Maths: Prime

Friday, 27 October 2023 3:24 PM

Q→ N boxes

Initially all boxes are closed.

Round 1: open every box.

Round 2: every 2nd box change the state.

Round 3: every 3rd box change the state.

Find no of open boxes in the end.

$b_1, b_2, b_3, \ldots, b_n$

1, 2, 3, 4, 5, 6, ... N

$N \to$?

N=6

b_1 b_2 b_3 b_4 b_5 b_6

1 2 3 4 5 6

3→ 6 → 9 → 12 → 15 → 18

4→ 8 → 12 → 16 → 20 → 24

5→ 10 → 15 → 20 → 25 → 30

6→ 12 → 18 → 24 → 30 → 36

ans = 2

N=10 ans = 3

1 2 3 4 5 6 7 8 9 10

2→ 4 6 8 10

3→ 6 9

4→ 8

5→ 10

6→ 12

7→ 14

8→ 16

9→ 18

10→ 20

observation → No. of state of box is changed = no. of times a box is touched

odd times → open box.

even times → closed box.

→ state changes → factors of box.

N=10 ans = 3

1 2 3 4 5 6 7 8 9 10

2→ 4 6 8 10

3→ 6 9

4→ 8

5→ 10

6→ 12

7→ 14

8→ 16

9→ 18

10→ 20

1, 4, 9, 16, 25, ...

ans = count of no. with odd factors from 1 to N.

30→ 1 2 3 5 6 10 15 30

30→ 1 2 3 5 6 10 15 30

1 2 3 4 5 6 7 8 9 10

perfect square

ans = count of perfect sq from 1 to N.

N=30

$\sqrt{N} = 5.4$

$\lfloor \sqrt{N} \rfloor = 5$

1² < 10

2² < 10

3² < 10

4² < 10

5² > 10

ans = $\lfloor \sqrt{N} \rfloor$

Q→ Find no of perfect square in range 1 to N.

$\sqrt{N} = 5.4$

$\lfloor \sqrt{N} \rfloor = 5$

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

$\sqrt{64} = 8$

$\sqrt{81} = 9$

$\sqrt{100} = 10$

$\sqrt{121} = 11$

$\sqrt{144} = 12$

$\sqrt{169} = 13$

$\sqrt{196} = 14$

$\sqrt{225} = 15$

$\sqrt{256} = 16$

$\sqrt{289} = 17$

$\sqrt{324} = 18$

$\sqrt{361} = 19$

$\sqrt{400} = 20$

$\sqrt{441} = 21$

$\sqrt{484} = 22$

$\sqrt{529} = 23$

$\sqrt{576} = 24$

$\sqrt{625} = 25$

$\sqrt{676} = 26$

$\sqrt{729} = 27$

$\sqrt{784} = 28$

$\sqrt{841} = 29$

$\sqrt{900} = 30$

$\sqrt{961} = 31$

$\sqrt{1000} = 32$

$\sqrt{1024} = 32$

$\sqrt{1089} = 33$

$\sqrt{1156} = 34$

$\sqrt{1225} = 35$

$\sqrt{1296} = 36$

$\sqrt{1369} = 37$

$\sqrt{1400} = 38$

$\sqrt{1521} = 39$

$\sqrt{1600} = 40$

$\sqrt{1681} = 41$

$\sqrt{1764} = 42$

$\sqrt{1849} = 43$

$\sqrt{1900} = 44$

$\sqrt{2025} = 45$

$\sqrt{2116} = 46$

$\sqrt{2209} = 47$

$\sqrt{2304} = 48$

$\sqrt{2401} = 49$

$\sqrt{2500} = 50$

$\sqrt{2601} = 51$

$\sqrt{2700} = 52$

$\sqrt{2800} = 53$

$\sqrt{2900} = 54$

$\sqrt{3000} = 55$

$\sqrt{3100} = 56$

$\sqrt{3200} = 57$

$\sqrt{3300} = 58$

$\sqrt{3400} = 59$

$\sqrt{3500} = 60$

$\sqrt{3600} = 61$

$\sqrt{3700} = 62$

$\sqrt{3800} = 63$

$\sqrt{3900} = 64$

$\sqrt{4000} = 65$

$\sqrt{4100} = 66$

$\sqrt{4200} = 67$

$\sqrt{4300} = 68$

$\sqrt{4400} = 69$

$\sqrt{4500} = 70$

$\sqrt{4600} = 71$

$\sqrt{4700} = 72$

$\sqrt{4800} = 73$

$\sqrt{4900} = 74$

$\sqrt{5000} = 75$

$\sqrt{5100} = 76$

$\sqrt{5200} = 77$

$\sqrt{5300} = 78$

$\sqrt{5400} = 79$

$\sqrt{5500} = 80$

$\sqrt{5600} = 81$

$\sqrt{5700} = 82$

$\sqrt{5800} = 83$

$\sqrt{5900} = 84$

$\sqrt{6000} = 85$

$\sqrt{6100} = 86$

$\sqrt{6200} = 87$

$\sqrt{6300} = 88$

$\sqrt{6400} = 89$

$\sqrt{6500} = 90$

$\sqrt{6600} = 91$

$\sqrt{6700} = 92$

$\sqrt{6800} = 93$

$\sqrt{6900} = 94$

$\sqrt{7000} = 95$

$\sqrt{7100} = 96$

$\sqrt{7200} = 97$

$\sqrt{7300} = 98$

$\sqrt{7400} = 99$

$\sqrt{7500} = 100$

$\sqrt{7600} = 101$

$\sqrt{7700} = 102$

$\sqrt{7800} = 103$

$\sqrt{7900} = 104$

$\sqrt{8000} = 105$

$\sqrt{8100} = 106$

$\sqrt{8200} = 107$

$\sqrt{8300} = 108$

$\sqrt{8400} = 109$

$\sqrt{8500} = 110$

$\sqrt{8600} = 111$

$\sqrt{8700} = 112$

$\sqrt{8800} = 113$

$\sqrt{8900} = 114$

$\sqrt{9000} = 115$

$\sqrt{9100} = 116$

$\sqrt{9200} = 117$

$\sqrt{9300} = 118$

$\sqrt{9400} = 119$

$\sqrt{9500} = 120$

$\sqrt{9600} = 121$

$\sqrt{9700} = 122$

$\sqrt{9800} = 123$

$\sqrt{9900} = 124$

$\sqrt{10000} = 125$

Should → $TC = O(N)$

$SC = O(1)$

Q→ with 2 factors in range 1 to N → $\boxed{\frac{N}{2}}$

Prime No → 1 → N. with exactly 2 factors it's itself.

Q→ For a given no. N find count of factors of N.

N=6 ans = 4

1, 2, 3, 6

Brute → $\text{ans} = N$

for(i = 1; i <= N; i++)

if(N % i == 0) ans++;

$TC = O(N)$

$30 = 2 \times 15$

$= 3 \times 10$

$= 5 \times 6$

action ans

N=30 → 1 2 3 5 6 10 15 30

$N = 4 \times 9$ $\boxed{4 \times 9}$

it will be root if it will be min.

$\sqrt{N} = \sqrt{4}$

$N = 4 \times 4 \Rightarrow \boxed{N = \sqrt{N}}$

→ if any no is in range it will have another factor N/i. $\boxed{2 \le \sqrt{N}}$

1 × N

for prime to n × 1 always.

N % 1 == 0

N % 0

2^2

for(i = 2; i <= N; i++)

if(N % i == 0)

break prime

$TC = O(\sqrt{N})$

action false;

action true;

---

ans = 0;

for(i = 1; i <= N; i++)

if(N % i == 0) ans++;

if(i*i == N) ans += 1;

else ans += 2;

action ans;

$TC = O(\sqrt{N})$

$SC = O(1)$

Q→ Find all prime no. from 1 to N!

V_1 → from 1 to N check if it prime → $TC = O(N \times \sqrt{N})$

N=15

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

prime

$\text{isPrime}[N+1] = 1$ // N isPrime[1] = 0;

isPrime[0] = isPrime[1] = 0;

for(i = 2; i <= N; i++)

if(isPrime[i] == 1)

for(j = 2*i; j <= N; j += i)

isPrime[j] = 0;

$TC = O(N \log(\log(N)))$

$SC = O(N)$

Q→ Find all prime no. from 1 to N!

V_2 → from 1 to N check if prime → $TC = O(N \times \sqrt{N})$

N=15

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

prime

$\text{isPrime}[N+1] = 1$ // N isPrime[1] = 0;

isPrime[0] = isPrime[1] = 0;

for(i = 2; i <= N; i++)

if(isPrime[i] == 1)

for(j = i^2; j <= N; j += i)

isPrime[j] = 0;

$i^2 \to 4$

$i \to 2$

$i^2 \to 9$

$i \to 3$

$i^2 \to 25$

$i \to 5$

$i^2 \to 49$

$i \to 7$

$i^2 \to 121$

$i \to 11$

$i^2 \to 169$

$i \to 13$

$i^2 \to 289$

$i \to 17$

$i^2 \to 361$

$i \to 19$

$i^2 \to 529$

$i \to 23$

$i^2 \to 841$

$i \to 29$

$i^2 \to 961$

$i \to 31$

$i^2 \to 1089$

$i \to 33$

$i^2 \to 1156$

$i \to 34$

$i^2 \to 1225$

$i \to 35$

$i^2 \to 1296$

$i \to 36$

$i^2 \to 1369$

$i \to 37$

$i^2 \to 1400$

$i \to 38$

$i^2 \to 1521$

$i \to 39$

$i^2 \to 1600$

$i \to 40$

$i^2 \to 1681$

$i \to 41$

$i^2 \to 1764$

$i \to 42$

$i^2 \to 1849$

$i \to 43$

$i^2 \to 1900$

$i \to 44$

$i^2 \to 2025$

$i \to 45$

$i^2 \to 2116$

$i \to 46$

$i^2 \to 2209$

$i \to 47$

$i^2 \to 2304$

$i \to 48$

$i^2 \to 2401$

$i \to 49$

$i^2 \to 2500$

$i \to 50$

$i^2 \to 2601$

$i \to 51$

$i^2 \to 2700$

$i \to 52$

$i^2 \to 2800$

$i \to 53$

$i^2 \to 2900$

$i \to 54$

$i^2 \to 3000$

$i \to 55$

$i^2 \to 3100$

$i \to 56$

$i^2 \to 3200$

$i \to 57$

$i^2 \to 3300$

$i \to 58$

$i^2 \to 3400$

$i \to 59$

$i^2 \to 3500$

$i \to 60$

$i^2 \to 3600$

$i \to 61$

$i^2 \to 3700$

$i \to 62$

$i^2 \to 3800$

$i \to 63$

$i^2 \to 3900$

$i \to 64$

$i^2 \to 4000$

$i \to 65$

$i^2 \to 4100$

$i \to 66$

$i^2 \to 4200$

$i \to 67$

$i^2 \to 4300$

$i \to 68$

$i^2 \to 4400$

$i \to 69$

$i^2 \to 4500$

$i \to 70$

$i^2 \to 4600$

$i \to 71$

$i^2 \to 4700$

$i \to 72$

$i^2 \to 4800$

$i \to 73$

$i^2 \to 4900$

$i \to 74$

$i^2 \to 5000$

$i \to 75$

$i^2 \to 5100$

$i \to 76$

$i^2 \to 5200$

$i \to 77$

$i^2 \to 5300$

$i \to 78$

$i^2 \to 5400$

$i \to 79$

$i^2 \to 5500$

$i \to 80$

$i^2 \to 5600$

$i \to 81$

$i^2 \to 5700$

$i \to 82$

$i^2 \to 5800$

$i \to 83$

$i^2 \to 5900$

$i \to 84$

$i^2 \to 6000$

$i \to 85$

$i^2 \to 6100$

$i \to 86$

$i^2 \to 6200$

$i \to 87$

$i^2 \to 6300$

$i \to 88$

$i^2 \to 6400$

$i \to 89$

$i^2 \to 6500$

$i \to 90$

$i^2 \to 6600$

$i \to 91$

$i^2 \to 6700$

$i \to 92$

$i^2 \to 6800$

$i \to 93$

$i^2 \to 6900$

$i \to 94$

$i^2 \to 7000$

$i \to 95$

$i^2 \to 7100$

$i \to 96$

$i^2 \to 7200$

$i \to 97$

$i^2 \to 7300$

$i \to 98$

$i^2 \to 7400$

$i \to 99$

$i^2 \to 7500$

$i \to 100$

$i^2 \to 7600$

$i \to 101$

$i^2 \to 7700$

$i \to 102$

$i^2 \to 7800$

$i \to 103$

$i^2 \to 7900$

$i \to 104$

$i^2 \to 8000$

$i \to 105$

$i^2 \to 8100$

$i \to 106$

$i^2 \to 8200$

$i \to 107$

$i^2 \to 8300$

$i \to 108$

$i^2 \to 8400$

$i \to 109$

$i^2 \to 8500$

$i \to 110$

$i^2 \to 8600$

$i \to 111$

$i^2 \to 8700$

$i \to 112$

$i^2 \to 8800$

$i \to 113$

$i^2 \to 8900$

$i \to 114$

$i^2 \to 9000$

$i \to 115$

$i^2 \to 9100$

$i \to 116$

$i^2 \to 9200$

$i \to 117$

$i^2 \to 9300$

$i \to 118$

$i^2 \to 9400$

$i \to 119$

$i^2 \to 9500$

$i \to 120$

$i^2 \to 9600$

$i \to 121$

$i^2 \to 9700$

$i \to 122$

$i^2 \to 9800$

$i \to 123$

$i^2 \to 9900$

$i \to 124$

$i^2 \to 10000$

$i \to 125$

$\log i$ → $\left(\frac{N}{2} + \frac{N}{3} + \frac{N}{5} + \frac{N}{7} + \frac{N}{11} + \frac{N}{13} + \frac{N}{17} + \frac{N}{19} + \frac{N}{23} + \frac{N}{29} + \frac{N}{31} + \frac{N}{37} + \frac{N}{41} + \frac{N}{43} + \frac{N}{47} + \frac{N}{53} + \frac{N}{59} + \frac{N}{61} + \frac{N}{67} + \frac{N}{71} + \frac{N}{73} + \frac{N}{79} + \frac{N}{83} + \frac{N}{89} + \frac{N}{97} + \frac{N}{101} + \frac{N}{103} + \frac{N}{107} + \frac{N}{109} + \frac{N}{113} + \frac{N}{127} + \frac{N}{131} + \frac{N}{137} + \frac{N}{139} + \frac{N}{149} + \frac{N}{151} + \frac{N}{157} + \frac{N}{163} + \frac{N}{167} + \frac{N}{173} + \frac{N}{179} + \frac{N}{181} + \frac{N}{191} + \frac{N}{193} + \frac{N}{197} + \frac{N}{199} + \frac{N}{211} + \frac{N}{223} + \frac{N}{227} + \frac{N}{229} + \frac{N}{233} + \frac{N}{239} + \frac{N}{241} + \frac{N}{251} + \frac{N}{257} + \frac{N}{263} + \frac{N}{269} + \frac{N}{271} + \frac{N}{277} + \frac{N}{281} + \frac{N}{283} + \frac{N}{293} + \frac{N}{307} + \frac{N}{311} + \frac{N}{313} + \frac{N}{317} + \frac{N}{331} + \frac{N}{337} + \frac{N}{347} + \frac{N}{349} + \frac{N}{353} + \frac{N}{359} + \frac{N}{367} + \frac{N}{373} + \frac{N}{379} + \frac{N}{383} + \frac{N}{389} + \frac{N}{397} + \frac{N}{401} + \frac{N}{409} + \frac{N}{419} + \frac{N}{421} + \frac{N}{431} + \frac{N}{433} + \frac{N}{439} + \frac{N}{443} + \frac{N}{449} + \frac{N}{457} + \frac{N}{461} + \frac{N}{463} + \frac{N}{467} + \frac{N}{479} + \frac{N}{487} + \frac{N}{491} + \frac{N}{499} + \frac{N}{503} + \frac{N}{509} + \frac{N}{521} + \frac{N}{523} + \frac{N}{541} + \frac{N}{547} + \frac{N}{557} + \frac{N}{563} + \frac{N}{569} + \frac{N}{571} + \frac{N}{577} + \frac{N}{587} + \frac{N}{593} + \frac{N}{599} + \frac{N}{601} + \frac{N}{607} + \frac{N}{613} + \frac{N}{617} + \frac{N}{619} + \frac{N}{631} + \frac{N}{641} + \frac{N}{643} + \frac{N}{647} + \frac{N}{653} + \frac{N}{659} + \frac{N}{661} + \frac{N}{673} + \frac{N}{677} + \frac{N}{683} + \frac{N}{691} + \frac{N}{701} + \frac{N}{709} + \frac{N}{719} + \frac{N}{727} + \frac{N}{733} + \frac{N}{739} + \frac{N}{743} + \frac{N}{751} + \frac{N}{757} + \frac{N}{761} + \frac{N}{769} + \frac{N}{773} + \frac{N}{787} + \frac{N}{797} + \frac{N}{799} + \frac{N}{803} + \frac{N}{809} + \frac{N}{811} + \frac{N}{821} + \frac{N}{823} + \frac{N}{827} + \frac{N}{829} + \frac{N}{839} + \frac{N}{853} + \frac{N}{857} + \frac{N}{859} + \frac{N}{863} + \frac{N}{877} + \frac{N}{881} + \frac{N}{883} + \frac{N}{887} + \frac{N}{907} + \frac{N}{911} + \frac{N}{919} + \frac{N}{929} + \frac{N}{937} + \frac{N}{941} + \frac{N}{947} + \frac{N}{953} + \frac{N}{967} + \frac{N}{971} + \frac{N}{977} + \frac{N}{983} + \frac{N}{991} + \frac{N}{997}\right)$

$TC = O(N \log(\log(N)))$

$N \times \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \frac{1}{13} + \frac{1}{17} + \frac{1}{19} + \frac{1}{23} + \frac{1}{29} + \frac{1}{31} + \frac{1}{37} + \frac{1}{41} + \frac{1}{43} + \frac{1}{47} + \frac{1}{53} + \frac{1}{59} + \frac{1}{61} + \frac{1}{67} + \frac{1}{71} + \frac{1}{73} + \frac{1}{79} + \frac{1}{83} + \frac{1}{89} + \frac{1}{97} + \frac{1}{101} + \frac{1}{103} + \frac{1}{107} + \frac{1}{109} + \frac{1}{113} + \frac{1}{127} + \frac{1}{131} + \frac{1}{137} + \frac{1}{139} + \frac{1}{149} + \frac{1}{151} + \frac{1}{157} + \frac{1}{163} + \frac{1}{167} + \frac{1}{173} + \frac{1}{179} + \frac{1}{181} + \frac{1}{191} + \frac{1}{193} + \frac{1}{197} + \frac{1}{199} + \frac{1}{211} + \frac{1}{223} + \frac{1}{227} + \frac{1}{229} + \frac{1}{233} + \frac{1}{239} + \frac{1}{241} + \frac{1}{251} + \frac{1}{257} + \frac{1}{263} + \frac{1}{269} + \frac{1}{271} + \frac{1}{277} + \frac{1}{281} + \frac{1}{283} + \frac{1}{293} + \frac{1}{307} + \frac{1}{311} + \frac{1}{313} + \frac{1}{317} + \frac{1}{331} + \frac{1}{337} + \frac{1}{347} + \frac{1}{349} + \frac{1}{353} + \frac{1}{359} + \frac{1}{367} + \frac{1}{373} + \frac{1}{379} + \frac{1}{383} + \frac{1}{389} + \frac{1}{397} + \frac{1}{401} + \frac{1}{409} + \frac{1}{419} + \frac{1}{421} + \frac{1}{431} + \frac{1}{433} + \frac{1}{439} + \frac{1}{443} + \frac{1}{449} + \frac{1}{457} + \frac{1}{461} + \frac{1}{463} + \frac{1}{467} + \frac{1}{479} + \frac{1}{487} + \frac{1}{491} + \frac{1}{499} + \frac{1}{503} + \frac{1}{509} + \frac{1}{521} + \frac{1}{523} + \frac{1}{541} + \frac{1}{547} + \frac{1}{557} + \frac{1}{563} + \frac{1}{569} + \frac{1}{571} + \frac{1}{577} + \frac{1}{587} + \frac{1}{593} + \frac{1}{599} + \frac{1}{601} + \frac{1}{607} + \frac{1}{613} + \frac{1}{617} + \frac{1}{619} + \frac{1}{631} + \frac{1}{641} + \frac{1}{643} + \frac{1}{647} + \frac{1}{653} + \frac{1}{659} + \frac{1}{661} + \frac{1}{673} + \frac{1}{677} + \frac{1}{683} + \frac{1}{691} + \frac{1}{701} + \frac{1}{709} + \frac{1}{719} + \frac{1}{727} + \frac{1}{733} + \frac{1}{739} + \frac{1}{743} + \frac{1}{751} + \frac{1}{757} + \frac{1}{761} + \frac{1}{769} + \frac{1}{773} + \frac{1}{787} + \frac{1}{797} + \frac{1}{799} + \frac{1}{803} + \frac{1}{809} + \frac{1}{811} + \frac{1}{821} + \frac{1}{823} + \frac{1}{827} + \frac{1}{829} + \frac{1}{839} + \frac{1}{853} + \frac{1}{857} + \frac{1}{859} + \frac{1}{863} + \frac{1}{877} + \frac{1}{881} + \frac{1}{883} + \frac{1}{887} + \frac{1}{907} + \frac{1}{911} + \frac{1}{919} + \frac{1}{929} + \frac{1}{937} + \frac{1}{941} + \frac{1}{947} + \frac{1}{953} + \frac{1}{967} + \frac{1}{971} + \frac{1}{977} + \frac{1}{983} + \frac{1}{991} + \frac{1}{997}\right)$

$\sum_{i=1}^{N} \frac{1}{i} = \log(N)$

$\therefore TC = O(N \log(N))$

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for(i = 1; i <= 5; i++)

for(j = 1; j <= 10; j++)

i, o(1)

Q→ Count set bits from all no. from 1 to N.

N=6

1 2 3 4 5 6

ans = 2 2 3 2 3 4

V_1 → find # factors → $TC = O(N \times \sqrt{N})$

N=10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9