Maths: Combinatorics

Monday, 1 November 2021 8:57 PM

Q → 3 T/F questions.

In how many ways we can answer them.

T/F

TTF, TFF, TTT, TFT, FTF, FFF, FTT, FFT,

T/F / —

$2 * 2 * 2 = 8$ ways

$3 * 3 * 3 = 27$ ways

Pune —Flight / Train / Car→ Delhi —Train / Car→ Agra

$3 * 2 = 6$ ways

In how many ways we can go from pune to agra?

Multiplication Principle

Task A & Task B

$n * m$

Blue: $B_1\ B_2\ B_3\ B_4\ B_5$ Red: $R_1\ R_2\ R_3$ green: $G_1\ G_2$

Ways to select a blue or a red ball = 8

Ways to select a blue ball = 5

Ways to select a red ball = 3

Addition Principle

Task A or Task B

$n + m$

Task 1 → Ways to select a red or green ball = 5

Task 2 → Ways to select a blue or green ball = 7

Not mutually exclusive

Ways to do task1 or task 2 → $\cancel{4*6}$ 10

Task 1&2 → $3 * 7 + 2 * 6$

Combinations → $\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}}$

$n! = 1*2*3 \ldots *n$

#ways to select r objects from n objects.

$B_1\ \cancel{B_2}\ B_3\ B_4\ \cancel{B_5}$ ways to select 2 blue balls $= {}^5C_2 = 10$

one blue ball = 5
second blue ball = 4

$\{B_2, B_5\} = \{B_5, B_2\}$

$\frac{5*4}{2} = 10$

Properties

$\boxed{x! = 1*2*3*\ldots(x-1)*x \quad x! = (x-1)!*x}$

1) $\boxed{{}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}}$

$\frac{n!}{r!\,(n-r)!} + \frac{n!}{(r+1)!\,(n-r-1)!} = \frac{n!}{r!\,(n-r-1)!}\left(\frac{1}{(n-r)} + \frac{1}{(r+1)}\right)$

$= \frac{n!}{r!\,(n-r-1)!}\left(\frac{(\cancel{r}+1) + (n-\cancel{r})}{(n-r)(r+1)}\right)$

$= \frac{n!\,(n+1)}{(r+1)!\,(n-r)!} = \frac{(n+1)!}{(r+1)!\,((n+1)-(r+1))!}$

$= {}^{n+1}C_{r+1}$

2) $B_1\ B_2\ B_3\ B_4\ B_5$

${}^5C_3 = 10$ → 3 Blue ← Bag 1

1 way → 2 Blue ← Bag 2

#ways $= {}^5C_2 = 10 * 1 = 10$

$10 + 10$

$\boxed{{}^nC_r = {}^nC_{n-r}}$

3) $\boxed{{}^nC_0 = 1}$

4) $\boxed{{}^nC_1 = n}$ $\frac{n!}{(1)!\,(n-1)!} = n$

$\boxed{0! = 1}$

5) $\boxed{{}^nC_n = 1}$ $\frac{\cancel{n!}}{\cancel{n!}\,(n-n)!} = 1$

Q → Ways to select any no. of objects out of n objects = ? $= 2^n$

$0 \le r \le n$

✓/✗ ✓/✗ ✓/✗

[k, r, b]

[ ]

[b]
[r]
[k] } 1

[k, r]
[r, b]
[k, b] } 2

[k, r, b]

n = 3

2^3

$\boxed{\#\text{subsets} = 2^n}$

$\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots {}^nC_n = 2^n}$

Q → $\boxed{{}^nC_r \% m}$ I/P → n, r, m

n = 4
r = 2
m = 10

Ans = 6

$\boxed{0 \le r \le n}$

${}^nC_0 = 1$ ${}^nC_n = 1$

${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

${}^nC_r = ({}^{n-1}C_{r-1} + {}^{n-1}C_r) \% m$

$< m$ $< m$

$TC = O(N^2) \to O(N*R)$

$SC = O(N^2) \to O(R)$

n \ r	0	1	2	3	4
1	1	1			
2	1	2	1		
3	1	3	3	1	
4	1	4	6	4	1

Pascal's Triangle

2m

$i! = ((i-1)*i) \% p$

m → p (prime)

${}^nC_r \% p$ $\frac{n!}{(n-r)!\,r!} \% p$

$(n! \% p) * ((n-r)! \% p)^{p-2} \% p * (r! \% p)^{p-2} \% p$

Inverse mod

$n! \to O(N)$

$O(\log(P))$

```
for( i=1 ; i <=n ; i++) {
    → nCr[i][0]=1;    nCr[i][i] = 1;
    for(r = 1 ; r < i ; r++) {
        → nCr[i][r] = (nCr[i-1][r-1] + nCr[i-1][r])%m;
    }
}
```

int → 4 bytes

char → 1 bytes

1s → $\boxed{10^8 \text{ steps}}$

$TC = O(N^2) \to N < 10^5$ ✗

space → $\boxed{10^7}$