



time



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Research Article

Can old sleepy galaxies have huge black holes in them without dark matter?

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Eappen and Kroupa wrote this!

In a nutshell:

The formation of supermassive black holes (SMBHs) in early-type galaxies (ETGs) is a key challenge for galaxy formation theories. Using the monolithic collapse models of ETGs formed in Milgromian Dynamics (MOND) from Eappen et al. (2022, MNRAS, 516, 1081, <https://doi.org/10.1093/mnras/stab229>, arXiv: 2209.00024 [astro-ph.GA]), we investigate the conditions necessary to form SMBHs in MOND under the hypothesis that ETGs are formed by a single SMBH through rapid central collapse. We study the evolution of the gravitational potential and gas inflow rates in the model relics with a total stellar mass ranging from $0.1 \times 10^{11} M_\odot$ to $0.7 \times 10^{11} M_\odot$. The gravitational potential shows rapid inward migration of the gas, fueling the growth of the central SMBH. These conditions suggest efficient central gas accumulation capable of fuelling SMBH formation. We further examine the $M_{\text{BH}} - \sigma$ relation by assuming that a fraction of the central stellar mass collapses to a black hole formed. Black hole masses derived from $10\% - 100\%$ of the central mass are comparable with the observed relation, particularly at high central velocity dispersions ($\sigma > 200 \text{ km/s}$). This highlights the necessity of substantial inner mass collapse to produce SMBHs consistent with observations. Our results demonstrate that MOND dynamics, through the rapid evolution of the gravitational potential and sustained gas inflow, provide a favorable environment for SMBH formation in ETGs. These findings support the hypothesis that MOND can naturally account for the observed SMBH-galaxy scaling relations without invoking cold dark matter, suggesting the importance of MOND dynamics in determining final SMBH properties.

Keywords: Galaxy: evolution; galaxy: formation; galaxies: elliptical and lenticular; cD; Galaxies: fundamental parameters; black holes

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Background:

Early-type galaxies (ETGs), including ellipticals and lenticulars, are compact spheroid systems characterized by old stellar populations, minimal gas content, and little to no ongoing star formation (Kormendy et al. 2009). These galaxies have been laboratories for understanding galaxy evolution and the co-evolution of supermassive black holes (SMBHs). The tightness of the scaling relations between SMBHs and galaxy properties, such as the $M_{\text{BH}} - \sigma$ (black hole mass versus central velocity dispersion) and $M_{\text{BH}} - M_{\text{bul}}$ (black hole mass versus bulge mass) relations, underscore the profound connection between SMBHs and the structural and dynamical properties of ETGs (McConnell & Ma 2013).

SMBHs with masses exceeding $10^6 M_\odot$ are the defining feature of ETGs (Kormendy & Richstone 1995; Kormendy 2001). Observations of high-redshift quasars show that these objects formed early in the universe's history, with some SMBHs already reaching $10^9 M_\odot$ less than a billion years after the Big-Bang (Pan et al. 2003; Mortlock et al. 2011; Vanzella et al. 2013). Some of the most massive ETGs in their host galaxies also formed rapidly, with star formation timescales (SF) shorter than 1 Gyr (Thomas et al. 2005; McConnell et al. 2015; Liu et al. 2016; Yan, Jerábková, & Kroupa 2021). Explaining how stellar-mass black hole seeds grow into SMBHs within these

short timescales remains a major challenge. Continuous gas accretion at the Eddington limit for nearly 1 Gyr is required to achieve the observed SMBH masses. While this scenario is plausible, the rapid cessation of gas inflows in these galaxies after star formation, as suggested by Kroupa et al. (2020), provides an alternative pathway for the formation of supermassive black holes (SMBHs). Kroupa et al. (2020) proposed that SMBHs can form through the merger and coalescence of stellar-mass black holes within dense star clusters. These clusters form in the central regions of collapsing gas clouds during galaxy formation, where high gas densities and stellar densities drive the central cluster of stellar remnants into a relativistic regime. This process leads to catastrophic collapse through gravitational wave emission, which a mechanism naturally accounts for the observed correlations between SMBH mass and spheroid mass.

The Milgromian Dynamics (MOND) paradigm offers a competing pathway for SMBH formation. First introduced by Milgrom (1983), MOND modifies the laws of gravity at low accelerations ($a \ll 1/a_0 = 1.2 \times 10^{-10} \text{ m/s}^2$ is Milgrom's critical acceleration), eliminating the need for dark matter in galaxies. MOND has successfully predicted numerous galaxy scaling relations, including flat rotation curves (Sab落 & McGaugh 1979; Sanders & McGaugh 2002), the Baryonic Tully-Fisher Relation (BTFR; Lelli et al. 2017), and the Revised Tully-Fisher Relation (RAR; Lelli et al. 2017). Comparative studies have shown that MOND is competitive with the Lambda Cold Dark Matter (Λ CDM) model in explaining galaxy dynamics and scaling relations (Bamkin & Liao 2022). This issue is particularly relevant given that dark matter particles have not been experimentally verified, and

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The biggest black holes are also called supermassive black holes!

the dynamical friction test further questions their existence (Oehm & Kroupa 2024). Within MOND, ETGs are thought to form via the monolithic collapse of primordial pre-galactic gas clouds, producing high stellar densities and rapid star formation (Eappen et al. 2022). Stellar-mass black holes forming in dense central clusters due to the collapse of stars may merge and coalesce to create SMBHs, as suggested by Kroupa et al. (2020).

Supermassive black hole properties have been linked to the motions at the center of the galaxy. To test if supermassive black holes could form in early-type galaxies under the MOND model, they check whether the central density is linked to how much gas flows in and how varied the velocities are. (this is introducing the method!)

In this study, we investigate Φ_{central} and gas inflow rates in ETG simulations in QUMOND mode using the PoR code. We aim to determine whether these systems create the necessary conditions for SMBH formation and whether resulting SMBH masses are consistent with the observed $M_{\text{BH}} - \sigma$ scaling relation. To achieve this, we analyse the evolution of the central gravitational potential and gas inflow rates during the early phases of galaxy formation. Additionally, based on the work of Kroupa et al. (2020), we evaluate SMBH masses by assuming that a fraction of the central stellar mass collapses into the SMBH. Finally, we compare the simulated SMBH masses to observed scaling relations to assess whether MOND-modelled ETGs can host SMBHs consistent with empirical data. This work builds on the foundation of Eappen et al. (2022) and aims to connect the dynamics of MOND-modelled ETGs with SMBH formation, providing new insights into galaxy evolution in alternative gravity frameworks.

mass of the stellar particles within a 500 kpc radius. This approach ensures that the centre is dynamically tracked throughout the simulation, independent of any fixed box coordinates.

The simulations are further constrained by a maximum spatial resolution of 0.24 kpc (which is the physical size of an individual grid cell at the highest level of refinement). However, we note that accurately resolving potential gradients typically requires several grid cells. Thus, the effective resolution of our simulation is somewhat larger than the cell size, approximately ≈ 1 kpc in the first few steps of refinement. The ability to refine (AMR) capabilities of the PoR code. Improving the spatial resolution would provide a more accurate simulation, but is currently infeasible due to the significant computational cost associated with higher resolution MOND simulations. While the PoR code in QUMOND mode – the quasi-linear formulation of MOND (Milgrom 2010) in which the modified gravitational field is derived via a two-step Poisson solver indeed requires at most twice the computational time compared to Newtonian N-body, a primary limitation for increasing refinement levels in our simulation was the available memory capacity. Higher refinement would substantially increase memory requirements due to the number of cells and particles, and future work will aim to address this with improved computational resources. Despite these limitations, the model allows us to explore key aspects of SMBH formation and galaxy evolution within the framework of MOND. A detailed description of the model formation process, including the initial conditions and simulation setup, can be found in Eappen et al. (2022).

Gravitational Potential Equations

The gravitational potential at the centre of the model, Φ_{central} , is calculated in the simulation by solving the MOND-modified Poisson equation. In MOND, the gravitational potential Φ is governed by the equation:

$$\Delta \Phi = 4\pi G \rho_b + \frac{\tilde{v}(\nabla \Phi)}{a_0} \quad (1)$$

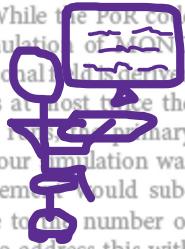
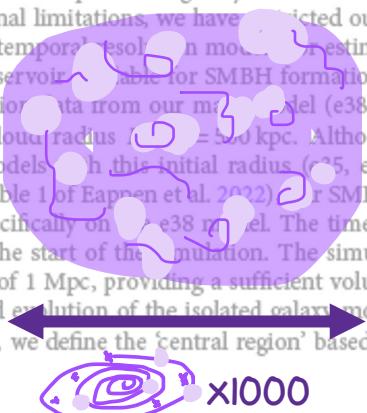
where $\Delta \Phi$ is the Laplacian of the gravitational potential, ρ_b is the baryonic matter density, G is the gravitational constant, a_0 is the MOND acceleration constant, ϕ is the Newtonian potential, and $\tilde{v}(y)$ is the MOND transition function, with $y = |\phi|/a_0$.

To figure out the gravitational pull in a certain area, the first step is to see how much normal matter is packed into that space. This is called baryonic density – “baryonic” just means ordinary stuff like stars, planets, and even people, all made of protons and neutrons.

$$\Delta \Phi = 4\pi G \rho_b \quad (2)$$

From the Newtonian potential, the code computes the phantom dark matter density (ρ_{ph}) that arises due to the MOND modification. This density is given by:

$$\frac{1}{4\pi G} \nabla^2 \left[\frac{(\nabla \Phi)^2}{a_0} \right] \quad (3)$$



The total gravitational potential Φ is then recalculated by solving the Poisson equation, which incorporates both ρ_b and ρ_{ph} :

$$\Delta \Phi = 4\pi G(\rho_b + \rho_{\text{ph}}) \quad (4)$$

Φ_{central} is extracted from the simulation output at specific radii, such as 1, 0.8, and 0.6 kpc, at each time step (The simulation outputs are saved at intervals of 10 Myr, allowing us to follow the evolution of the galaxy's potential well over time). For the period of rapid collapse ($\approx 3\text{--}5$ Gyr), this corresponds to a temporal resolution of 100 snapshots per Gyr). This value represents the depth of the gravitational potential well at the galaxy's centre and evolves as the galaxy forms and stabilises.

Physically, Φ_{central} reflects the gravitational strength in the galaxy's core. A deeper potential well corresponds to a denser and more massive central region. The evolution of Φ_{central} over time provides insight into the baryonic collapse and the formation of a dense core, processes that are critical for driving gas inflows and forming a SMBH. Additionally, the differences in Φ_{central} at various radii, such as between 1 and 0.6 kpc, reveal the steepness of the potential gradient, which influences the dynamics of gas inflows and central star formation.

By solving the MOND-modified Poisson equation and extracting gravitational potential values at central radii, the simulation provides a detailed measure of Φ_{central} , which is fundamental to understanding galaxy evolution and the conditions for SMBH formation in MOND.

How to calculate how much gas falls to the centre?

The inflow rates at different radii ($r = 1$ kpc, 0.8 kpc, 0.6 kpc) were computed using data from the simulation. The inflow rate (\dot{M}_{gas}) was calculated as the rate of change of gas mass (M_{gas}) within a given radius over a discrete time interval. For each radius, the inflow rate is given by:

How much gas is falling into the centre can be calculated at each radius and timestep. It is equal to how much mass has been gained or lost over what timeframe at that point in space. $\dot{M}_{\text{gas}}(r, t) = \frac{M_{\text{gas}}(r, t) - M_{\text{gas}}(r, t - \Delta t)}{\Delta t}$, (5)

where $M_{\text{gas}}(r, t)$ is the inflow rate at radius r and time t , Δt is the time interval between consecutive snapshots, Δt is the time interval between consecutive snapshots. The rate is calculated by summing the contributions from individual cells within the specified radius. For each cell, the gas mass is obtained by multiplying the cell volume by its gas density. Cells whose centres lie within the selected radius are included in full. Since the AMR grid structure results in varying cell resolutions, this calculation automatically accounts for local resolution differences. We note that we do not perform partial cell volume calculations for boundary cells; cells are either fully included or excluded based on their centre position.

The simulation data provides $M_{\text{gas}}(r, t)$ for discrete time points. Using the differences between consecutive snapshots, the inflow rate is computed for the midpoints of these time intervals to align with the calculated rates.

How to calculate the black hole mass?

This analysis estimates the black hole mass (M_{BH}) as a fraction of the central stellar mass and computes the central velocity dispersion (σ) for galaxies within the simulation. The results are compared to the observed $M_{\text{BH}} - \sigma$ scaling relation from McConnell & Ma (2013).

$$r = \sqrt{x^2 + y^2 + z^2} \quad (3)$$

The central stellar mass (M_{central}) is calculated by summing the masses of all stellar particles formed from the gas located within a defined radius ($r_{\text{central}} = 1$ kpc) of the galaxy centre. The distance of each particle from the central region is given as:

$$(6)$$

where x, y, z are the coordinates of the i -th particle, and m_i is the mass of the i -th particle. For a given radius r , the central stellar mass is given as:

$$(7)$$

where m_i is the mass of the i -th particle in the central region.

The black hole mass is estimated as a fraction of the central stellar mass. For a given fraction f , the black hole mass is given by:

Black hole mass \propto Central star Mass

Fractions ranging from $f = 0.1$ to $f = 1.0$ are used to explore the sensitivity of the black hole mass to the SMBH mass function (IMF). The mass function depends on the star-forming gas and is top-heavy under the relevant physical conditions such that the majority of the cluster's mass ends up being in supermassive black holes (McConnell et al. 2013; Bell et al. 2018; Kroupa et al. 2020; Kroupa et al. 2024).

The 3D central velocity dispersion (σ_{total}) is calculated using the particle velocities in the central region. For each velocity component (v_x, v_y, v_z), the dispersion is computed as:

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^N (v_{x,i} - \bar{v}_x)^2}, \quad (9)$$

where N is the number of particles, $v_{x,i}$ is the velocity of the i -th particle, and \bar{v}_x is the mean velocity in the x -direction. Similar expressions are used for σ_y and σ_z . The total 3D central velocity dispersion is:

Velocity dispersion in a direction measures how much the measurement differs from the average velocity. $\sigma = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$, (10)

values are compared to the observed $M_{\text{BH}} - \sigma$ relation from McConnell & Ma (2013).

$$\log_{10}(M_{\text{BH}}) = 0.55 + 0.20 \log_{10}(\sigma_{\text{total}}), \quad (11)$$

where σ is in km/s. This comparison provides insight into whether the simulated galaxies reproduce the observed scaling relation between black hole mass and central velocity dispersion, thereby validating the model's predictions for SMBH formation.

Results!

Is the gravitational potential well deep enough for stuff to not escape?

We analyse the evolution of the gravitational potential difference at the centre of our model ETG (e38), which forms a total stellar mass of $0.6 \times 10^{11} M_{\odot}$. Fig. 1 shows the evolution of the relative gravitational potential difference relative to the potential

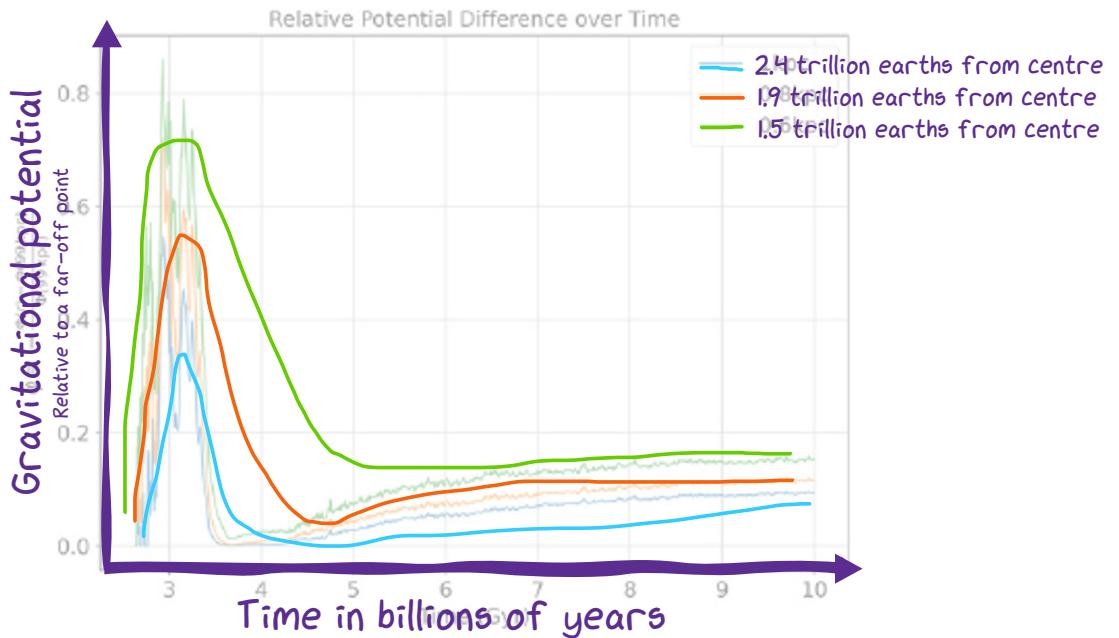


Figure 1. Time evolution of the relative gravitational potential difference between the central region (at radii of 1, 0.8, and 0.6 kpc) and a reference point at 99 kpc. The potential deepens rapidly during the collapse phase ($\approx 3\text{--}5$ Gyr), stabilising thereafter. The use of potential difference avoids dependence on absolute values and reflects the physical depth of the potential well relative to the galaxy outskirts.

at a radius of 99 kpc between three central radii (1 kpc, 0.8 kpc, and 0.6 kpc) over 10 Gyr.

The reference radius at 99 kpc corresponds to the outskirts of our simulated system, providing a consistent frame of reference for measuring the depth of the central potential well. While an ideal reference would be at infinity, the finite simulation volume necessitates this practical choice. The potential difference highlights the deepening of the gravitational well relative to the galaxy outskirts.

Within the first 5 billion years the gravitational well rapidly gets deeper, allowing mass to fall, particularly when closer to the centre. These are ingredients to create Black holes!

Beyond 5 Gyr, the potential difference stabilises, indicating that the system transitions to a more dynamically settled state. Notably, the deepest potential difference is seen at smaller radii (0.6 kpc), emphasising the concentration of baryonic mass toward the centre and suggesting that any gas reaching these inner regions could contribute to black hole growth.

To assess the role of MOND dynamics in the evolution of the central potential, we computed the acceleration profile at $t = 4.7$ Gyr, corresponding to the peak of early infall collapse (Fig. 3). The accelerations within the inner ≈ 2 kpc exceed the MOND acceleration scale a_0 , indicating that the dynamics in the central potential well are too fast to be part of the MOND model. MOND is more likely seen at larger radii to help transport mass to the centre.

A compact observational example is the compact ETG NGC 1277 in the Perseus Cluster, with a total stellar mass of $1.2 \times 10^{11} M_\odot$ and an effective radius of approximately 1.2 kpc. This

galaxy hosts a SMBH with a mass of $\approx 1.7 \times 10^{10} M_\odot$, accounting for nearly 14% of its total stellar mass – far above typical scaling relations. The gravitational potential difference in NGC 1277 is similarly deep, driven by its compact stellar core and massive black hole (Trujillo et al. 2011; van den Bosch et al. 2012; Yldrm et al. 2017; Ferré-Mateu et al. 2020). This is comparable to the potential difference in our model after ≈ 5 Gyr, supporting the notion that compact ETGs naturally develop deep gravitational wells.

This comparison provides a first observational test of the physical environment in our simulated system. It also highlights the expected properties of real compact ETGs. The rapid collapse of the central potential in both cases underlines the importance of compact stellar cores in establishing conditions for the formation and growth of SMBHs. The evolution of the potential difference in our model further supports the idea that compact ETGs can host over-massive SMBHs as a consequence of their early, rapid formation and subsequent stabilisation.

Does enough gas fall into the centre to make a supermassive black hole?

The gas inflow rates at the same radii (1, 0.8, and 0.6 kpc) are illustrated in Fig. 3 for the critical epoch between 4 and 6 Gyr, corresponding to the period of most rapid potential evolution. At a radius of 1 kpc, gas inflow peaks at rates exceeding $2 \times 10^{10} M_\odot/\text{Gyr}$, while smaller radii exhibit similarly high but slightly delayed peaks. These inflows correspond to periods of rapid cooling and contraction of gas toward the centre. The sustained high inflow rate over about half a Gyr suggests that sufficient mass can be continually infalling inwards to fuel the growing central SMBH. Notably, around $t \approx 4.7$ Gyr, the central regions experience an outflow phase, as shown in Fig. 3, driven by feedback mechanisms inherent to the model (Eappen et al. 2022), leading to gas heating and temporary disruption of inflows. After 5 Gyr,

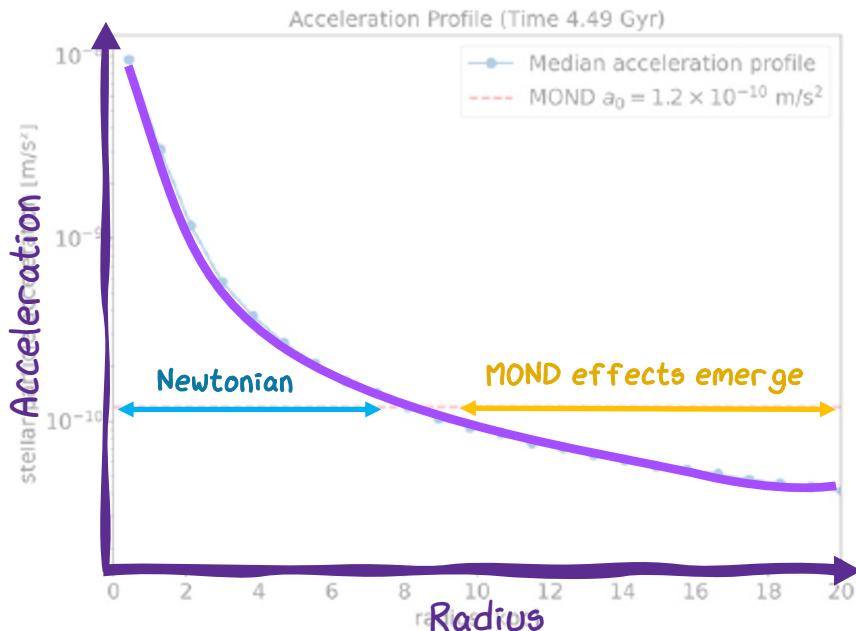


Figure 2. Radial profile of the median stellar particle acceleration at $t = 4.49$ Gyr. The red dashed line indicates the MOND acceleration scale $a_0 = 1.2 \times 10^{-10} \text{ m/s}^2 \approx 3.8 \text{ pc/Myr}^2$. Accelerations in the inner $\approx 3\text{--}4$ kpc exceed a_0 , indicating that the central region is in the Newtonian regime. MOND effects emerge at larger radii (≈ 8 kpc), influencing the global gravitational collapse.

outflow rates decline and stabilise, consistent with the exhaustion of central gas reservoirs. This reduction implies that star formation and black hole accretion activity slow significantly, transitioning the system into a quiescent phase.

There is a lot of interesting activity in early time after collapse, how relevant this is to black hole growth requires better resolution simulations.

These findings underscore the importance of the early epochs (4–5 Gyr to 5–15 Gyr after star formation; see Fig. 1) as a critical window for black hole growth, during which high inflow rates enable the formation of a massive central black hole. We note that during the time ≈ 3.7 – 4.7 Gyr there is a slowly rising inflow rate from $0.1 \times 10^{-10} M_\odot/\text{Gyr}$ to $1.1 \times 10^{-10} M_\odot/\text{Gyr}$. The inflow rates exhibit substantial variability after about 4.5 Gyr with fluctuations between strong inflow and outflow on timescales comparable to our snapshot interval driving supernova activity. Given the limited spatial and temporal resolution of our simulation, and the inherent variability in the gas inflow rates, our conclusions regarding the viability of SMBH seed formation remain qualitative. Higher resolution simulations would be required to fully resolve the detailed inflow behaviour at sub-kpc scales relevant for SMBH feeding.

The trends observed in our model align closely with theoretical predictions and observations of ETGs in the literature. For instance, Kroupa et al. (2020) proposed that SMBHs form through the coalescence of stellar-mass black holes in dense star clusters. These in turn, originating from galaxy-scale stellar systems, rely on significant gas inflows to maintain their density and fuel black hole growth. In our simulation, peaks reaching above $2 \times 10^{10} M_\odot/\text{Gyr}$ at 1 kpc provide the necessary conditions for this process, allowing central gas densities to remain high enough to support the formation of the massive clusters. Furthermore, the slight delay in peak inflows at smaller radii (e.g. 0.6 kpc) highlights the gradual inward transport of gas, a feature consistent with the cooling flows observed in present day dense early environments.

Observational studies of high-redshift quasars (e.g. Venemans et al. 2013) have revealed black holes with masses exceeding $10^9 M_\odot$ by $z \approx 6$, implying efficient accretion mechanisms in the

first billion years. This comparison reinforces the notion that the early, gas-rich environment in our simulated ETG is consistent with the conditions required for forming and growing central SMBHs observed in compact ETGs and high-redshift quasars.

Do we see the relationship between the size of the supermassive black hole and the spread of velocities?

3.3 Would the $M_{\text{BH}} - \sigma$ scaling relation emerge?

The relationship between M_{BH} and σ is explored in the context of supermassive black hole formation. The contribution of the central stellar mass within 1, 0.8, and 0.6 kpc contributes to black hole formation. This is based on the model of Kroupa et al. (2020) in which the mean mass star formation in the innermost regions of the forming spheroidal galaxy, each 10 Myr. The calculated black hole masses based on $M_{\text{BH}} = 10^{10}, 10^{10.5}, 10^{11}$ of the central stellar mass, are plotted alongside the observed $M_{\text{BH}} - \sigma$ relation for the central $M_{\text{BH}} > 10^{10.5} M_\odot$ masses. The models e35, e26, e37, e38 and e39 from Lappan et al. (2022) include the models e35, e26, e37, e38 and e39 from Lappan et al. (2022) as well as the observed ETGs from McConnell & Ma (2013) and Saito et al. (2014).

Black hole masses derived from lower contributions of the central stellar mass (10–50%) and at larger central velocity dispersion exhibit good agreement with the observed $M_{\text{BH}} - \sigma$ relation across all radii. This suggests that if a significant fraction of the central stellar mass contributes to the black hole's formation, then the resulting masses are consistent with those observed in real ETGs. This indicates that a minimum threshold of stellar mass contribution is necessary to match observed SMBH properties. However they can only see an upper limit of mass available with the resolution used.

Therefore there must be a minimum stellar mass contribution threshold.

However they can only see an upper limit of mass available with the resolution used.

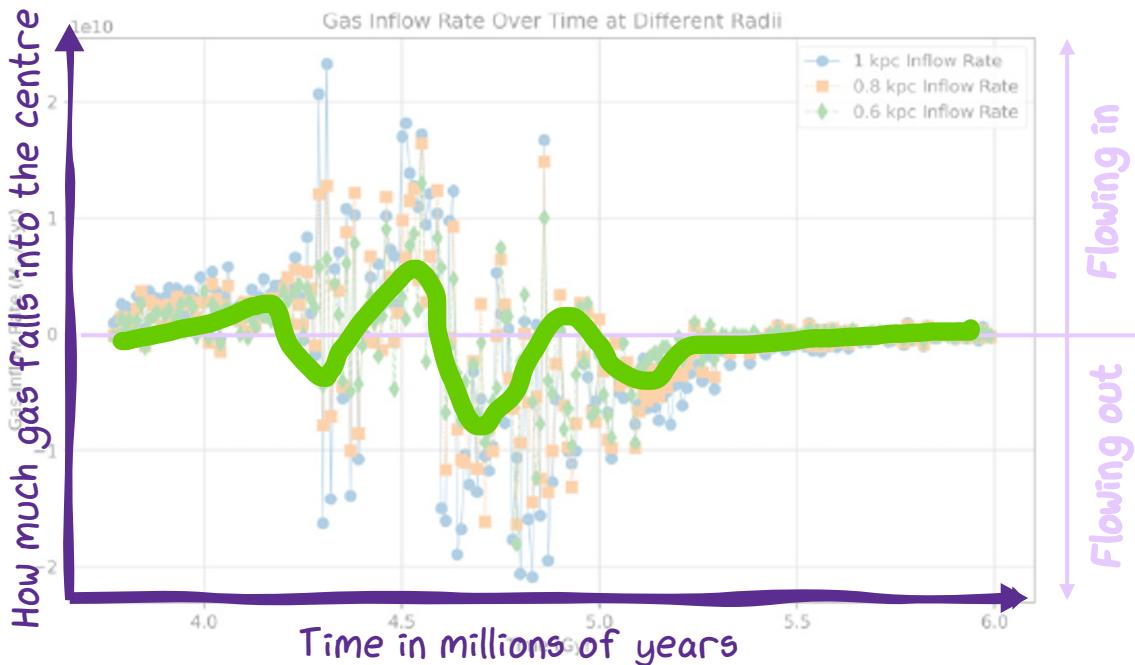


Figure 3. Gas Inflow rate over time at different radii (1, 0.8, and 0.6 kpc). The plot illustrates fluctuations in the Inflow rate of gas (in M_{\odot}/Gyr) during the time range from 4 to 6 Gyr. Significant variability is observed, particularly in the interval between 4.0 and 5.0 Gyr, with notable peaks and troughs, indicative of dynamic processes affecting gas Inflow at these radii.

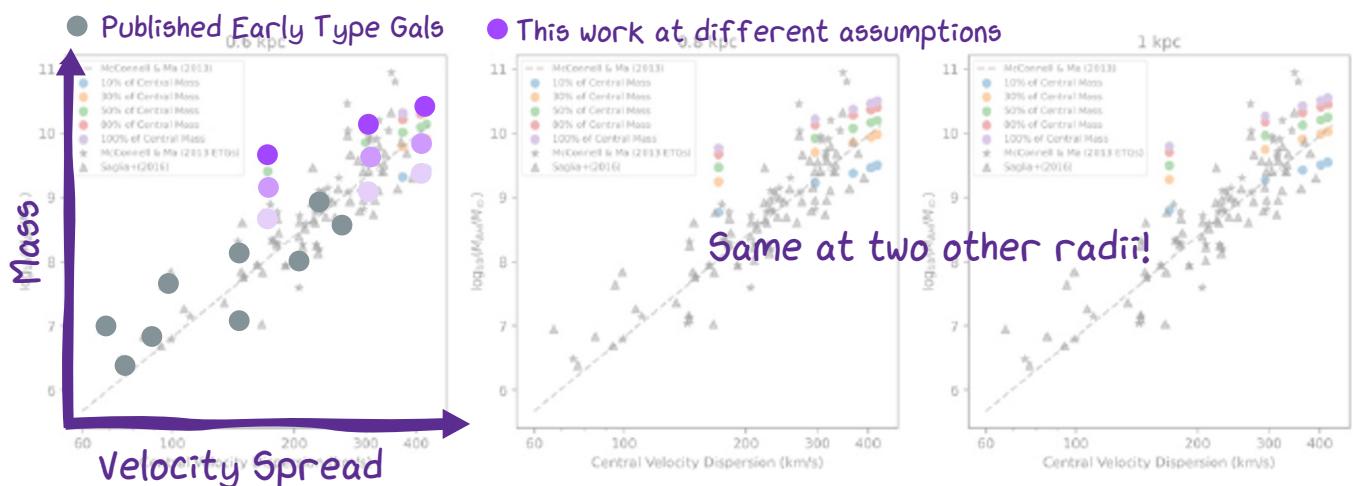


Figure 4. The calculated black hole masses for the model relics (e35, e36, e37, e38 and e39) from Eappen et al. (2022), based on 10%, 30%, 50%, 80%, and 100% of the central stellar mass, are plotted alongside the observed $M_{\text{BH}} - \sigma$ relation from McConnell & Ma (2013). The plotted black stars and black triangles are the ETGs from McConnell & Ma (2013) and Saglia et al. (2016) respectively.

The analysis highlights the capability of MOND dynamics to support the formation of SMBHs in ETGs formed via monolithic collapse. The rapid deepening of the central gravitational potential during the first 0.5 Gyr after begin of the collapse, coupled with high gas inflow rates, creates a conducive environment for black hole seeding and growth.

mechanisms governing the formation and growth of SMBHs in compact systems.

The evolution of the central gravitational potential reveals a sharp decline during the first 0.5 Gyr after the start of the collapse, corresponding to the rapid collapse of baryonic matter and the formation of the stellar population. This phase is marked by the deepening of the central potential well. The deeper gravitational potential at smaller radii (e.g. 0.6 kpc) underscores the concentration of baryonic matter and the resulting conditions necessary for sustained gas inflows and SMBH formation.

Gravitational potential decreases severely very quickly after the collapse of matter

Deep potential wells close to the centre confirm conditions for gas inflows and properties for the formation of huge black holes.

Gas inflow rates during the initial epoch between 2 and 1.5 Gyr after the collapse starts exhibit peaks exceeding $2 \times 10^{-9} M_{\odot}/\text{Gyr}$ at 1 kpc, with similarly high but slightly delayed peaks at 0.8 and 0.6 kpc. The subsequent inflow rates and their decline rates after 4.7 Gyr reflects stellar activity and the depletion

Quick take away messages:

In this study, we investigated the evolution of ETGs within a MOND framework, focusing on the central gravitational potential, gas inflow rates, and the relationship between black hole mass and central velocity dispersion. Using simulations, we analysed the gravitational and dynamical properties of a model galaxy with a total stellar mass of $0.6 \times 10^{11} M_{\odot}$, providing insights into the

of gas reservoir, transitioning the system into a quiescent phase. Variations in the adopted feedback prescriptions could significantly influence the inflow dynamics and star formation rates, potentially altering the efficiency of central mass accumulation. Exploring these effects will be the subject of future work. This early period of high inflow rates is crucial for fuelling the growth of the central SMBH and forming a dense stellar core, consistent with the analytical results by Kroupa et al. (2020).

Finally, the relationship between M_{BH} and σ was examined by assuming various fractions (10%, 30%, 50%, 80%, and 100%) of the central stellar mass contribute to the black hole. Black hole masses derived from the simulations align closely with the observed $M_{\text{BH}} - \sigma$ relation (Ferré-Mateu et al. 2017). The modelled dispersions ($\sigma > 200$ km/s). The results suggest that the combination of a deep gravitational potential, early gas inflow rates, and significant central mass contributions is essential for forming SMBHs consistent with those observed in real galaxies. A more accurate estimate of the SMBH formation potential would require resolving sub-parsec scales and incorporating detailed accretion physics. While our current resolution limits such analysis, the observed collapse and mass assembly predicted by KFPP is a useful framework for assessing early SMBH formation in MOND.

Overall, our findings demonstrate that ETGs formed within the MOND framework have the potential to naturally reproduce many of the observed properties of ETGs, including their dense cores, quiescent phases, and the $M_{\text{BH}} - \sigma$ relation. These results underscore the importance of early gas inflows and gravitational potential evolution in shaping the growth of SMBHs and the structural properties of ETGs, offering a viable pathway for understanding their formation and evolution.

However, we recognise certain limitations in this work. The radii typically around 0.1 kpc or smaller, are not fully resolved in our simulations due to computational limitations. As a result, the work presented here provides only a basic estimate of the conditions for SMBH formation and growth in MOND. This is a simplified approach that does not account for complex processes such as detailed feedback mechanisms, the actual coalescence of the cluster of stellar remnants to a SMBH seed, relativistic effects, or accretion dynamics at smaller scales. Nonetheless, this work offers a foundational framework for future MOND cosmological simulations, highlighting the importance of early gas inflows and gravitational potential evolution in shaping compact ETGs and their central SMBHs. Future efforts to incorporate finer resolutions and additional physics into MOND-based simulations will be critical for achieving a more comprehensive understanding of these processes.

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Data Availability. None.

Where did all these ideas come from?:

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People have been building the puzzle pieces of this paper since 1979!

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(Jayde Willingham aka me!)