Basic Maths 1

17/3 = 5

17 % 3 = 2

33 % 5 = 3

{ "%" – modulus

a % m → remainder

a / b

a % m

∞

min max

\# 0 ⟷ m−1

a / m−1

x % 33 = 32

34

x % 16 = 18

0–15

18
16
2

a < m

0 – m−1

6573249988 % 37

0 – 36

0 1 2 3 4 5 6 7 8 9 10 11

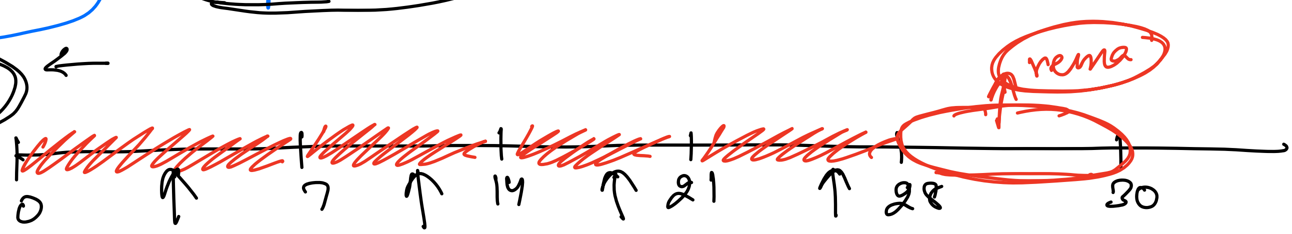
%4 0 1 2 3 0 1 2 3 0 1 2 3 . . .

2 * 3

multiplication – repetitive addition

division – repetitive sub

30 % 7 ←



$30 \% 7 = 30 - $ largest multiple of 7 $<= 30$

$30 - 28$

$<= 30$

-28

$<= 30$

-35

$-30 \% 7 = -30 - (-35)$

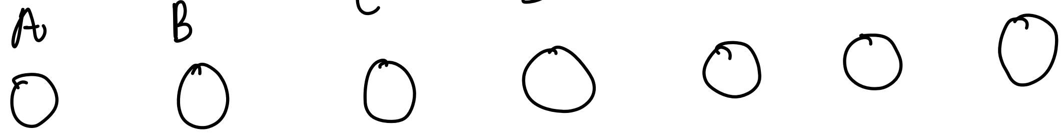
$-30 \% 7 = 5$

remainder can −ve

7 balls

1 ball is golden ball

A B C D E F G



1 2 3 4 5 6 7

12 11 10 9 8

13 14 15 16 17 18 19

24 23 22 21 20

1000th ball

A B C D E F G F E D C B 1000th

1 2 3 4 5 6 7 8 9 10 11 12

13 14 16 16 17 18 19 20 21 22 23 24

28 — — — — — — —

4) rem = 1000 % 12 → 4th number

997 998 999 1000

96

996

1000th a % 8

A B C D E

1 2 3 4 5

8 7 6

9 10 11 12 13

16 15 14

x

E D C B 8 5

A B C D

1 2 3 4 5 6 7 8

1000 % 8

A B C D E D C B

9 10 11 12 13 14 15 16

0

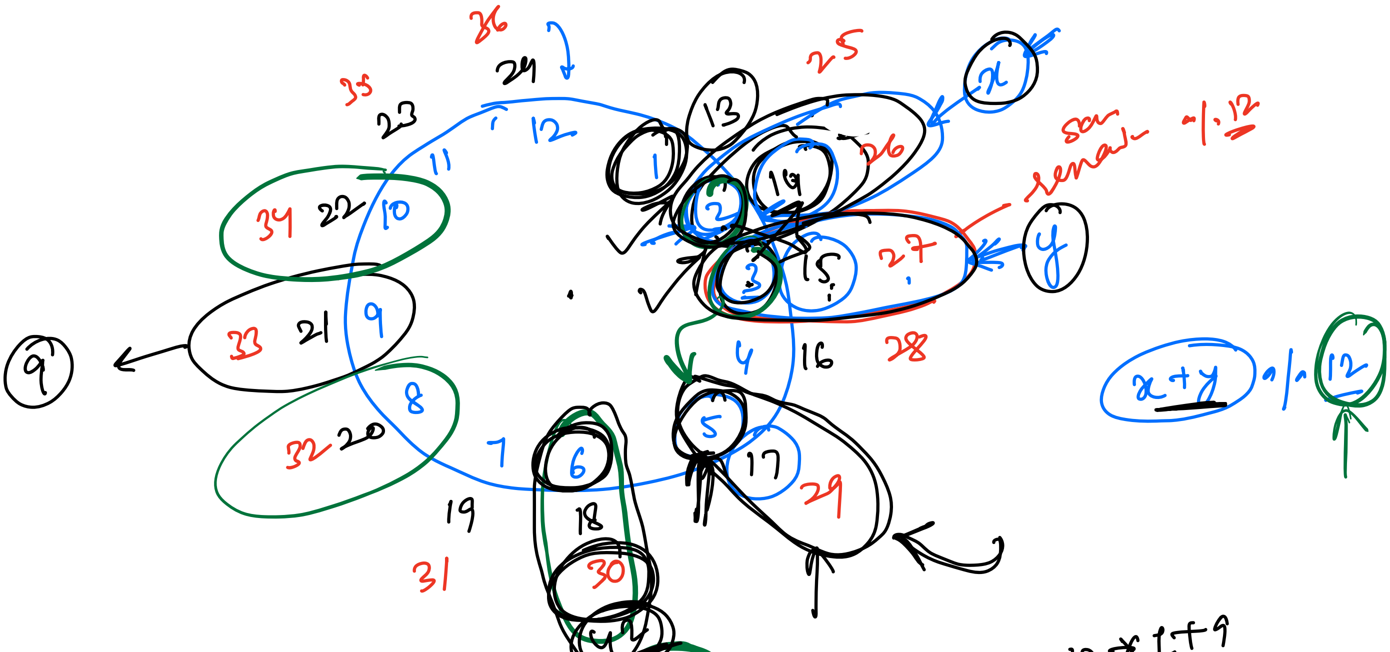
a % b

A . . .

17 18 19 . .

1000

*



$23 = 12 * 1 + 9$

$8 \% 12$
$10 \% 12$

$8 + 10 = 18 \% 12 = 6$

a b

$\boxed{(a+b) \% m = (a \% m + b \% m) \% m}$

$m = 12$

$a = 12n + r_1$

$b = 12m + r_2$

$(a+b) \% 12 = 12(n+m) + (r_1 + r_2) \% m$

0–11 0 0–11 0–11

% 12

a
b

$a = bq + r$

$7 = 12$

0 – m – 1

$r_1 = 1$
$r_2 = 2$
$3 < 12$
% 12

$a = 8$
$b = 10$

0–22

0–11 0–11 0–21

$(a+b) \% 12 = (a \% 12 + b \% 12) \% 12$

8 10 0–11

18

$(8+10) \% 12$
$18 \% 12$
6

2m–1

$13 \% 12$

1

$(12) \quad (a*b)\%m$

$a = 12n + r_1 \rightarrow a\%12$

$b = 12m + r_2 \rightarrow b\%12$

$r1 * 12 * m \rightarrow 0$

$a*b = (12n + r_1) * (12m + r_2)$

$(a*b)\%12 = \cancel{12n(12m)} + \cancel{12n(r_2)} + \cancel{r_1 * 12m} + r_1 * r_2$

$0\%12 \quad 0\%12 \quad 0$

(2) $\boxed{(a*b)\%m = (a\%m \; * \; b\%m)\%m}$

$r1 * r2 \quad 0\text{-}m\text{-}1 \quad 0\text{-}m\text{-}1 \quad (m-1)^2$

$(r1 * r2)\%m$

$0 - m-1$

$(a-b)\%m = (a\%m - b\%m)\%m$

$(33-14)\%12 = (33\%12 - 14\%12)\%12$

$= (9 - 2)\%12$

L.H.S

$(19)\%12$

$= 7$

7

$(a\%m - b\%m + m)\%m$

$+m$

$m\%m = 0$

$(14 - 33)\%12 = (14\%12 - 33\%12)\%12$

$= (2 - 9^{+12})\%12$

$(-19)\%12$

$= (-7^{+12})\%12 = -7$

5

5

$12 \quad (-24)$

$-19 \% 12 = -19 - (\ \ )$

$= 5$

$$\boxed{(a-b) \% m = (a \% m - b \% m + m) \% m}$$

$36 \% 12 = 0$ $\quad 33 \% 12 = 9$

$(14 - 33) \% 12$

$(14 \% 12 - 33 \% 12) \% 12$

$2 - 9$

$-7 \% 12$

5

$-7 \% 12 = -7 - (-12)$

$= 5$

$(14 + 33) \% 12 = 11$

$(14 \% 12 + 33 \% 12)$

$2 + 9 + 12$

$23 \% 12 = 11$

$(-7 + 12) \% 12$

$5 \% 12$

0

$-7 \% 12 = -7 + 12$

$+m?$

$+m + m + m$

$(r1 - r2) \% m$

$0 - m-1 \qquad 0 - m-1$

$0 - (m-1) + m$

-7

-8

-9

-10

$-(m-1) + m = 1$

Q: sum of all integers in an array

take modulo p

$10^9 \quad 10^9 \quad 10^5$

10^{14}

int

$2^{31} \quad (\approx 10^9)$

sum += arr[i] overflow

10^9

$0 - 10^9 - 1$

sum % p

sum = (sum % m + arr[i] % m) % m

$10^9 \quad 10^9 \quad 10^9 \quad 10^9 \quad \ldots \ldots$ } 10^5 values

Total sum $= 10^{14}$ ← int ?

$(10^9 + 10^9) \% p$ — $p = 10^9+7$

$0 - p-1$

congruence relation

x & y are congruent to each other w.r.t n

$$x \% n = y \% n$$

$$\boxed{x \equiv y \pmod{n}}$$

$n = 12$

$26, 14$ → 2, 2

$$26 \equiv 14 \pmod{12}$$

if $x \equiv y \pmod{n}$

$\Rightarrow$ $(x-y)$ divisible by n

$$x \% n = y \% n$$

$$(x \% n - y \% n) \% n = (0) \% n$$

$$(x-y) \% n = 0$$

vice versa ?

$$a \equiv b \pmod{n}$$
$$c \equiv d \pmod{n}$$

$$(a+c) \equiv (b+d) \pmod{n}$$
$$(a*c) \equiv (b*d) \pmod{n}$$

$$(37^{103} - 1) \,\%\, 12$$

$$\left((37)^{103} \,\%\, 12 - \underbrace{(1 \,\%\, 12)}_{1} \right) \,\%\, 12$$

$$37 * 37 * 37 * 37 \ldots$$

$$\underbrace{(37 \,\%\, 12)}_{1} * \underbrace{(37 \,\%\, 12)}_{1} * \underbrace{(37 \,\%\, 12)}_{1} \ldots$$

$$\left((1)^{103} - 1 \right) \,\%\, 12$$

$$= 0 \,\%\, 12 = 0$$

Q Given with N array elements, find if there exists a pair whose sum is divisible by K.

[3 7 5 13 4 6 9]

K=10

K=8

3 & 5
13 & 3
9 & 7

B.F → consider all pairs − $O(n^2)$

$x = bq + r$ (div, rem, divid, quo)

K

$arr[i] = K*n + r_1$ (rem)

$arr[j] = k*m + r_2$

$= \underbrace{k(n+m)}_{0} + r_1 + r_2$ remainder

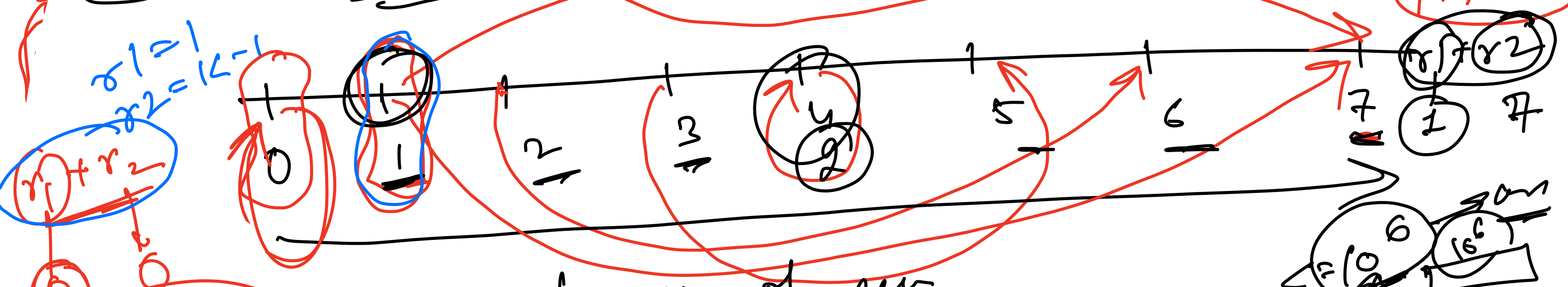
3 7 5 13 4 6 9

3 7 5 5 4 6 1 (9%8) K=8

1+7=8

reverse

$r_1 = 1$, $r_2 = k-1$



store the frequency of rem

hashmap, array

0 − k − 1

4 4

0	1	2	3	4	5	6	7
0	1	0	1	1	2	1	1

input

>=2

>=2

S.C : O(k)

T.C : O(n)

key, fre

val

m

(int, int)
key val

0

m[rem]++; 7

m[7]++;

H.M

m

1)

for i = 0 → n

m[a[i] % k] ++;

if(m[0] >= 2) return true;

i = 1 → i <= k/2

if(m[i] >= 1 && m[k-i] >= 1) return true;

if(i == k-i && m[i] >= 2) return

return false;

0 — 0
1 — k-1
2 — k-2
3 — k-3
i — k-i
k/2

i k-i

r1 + r2 = 0 = k

0 0

1 k-1
2 k-2

~~fre~~ >=2

0
k/2
all other remain

fre >=1

$a = 5n + r1$

$b = 5m + r2$

a+b
div
5

r1 + r2

$5n + 5m + r1 + r2$

$5(n+m) + r1 + r2$

5

(2) Given with an array of size N, find if there exists a subarray with sum divisible by K

K

size

{7 5 3 7}

N=4

{3 7 14}

N=3

B.F

find all subarrays & then find the sum

$n^2 * n$ $= O(n^3)$

O(1)

l–r

prefix sum array

pf[r] – pf[l-1]

$n^2 * (1) = O(n^2)$

Sum of all subarray % n

0

{7 5 3 7}

Pf: 7 12 15 22

N=4

% N

pf[i] = sum from 0 – i

0 % N

N = 4

pf[i]

i=0

⇒ {3 ~~0~~ 1 3 2}

0 → i

pf[1] = sum 0 – 1 % N

No value comes out

l != 0

l – r

(pf[r] – pf[l-1]) % N = 0

pf[r] % N − pf[l−1] % N = 0

pf[r] % N = pf[l−1] % N

If there are value ~~duplicate~~ remainder in pf arr

⇒ subarr ✓

pf sum array → O(n)

remainder % N → O(n)

if zero appears return true O(n)

if duplic return true O(n)

O(n) → O(1) T.C

n → 0 − N−1 ← % N

N

0 − 4

N = 5

1	~~0~~	2	3	4

if +

0 − n−1 = n

size n = n

return true; O(1)

N ⟸ pf :

remaining array

N = 5

% N

0 – 4 — 5

0 1 2 3 4

0

duplicates ?

true

true

false

[0 | | | |]

N = 5

return true;

[7 5 3 7]   N = 4

pf[i] = 7 12 15 22

[0-0] [0-1] [0-2] [0-3]

subarray sum from 0-i

% 4 % 4 % 4 % 4

→ 3 ~~0~~ 3 2

divisible N

% n = 0

0 N

1st condit true

lena pf → 0

if there is no zero then is no subarray start from 0 whose sum is div by N

start end subarray

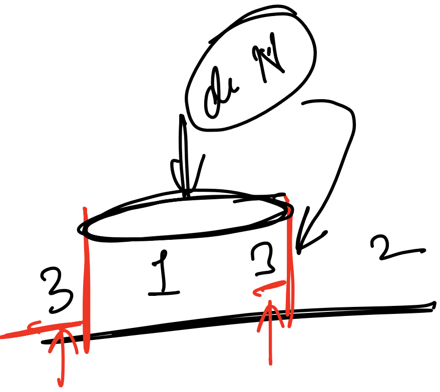
l – r

pf[r] - pf[l-1]

divisib % N = 0

$$(pf[r] \% N - pf[l-1] \% N) \% N = 0$$

$$pf[r] \% N - pf[l-1] \% N = 0$$

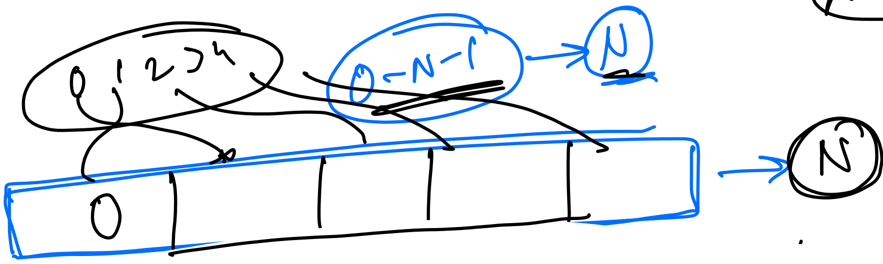


$$pf[r] \% N = pf[l-1] \% N$$

remai of sum of subar 0-r

remainder of sum of sub 0-l-1

N=5



0 — true
duplicate — true
else false