Matrix expo

1) What is a matrix $\Rightarrow$ 2d array
2) Matrix multiplication
3) $a^n \Rightarrow \log n$

$a^{n/2}$

n odd

n even

$a^n \Rightarrow a^{n/2} \times a^{n/2}$ n even

$a^n \Rightarrow a^{n/2} \times a^{n/2} \times a$ n odd

```
int pow(int a, int n){
    if (n==0) return 1
    int p = pow(a, n/2)
    if( n is even )
        return p*p
    else
        return a*p*p
}
```

$x \times 1 = x$

TC: log(n)

A^n

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$A^{20} = A^{10} \times A^{10}$

$A^{10} = A^5 \times A^5$

$A^5 = A \times A^2 \times A^2$

```
matrix pow (matrix a, int n) {
    if (n==0)      return identity matrix
    matrix p = pow(a, n/2)
    if (n is even)
        return p*p
    else
        return a*p*p
}
```

$A \times I = A$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Q1 Fibonacci no.s.

$$F(n) = F(n-1) + F(n-2)$$

Using matrix expo. $\Rightarrow$ $\log n$

$$F(n) = F(n-1) + F(n-2)$$

$$F(n) = 1 * F(n-1) + 1 * F(n-2)$$
$$F(n-1) = 1 * F(n-1) + 0 * F(n-2)$$

$$\underbrace{\begin{bmatrix} F(n) \\ F(n-1) \end{bmatrix}}_{M(n)} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}_{x} \times \underbrace{\begin{bmatrix} F(n-1) \\ F(n-2) \end{bmatrix}}_{M(n-1)}$$

$F(1) = 1$ $\quad$ $F(2) = 1$

$$M(2) = \begin{matrix} F_2 \\ F_1 \end{matrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$M(n) = x\, M(n-1)$$

$$M(n-1) = x\,M(n-2)$$

$$M(n) = x^2 M(n-2)$$

$$M(n-2) = x\,M(n-3)$$

$$M(n) = x^3 M(n-3)$$

$\vdots$

$$M(n) = x^k M(n-k)$$

$$M(2) = \begin{bmatrix} F(2) \\ F(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$n-k = 2$ $\qquad$ $k = n-2$

$$M(n) = x^{n-2}\,M(2)$$

$$M(n) = x^{n-2} \begin{bmatrix} F(2) \\ F(1) \end{bmatrix}$$

$$M(n) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n-2} \begin{bmatrix} F(2) \\ F(1) \end{bmatrix}$$

$$\begin{bmatrix} F(n) \\ F(n-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n-2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Calc using pow function for matrices.

TC: $\log(n)$

$n = 4$

$$\begin{matrix} F(4) \\ F(3) \end{matrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix} \times \begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{matrix} F(4) \\ F(3) \end{matrix} = \begin{matrix} 2 & 1 \\ 1 & 1 \end{matrix} \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{matrix} F(4) \\ F(3) \end{matrix} = \begin{matrix} 3 \\ 2 \end{matrix}$$

1 1 2 3 5

1 2 3 4

$$F(n) = F(n-1) + F(n-2)$$

We can extend it to all linear recurrences.

$$F(n) = F(n-1) + 3F(n-2) + 2F(n-3)$$

$$\begin{bmatrix} F(n) \\ F(n-1) \\ F(n-2) \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} F(n-1) \\ F(n-2) \\ F(n-3) \end{bmatrix}$$

$M(n)$ $\quad x \quad$ $M(n-1)$

Q: Find the sum of all fibonacci nos uptill N.

Sumonacci

$$F(n) = F(n-1) + F(n-2)$$

$$S(n) = F(1) + F(2) + F(3) + \dots + F(N)$$

$$S(n-1) = F(1) + F(2) + \dots F(n-1)$$

$$S(n) - S(n-1) = F(n)$$

$$S(n) = S(n-1) + F(n)$$

$$S(n) = S(n-1) + F(n-1) + F(n-2)$$

$$\begin{bmatrix} S_n \\ F_n \\ F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} S_{n-1} \\ F_{n-1} \\ F_{n-2} \end{bmatrix}$$

$$\begin{bmatrix} S_N \\ F_N \\ F_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} S_{N-1} \\ F_{N-1} \\ F_{N-2} \end{bmatrix}$$

$1+1+2+3$

$$m(n) = x^{n-2} \quad m_2$$

$$m(4) = \begin{bmatrix} 1 & 3 & 2 \\ & & \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \qquad 7$$

$\underline{\underline{Q}}$ $f_n = 4 f_{n-1} + 2 g_{n-1}$

$$g_n = 3 g_{n-1} + 2^{n-1}$$

Calc f_n

$$\begin{bmatrix} f_n \\ g_n \\ 2^n \end{bmatrix} = \begin{bmatrix} 4 & 2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} f_{n-1} \\ g_{n-1} \\ 2^{n-1} \end{bmatrix}$$

$$
\begin{matrix} f_n \\ g_n \\ 2^n \end{matrix} = \begin{matrix} 4 & 2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{matrix} \quad \begin{matrix} f_{n-1} \\ g_{n-1} \\ 2^{n-1} \end{matrix}
$$

Q $f_n = f_{n-1} + 3f_{n-2} + 5f_{n-4}$

$$
\begin{matrix} f_n \\ f_{n-1} \\ f_{n-2} \\ f_{n-3} \end{matrix} = \begin{bmatrix} 1 & 3 & 0 & 5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} f_{n-1} \\ f_{n-2} \\ f_{n-3} \\ f_{n-4} \end{matrix}
$$

$$\begin{matrix} F(n) \\ F(n-1) \end{matrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n-2} \begin{matrix} F_2 \\ F_1 \end{matrix}$$

$$F(n) = a F_2 + b F_1$$

$$F(20) = 1000$$

$$F(20) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{18} \begin{matrix} F_2 \\ F_1 \end{matrix}$$

1000

$$F(30) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{28} \begin{matrix} F_2 \\ F_1 \end{matrix}$$

5000

$$1000 = 500 F_2 + 300 F_1$$

$$5000 = 2000 F_2 + 1000 F_1$$

$$F_2 \qquad F_1$$

$$F_{50} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{48} \begin{bmatrix} F_2 \\ F_1 \end{bmatrix}$$

<done>