

Homework 12

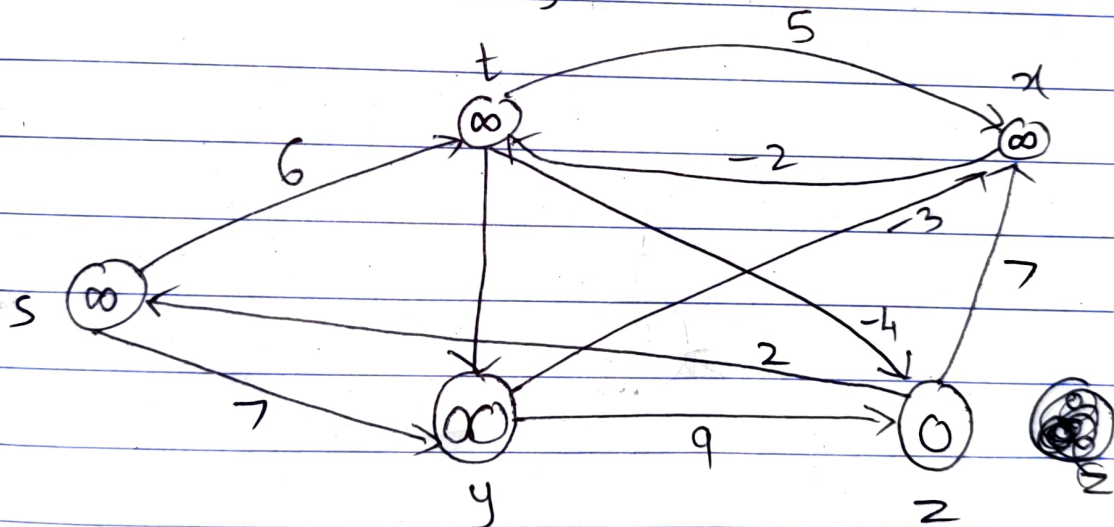
Q1 CLRS Exercise 24.1-1

→

List of all edges $\rightarrow (t, x), (t, y), (t, z),$
 $(x, t), (y, x), (y, z), (z, x), (z, s), (s, t),$
 (s, y)

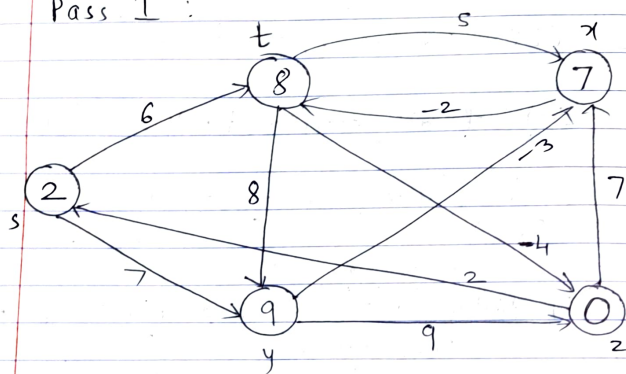
\nearrow (source)

Initially, $d(z) = 0$ and remaining all will be infinity.

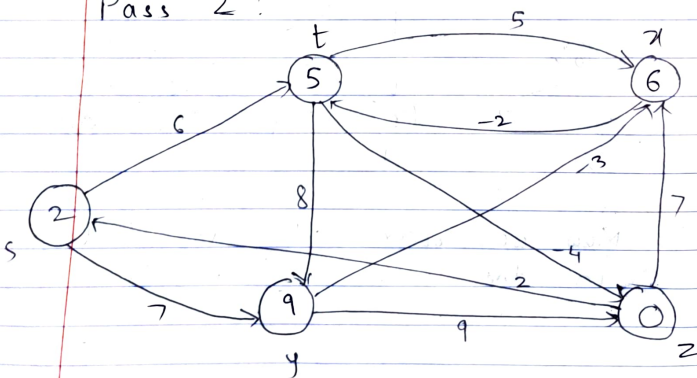


Now we relax each edge 4 times in the same given order.

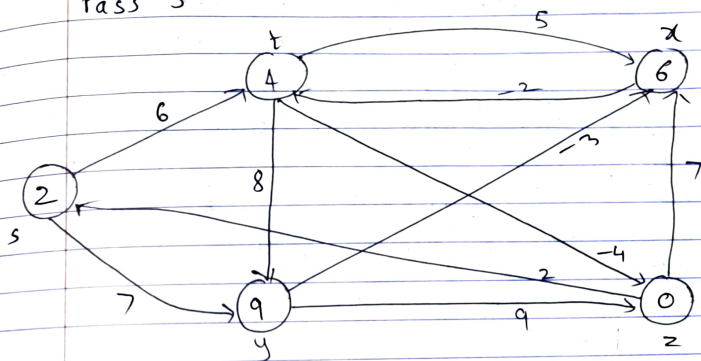
Pass 1 :



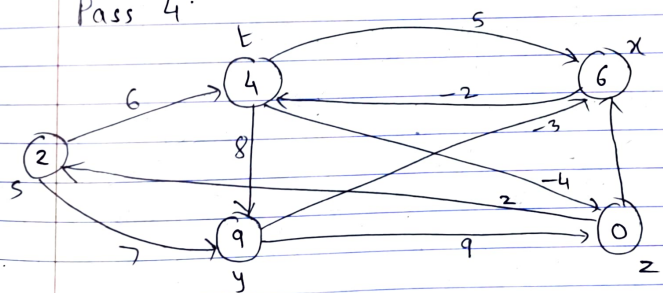
Pass 2 :



Pass 3 :

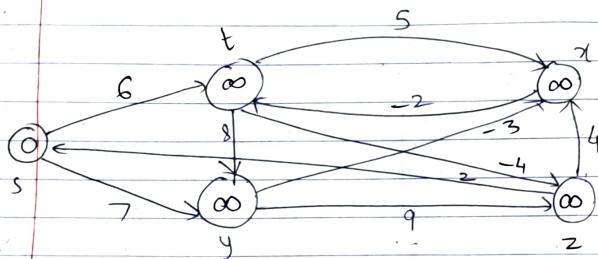


Pass 4 :

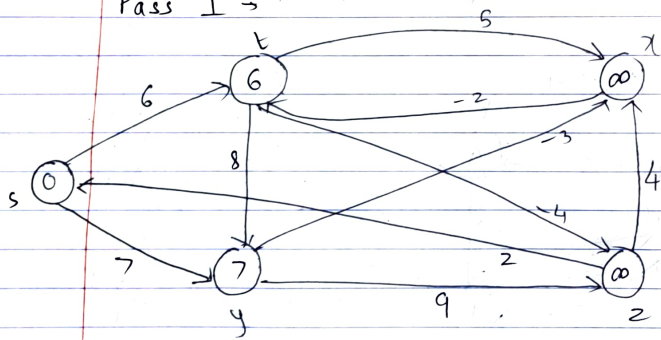


	s	t	x	y	z
d	2	4	6	9	0
π	z	x	y	s	NIL

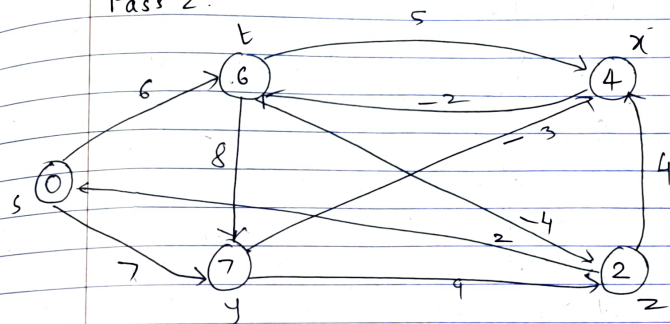
Now with $c(z, x) = 4$, and source s



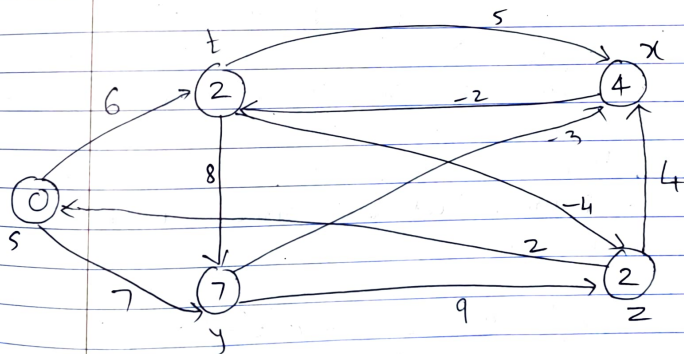
Pass 1 →



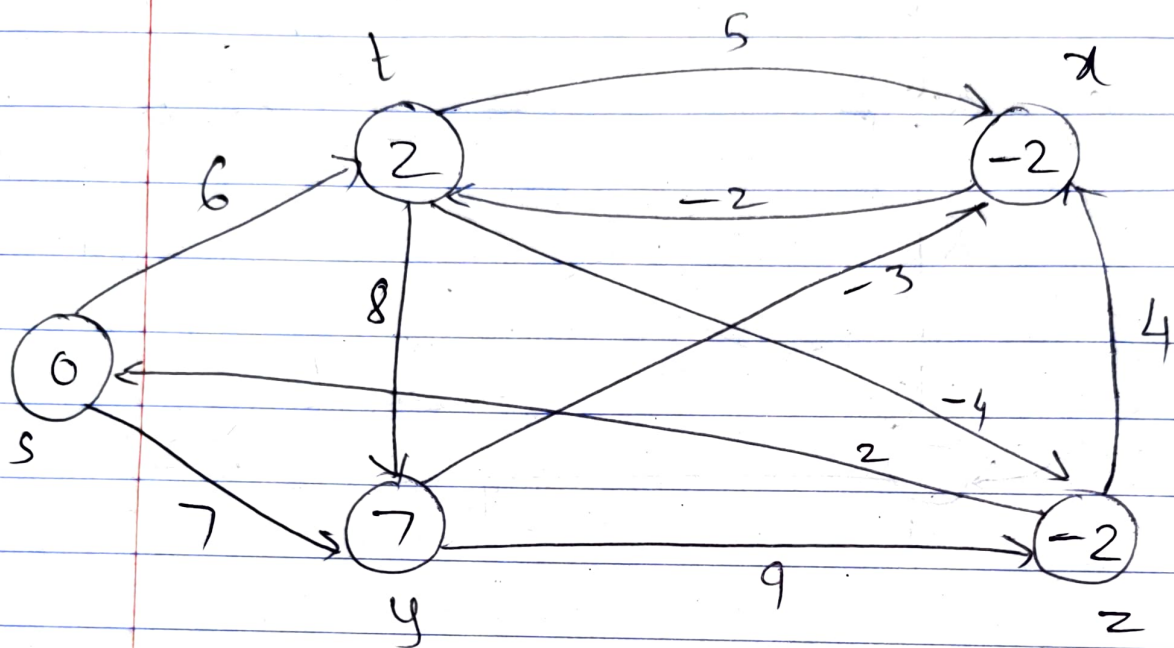
Pass 2:



Pass 3:



Pass 4 →

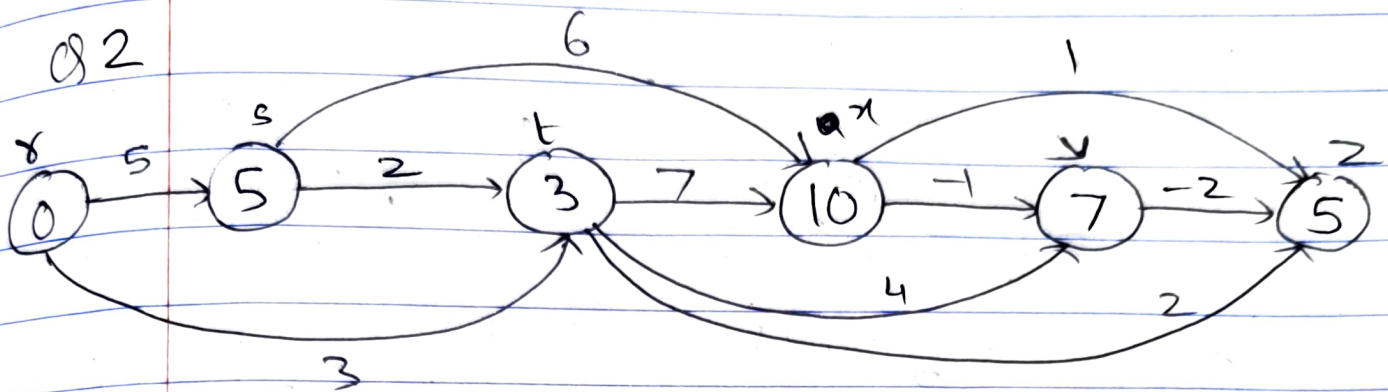


	s	t	x	y	z
d	0	2	-2	7	-2
π	NIL	x	y	s	t

Here,

Bellman Ford will return FALSE
because

$$x.d = 4 > z.d + c(z, x) = -2 + 4$$



d-table

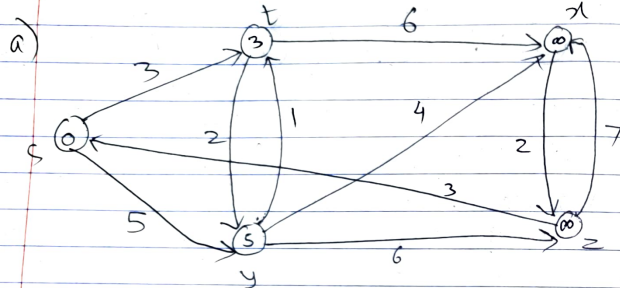
	x	s	t	x	y	z
x	0	∞	∞	∞	∞	∞
s	0	5	3	∞	∞	∞
t	0	5	3	10	7	5
x	0	5	3	10	7	5

→ values

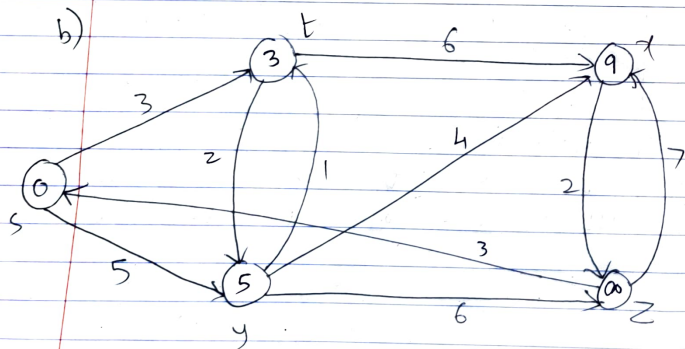
x	s	t	x	y	z
NIL	x	x	t	t	t

3.

Source 's'

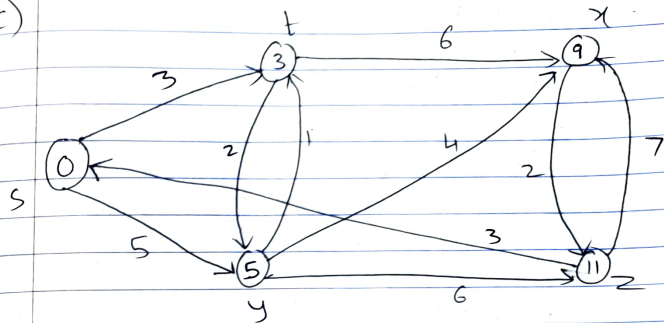


$$S = \{s\}$$



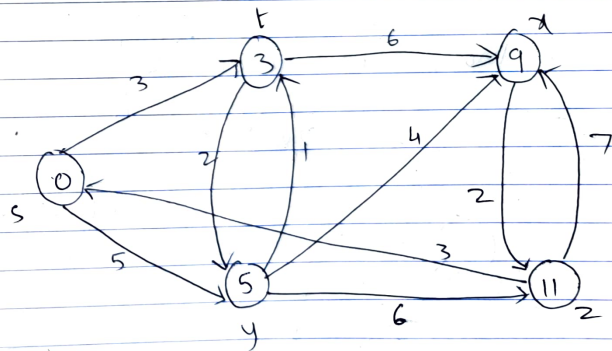
$$S = \{s, t\}$$

c)



$$S = \{s, t, y\}$$

d)



$$S = \{s, t, y, x\}$$

∴ Path

= $s \rightarrow t \rightarrow y \rightarrow x \rightarrow z$

dual values

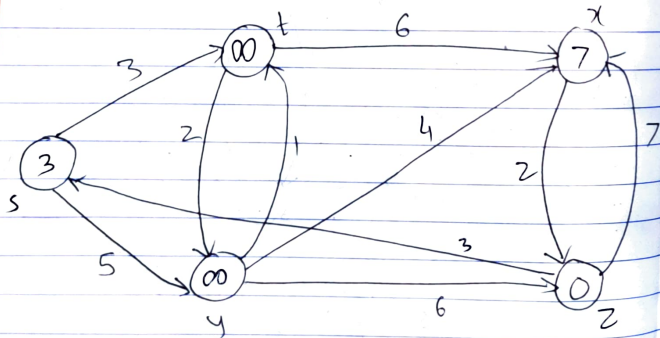
s	t	y	x	y	z
0	3	9	5	11	

→ values

s	t	x	y	z
NIL	s	y	t	x

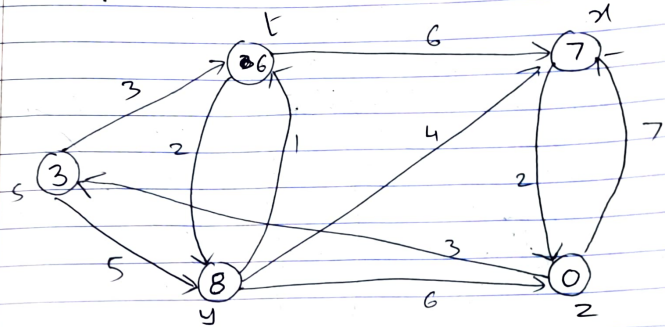
Now, new source 'z'

a)



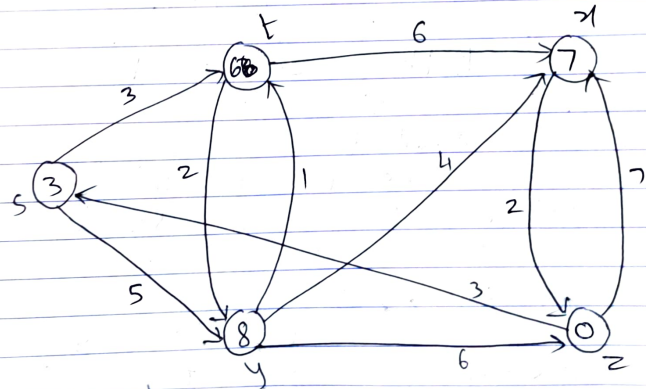
$s = \{z\}$

b)

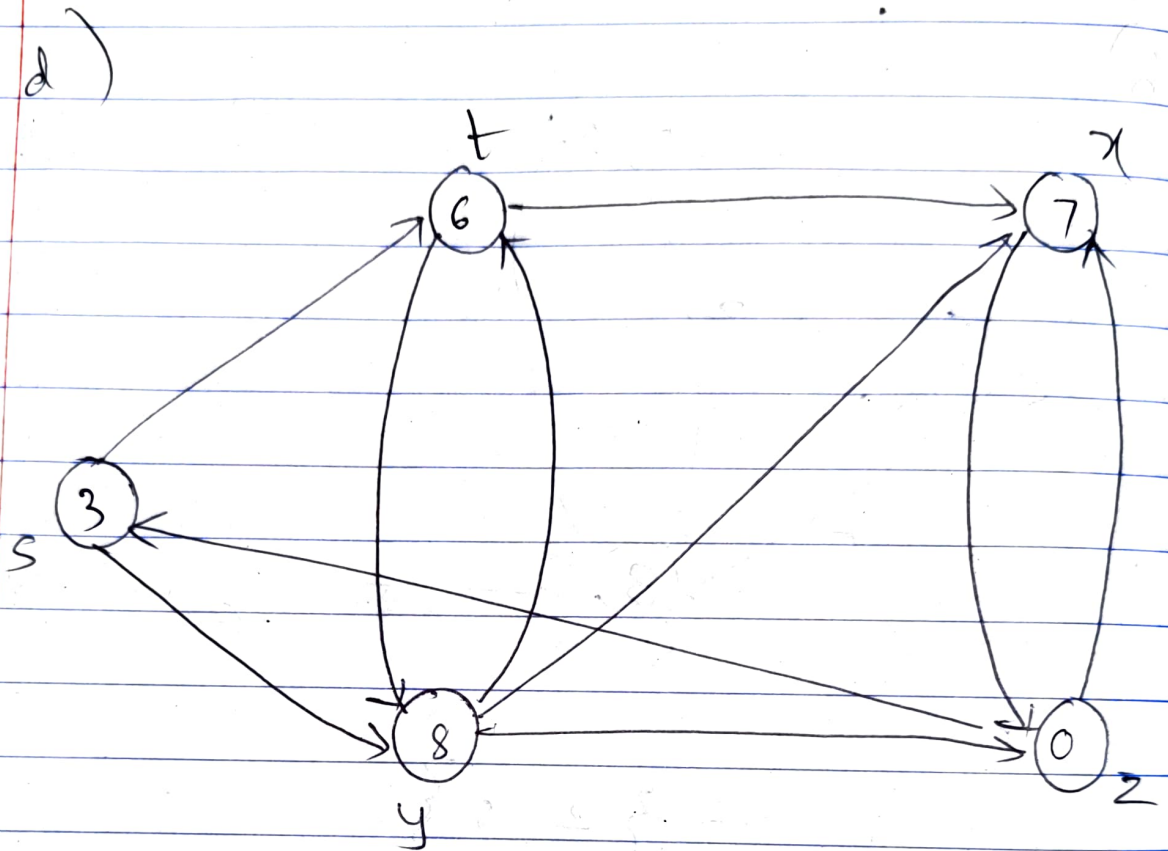


$s = \{z, s\}$

c)



$s = \{z, s, t\}$



$$S = \{ z, s, t, x \}$$

\therefore Path $\Rightarrow z \rightarrow s \rightarrow t \rightarrow x \rightarrow y$

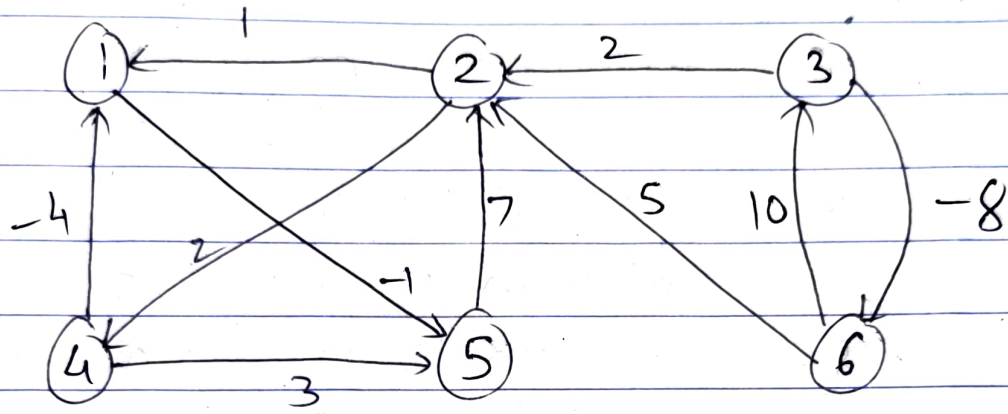
d values

	s	t	x	y	z
distances \rightarrow	3	6	7	8	0

π values

	s	t	x	y	z
π values \rightarrow	z	s	t	x	NIL

Q4



$D^{(k)}$ matrices \rightarrow

$k=1$

0	∞	∞	∞	-1	∞
1	0	∞	2	0	∞
∞	2	0	∞	∞	-8
-4	∞	∞	0	-5	∞
∞	7	∞	∞	0	∞
∞	5	10	∞	∞	0

$k=2 \Rightarrow$

$$\begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix}$$

$k=3 \Rightarrow$

$$\begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix}$$

$k=4 \Rightarrow$

$$\begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$

$k=5 \Rightarrow$

$$\begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$

$$k = 6$$

$$\begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 6 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$