**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.
8. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Mean= 38

SD= 6

Z score= (value-mean)/SD

Z score for 44= (44-38)/6 =1 =>84.13%

People above 44 age = 100-84.13 age= 15.87%= 63 out of 400

Z score for 38= (38-38)/6= 0 =>50%

Hence people between 38 or 44 age= 84.13-50=34.13% =137 out of 400

Hence a training program for employees under the age of 30 at the center would be expected to attract above 36 employees -TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

The difference between 2X1 and X1 +X2 is N(0,6ro^2)

According to the Central Limit Theorem, any large sum of independent, identicallydistributed (iid) random variables is approximately Normal.

The Normal distribution is defined by two parameters, the mean, https://tex.z-dn.net/?f=%5Cmu, and the variance, https://tex.z-dn.net/?f=%5Csigma%5E%7B2%7D and written as https://tex.z-dn.net/?f=X%20%5Csim%20N(%5Cmu%2C%20%5Csigma%5E2).

Given https://tex.z-dn.net/?f=X_1%20%5Csim%20N(%5Cmu%2C%20%5Csigma%5E2)~~%5Cmbox%7Band%7D~~%20X_2%20%5Csim%20N(%5Cmu%2C%20%5Csigma%5E2)  are two independent identically distributed random variables.

From the properties of normal random variables,

if https://tex.z-dn.net/?f=X%20%5Csim%20N(%5Cmu_1%2C%20%5Csigma_1%5E2) and https://tex.z-dn.net/?f=Y%20%5Csim%20N(%5Cmu_2%2C%20%5Csigma_2%5E2) are two independent identically distributed random variables then

* the sum of normal random variables is given by

https://tex.z-dn.net/?f=X%20%2B%20Y%20%5Csim%20N(%5Cmu_1%20%2B%20%5Cmu_2%2C%20%5Csigma_1%5E2%20%2B%20%5Csigma_2%5E2%20),

* and the difference of normal random variables is given by

https://tex.z-dn.net/?f=X%20-%20Y%20%5Csim%20N(%5Cmu_1%20-%20%5Cmu_2%2C%20%5Csigma_1%5E2%20%2B%20%5Csigma_2%5E2%20)

* When  https://tex.z-dn.net/?f=Z%20%3D%20aX, the product of X is given by

https://tex.z-dn.net/?f=Z%20%5Csim%20N(a%5Cmu_1%20%2C%20a%5E2%5Csigma_1%5E2%20)

* When  https://tex.z-dn.net/?f=Z%20%3D%20aX%20%2B%20bY, the linear combination of X and Y is given by

https://tex.z-dn.net/?f=Z%20%5Csim%20N(a%5Cmu_1%20%2B%20b%5Cmu_2%2C%20a%5E2%5Csigma_1%5E2%20%2B%20b%5E2%5Csigma_2%5E2%20)

Given to find, https://tex.z-dn.net/?f=2X_1

Thus, following the property of multiplication, we get

https://tex.z-dn.net/?f=2X_1%20%5Csim%20N(2%5Cmu%2C%202%5E2%5Csigma%5E2)%5Cimplies2X_1%20%5Csim%20N(2%5Cmu%2C%204%5Csigma%5E2)

and following the property of addition,

https://tex.z-dn.net/?f=X_1%2BX_2%5Csim%20N(%5Cmu%20%2B%20%5Cmu%2C%20%5Csigma%5E2%20%2B%20%5Csigma%5E2%20)%20%5Csim%20N(2%5Cmu%2C%202%5Csigma%5E2%20)

And the difference between the two is given by

https://tex.z-dn.net/?f=2X_1-(X_1%2BX_2)%20%5Csim%20N(2%5Cmu%20-%202%5Cmu%2C%202%5Csigma_1%5E2%20%2B%204%5Csigma_2%5E2%20)%5Csim%20N(%200%2C6%20%5Csigma%5E2)

The mean of https://tex.z-dn.net/?f=2X_1 and https://tex.z-dn.net/?f=X_1%2BX_2 is same but the var(https://tex.z-dn.net/?f=%5Csigma%5E2) of  https://tex.z-dn.net/?f=2X_1 is 2 times more than the variance of https://tex.z-dn.net/?f=X_1%2BX_2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

                                         Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576)+100= 151.5

Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

import numpy as np

from scipy import stats

from scipy.stats import norm

# Mean profits from two different divisions of a company = Mean1 + Mean2

Mean = 5+7

print('Mean Profit is Rs', Mean\*45,'Million')

Mean Profit is Rs 540 Million

# Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2

SD = np.sqrt((9)+(16))

print('Standard Deviation is Rs', SD\*45, 'Million')

Standard Deviation is Rs 225.0 Million

# A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

# B. Specify the 5th percentile of profit (in Rupees) for the company

# To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table, 5 percentile = -1.645

X= 540+(-1.645)\*(225)

print('5th percentile of profit (in Million Rupees) is',np.round(X,))

5th percentile of profit (in Million Rupees) is 170.0

# C. Which of the two divisions has a larger probability of making a loss in a given year?

# Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(5,3)

0.9772498680518208

# Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(7,4)

0.9986501019683699

# the probability of division by 2

0.9772498680518208