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# Task: 5: Mie Theory AND RADIATIVE TRASFER

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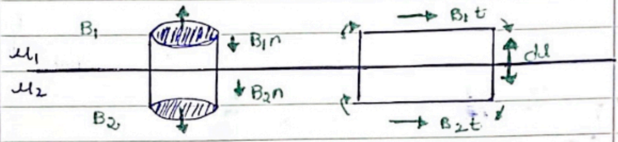
Task: 5: Derive the boundary conditions for the Maxwell's Equations for the Magnetic field.

Answer:




Normal component                Tangential component

We know the following maxwell's equations that are related to magnetic field.

1) $\operatorname{div} B = 0$ : 1st Maxwell Equation

2) $\nabla \times H = J + \frac{dE}{dt}$ : 4th Maxwell Equation

Boundary conditions for the Normal component of Magnetic field:

→ Gauss Theorem for Magnetic field is given as follows:

$$\oint \vec{B} \cdot \vec{ds} = \int \operatorname{div} B \cdot dv = \Phi \text{ (flux)}$$

from the 1st maxwell equation:

$$\operatorname{div} B = 0$$

We can compute the above equation

when $\text{div} B = 0$

as: $\int \text{div} B \cdot \partial v = 0$

That means:

$$\oint \vec{B} \cdot \partial \vec{s} = \int \text{div} B \, \partial v = \phi = 0$$

From the figure above, we can also compute the value of $\oint \vec{B} \cdot \partial \vec{s}$:

$$s[B_{2n} - B_{1n}] = \phi = 0$$

$$B_{2n} - B_{1n} = 0$$

$$\boxed{B_{2n} - B_{1n} = 0}$$

Boundary condition for Normal component.

Boundary condition for Tangential component:

for this, we will use the 4th maxwell Equation:

$$\nabla \times H = J + \frac{dE}{\partial t}$$

$J = 0$ [current density is zero for the given situation]

Now, from the figure,

taking a closed loop integral over a finite length l for field $\vec{H}$ is same as taking surface integral over $\vec{\nabla}\times\vec{H}$

Hence, $\oint_l \vec{H}\cdot d\vec{l} = \int \nabla\times\vec{H}\cdot d\vec{s}$

from the above equation we know that

$$\nabla\times H = \vec{J} + \frac{\partial E}{\partial t}$$

$$\nabla\times H = \frac{\partial E}{\partial t}$$ as there no charge flow in the given situation

Hence: $\int \nabla\times\vec{H}\cdot d\vec{s} = \int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{s}$

$$= \frac{\partial}{\partial t}\int \vec{E}\cdot d\vec{s}$$

$$= 0$$ [ No varying Electric field ]

Hence: $\int \nabla\times\vec{H}\cdot d\vec{s} = 0$

Now: we know that $B = \mu H$

where $\mu$ is magnetic permeability for different mediums.

Now: $\oint_l \vec{H}\cdot d\vec{l} = 0$

$$\frac{1}{\mu}\oint \vec{B}\cdot d\vec{l} = 0$$

from the figure we can compute $\oint \vec{B} \cdot d\vec{l}$ as,

$$B_{1t}\, l + B_{1n}\, \partial l - B_{2t}\, l - B_{2n}\, \partial l = 0$$

as $\partial l \approx 0$

$$\oint \vec{B} \cdot \partial \vec{l} : B_{1t}\, l - B_{2t}\, l = 0$$

as the permeability is different in these two mediums:

$$\frac{1}{\mu} \oint \vec{B} \cdot \partial l = \frac{1}{\mu_1} B_{1t}\, l - \frac{1}{\mu_2} B_{2t}\, l = 0$$

$$\frac{B_{1t}}{\mu_1} l - \frac{B_{2t}}{\mu_2} l = 0$$

$$\boxed{\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = 0}$$

Boundary condition for the tangential component of the magnetic field.