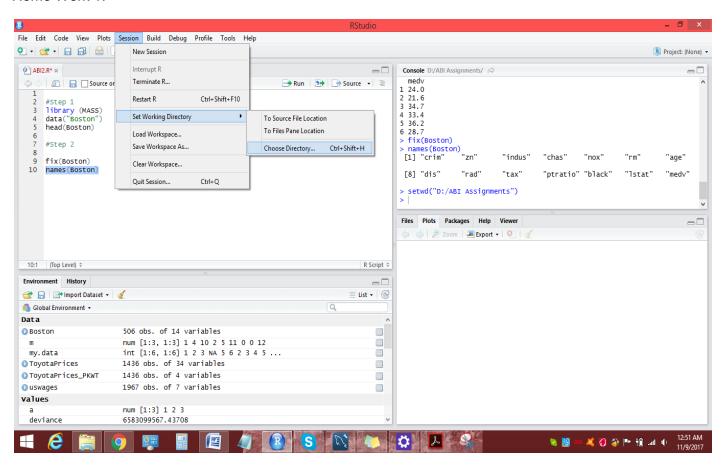
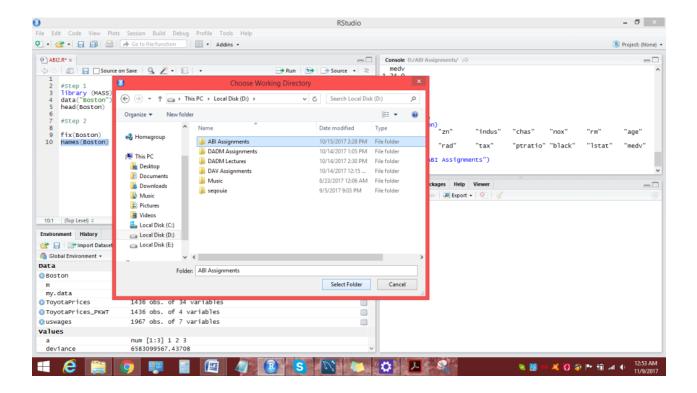
Regression with interaction

Step 1 You need create the working directory and connect Boston data file as you did this in Home Work 1.





Step 2 include interaction terms in a linear model using the Im() function. The syntax Istat:black tells R to include an interaction term between Istat and black. The syntax Istat*age simultaneously includes Istat, age, and the interaction term Istat*age as predictors; it is a shorthand for Istat+age+Istat:age.

COMMAND:-

library (MASS) data("Boston") head(Boston)

fix(Boston) names(Boston)

summary(lm(medv~lstat:black, data=Boston))

summary(Im(medv~Istat*age, data = Boston))

Output:-

```
> summary(lm(medv~lstat:black, data=Boston))
lm(formula = medv ~ lstat:black, data = Boston)
Residuals:
                  Median
     Min
              10
                                30
                                        Max
-21.9377 -3.4993 -0.9214
                            2.7866 26.0088
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 30.4706654 0.6669783 45.69 <2e-16 ***
lstat:black -0.0018569 0.0001332 -13.95 <2e-16 ***
signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 7.82 on 504 degrees of freedom
Multiple R-squared: 0.2784, Adjusted R-squared: 0.277
F-statistic: 194.5 on 1 and 504 DF, p-value: < 2.2e-16
> fix(Boston)
> names(Boston)
[1] "crim"
[7] "age"
                        "indus"
                                  "chas"
                                            "nox"
                                                      "rm"
               "dis"
                                            "ptratio" "black"
                        "rad"
                                  "tax"
[13] "Istat"
              "medv"
> summary(lm(medv~lstat*age, data = Boston))
call:
lm(formula = medv ~ lstat * age, data = Boston)
Residuals:
            1Q Median
   Min
                            3Q
-15.806 -4.045 -1.333
                        2.085 27.552
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 36.0885359 1.4698355 24.553 < 2e-16 ***
            -1.3921168 0.1674555 -8.313 8.78e-16 ***
Istat
            -0.0007209 0.0198792 -0.036 0.9711
age
lstat:age    0.0041560    0.0018518    2.244    0.0252 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.149 on 502 degrees of freedom
Multiple R-squared: 0.5557, Adjusted R-squared: 0.5531
F-statistic: 209.3 on 3 and 502 DF, p-value: < 2.2e-16
>
```

Step 3 (0.5 mark) Use command dim to check how many observations and variables are in the Boston file. Find the names of variables in this file.

COMMAND:-

dim(Boston)
names(Boston)

OUTPUT:-

```
> dim(Boston)
[1] 506 14
> names(Boston)
[1] "crim" "zn" "indus" "chas" "nox" "rm"
[7] "age" "dis" "rad" "tax" "ptratio" "black"
[13] "lstat" "medv"
> |
```

There are 506 such observations.

Step 4 (0.5 mark) Run the multivariate regression of medv against lstat and age with interaction.

COMMAND:-

Im.fit =Im(medv~Istat*age, data = Boston)
attach(Boston)
Im.fit= Im(medv~Istat*age, data = Boston)
summary(Im.fit)

```
> lm.fit =lm(medv~lstat*age, data = Boston)
> attach(Boston)
The following objects are masked from Boston (pos = 3):
    age, black, chas, crim, dis, indus, lstat, medv, nox,
   ptratio, rad, rm, tax, zn
> lm.fit= lm(medv~lstat*age, data = Boston)
> summary(1m.fit)
lm(formula = medv ~ lstat * age, data = Boston)
Residuals:
Min 1Q Median 3Q Max
-15.806 -4.045 -1.333 2.085 27.552
           1Q Median
                                 Max
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 36.0885359 1.4698355 24.553 < 2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.149 on 502 degrees of freedom
Multiple R-squared: 0.5557, Adjusted R-squared: 0.5531
F-statistic: 209.3 on 3 and 502 DF, p-value: < 2.2e-16
> |
```

Step 5 (0.5 mark) Is interaction term significant or not. Is the answer difference for confidence probability 5% and 1%? Formulate the appropriate hypotheses and make a conclusion based on the relevant p-values.

Yes, The interaction term is significant here because the p-value is very small (0.0252) and less than 0.05.

Yes, the answer is different for confidence probability 5% and 1% because P-value is very small so it rejects null hypothesis. If P-value is greater than 5% we accept null hypothesis.

Step 6 (1 Mark) Implement the multiple linear regression of medv against lstat and age without interaction (or just see the results in HW1). Compare these two models with and without interact-ion.

COMMAND:-

lm.fit =lm(medv~lstat+age ,data=Boston)
summary (lm.fit)

```
> lm.fit =lm(medv~lstat+age ,data=Boston )
> summary (1m.fit)
call:
lm(formula = medv ~ lstat + age, data = Boston)
Residuals:
            1Q Median
   Min
                            30
                                   Max
-15.981 -3.978 -1.283
                        1.968 23.158
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
                      0.73085 45.458 < 2e-16 ***
(Intercept) 33.22276
                       0.04819 -21.416 < 2e-16 ***
Istat
           -1.03207
age
            0.03454
                       0.01223 2.826 0.00491 **
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ''
Residual standard error: 6.173 on 503 degrees of freedom
Multiple R-squared: 0.5513,
                              Adjusted R-squared: 0.5495
              309 on 2 and 503 DF, p-value: < 2.2e-16
F-statistic:
```

P-value is drastically changing if we put interaction effect in the model.

Without Interaction: - On Individual Basis, the Istat will still reject null hypothesis, but age will have it changed from being rejected without the interaction variable to being accepted with the interaction variable.

Step 7 (0.5 mark) Compare the residual standard errors for these two models.

The Residual Standard Error for Model 1 with Interaction - 6.149 on 502 degrees of freedom And the Residual standard error for Model 2 without Interaction - 6.173 on 503 degrees of freedom. As we can see that the Residual standard error for both the models are same, although the model without interaction will perform better but there will not be any significant impact on the model.

Nonlinear transform of predictors (3 marks)

Step 1 (1 mark) The lm() function can also accommodate non-linear transformations of the predictors. For instance, given a predictor *X*, we can create a predictor *X*² using I(X^2). The function I() is needed since the ^ has a special meaning I() in a formula; wrapping as we do allows the standard usage in R, which is to raise X to the power 2. We now perform a regression of medy onto Istat and Istat².

COMMAND:-

```
Im.fit1 = Im(medv~lstat + I(lstat^2))
summary(Im.fit1)
```

OUTPUT:-

```
> lm.fit1 = lm(medv~lstat + I(lstat^2))
> summary(lm.fit1)
lm(formula = medv ~ lstat + I(lstat^2))
Residuals:
            1Q Median
    Min
                               3Q
                                      Max
-15.2834 -3.8313 -0.5295
                           2.3095 25.4148
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 42.862007   0.872084   49.15   <2e-16 ***
lstat -2.332821 0.123803 -18.84 <2e-16 ***
I(lstat^2) 0.043547 0.003745 11.63 <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 5.524 on 503 degrees of freedom
Multiple R-squared: 0.6407, Adjusted R-squared: 0.6393
F-statistic: 448.5 on 2 and 503 DF, p-value: < 2.2e-16
```

Step 2 (1 mark) Build the single regression of medv (results can be taken from HW1) against lstat. Compare linear and quadratic model using the adjuster R-squared.

COMMAND:-

```
lm.fit = lm(medv~lstat, data = Boston)
summary(lm.fit)
```

```
> lm.fit = lm(medv~lstat, data = Boston)
> summary(1m.fit)
lm(formula = medv ~ lstat, data = Boston)
Residuals:
   Min
            10 Median
                            30
                                  Max
-15.168 -3.990 -1.318
                         2.034 24.500
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 34.55384 0.56263
                                61.41 <2e-16 ***
                                        <2e-16 ***
lstat -0.95005
                       0.03873 -24.53
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.216 on 504 degrees of freedom
Multiple R-squared: 0.5441,
                              Adjusted R-squared: 0.5432
F-statistic: 601.6 on 1 and 504 DF, p-value: < 2.2e-16
```

#Adjusted R-square for linear model is 0.5432 and for Quadratic model is 0.6393.

In the Quadratic model we see the adjusted R-squared value of 0.6393 is higher than the adjusted R-squared value of 0.5432 in linear model. That means, introduction of a new predictor term lstat^2 improves the model more than would be expected by chance. Also, since the value 0.6393 in quadratic model is closer to 1 and, 0.5432 in linear model is closer to 0, this indicates that the Quadratic model has a better fit.

Step 3 (1 mark) Use the anova() function to further quantify the extent to which the quadratic fit is superior to the linear fit. The anova() function performs a hypothesis test comparing the two models. The null hypothesis is that the two models fit the data equally well, and the alternative hypothesis is that the full model is superior.

```
COMMAND:-
lm.fit = lm(medv~lstat)
anova(lm.fit, lm.fit1)
```

OUTPUT:-

#Quadratic Fit is superior model than linear fit because F-statstic is 135.2 and the associated p-value is highly significant.

Classification: Logistic regression (9 marks)

We will begin by examining some numerical and graphical summaries of the Smarket data, which is part of the ISLR library. This data set consists of percentage returns for the S&P 500 stock index over 1, 250 days, from the beginning of 2001 until the end of 2005. For each date, we have recorded the percentage returns for each of the five previous trading days, Lag1 through Lag5. We have also recorded Volume (the number of shares traded on the previous day, in billions), Today (the percentage return on the date in question) and Direction (whether the market was Up or Down on this date).

Step 1 (0.5 mark) Open library (ISLR) and check the name of variables in Smarket file and provide the summary statistics for al variables.

COMMAND:-

library(ISLR) names(Smarket) summary(Smarket) dim(Smarket)

```
> library(ISLR)
> names(Smarket)
                "Lag1"
                                                     "Lag4"
[1] "Year"
                             "Lag2"
                                         "Lag3"
[6] "Lag5"
                "volume"
                                         "Direction"
                             "Today"
> summary(Smarket)
      Year
                     Lag1
                                          Lag2
                Min.
                       :-4.922000
                                    Min.
                                            :-4.922000
        :2001
 1st Qu.:2002
                1st Qu.:-0.639500
                                    1st Qu.:-0.639500
                Median : 0.039000
                                    Median : 0.039000
 Median :2003
 Mean
        :2003
                Mean
                       : 0.003834
                                           : 0.003919
                                    Mean
 3rd Qu.:2004
                3rd Qu.: 0.596750
                                     3rd Qu.: 0.596750
 Max.
        :2005
                Max. : 5.733000
                                    Max. : 5.733000
      Lag3
                          Lag4
                                               Lag5
                            :-4.922000
 Min.
        :-4.922000
                     Min.
                                         Min.
                                                 :-4.92200
 1st Qu.:-0.640000
                     1st Qu.:-0.640000
                                         1st Qu.:-0.64000
 Median : 0.038500
                     Median: 0.038500
                                         Median: 0.03850
 Mean
        : 0.001716
                     Mean
                           : 0.001636
                                          Mean
                                                 : 0.00561
 3rd Qu.: 0.596750
                     3rd Qu.: 0.596750
                                          3rd Qu.: 0.59700
       : 5.733000
                            : 5.733000
                                                : 5.73300
                     Max.
                                          Max.
     volume
                      Today
                                      Direction
 Min.
        :0.3561
                  Min.
                          :-4.922000
                                      Down:602
 1st Qu.:1.2574
                  1st Qu.:-0.639500
                                      Up :648
                  Median: 0.038500
 Median :1.4229
 Mean
        :1.4783
                  Mean.
                         : 0.003138
 3rd Qu.:1.6417
                  3rd Qu.: 0.596750
        :3.1525
                         : 5.733000
 Max.
                  Max.
> dim(Smarket)
[1] 1250
```

Step 2 (0.5 mark) Use the **cor()** function produces a matrix that contains all of the pairwise correlations among the predictors in a data set. Use the parameter [,9] because the **Direction (#9)** variable is qualitative.

COMMAND:-

cor(Smarket [-9])

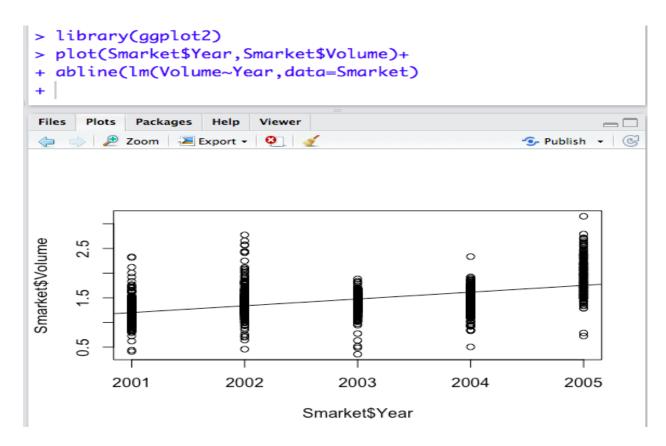
OUTPUT:-

```
> cor(Smarket [-9])
            Year
                         Lag1
                                      Lag2
                                                   Lag3
      1.00000000 0.029699649 0.030596422 0.033194581
Year
                                                         0.035688718
      0.02969965 1.000000000 -0.026294328 -0.010803402 -0.002985911
Lag1
      0.03059642 -0.026294328 1.000000000 -0.025896670 -0.010853533
Lag2
Lag3
      0.03319458 -0.010803402 -0.025896670 1.000000000 -0.024051036
      0.03568872 -0.002985911 -0.010853533 -0.024051036 1.000000000
Lag4
      0.02978799 -0.005674606 -0.003557949 -0.018808338 -0.027083641
Lag5
Volume 0.53900647 0.040909908 -0.043383215 -0.041823686 -0.048414246
Today 0.03009523 -0.026155045 -0.010250033 -0.002447647 -0.006899527
              Lag5
                                       Today
                        Volume
       0.029787995 0.53900647 0.030095229
Year
      -0.005674606 0.04090991 -0.026155045
Lag1
      -0.003557949 -0.04338321 -0.010250033
Lag2
      -0.018808338 -0.04182369 -0.002447647
Lag3
      -0.027083641 -0.04841425 -0.006899527
Lag4
Lag5
       1.000000000 -0.02200231 -0.034860083
volume -0.022002315 1.00000000 0.014591823
Today -0.034860083 0.01459182 1.000000000
>
```

Step 3 (1 mark) Explain why volume is correlated with year. Illustrate this graphically.

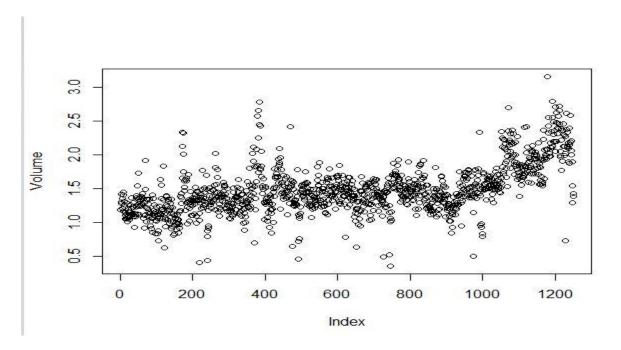
#There is a substantial correlation between Volume and Year.

#By plotting the graph we can see that the volume is increasing over time.



COMMAND:-

attach(Smarket) plot(Volume)



Step 4 (1 mark) Fit a logistic regression model in order to predict Direction using Lag1 through Lag5 and Volume. The glm() function fits *generalized linear models*, a class of models that includes logistic regression. The syntax of the glm() function is similar to that of lm(), except that we must pass in the argument family=binomial in order to tell R to run a logistic regression rather than some other type of generalized linear model.

COMMAND:-

glm.fit = glm(Direction~Lag1+Lag2+Lag3+Lag4+Lag5+Volume, data = Smarket, family = binomial) summary(glm.fit)

```
> glm.fit = glm(Direction~Lag1+Lag2+Lag3+Lag4+Lag5+Volume , data = Smarke
t , family = binomial)
> summary(glm.fit)
call:
glm(formula = Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 +
    volume, family = binomial, data = Smarket)
Deviance Residuals:
  Min
           10 Median
                          3Q
                                 Max
-1.446 -1.203
               1.065 1.145
                              1.326
Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.126000 0.240736 -0.523
                                         0.601
Lag1
         -0.073074
                     0.050167 -1.457
                                         0.145
Lag2
           -0.042301 0.050086 -0.845
                                         0.398
           0.011085 0.049939 0.222
                                         0.824
Lag3
Lag4
            0.009359 0.049974
                               0.187
                                         0.851
Lag5
            0.010313 0.049511
                                0.208
                                         0.835
          0.135441 0.158360 0.855
volume
                                         0.392
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1731.2 on 1249 degrees of freedom
Residual deviance: 1727.6 on 1243 degrees of freedom
AIC: 1741.6
Number of Fisher Scoring iterations: 3
>
```

Step 5 (1 mark) Analyze the logistic Table in order our ability to predict trading Volume using the information about lags.

The negative coefficient for this predictor suggests that if the market had a positive return yesterday, then it is less likely to go up today.

The smallest p-value here is associated with Lag1.

At a value of 0.15, the p-value is still relatively large, and so there is no clear evidence of a real association between Lag1and Direction.

Step 6 (0.5 mark) Use the coef() function in order to access just the coefficients for this fitted model. Check the consistency with the previous step.

COMMAND:coef(glm.fit)

OUTPUT:-

```
> coef(glm.fit)
(Intercept) Lag1 Lag2 Lag3 Lag4
-0.126000257 -0.073073746 -0.042301344 0.011085108 0.009358938
Lag5 Volume
0.010313068 0.135440659
```

INFERENCE:

The results for the coefficients for the model are consistent with the previous step as the values of the coefficients are same.

Step 7 (0.5 mark) Use the summary() function to access particular aspects of the fitted model, such as the p-values for the coefficients.

COMMAND:-

summary(glm.fit)\$coef summary (glm.fit)\$coef[,4]

```
> summary (glm.fit )$coef
                Estimate Std. Error
                                       z value Pr(>|z|)
(Intercept) -0.126000257 0.24073574 -0.5233966 0.6006983
Lag1
            -0.073073746 0.05016739 -1.4565986 0.1452272
            -0.042301344 0.05008605 -0.8445733 0.3983491
Lag2
Lag3
             0.011085108 0.04993854 0.2219750 0.8243333
Lag4
             0.009358938 0.04997413 0.1872757 0.8514445
             0.010313068 0.04951146 0.2082966 0.8349974
Lag5
             0.135440659 0.15835970 0.8552723 0.3924004
Volume
> summary (glm.fit )$coef[,4]
(Intercept)
                   Lag1
                               Lag2
                                           Lag3
                                                       Lag4
  0.6006983
              0.1452272
                          0.3983491
                                      0.8243333
                                                  0.8514445
       Lag5
                 volume
  0.8349974
              0.3924004
>
```

Step 8 (0.5 mark) Use the predict() function, which can be used to predict the probability that the market will go up, given values of the predictors. The type="response" option tells R to output probabilities of the form P(Y = 1|X), as opposed to other information such as the logit. If no data set is supplied to the predict() function, then the probabilities are computed for the training data that was used to fit the logistic regression model. Print the first ten probabilities.

```
COMMAND:-
glm.probs = predict(glm.fit , type = "response")
glm.probs[1:10]
```

OUTPUT:-

Step 9 (0.5 mark) Use contrasts() function, which indicates that R has created a dummy variable with a 1 for Up, to check if these values correspond to the probability of the market going up, rather than down.

COMMAND:-

contrasts(Direction)

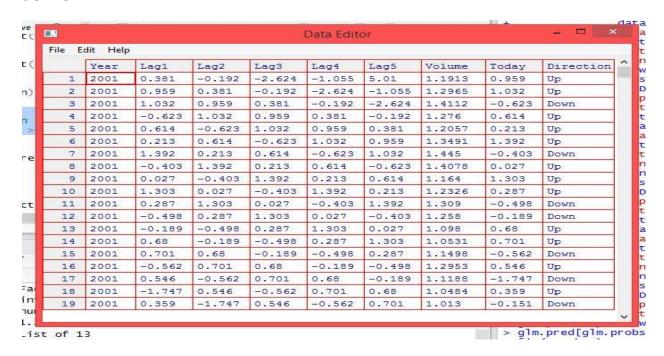
```
> contrasts(Direction)
Up
Down 0
Up 1
> |
```

Step 10 (0.5 mark) Convert these predicted probabilities into class labels, Up or Down, in order to make our prediction more transparent. Check the result with command fix.

COMMAND:-

glm.pred=rep ("Down " ,1250) glm.pred[glm.probs >.5]=" Up" fix(Smarket)

OUTPUT:-



Step 11 (0.5 mark) Given these predictions, use the table() function to produce a confusion matrix in order to determine how many observations were correctly or incorrectly classified.

COMMAND:-

table(glm.pred ,Direction) summary(Direction)

(507+145)/1250

Step 12 (0.5 mark) Calculate the proportion of correct predictions using command mean. Check the results manually using the information from the confusion table.

COMMAND:-

mean(glm.pred==Direction)

OUTPUT:-

```
> mean(glm.pred==Direction)
[1] 0.5216
> |
```

Step 13 (0.5 mark) The statistics in Step 13 corresponds to the training set equal to the total set of observation. Create the new training set for records from 2001 to 2004. We will then use this vector to create a held out data set of observations from 2005. How many records remains in the test set?

```
COMMAND:-
```

```
train =(Year<2005)
Smarket.2005= Smarket [!train ,]
dim(Smarket.2005)

OUTPUT:-
> train =(Year <2005)
> Smarket.2005= Smarket [!train ,]
> dim(Smarket.2005)
[1] 252 9
```

There are 252 observations and 9 variables left.

```
Step 14 (1 mark) Implement command
>Direction.2005= Direction [! train]
```

The object train is a vector of 1, 250 elements, corresponding to the observations in our data set. The elements of the vector that correspond to observations that occurred before 2005 are set to TRUE, whereas those that correspond to observations in 2005 are set to FALSE. The object train is a *Boolean* vector, since its elements are TRUE and FALSE.

Fit a logistic regression model using only the subset of the observations that correspond to dates before 2005, using the subset argument. Compute the predictions for 2005 and compare them to the actual movements of the market over that time period. Interpret the results.

COMMAND:-

```
> Direction. 2005= Direction[!train]
> glm.fit=glm(Direction~Lag1+Lag2+Lag3+Lag4+Lag5+Volume ,
+
              data=Smarket, family =binomial, subset =train
> glm.probs=predict(glm.fit, Smarket.2005,type="response")
> glm.pred=rep ("Down",252)
> glm.pred[glm.probs >0.5]="Up"
> table(glm.pred, Direction.2005)
        Direction. 2005
glm.pred Down Up
    Down 77 97
           34 44
> mean(glm.pred==Direction.2005)
[1] 0.4801587
> mean(glm.pred!= Direction.2005)
[1] 0.5198413
```

COMMAND:-

```
glm.fits=glm(Direction~Lag1+Lag2,data=Smarket,family=binomial,
       subset =train)
glm.probs = predict(glm.fits,Smarket.2005,type = "response")
glm.pred = rep("Down", 252)
glm.pred[glm.probs>.5]="Up"
table(glm.pred,Direction.2005)
mean(glm.pred == Direction.2005)
106/(106+76)
predict(glm.fits,newdata=data.frame(Lag1=c(1.2,1.5),Lag2=c(1.1,-0.8)),
    type="response")
OUTPUT:-
> glm.fits=glm(Direction~Lag1+Lag2,data=Smarket ,family =binomial ,
                subset =train )
> glm.probs = predict(glm.fits,Smarket.2005,type = "response")
> glm.pred = rep("Down", 252)
> glm.pred[glm.probs>.5]="Up"
> table(glm.pred,Direction.2005)
         Direction. 2005
alm. pred Down Up
     Down
            35 35
            76 106
     Up
> mean(glm.pred == Direction.2005)
 [1] 0.5595238
> 106/(106+76)
[1] 0.5824176
> predict(glm.fits,newdata=data.frame(Lag1=c(1.2,1.5),Lag2=c(1.1,-0.8)),
           type="response")
0.4791462 0.4960939
```

Now the results appear to be a little better: 56% of the daily movements have been correctly predicted. The results suggest that there 52% test error rate which shows that it is worse than random guessing. Also the p-value of all the predictors is highly insignificant which can be seen using summary command. So, we also tried by removing the variables that are not helpful in predicting Direction, thereby making a more effective model. As these variables might cause Deterioration in the test error rate so, we remove such variables to yield an improvement. After the removal of certain variables it can be seen that there has been better prediction of the Direction which 56%. To predict the values of Lag1 and Lag2 on a particular day for a particular value of Lag1 and Lag2 we use predict() Function.