Exercise 1 – *Bayesian inference and Data assimilation*

Due by: Tuesday, 25 April 2023, 23:59 (CEST)
Please refer to the assignment submission guideline on Moodle.

Problem 1 In this exercise, you are asked to implement Euler scheme to simulate a nonlinear dynamics. Consider the *Van der Pol* oscillator, given by

$$\dot{x}(t) = y(t) \tag{1a}$$
$$\dot{y}(t) = -x(t) + \big(1 - (x(t))^2\big)y(t) \tag{1b}$$

from initial value $x(0) = y(0) = 0.1$. Implement Euler scheme for this system up to time $t = 30$ with uniform time discretisation.

- Plot $(x(t), y(t))$ for $0 \leq t \leq 30$ with $\Delta t = 0.01$.
- Change the time step to $\Delta t = 0.2$ and $\Delta t = 0.3$, then plot the results.
- **(Optional, not to be marked)** Play further with the time step until it finds numerical instability. Also simulate the oscillator with different initial values.

Problem 2 In this exercise, the nonlinear term is removed to obtain analytic solution. Consider the harmonic oscillator:

$$\dot{x}(t) = y(t) \tag{2a}$$
$$\dot{y}(t) = -x(t) \tag{2b}$$

- Simulate the harmonic oscillator from initial value $x(0) = y(0) = 1.0$ using Euler scheme with $\Delta t = 0.01$. Obtain the values of $y(5)$ and $y(10)$.
- Given initial condition $x(0) = x_0$ and $y(0) = y_0$, $y(t)$ is given by

  $$y(t) = -x_0 \sin(t) + y_0 \cos(t) \tag{3}$$

  Solve x_0 and y_0 from the values of $y(5)$ and $y(10)$ you obtained in the previous step. Is it reasonably close to the true value?
- Increase the time step Δt to 0.1 and repeat the procedure. How is the result different from the previous one?
- **(Optional, not to be marked)** Play with the time step. Decrease Δt towards zero and plot the relationship of Δt vs the error $|x_0 - 1|$.