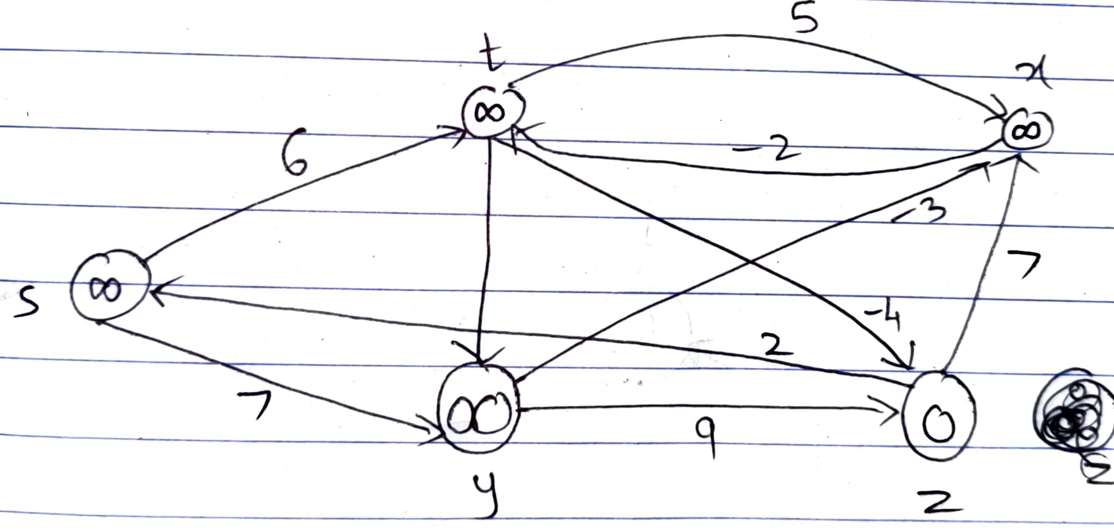
Homework 12

Q1 CLRS Exercise 24-1-1

→

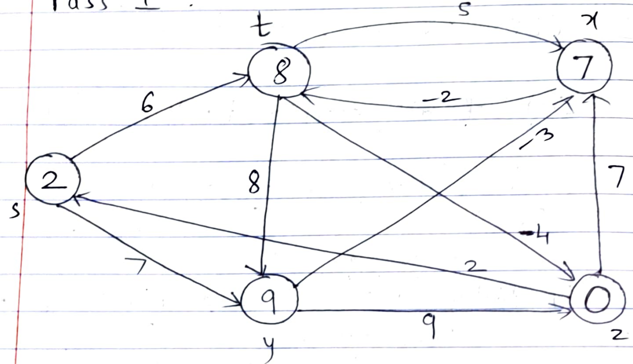
List of all edges → $(t,x), (t,y), (t,z), (x,t), (y,x), (y,z), (z,x), (z,s), (s,t), (s,y)$

Initially, $d(z) = 0$ (source) and remaining all will be infinity.

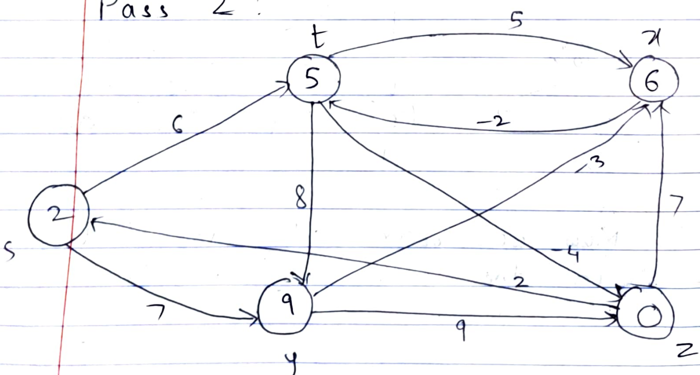


Now we relax each edge 4 times in the same given order.

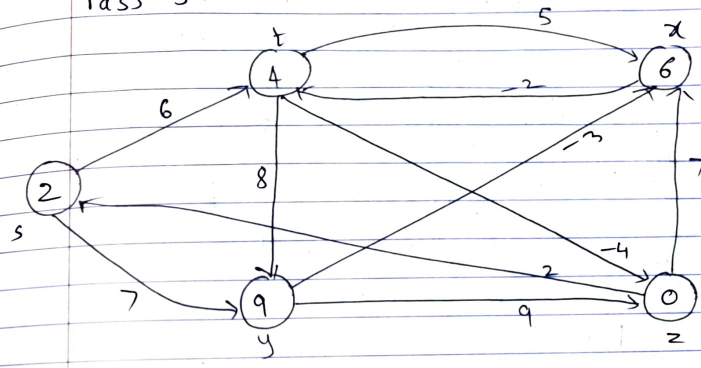
Pass 1 :



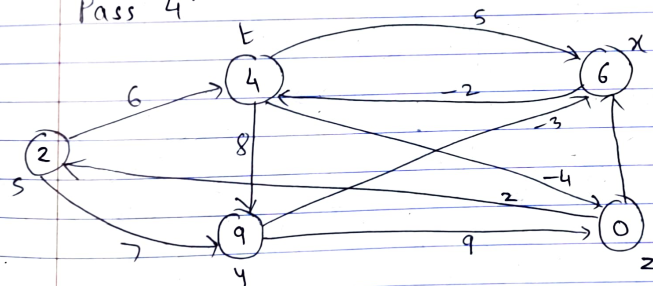
Pass 2 :



Pass 3 :

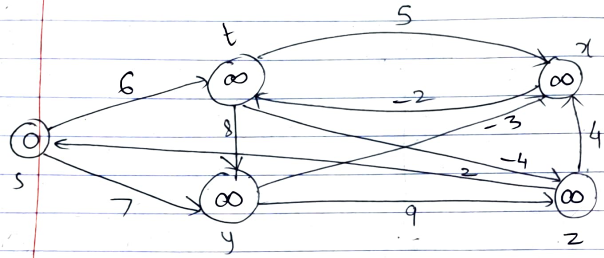


Pass 4 :

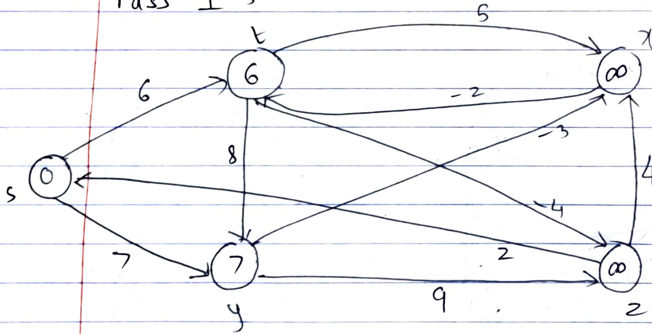


	s	t	x	y	z
d	2	4	6	9	0
π	z	x	y	s	NIL

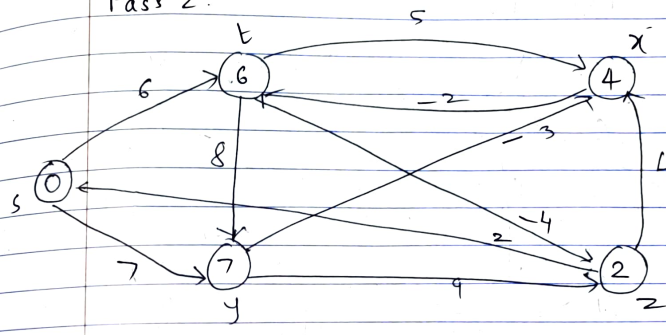
Now with $c(z, x) = 4$, and source s



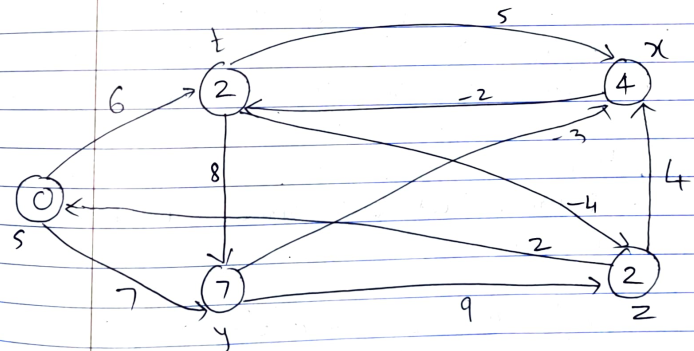
Pass I →



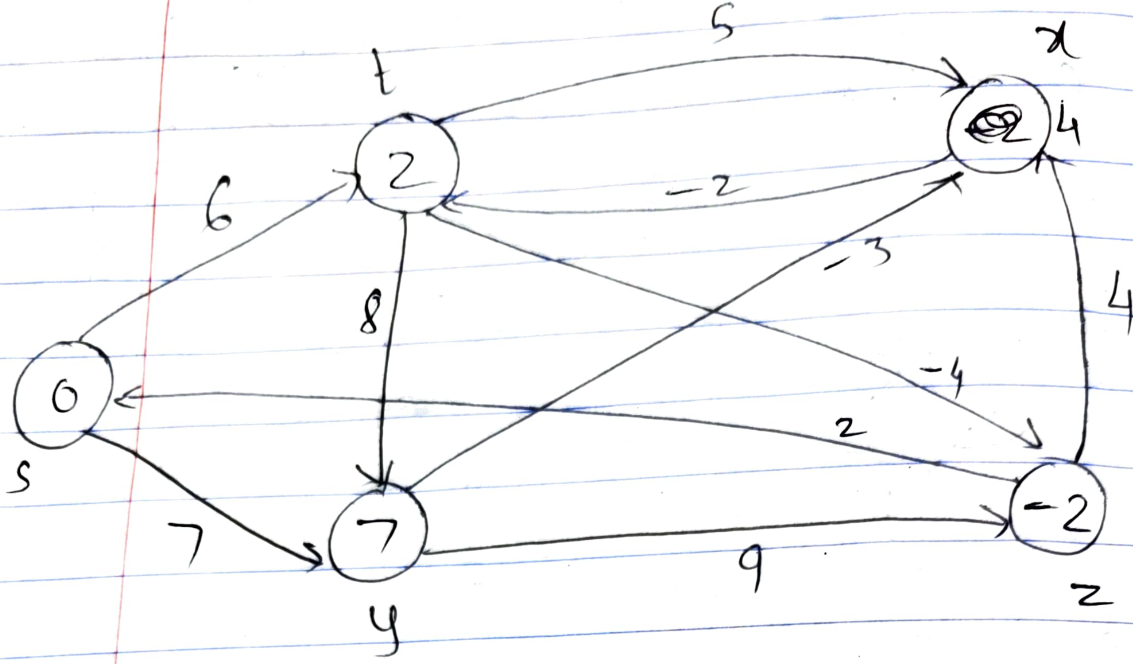
Pass 2:



Pass 3:



Pass 4 →

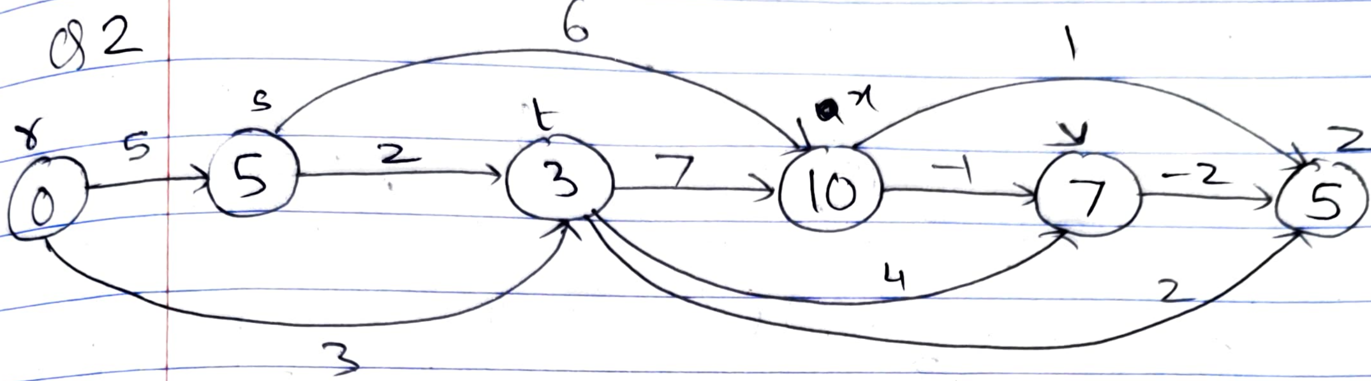


	s	t	x	y	z
d	0	2	4	7	-2
π	NIL	x	y	s	t

Here,
Bellman Ford will return FALSE
because

$$x.d = 4 > z.d + c(z,x) = -2+4$$

Q2



d-table

	r	s	t	x	y	z
r	0	∞	∞	∞	∞	∞
r	0	5	3	∞	∞	∞
s	0	5	3	11	∞	∞
t	0	5	3	10	7	5
x	0	5	3	10	7	5

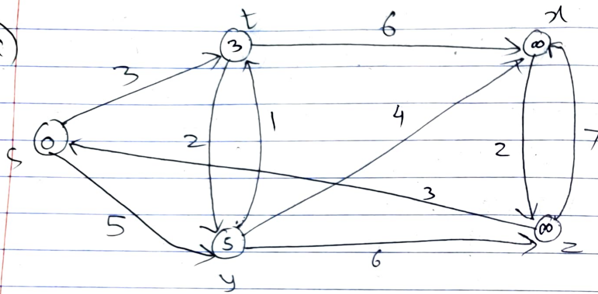
π - values

r	s	t	x	y	z
NIL	r	r	t	t	t

3.

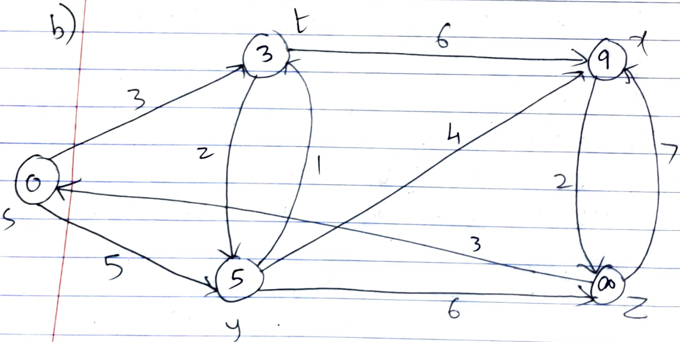
Source 's'

a)



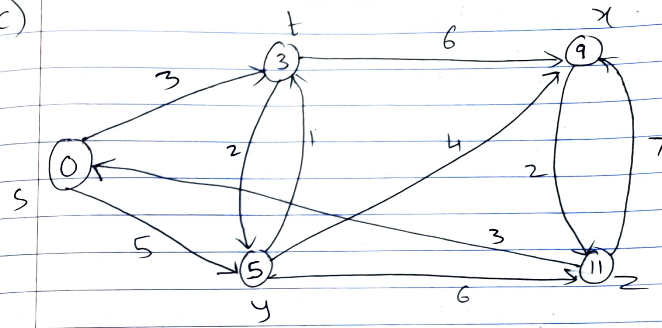
$S = \{s\}$

b)



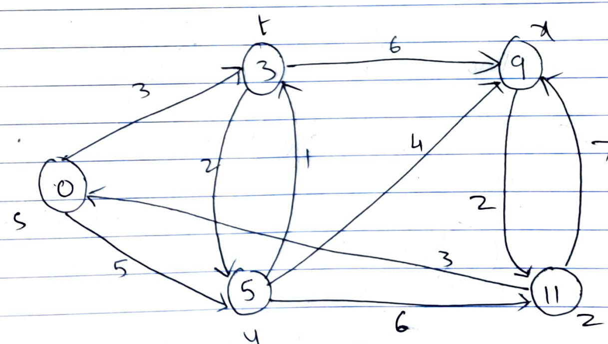
$S = \{s, t\}$

c)



$S = \{s, t, y\}$

d)



$S = \{s, t, y, x\}$

∴ Path

= $s \rightarrow t \rightarrow y \rightarrow x \rightarrow z$

d values

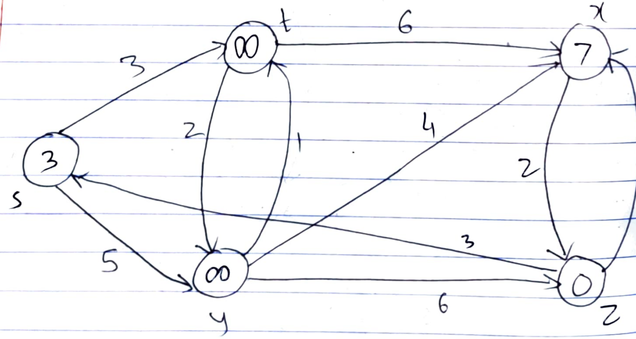
s	t	x	y	z
0	3	9	5	11

π values

s	t	x	y	z
NIL	s	y	t	x

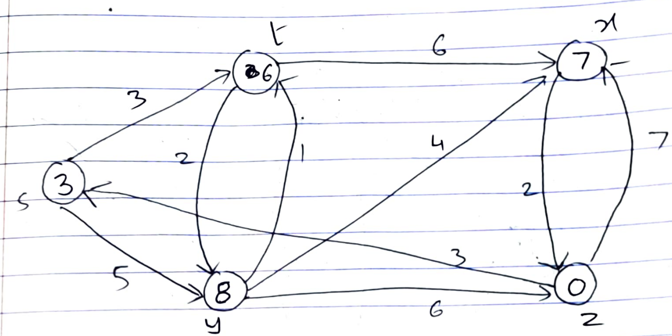
Now, new source 'z'

a)



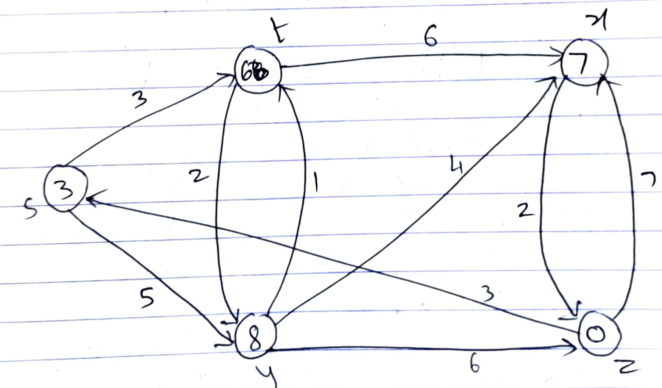
$S = \{z\}$

b)



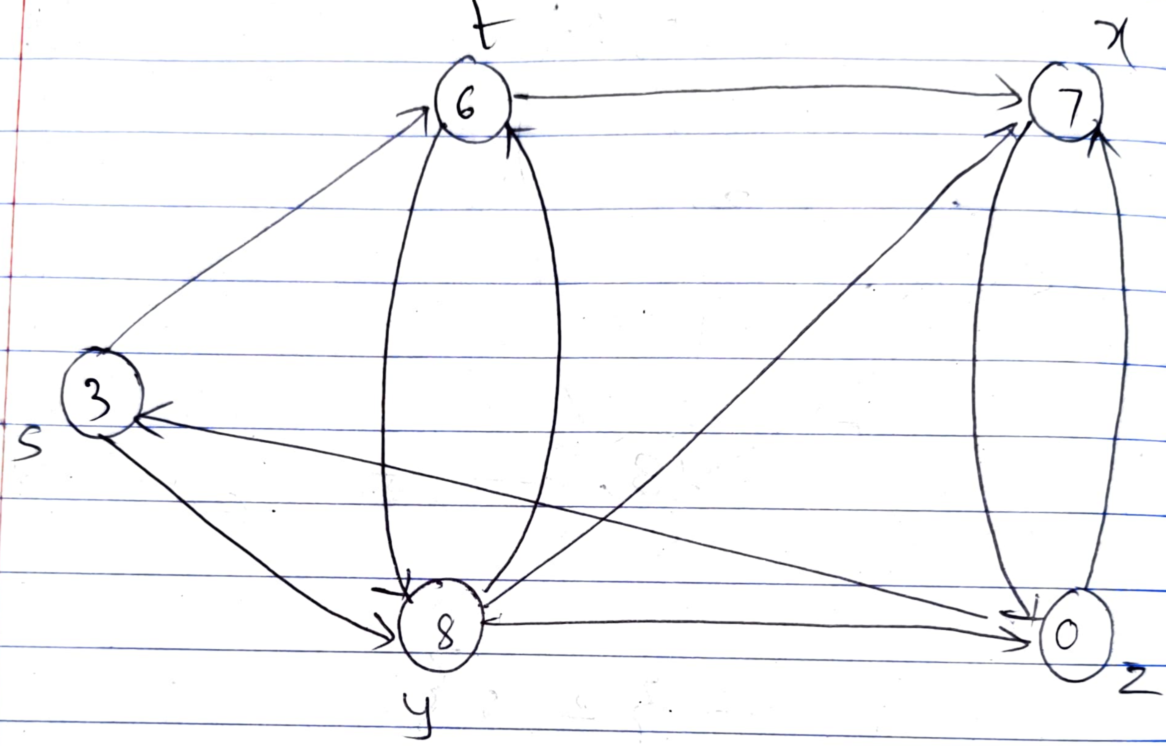
$S = \{z, s\}$

c)



$S = \{z, s, t\}$

d)



$S = \{ z, s, t, x \}$

$\therefore$ Path $\Rightarrow$ $z \rightarrow s \rightarrow t \rightarrow x \rightarrow y$

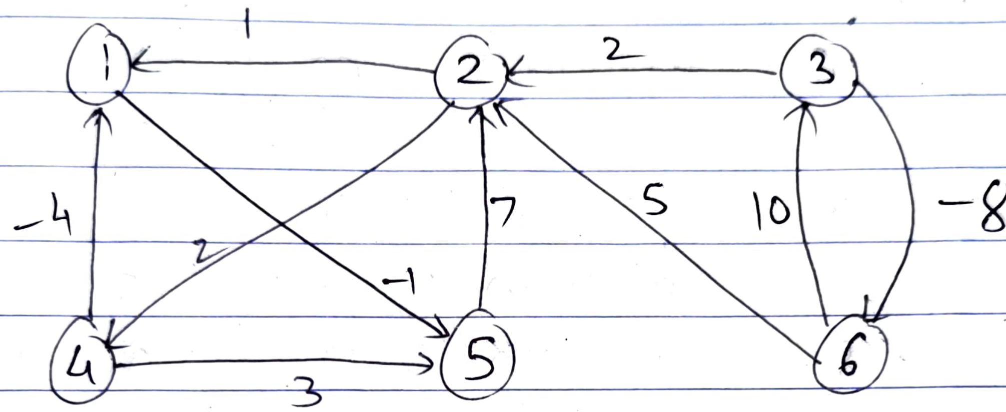
d values

	s	t	x	y	z
distances →	3	6	7	8	0

π values

	s	t	x	y	z
π values →	z	s	t	x	NIL

Q4



$D^{(k)}$ matrices :→

k=1

$$\begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix}$$

$k = 2 \Rightarrow$

$$\begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix}$$

$k = 3 \Rightarrow$

$$\begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix}$$

$k = 4 \Rightarrow$

$$\begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$

$k = 5 \Rightarrow$

$$\begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$

$k = 6$

$$\begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$