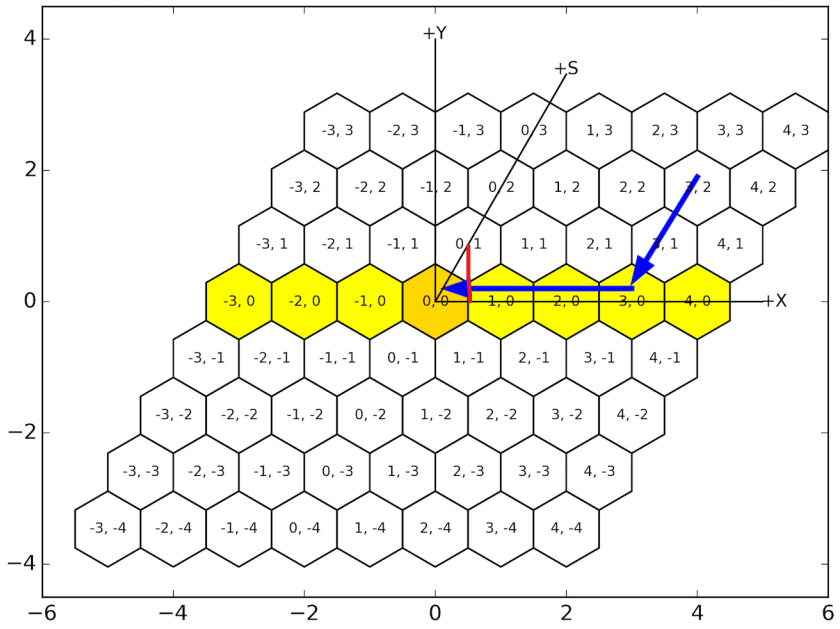
Hexagonal Grids

M.Lampton UCB SSL 26 Feb 2018 Draft 3

Purpose: Given an infinite plane grid of hexagons and a location (x,y) in the global or laboratory frame, what are that point's local (x, y) coordinates within its enclosing hexagon? This question arises in evaluating the optical behavior of hexagonal array lenses.

Coordinate System: I consider an infinite grid of regular unit hexagons in their 'Pointy Top' orientation as shown in the figure below. Each has a horizontal column index m and a row index n that increases along the slant axis S. The m, n addresses are shown. The blue arrows are the two reduction steps.



Rule 1: The pattern is periodic on S; adding any integer to S does not change the local (x,y).

Rule 2: The pattern is periodic in X; adding any integer to X does not change the local (x,y).

Python Implementation:

```
root3 = math.sqrt(3.0)
unitSlant = np.array([0.5, 0.5*root3])
unitHoriz = np.array([1.0, 0.0])

def getXYcenter(m, n):
    return m*unitHoriz + n*unitSlant

def getTopBot(xy):
    a = math.fabs(xy[0])
    b = math.fmod(a, 1.0)
    arg = b if b<0.5 else 1.-b
    top = (1.-arg)/root3
    bot = (arg-1.)/root3
    return top, bot
```

```
def reduce(xy):
    top, bot = getTopBot(xy)
    n = 0
    while xy[1] > top:
        xy -= unitSlant
        top, bot = getTopBot(xy)
        n += 1
    while xy[1] < bot:
        xy += unitSlant
        top, bot = getTopBot(x)
        n -= 1
    m = int(round(xy[0]))
    xy[0] -= m
    return xy[0], xy[1], m, n
```