

Q2]

**A) An algorithm with complexity  $kn\log(kn)$**

Using merge sort:

- We implement the combining phase of merge sort for  $k$  different arrays where total number of elements is  $k*n$
- Therefore, on each iteration, we will have half the number of arrays we had in the previous iteration.
- Imagine a tree where the  $k$  arrays merge together, 2 at a time using merge sort.
- The length of the tree will be  $\log(nk)$
- Also, we know that the merging phase of merge sort takes  $O(n)$ . Here, we have a  $k*n$  elements. Therefore, merge sort will take  $O(kn)$  for merging in total
- Therefore, the total time complexity:  $O(k*n*\log(nk))$

**B) Second algorithm for  $kn(\log k)$**

Using Min-Heap:

- Select the first elements from all the  $k$  arrays and add them to a min heap.
- Since the  $k$  arrays are sorted, the first elements selected from all the arrays are the lowest elements in their respective array.
- Thus, the root of the element in the min-heap is the smallest element among all the  $k$  arrays. Extract the root and save it in the output array of size  $k*n$  (total number of elements) and heapify the min heap.
- Now, add the next element in the min-heap from the array where the root element extracted from the min heap in the last iteration belonged to.
- Thus, keep extracting the root from the min-heap as you keep adding the elements one by one from the  $k$  arrays
- Eventually, we will have a sorted output array of  $kn$  elements.
- Insertion / Deletion operation in minheap requires  $O(\log k)$  time. Since we have  $kn$  elements, the total time required will be  $O(kn\log(k))$