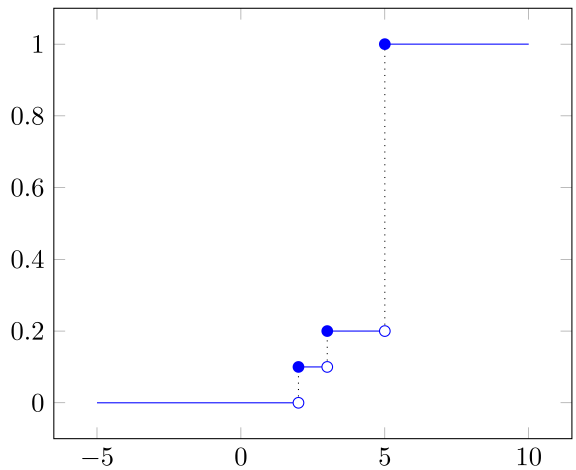
1. Show that
$$\mathbb{P}(X = x) = F(x^+) - F(x^-).$$

2. Let X be such that $\mathbb{P}(X = 2) = \mathbb{P}(X = 3) = 1/10$ and $\mathbb{P}(X = 5) = 8/10$. Plot the CDF F. Use F to find $\mathbb{P}(2 < X \leq 4.8)$ and $\mathbb{P}(2 \leq X \leq 4.8)$.

x	$\mathbb{P}$(X=x)	$F_X(x)$
2	1/10	1/10
3	1/10	2/10
5	8/10	10/10



$$\begin{aligned}
\mathbb{P}(2 < X \leq 4.8) &= \mathbb{P}(X > 2) \cdot \mathbb{P}(X \leq 4.8) \\
&= \big\{1 - \mathbb{P}(X \leq 2)\big\} \cdot \mathbb{P}(X \leq 4.8) \\
&= \big\{1 - F_X(2)\big\} \cdot F_X(4.8) \\
&= \big\{1 - 1/10\big\} \cdot 2/10 \\
&= 18/100
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(2 \leq X \leq 4.8) &= \mathbb{P}(X \geq 2) \cdot \mathbb{P}(X \leq 4.8) \\
&= \big\{\mathbb{P}(X > 2) + \mathbb{P}(X = 2)\big\} \cdot \mathbb{P}(X \leq 4.8) \\
&= \big\{1 - \mathbb{P}(X \leq 2) + \mathbb{P}(X = 2)\big\} \cdot \mathbb{P}(X \leq 4.8) \\
&= \big\{1 - F_X(2) + \mathbb{P}(X = 2)\big\} \cdot F_X(4.8) \\
&= \big\{1 - 1/10 + 1/10\big\} \cdot 2/10 \\
&= 2/10
\end{aligned}$$

3. Prove Lemma 2.15.

4. Let X have probability density function

$$f_X(x) = \begin{cases} 1/4 & : 0 < x < 1 \\ 3/8 & : 3 < x < 5 \\ 0 & : otherwise. \end{cases}$$

(a) Find the cumulative distribution function of X.

(b) Let $Y = 1/X$. Find the probability density function $f_Y(y)$ for Y

Hint: Consider three cases: $\frac{1}{5} \leq y \leq \frac{1}{3}$, $\frac{1}{3} \leq y \leq 1$, and $y \geq 1$

$x \leq 0$

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt = \int_{-\infty}^{x} 0dt = 0; x \leq 0$$

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt = \int_{-\infty}^{x} 0dt = 0; 0 \leq x \leq 1$$

$$\begin{aligned} F_X(x) &= \int_{-\infty}^{x} f_X(t)dt; 0 \leq x \leq 1 \\ &= \int_{-\infty}^{x} 0dt \end{aligned}$$

$$\begin{aligned} F_X(x) &= \int_{-\infty}^{x} f_X(t)dt \quad x \leq 0 \\ F_X(x) &= \int_{-\infty}^{x} f_X(t)dt \quad 0 \leq x \leq 1 \\ F_X(x) &= \int_{-\infty}^{x} f_X(t)dt \quad 1 \leq x \leq 3 \\ F_X(x) &= \int_{-\infty}^{x} f_X(t)dt \quad 3 \leq x \leq 5 \end{aligned}$$

$$F_X(x) = \begin{cases} 0 & : x \leq 0 \\ \frac{x}{4} & : 0 \leq x \leq 3 \\ \frac{5x-9}{4} & : 3 \leq x \leq 5 \\ 1 & : x \geq 5. \end{cases}$$