Mathematical Model and Method for Diagnosing the Operability of Information and Control Systems

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***Abstract* — This paper focuses on the development of mathematical methods for diagnosing the operability of complex structured information and control systems with arbitrary topology in real time. We describe a model based on graph analytics which allows us to analyze various options for changing the state of information flows as they pass through the system's structural modules. As a result, it is possible to calculate the failure coefficients for the structural modules of information and control systems and the connections between them. In order to detect and forecast failures, a software application has been developed using the mathematical methods proposed in this paper. Conclusions are drawn about increasing the probability of failure-free operation of the system when using the proposed methods for diagnosing and predicting failures.**

Keywords—information and control systems, graphical and analytical model, mathematical methods of diagnostics, structural module of the system, failure coefficient for the structural module.

I. Introduction

At the present time, the growth of industry and the development of information technologies require the use of information and control systems in all spheres of human life. The reliability and fault tolerance of such systems are defined by processing speed and analysing "big data" as well as the system's response to the control actions. These data are information flows of ICS (information and control system). Therefore, there is the pressing task of creating methods for diagnosing the operability of an ICS, and furthermore, detecting and localizing failures in real time.

II. Analysis of Recent Research

As is known [1-3], one of the main directions of ICS diagnostics is based on the modelling of processes and information flows occurring in the system. These include analytical models, neural network models, fuzzy logic, statistical methods, expert systems, pattern recognition, principal component analysis (PCA) models, qualitative trend analysis (QTA) models, signed direct graph (SDG) models, etc.

Large-scale critical infrastructure systems (CIS) represented by a digraph G(V, E) are considered in paper [4]. In this digraph a set of elements, i.e., sources, storage elements and intersection nodes are represented by $v \in V$ vertices connected by $e \in E$ links. There are several approaches for diagnosis and fault-tolerant control of CIS presented in this paper. These include the following:

- system analysis to understand the weaknesses and risks in case of fault occurrence;
- fault diagnosis based on analytical redundancy relations;
- fault tolerant control schemes and assessment of fault tolerances;
- inclusion of health-aware mechanisms in the CIS control system.

Paper [5] discusses Support Vector Machine Algorithm (SVM) and k Nearest Neighbors Algorithm (kNN), which are used in data mining systems. This paper proposes a two-stage classification method based on the combined use of SVM and kNN algorithms and providing classification accuracy for complex multidimensional data.

The following major and well-established technologies are considered in work [6]. These can be classified into:

- Hardware redundancy-based fault diagnosis;
- Signal processing-based fault diagnosis;
- Statistical data based fault diagnosis;
- Analytical model based fault diagnosis;
- Knowledge-based fault diagnosis.

The focus of the research is devoted to processing data-based fault diagnosis and implementing analytical model-based methods. These methods imply the statistical data-based framework and are called data-driven design of fault diagnosis systems.

III. Aim of Research

The purpose of this research is to improve the fault tolerance and reliability of ICS by creating a methodology for diagnosing the system's performance in real time. This is based on the analysis of changes in the state of information flows when passing through the system's structural modules.

IV. Materials and Methods

A. Graphical and Analytical Model of ICS

As is known, an ICS of any topology can be represented as a graphical and analytical model (GAM), conventionally divided into a set of structural modules M with links between them L:

$$M=\{m_1, m_2, \ldots, m_i, \ldots, m_{|M|}\}; \quad (1)$$

$$L=\{l_1, l_2, \ldots, l_j, \ldots, l_{|L|}\}. \quad (2)$$

Therefore, ICS can be represented as a graph G, where |M| is the number of graph vertices corresponding to the number of structural modules of the ICS;

|L| is the number of graph edges corresponding to the number of connections between the structural modules of the ICS.

We use the configuration matrix of graph G to describe the topology of the ICS. In this matrix, the rows correspond to the set of ICS structural modules M, and the columns correspond to the set of connections between them L.

$$M \begin{array}{c} \\ m_1 \\ m_2 \\ \cdots \\ m_i \\ \cdots \\ m_{|M|} \\ \\ \end{array} \quad G = \begin{vmatrix} & & & \boldsymbol{L} & & \\ \boldsymbol{l_1} & \boldsymbol{l_2} & \cdots & \boldsymbol{l_j} & \cdots & \boldsymbol{l_{|L|}} \\ g_{11} & g_{12} & \cdots & g_{1j} & \cdots & g_{1|L|} \\ g_{21} & g_{22} & \cdots & g_{2j} & \cdots & g_{2|L|} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ g_{i1} & g_{i2} & \cdots & g_{ij} & \cdots & g_{i|L|} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ g_{|M|1} & g_{|M|2} & \cdots & g_{|M|j} & \cdots & g_{|M||L|} \end{vmatrix} \quad (3)$$

Each element of matrix G can take a value from the 3-digit set E3 =\{0, 1, 2\} ($g_{ij} \in E_3$). E3 values are interpreted as follows:

if $g_{ij} = 0$, then structural module m_i has no relation to l_j;
if $g_{ij} = 1$, then structural module m_i has an incoming connection l_j;
if $g_{ij} = 2$, then structural module m_i has an outgoing connection l_j.

Therefore, the number of structural module connections m_i is calculated as follows:

$$nL_i = \sum_{j=1}^{|L|}(g_{ij} > 0). \quad (4)$$

Accordingly, the number of incoming and (outgoing connections of structural module m_i is calculated using the following formulas:

$$nLI_i = \sum_{j=1}^{|L|}(g_{ij} == 1); \quad (5)$$

$$nLO_i = \sum_{j=1}^{|L|}(g_{ij} == 2). \quad (6)$$

B. Description of GAM's Structural Module

We accept the following constraints for our model:

- Each structural module m_i can have any number of incoming or outgoing connections, but it must have at least one incoming or outgoing connection, i.e. $nL_i \geq 1$.
- Each connection l_j links a single pair of structural modules (m_{k_1}, m_{k_2}), where k_1 is a structural module m_i which does not have a connection l_j or 0, if this is an incoming connection only; k_2 is a structural module m_i which has an incoming connection l_j or 0, if this is an outgoing connection only.

Thus, the correctness of configuration matrix G is checked as follows:

$$\exists i \in [1; |M|] \Rightarrow \sum_{j=1}^{|L|} g_{ij} > 0; \quad (7)$$

$$\exists j \in [1; |L|] \Rightarrow 0 < \sum_{i=1}^{|M|} g_{ij} \leq 3. \quad (8)$$

In general, any ICS structural module m_i can be represented as a diagram shown in Fig. 1:

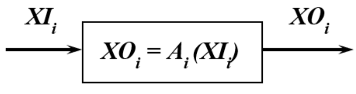

Fig. 1. Structural module of ICS m_i

Fig.1. has the following designations:

XI_i is a set of input parameters of ICS structural module m_i;
XO_i is a set of output parameters of ICS structural module m_i;
A_i is a set of algorithms for converting input parameters into output parameters for ICS structural module m_i.

C. Description of GAM Connections

We accept for our model that a correctly working connection l_j between two structural modules (m_{k_1}, m_{k_2}) transmits the information flow without distortion:

$$XO_j = X_j = XI_j. \quad (9)$$

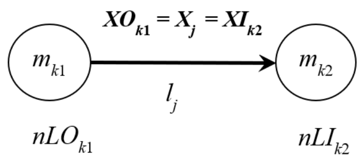

Fig. 2. Connection between structural modules of ICS

Fig. 2 has the following designations:

l_j is a connection between structural modules of ICS m_{k1} and m_{k2};

X_j is a set of parameters transmitted over connection l_j;

XI_{k2} is a set of input parameters for ICS structural module m_{k2} transmitted over connection l_j;

XO_{k1} is a set of output parameters for ICS structural module m_{k1} transmitted over connection l_j;

nLO_{k1} is a set of output connections for ICS structural module m_{k1};

nLI_{k2} is the number of input connections for ICS structural module m_{k2}.

D. Destribution of Connections through GAM Structural Modules

Based on matrix G, we define an ascending-ordered set of connection numbers for structural module m_i, such as:

$$J_i = \{j_1, j_2, \ldots, j_\gamma, \ldots, j_{|J_i|=nL_i} | \exists j_\gamma \in [1; |L|] \Rightarrow g_{ij_\gamma} > 0\}. \quad (10)$$

Accordingly, successively increasing sets of numbers of input and output connections for structural module m_i are defined as follows:

$$JI_i = \{jI_1, jI_2, \ldots, jI_\mu, \ldots, jI_{|JI_i|=nLI_i} | \exists jI_\mu \in [1; |L|] \Rightarrow g_{i,jI_\mu} == 1\}; \quad (11)$$

$$JO_i = \{jO_1, jO_2, ..., jO_\eta, ..., jO_{|JO_i|=nLO_i} \mid \exists jO_\eta \in [1;|L|] \Rightarrow g_{i,jO_\eta} == 2\}; \quad (12)$$

As a result, we define sets of input and output connections $(JI, JO)_I$ based on configuration matrix G for each system's structural module m_i.

Consequently, a set of input and output parameters (XI_i and XO_i) of system's structural module m_i can be grouped accordingly by input and output connections (JI_i and JO_i) for this module.

$$XI_i = \{XI_{jI_1}, XI_{jI_2}, ..., XI_{jI_\mu}, ..., XI_{jI_{/JI_i|}}\}; \quad (13)$$

$$XO_i = \{XO_{jO_1}, XO_{jO_2}, ..., XO_{jO_\eta}, ..., XO_{jO_{/JO_i|}}\}. \quad (14)$$

E. Calculation of Failure Coefficients in the Operation of GAM's Structural Modules and Connections

For ICS structural modules m_i and connections between them l_j we can calculate failure coefficients of structural module operation $Km_i \in [0,1]$ as well as failure coefficients of connection operability $Kl_j \in [0,1]$ in terms of passing data flows through them.

In the event of temporary inoperability or shutdown of ICS structural module m_i, due to the need for restoration, disconnection / loss of power or in case of preventive maintenance, repair, modernization, etc., we accept $Km_i = -1$.

All connections l_j interacting with given module m_i, are also considered disabled and have $Kl_j = -1$.

Accordingly, in case of temporary inoperability of connection l_j, its failure coefficient is Kl_j=-1. If this connection is the only connection of ICS structural module m_i to the system, then the failure coefficient becomes Km_i=-1. Additionally, if all input and/or output connections of ICS structural module m_i are inoperable, then the failure coefficient of the structural module becomes $Km_i = -1$.

For our diagnostic model, we assume that for a working connection l_j, the failure coefficient of its operability $Kl_j = 0$, in case after the expiration of the maximum time interval $\Delta Tmax_j$ specified in the system for the session of receiving and transmitting data, all data X_j, transmitted over connection l_j will be transmitted completely and without distortion.

Accordingly, the failure coefficient of the connection is $Klj = 1$, if for a working connection l_j after $\Delta Tmax_j$ expires, none of the information parameters are transmitted.

In order to calculate the failure coefficient for the connection operation Kl_j, we define the following parameters:

$a_r = 0$ is a weight coefficient of the reliably transmitted parameter over connection l_j;

$a_u = 0.5$ is a weight coefficient of an invalidly transmitted parameter over connection l_j;

$a_m = 1$ is a weight coefficient of the parameter which was not transmitted over connection l_j;

$|X_j|$ is the number of information parameters transmitted over connection l_j;

$nX_j(t)$ is the number of information parameters generated for transmission over connection l_j at time t;

$n_u\ X_j(t)$ is the number of information parameters transmitted unreliably over connection l_j at time t;

$n_m\ X_j(t)$ is the number of information parameters which were not transmitted over connection l_j at time t;

Then the failure coefficient of connection operability l_j at time t is calculated as follows:

$$Kl_j(t) = \frac{a_u \cdot n_u X_j(t) + a_m \cdot n_m X_j(t)}{nX_j(t)}. \quad (15)$$

In order to calculate the failure coefficient for the structural module Km_i we determine the following parameters:

$xI_{i\mu} \in XI_i$ is an input information parameter of the ICS structural module m_i;

$xO_{i\eta} \in XO_i$ is an output information parameter of the ICS structural module m_i;

$d(x)$ is a domain of values for information parameter x;

$d_e(x)$ is a domain of exceptions (failures) for information parameter x when algorithm A_i is passing inside ICS structural module m_i. At the same time, structural module m_i remains operational, but the value of information parameter x is missing as it cannot be determined;

$d_A(x)$ is a domain of algorithm failures A_i inside ICS structural module m_i for information parameter x. In this case, structural module m_i becomes temporarily inoperable;

$d_r(x) = d(x)\ U\ d_e(x)$ is a domain of reliable values of information parameter x;

$d_u(x) = \neg(d(x)\ U\ d_e(x))$ is a domain of unreliable values of information parameter x;

$nXI_i = |XI_i|$ is the number of input parameters of ICS structural module m_i;

$nXO_i = |XO_i|$ is the number of output parameters of ICS structural module m_i;

nD_i, nDe_i, nDA_i, nDr_i, nDu_i are the number of output parameters of ICS structural module m_i in the corresponding domains;

$a_r = 0$ is a weight coefficient of the output parameter for ICS structural module m_i in domain $d(x)$;

$a_u = 0.5$ is a weight coefficient of the output parameter for ICS structural module m_i in domain $d_u(x)$;

$a_m = 1$ is a weight coefficient of the output parameter for ICS structural module m_i in domain $d_e(x)$.

Then the failure coefficient of ICS structural module m_i at time t is calculated as follows:

$$Km_i(t) = (nDA_i(t))?(-1): \frac{a_u \cdot nDu_i(t) + a_m \cdot nDe_i(t)}{nXO_i(t)}. \quad (16)$$

V. Results and Discussion

Fig. 3 shows a directed graph providing an example of a typical block diagram for a certain ICS.

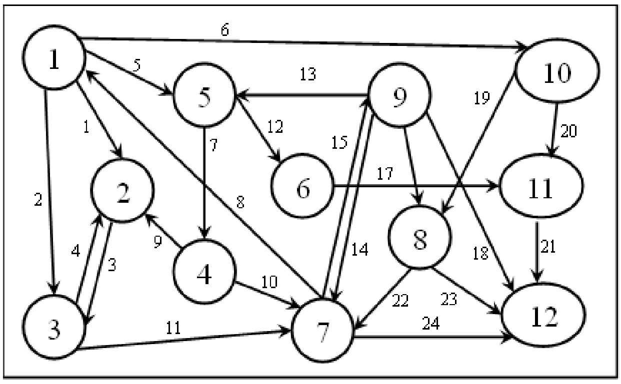

Fig. 3. An example of a typical structural diagram for an ICS

In this example, a certain ICS is represented as a directed graph *G*, where the vertices of the graph are interpreted as structural modules of ICS, and graph edges are interpreted as information flows with input and output data.

The ICS structural modules are described by a set $M=\{m_1, m_2, \ldots, m_i, \ldots, m_{|M|}\}$. The number of structural modules of ICS (vertices of the graph) is |M|=12.

The edges of the graph are described by a set $L=\{l_1, l_2, \ldots, lj, \ldots, l_{|L|}\}$. The number of edges between ICS structural modules is |L| =24.

The configuration matrix of graph *G* is shown in Fig. 4 and Fig. 5.

	1	2	3	4	5	6	7	8	9	10	11	12
1	2	2			2	2		1				
2	1		2	1					1			
3		1	1	2							2	
4							1		2	2		
5					1		2					2
6												1
7								2		1	1	
8												
9												
10						1						
11												
12												

Fig. 4. Configuration matrix of graph *G* (part 1 of 2)

	13	14	15	16	17	18	19	20	21	22	23	24
1												
2												
3												
4												
5	1											
6					2							
7		1	2							1		2
8				1			1			2	2	
9	2	2	1	2		2						
10							2	2				
11					1			1	2			
12						1			1		1	1

Fig. 5. Configuration matrix of graph *G* (part 2 of 2)

The configuration matrix consists of 12 rows (graph vertices) and 24 columns (graph edges).

The software application developed within the framework of this research allows us to do the following:

- Configure block diagram for ICS of any topology;
- Display mnemonic diagrams with the current values of the failure coefficients operation for the structural modules and connections of the system;
- Display graphs with a retrospective of the values in the failure coefficients for the structural modules and connections of the system.

The method described in this paper calculates the values of the failure coefficients for the operation of ICS structural modules and connections. It works on the basis of the analysis of changes in the state of information flows passing through the system in real time and allows us to monitor the current system's state and predict failures in the system.

VI. Conclusions

The method of failure detection and localization in distributed information and control systems considered in the article is based on the analysis of changes in the state of information flows passing through the system. It allows us to perform automatic detection and localization of failures in the system's structural modules during its operation in real time.

Minimization of hidden failure detection period [7] increases the probability of the system's failure-free operation by an average of 7% ... 8%.

Application of methods for automatic failure diagnostics of a particular system's structural modules is a good prerequisite for creating methods for automatic self-recovery of distributed information and control systems after reversible failures in real time.

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