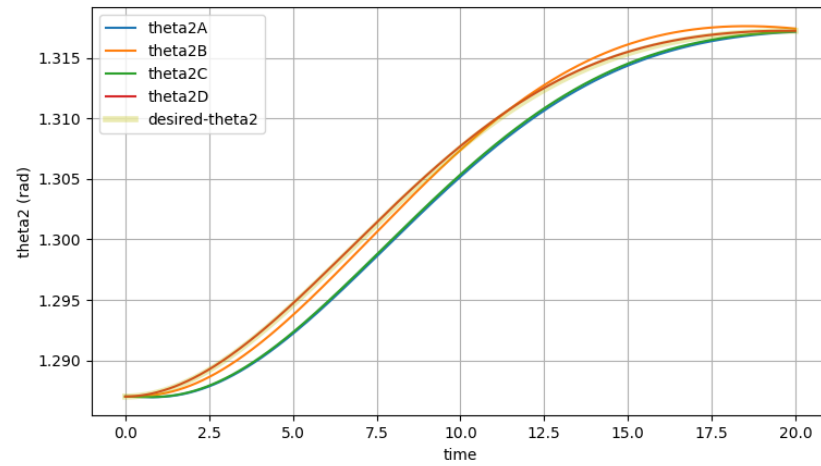


### Problem 3

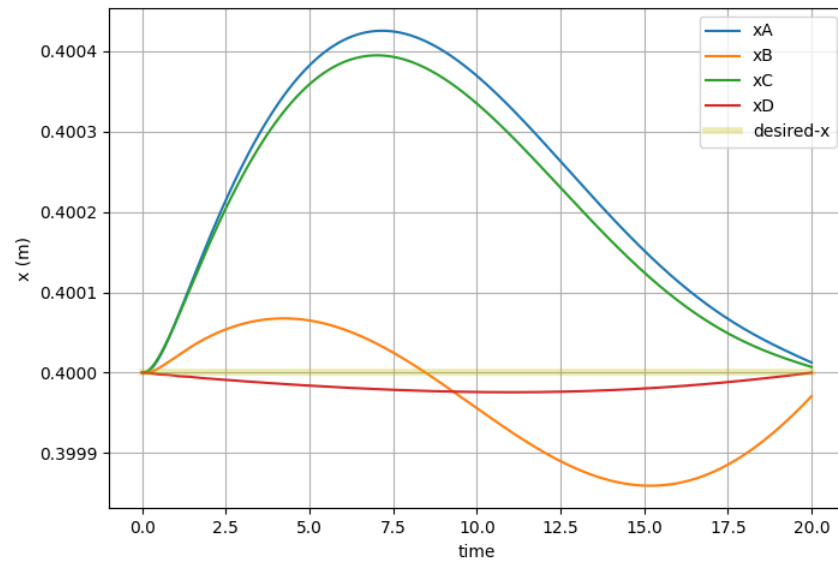
Problem3: End Effector motion from A SCARA Robot using VARIOUS Controller



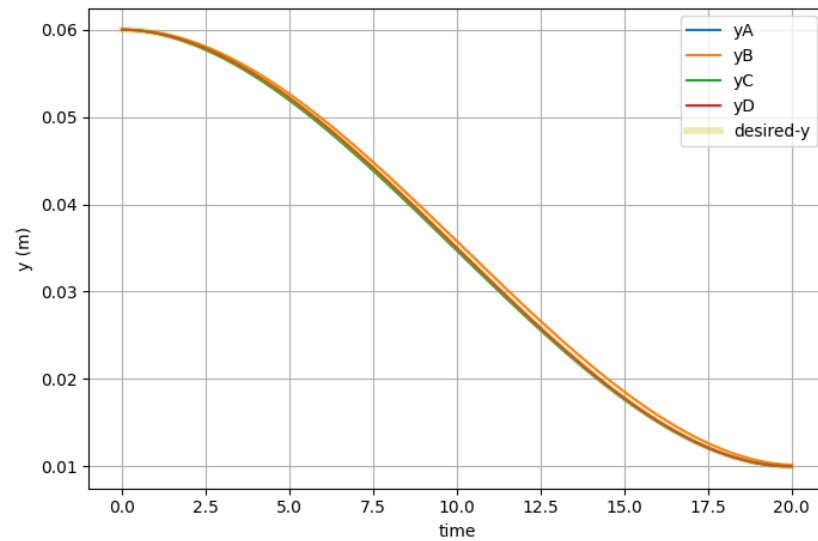
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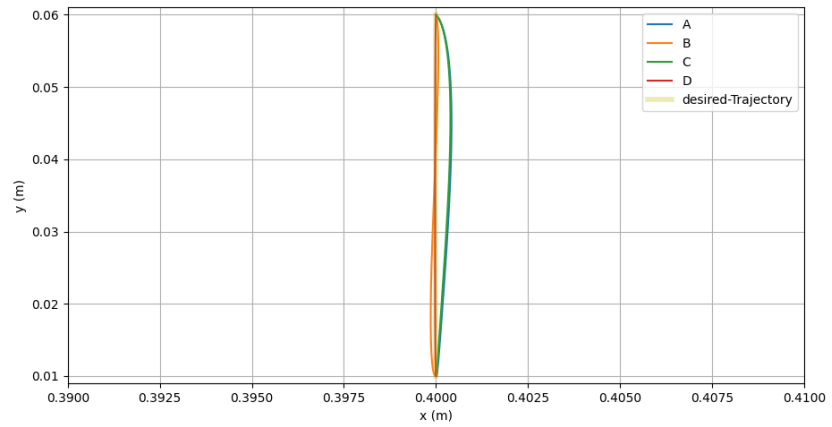
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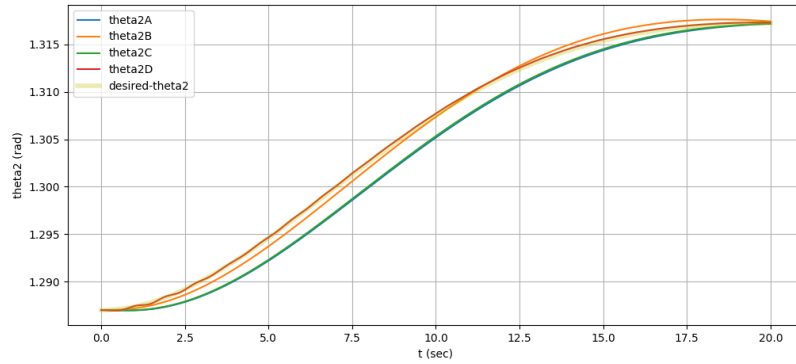


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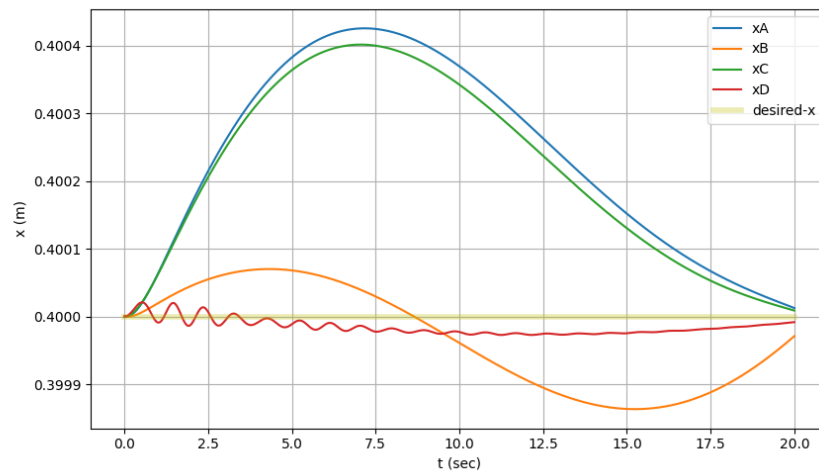


## Problem 4

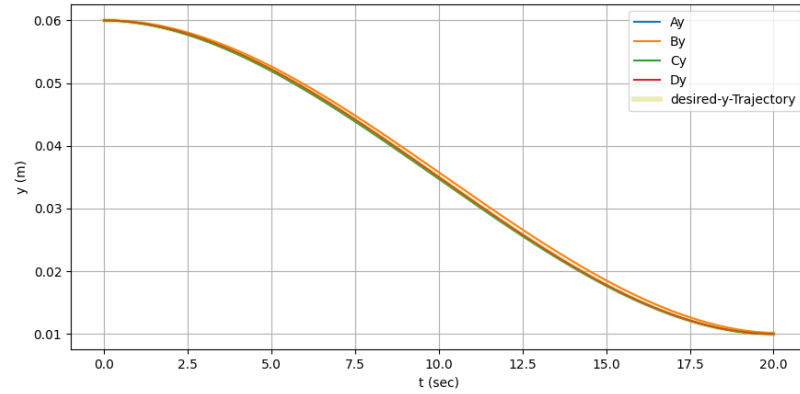
Problem4: End Effector motion from A SCARA Robot using VARIOUS Controller, with 20% error in link 1 and 2



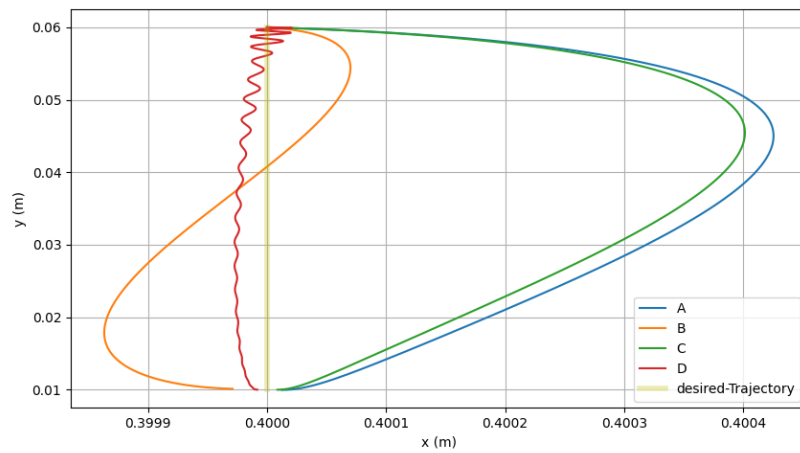
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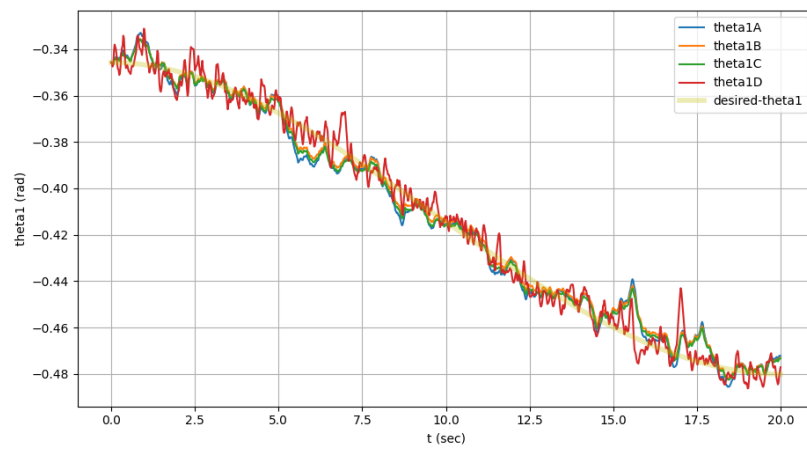


Problem4: End Effector motion from A SCARA Robot using VARIOUS Controller, with 20% error in link 1 and 2

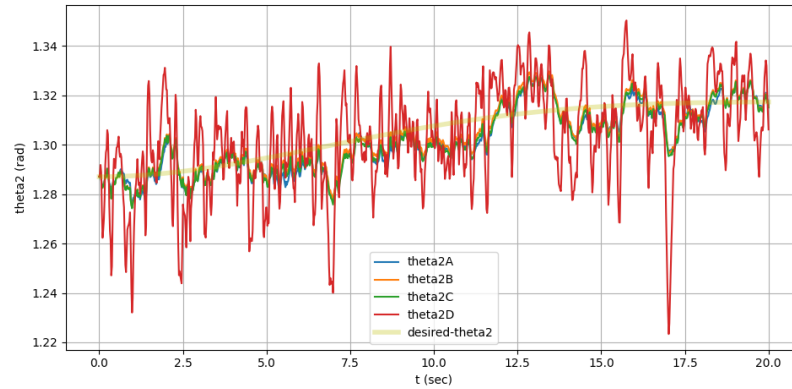


## Problem 5

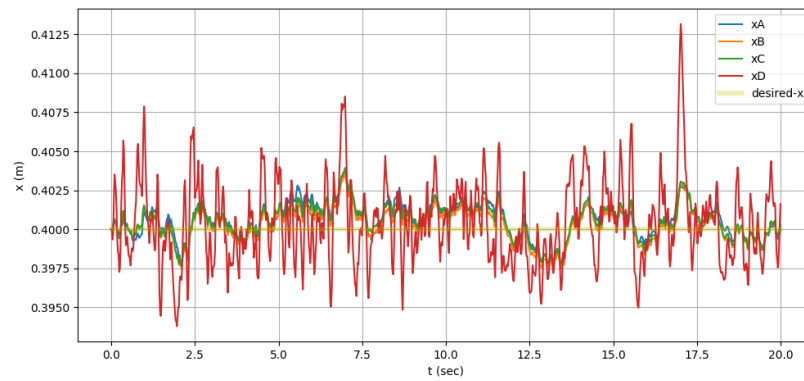
Problem5: End Effector motion from A SCARA Robot using VARIOUS Controller, with disturbances, mean = 0 and SD = 0.1



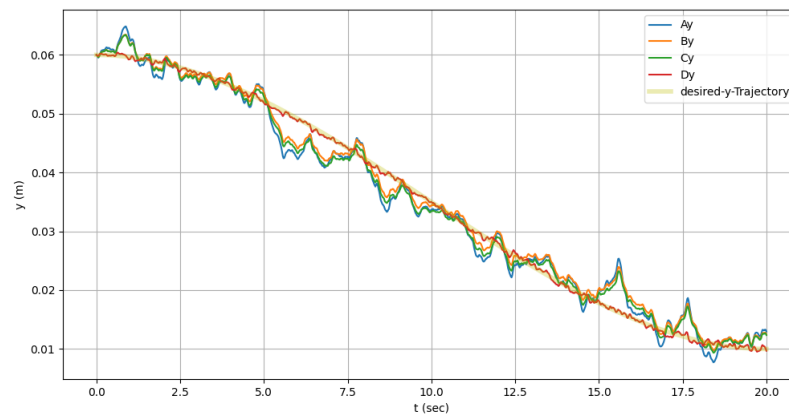
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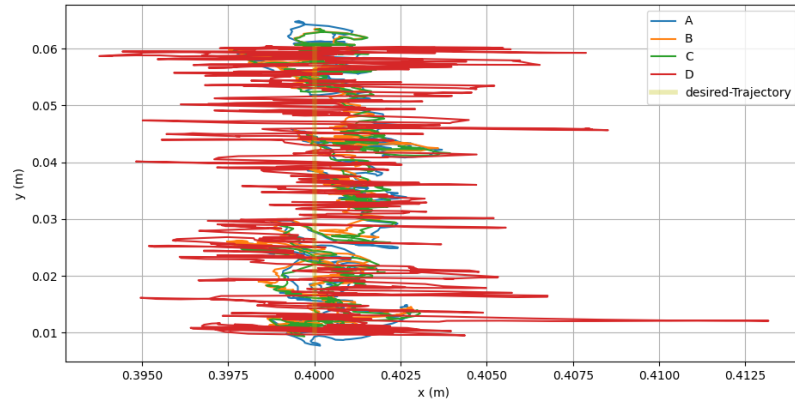


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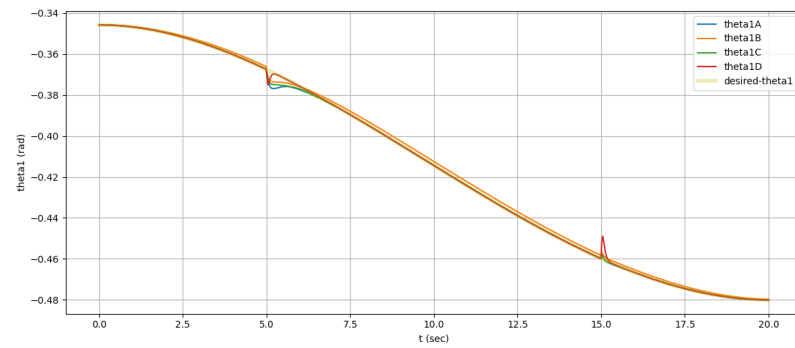


Problem5: End Effector motion from A SCARA Robot using VARIOUS Controller, with disturbances, mean = 0 and SD = 0.1

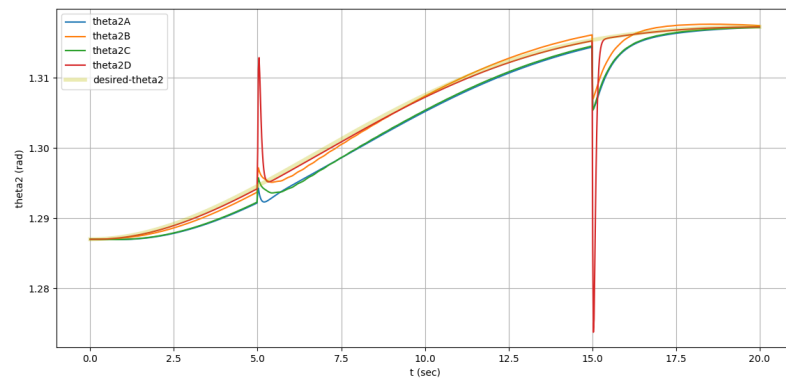


Problem 6

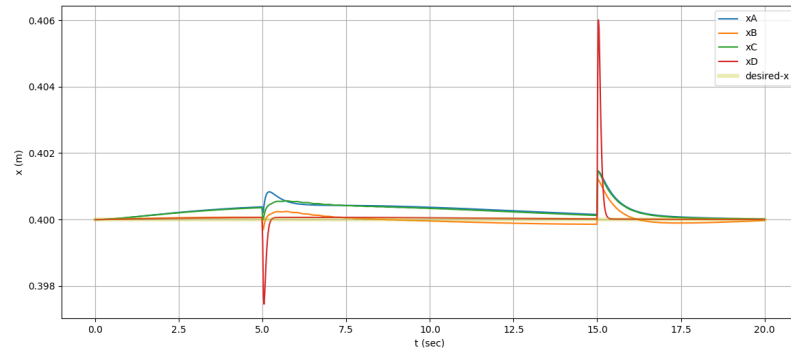
Problem6: End Effector motion from A SCARA Robot using VARIOUS Controller, with -1 Nm impulse at Joint 1 @5 sec and -1 Nm impulse at Joint 2 @15 sec



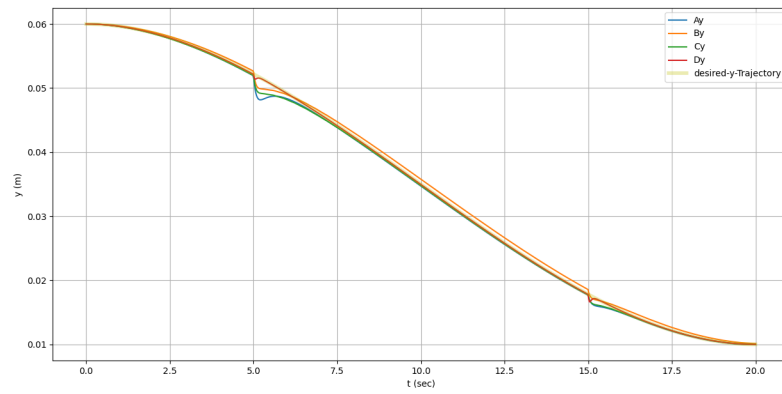
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