

Final_Report

1. Project Description

Moir Foster was tasked with designing an experiment that measured helmet foam durability over long periods of time. The experiment contained three different chemical compositions of foam with two different porosity levels, to create six types of foam. Each of these foams were compressed at different frequencies, strains, and stresses over a period of time. Dr. Foster has hired **enter group name here.** to determine which foam type has the most consistent static stiffness over time with impact. The sample contains 71 different foam samples across the 6 combinations.

1.1 Research Questions

Question 1: Which combination of porosity and chemical index is best to predict static stiffness over time after accounting for all other variables?

1.2 Variables

To ensure that we are extracting the information needed, we created two new variables to be used in our analysis. First, we created a time variable that was calculated by taking the number of squishes performed on a foam sample in that current cycle and dividing it by the frequency of those squishes. Next, we created a relative static stiffness variable to act as our response variable. This was done by taking every measurement of static stiffness for a given foam sample and dividing it by the initial value of static stiffness for that foam sample. This helps with seeing how the static stiffness changes over time.

Below is a table of the variables we considered in our analysis:

Table 1: Variables used in Data Analysis

Name	Type	Notes
Sample Code	Categorical	Unique identifier for sample of foam
Strain	Numerical	Amount of strain tested on foam sample in kPa
Relative Porosity	Numerical	The measured porosity of the foam sample after the initial squish
Frequency	Numerical	The frequency at which the sample foam was squished
Amplitude	Numerical	The amplitude at which the sample foam was squished
Number of Squishes	Numerical	Number of squishes performed on foam at given time
Chemical Index	Categorical	Chemical index of given foam sample (Low, Medium, High)
Porosity	Categorical	Porosity of given foam sample (Low, High)
Relative Static Stiffness	Numerical	Static stiffness of given foam sample relative to the initial value of static stiffness
Time	Numerical	Time recorded at each measurement

2. Exploratory Data Analysis (EDA)

To explore what needs to be done for modeling, we perform some exploratory data analysis.

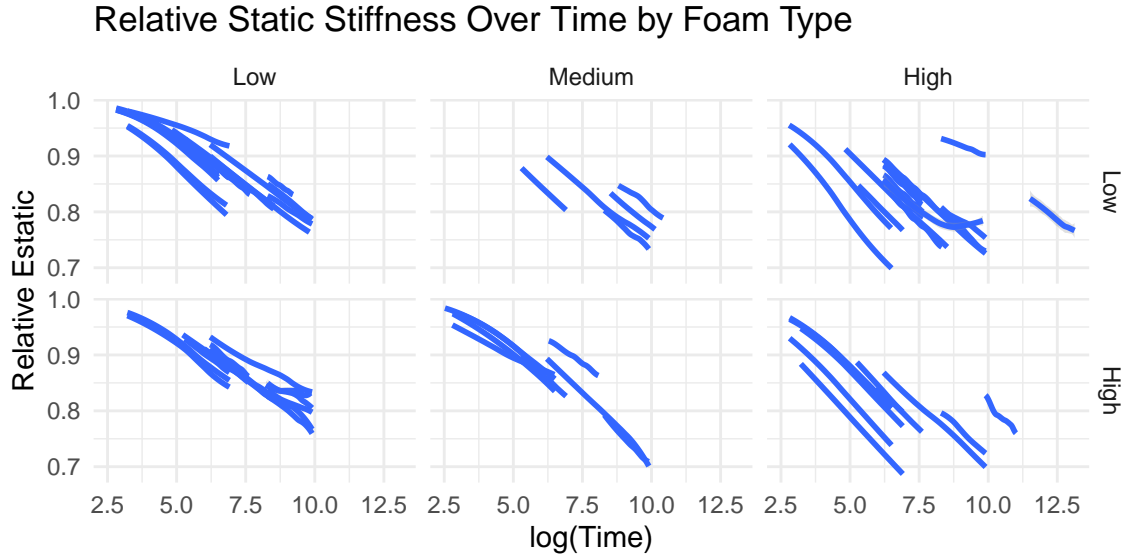


Figure 1: $\log(\text{Time})$ vs. Relative Static Stiffness for each Foam Type

Figure 1 shows relative static stiffness over $\log(\text{Time})$. We are using $\log(\text{Time})$ because the relationship between time and static stiffness was not linear. See Figure 6 in the appendix for more information. From this figure, we can see that there is a negative relationship between $\log(\text{Time})$ and relative static stiffness. It also appears that, depending on the foam type, there might be a different relationship. Some slopes are steeper than others, and some of the starting values of relative static stiffness are different than others which led us to believe that in the model we would need unique intercepts and slopes based on the foam type.

Next, we decided to look at the chemical index and porosity of the foams versus the relative static stiffness.

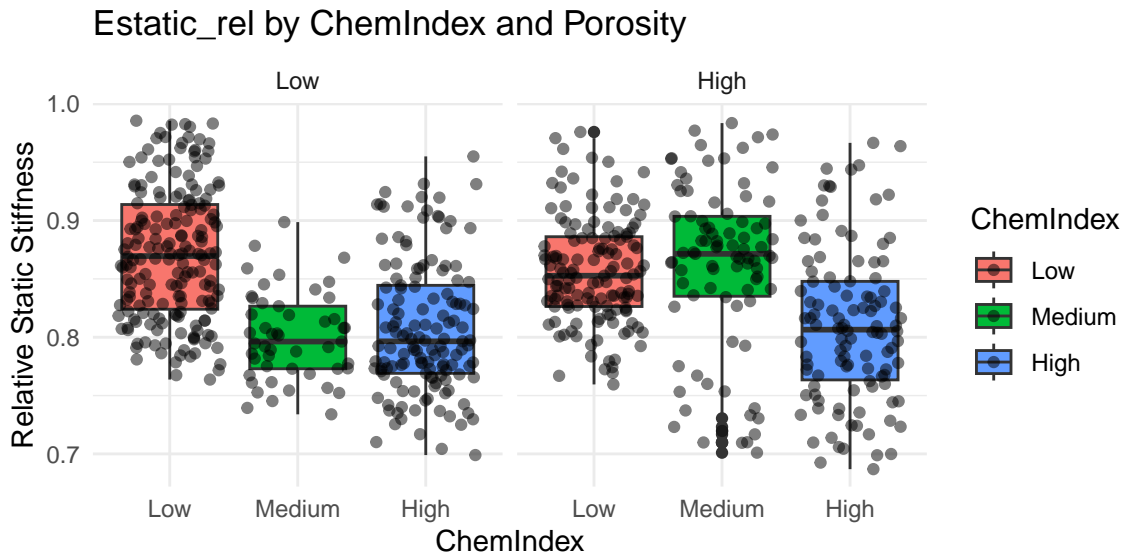


Figure 2: Chemical Index vs. Relative Static Stiffness for each level of Porosity

In Figure 2, the relationship between chemical index and relative static stiffness is different for the two levels of foam porosity. This indicates that there is likely a significant interaction between chemical index and porosity.

Next, we look at a correlation plot between all numerical variables.

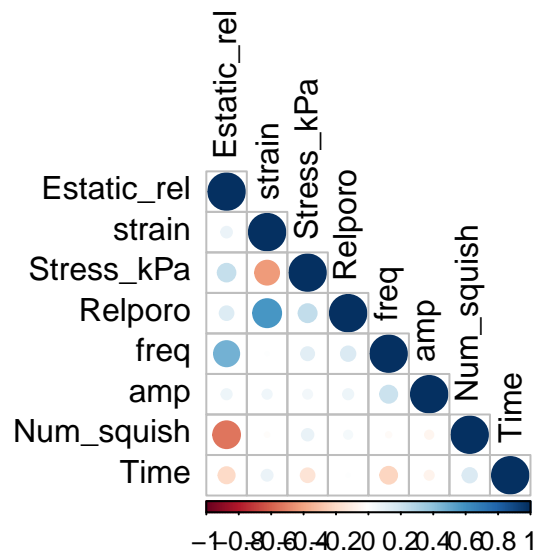


Figure 3: Correlation Matrix

Figure 3 gives us a good idea of what variables might need to be included in our final model.

As we can see, number of squishes and frequency are highly correlated with relative static stiffness. This is obvious, as relative static stiffness is going to have a relationship with time as we saw in Figure 1. We also see that relative porosity and strain are correlated, as well as stress and strain.

Lastly, we look at how frequency is related to relative static stiffness over time.

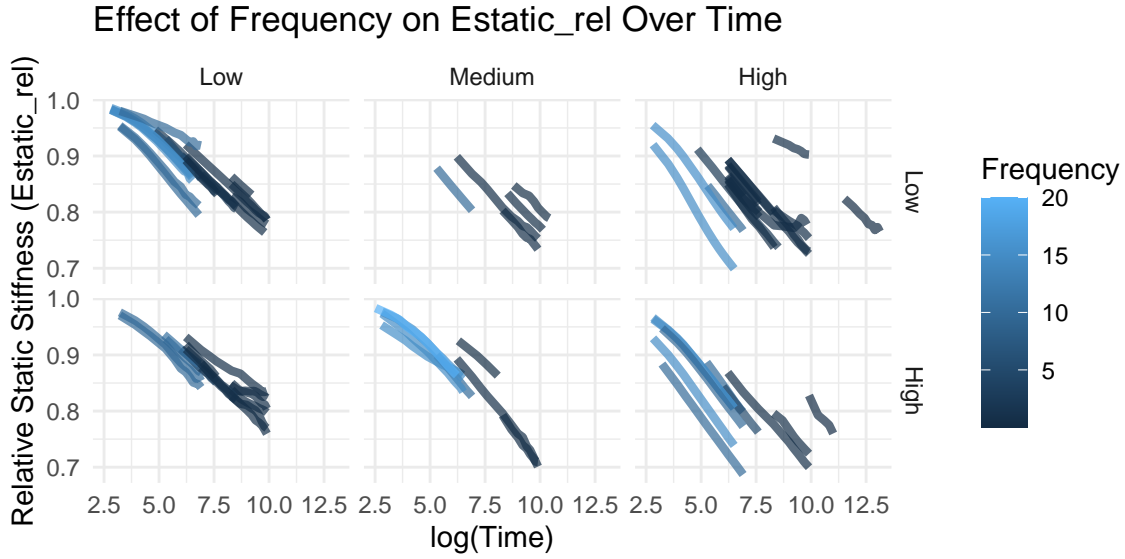


Figure 4: Frequency vs. Relative Static Stiffness over Time

In Figure 4, we can see that for lower frequencies, there is a higher relative static stiffness level recorded. This trend is the same across the different foam types. Frequency is a possible explanatory variable that we will consider in our final model.

3. Statistical Analysis

We will be using a mixed model for our analysis. A mixed model is used when there is variation

The linear mixed effects model is defined as:

$$\begin{aligned}
Estatic_rel_{ij} = & \underbrace{\beta_0}_{\text{fixed intercept}} + \underbrace{a_j}_{\text{random intercept for SampleCode}} \\
& + \beta_1 (\log(\text{Time}) \cdot \text{Porosity})_{ij} \\
& + \beta_2 (\log(\text{Time}) \cdot \text{Porosity} \cdot \text{ChemIndex})_{ij} \\
& + \beta_3 \cdot \text{freq}_{ij} + \beta_4 \cdot \text{numSquish}_{ij} + \varepsilon_{ij}
\end{aligned}$$

where:

β_0 is the fixed intercept $a_j \sim \mathcal{N}(0, \sigma_{\text{SampleCode}}^2)$ is the random intercept for group j (SampleCode) $\varepsilon_{ij} \sim \mathcal{N}(0, \sigma^2)$ is the residual error.

```

Linear mixed model fit by REML ['lmerMod']
Formula: Estatic_rel ~ log(Time):Porosity * ChemIndex + freq + Num_squish +
  (log(Time) | SampleCode)
Data: df_clean

```

REML criterion at convergence: -5086.3

Scaled residuals:

Min	1Q	Median	3Q	Max
-6.0458	-0.3427	-0.0347	0.3885	6.4327

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
SampleCode	(Intercept)	2.735e-03	0.052299	
	log(Time)	7.268e-05	0.008525	-0.75
Residual		1.217e-05	0.003489	

Number of obs: 700, groups: SampleCode, 71

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	1.116e+00	1.113e-02	100.208
ChemIndexMedium	2.858e-02	1.799e-02	1.589
ChemIndexHigh	2.821e-02	1.446e-02	1.950
freq	-3.864e-03	7.428e-04	-5.202
Num_squish	-7.914e-07	1.258e-07	-6.293
log(Time):PorosityLow	-3.231e-02	1.875e-03	-17.228
log(Time):PorosityHigh	-3.186e-02	1.990e-03	-16.009
log(Time):PorosityLow:ChemIndexMedium	-6.164e-03	3.652e-03	-1.688
log(Time):PorosityHigh:ChemIndexMedium	-4.527e-03	3.360e-03	-1.347

```
log(Time):PorosityLow:ChemIndexHigh    -1.055e-02  2.651e-03  -3.979
log(Time):PorosityHigh:ChemIndexHigh    -1.298e-02  2.968e-03  -4.373
```

Correlation of Fixed Effects:

```
(Intr) ChmInM ChmInH freq    Nm_sqs lg(T):PL lg(T):PH l(T):PL:CIM
ChemIndxMdm -0.452
ChemIndxHgh -0.613  0.367
freq        -0.413 -0.063  0.025
Num_squish   0.297  0.011 -0.035 -0.153
lg(Tm):PrsL -0.611  0.347  0.438 -0.010 -0.231
lg(Tm):PrsH -0.626  0.319  0.416  0.115 -0.227  0.431
l(T):PL:CIM  0.234 -0.603 -0.218  0.139 -0.005 -0.490  -0.184
l(T):PH:CIM  0.357 -0.644 -0.243 -0.112  0.006 -0.223  -0.569   0.373
l(T):PL:CIH  0.363 -0.252 -0.665  0.108  0.030 -0.680  -0.268   0.359
l(T):PH:CIH  0.416 -0.210 -0.595 -0.146  0.030 -0.257  -0.650   0.111
l(T):PH:CIM l(T):PL:CIH
```

ChemIndxMdm

ChemIndxHgh

freq

Num_squish

lg(Tm):PrsL

lg(Tm):PrsH

l(T):PL:CIM

l(T):PH:CIM

l(T):PL:CIH 0.147

l(T):PH:CIH 0.390 0.379

fit warnings:

Some predictor variables are on very different scales: consider rescaling

	numDF	denDF	F-value	p-value
(Intercept)	1	622	49403.47	<.0001
ChemIndex	2	67	4.69	0.0124
freq	1	67	30.35	<.0001
Num_squish	1	622	388.58	<.0001
log(Time):Porosity	2	622	560.63	<.0001
log(Time):Porosity:ChemIndex	4	622	6.47	<.0001

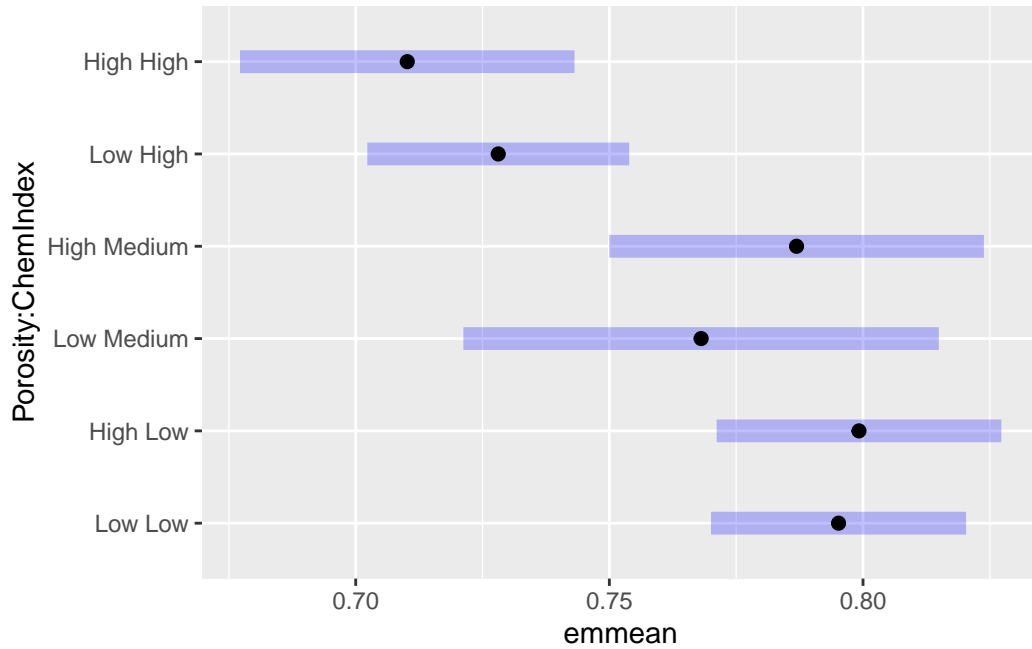


Figure 5: Emmeans Comparison by Porosity*ChemIndex

4. Recommendations

Question 1: Which combination of porosity and chemical index is best at keeping static stiffness stable over time, after accounting for all other variables?

5. Resources

6. Additional Considerations

Technical Appendix

Here we can see a clear nonlinear relationship between relative static stiffness and time. A logarithmic transformation is needed. See Figure 7.

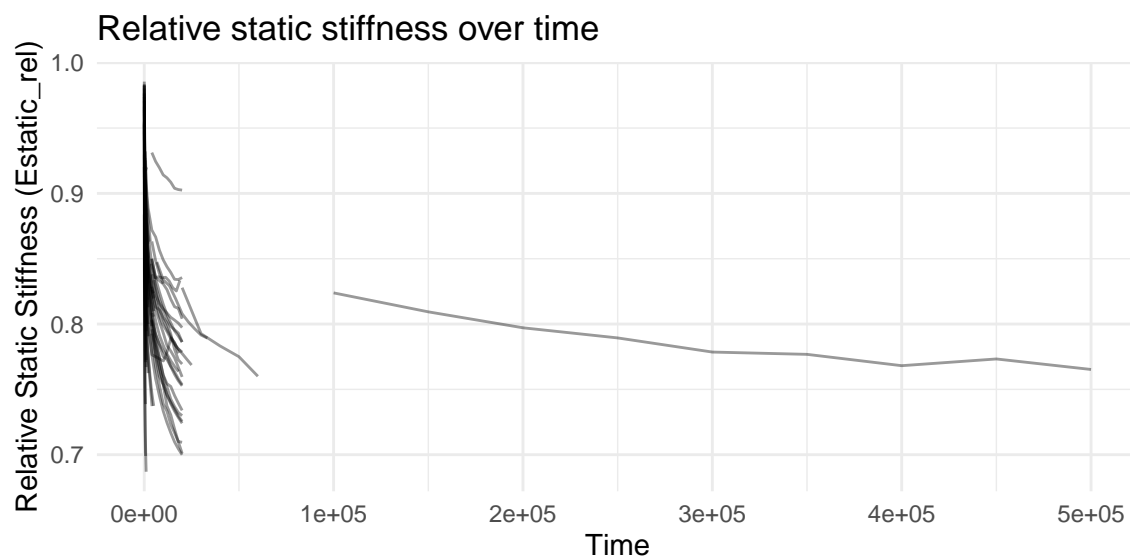


Figure 6: Time vs. Relative Static Stiffness

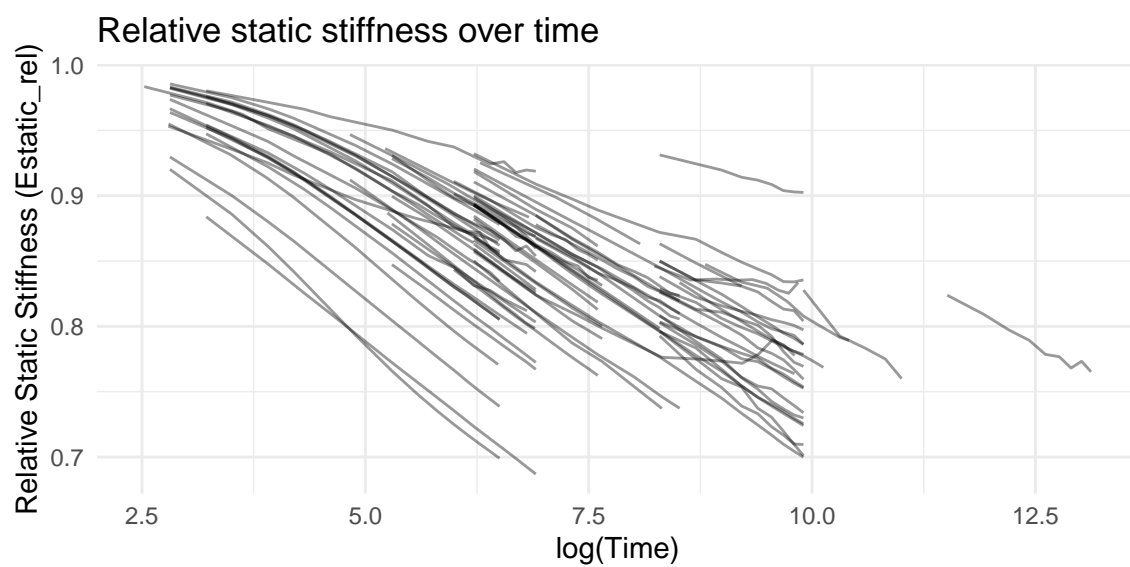
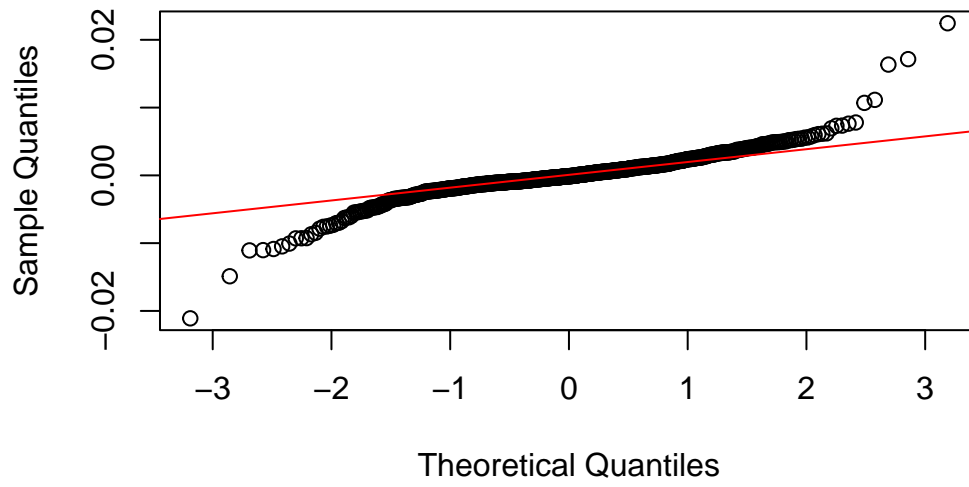
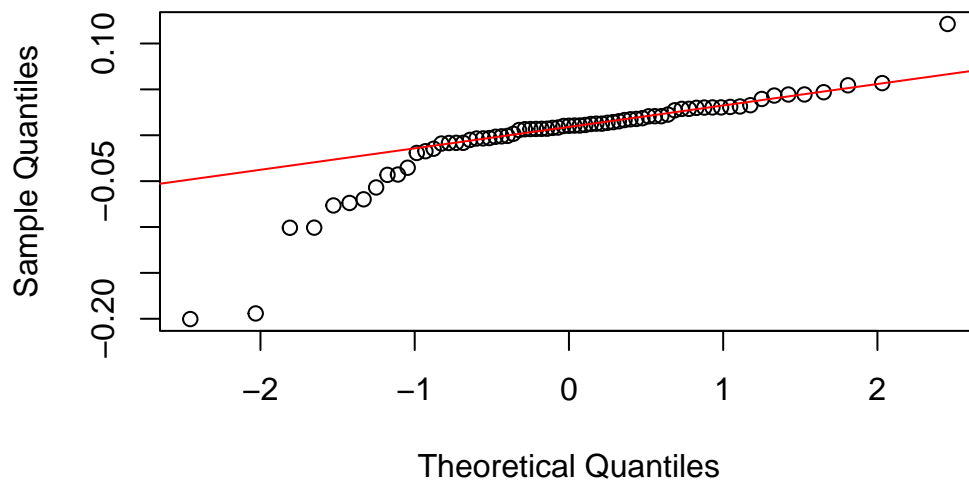


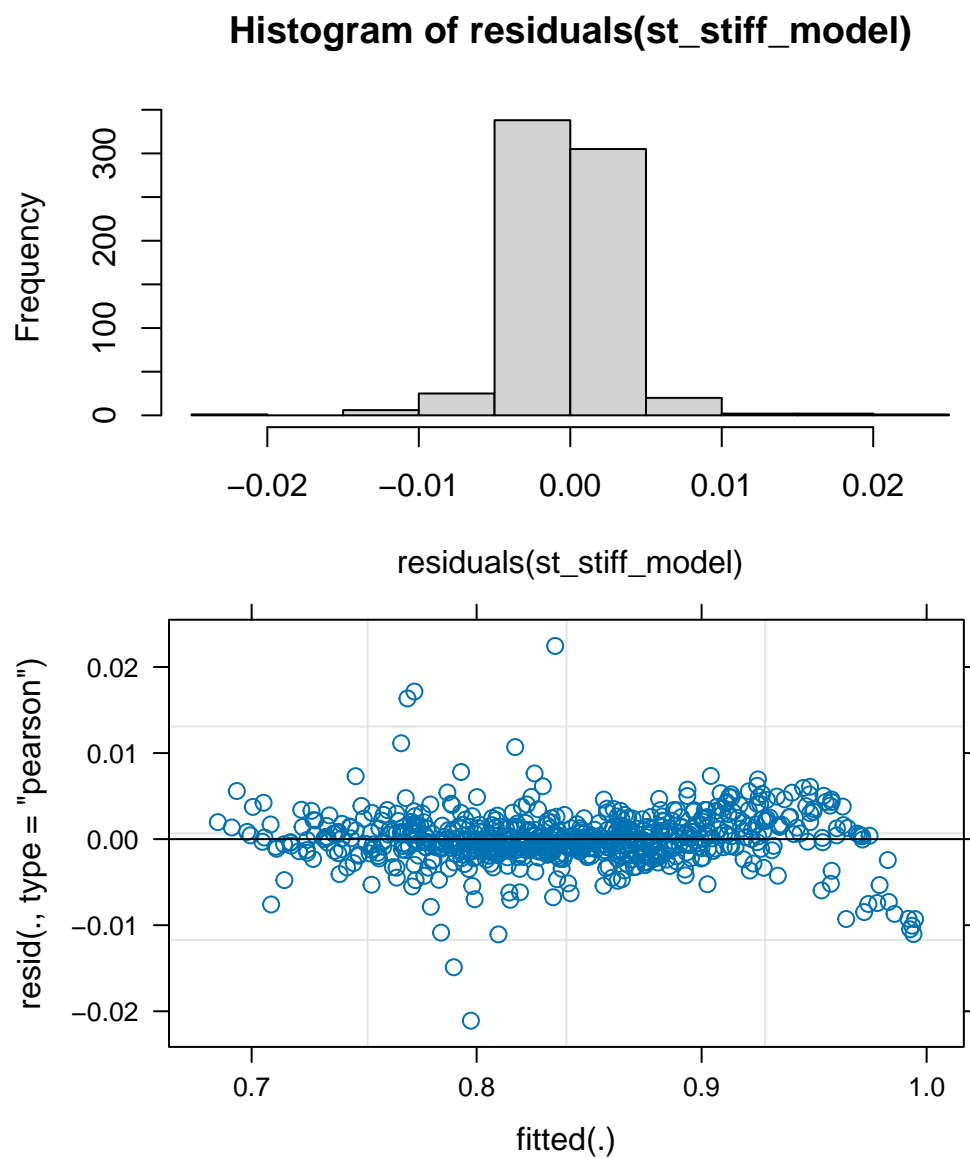
Figure 7: $\log(\text{Time})$ vs. Relative Static Stiffness

Normal Q-Q Plot: Residuals



Normal Q-Q Plot: Random Intercepts





R Script

```
# clean up & set default chunk options
rm(list = ls())
knitr::opts_chunk$set(echo = FALSE)

# packages
library(tidyverse) # for example
```

```

library(mosaic)      # for example
library(ggformula)  # for example
library(lme4)
library(ggplot2)
library(emmeans)
library(nlme)
library(readxl)
library(tinytex)
library(xtable)
library(knitr)
library(ggthemes)
library(GGally)
library(kableExtra)
library(emmeans)
library(Stat2Data)
library(corrplot)

# read in data

# use this space to do any data processing you need

# load data
df <- read_xlsx("Foster_Mines_Capstone_Data_v2.xlsx")

#Pivot all cycle-related columns into long format
df_long <- df %>%
  pivot_longer(
    cols = matches("(^Cyc_[A-Z]$)|(_Cyc_[A-Z]$)"),
    names_to = "FullName",      # give it a neutral name
    values_to = "Value"
  )

#Extract the cycle (e.g., Cyc_A) and variable type (e.g., E1_MPA)
df_tidy <- df_long %>%
  mutate(
    Cycle = str_extract(FullName, "Cyc_[A-Z]$"),
    VariableType = str_remove(FullName, "_Cyc_[A-Z]$"),
    VariableType = ifelse(VariableType == "Cyc", "Cycle_time", VariableType)
  ) %>%
  select(-FullName) %>%
  pivot_wider(

```

```

    names_from = VariableType,
    values_from = Value
  ) %>%
  arrange(SampleCode, Cycle)

head(df_tidy)

#Pivot the actual cycle time columns only
df_lt <- df_tidy %>%
  pivot_longer(
    cols = matches("^Cyc_[A-Z]$"), # Cyc_A to Cyc_N
    names_to = "Cycle_column",      # Temporary name to avoid collision
    values_to = "Num_squish"
  )

#Remove NA times
df_clean <- df_lt %>%
  filter(!is.na(Num_squish)) %>%
  select(-Cycle) %>%                #Drop the existing "Cycle" column and rename
  rename(Cycle = Cycle_column)

df_clean <- df_clean |>
  mutate(
    ChemIndex = as.numeric(str_split_fixed(SampleCode, "_", 4)[,2]),
    Porosity = as.numeric(str_split_fixed(SampleCode, "_", 4)[,3]))

df_clean <- df_clean %>%
  group_by(SampleCode) %>%
  mutate(
    Estatic_initial = first(Estatic_MPA), # First Estatic value per SampleCode
    Estatic_rel = Estatic_MPA / Estatic_initial # Relative to initial
  ) %>%
  ungroup()

df_clean <- df_clean |>
  select(-E1_MPA, -tandelta, -NVP)

df_clean <- df_clean %>%
  mutate(Porosity = case_when(
    Porosity == 71 ~ "Low",
    Porosity == 81 ~ "High"
  ))

```



```

variable.desc$Notes <- c("Unique identifier for sample of foam", "Amount of strain tested on
knitr::kable(variable.desc, format = "latex", booktabs = TRUE,
              col.names = c("Name", "Type", "Notes"),
              longtable = FALSE) %>%
  kable_styling( full_width = FALSE) %>%
  column_spec(3, width = "8cm") %>%
  add_header_above(c(" " = 3))
ggplot(df_clean, aes(x = log(Time), y = Estatic_rel, group = SampleCode)) +
  geom_smooth(alpha = 0.3) +
  # stat_summary(aes(group = 1), fun = mean, geom = "line", color = "black", size = 1) +
  facet_grid(Porosity ~ ChemIndex) +
  labs(title = "Relative Static Stiffness Over Time by Foam Type",
       x = "log(Time)",
       y = "Relative Estatic") +
  theme_minimal()
# grey lines are individual measurements

ggplot(df_clean, aes(x = ChemIndex, y = Estatic_rel, fill = ChemIndex)) +
  geom_boxplot() +
  geom_jitter(alpha = 0.5) +
  facet_wrap(~ Porosity) +
  labs(title = "Estatic_rel by ChemIndex and Porosity",
       y = "Relative Static Stiffness") +
  theme_minimal()

df_numeric <- df_clean %>%
  select(Estatic_rel, strain, Stress_kPa, Relporo, freq, amp, Num_squish, Time) # Adjust bas

# Compute the correlation matrix
corr_matrix <- cor(df_numeric, use = "complete.obs")

# Plot the correlation matrix
corrplot(corr_matrix, method = "circle", type = "lower", tl.col = "black")

ggplot(df_clean, aes(x = log(Time), y = Estatic_rel, color = freq)) +
  geom_line(aes(group = SampleCode), alpha = 0.7, linewidth = 1.5) +
  facet_grid(Porosity ~ ChemIndex) +
  labs(
    title = "Effect of Frequency on Estatic_rel Over Time",
    x = "log(Time)",
    y = "Relative Static Stiffness (Estatic_rel)",
    color = "Frequency"
  )

```

```

) +
  theme_minimal()
emmean <- emmeans(st_stiff_model, ~ Porosity * ChemIndex)
plot(emmean)

ggplot(df_clean, aes(x = Time, y = Estatic_rel)) +
  geom_line(aes(group = SampleCode), alpha = 0.4) +
  labs(
    title = "Relative static stiffness over time",
    x = "Time",
    y = "Relative Static Stiffness (Estatic_rel)",
  ) +
  theme_minimal()

ggplot(df_clean, aes(x = log(Time), y = Estatic_rel)) +
  geom_line(aes(group = SampleCode), alpha = 0.4) +
  labs(
    title = "Relative static stiffness over time",
    x = "log(Time)",
    y = "Relative Static Stiffness (Estatic_rel)",
  ) +
  theme_minimal()
# model diagnostic plots
qqnorm(residuals(st_stiff_model),
       main = "Normal Q-Q Plot: Residuals")
qqline(residuals(st_stiff_model), col = "red")

qqnorm(ranef(st_stiff_model)$SampleCode[[1]],
       main = "Normal Q-Q Plot: Random Intercepts")
qqline(ranef(st_stiff_model)$SampleCode[[1]], col = "red")

hist(residuals(st_stiff_model))

plot(st_stiff_model)

```