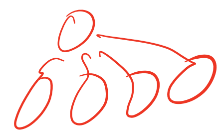
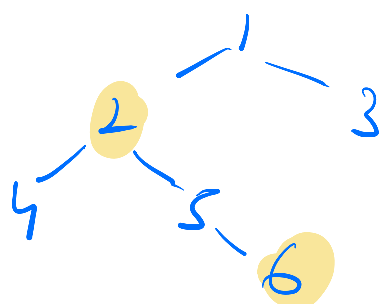
DP on trees

We define functions for nodes of trees, which we recursively calculate based on children of a nodes.
One of the states in the DP is node i, denoting we are solving for subtree of node i.



Lets see some examples

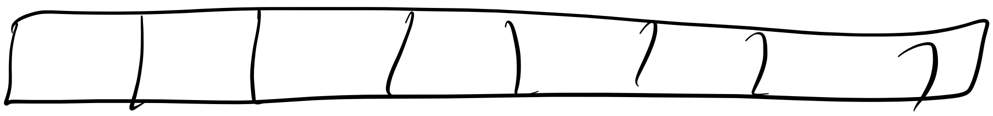
Q1 Given a tree, N nodes 1-N.
Each node has C_i coins. Choose a set of nodes such that
1) No 2 nodes are adjacent
2) Sum of coins is max
Return this max sum of coins.



If parent is taken ⇒

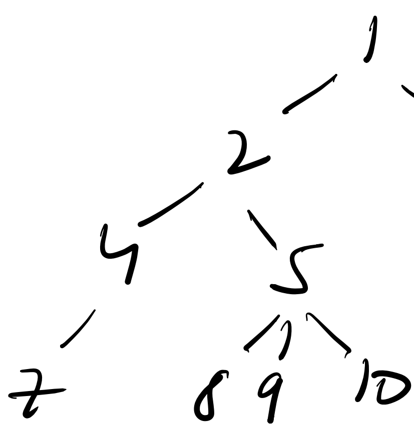
Cant take child

Have we seen something like this before



House Robber

$$dp(i) \Rightarrow \max(dp(i-1),\ coins[i] + dp(i-2))$$



dp[node][can_this_be_taken]

1

2 scenarios

you take 1

coins[1] + Σ dp[child][false]

you dont take 1

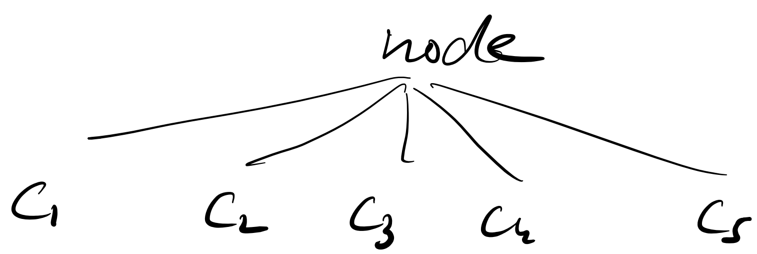
Σ dp[child][true]

Generalise

dp(node, true) → coins[i] + Σ dp[child][false]

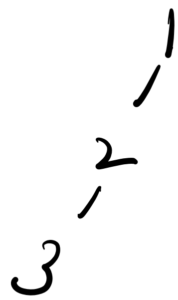
→ Σ dp[child][true]

dp(node, false) → Σ dp[child][true]



```
int maxCoins (vector <int> adj [N], int coins[]){
    int dp [N][2]
    // i,0 → false        i,1 → true
    ans = dfs (1, 1)
    return ans.
}

int dfs ( int node, par, int can_take ){
    if( dp[node][can_take] in cache)
        return dp[node][can_take]

    ans = 0
    if( can_take == 0 ){
        for( j = 0; j < adj[i].size; j++ ){
            cur_child = adj[i][j]
            if( cur_child != par)
                ans += dfs( cur_child, 1)
        }
    }
```

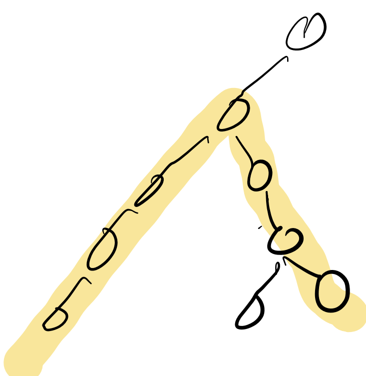
```
else {
    ans1 = coins[node]
    for(j=0; j< adj[i].size; j++){
        cur_child = adj[i][j]
        ans1 += dfs(cur_child, 0)
    }

    ans2 = 0
    for(j=0; j< adj[i].size; j++){
        cur_child = adj[i][j]
        ans2 += dfs(cur_child, 1)
    }

    ans = max(ans1, ans2)
}
dp[node][can_take] = ans
return ans

}
```



adj(1) 2
adj(2) 1 3
adj(3) 2

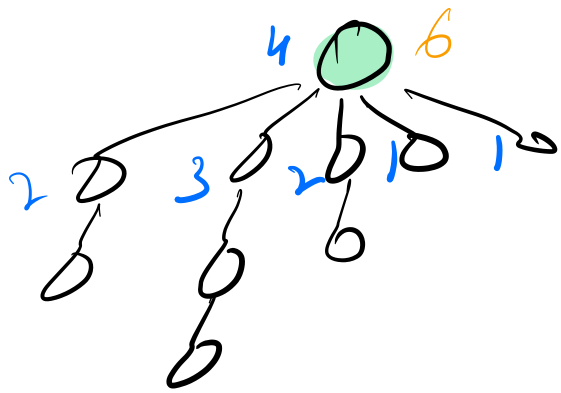
Q2 Given a tree, find the diameter

Diameter ⇒ Longest path b/w any 2 nodes.



When standing at node i ⇒

- Longest path starts at i & goes into subtree $f(i)$
- Longest path passing through i $g(i)$

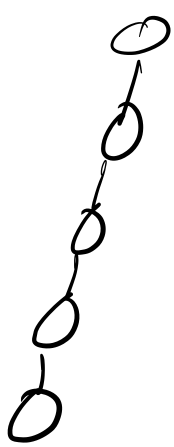


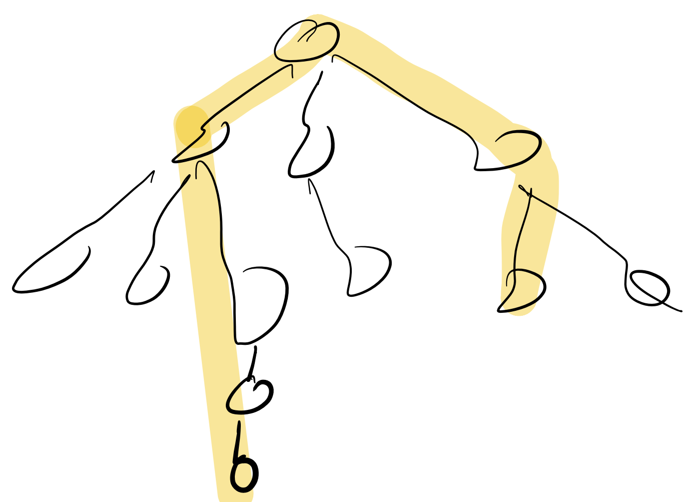
$$f(i) = 1 + \max(f(\text{child}))$$

$$g(i) = 1 + \text{sum of 2 biggest in } \{f_{c_1}, f_{c_2}, f_{c_3}, \ldots\}$$

$f(i) \Rightarrow$ One arm maximum

$g(i) \Rightarrow$ Two arm maximum.





Diameter $\Rightarrow$ max(all f values &&
all g values)

node

2 biggest f values.

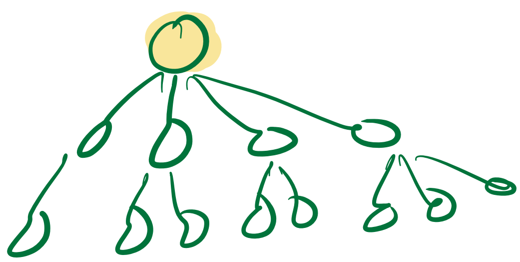
f_values in a arraylist
sort this arraylist
get the last & second last

$$(1+f_{c_1}) * (1+f_{c_2}) * (1+f_{c_3})$$

Q3 Given a tree, find no of diff subtrees

S(node) ⇒ Subtree rooted at node

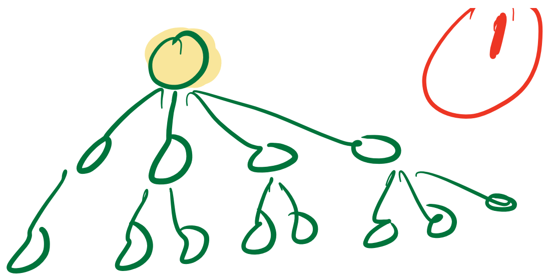
f(node) ⇒ no of subtrees of S(node) which includes node



$$f(\text{node}) = \prod (1 + f(\text{child}))$$

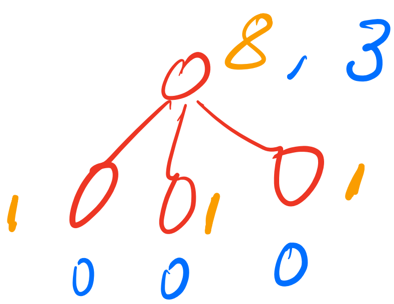
mult

g(node) = no of subtrees not rooted at node



$$g(i) = \sum f(\text{child}) + g(\text{child})$$

$$\text{ans} = \boxed{f(1) + g(1)}$$



$= 11$

{done}