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ME 639 - Intro to Robotics

Assignment 6 and 7

problem: ① A straight line movement from

A (0.4, 0.06, 0.1) to B(0.40, 0.01, 0.1)

⇒ Derive desired end point trajectory, position, velocity and acceleration w.r.t time.

Answer: starting point,

$$A \Rightarrow (x_0, y_0, z_0) = (0.4, 0.06, 0.1)$$

end point $B \Rightarrow (x_f, y_f, z_f) = (0.4, 0.01, 0.1)$.

⇒ y co-ordinate of end point is changing, we can consider that during the motion x and z coordinates are constant.

⇒ So, we will define trajectory for y coordinate only.

⇒ Using cubic polynomial for trajectory, we can find trajectory as,

$$y(t) = y_0 + 3(y_f - y_0)\frac{t^2}{T^2} + (-2)(y_f - y_0)\frac{t^3}{T^3}$$

T is total time to reach from A to B.

so, for our trajectory,

$$y(t) = 0.06 + \frac{(-0.15)t^2}{T^2} + \frac{(0.1)t^3}{T^3} \quad m$$

$$\dot{y}(t) = \frac{-0.3t}{T^2} + \frac{0.3t^2}{T^3} \quad m/sec$$

$$\ddot{y}(t) = \frac{-0.3}{T^2} + \frac{(0.6)t}{T^3} \quad m/sec^2$$

⇒ Plots and python files are submitted with PDF

problem: (2) : Selected manipulator: SCARA.

⇒ Phython file and desired trajectory in joint space are attached with submission

⇒ θ_1, θ_2 and d are calculated using inverse kinematics.

⇒ $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{d}$, are calculated using following equations:

⇒ For SCARA Robot,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -l_2\sin(\theta_1+\theta_2) - l_1\sin\theta_1, & -l_2\sin(\theta_1+\theta_2), & 0 \\ +l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1, & l_2\cos(\theta_1+\theta_2), & 0 \\ 0, & 0, & 1 \end{bmatrix} \times \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d} \end{bmatrix}$$

so,

$$J_{3\times3} = \begin{bmatrix} -l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2), & -l_2\sin\theta_1+\theta_2, & 0 \\ +l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2), & l_2\cos(\theta_1+\theta_2), & 0 \\ 0, & 0, & 1 \end{bmatrix}$$

Now, for inverse kinematics,

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d} \end{bmatrix} = J^{+} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

so, $J^{+} = \begin{bmatrix} \frac{\cos(\theta_1+\theta_2)}{l_1\sin\theta_2}, & \frac{\sin(\theta_1+\theta_2)}{l_1\sin\theta_2}, & 0 \\ \frac{-(l_2\cos\theta_1+\theta_2+l_1\cos\theta_1)}{l_1l_2\sin\theta_2}, & \frac{-l_2\sin\theta_1+\theta_2-l_1\sin\theta_1}{l_1l_2\sin\theta_2}, & 0 \\ 0 & 0, & 1 \end{bmatrix}$

for our case, $\dot{x} = 0$ and $\dot{z} = 0$,

so,

$$\dot{\theta}_1 = \frac{\sin(\theta_1+\theta_2)}{l_1\sin\theta_2}\dot{y}$$

$$\dot{\theta}_2 = \frac{-(l_2\sin(\theta_1+\theta_2)+l_1\sin\theta_1)}{l_1l_2\sin\theta_2}\dot{y}$$

→ Now, acceleration,

$$\ddot{\theta}_1 = \frac{d}{dt}\left(\frac{\sin(\theta_1+\theta_2)}{l_1\sin\theta_2}\dot{y}\right)$$

$$= \ddot{y}\frac{\sin(\theta_1+\theta_2)}{l_1\sin\theta_2} + \frac{d}{dt}\left(\frac{\sin(\theta_1+\theta_2)}{l_1\sin\theta_2}\right)\dot{y}$$

$$\ddot{\theta}_1 = \ddot{y}\frac{\sin(\theta_1+\theta_2)}{l_1\sin\theta_2} + \left(\frac{S_2C_{12}(\dot{\theta}_1+\dot{\theta}_2) - S_{12}C_2\dot{\theta}_2}{l_1\sin^2\theta_2}\right)$$

$$\ddot{\theta}_2 = -\ddot{\theta}_1 + \frac{d}{dt}\left(\frac{l_1\sin\theta_1}{l_1l_2\sin\theta_2}\dot{y}\right)$$

$$= -\ddot{\theta}_1 \;\bar{\cancel{+}}\; \frac{d}{dt}\left(\frac{\sin\theta_1}{l_2\sin\theta_2}\right)\dot{y}$$

$$\cancel{+} - \ddot{y}\frac{\sin\theta_1}{l_2\sin\theta_2}$$

$$= -\ddot{\theta}_1 - \frac{\sin\theta_1}{l_2\sin\theta_2}\ddot{y} - \left(\frac{\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2}{l_2\sin^2\theta_2}\right)$$

$$\ddot{\theta}_2 = -\ddot{\theta}_1 - \frac{\sin\theta_1}{l_2\sin\theta_2}\ddot{y} + \left(\frac{\sin(\theta_1-\theta_2)}{l_2\sin^2\theta_2}\right)\dot{y}$$

problem (3) Implement four controllers for above desired trajectory.

(a) PD control that might yield a small steady state error with damping ratio 1.

Dynamics.

$$M = \begin{bmatrix} \alpha+\beta+2r\cos\theta_2, & \beta+2r\cos\theta_2, & 0 \\ \beta+r\cos\theta_2 & , \beta & , 0 \\ 0 & , 0 & , m_3 \end{bmatrix}$$

$$C = \begin{bmatrix} -r\sin\theta_2\dot{\theta}_2, & -r\sin\theta_2(\dot{\theta}_1+\dot{\theta}_2), & 0 \\ r\sin\theta_2\dot{\theta}_1, & 0 & , 0 \\ 0 & , 0 & , 0 \end{bmatrix}$$

$$q = \begin{bmatrix} 0 \\ 0 \\ m_3 g \end{bmatrix}, \quad x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ d \end{bmatrix}, \quad F = \begin{bmatrix} \tau_1 \\ \tau_2 \\ F_3 \end{bmatrix}$$

$$\ddot{x}M + C\dot{x} + q = F'$$

$$J = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$$

→ Considering motor dynamics,

$$B = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix}$$

$$\ddot{x}(M+J) + (C+B)\dot{x} + q = F$$

$\Rightarrow$ Control law,

$\tau_1 = 2\Delta\theta_1 + 1\cdot\Delta\dot{\theta}_1$

$\tau_2 = 2\Delta\theta_2 + 1\cdot\Delta\dot{\theta}_2$

$\tau_3 = 50\cdot\Delta d + 0.001\,\Delta\dot{d}$

for τ_3, we can add $m_3\cdot g$ to compensate for gravity.

(b) PD control with feedforward control.

$\tau_1 = 2\Delta\theta_1 + \Delta\dot{\theta}_1 + (\alpha + \beta + 2\gamma\cos\theta_{2d} + J_1)\ddot{\theta}_1$

$\tau_2 = 2\Delta\theta_2 + \Delta\dot{\theta}_2 + (\beta + J_2)\ddot{\theta}_{2d}$

$\tau_3 = 50\,\Delta d + 0.001\,\Delta\dot{d} + m_3\cdot\ddot{d}_d$ $\to 0$

$\approx 50\,\Delta d + 0.001\Delta\dot{d}$.

(c) PD along with feedforward disturbance cancellation.

$\tau_1 = 2\Delta\theta_1 + \Delta\dot{\theta}_1 + (\alpha + \beta + 2\gamma\cos\theta_{2d} + J_1)\ddot{\theta}_{1d} +$

$(\beta + 2\gamma\cos\theta_{2d})\ddot{\theta}_{2d} - \gamma\sin\theta_{2d}\,\dot{\theta}_{2d}\,\dot{\theta}_{1d}$

$- \gamma\sin\theta_{2d}(\dot{\theta}_{1d} + \dot{\theta}_{2d})\cdot\dot{\theta}_{2d}$

$\tau_2 = \cancel{2\Delta\theta_2} + 2\Delta\theta_2 + \Delta\dot{\theta}_2 + (\beta + J_2)\ddot{\theta}_{2d}$

$(\beta + \gamma\cos\theta_{2d})\ddot{\theta}_{1d} + \gamma\sin\theta_{2d}\,\dot{\theta}_{1d}\,\dot{\theta}_{1d}$

$\tau_3 = 50\,\Delta d + 0.001\,\Delta\dot{d} + m_3 g$

(4) Multivariable control:

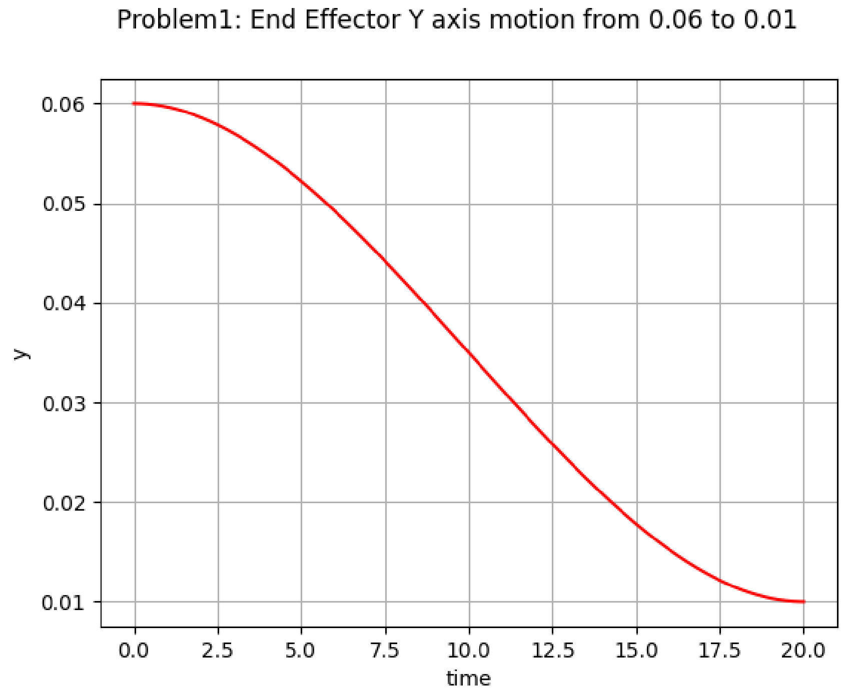
$$U = \begin{bmatrix} K_{P1} & 0 & 0 \\ 0 & K_{P2} & 0 \\ 0 & 0 & K_{P3} \end{bmatrix} \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \Delta d \end{bmatrix} + \begin{bmatrix} K_{d1} & 0 & 0 \\ 0 & K_{d2} & 0 \\ 0 & 0 & K_{d3} \end{bmatrix} \begin{bmatrix} \Delta\dot{\theta}_1 \\ \Delta\dot{\theta}_2 \\ \Delta\dot{d} \end{bmatrix}$$

$$+ M \begin{bmatrix} \ddot{\theta}_{1d} \\ \ddot{\theta}_{2d} \\ \ddot{d}_d \end{bmatrix} + C \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} + G$$

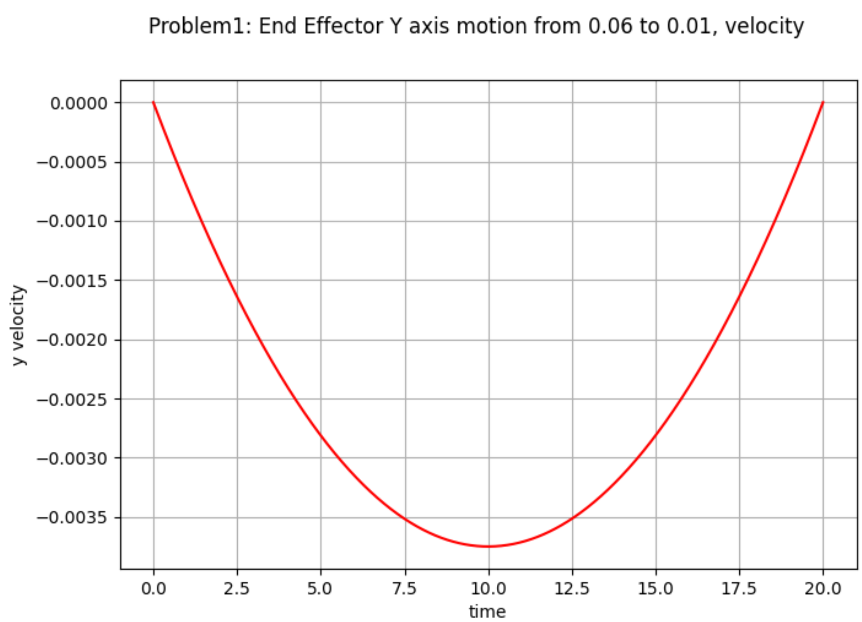
⇒ codes and plots are attached with submission.

⇒ problem (4)(5)(6): codes and plots are attached.

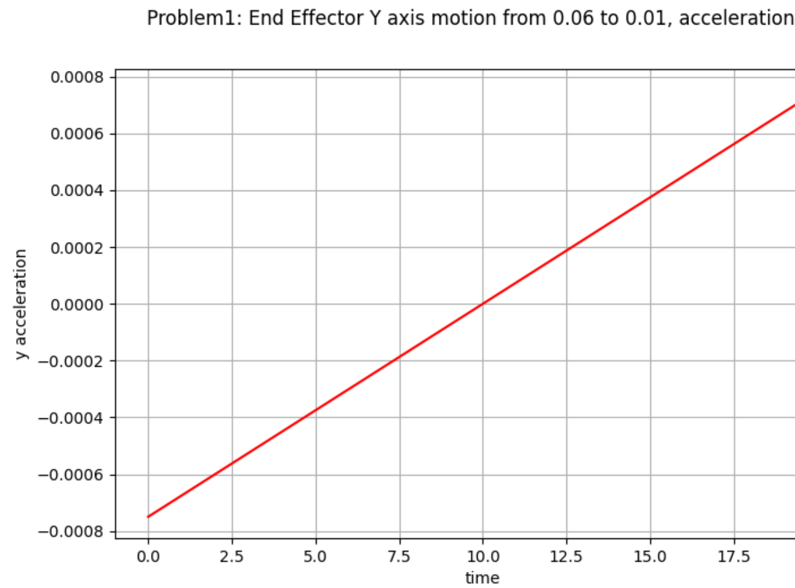
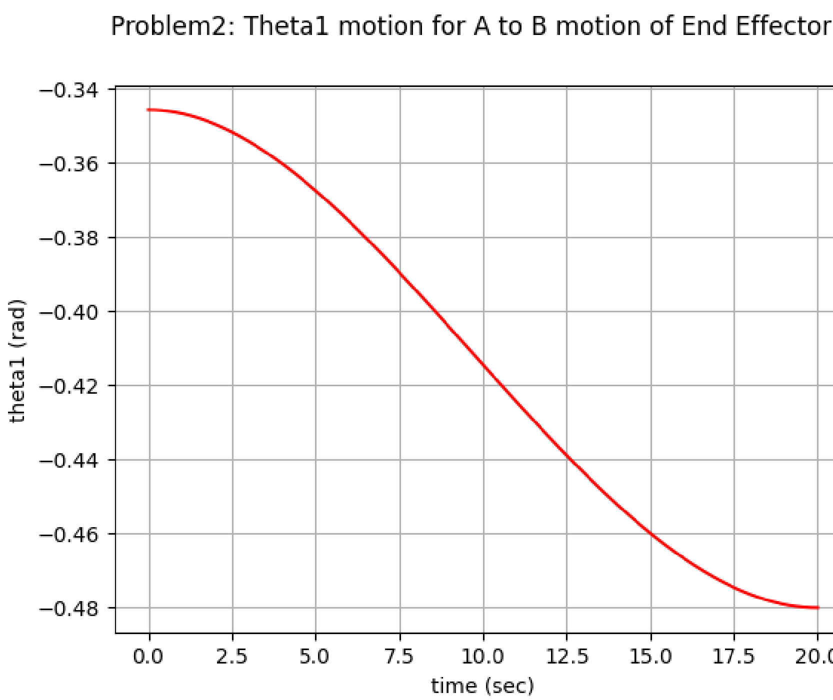
Problem 1

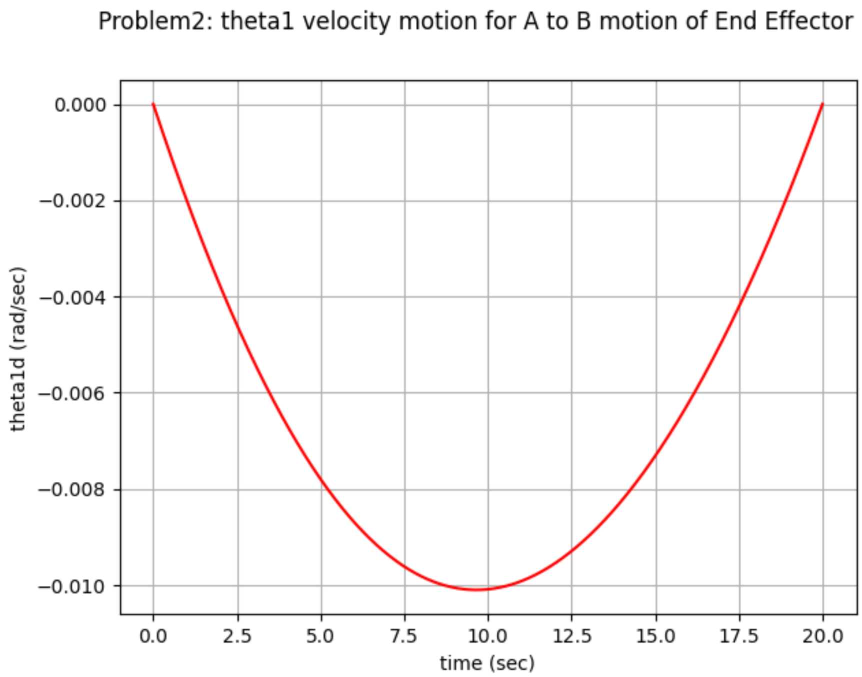
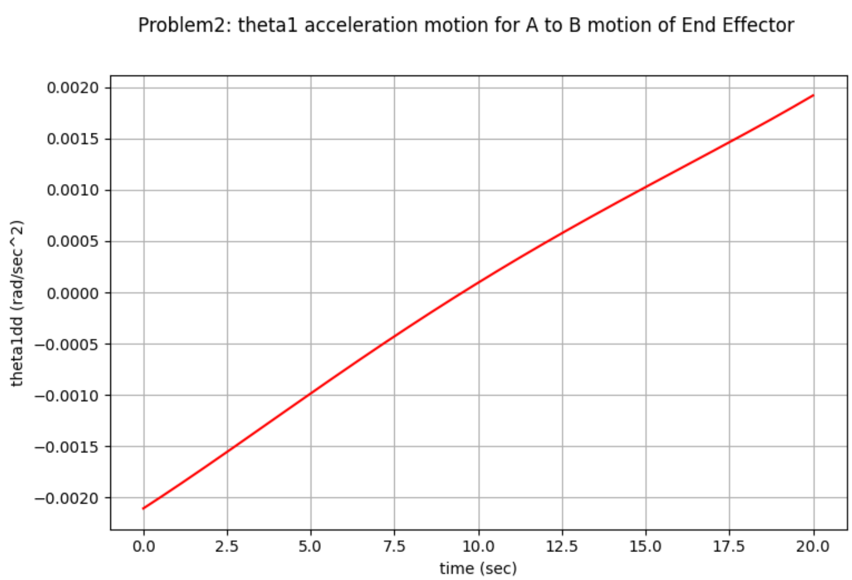


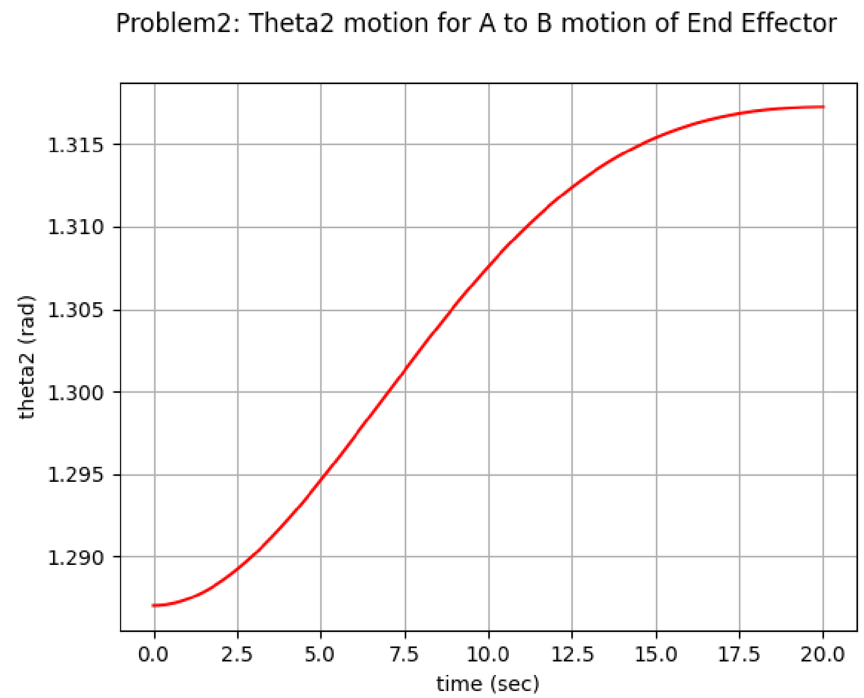
Problem1: End Effector Y axis motion from 0.06 to 0.01
y
0.06
0.05
0.04
0.03
0.02
0.01
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time

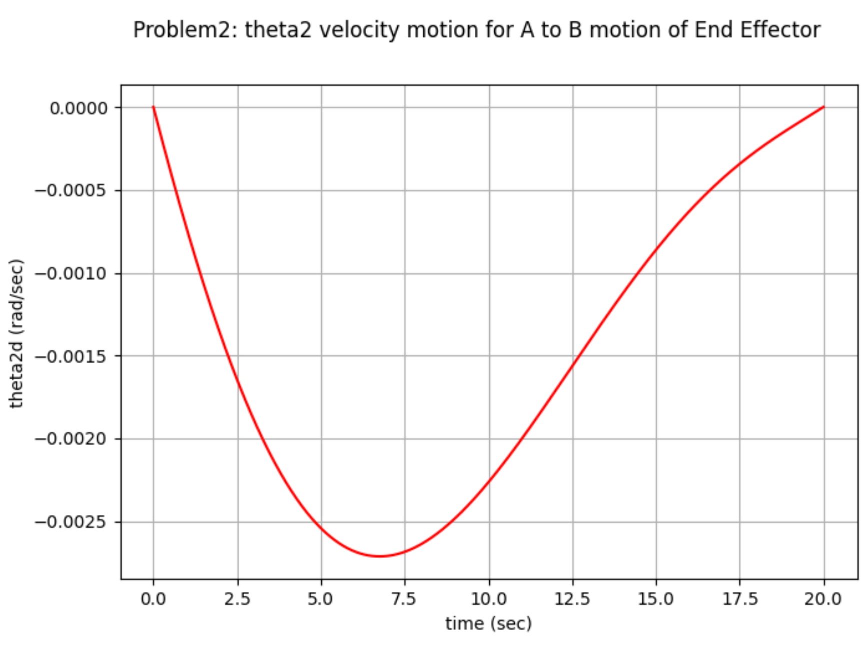


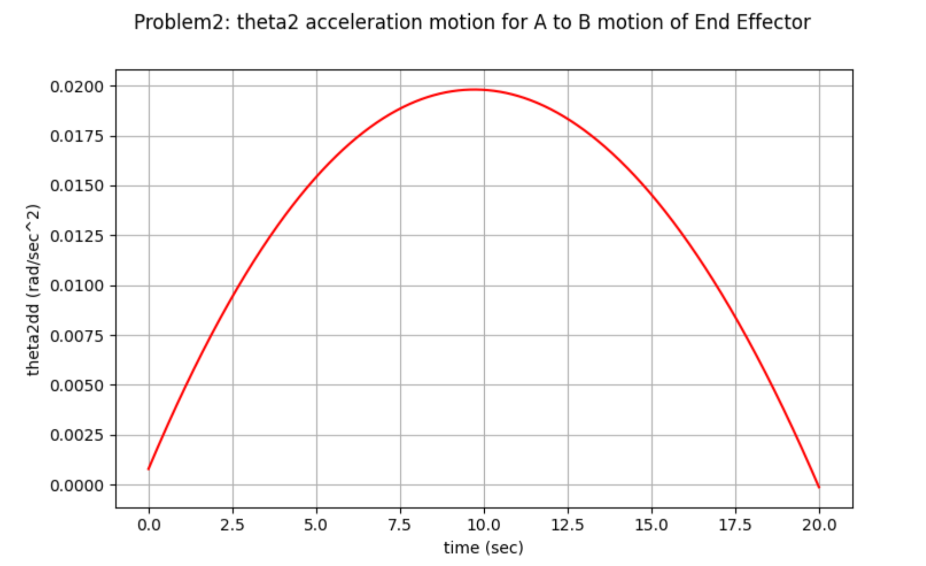
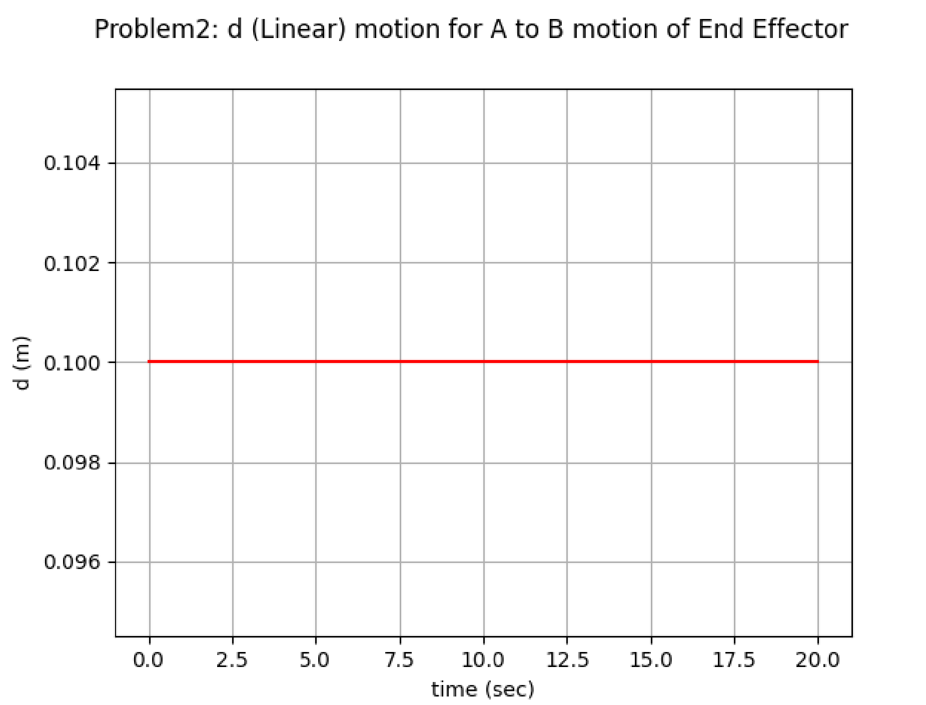
Problem1: End Effector Y axis motion from 0.06 to 0.01, velocity
y velocity
0.0000
−0.0005
−0.0010
−0.0015
−0.0020
−0.0025
−0.0030
−0.0035
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time

	Problem1: End Effector Y axis motion from 0.06 to 0.01, acceleration y acceleration 0.0008 0.0006 0.0004 0.0002 0.0000 −0.0002 −0.0004 −0.0006 −0.0008 0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 time
Problem 2	Problem2: Theta1 motion for A to B motion of End Effector theta1 (rad) −0.34 −0.36 −0.38 −0.40 −0.42 −0.44 −0.46 −0.48 0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 time (sec)

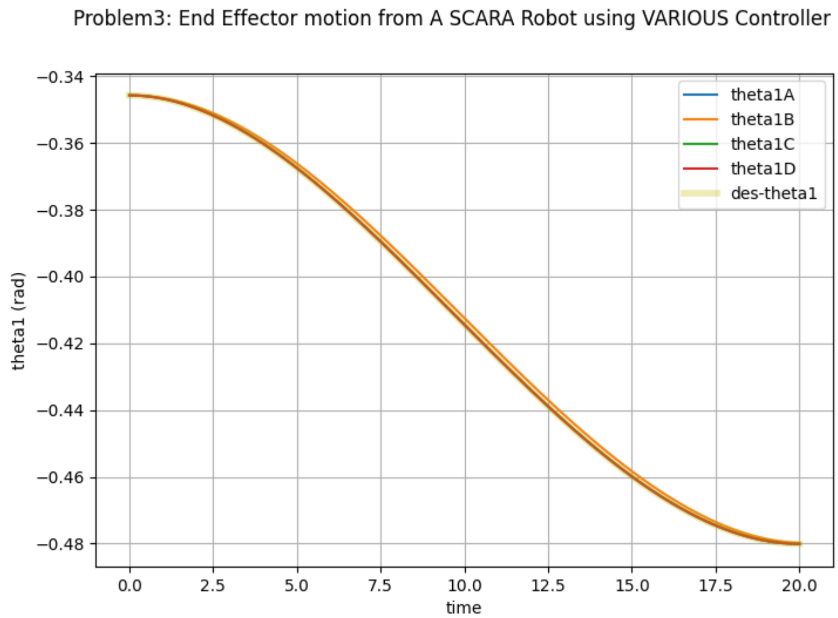
	Problem2: theta1 velocity motion for A to B motion of End Effector theta1d (rad/sec) time (sec) 0.000 −0.002 −0.004 −0.006 −0.008 −0.010 0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0
	Problem2: theta1 acceleration motion for A to B motion of End Effector theta1dd (rad/sec^2) time (sec) 0.0020 0.0015 0.0010 0.0005 0.0000 −0.0005 −0.0010 −0.0015 −0.0020 0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0

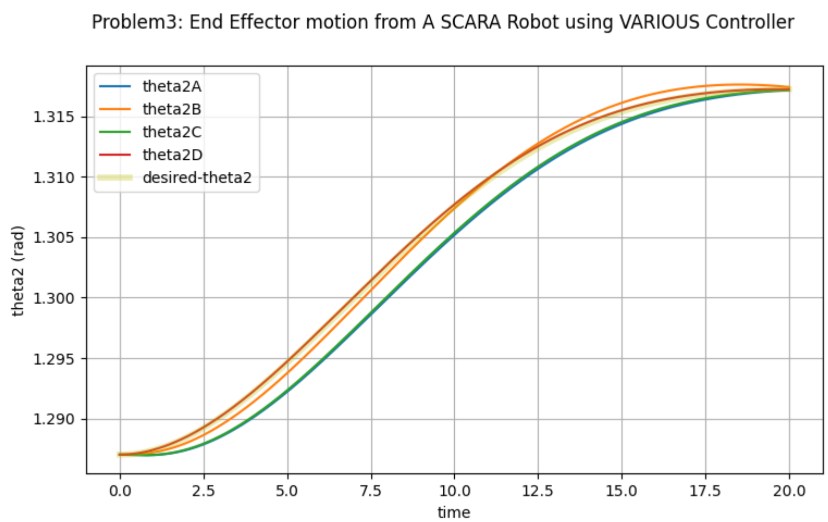

Problem2: Theta2 motion for A to B motion of End Effector
theta2 (rad)
1.315
1.310
1.305
1.300
1.295
1.290
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time (sec)

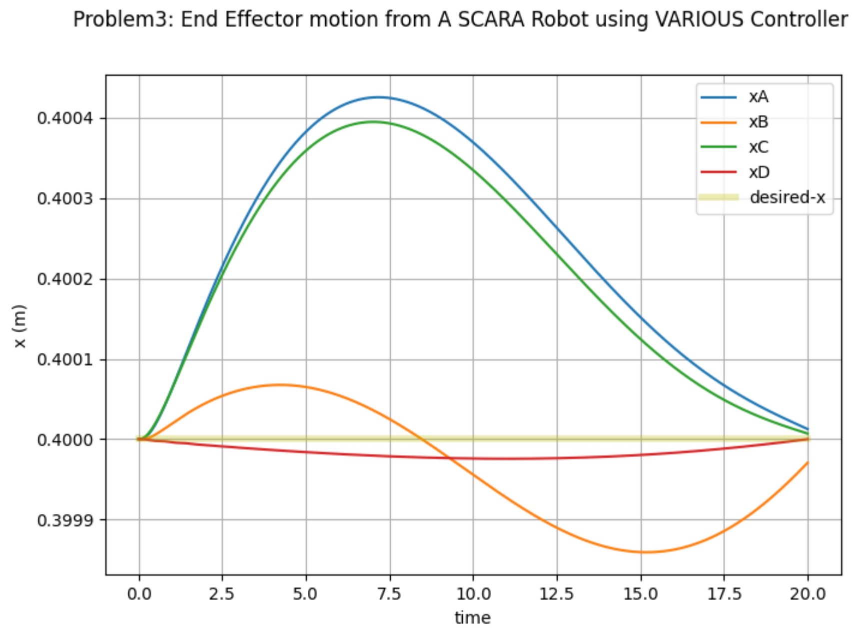

Problem2: theta2 velocity motion for A to B motion of End Effector
theta2d (rad/sec)
0.0000
−0.0005
−0.0010
−0.0015
−0.0020
−0.0025
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time (sec)

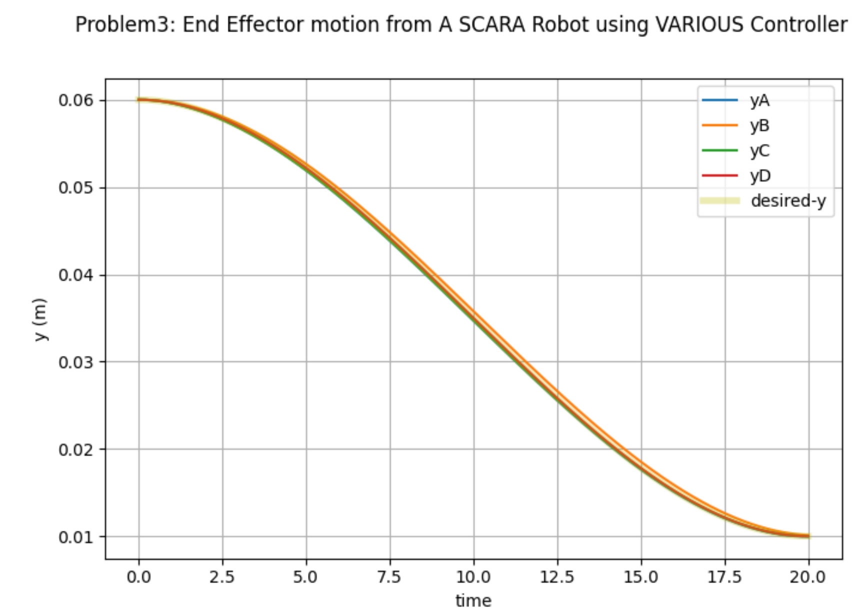
	Problem2: theta2 acceleration motion for A to B motion of End Effector theta2dd (rad/sec^2) 0.0000 0.0025 0.0050 0.0075 0.0100 0.0125 0.0150 0.0175 0.0200 0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 time (sec)
	Problem2: d (Linear) motion for A to B motion of End Effector d (m) 0.096 0.098 0.100 0.102 0.104 0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 time (sec)

Problem 3

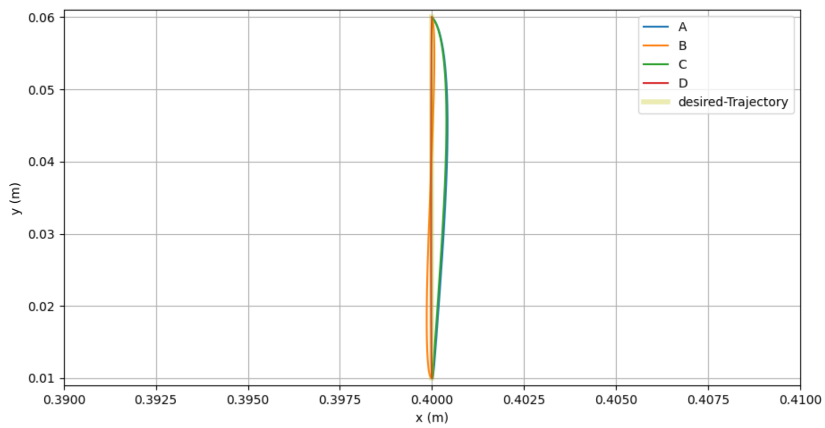

Problem3: End Effector motion from A SCARA Robot using VARIOUS Controller
theta1A
theta1B
theta1C
theta1D
des-theta1
theta1 (rad)
−0.34
−0.36
−0.38
−0.40
−0.42
−0.44
−0.46
−0.48
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time


Problem3: End Effector motion from A SCARA Robot using VARIOUS Controller
theta2A
theta2B
theta2C
theta2D
desired-theta2
theta2 (rad)
1.315
1.310
1.305
1.300
1.295
1.290
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time


Problem3: End Effector motion from A SCARA Robot using VARIOUS Controller
xA
xB
xC
xD
desired-x
x (m)
0.4004
0.4003
0.4002
0.4001
0.4000
0.3999
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time

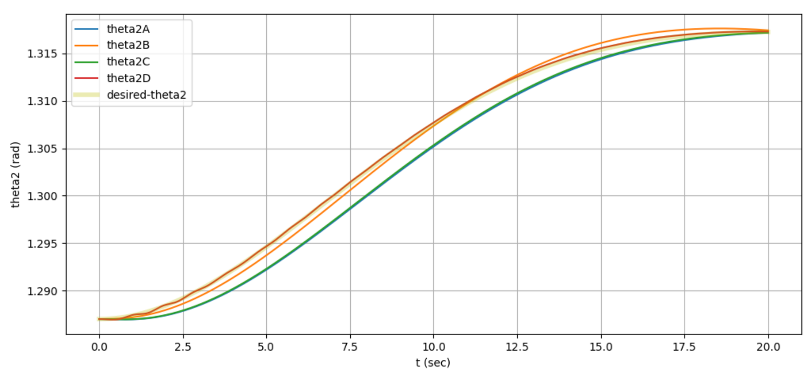

Problem3: End Effector motion from A SCARA Robot using VARIOUS Controller
yA
yB
yC
yD
desired-y
y (m)
0.06
0.05
0.04
0.03
0.02
0.01
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time

Problem3: End Effector motion from A SCARA Robot using VARIOUS Controller

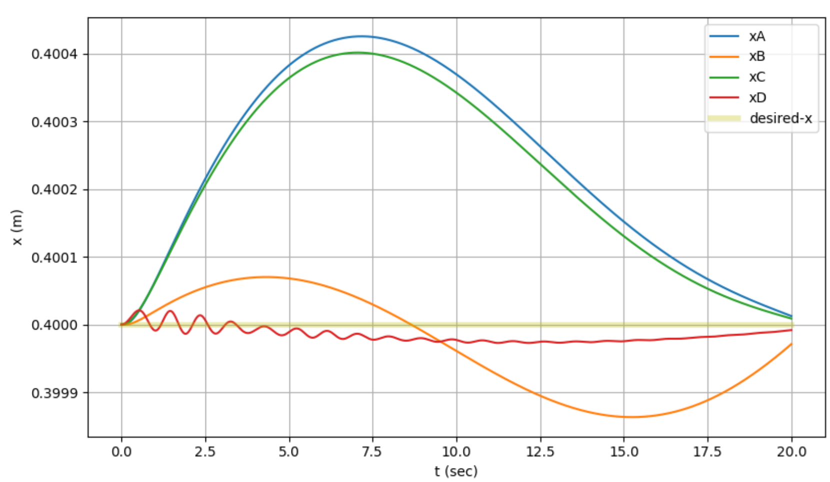


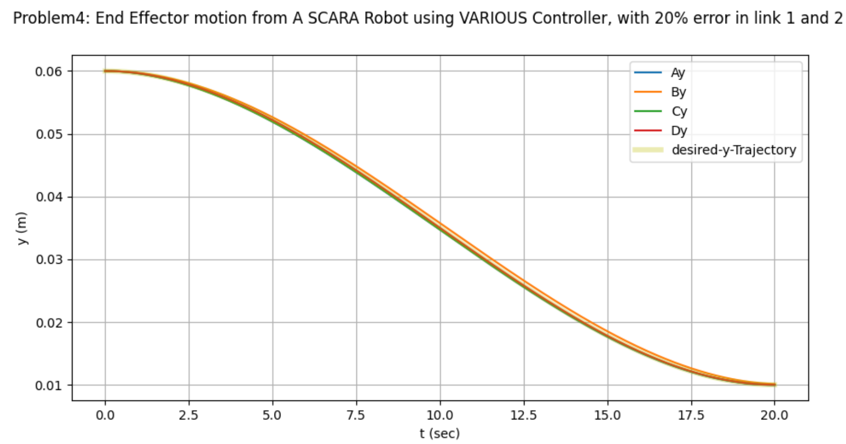
Problem 4

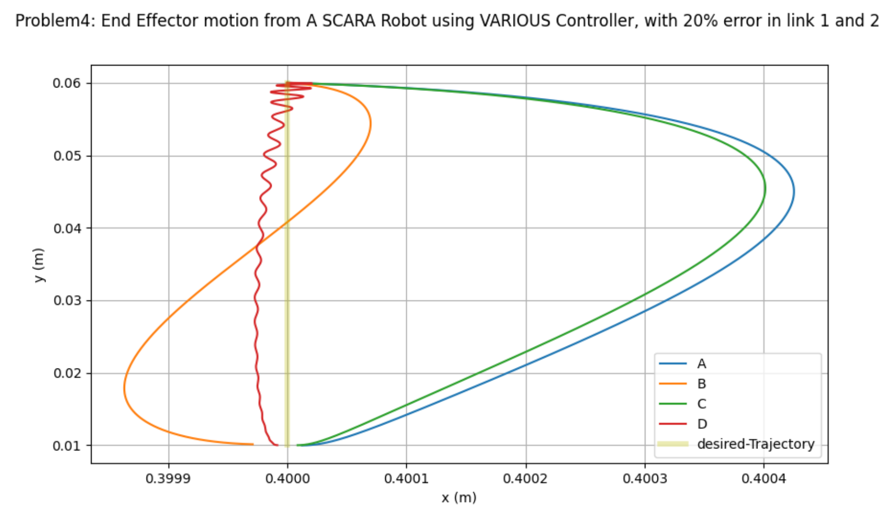
Problem4: End Effector motion from A SCARA Robot using VARIOUS Controller, with 20% error in link 1 and 2



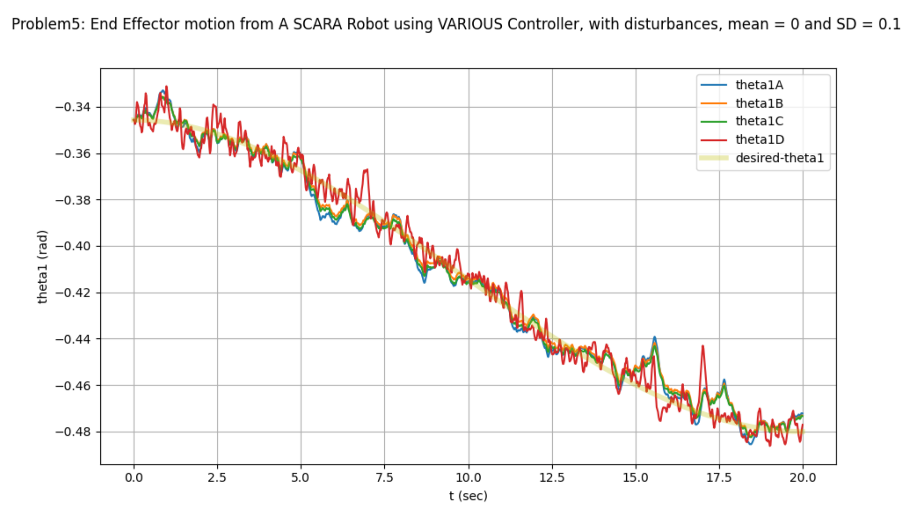
Problem4: End Effector motion from A SCARA Robot using VARIOUS Controller, with 20% error in link 1 and 2

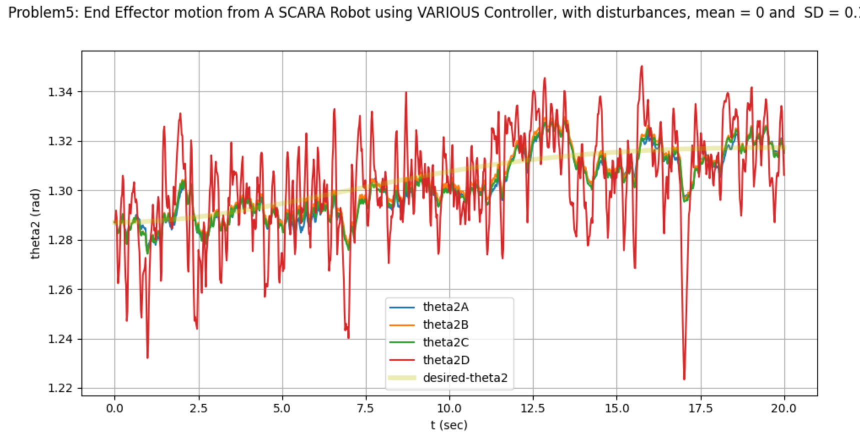


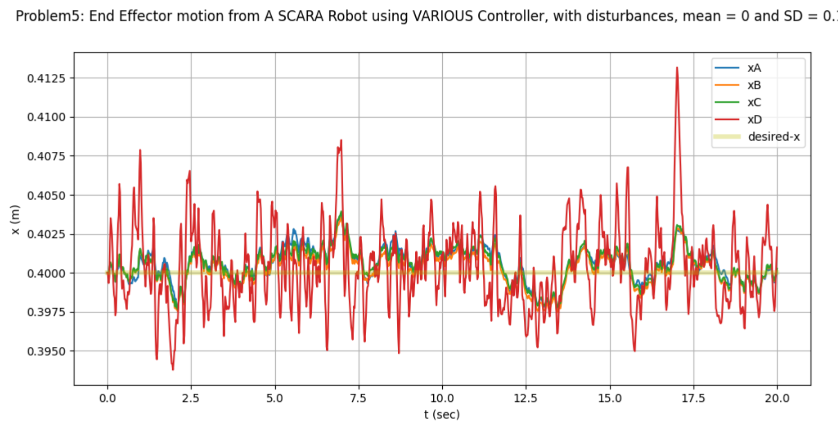

Problem4: End Effector motion from A SCARA Robot using VARIOUS Controller, with 20% error in link 1 and 2
Ay
By
Cy
Dy
desired-y-Trajectory
y (m)
t (sec)

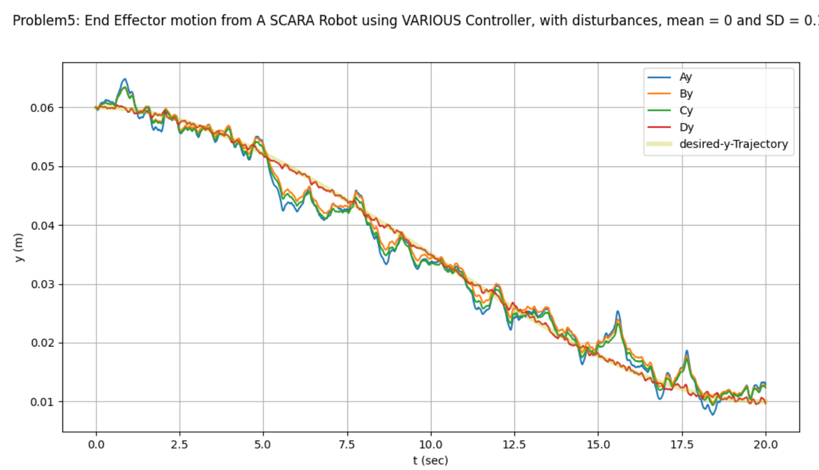

Problem4: End Effector motion from A SCARA Robot using VARIOUS Controller, with 20% error in link 1 and 2
A
B
C
D
desired-Trajectory
y (m)
x (m)

Problem 5


Problem5: End Effector motion from A SCARA Robot using VARIOUS Controller, with disturbances, mean = 0 and SD = 0.1
theta1A
theta1B
theta1C
theta1D
desired-theta1
theta1 (rad)
t (sec)


Problem5: End Effector motion from A SCARA Robot using VARIOUS Controller, with disturbances, mean = 0 and SD = 0.1
theta2 (rad)
t (sec)
theta2A
theta2B
theta2C
theta2D
desired-theta2


Problem5: End Effector motion from A SCARA Robot using VARIOUS Controller, with disturbances, mean = 0 and SD = 0.1
x (m)
t (sec)
xA
xB
xC
xD
desired-x


Problem5: End Effector motion from A SCARA Robot using VARIOUS Controller, with disturbances, mean = 0 and SD = 0.1
y (m)
t (sec)
Ay
By
Cy
Dy
desired-y-Trajectory

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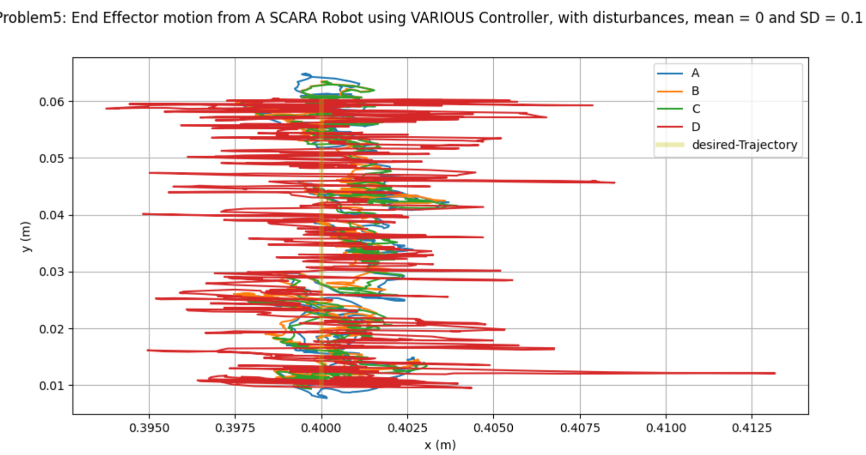
<tr><td></td><td>
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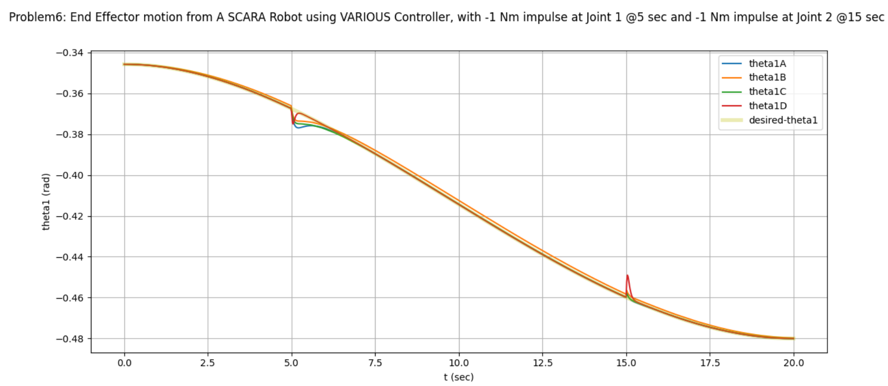
<tr><td rowspan="2">Problem 6</td><td>
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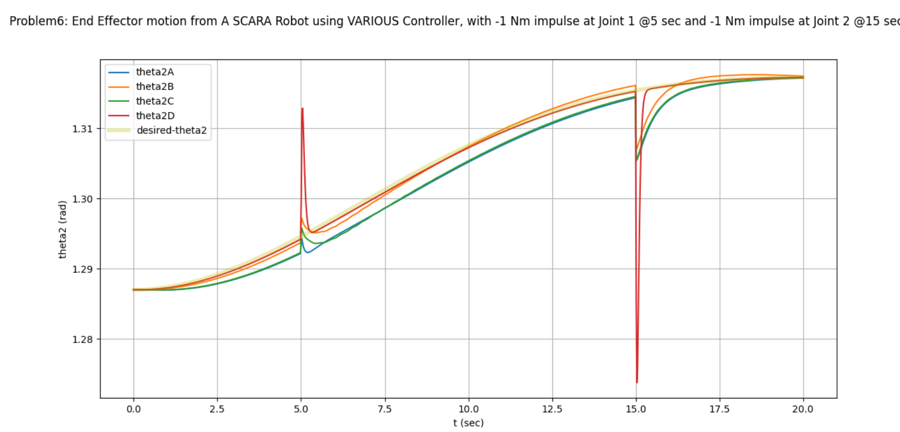
<tr><td>
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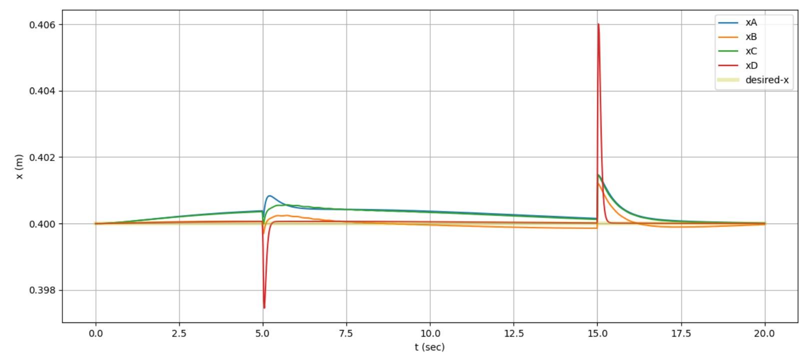
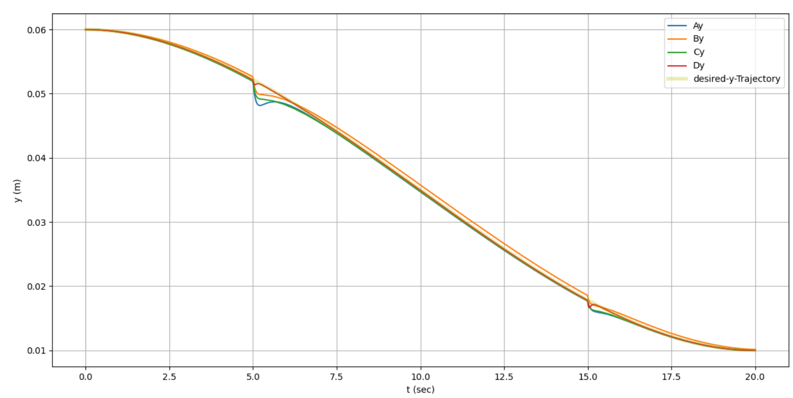
Problem6: End Effector motion from A SCARA Robot using VARIOUS Controller, with -1 Nm impulse at Joint 1 @5 sec and -1 Nm impulse at Joint 2 @15 sec



Problem6: End Effector motion from A SCARA Robot using VARIOUS Controller, with -1 Nm impulse at Joint 1 @5 sec and -1 Nm impulse at Joint 2 @15 sec



Problem6: End Effector motion from A SCARA Robot using VARIOUS Controller, with -1 Nm impulse at Joint 1 @5 sec and -1 Nm impulse at Joint 2 @15 sec