Polynomials-2.

Exercise-2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

→ Polynomial in one variable, x. ~~Ans~~

(ii) $y^2 + \sqrt{2}$

→ Polynomial in one variable, y. ~~Ans~~

(iii) $3\sqrt{t} + t\sqrt{2}$

→ $3\sqrt{t} + t\sqrt{2}$ is not a polynomial as power of t in $\sqrt{t}$ is not a whole number. ~~Ans~~

(iv) $y + \frac{2}{y}$

→ $y + 2y^{-1}$

$y + 2y^{-1}$ is not a polynomial as power of y in negative number. ~~Ans~~

(v) $x^{10} + y^3 + t^{50}$

→ $x^{10} + y^3 + t^{50}$ is not a polynomial in one variable but a polynomial in three variables x, y and t. ~~Ans~~

(2) write the coefficients of x^2 in each of the following.

(i) $2 + x^2 + x$

$\longrightarrow$ coefficient of $x^2 = 1$ Ans

(ii) $2 - x^2 + x^3$

$\longrightarrow$ coefficient of $x^2 = -1$ Ans

(iii) $\frac{\pi}{2} x^2 + x$

$\longrightarrow$ coefficient of $x^2 = \frac{\pi}{2}$ Ans

(iv) $\sqrt{2}x - 1$

$\longrightarrow$ coefficient of $x^2 = 0$ Ans

(3) Give one example each of a binomial of degree 35, and of a monomial of degree 100.

$\longrightarrow$ $x^{35} + 6$ is a binomial of degree 35.

$3y^{100}$ is a monomial of degree 100. Ans

(4) write the degree of each of the following polynomials.

(i) $5x^3 + 4x^2 + 7x$

$\longrightarrow$ Degree is 3 as x^3 is the highest power. Ans

(ii) $4 - y^2$

$\longrightarrow$ Degree is 2 as y^2 is the highest power. Ans

(iii) $5t - \sqrt{7}$

$\longrightarrow$ Degree is 1 as t is the highest power. Ans

(iv) 3

→ Degree is 0 as x^0 is the highest power. Ans

(5) Classify the following as linear, quadratic and cubic polynomials.

(i) $x^2 + x$

= Quadratic polynomial. Ans

(ii) $x - x^3$

= cubic polynomial. Ans

(iii) $y + y^2 + 4$

→ Quadratic polynomial. Ans

(iv) $1 + x$

= Linear polynomial. Ans

(v) $3t$

= Linear polynomial. Ans

(vi) r^2

= Quadratic polynomial. Ans

(vii) $7x^3$

= cubic polynomial. Ans

Exercise-2.2

(1.) Find the value of the polynomial $5x - 4x^2 + 3$ at

(i) $x = 0$

$\longrightarrow$ Let $P(x) = 5x - 4x^2 + 3$, at $x = 0$

$P(0) = 5 \times 0 - 4(0)^2 + 3$

$= 0 - 0 + 3$

$= 3$ Ans

(ii) $x = -1$

$\longrightarrow$ Let $P(x) = 5x - 4x^2 + 3$, at $x = -1$

$P(-1) = 5 \times (-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3$

$= -9 + 3$

$= -6$ Ans

(iii) $x = 2$

$\longrightarrow$ Let $P(x) = 5x - 4x^2 + 3$, at $x = 2$

$P(2) = 5 \times 2 - 4(2)^2 + 3$

$= 10 - 4 \times 4 + 3$

$= 10 - 16 + 3$

$= 13 - 16$

$= -3$ Ans

(2.) Find $P(0)$, $P(1)$ and $P(2)$ for each of the following polynomials.

(i) $P(y) = y^2 - y + 1$

$\longrightarrow$ $P(0) = (0)^2 - 0 + 1$

$= 1$ Ans

$P(1) = (1)^2 - \cancel{1} + \cancel{1}$

$= 1$ Ans

$$P(2) = (2)^2 - 2 + 1$$
$$= 4 - 2 + 1$$
$$= 3 \text{ Ans}$$

(ii) $P(t) = 2 + t + 2t^2 - t^3$

$\rightarrow$ $P(0) = 2 + 0 + 2(0)^2 - (0)^3$
$$= 2 \text{ Ans}$$
$$P(1) = 2 + 1 + 2(1)^2 - (1)^3$$
$$= 2 + \cancel{1} + 2 - \cancel{1}$$
$$= 4 \text{ Ans}$$
$$P(2) = 2 + 2 + 2(2)^2 - (2)^3$$
$$= 4 + 2 \times 4 - 8$$
$$= 4 + \cancel{8} - \cancel{8}$$
$$= 4 \text{ Ans}$$

(iii) $P(x) = x^3$

$\rightarrow$ $P(0) = (0)^3$
$$= 0 \text{ Ans}$$
$$P(1) = (1)^3$$
$$= 1 \text{ Ans}$$
$$P(2) = (2)^3$$
$$= 8 \text{ Ans}$$

(iv) $P(x) = (x-1)(x+1)$

$\rightarrow$ $P(x) = x^2 - 1$
$$P(0) = (0)^2 - 1 = -1 \text{ Ans}$$
$$P(1) = (1)^2 - 1 = 1 - 1 = 0 \text{ Ans}$$
$$P(2) = (2)^2 - 1$$
$$= 4 - 1 = 3 \text{ Ans}$$

③ verify whether the following are zeroes of the polynomial indicated against them.

(i) $P(x) = 3x+1$, $x = -\frac{1}{3}$

$\longrightarrow P\left(-\frac{1}{3}\right) = \cancel{3} \times -\frac{1}{\cancel{3}} + 1$

$= -\cancel{1} + \cancel{1}$

$= 0$ Ans

(ii) $P(x) = 5x - \pi$, $x = \frac{4}{5}$

$\longrightarrow P\left(\frac{4}{5}\right) = \cancel{5} \times \frac{4}{\cancel{5}} - \pi$

$= 4 - \pi$ Ans

(iii) $P(x) = x^2 - 1$, $x = 1, -1$.

$\longrightarrow P(1) = (1)^2 - 1$

$= 1 - 1$

$= 0$ Ans

$P(-1) = (-1)^2 - 1$

$= 1 - 1$

$= 0$ Ans

(iv) $P(x) = (x+1)(x-2)$, $x = -1, 2$.

$\longrightarrow P(-1) = (-1+1)(-1-2)$

$= 0 \times (-3)$

$= 0$ Ans

$P(2) = (2+1)(2-2)$

$= 3 \times 0$

$= 0$ Ans

(v) $P(x) = x^2$, $x = 0$

$\longrightarrow P(0) = (0)^2$

$= 0$ Ans

(vi) $P(x) = lx + m$; $x = \frac{-m}{l}$

$\longrightarrow P\left(\frac{-m}{l}\right) = \cancel{l} \times \frac{-m}{\cancel{l}} + m$

$= \cancel{-m} + \cancel{m}$

$= 0$ Ans

(vii) $P(x) = 3x^2 - 1$, $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

$\longrightarrow P\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1$

$= \cancel{3} \times \frac{1}{\cancel{3}} - 1$

$= 1 - 1$

$= 0$ Ans

(viii) $P(x) = 2x + 1$, $x = \frac{1}{2}$

$\longrightarrow P\left(\frac{1}{2}\right) = \cancel{2} \times \frac{1}{\cancel{2}} + 1$

$= 1 + 1$

$= 2$ Ans

4. Find the zero of the polynomial in each of the following cases.

(i) $P(x) = x + 5$

$\longrightarrow P(x) = 0$

$x + 5 = 0$

$x = -5$ Ans

(ii) $P(x) = x - 5$

$\longrightarrow P(x) = 0$

$x - 5 = 0$

$x = 5$ Ans.

(iii) $P(x) = 2x + 5$

$\longrightarrow P(x) = 0$

$2x + 5 = 0$

$2x = -5$

$x = \frac{-5}{2}$ Ans

~~(vii) cx+d~~

(vii) $P(x) = cx + d$, $c \neq 0$, c, d are real numbers.

$\longrightarrow P(x) = 0$

$cx + d = 0$

$cx = -d$

$x = \frac{-d}{c}$ Ans

(iv) $P(x) = 3x - 2$

$\longrightarrow P(x) = 0$

$3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$ Ans

(v) $P(x) = 3x$

$\longrightarrow P(x) = 0$

$3x = 0$

$x = \frac{0}{3}$

$x = 0$ Ans

(vi) $P(x) = ax$, $a \neq 0$

$\longrightarrow ax = 0$

$x = \frac{0}{a}$

$x = 0$ Ans

Exercise-2.3

(1) Find the remainder when x^3+3x^2+3x+1 is divided by

(i) $x+1$

$\rightarrow$ $x+1)\ x^3+3x^2+3x+1\ (x^2+2x+1$

$\quad -\cancel{x^3}+x^2$

$\quad 2x^2+3x+1$

$\quad -\cancel{2x^2}+2x$

$\quad x+1$

$\quad -x+1$

$\quad 0$

$\therefore$ Remainder $= 0$ Ans

(ii) $x-\frac{1}{2}$

$\rightarrow$ $2x-1$

$2x-1)\ x^3+3x^2+3x+1\ \left(\frac{1}{2}x^2+\frac{7x}{4}+\frac{19}{8}\right.$

$\quad -\cancel{x^3}-\frac{1}{2}x^2$ (+)

$\quad \frac{7}{2}x^2+3x+1$

$\quad -\frac{7}{2}x^2-\frac{7}{4}x$ (+)

$\quad \frac{19}{4}x+1$

$\quad -\frac{19}{4}x-\frac{19}{8}$ (+)

$\quad \frac{27}{8}$

$\therefore$ Remainder $= \frac{27}{8}$ Ans

(iii) x

$\longrightarrow$ $x\,)\ \cancel{x^3}+3x^2+3x+1\ (\,x^2+3x+3$

$$-\cancel{x^3}$$

$$\cancel{3x^2}+3x+1$$

$$-\cancel{3x^2}$$

$$\cancel{3x}+1$$

$$-\cancel{3x}$$

$$1$$

$\therefore$ Remainder $= 1$ Ans

(iv) $x+\pi$

$\longrightarrow$ $x+\pi\,)\ \cancel{x^3}+3x^2+3x+1\ (\,x^2+(3-\pi)x+(3-3\pi+\pi^2)$

$$-\cancel{x^3} \underset{-}{+} \pi x^2$$

$$\cancel{(3-\pi)x^2}+3x+1$$

$$-\cancel{(3-\pi)x^2} \underset{-}{+} (3\pi-\pi^2)x$$

$$\cancel{(3-3\pi+\pi^2)x}+1$$

$$-\cancel{(3-3\pi+\pi^2)x} \underset{-}{+} \pi^3-3\pi^2+3\pi$$

$$-\pi^3+3\pi^2-3\pi+1$$

$\therefore$ Remainder $= -\pi^3+3\pi^2-3\pi+1$ Ans

(v) $5+2x$

$$2x+5\overline{)\,x^3+3x^2+3x+1\,}\left(\frac{1}{2}x^2+\frac{1}{4}x+\frac{7}{8}\right.$$

$$\cancel{x^3}+\frac{5}{2}x^2$$
$$-\quad -$$

$$\frac{1}{2}x^2+3x+1$$

$$\cancel{\frac{1}{2}x^2}+\frac{5}{4}x$$
$$-\quad -$$

$$\frac{7}{4}x+1$$

$$\cancel{\frac{7}{4}x}+\frac{35}{8}$$
$$-\quad -$$

$$-\frac{27}{8}$$

$\therefore$ Remainder $= \frac{-27}{8}$ ~~Ans~~

(2) Find the remainder when x^3-ax^2+6x-a is divided by $x-a$.

$$x-a\overline{)\,x^3-ax^2+6x-a\,}\left(x^2+6\right.$$

$$\cancel{x^3}-ax^2$$
$$-\quad +$$

$$6x-a$$

$$\cancel{6x}-6a$$
$$-\quad +$$

$$5a$$

$\therefore$ Remainder $= 5a$ ~~Ans~~

3. ~~Find the~~ check whether $7+3x$ is a factor of $3x^3+7x$.

$\rightarrow$ $p(x) = 3x^3 + 7x$

$$7+3x=0$$
$$3x=-7$$
$$x=-\frac{7}{3}$$

$$p\left(-\frac{7}{3}\right) = 3\left(-\frac{7}{3}\right)^3 + 7\times -\frac{7}{3}$$

$$= \cancel{3}\times\frac{-343}{\cancel{27}_{9}} - \frac{49}{3}$$

$$= \frac{-343}{9} - \frac{49}{3}$$

$$= \frac{-343-147}{9}$$

$$= \frac{-490}{9}$$

$$R \neq 0$$

$7+3x$ is not a factor of $p(x)$ ~~Ans~~

Exercise-2.4

① Determine which of the following polynomials has $(x+1)$ a factor.

(i) x^3+x^2+x+1

$\rightarrow$ $P(x)=x^3+x^2+x+1$

$\begin{cases} x+1=0 \\ x=-1 \end{cases}$

$P(-1)=(-1)^3+(-1)^2-\cancel{1}+\cancel{1}$

$=-\cancel{1}+\cancel{1}$

$=0$

$x+1$ is a factor of $P(x)$. Ans

(ii) $x^4+x^3+x^2+x+1$

$\rightarrow$ $P(x)=x^4+x^3+x^2+x+1$

$\begin{cases} x+1=0 \\ x=-1 \end{cases}$

$P(-1)=(-1)^4+(-1)^3+(-1)^2-\cancel{1}+\cancel{1}$

$=1-\cancel{1}+\cancel{1}$

$=1$

$P(-1)\neq 0$

$\therefore$ $(x+1)$ is not a factor of $P(x)$. Ans

(iii) $x^4+3x^3+3x^2+x+1$

$\rightarrow$ $P(x)=x^4+3x^3+3x^2+x+1$

$\begin{cases} x+1=0 \\ x=-1 \end{cases}$

$P(-1)=(-1)^4+3(-1)^3+3(-1)^2-\cancel{1}+\cancel{1}$

$=+1-\cancel{3}+\cancel{3}$

$P(-1)\neq 0$

$\therefore$ $(x+1)$ is not a factor of $P(x)$ Ans

(iv) $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

$\begin{cases} x+1=0 \\ x=-1 \end{cases}$

$\rightarrow$ $P(x)=x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

$P(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

$=-\cancel{1}-\cancel{1}+\cancel{2}+\sqrt{2}+\sqrt{2}$

$=2\sqrt{2}$

$P(-1)\neq 0$

$\therefore$ $(x+1)$ is not a factor of $P(x)$ Ans

② Use the factor theorem to determine wether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $P(x) = 2x^3 + x^2 - 2x - 1, \quad g(x) = x + 1$

$\rightarrow P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= \cancel{-2} + \cancel{1} + \cancel{2} - \cancel{1}$

$= 0$

$R = 0$

$\left\{\begin{array}{l} g(x = x+1 \\ x+1 = 0 \\ x = -1 \end{array}\right.$

$\therefore$ $g(x)$ is a factor of $P(x)$. Ans

(ii) $P(x) = x^3 + 3x^2 + 3x + 1, \quad g(x) = x + 2$

$\rightarrow P(-2) = (-2)^3 + 3(-2)^2 + 3\times(-2) + 1$

$= -8 + 3\times 4 - 6 + 1$

$= -8 + 12 - 6 + 1$

$= -14 + 13$

$= -1$

$R \neq 0$

$\left\{\begin{array}{l} g(x) = x+2 \\ x+2 = 0 \\ x = -2 \end{array}\right.$

$\therefore$ $g(x)$ is not factor of $p(x)$ Ans

(iii) $P(x) = x^3 - 4x^2 + x + 6, \quad g(x) = x - 3$

$\rightarrow P(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 4\times 9 + 3 + 6$

$= 36 - 36$

$= 0$

$R = 0$

$\left\{\begin{array}{l} g(x) = x-3 \\ x-3 = 0 \\ x = 3 \end{array}\right.$

$\therefore$ $g(x)$ is a factor of $P(x)$ Ans

(3) Find the value of k, if x-1 is a factor of P(x) in each of the following cases:

(i) $P(x) = x^2 + x + k$

$\begin{cases} x-1=0 \\ x=1 \end{cases}$

$\rightarrow P(1) = (1)^2 + 1 + k$

$0 = 1 + 1 + k$

$0 = 2 + k$

$\boxed{k = -2}$ Ans

(ii) $P(x) = 2x^2 + kx + \sqrt{2}$

$\begin{cases} x-1=0 \\ x=1 \end{cases}$

$\rightarrow P(1) = 2(1)^2 + k \times 1 + \sqrt{2}$

$0 = 2 + k + \sqrt{2}$

$-2 - \sqrt{2} = k$

$k = -(2 + \sqrt{2})$ Ans

(iii) $P(x) = kx^2 - \sqrt{2}x + 1$

$\begin{cases} x-1=0 \\ x=1 \end{cases}$

$\rightarrow P(1) = k(1)^2 - \sqrt{2} \times 1 + 1$

$0 = k - \sqrt{2} + 1$

$k = \sqrt{2} - 1$ Ans

(iv) $P(x) = kx^2 - 3x + k$

$\begin{cases} x-1=0 \\ x=1 \end{cases}$

$\rightarrow P(1) = k(1)^2 - 3 \times 1 + k$

$0 = k - 3 + k$

$\cancel{-}2k = \cancel{-}3$

$k = \frac{3}{2}$ Ans

④ Factorise:

(i) $12x^2 - 7x + 1$

$\rightarrow 12x^2 - 4x - 3x + 1$

$= 4x(3x-1) - 1(3x-1)$

$= (3x-1)(4x-1)$ Ans

(ii) $2x^2 + 7x + 3$

$\rightarrow 2x^2 + 6x + x + 3$

$= 2x(x+3) + 1(x+3)$

$= (x+3)(2x+1)$ Ans

(iii) $6x^2 + 5x - 6$

$\rightarrow 6x^2 + 9x - 4x - 6$

$= 3x(2x+3) - 2(2x+3)$

$= (2x+3)(3x-2)$ Ans

(iv) $3x^2 - x - 4$

$\rightarrow 3x^2 - 4x + 3x - 4$

$= x(3x-4) + 1(3x-4)$

$= (3x-4)(x+1)$ Ans

⑤ Factorise:

(i) $x^3 - 2x^2 - x + 2$

$\rightarrow$ By trial and error method

$P(x) = x^3 - 2x^2 - x + 2$

$P(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$= -1 - 2\times 1 + 1 + 2$

$= \cancel{-1} \cancel{-2} \cancel{+1} \cancel{-2}$

$P(-1) = 0$

$x = -1$

$(x+1)$ is a factor of $P(x)$

$$
\begin{array}{r|l}
x+1 & x^3 - 2x^2 - x + 2 \;(x^2 - 3x + 2 \\
 & \underline{-x^3 \pm x^2} \\
 & -3x^2 - x + 2 \\
 & \underline{\mp 3x^2 \mp 3x} \\
 & 2x + 2 \\
 & \underline{-2x \pm 2} \\
 & 0
\end{array}
$$

$= x^2 - 3x + 2$

$= x^2 - 2x - x + 2$

$= x(x-2) - 1(x-2)$

$= (x-2)(x-1)$

$= (x+1)(x-1)(x-2)$ Ans

(ii) $x^3 - 3x^2 - 9x - 5$

→ By trial and error method

$P(x) = x^3 - 3x^2 - 9x - 5$

$P(-1) = (-1)^3 - 3(-1)^2 - 9 \times (-1) - 5$

$= -1 - 3 \times 1 + 9 - 5$

$= -1 - 3 + 9 - 5$

$= 9 - 9$

$P(-1) = 0$

$x = -1$

$(x+1)$ is a factor of $P(x)$

$$
\begin{array}{r|l}
x+1 & x^3 - 3x^2 - 9x - 5 \;(x^2 - 4x - 5 \\
 & \cancel{x^3} \pm x^2 \\
 & \hline -4x^2 - 9x - 5 \\
 & \cancel{-4x^2} - 4x \\
 & \hline -5x - 5 \\
 & -5x - 5 \\
 & \hline 0
\end{array}
$$

$= x^2 - 4x - 5$

$= x^2 - 5x + x - 5$

$= x(x-5) + 1(x-5)$

$= (x-5)(x+1)$

$= (x+1)(x-1)(x-5)$ Ans

(iii) $x^3 + 13x^2 + 32x + 20$

→ By trial and error method

$P(x) = x^3 + 13x^2 + 32x + 20$

$P(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33$

$P(-1) = 0$

$x = -1$

$(x+1)$ is a factor of $P(x)$

$$
\begin{array}{r l}
x+1 \,\big) & x^3 + 13x^2 + 32x + 20 \,\big(\, x^2 + 12x + 20 \\
& \underline{-\cancel{x^3} \pm x^2} \\
& \quad 12x^2 + 32x \\
& \quad \underline{-\cancel{12x^2} \pm 12x} \\
& \qquad 20x + 20 \\
& \qquad \underline{-\cancel{20x} \pm 20} \\
& \qquad\qquad 0
\end{array}
$$

$= x^2 + 12x + 20$

$= x^2 + 10x + 2x + 20$

$= x(x+10) + 2(x+10)$

$= (x+10)(x+2)$

$= (x+1)(x+10)(x+2)$ Ans

(iv) $2y^3 + y^2 - 2y - 1$

→ By trial and error method

$P(y) = 2y^3 + y^2 - 2y - 1$

$P(-1) = 2(-1)^3 + (-1)^2 - 2\times(-1) - 1$

$= \cancel{-2} \cancel{+1} \cancel{+2} \cancel{-1}$

$P(-1) = 0$

$y = -1$

$(y+1)$ is a factor of $P(y)$

$$y+1)\ 2y^3 + y^2 - 2y - 1\ (2y^2 - y - 1$$
$$\cancel{2y^3} + 2y^2$$
$$\underline{-\quad -}$$
$$\cancel{-y^2} - 2y - 1$$
$$\cancel{\mp y^2} \mp y$$
$$\cancel{-y} - 1$$
$$\cancel{\mp y \mp 1}$$
$$0$$

$= 2y^2 - y - 1$

$= 2y^2 - 2y + y - 1$

$= 2y(y-1) + 1(y-1)$

$= (y-1)(2y+1)$

$= (y+1)(y-1)(2y+1)$ Ans

Exercise-2.5

① Use suitable identities to find the following products:

ⓘ $(x+4)(x+10)$

→ Using identity,

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(x+4)(x+10) = x^2+(4+10)x+4\times10$

$= x^2+14x+40$ Ans

(ii) $(x+8)(x-10)$

→ Using identity,

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(x+8)(x-10) = x^2+(8-10)x+8\times(-10)$

$= x^2-2x-80$ Ans

(iii) $(3x+4)(3x-5)$

→ Using identity,

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(3x+4)(3x-5) = (3x)^2+(4-5)3x+4\times-5$

$= 9x^2-1\times3x-20$

$= 9x^2-3x-20$ Ans

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

→ Using identity,

$(x+a)(x-a) = x^2-a^2$

$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right) = (y^2)^2-\left(\frac{3}{2}\right)^2$

$= y^4-\frac{9}{4}$ Ans

(v) $(3-2x)(3+2x)$

$\longrightarrow$ using identity,

$(x-a)(x+a) = x^2 - a^2$

$(3-2x)(3+2x) = (3)^2 - (2x)^2$

$= 9 - 4x^2$. Ans

(2) Evaluate the following products without multiplying directly:

(i) 103×107

$\longrightarrow$ $(100+3)(100+7)$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$(100+3)(100+7) = (100)^2 + (3+7)100 + 3 \times 7$

$= 10000 + 10 \times 100 + 21$

$= 10000 + 1000 + 21$

$= 11021$ Ans

(ii) 95×96

$\longrightarrow$ $(100-5)(100-4)$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$(100-5)(100-4) = (100)^2 + (-5-4) \times 100 + (-5) \times (-4)$

$= 10000 - 9 \times 100 + 20$

$= 10000 - 900 + 20$

$= 9120$ Ans

(iii) 104×96

$\longrightarrow$ $(100+4)(100-4)$

$(x+a)(x-a) = x^2 - a^2$

$= (100)^2 - (4)^2$

$= 10000 - 16$

$= 9984$ Ans

(3.) Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

$\longrightarrow (3x)^2 + 2(3x).(y) + (y)^2$

$= (a)^2 + 2ab + (b)^2 = (a+b)^2$

$= (3x+y)^2$

$= (3x+y)(3x+y)$ Ans

(ii) $4y^2 - 4y + 1$

$\longrightarrow (2y)^2 - 2 \times 2y \times 1 + (1)^2$

$= (a)^2 - 2ab + (b)^2 = (a-b)^2$

$= (2y-1)^2$

$= (2y-1)(2y-1)$ Ans

(iii) $x^2 - \frac{y^2}{100}$

$\longrightarrow (x)^2 - \left(\frac{y}{10}\right)^2$

$\Rightarrow (x)^2 - (a)^2 = (a+b)(a-b)$

$\Rightarrow (x)^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$ Ans

(4.) Expand each of the following, using suitable identities.

(i) $(x+2y+4z)^2$

$\longrightarrow (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(x+2y+4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2 \times x \times 2y + 2 \times 2y \times 4z$
$+ 2 \times 4z \times x$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$ Ans

(ii) $(2x - y + z)^2$

$\longrightarrow (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (2x)^2 + (-y)^2 + (z)^2 + 2 \times 2x \times (-y) + 2 \times (-y) \times z$
$+ 2 \times z \times 2x$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$ Ans

(iii) $(-2x + 3y + 2z)^2$

$\longrightarrow (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2 \times (-2x) \times 3y + 2(3y)(2z)$
$+ 2(2z)(-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$ Ans

(iv) $(3a - 7b - c)^2$

$\longrightarrow (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2 \times 3a \times (-7b) + 2(-7b) \times (-c)$
$+ 2(-c) \times 3a$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$ Ans

(v) $(-2x + 5y - 3z)^2$

$\longrightarrow (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \times (-2x)(5y) + 2(5y)(-3z)$
$+ 2(-3z)(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$ Ans

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

$\longrightarrow (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + \cancel{2} \times \frac{1}{\cancel{4}_2}a \times \left(-\frac{1}{2}b\right) + \cancel{2}\left(-\frac{1}{\cancel{2}}b\right) \cdot 1$
$+ \cancel{2} \times 1 \times \frac{1}{\cancel{4}_2}a$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$ Ans

(5) Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$\longrightarrow (2x)^2 + (3y)^2 + (4z)^2 + 2(2x)(3y) - 2(3y)(4z) - 2(4z)(2x)$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (a+b+c)^2$

$= (2x + 3y - 4z)^2$

$= (2x+3y-4z)(2x+3y-4z)$ Ans

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$\longrightarrow (\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 - 2(\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) - 8(\sqrt{2}x)(2\sqrt{2}z)$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (a+b+c)^2$

$= (\sqrt{2}x - y - 2\sqrt{2}z)^2$

$= (\sqrt{2}x - y - 2\sqrt{2}z)(\sqrt{2}x - y - 2\sqrt{2}z)$ Ans

(6) write the following cubes in expanded form:

(i) $(2x+1)^3$

$\longrightarrow (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$= (2x)^3 + (1)^3 + 3(2x)^2(1) + 3(2x)(1)^2$

$= 8x^3 + 1 + 3 \times 4x^2 \times 1 + 3 \times 2x$

$= 8x^3 + 1 + 12x^2 + 6x$ Ans

(ii) $(2a-3b)^3$

$\longrightarrow (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$= (2a)^3 + (-3b)^3 + 3(2a)^2(-3b) + 3(2a)(-3b)^2$

$= 8a^3 - 27b^3 + 3 \times 4a^2 \times (-3b) + 3 \times 2a \times 9b^2$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$ Ans

(iii) $\left[\frac{3}{2}x+1\right]^3$

$\longrightarrow (a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$= (\frac{3}{2}x)^3 + (1)^3 + 3(\frac{3}{2}x)^2(1) + 3\times\frac{3}{2}x\times(1)^2$

$= \frac{27}{8}x + 1 + 3\times\frac{9}{4}x^2 + \frac{9}{2}x$

$= \frac{27}{8}x + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$ Ans

(iv) $\left[x-\frac{2}{3}y\right]^3$

$\longrightarrow (a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$= (x)^3 + (\frac{-2}{3}y)^3 + 3(x)^2(\frac{-2}{3}y) + 3(x)(\frac{-2}{3}y)^2$

$= x^3 - \frac{8}{27}y^3 + \cancel{3}\times x^2\times(\frac{-2}{\cancel{3}}y) + \cancel{3}\times x\times\frac{4}{\cancel{9}_3}y^2$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}y^2x$ Ans

(7.) Evaluate the following using suitable identities.

(i) $(99)^3$

$\longrightarrow (100-1)^3$

$\Rightarrow (a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$= (100)^3 + (-1)^3 + 3(100)^2(-1) + 3(100)(-1)^2$

$= 1000000 - 1 + 3\times10000\times-1 + 3\times100$

$= 1000000 - 1 - 30000 + 300$

$= 1000300 - 30001$

$= 970299$ Ans

(ii) $(102)^3$

$\rightarrow (100+2)^3$

$\Rightarrow (a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$= (100)^3 + (2)^3 + 3(100)^2(2) + 3(100)(2)^2$

$= 1000000 + 8 + 3\times10000\times2 + 300\times4$

$= 1000000 + 8 + 30000\times2 + 1200$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$ Ans

(iii) $(998)^3$

$\rightarrow (1000-2)^3$

$\Rightarrow (a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$= (1000)^3 + (-2)^3 + 3(1000)^2(-2) + 3(1000)(-2)^2$

$= 1000000000 - 8 + 3\times1000000\times(-2) + 3000\times4$

$= 1000000000 - 8 - 6000000 + 12000$

$= 994011992$ Ans

(8) Factorise each of the following:

(i) $8a^3+b^3+12a^2b+6ab^2$

$\rightarrow (2a)^3 + (b)^3 + 3(2a)^2(b) + 3(2a)(b)^2$

$= a^3+b^3+3a^2b+3ab^2$

$= (a+b)^3$

$= (2a+b)^3$

$= (2a+b)(2a+b)(2a+b)$ Ans

(ii) $8a^3-b^3-12a^2b+6ab^2$

$\rightarrow (2a)^3 - (b)^3 - 3(2a)^2b + 3(2a)(b)^2$

$= a^3-b^3-3a^2b+3ab^2$

$= (a-b)^3$

$= (2a-b)^3$

$= (2a-b)(2a-b)(2a-b)$ Ans

(iii) $27 - 125a^3 - 135a + 225a^2$

$\longrightarrow (3)^3 - (5a)^3 - 3(3)^2(5a) + 3\times3\times(5a)^2$

$= a^3 - b^3 - 3a^2b + 3ab^2$

$= (a-b)^3$

$= (3-5a)^3$

$= (3-5a)(3-5a)(3-5a)$ Ans

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$\longrightarrow (4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$

$= a^3 - b^3 - 3a^2b + 3ab^2$

$= (a-b)^3$

$= (4a-3b)^3$ Ans

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$\longrightarrow (3p)^3 - (\frac{1}{6})^3 - 3(3p)^2(\frac{1}{6}) + 3\times3p\times(\frac{1}{6})^2$

$= a^3 - b^3 - 3a^2b + 3ab^2$

$= (a-b)^3$

$= (3p-\frac{1}{6})^3$

$= (3p-\frac{1}{6})(3p-\frac{1}{6})(3p-\frac{1}{6})$ Ans

(9) verify: (i) $x^3+y^3 = (x+y)(x^2-xy+y^2)$

$\longrightarrow$ R.H.S. $= (x+y)(x^2-xy+y^2)$

$= x(x^2-xy+y^2) + y(x^2-xy+y^2)$

$= x^3 - \cancel{x^2y} + \cancel{xy^2} + \cancel{yx^2} - \cancel{xy^2} + y^3$

$= x^3 + y^3$

L.H.S. = R.H.S. proved

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$\rightarrow$ R.H.S. $= (x-y)(x^2 + xy + y^2)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + \cancel{x^2y} + \cancel{xy^2} - \cancel{x^2y} - \cancel{xy^2} - y^3$

$= x^3 - y^3$

L.H.S. = R.H.S. proved

(10) Factorise each of the following:

(i) $27y^3 + 125z^3$

$\rightarrow (3y)^3 + (5z)^3$

$\Rightarrow x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$= (3y + 5z)((3y)^2 - 3y \times 5z + (5z)^2)$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$ Ans

(ii) $64m^3 - 343n^3$

$\rightarrow (4m)^3 - (7n)^3$

$\Rightarrow x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$= (4m - 7n)((4m)^2 + 4m \times 7n + (7n)^2)$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$ Ans

(11) Factorise: $27x^3 + y^3 + z^3 - 9xyz$

$\rightarrow 27x^3 + y^3 + z^3 - 9xyz$

$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)((3x)^2 + (y)^2 + (z)^2 - 3x \times y - y \times z - z \times 3x)$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$ Ans

(12) verify that:

$x^3+y^3+z^3-3xyz = \frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

→ $R.H.S. = \frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

$= \frac{1}{2}(x+y+z)[x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx]$

$= \frac{1}{2}(x+y+z)[2x^2+2y^2+2z^2-2xy-2yz-2zx]$

$= \frac{1}{\cancel{2}}(x+y+z)\times\cancel{2}(x^2+y^2+z^2-2xy-2yz-2zx)$

$= (x+y+z)(x^2+y^2+z^2-2xy-2yz-2zx)$

$= x^3+y^3+z^3-3xyz$ verify.

(13) If $x+y+z=0$, show that $x^3+y^3+z^3=3xyz$.

→ $x+y+z=0$ (Given)

from identity,

$x^3+y^3+z^3-3xyz = (x+y+z)(x^3+y^3+z^3-xy-yz-zx)$

$x^3+y^3+z^3-3xyz = 0\,(x^3+y^3+z^3-xy-yz-zx)$

$x^3+y^3+z^3-3xyz = 0$

$\boxed{x^3+y^3+z^3=3xyz}$ proved

(14) without actually calculating the cubes, find the value of each of the following.

(i) $(-12)^3+(7)^3+(5)^3$

→ $x^3+y^3+z^3 = 3xyz$

$= 3\times(-12)\times7\times5$

$= -1260$ Ans

$x+y+z$
$=-12+7+5$
$=0$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

$\longrightarrow x^3 + y^3 + z^3 = 3xyz$

$= 3 \times 28 \times (-15) \times (-13)$

$= 16380$ Ans

$x + y + z$
$= 28 + (-15) + (-13)$
$= 28 - 15 - 13$
$= 0$

(15) Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) $\boxed{\text{Area: } 25a^2 - 35a + 12}$

$\longrightarrow$ Area $= 25a^2 - 35a + 12$

$= 25a^2 - 20a - 15a + 12$

$= 5a(5a - 4) - 3(5a - 4)$

Area $= \underset{l}{(5a-4)}\underset{b}{(5a-3)}$

length $= 5a - 4$, breadth $= 5a - 3$ Ans

(ii) $\boxed{\text{Area: } 35y^2 + 13y - 12}$

$\longrightarrow$ Area $= 35y^2 + 13y - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

Area $= \underset{l}{(5y+4)}\underset{b}{(7y-3)}$

length $= 5y + 4$, breadth $= 7y - 3$ Ans

(16) What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) $\boxed{\text{Volume: } 3x^2 - 12x}$

$\longrightarrow$ $\text{Volume} = 3x^2 - 12x$

$= 3x(x-4)$

$\text{Volume} = \underset{l}{3} \times \underset{b}{x} \times \underset{h}{(x-4)}$

$l = 3,\ b = x,\ h = x-4$ Ans

(ii) $\boxed{\text{Volume: } 12ky^2 + 8ky - 20k}$

$\longrightarrow$ $\text{Volume} = 12ky^2 + 8ky - 20k$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 + 5y - 3y - 5)$

$= 4k[y(3y+5) - 1(3y+5)]$

$V = \underset{l}{4k}\ \underset{b}{(3y+5)}\ \underset{h}{(y-1)}$

$l = 4k,\ b = 3y+5,\ h = y-1$ Ans