Analysis of On-line Routing and Spectrum Allocation in Spectrum-sliced Optical Networks

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***Abstract*—The orthogonal frequency division multiplexing (OFDM) technology provides an opportunity for efficient resource utilization in optical networks. It allows allocation of multiple sub-carriers to meet traffic demands of varying size. Utilizing OFDM technology, a spectrum efficient and scalable optical transport network called SLICE was proposed recently. The SLICE architecture enables sub-wavelength, super-wavelength resource allocation and multiple rate data traffic that results in efficient use of spectrum. However, the benefit is accompanied by additional complexities in resource allocation. In SLICE architecture, in order to minimize utilized spectrum, one has to solve the *routing and spectrum allocation* (RSA) problem, a generalization of the *routing and wavelength allocation* (RWA) problem. In this paper, we focus our attention to the on-line version of RSA problem and provide an algorithm for the *ring network* with a *competitive ratio* of $\min\{O(\log(d_{max})), O(\log(k))\}$ where k is the total number of requests and d_{max} is the maximum demand in terms of the number of sub-carriers. Moreover, we provide a heuristic for the network with arbitrary topology and measure the effectiveness of the heuristic with extensive simulation.**

I. Introduction

It is being increasingly recognized by the optical network designers that in order to meet the challenges posed by the explosive growth of the network traffic, the networks must be operated in the most innovative and efficient manner. The traditional WDM network operates at the granularity of a wavelength, which may lead to inefficient use of resources as some connection requests may not have enough traffic to utilize the full capacity of a wavelength. However such wastage of networking resources can be avoided if the optical network can be made to operate at a finer grain (i.e., sub-wavelength level) instead of the current practice of course grain operation (i.e., wavelength level). Recent introduction of Orthogonal Frequency Division Multiplexing (OFDM) technology in optical networks [1] offers an opportunity for operating optical networks at a much finer grain than what is currently possible. The advantages offered by the OFDM in terms of flexibility and scalability originate from the unique multicarrier nature of this technology [1].

Utilizing the OFDM technology, a spectrum efficient and scalable optical transport network called *spectrum-sliced elastic optical path network (SLICE)* was proposed recently [2]. Just as the ability to operate at a granularity finer than a wavelength (i.e., a sub-wavelength) will enable the network operator to manage resources more efficiently, the same is true if the operator is provided with capability to operate at super-wavelength granularity. Such a capability will be useful for the network operator to meet large traffic demand. The goal of SLICE architecture is to allocate variable sized optical bandwidths that matches with the user traffic demands. It achieves that goal by *slicing off* spectral resources of a route and allocating only the requested amount to establish an end-to-end optical path.

Although the sub-wavelength (sub-carrier) level allocation capability of SLICE leads to more effective resource utilization, it also leads to additional complexities in network control and management. First, if a call requests for d sub-carriers, the network controller must allocate d *consecutive* sub-carriers to this request. Second, if the paths corresponding to two requests R_1 and R_2 share a fiber link, not only the set of carriers allocated to R_1 and R_2 must be disjoint, in order to avoid interference, they must be *separated* from each other in the spectrum domain by a few carriers, known as *guard carriers* or *guard bands*. The first and the second constraints are known as the *sub-carrier consecutiveness constraint* and the *guard-carrier constraint* respectively [3]. The introduction of the *sub-carrier consecutiveness constraint* significantly increases the complexity of the Routing and Spectrum Assignment (RSA) problem that needs to be solved in SLICE. The RSA problem may be informally defined as follows: Given a network topology and a set of call requests with varying demands (in terms of the number of sub-carriers) find a route for each request and allocate a number of sub-carriers to each request (equal to their requested demand), so that *the utilized part of the spectrum* span *is minimized*. It may be noted that if the demand of each request is one sub-carrier, then the RSA problem reduces to the Routing and Wavelength Assignment (RWA) problem, which has been studied extensively. One can conceive of two different versions of the RSA problem - *off-line* and *on-line*. In the off-line version all the requests are known ahead of time before path and spectrum allocation for any request is carried out. In the on-line version, the requests come in a sequence and path and spectrum allocation for a request has to carried out at the time of arrival of that request. Because the off-line version has the luxury of knowing all the requests, it can carry out better optimization of utilized spectrum span than its on-line counterpart.

In this paper we study the on-line version of RSA problem. Previous studies on the on-line version of the RSA problem,

[4]–[9] primarily focus on the development of efficient heuristics for the problem. The effectiveness of these heuristics are primarily evaluated through simulation. To the best of our knowledge, very little analytical results are available in the literature regarding the performance of these heuristics. In this paper we present analytical results relating to the on-line version of the RSA problem when the network topology is a ring. The performance of an on-line algorithm is measured in terms of the metric *competitive ratio*. In this metric, the performance of an on-line algorithm is compared with the performance of an optimal off-line algorithm that knows the sequence of requests in advance. The maximum ratio between their respective performances, taken over all sequences, is known as the *competitive ratio* of the algorithm [10].

In this paper, we provide an algorithm for the on-line version of the RSA problem for the ring network with a competitive ratio of $\min\{O(\log(d_{max})), O(\log(k))\}$ where k is the total number of requests, $d_{max} = \max_{1 \leq i \leq k} d_i$, and d_i is the demand in terms of the number of sub-carriers associated with request R_i. Moreover, we provide a heuristic for the network with arbitrary topology and measure the effectiveness of the heuristic with extensive simulation.

The rest of the paper is organized as follows. We discuss related works in section II. In section III we introduce definitions and notations. We present problem statement for the on-line RSA problem in section IV. Analytical results for on-line RSA in rings is presented in section V. A heuristic and experimental results for the arbitrary network topology is presented in section VI. Section VII concludes the paper.

II. Related Work

Utilizing the optical OFDM technology, the SLICE architecture proposes a novel scheme for slicing off the spectral resources of a route, resulting in more efficient utilization [2]. The fact that the sub-carriers in the SLICE architecture have to be assigned in a contiguous manner, led to the formulation of the RSA problem. To the best of our knowledge, the RSA problem was originally introduced in [4], [11], [12]. Since then a few other papers, [3], [13] have also studied the RSA problem. In most of these studies [3], [12]–[14], the authors propose an integer linear program based solution and a heuristic solution for the off-line RSA problem. Based on the experimental results, the authors claim effectiveness of their heuristics.

The on-line version of RSA problem has been studied in [4]–[9]. In all of these papers, the objective of the on-line RSA problem is to maximize the number of requests that can be satisfied and minimize the blocking probability. In this version of on-line RSA problem, the number of available spectrum sub-carriers is limited. The authors of these papers proposed heuristic solutions mainly by modifying the Dijkstra shortest path algorithm or using K-shortest path algorithm accompanying with the First-Fit algorithm. To the best of our knowledge none of these papers consider the objective of minimizing the utilized spectrum while satisfying all the requests. It may be the case that all the requests should be satisfied while the utilized spectrum is minimized. In this paper we propose a new heuristic for arbitrary network graphs. We also modify the K-shortest path approach for this version of the on-line RSA and through simulations we evaluate their performance.

Most of the studies both on on-line and off-line RSA do not present any analytical results for the RSA problem, even for the simplest optical network topologies such as rings. The ring topology is of particular importance in the optical domain because of its application in metro networks and in some long haul networks. A major thrust of our effort is to present analytical results for the on-line RSA for optical networks with ring structure.

III. Definitions and Notations

Spectrum Slice/Interval: A number of consecutive sub-carriers from a_i to b_i denoted by $[a_i, b_i]$, that is allocated to a specific request R_i to establish a connection between (s_i, t_i) with d_i sub-carriers. The length of this slice is $b_i - a_i + 1 = d_i$.
Spectrum Span/Spread: The total amount of spectrum used for allocating a slice to all the requests; If $R_i, 1 \leq i \leq k$ is allocated the spectrum interval $[a_i, b_i]$ then the spectrum span is $[\min_{1 \leq i \leq k} a_i, \max_{1 \leq i \leq k} b_i]$.
Chromatic Number: The Chromatic Number, $\chi(G)$, of a graph $G = (V, E)$ is the fewest number of colors necessary to color the nodes of the graph, such that no two adjacent nodes have the same color.
Interval Chromatic Number (ICN): Consider a weighted graph $G^* = (V, E, w)$ with a strictly positive integer weight $w(v)$ associated with each node $v \in V$. An *interval t-coloring* of $G^* = (V, E, w)$ is a function c from V to {1,2, …, t} such that $c(x) + w(x) - 1 \leq t$ and if both $c(x) \leq c(y)$ and $(x, y) \in E$ then $c(x) + w(x) - 1 < c(y)$. We can view an interval coloring c of G^* as assigning an interval $[c(v), \ldots, c(v) + w(v) - 1]$ of $w(v)$ consecutive colors to each vertex v so that the intervals of colors assigned to two adjacent vertices (i.e., the pair of nodes that has an edge between them) do not overlap. If interval t-coloring is feasible for a graph G^* then G^* is said to be *interval t-colorable*. The interval chromatic number of G^*, denoted by $\chi_{int}(G^*)$ is the *least* t such that G^* has a interval t-coloring [15].
Interval Graph: Let $\mathcal{F}$ be a family of non-empty sets. The *intersection graph* of $\mathcal{F}$ is obtained by representing each set in $\mathcal{F}$ by a node and connecting the two nodes with an edge, if and only if the corresponding sets intersect. The intersection graph of a family of intervals on a linearly ordered set (such as the real line) is called *Interval Graph*.
Path Intersection Graph: Consider a graph $G = (V, E)$ and a set of paths $\mathcal{P} = \{P_1, \ldots, P_k\}$, where each P_i is a path between a node pair (s_i, t_i), $\forall i, 1 \leq i \leq k$. A graph $G' = (V', E')$ is a Path Intersection Graph corresponding to $\mathcal{P}$, if each vertex $p_i \in V'$ corresponds to a path $P_i \in \mathcal{P}$ and two nodes p_i and p_j in V' have an edge between them, if the corresponding paths P_i and P_j in $\mathcal{P}$ have at least one common edge in E.

IV. Problem Formulation

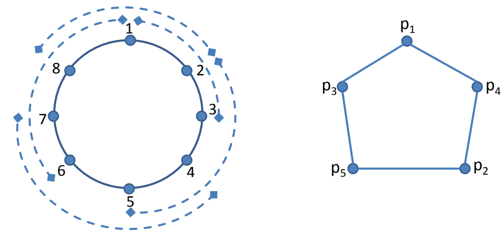
In this section we provide a formal statement of the on-line routing and spectrum allocation problem.

On-line Routing and Spectrum Allocation (RSA) Problem: A graph $G = (V, E)$ representing the network topology is given. The connection requests arrive in a sequence one by one where k is the total number of requests. The ith connection request is denoted by a triple $R_i = (s_i, t_i, d_i), 1 \leq i \leq k$, where s_i represents a source node, t_i represents a destination node, and d_i represents the demand between s_i and t_i in terms of sub-carriers. Once a request R_i arrives without knowledge of the future requests, assign a path P_i from s_i to t_i and assign a spectrum interval $I_i = [a_i, b_i]$ of length d_i to P_i, such that for every pair of requests i and $j, j \leq i$ the intervals I_i and I_j do not overlap if the corresponding paths P_i and P_j share an edge between them in $G = (V, E)$. Moreover, if the paths P_i and P_j overlap, not only the corresponding intervals I_i and I_j must be non-overlapping, these two intervals must be separated by a fixed number of sub-carriers, known as the *guard band*. The objective is to minimize *spectrum span*, $\mathcal{I} = [\min_{1 \leq i \leq k} a_i, \max_{1 \leq i \leq k} b_i]$. Without loss of generality, we number the first available sub-carrier one and the rest are numbered accordingly.

We note that guard-band constraint can be satisfied by increasing the demand values by guard-band value g. In other words, in an instance of RSA problem, RSA_1 with guard-band $g_1 > 0$ and requests $\{R_i = (s_i, t_i, d_i) | 1 \leq i \leq k\}$, we can increase the demand values in each request by g_1 and consider another instance of RSA, RSA_2 where guard-band $g_2 = 0$ and for every request R_i in RSA_1, request $R'_i = (s_i, t_i, d_i + g_1)$ is added to RSA_2. Then the optimal solution of RSA_2 can be used to create the optimal solution of RSA_1 by removing the last g_1 sub-carriers from each spectrum slice assigned to each request (for the proof, reader is referred to the proof of the Observation 4 in [16]). As a result, from this point onward we assume that guard-band is zero.

The RSA problem has two distinct components - the routing component and the spectrum allocation component. When routing is given and the paths for the requests are known then interval chromatic number (ICN) of the intersection graph of request paths finds the solution of the SA problem. Let $G' = (V', E', w)$ be the weighted path intersection graph of paths of all requests where $V' = \{p_1, p_2, \ldots, p_k\}$ and each node p_i corresponds to the path of request R_i and the weight of p_i is d_i; i.e., $w(p_i) = d_i$. Let $\chi_{int}(G')$ be the ICN of graph G'. In computation of $\chi_{int}(G')$, each node $p_i \in V'$ is assigned an interval $[a_i, b_i]$ of colors with length $w(p_i) = d_i$ where the intervals of two adjacent vertices do not intersect and total number of distinct colors used is minimum. Therefore, interval $[a_i, b_i]$ can be allocated to the path P_i in G and no two paths with common edge intersect in their spectrum intervals. Hence, the spectrum span of $\chi_{int}(G')$ is sufficient for the spectrum allocation of requests in G with predefined set of paths $\mathcal{P}$. Moreover, $\chi_{int}(G')$ is the minimum spectrum span needed in the SA problem; otherwise, it contradicts with $\chi_{int}(G')$ being the minimum interval chromatic number of G'. It is known that computation of ICN of interval graphs is NP-complete (Problem SR2 in [17]).

Fig. 1 shows an example of SA instance where the network graph is a ring with 8 nodes and requests are $\{R_1 = (1, 3, 15), R_2 = (1, 6, 6), R_3 = (2, 5, 6), R_4 = (2, 8, 6), R_5 = (4, 7, 12)\}$. Dashed lines show the paths for the requests. Fig. 1(b) depicts G', the path intersection graph of these paths where $w(p_1) = 15, w(p_2) = 6, w(p_3) = 6, w(p_4) = 6$ and $w(p_5) = 12$. In this example, $\chi_{int}(G')$ is 24 where the requests R_1 to R_5 are assigned intervals $[1, 15]$, $[13, 18]$, $[16, 21]$, $[19, 24]$ and $[1, 12]$ respectively.



(a) (b)

Fig. 1. (a) An example of SA instance where the network graph is a ring (b) Path intersection graph G' of SA instance in (a)

V. On-line Routing and Spectrum Allocation Problem in Rings

Theorem 1: RSA problem (the off-line case) is NP-Complete when the optical network topology is a Ring.

Proof: If the demands of the requests in the off-line RSA instance are all equal to one, then RSA problem becomes RWA problem. In [18], it is proven that the RWA problem for optical networks with a ring topology is NP-complete. Since RWA problem is a special case of the RSA problem, it follows that the RSA problem for optical networks with a ring topology is also NP-complete.

Next, we propose an on-line algorithm for RSA problem when network topology is a ring. In this algorithm, first we use cut-one-link approach and after removing one link the induced graph is a chain. In the chain for every request there exists just one path. Therefore routing is trivial. For the spectrum assignment, we use First-Fit technique that finds the first free spectrum interval fit the demand of the current request. The steps of the algorithms are explained in Algorithm 1.

Algorithm 1 On-line RSA in Ring

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Remove an edge e ∈ E randomly; Let G_p be the induced chain;
while A new request arrives do
    Find the path for the request in graph G_p;
    Compute the first free spectrum interval fit the demand of the current request
end while
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Theorem 2: Algorithm 1 has competitive ratio of $\min\{O(\log(d_{max})), O(\log(k))\}$ where k is total number of requests and $d_{max} = \max_{1 \leq i \leq k} d_i$.

Proof: In order to compute the competitive ratio we need to compare the spectrum span of Algorithm 1 with the optimal spectrum span of off-line RSA where the sequence of requests is known in advance. After removing one edge randomly from $G = (V, E)$ in Algorithm 1, the induced graph G_p is a chain. Let OPT and OPT_p be the optimal spectrum span in RSA problem when network graph is G and G_p, respectively, and $\mathcal{I}$ be the size of the spectrum computed by Algorithm 1. Clearly, the intersection graph of paths of the requests in G_p is an interval graph (a path from node i to node j in G_p can be interpreted as an interval from i to j). Let G'_p be the path intersection graph. Therefore, minimum spectrum needed to satisfy requests in G_p is equivalent to the $\chi_{int}(G'_p)$. Based on the paper [10], First-Fit algorithm will have competitive ratio of $\min\{O(\log(d_{max})), O(\log(\chi_{G'_p}))\}$ for on-line interval coloring in G'_p. Also it is obvious $\chi_{G'_p} \leq k$ (i.e., chromatic number of G'_p is at most as large as the number of nodes in G'_p that is number of requests). Hence, we have (1) $\mathcal{I} \leq \min\{O(\log(d_{max})), O(\log(k))\} \cdot OPT_p$. We denote the set of paths in the optimal solution of RSA (off-line) when network graph is G by $\mathcal{P}_{OPT}$. The paths in $\mathcal{P}_{OPT}$ can be partitioned into two subsets, $\mathcal{P}_e^1$ and $\mathcal{P}_e^2$ such that $\mathcal{P}_e^1$ is the set of paths that include edge e and the paths in $\mathcal{P}_e^2$ do not include edge e. Let OPT_e^1 and OPT_e^2 be the ICN of the intersection graph of paths in $\mathcal{P}_e^1$ and $\mathcal{P}_e^2$ respectively. Then we have (2) $OPT \geq \max(OPT_e^1, OPT_e^2)$. Since all the paths in $\mathcal{P}_e^1$ have intersection in edge e, their intervals do not intersect. Clearly, (3) $OPT_p \leq OPT_e^1 + OPT_e^2$. The reason is that in the worst case, all requests that were routed through edge e in $\mathcal{P}_{OPT}$ are routed the other way in G_p and now they at most need OPT_e^1 spectrum span not intersecting the spectrum allocated to the paths in $\mathcal{P}_e^2$. Therefore, using relations in (2) and (3) we have $OPT_p \leq 2OPT$. Also, based on relation (1) we can conclude $\mathcal{I} \leq \min\{O(\log(d_{max})), O(\log(k))\} \cdot OPT$.

VI. A Heuristic and Results for General Graphs

In this section first we present our heuristic for on-line RSA problem in general graphs. Then we present the results of our extensive simulation that demonstrate the efficacy of our heuristic for the on-line RSA problem by comparing it against (i) the optimal solution and (ii) the solution obtained by executing the heuristic based on K-shortest path and First-Fit technique.

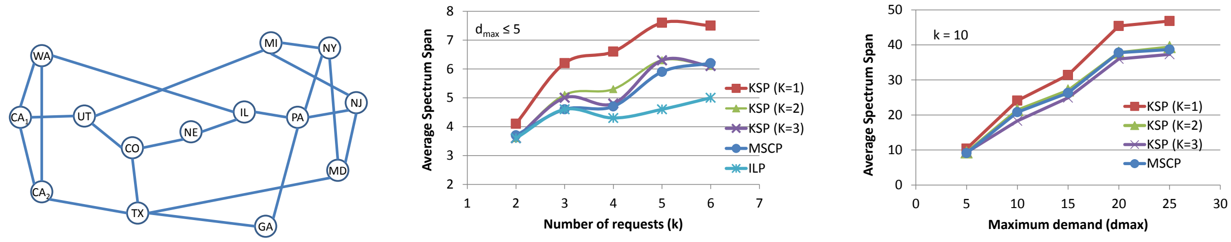
Minimum Sub-Carrier Path Heuristic (MSCP): The main idea in our heuristic is that it tries to find disjoint paths for routing the requests to increase the reuse of sub-carriers in spectrum allocation. Of this concern, we define a new weight function on the edges (fibers) of the network where weight of an edge $e \in E$, $w(e)$ will be largest sub-carrier number that is used in that edge. We also define the weight of a path, P from node s to node t to be $\max_{e \in P}\{w(e)\}$. For each new request, $MSCP$ selects the path with minimum weight. The minimum weight path can be computed by modifying the distance function in Dijkstra algorithm so that it considers the new weight function as the distance. After finding the path, $MSCP$ uses First-Fit algorithm to find the first available spectrum slice with the length of the request demand in all the edges of the path. Then, $MSCP$ updates the weight of every edge in the path to the largest sub-carrier so far used in that edge. For each request, time complexity of minimum-weight path computation is $O(|V|^2)$ and First-Fit algorithm takes $O(k|V|^2)$ where k is the number of requests. Hence, time complexity of $MSCP$ is $O(k^2|V|^2)$.

K-Shortest Path Heuristic (KSP): In this heuristic, initially K shortest paths are computed between every pair of nodes in the network using [19] algorithm with $O(k|V|^3)$. When a request R_i arrives, for every path in the K shortest paths between s_i and t_i we compute First-Fit algorithm to find the first available spectrum slice $[a_i, b_i]$ with the length of d_i. Then we select the path whose first available spectrum slice $[a_i, b_i]$ has the smallest b_i. This algorithm takes $O(Kk^2|V|^2)$ for satisfying all the k requests.

We perform our experiments on the NSFnet (Fig. 2(a)) and the fiber network of Level-3 that spans Europe (Fig. 3(a)) [20]. We view the NSFnet and Level-3 networks as examples of a small and a large network respectively.

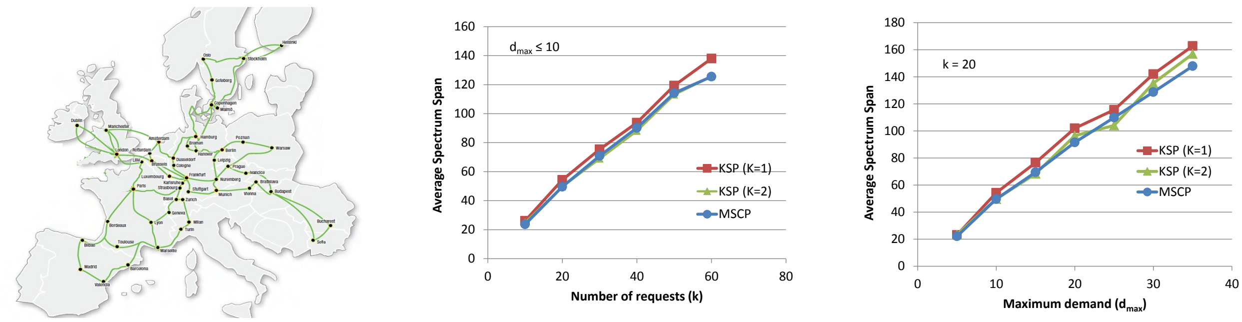
In Fig. 2(b), we present the results obtained from ILP, $MSCP$ and KSP when executed on the NSFnet. We find the optimal solution of the RSA problem (off-line) by solving an ILP using the software package CPLEX. Since computing the optimal solution by ILP takes considerable amount of time, we need to do this set of experiments for small number of requests. In this set of experiments, the number of requests, k, is varied from 2 to 6 with step of one. For each value of k, we generate 10 instances. In each instance we generate k requests randomly and consider them one at a time. For this set of experiments all the demand values are at most 5, (i.e., $d_{max} \leq 5$). The average spectrum span computed by each of the three methods is shown in Fig. 2(b). It may be observed that the average spectrum span of $MSCP$ is closest to the ILP almost in all cases. The ratio of the average spectrum span of $MSCP$ to ILP is at most 1.28 demonstrating the closeness of the $MSCP$ to the optimal. The results in these experiments also show that $MSCP$ works better than KSP algorithm in almost all the cases even when number of paths in KSP is $K = 3$. We repeat similar experiments for larger value of k, where $k = 10$ and we change the value of d_{max} from 5 to 25. The result of these experiments is depicted in Fig. 2(c). It can be observed that spectrum span in $MSCP$ is at least 12% smaller than the span in KSP where $K = 1$ and it is even smaller than the one in KSP where $K = 2$. When $K = 3$ in KSP, KSP needs smaller spectrum span than $MSCP$ but its time complexity is at least 3 times $MSCP$.

We perform our next set of experiments on the Level-3 network shown in Fig. 3(a). In these experiments, first, we vary k from 10 to 60 with step of 10. For a specific value of k we generate 10 instances. In all these instances, the maximum demand is limited to 10 (i.e., $d_{max} \leq 10$). The average utilized spectrum span is shown in Fig. 3(b). These results show that $MSCP$ efficacy with respect to utilized spectrum span is almost the same as KSP when $K = 2$. We also conduct experiments for the case that values of d_{max} is varied from 5 to 25 with step of 5, while keeping the number of requests k constant at 20. We compute the average spectrum span over 10 random instances for each value of d_{max}. The results are shown in Fig. 3(c). According to these results, $MSCP$'s performance is better than KSP when $K = 2$ especially for larger values of d_{max}. According to the last experiments we may conclude that $MSCP$ outperforms KSP when $K = 2$ for larger values of d_{max}.



(a) (b) (c)

Fig. 2. (a) The 14-node NSF Network, (b) The average spectrum span in NSF Network for different values of k where $d_{max} \leq 5$, (c) different values of d_{max} where $k = 10$



(a) (b) (c)

Fig. 3. (a) Level-3 network over Europe, (b) The average spectrum span in Level-3 network for $d_{max} \leq 10$, (c) $k = 20$

VII. Conclusion

In this paper we study *on-line* version of *Routing and Spectrum Allocation* problem in OFDM-based optical networks. We propose an algorithm for the *ring network* with a *competitive ratio* of $\min\{O(\log(d_{max})), O(\log(k))\}$ where k is the total number of requests and d_{max} is the maximum demand. In addition, we provide a heuristic for networks with arbitrary topology and measure its effectiveness with extensive simulation. In future, we plan to develop efficient algorithms for on-line RSA in networks with tree and grid topologies. We also would like to extend our results to the case that different modulation models can be used.

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