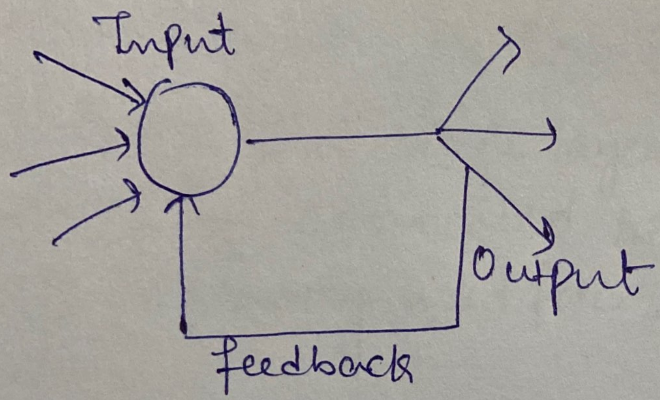
Single layer feedback



When outputs can be directed back as inputs to the same layer or preceeding layer nodes , then it results in feedback networks.

Recurrent networks are feed back networks with closed loop. Above figure shows a single recurrent network having single neuron with feedback to itself.

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Hopfield Network (Unit 5)

It was invented by Dr. John J. Hopfield in 1982.

It Consists of a single layer which Contains one or more fully connected recurrent neurons.

It is commonly used for auto association & optimization tasks.

In this network, i/p & o/p patterns are discrete vector, which can be either binary (0,1) or bipolar (+1, -1) in nature.

The network has symmetrical weights with no self connections. i.e $w_{ij} = w_{ji}$ and $w_{ii} = 0$.

It depends on Training Algo & Testing Algo.

Training Algo

For set of Binary pattern $S(P)$, $p = 1$ to P

where $S(p) = (s_1(p), s_2(p), \ldots, s_i(p), \ldots s_n(P))$

weight matrix is given by

$$w_{ij} = \sum_{p=1}^{P} [2s_i(p) - 1][2s_j(p) - 1], \quad (i \neq j)$$

For Bipolar Pattern

$$w_{ij} = \sum_{P=1}^{P} s_i^T(p)\, s_j(p), \quad \text{for } i \neq j$$

~~& weight here~~ weight having no self connection.

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Testing Algo

1. Initialize weights which are obtained from training algo.
2. Perform step 3-8 if activation of the n/w are not consolidated
3. For each input vector X, perform steps 4 to 7
4. Make initial activation of the networks equal to external vector as follows.

$$y_i = x_i \text{ for } i = 1 \text{ to } n.$$

5. For each unit y_i; perform steps 6-8.
6. Calculate the net input of the n/w.

$$y_{in} = x_i + \sum_j y_i w_{ji}$$

7. Apply activation as follows over the net input to calculate the O/P

$$y_i = \begin{cases} 1 & \text{if } y_{in} > \theta_i \\ y_i & \text{if } y_{in} = \theta_i \\ 0 & \text{if } y_{in} < \theta_i \end{cases}$$

where θ_i is threshold (given, if not then $\theta = 0$)

8. Broadcast this output y_i to all other units.
9. Finally test the networks for convergence.

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Construct a Hopfield network with O/P vector [1 1 1 -1], Test discrete Hopfield n/w with missing entries in first & ~~secon~~ second components of stored vector.

Sol^n Input vector $(x) = [1 \ 1 \ 1 \ -1]$

Now find weight

$$w = \Sigma S^T(P) S(P)$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

Cond^n → weight matrix with no self connection.

$$= \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix}$$

Binary representation of given input vector is [1 1 1 0]

Missing entries [0 0 1 0]

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Testing Algo:

S1. $\begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix}$

2. Input vector is $x = [0\ 0\ 1\ 0]$

3. for this vector $y = [0\ 0\ 1\ 0]$

4. Choosing unit y_1 for updating it activation.

$$y_{in_1} = x_1 + \sum_{j=1}^{4} y_i w_{ji} \qquad \left(\begin{matrix} i=1 \\ j=1 \text{ to } 4 \end{matrix}\right) \quad \begin{matrix} w_{11} \\ w_{21} \\ w_{31} \\ w_{41} \end{matrix}$$

$$\sum_{j=1}^{4} = 0 + [0\ 0\ 1\ 0] \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$= 0 + 1$$

apply step 7 (activation) $\theta = 0$

$$y_1 = 1 \text{ if } y_{in} > \theta$$

$$y_1 = 1$$

Broadcasting y_1 to all other units we get

$$y = [1\ 0\ 1\ 0]$$

Check y = given i/p vector $[1\ 1\ 1\ -1]$

$[1\ 0\ 1\ 0] \neq [1\ 1\ 1\ -1]$ it means no convergence takes place

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gain ~~Stt~~ Run the network.

Now choose unit y_4 for updating its activation

$$y_{in4} = x_4 + \sum_{j=1}^{4} y_j w_{i4}$$

$$= 0 + \underbrace{[1\ 0\ 1\ 0]}_{\text{updated } y} \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$$= 0 - 1 - 1$$
$$= -2.$$

Apply step 7.

$$y = -2 < 0$$
$$\therefore y = 0$$

Broadcast

$y = [1\ 0\ 1\ 0]$ ~~no~~

$y \neq$ given i/p vector

Again Repeat

Choose unit y_2

$$y_{in2} = \sum_{j=1}^{4} y_j \cdot w_{i2}$$

$$= 0 + [1\ 0\ 1\ 0] \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

$$= 0 + 2$$

$y_{in2} = 2.$

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Apply step 7

$y = 2 > \theta$

$\therefore y = 1$

Broadcast

$y = [1 \ 1 \ 1 \ 0]$

$[1 \ 1 \ 1 \ 0] = [1 \ 1 \ 1 \ -1]$

Convergence Done

$\boxed{-1 \text{ or } 0}$ — both are same

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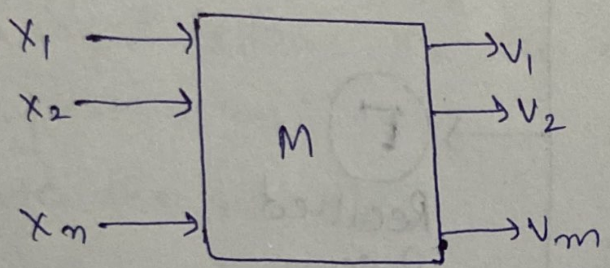
Associative Memories

This work on the basis of pattern associations which means they can store different patterns & at the time of giving an O/P they can produce one of the stored patterns by matching them with the given input pattern. These type of memories are also called Content Addressable memory (CAM)



Block diagram of Associative Memory

$$X = [x_1, x_2, ---, x_n]^t$$
$$V = [V_1, V_2, ---, V_m]^t$$

Above figure shows how an associative memory performing an associative mapping of an input vector X into an output vector V.

$$V = M[X]$$

where M = denotes a general non linear diagonal operator & it is different for each type of memory model used.

The Computation of M is done with the help of "recording or storage Algorithm". The mapping of V to X is called retrieval. The storage algorithm depends on which type of memory is designed.

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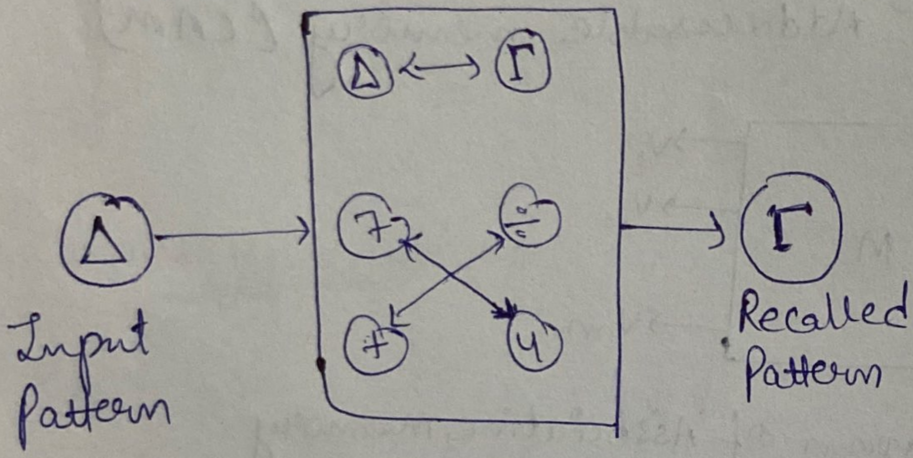
It is not necessary that Recall always provide desired solution prototype. Sometimes it provide undesired prototype.

Example

An associative memory is a storehouse of associated patterns which are encoded in some form



Related Terms

① ~~En~~ Encoding or Memorization

Building an associative memory means, constructing a connection weight matrix 'w' such that when an input pattern is presented and the stored pattern associated with the input pattern is retrieved. This process of constructing the connection weight matrix is called encoding. During encoding, for an associated pattern pair (x_k, y_k), the weight values of correlation matrix w_k are computed as

$$(w_{ij}) = (x_i)(y_j) \text{ where}$$

x_i represents the i^{th} component of Pattern x.

and y_j represents the j^{th} component of pattern y.

for $i = 1, 2, ----, m$.

and $j = 1, 2, ---, n$.

Retrieval or Recollection

After memorization, the process of retrieving a stored pattern, given an input pattern is called decoding.

Given an input pattern X, the decoding or recollection is accomplished by:

→ first compute the net i/p:

$$\text{net i/p} = \sum_{i=1}^{m} x_i w_{ij}$$

Then determine the units o/p using a bipolar o/p function:

$$y = \begin{cases} +1 & \text{if input} \geq \theta \\ -1 & \text{otherwise} \end{cases}$$

where θ is the threshold value

Errors and Noise

The input pattern may contain errors & noise, or may be an incomplete version of some previously encoded pattern.

When a corrupted i/p pattern is presented, the network will retrieve the stored pattern that is closest to actual input pattern.

- The presence of noise or errors results only in degradation in the performance of network.

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Performance Measures

"Memory capacity" & "Content addressability" are the measures of associative memory performance for correct retrieval. These two performance measures are related to each other.

Memory capacity :- Memory capacity refers to the maximum no of associated pattern pairs that can be stored & correctly retrieved.

Content addressability :- It is the ability of the network to retrieve the correct stored patterns.

Associative Memory Classes

There are two classes of associative memory.

1. Auto Associative Memory
2. Hetero Associative Memory.

Auto Associative Memory

This is a single layer neural network in which the input training vector & the output target vectors are the same

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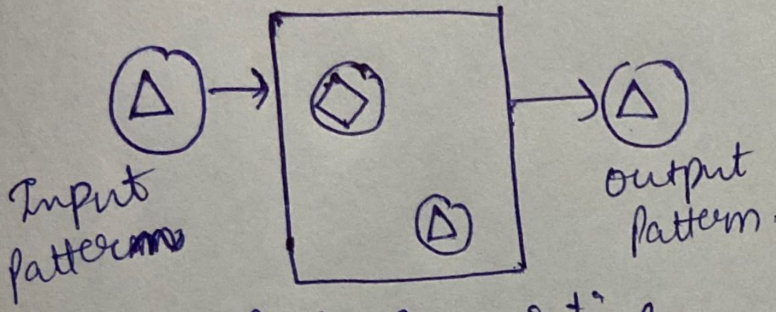
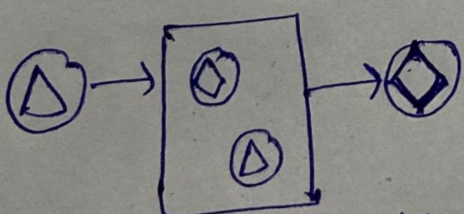
is also used to retrieve a previously stored patterns that most closely resembles the current pattern.



Auto Associative

Hetero Associative Memory

This is also a single layer neural network. In this network the input training vector & the O/P target vectors are not the same. It is static in nature. There would be no non linear & delay operations.



Hetero Associative

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