A Continuous Time Markov Model for Unlicensed Spectrum Access

Usama Mir, Leila Merghem-Boulahia, Moez Esseghir and Dominique Gaïti
ICD/ERA, CNRS UMR 6279, Université de Technologie de Troyes,
12 Rue Marie Curie, 10010 Troyes Cedex, France
{usama.mir, leila.merghem_boulahia, moez.esseghir, dominique.gaiti}@utt.fr

Abstract— **Recent static spectrum allocations have created an opportunity to develop novel solutions that can efficiently share the available spectrum both in licensed and unlicensed bands. Considering the relative rarity of solutions for unlicensed spectrum access, in this paper, we propose a scheme, where the cognitive radio (CR) devices (equipped with agents) interact with their neighbors to form several coalitions over the unlicensed bands. These types of coalitions can provide a less-conflicted spectrum access as the agents mutually agree for spectrum sharing. Further, we present a continuous time Markov chain (CTMC) with queuing to model the user interactions with the movement of spectrum access process from one state to another and derive the important performance metric as the blocking probability. Numerical results are presented to observe the impact of forming multiple coalitions on the blocking probability.**

Keywords— **Cognitive radios, continuous time Markov chains, unlicensed spectrum access, multiagent coalition formation.**

I. Introduction

Modern day wireless networks have provided us access to multimedia and communication services which are available anywhere and can be accessed from any place using our cellular phones or PDAs. However, the usage of these diverse wireless services depends upon the efficient access to the available radio frequency (RF) spectrum. Reports from the Federal Communications Commission (FCC) [1][14] indicate that current spectrum usage is static where the spectrum is either assigned to the licensed holders for restrictive use in a particular geographical location or inefficiently utilized in unlicensed bands. This static and inefficient regime causes spectrum scarcity problem that needs to be addressed with an effective reform [9].

The scarcity problem is temporary and can be alleviated if the spectrum access is performed efficiently. One important step towards efficient spectrum access is the development of cognitive radio (CR) [10] technology, which senses the nearby spectrum portions (or bands), tries to use them and may vacate the spectrum when a licensed (or primary) user appears [8]. Basically, a CR (or a secondary) user does not own a license for its spectrum usage and it can access the spectrum either opportunistically or by coexisting with the neighboring licensed users. This kind of access is called licensed sharing and a rather large number of solutions already exist in the literature [5][6][19][20]. On the other hand, spectrum access for the unlicensed bands, where there is no primary user and all the CR users (having equal rights) try to use the spectrum, is less addressed in literature [13][16][22]. One important concern that remains unanswered in the existing literature on unlicensed bands is the possibility of users' conflicts that can occur if the spectrum access is performed without cooperation. Thus, a mechanism must be ensured where the users can mutually agree to share their acquired spectrum with the neighboring users and providing this type of spectrum access is the objective of our work.

In fact, in this paper, we propose a solution for the unlicensed bands where the secondary users (SUs) equipped with agents [12] cooperate with each other to form coalitions [15]. The cooperative coalition formation can provide a less-conflicted spectrum access [18] as the coalitions are formed, when the agents mutually agree for spectrum sharing. Essentially, our work is divided in two main parts: First, we provide a multiagent (MA) coalition formation algorithm by detailing the message exchanges between the SU agents. The agents directly cooperate with each other to form several coalitions over the unlicensed bands. Second, we present a continuous time Markov chain (CTMC) model [5][22] with queuing to depict the user interactions with movement of spectrum access process from one state to another. We start our model by proposing a 2-SUs CTMC with queuing and extend it for *N* users. The queuing system can increase system performance by reducing the blocking probability, which will be shown by the numerical results.

The rest of the paper is shaped as follows: A brief related work is provided in Section II. System model is presented in Section III. Section IV explains our coalition formation framework along with the algorithm. The CTMC model with queuing is described in Section V. Numerical results are presented in Section VI. Finally, some conclusions and future perspectives are given in Section VII.

II. Related Work

In recent years, several authors have addressed spectrum access concerns under unlicensed bands using game-theoretical approaches [7][13][16][21][23] and CTMC modeling [22]. A repeated game is proposed in [13], where two independent networks share the unlicensed spectrum and Nash equilibrium condition is achieved showing the improvement in overall spectrum usage. Another similar game is presented in [16] where SUs show selfish behaviors in order to improve their individual spectrum usage. The authors of [21] propose two different game-theoretical algorithms to improve individual and global utility functions of respected SUs (in terms of their spectrum usage). Slightly different algorithm is presented in [7], where the wireless network is divided into several clusters and the global utility of every cluster (in terms of power allocation) is maximized subject to the total utility of each SU in that cluster. The work in [23] proposes a coalition formation game based on a Markov model to analyze the selfish user interactions over an interference channel. On contrary, a well-known CTMC model for unlicensed spectrum access is proposed in [22] to achieve fair spectrum assignments amongst the SUs. The parameters such as spectrum utilization time and blocking probabilities are derived using the CTMC model.

Although, existing schemes for unlicensed access are focused on improving spectrum efficiency, still there are a few concerns, which remain unanswered. For example, if multiple SUs try to access the available spectrum, their access is not coordinated with the already presented SUs causing unnecessary conflicts. Though, some game-theoretical approaches address this concern, however, these solutions do not consider how the players should interact for spectrum sharing [4]. Another issue is related to the Markov chains where most of the models are developed for a few users without focusing on *N* users.

Therefore, we address the above mentioned issues by allowing users to form coalitions via mutual interactions. Moreover, we model the spectrum access process in continuous time via CTMC for *N* users.

III. BASIC MODEL OF UNLICENSED SPECTRUM ACCESS

We depict the example deployment of several SUs over unlicensed bands in Fig. 1. In this figure, the environment is distributed where spectrum is like an open pool and all the users are SUs (or cognitive radios) i.e. none of them has exclusive spectrum access right/license. The cooperative agents are deployed over each of the SUs to form coalitions (CGs: coalitional groups) with the neighboring SUs. We also assume that the agents are arbitrarily distributed such that each SU should have one agent and the SUs transmit using a combination of radio parameters (e.g. power and modulation) which are predefined in their internal hardware. An SU can be a member of several coalitions simultaneously and it cooperates with the neighbors using contract net protocol (CNP) [17], which allows a user to exchange a series of messages in order to form coalitions. The entire spectrum is assumed to be uniform i.e. all its bands (or channels) have the same characteristics and nature. We define two types of users in our system: (1) the *acquiring* users, which are first to acquire the 'idle' spectrum or the users which are already using the available spectrum and (2) the *requesting* users, who want to utilize the spectrum but they sense (or detect)[1] the presence of acquiring users on the spectrum. These *requesting* users then have to cooperate with the *acquiring* users for spectrum access. The SUs arrival and departure rates follow Poisson distribution with means λ s^{-1} and μ s^{-1} (s^{-1} = per second), respectively. Note that Fig. 1 only serves as an example to illustrate our proposed approach and spectrum sharing with multiple users will be examined in later sections. The words SU, CR, user, and agent are used interchangeably. Similarly, the words spectrum, portion and band will be used conversely throughout the rest of the paper.

[1] Note that, in essence, spectrum and user sensing (or detection) is beyond the scope of our work, however, several existing techniques such as matched filter sensing [3], cyclostationary sine waves detection [2] and user's energy detection [11] can be used.

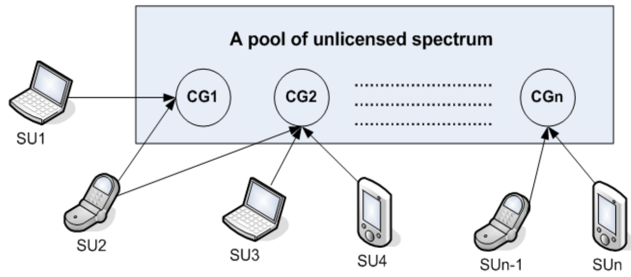

Fig. 1. An example of spectrum access over unlicensed bands via coalition formation

IV. COALITION FORMATION PROCESS

In this section, we present our algorithm based on bottom-up coalition formation [15] for unlicensed spectrum access. At first, the bottom-up coalition formation process starts when the *requesting* SU agents send *call for proposal (CfP)* messages to the nearby *acquiring* SUs. The CfP message indicates the time for which spectrum is needed by the *requesting* SU along with its ID (which helps an *acquiring* SU to reply back to the *requesting* SU). The *acquiring* SUs examine the received CfPs on FIFO bases and send their *proposals* to the *requesting* SUs. The proposal message contains the amount of spectrum available for sharing. It is also possible at the reception of CfP(s) the spectrum is completely occupied. In this case, the *acquiring* SU saves the CfP(s) in its temporary queue (i.e. the concept of queuing) and sends the proposal(s) when the acquired spectrum becomes available. The process ends when the *requesting* SU *accepts* the proposal(s), which better serves its spectrum usage.

Having received the CfP message(s), an *acquiring* agent uses the algorithm below to determine which SU should be granted entrance in its coalition. If there are some spectrum portions yet to be utilized (i.e. the *SpectrumAvailabilityStatus* is *"Yes"*), then the *acquiring* SU starts handling the received CfP(s). When the *acquiring* SU receives only one CfP, it sends its proposal to the requesting SU; otherwise, it examines multiple CfPs on FIFO bases and sends the proposals accordingly. The coalition is formed when the *requesting* SU accepts the proposal. In other case, if the spectrum is not available (or the *SpectrumAvailabilityStatus"* is *"No"*), the *acquiring* SU saves the received CfPs in its temporary queue and handles them (on FIFO bases) when it gets free.

V. CTMC MODELS FOR THE UNLICENSED SPECTRUM ACCESS

In this section, we model MA coalition formation based spectrum access process as a continuous time Markov chain (CTMC) with queuing. This queuing CTMC allows an *acquiring* SU to save the received CfPs in a temporary queue (when it does not have any spectrum to share) and handle them when the spectrum becomes available. Following the simplified method proposed in [5], we start by a 2-SUs CTMC and extend it for multiple users.

Coalition Formation Algorithm

Let $Spec_{currenta}$ is the spectrum acquired by an SU denoted by 'su_a' and $List_{CfPa}$ is a temporary array for saving the received CfPs.

If SpectrumAvailabilityStatus = "Yes" do
If received message(s) = CfP
Add each received CfP in $List_{CfPa}$
If $List_{CfPa}$= {i}, where i is the only element in $List_{CfPa}$
/su_a sends proposal to the requesting secondary user su_i with the amount of spectrum available for sharing*/*
Send proposal to su_i
Else
Arrange CFPs using FIFO method
Send proposals based on the available spectrum and the required spectrum of the requesting users
End If
End If

*/*Handling accept*/*
If received message(s) = Accept
Build a coalition and start spectrum sharing with the selected secondary user(s)

/ $Spec_{assignedi}$ is the amount of spectrum assigned to a secondary user su_i*/*
$Spec_{currenta} = Spec_{currenta} - Spec_{assignedi}$
If $Spec_{currenta} = 0$ Then
Set SpectrumAvailabilityStatus = "No"
End If
End If
Else
Temporarily save each CfP in $List_{CfPa}$.
Arrange the CfP(s) (on FIFO bases) and send proposal(s), when "SpectrumAvailabilityStatus" becomes "yes"
End If

A. 2-SUs CTMC

The 2-SUs CTMC with queuing can be modeled as a six states chain shown in Fig. 2. For explanation purpose, we denote the arrivals and departures of two users SU_i and SU_k with (λ_i s^{-1}, λ_k s^{-1}) and (μ_i s^{-1}, μ_k s^{-1}) such that $\lambda_i = \lambda_k = \lambda$ s^{-1} and $\mu_i = \mu_k = \mu$ s^{-1} respectively. The first state is the 'idle' state where no SU exists on the spectrum. The CTMC goes to state *i* and/or *k* with rates λ_i s^{-1} and λ_k s^{-1} when both the users can individually access the spectrum. The CTMC can return to 'idle' state with rate $\mu_{i\ or\ k}$ s^{-1}, if any of the users completes its spectrum usage. In other case, let us suppose that SU_i has acquired the whole spectrum, so it is the *acquiring* user. Now, assume that SU_k arrives in the system, performs spectrum sensing and detects the presence of SU_i on the sensed spectrum. In this case, rather than trying to access the spectrum at the same time as SU_i and to avoid any unnecessary conflicts, SU_k sends a CfP message to SU_i. If SU_i has the available spectrum to share, CTMC moves to state *(i, k)* with λ_k where the coalition is formed between the users (after the exchange of proposal and accept messages). Otherwise, CTMC goes to state *kw* with λ'_k such that $\lambda'_k = \lambda$s^{-1}. In *kw*, SU_k has to wait until the spectrum becomes available (or free) for sharing. When available, CTMC moves to state *(i, k)* with μ'_i (such that $\mu'_i = \mu$ s^{-1}) where both the users access the spectrum under a coalition. It is also possible that SU_i has completed its utilization and it wants to leave the spectrum, therefore, CTMC moves from state *kw* to *k* with μ_i where SU_k uses the spectrum individually. CTMC works similarly if SU_k is the *acquiring* user and SU_i is the *requesting* user.

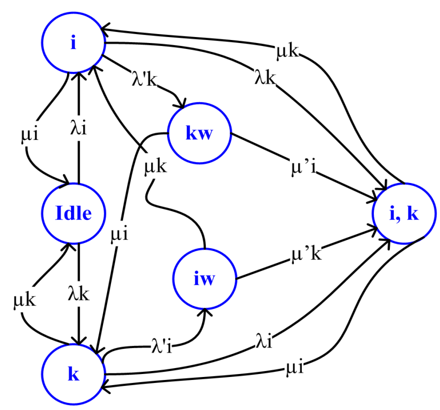

Fig. 2. 2-SUs CTMC with queuing

The infinitesimal generator matrix 'G_2' (for Fig. 2) which characterizes various state transitions of 2-SUs CTMC is shown in Fig. 3. The balance equations with the rate of flow in equaling the rate of flow out are as follows:

$$\pi_{idle} = \frac{\pi_i \mu_i + \pi_k \mu_k}{(\lambda_i + \lambda_k)} \tag{1}$$

$$\pi_i = \frac{\pi_{idle}\lambda_i + \pi_{i,k}\mu_k + \pi_{iw}\mu_k}{(\lambda_k + \lambda'_k + \mu_i)} \tag{2}$$

$$\pi_k = \frac{\pi_{idle}\lambda_k + \pi_{i,k}\mu_i + \pi_{kw}\mu_i}{(\lambda_i + \lambda'_i + \mu_k)} \tag{3}$$

$$\pi_{i,k} = \frac{\pi_i \lambda_k + \pi_k \lambda_i + \pi_{iw}\mu'_i + \pi_{kw}\mu'_k}{(\mu_i + \mu_k)} \tag{4}$$

$$\pi_{iw} = \frac{\lambda'i}{(\mu_k + \mu'_k)}\pi_k \tag{5}$$

$$
G_2 = \begin{array}{c|cccccc}
\text{States} & 0 & i & k & iw & kw & (i,k) \\
\hline
0 & -(\mu_i+\mu_k) & \lambda_i & \lambda_k & 0 & 0 & 0 \\
i & \mu_i & -(\lambda_i+2\mu_k) & 0 & 0 & \lambda'_k & \lambda_k \\
k & \mu_k & 0 & -(\lambda_k+2\mu_i) & \lambda'i & 0 & \lambda i \\
iw & 0 & \mu_k & 0 & -\lambda'_i & 0 & \mu'_k \\
kw & 0 & 0 & \mu_i & 0 & -\lambda'_k & \mu'_i \\
(i,k) & 0 & \mu_k & \mu_i & 0 & 0 & -(\lambda_i+\lambda_k+\mu'_i+\mu'_k)
\end{array}
$$

Fig. 3. infinitesimal generator matrix for 2-SUs CTMC

$$\pi_{kw} = \frac{\lambda'_k}{(\mu_i+\mu'_i)}\pi_i \qquad (6)$$

$$\pi_0+\pi_i+\pi_k+\pi_{iw}+\pi_{kw}+\pi_{i,k}=1 \qquad (7)$$

where π is the stationary probability of being in any of the states *{idle, i, k, iw, kw, (i, k)}*. Supposing, $\lambda_i = \lambda_k = \lambda'_i = \lambda'_k = \lambda$ and $\mu_i = \mu_k = \mu'_i = \mu'_k = \mu$ and solving the above equations, we get the following stationary probability values:

$$
\left.
\begin{aligned}
\pi_{idle} &= \frac{\mu}{\lambda}\left(\frac{1}{2+\frac{5\lambda}{2\mu}+\frac{\mu}{\lambda}}\right) \\
\pi_i = \pi_k = \pi &= \frac{1}{2+\frac{5\lambda}{2\mu}+\frac{\mu}{\lambda}} \\
\pi_{iw} = \pi_{kw} = \pi_w &= \frac{\lambda}{2\mu}\left(\frac{1}{2+\frac{5\lambda}{2\mu}+\frac{\mu}{\lambda}}\right) \\
\pi_{i,k} &= \frac{3\lambda}{2\mu}\left(\frac{1}{2+\frac{5\lambda}{2\mu}+\frac{\mu}{\lambda}}\right)
\end{aligned}
\right| \qquad (8)
$$

An important performance metric for access networks estimated using CTMC models is the blocking probability (P_b). In 2-SUs CTMC, the third SU is blocked, because the coalition is only possible at maximum between two users, i.e. between SU_i and SU_k in Fig. 2. Even when, one SU is using the spectrum and the other is waiting, the third user will not be allowed to enter in the waiting state. Formally,

$$P_{b\,(2\text{-SUs})} = \pi_{i,\,k} \qquad (9)$$

B. N-SUs CTMC

The N-SUs CTMC can be drawn similar to 2-SUs CTMC and is shown in Fig. 4[2], such that $\lambda_i = \lambda_j = \ldots\ldots \lambda_N = \lambda$ s^{-1} and $\mu_i = \mu_j = \ldots\ldots \mu_N = \mu$ s^{-1}, respectively. It can be seen from the figure that we need at least one *acquiring* SU to form a coalition and after that several *requesting* SUs can join the coalition. Moreover, *m= {1, 2… N}* represents the size of a coalition, e.g. when *m=3*, then any SU *i* can form a coalition with any other SUs *j* and *k* as *(i, j, k)*. The number of states (S_N) in N-SUs CTMC at each value of *m* follows a combinational series pattern and is given by:

$$S_{N\ (\text{queuing})} = 1+\sum_{m=1}^{N} C^m_N + \sum_{m=1}^{N-1} C^m_N \qquad (10)$$

$$\text{such that} \quad C^m_N = \frac{N!}{(N-m)!*m!}$$

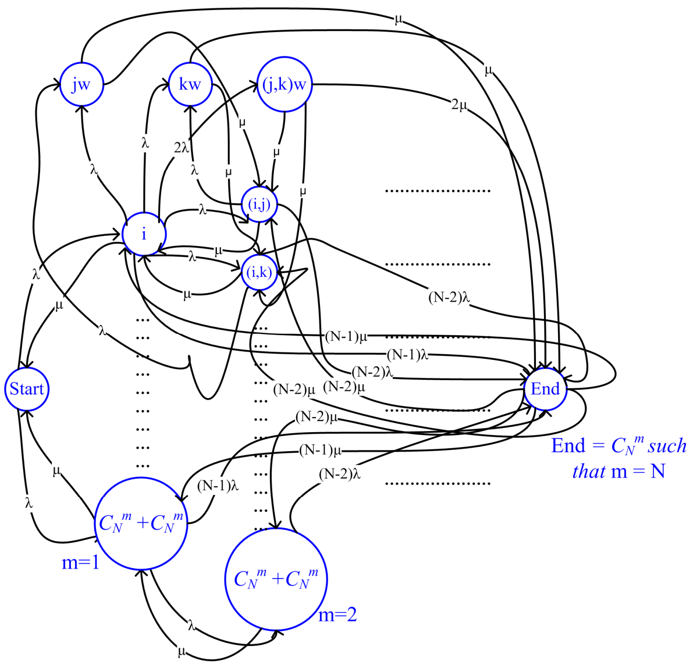

Fig. 4. N-SUs CTMC

We can combine equations (1-8) to obtain the stationary probabilities for *N* users in the following concise equation [22].

$$G'\pi = b \qquad (11)$$

[2] The figure representation looks fairly complex; however, the principal working of this figure is similar to our 2-SUs CTMC. In other words, Fig. 4 is an *N* users extension of Fig. 2.

where $\pi = \{\pi_{idle}, \pi_{i}, \ldots, \pi_{i,j} \ldots, \pi_{End}\}$, $G' = \begin{bmatrix} G^T \\ 1_{S_N \times 1} \end{bmatrix}$ (G can be constructed similar to Fig. 3) and $b = \begin{bmatrix} 0_{S_N \times 1} \\ 1 \end{bmatrix}$. The P_b for N-SUs can be derived similar to equation (9):

$$P_{b\,(\text{N-SUs})} = \pi_{(i,\,j,\,k,\ldots\ldots,\,N)} \qquad (12)$$

VI. NUMERICAL RESULTS

In this section, we present the numerical results for our CTMC models. We start by showing the blocking probability (P_b) for 2-SUs CTMC. Then, we depict the effect of increasing arrival and departure rates on P_b considering our CTMC with N SUs where several coalitions can be formed between the users. We also compare the N-SUs queuing CTMC with no-queuing (or without queuing) model which has been submitted for a journal publication. All the experiments are realized on a PC using MATLAB.

In Fig. 5, P_b is compared for without and with coalition spectrum access, respectively. This set of experiments is realized for our 2-SUs CTMC where we vary λ from 5 to 30 s^{-1} and fix $\mu = 10\ s^{-1}$ in order to observe several values of P_b. In case of without coalition, only one user is allowed to access the spectrum, as a result, P_b is $\dfrac{1}{(1+\mu/\lambda)}$. On the other hand, considering with coalition case, both the users can access the spectrum in a coalition, therefore, P_b is calculated using equation (9). This comparison between without and with coalition systems is clear from Fig. 5 which shows that the coalition formation process increases system performance by reducing the overall blockage.

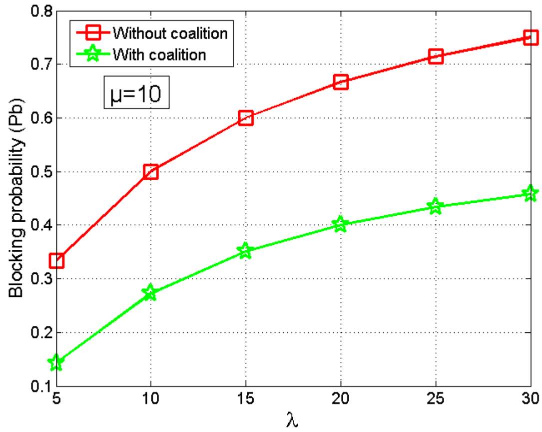

Fig. 5. Comparison between spectrum access without and with coalition in terms of P_b at $\mu = 10\ s^{-1}$ and different arrival rates for 2-SUs CTMC with queuing

In Fig. 6, we compare P_b at distinct arrival rates for N-SUs CTMC with queuing. It is noticeable that P_b continuously decreases, with increasing number of users (N), because the newly arriving SUs have various choices to form coalitions and they can simultaneously cooperate with multiple users for spectrum access. Thus, our N-SUs CTMC model reduces P_b by creating more spectrum access opportunities for the newly arriving SUs.

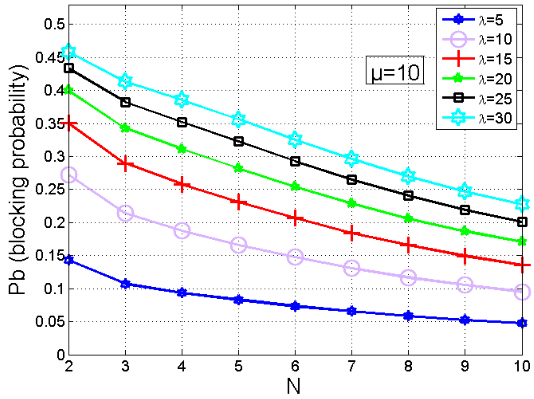

Fig. 6. Blocking probability at $\mu = 10\ s^{-1}$ and different arrival rates for N-SUs CTMC with queuing

In Fig. 7, we compare the blocking probability (P_b) for the queuing and without-queuing systems. Again the experiment is performed for N SUs with $\lambda = 20\ s^{-1}$ and $\mu = 10\ s^{-1}$, respectively. For without-queuing CTMC, a *requesting* SU's received CfP is rejected, in case the *acquiring* SU is occupied. On the other hand, with the queuing CTMC, the received CfP can stay in a temporary queue while occupied and is handled when the *acquiring* SU gets free. As a result, the queuing system causes lower blockage compared to the system without queuing, which can be seen, from Fig. 7. By looking at this figure, we can conclude that the newly arriving SUs have higher chances of accessing the spectrum if their messages are queued, rather than being immediately rejected.

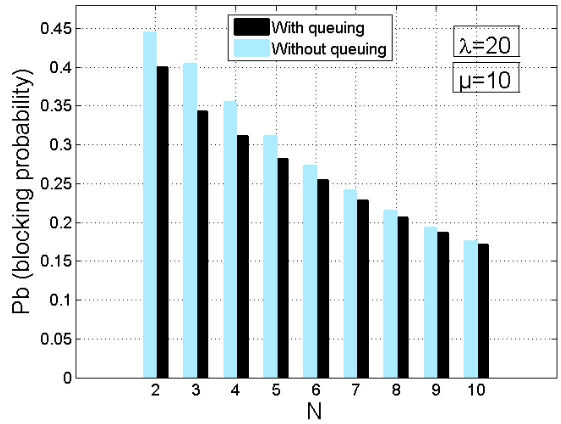

Fig. 7. Comparison between queuing and without-queuing CTMCs for N-SUs in terms of P_b with $\lambda = 20\ s^{-1}$ and $\mu = 10\ s^{-1}$

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