$^nC_r \Rightarrow \dfrac{n!}{r!\,(n-r)!}$

Choosing r items out of n distinct items.

Ways of calculating ^nC_r

1) $^nC_r = \boxed{^{n-1}C_r + {}^{n-1}C_{r-1}}$

Pascal formulae

	0	1	2	3	4	5	6
0	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0
2	1	2	1	0	0	0	0
3	1	3	3	1	0	0	0
4	1	4	6	4	1	0	0
5	1	5	10	10	5	1	0

Code for pascal triangle

(for n numbers [0, n]

int ${}^nC_r$ [n+1][n+1]

// set everything as 0

for (i=0 ; i ≤ n ; i++)

    ${}^nC_r$[i][0] = 0

// now set column as 1

for (i=1 ; i ≤ n ; i++) {

    for (j=1 ; j ≤ n ; j++) {

        ${}^nC_r[i][j] = ({}^nC_r[i-1][j] + {}^nC_r[i-1][j-1])$ % M

    }

}

TC: $O(n^2)$

SC: $O(n^2)$

what is ${}^{75}C_{48}$

${}^nC_r$[75][48]

$N \to 10^5$

150 % 20 ⟹ 10

2) Pre-calculate the factorials and the inverse of them.

$$^nC_r \Rightarrow \frac{n!}{r!\,(n-r)!}$$

$$n! \times \underbrace{\frac{1}{r!}}_{\text{inv of } (r!)} \times \frac{1}{(n-r)!} \rightarrow \text{Inv of } (n-r)!$$

int fact[n+1] fact(i) = i! % M
int invfact[n+1] invfact(i) = inverse of fact(i)

inverse of a wrt M
= pow(a, M-2, M)

fact[0] = 1
fact[1] = 1

```
for (i=2 ; i≤n ; i++) {
    fact[i] = (i × fact[i-1]) % M
}
```

```
for (i=0 ; i≤n ; i++) {
    invfact[i] = pow(fact[i], M-2, M)
}
```

$$a^{M-1} \% M = 1 \qquad M \text{ prime}$$

$$(a \times a^{M-2}) \% M = 1$$

$$(a \% M \times a^{M-2} \% M) \% M = 1$$

$$[a \times pow(a, M-2, M)] \% M = 1$$

n, r what is ${}^nC_r$

$n! \times invfact(r) \times invfact(n-r)$

$(fact[n] \times invfact[r] \times invfact[n-r])$
$\% M$

ans $= (fact[n] \times invfact[r]) \% M$

ans $= (ans \times invfact[n-r]) \% M$

$(a\%m \times b\%m \times c\%m) \% m$

$x\%m \Rightarrow [0, m-1]$

$[(m-1) \; (m-1) \; (m-1)] \% m$

$M = 10^9$

$\approx \boxed{10^{27}}$

2 at a time $\Rightarrow$ $\boxed{10^{18}}$ long

$\{[(a \times b) \% m] \times c\} \% m$

Hockey Stick Rule

$${}^{n}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + {}^{n+3}C_3 + \cdots$$
$$\cdots + {}^{n+r}C_r = {}^{n+r+1}C_r$$

$${}^{n}C_0 = 1 = {}^{n+1}C_0$$

$${}^{n}C_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$$

$$\underbrace{{}^{n+1}C_0 + {}^{n+1}C_1}_{} + {}^{n+2}C_2 + {}^{n+3}C_3 + \cdots + {}^{n+r}C_r = {}^{n+r+1}C_r$$

$$\underbrace{{}^{n+2}C_1 + {}^{n+2}C_2}_{} + {}^{n+3}C_3 + \cdots$$

$$\underbrace{{}^{n+3}C_2 + {}^{n+3}C_3}_{}$$

$$^{n+4}C_3$$

$$\underbrace{^{n+r}C_{r+1} + {}^{n+r}C_r}$$

$$^{n+r+1}C_r$$

```
int nCr ( int n, int r) {
    ans = (fact[n] x invfact[r]) % M
    ans = (ans x invfact[n-r]) % M
}
```

Q1 You are given Q queries of the form n, r. For each query print the number of ways to select either r from n
OR r/2 from n/2

Constraints ⇒ Q 10^5
n, r 10^5

```
fact[0] = 1
fact[1] = 1
for (i=2 ; i≤10^5; i++) {
    fact[i] = (i × fact[i-1]) % M
}


for (i=0 ; i≤10^5 ; i++) {
    invfact[i] = pow(fact[i], M-2, M)
}


for (i=0 ; i<q ; i++) {
        read(n, r)
```

// ans is $^{n}C_r + {}^{n/2}C_{r/2}$

ans = [nCr (n, r) +
nCr (n/2, r/2)] % M

print (ans)

}

Q2 Given an array of distinct no.s, find out no of ways of choosing K even no.s from the array.

Eg - {1, 2, 3, 4}   K = 1

ans = 2

N is till 10^5

K is till 10^5

we are only choosing from even no.s.

```
for (i=0 ; i<n ; i++) {
    if ( a[i] % 2  == 0)
        count ++
}
```

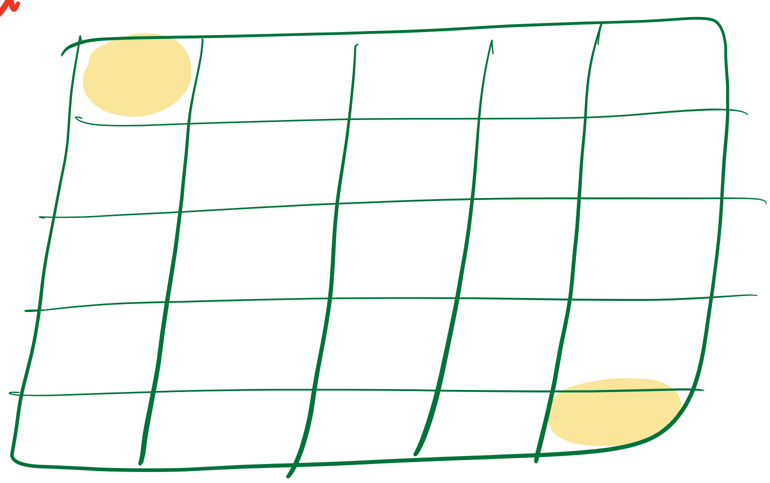
choose k from count

ans = $nCr(count, k)$

$^{count}C_k$

- How to figure out solutions

Walmart



$n \times m$

$$dp(i,j) = dp(i-1,j) + dp(i,j-1)$$

R $\quad$ D

$m-1$ $\quad$ $n-1$

D R D R D D R D D

$n+m-2$

$$^{n+m-2}C_{m-1}$$

$${}^{n+m-2}C_{n-1}$$

$${}^{n}C_{r} = {}^{n}C_{n-r}$$

{done}