Number Systems

Exercise-1.1

① Is zero a rational number? Can you write it in the form p/q where p and q are integers and $q \neq 0$?

→ yes. zero is a rational number.

Ex:- $\frac{0}{1}$, $\frac{0}{2}$ etc.

② Find six rational numbers between 3 and 4.

→ $n = 6$ $\quad x = 3 \quad y = 4$

(Digit) $\boxed{d = \frac{y-x}{n+1}} = \frac{4-3}{6+1} = \frac{1}{7}$

$x + d = 3 + \frac{1}{7} = \frac{22}{7}$

$x + 2d = 3 + \frac{2}{7} = \frac{23}{7}$

$x + 3d = 3 + \frac{3}{7} = \frac{24}{7}$

$x + 4d = 3 + \frac{4}{7} = \frac{25}{7}$

$x + 5d = 3 + \frac{5}{7} = \frac{26}{7}$

$x + 6d = 3 + \frac{6}{7} = \frac{27}{7}$

$= \frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$ Ans

There are infinite rational numbers in between 3 and 4.

3 and 4 can be represented as $21/7$ and $\frac{28}{7}$ respectively.

Therefore, six rational numbers between 3 and 4 are

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$ Ans

③ Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

→ There are infinite rational numbers in between $\frac{3}{5}$ and $\frac{4}{5}$.

$$\frac{3}{5} = \frac{3\times6}{5\times6} = \frac{18}{30}$$

$$\frac{4}{5} = \frac{4\times6}{5\times6} = \frac{24}{30}$$

Therefore, five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are.

$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$ Ans

④ State wether the following statements are true or false. Give reasons for your answers

(i) Every natural number is a whole number.

→ True, since the collection of whole numbers contains all natural numbers.

(ii) Every Integer is a whole number.

→ False, as integer may be negative but whole numbers are always positive.

(iii) Every rational number is a whole number.

→ False, as rational numbers may be fractional but whole numbers may not be.

Exercise - 1.2

① State wether the following statements are true or false. Justify your answers.

ⓘ Every irrational number is a real number.

→ True.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

→ False.

(iii) Every real number is an irrational number.

→ False.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

→ NO, EX:- $\sqrt{4}$, $\sqrt{9}$ etc.

3. Show how $\sqrt{5}$ can be represented on the number line.

→ To represent $\sqrt{5}$ on the number line we take a length of two units from 0 on the number line in positive direction and one unit perpendicular to it. The hypotenuse of the triangle thus formed is of length $\sqrt{5}$. Then with the help of a divider a length equal to the hypotenuse of $\sqrt{5}$ units can be cut on the number line.

Exercise-1.3

(1) write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$

$= 0.36$, terminating. Ans

(ii) $\frac{1}{11}$

$= 0.\overline{09}$, recurring non-terminating. Ans

(iii) $4\frac{1}{8} = \frac{33}{8}$

$= 4.125$, terminating. Ans

(iv) $\frac{3}{13}$

$= 0.\overline{230769}$, recurring non-terminating. Ans

(v) $\frac{2}{11}$

$= 0.\overline{18}$, non-terminating recurring. Ans

(vi) $\frac{329}{400}$

$= 0.8225$, terminating. Ans

(2) you know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

$\rightarrow \frac{2}{7} = 2 \times \frac{1}{7}$

$= 2 \times 0.\overline{142857}$

$= 0.\overline{285714}$ Ans

$\frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$ Ans

$\frac{4}{7} = 4 \times \frac{1}{7} = 0.\overline{571428}$ Ans

$\frac{5}{7} = 5 \times \frac{1}{7} = 0.\overline{714285}$ Ans

$\frac{6}{7} = 6 \times \frac{1}{7} = 0.\overline{857142}$ Ans

(3) Express the following in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$

→ Let $x = 0.666\ldots\ldots$ —(i)

multiply 10 on both sides

$10x = 6.666\ldots\ldots$ —(ii)

subtract (i) from eqn (ii) →

$10x - x = 6$

$9x = 6$

$x = \frac{\cancel{6}^{2}}{\cancel{9}_{3}}$

$x = \frac{2}{3}$ Ans.

(ii) $0.4\overline{7}$

→ Let $x = 0.4777\ldots\ldots$

multiply 10 on both sides

$10x = 4.777\ldots\ldots$ —(i)

multiply 10 on both sides

$100x = 47.777\ldots\ldots$ —(ii)

subtract (i) from (ii) —

$100x - 10x = 17 - 4$

$90x = 13$

$x = \frac{13}{90}$ Ans

(iii) $x = 0.\overline{001}$

→ $x = 0.001001 \ldots$ —(i)

multiply 1000 on both sides

$1000x = 1.001001 \ldots$ —(ii)

subtract (i) from (ii) —

$1000x - x = 1$

$999x = 1$

$x = \frac{1}{999}$ Ans

(4) Express $0.999\ldots$ in the form $\frac{p}{q}$. Are you surprised by your answer? with your teacher and classmates discuss why the answer makes sense.

→ let $x = 0.999 \ldots$ —(i)

multiply 10 on both sides

$10x = 9.999 \ldots$

$10x = 9 + x$

$9x = 9$

$x = 1$ Ans

(5) what can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? perform the division to check your answer.

→ The maximum number of digits in the repeating block is 16 (<17).

$\frac{1}{17} = 0.\overline{0588235294117647}$

The repeating block has 16 digits. Ans

(6) Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 guess what property q must satisfy?

→ $\frac{P}{q}$ (Rational numbers)

For terminating decimal expansion q must have a property that prime factor of q must be in the form $2^m \times 5^n$

Ex: (i) 1.5

$= \frac{15}{10}$

$= \frac{15}{2^1 \times 5^1}$

(ii) $\frac{2.53}{100}$

$= \frac{253}{2^2 \times 5^2}$

(7) Write three numbers whose decimal expansion are non-terminating non-recurring.

→ 7.3141141114111114 ....

0.1010020030000 4 ....

$\pi = 3.1416....$ Ans

(8) Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

→ $\frac{5}{7} = 0.71$ $\frac{9}{11} = 0.81$

$7)\ 50\ (0.71$
-49
$\times 10$
-7
3

$11)\ 90\ (0.81$
-88
20
-11
9

0.71 and 0.81....

irrational = 0.72072007200072....
0.73073007300073....
0.74074007400074.... Ans

(9) classify the following numbers as rational or irrational.

(i) $\sqrt{23}$
= irrational number Ans

(ii) $\sqrt{225} = 15$
= Rational number Ans

(iii) 0.3796
= Rational number Ans

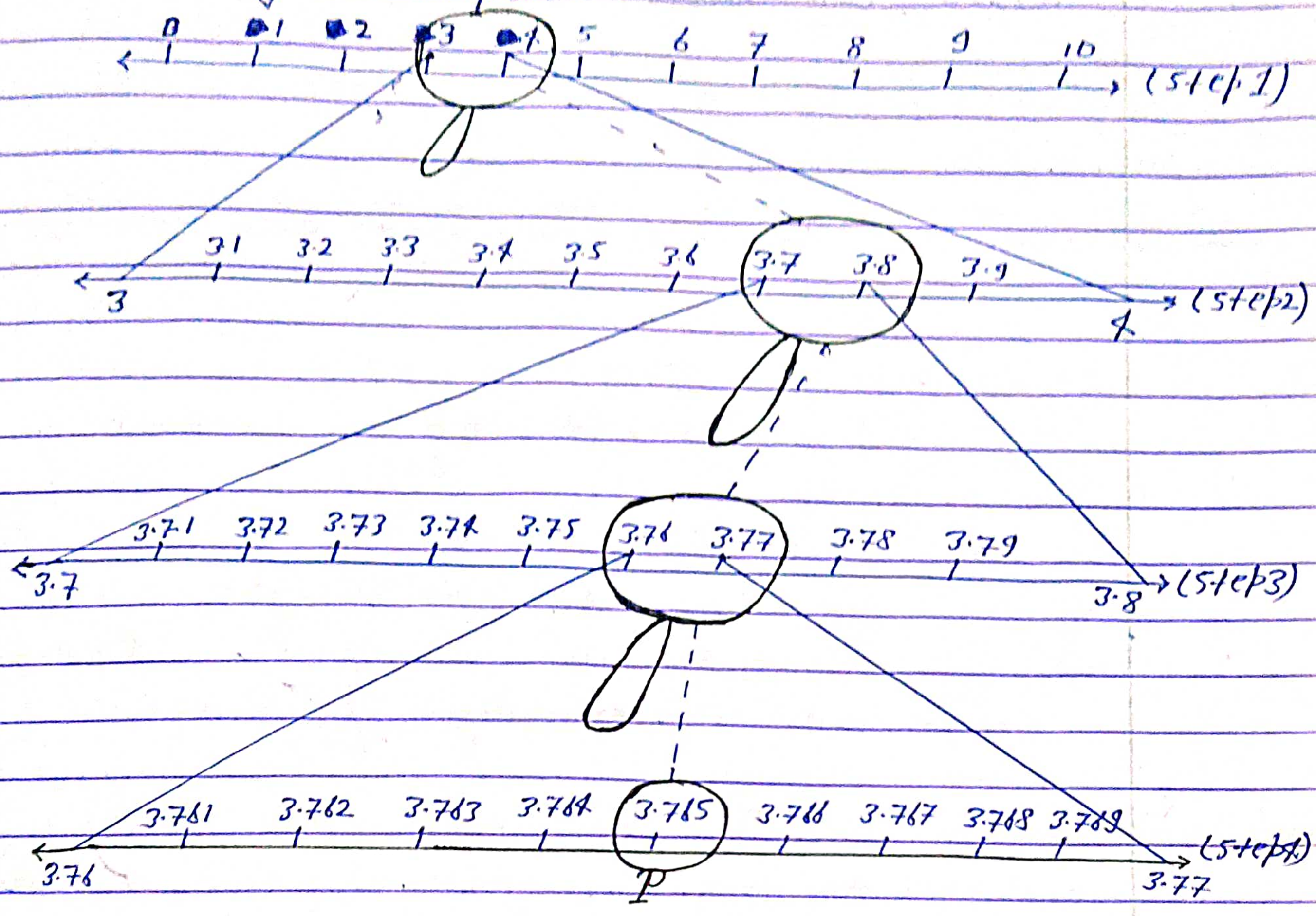
(iv) 7.478478....
= irrational number Ans

(v) 1.101001000100001....
= irrational number Ans

Exercise-1.4

① Visualise 3.765 on the number line, using successive magnification.

→ 3.765 can be visualised as in the following steps.



② visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

→ $4.\overline{26} = 4.2626$.

4.2626 can be visualised as in the following steps.

Exercise - 1.5

① classify the following numbers as rational or irrational.

(i) $2-\sqrt{5}$

$=$ irrational number. Ans

(ii) $(3+\sqrt{23})-\sqrt{23}$

$= 3+\cancel{\sqrt{23}}-\cancel{\sqrt{23}}$

$= 3$

Rational number. Ans

(iii) $\frac{2\cancel{\sqrt{7}}}{7\cancel{\sqrt{7}}}$

$= \frac{2}{7}$

Rational number. Ans

(iv) $\frac{1}{\sqrt{2}}$

$=$ irrational number. Ans.

(v) 2π

$=$ irrational number. Ans

② simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

$= 6+2\sqrt{3}+3\sqrt{2}+\sqrt{6}$ Ans

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

$= (3)^2-(\sqrt{3})^2$

$= 9-3$

$= 6$ Ans

(iii) $(\sqrt{5}+\sqrt{2})^2$

$= (\sqrt{5})^2 + (\sqrt{2})^2 + 2 \times \sqrt{5} \times \sqrt{2}$

$= 5 + 2 + 2\sqrt{10}$

$= 7 + 2\sqrt{10}$ Ans

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$= (\sqrt{5})^2 - (\sqrt{2})^2$

$= 5 - 2$

$= 3$ Ans

(3) Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

→ Let π be a rational number.

$\pi = \frac{c}{2R}$

$C = 2\pi R$

$\frac{C}{2R} = \pi$

$\pi = \frac{c}{d} \left(\frac{p}{q}\right)$

π = Rational number.

But it contradict the fact that π is an irrational number

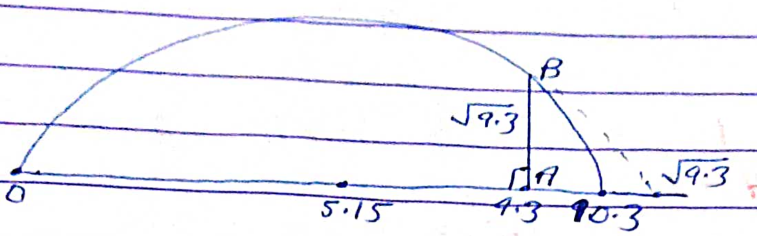
So our supposition is wrong

π is irrational number.

(4) Represent $\sqrt{9.3}$ on the number line.

→ To represent $\sqrt{9.3}$, draw a segment of 9.3 units on the number line. Let A represent 9.3. Extend it by 1 cm. Show point $\frac{10.3}{2}$

$= 5.15$ by on the number line. With 'O' as centre and radius 5.15 units, draw a semicircle. Draw AB perpendicular to OA to cut the hemisphere at B. The length AB is $\sqrt{9.3}$ units.



(5) Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

$= \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{\sqrt{7}}{7}$ Ans

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

$= \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$ Ans

(iii) $\dfrac{1}{\sqrt{5}+\sqrt{2}}$

$= \dfrac{1}{\sqrt{5}+\sqrt{2}} \times \dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \dfrac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}$

$= \dfrac{\sqrt{5}-\sqrt{2}}{5-2}$

$= \dfrac{\sqrt{5}-\sqrt{2}}{3}$ Ans

(iv) $\dfrac{1}{\sqrt{7}-2}$

$= \dfrac{1}{\sqrt{7}-2} \times \dfrac{\sqrt{7}+2}{\sqrt{7}+2}$

$= \dfrac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$

$= \dfrac{\sqrt{7}+2}{7-4}$

$= \dfrac{\sqrt{7}+2}{3}$ Ans

Exercise-1.6

(1) Find:

(i) $64^{\frac{1}{2}}$

$= (8^2)^{\frac{1}{2}}$

$= 8$ Ans

(ii) $32^{\frac{1}{5}}$

$= (2^5)^{\frac{1}{5}}$

$= 2$ Ans

(iii) $125^{\frac{1}{3}}$

$= (5^3)^{\frac{1}{3}}$

$= 5$ Ans

(2) Find:-

(i) $9^{\frac{3}{2}}$

$= (3)^{2\times\frac{3}{2}}$

$= (3)^3$

$= 27$ Ans

(ii) $32^{\frac{2}{5}}$

$= (2)^{5\times\frac{2}{5}}$

$= (2)^2$

$= 4$ Ans

(iii) $16^{\frac{3}{4}}$

$= (2)^{4\times\frac{3}{4}}$

$= (2)^3$

$= 8$ Ans

(iv) $125^{-\frac{1}{3}}$

$= (5)^{3\times-\frac{1}{3}}$

$= (5)^{-1}$

$= \frac{1}{5}$ Ans

(3) simplify:-

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

$= (2)^{\frac{2}{3}+\frac{1}{5}}$

$= (2)^{\frac{13}{15}}$ Ans

(ii) $\left(\frac{1}{3^3}\right)^7$

$= (3^{-3})^7$

$= (3)^{-21}$ Ans

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

$= (11)^{\frac{1}{2}-\frac{1}{4}}$

$= (11)^{\frac{1}{4}}$ Ans

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

$= (56)^{\frac{1}{2}}$ Ans