

# EL9343 Midterm Exam (2021 Fall)

**Name:**

**NetID:**

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Answer ALL questions. Exam is closed book. No electronic aids. However, you are permitted two cheat sheets, two sides each sheet. Any content on the cheat sheet is permitted.

Multiple choice questions may have **multiple** correct answers. You will get **partial credits** if you only select a subset of correct answers, and will get zero point if you select one or more wrong answers.

## **Requirements for in-person students ONLY**

Please answer all questions on the question book. You should have enough space. Since we scan all the submissions in a batch, DON'T write on the back of the pages and don't tear off any page. Make sure you write your NetID on the bottom of each page (not your N number).

If you need extra scrape papers, we can give to you. However, it will not be graded.

## **Requirements for remote students ONLY**

Each student is required to open Zoom and turn on their video, making sure the camera captures your hands and your computer screen/keyboard. The whole exam will be recorded. To clearly capture the video of exam taking, it is recommended that: A student can use an external webcam connected to the computer, or use another device (smartphone/tablet/laptop with power plugged in). Adjust the position of the camera so that it clearly captures the keyboard, screen and both hands. During the exam, you should keep your video on all the time. If there is anything wrong with your Zoom connection, please reconnect ASAP. If you cannot, please email ASAP.

During the exam, if you need to use the restroom, please send a message using the chat function in Zoom, so we know you left.

Please use a separate page for each question. Clearly write the question numbers on top of the pages. When you submit, please order your pages by the question numbers.

Once exam is finished, please submit a single PDF on Gradescope, under the assignment named "Midterm Exam". DEADLINE for submission is 15 minutes after the exam ended. Please remember to parse your uploaded PDF file. Before you leave the exam, it's your responsibility to make sure all your answers are uploaded. If you have technical difficulty and cannot upload 10 minutes after exam ended, email a copy to your proctor and the professor.

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1. (20 points) True or False

- (a) **T or F:** The  $n$ -th Fibonacci sequence number  $Fib(n) = O(2^n)$ ;
- (b) **T or F:** To prove  $T(n) = o(f(n))$ , we need to find  $c > 0$  and  $n_0 > 0$ , such that  $T(n) < cf(n)$  when  $n > n_0$ ;
- (c) **T or F:** Randomized quicksort can avoid the absolute worst case running time of  $\Theta(n^2)$ ;
- (d) **T or F:** The complexity of insertion in a binary search tree with  $n$  nodes is  $O(\log n)$ ;
- (e) **T or F:** Radix sort can use comparison sort to sort each of the  $d$ -digits;
- (f) **T or F:** It is possible to use counting sort on an array of real numbers;
- (g) **T or F:** Tree height of a max heap with  $n$  numbers is always  $\Theta(\log n)$ ;
- (h) **T or F:** A pure random number generator can be used as the hash function;
- (i) **T or F:** Pre-order walk of a binary search tree outputs a sorted sequence;
- (j) **T or F:** One can find the maximum sub-array of an array with  $n$  elements within  $O(n \log n)$  time;

2. (4 points) Which of the following function pairs satisfy  $f(n) = \Omega(g(n))$ ?

- (a)  $f(n) = n$  and  $g(n) = \sqrt{n}$ ;
- (b)  $f(n) = n \log n$  and  $g(n) = 3n + \log n$ ;
- (c)  $f(n) = 2n$  and  $g(n) = \log n$ ;
- (d)  $f(n) = n^k$  and  $g(n) = k^n$ , where  $k > 1$  is a constant;
- (e) None of the above

3. (4 points) Which of the following statements regarding Divide-and-Conquer algorithms are true?

- (a) Divide-and-Conquer algorithms always run faster than iterative algorithms;
- (b) The master theorem cannot be used to solve all recurrences;
- (c) Heapsort is a divide-and-conquer algorithm;
- (d) In the maximum-subarray problem, combining solutions to the sub-problems is more complex than dividing the problem into sub-problems;
- (e) None of the above.

4. (4 points) For a given node  $v$  in a binary search tree, which of the following node can be its successor?

- (a) right child of  $v$ 's left child;
- (b) left child of  $v$ 's right child;
- (c)  $v$ 's right child;
- (d) parent of  $v$ 's parent node;
- (e) None of the above

5. (4 points) When handling collisions using chaining, which of the following statements about a hash table of  $m$  slots and stored  $n$  elements are true?

- (a) Worst-case insertion complexity is always  $\Theta(1)$ ;
- (b) The linked list at each slot is sorted;
- (c) Worst-case time to search is  $\Theta(n)$ ;
- (d) To have an average-case search time of  $O(1)$ ,  $n$  must be  $O(m)$ ;
- (e) In universal hashing, a new hash function is picked randomly each time when inserting;

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**6. (4 points)** Which of the following statements about sorting algorithms are true?

- (a) QuickSort runs in best-case time  $O(n)$ ;
- (b) HeapSort runs in worst-case time  $O(n \log n)$ ;
- (c) Radix sort is a comparison sort algorithm;
- (d) MergeSort runs in worst-case time  $O(n \log n)$ ;
- (e) None of the above

**7. (6 points)** Prove the following properties of asymptotic notation:

- (a) **(3 points)**  $n = \omega(\sqrt{n})$ ;
- (b) **(3 points)** If  $f(n) = \Omega(g(n))$ , and  $h(n) = \Theta(g(n))$ , then  $f(n) = \Omega(h(n))$ .

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8. (12 points) Solve the following recurrences:

- (a) (4 points) Use the iteration method to solve  $T(n) = T(\frac{n}{8}) + T(\frac{n}{3}) + 3n$ ;
- (b) (4 points) Use the substitution method to verify your solution for Question 8a.
- (c) (4 points) Solve the recurrence  $T(n) = 3T(\sqrt{n}) + (\log n)^2$ .

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9. (6 points) Illustrate the operation of merge-sort on the array [71, 25, 40, 7, 60, 13, 20, 80]

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**10. (6 points)** Given an array of  $[4, 11, 6, 12, 16, 3, 9, 5, 8, 15, 7]$ ,

- (a) **(3 points)** plot the initial max-heap built from the array;
- (b) **(3 points)** plot the max-heap after the maximum is extracted, then a new number 10 is inserted.

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**11. (6 points)** Design a  $\Theta(n)$  algorithm to find the minimum of an unordered array.

- (a) **(2 points)** write down the pseudocode;
- (b) **(4 points)** prove the correctness of your algorithm using Loop-Invariant.

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**12. (12 points)** Answer the following questions about Selection:

- (a) **(4 points)** To find the  $k$ -th order statistic of a list of  $n$  distinct numbers using randomized selection, what is the probability that the  $k$ -th order statistic is found right after the first call of randomized-partition(i.e., no need to do second partition)? what is the probability that the  $k$ -th order statistic is found right after the second call of randomized-partition?
- (b) **(8 points)** If there is a selection algorithm that can find any  $k$ -th order statistic of any array with  $n$  or less numbers with worst-case time complexity of  $O(n)$ , describe how to use this algorithm to find the  $k$ -th order statistic of any array of  $2n$  numbers with worst-case complexity of  $O(n)$ , where  $1 \leq k \leq \frac{n}{2}$  or  $\frac{3n}{2} \leq k \leq 2n$ . (*NOTE: the given selection algorithm cannot work with any array with more than  $n$  numbers, and you are not allowed to use any other  $O(n)$  Selection algorithm.*)



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**13. (12 points)** Answer the following questions about hash table with collision resolved by chaining.

- (a) **(3 points)** If the hash table has  $m$  slots numbered between 1 and  $m$ , and  $n$  keys are sequentially inserted into the hash table, the slot of each key is determined by a simple universal hash function  $h(\cdot)$  with value range of 1 to  $m$ . Let  $X_i$  be the position of the  $i$ -th ( $1 \leq i \leq n$ ) inserted key in its chain, what is the probability mass function of  $X_i$ , i.e., calculate  $P(X_i = k)$ ,  $\forall k \geq 0$ ?
- (b) **(4 points)** If the hash table has  $2m$  slots numbered between 1 and  $2m$ , and  $n$  keys are sequentially inserted into the hash table, the slot of each key is determined by the sum of two simple universal hash functions  $h_1(\cdot) + h_2(\cdot)$ , each with value range of 1 to  $m$ . What is the probability that a key is insert into the chain stored at hash table slot  $j$  ( $1 \leq j \leq 2m$ )?
- (c) **(5 points)** For the hash table in (b), what is the expected time complexity for unsuccessful search? what is the expected time complexity for successful search?

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