The General Term of a Binomial

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Recall: $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$

$= \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n-1} a^1 b^{n-1} + \binom{n}{n} a^0 b^n$

$t_1 = \binom{n}{0} a^n b^0$

$t_2 = \binom{n}{1} a^{n-1} b^1$

$t_3 = \binom{n}{2} a^{n-2} b^2$

$t_k = \binom{n}{k-1} a^{n-k+1} b^{k-1}$ (ugly!)

$\boxed{t_{r+1} = \binom{n}{r} a^{n-r} b^r}$

General term of $(a+b)^n$

$\boxed{t_{r+1} = \binom{n}{r} a^{n-r} b^r}$

where:

$r, n \in \mathbb{Z}^+$

$0 \leq r \leq n$

Ex: Given $(3x-4)^5$ determine:

a) The # of terms in this expansion

b) The general term

c) t_5

d) coefficient of term 3

e) term containing x^4

a) 6 terms (no middle term, only middle if odd # terms)

b) $t_{r+1} = \binom{5}{r}(3x)^{5-r}(-4)^r$

$= \binom{5}{r}(3)^{5-r}(x)^{5-r}(-1)^r(4)^r$ sep. to find coefficients

$= \binom{5}{r} 3^{5-r}(-1)^r 4^r x^{5-r}$ $0 \leq r \leq 5$

c) $t_5 = t_{4+1}$

$t_5 = \binom{5}{4} 3^1 (-1)^4 4^4 x^1$

$t_5 = 3840x$

d) $t_3 = t_{2+1}$

coefficient $t_3 = \binom{5}{2} 3^3 (-1)^2 4^2$

$= 4320$

e) For t_{r+1}, x^r

$5-r = 4$

$r = 1$

$0 \leq r \leq 5$ valid

$$t_{1+1} = \binom{5}{1} 3^4 (-1)^1 (4)^1 x^4$$
$$= -1620x^4$$

Ex: Given $\left(x^2 - \frac{2}{x}\right)^6$ determine:

a) General term
b) Term containing x^9
c) The coefficient of t_4
d) Term containing x^2
e) Constant term

a) $t_{r+1} = \binom{6}{r}(x^2)^{6-r}(-2x^{-1})^r$
$t_{r+1} = \binom{6}{r}(x)^{12-2r}(-1)^r(2)^r(x)^{-r}$
$t_{r+1} = \binom{6}{r}(-1)^r(2)^r(x)^{12-3r}$

[start @ x^{12} + ↓ exponent by 3 each time
x^{-6} will be smallest
$x^{12} \rightarrow x^{-6}$]

b) $12-3r = 9$
$3 = 3r$
$1 = r$
∴ Term 2 has x^9 $\longrightarrow$ $t_2 = t_{1+1}$
$t_2 = \binom{6}{1}(-1)^1(2)^1(x)^9$
$t_2 = -12x^9$

c) $t_4 = t_{3+1}$
coefficient $t_4 = \binom{6}{3}(-1)^3(2)^3$
$= -160$

d) $12-3r = 2$
$10 = 3r$
$\frac{10}{3} = r$
invalid (not int)

e) constant term, exp on x is 0 (* constant may not necessarily be last term if exponents go negative)
$12-3r = 0$
$12 = 3r$
$4 = r$
∴ Term 5 $\quad t_5 = \binom{6}{4}(-1)^4(2)^4$
$= 240$