DSA Midterm

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Q 1

a 1 True

b 2 True

c 3 True

d 4 True

e 5 True

f 6 True

g 7 True

h 8 ~~True~~ False → should give same value for same key

i 9 False

j 10 True

Q 2. (a), (c), (b)

Q 3. (b), (d),

Q 4. (b), (c), (d)

Q 5. (c), (d) → depends on the hash function too, here I marked (d) assuming that the hash function is sufficiently a ~~random~~ uniform ~~generator~~ distributor, (a) → always added at start

Q 6. (d), (b),

Q7

b] $f(n) = \Omega(g(n))$ and
$h(n) = \theta(g(n))$

To prove, $f(n) = \Omega(h(n))$

By definition,

① $f(n) \geq c_1 g(n)$ where $c_1 > 0$ &
$n > n_0$

also,

② $c_2 g(n) \leq h(n) \leq c_3 g(n)$ where
$c_2, c_3 > 0$;
$n > n_0$

From ① + ②
$c_3 f(n) \geq c_3 c_1 g(n) \geq c_1 h(n)$

Divide by c_1,

$c_1 c_3 f(n) \geq c_1 h(n)$

Let $k = c_1 \cdot c_3$

$\therefore$ We have the form

$$h(n) \leq k.\, f(n) \quad \text{where} \quad k > 0$$
$$\text{and} \quad n > n_0$$

$$\therefore \frac{1}{k} h(n) \leq f(n) \quad \text{where} \quad \frac{1}{k} > 0$$

$\therefore$ By definition,

$$\boxed{f(n) \text{ is } \Omega(h(n))}$$

a) To prove $n = \omega(\sqrt{n})$

To prove

i.e. $n > c\sqrt{n}$

For $n = 4$ and $c = 1.5$,

L.H.S $= 4$, R.H.S $= 1.5 \times 2 < 4$

$\therefore$ We find one c and n pair for which $n > c\sqrt{n}$

$$\therefore n = \omega(\sqrt{n})$$

~~Q13)~~

Q 8] $T(n) = T\left(\frac{n}{8}\right) + T\left(\frac{n}{3}\right) + 3n$

$$T(n/8) = T\left(\frac{n}{8^2}\right) + T\left(\frac{n}{8\cdot 3}\right) + \frac{3n}{8}$$

$$T\left(\frac{n}{3}\right) = T\left(\frac{n}{8\cdot 3}\right) + T\left(\frac{n}{3^2}\right) + \frac{3n}{3}$$

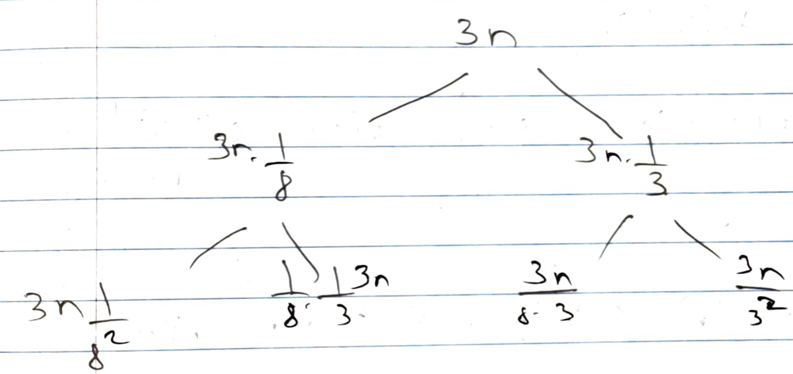
$$\therefore T(n) = T\left(\frac{n}{8^2}\right) + T\left(\frac{n}{8\cdot 3}\right) + T\left(\frac{n}{8\cdot 3}\right) + T\left(\frac{n}{3^2}\right) + \cancel{\tfrac{3}{8}}n\ 3n + \frac{11}{8}n$$

$$T\left(\frac{n}{8^2}\right) = T\left(\frac{n}{8^3}\right) + T\left(\frac{n}{8^2\cdot 3}\right) + \frac{3\cdot n}{8^2}$$

$$T\left(\frac{n}{3^2}\right) = T\left(\frac{n}{3^2\cdot 8}\right) + T\left(\frac{n}{3^3}\right) + \frac{3n}{3^2}$$

$$T\left(\frac{n}{8\cdot 3}\right) = T\left(\frac{n}{8^2\cdot 3}\right) + T\left(\frac{n}{8\cdot 3^2}\right) + \frac{3n}{8\cdot 3}$$

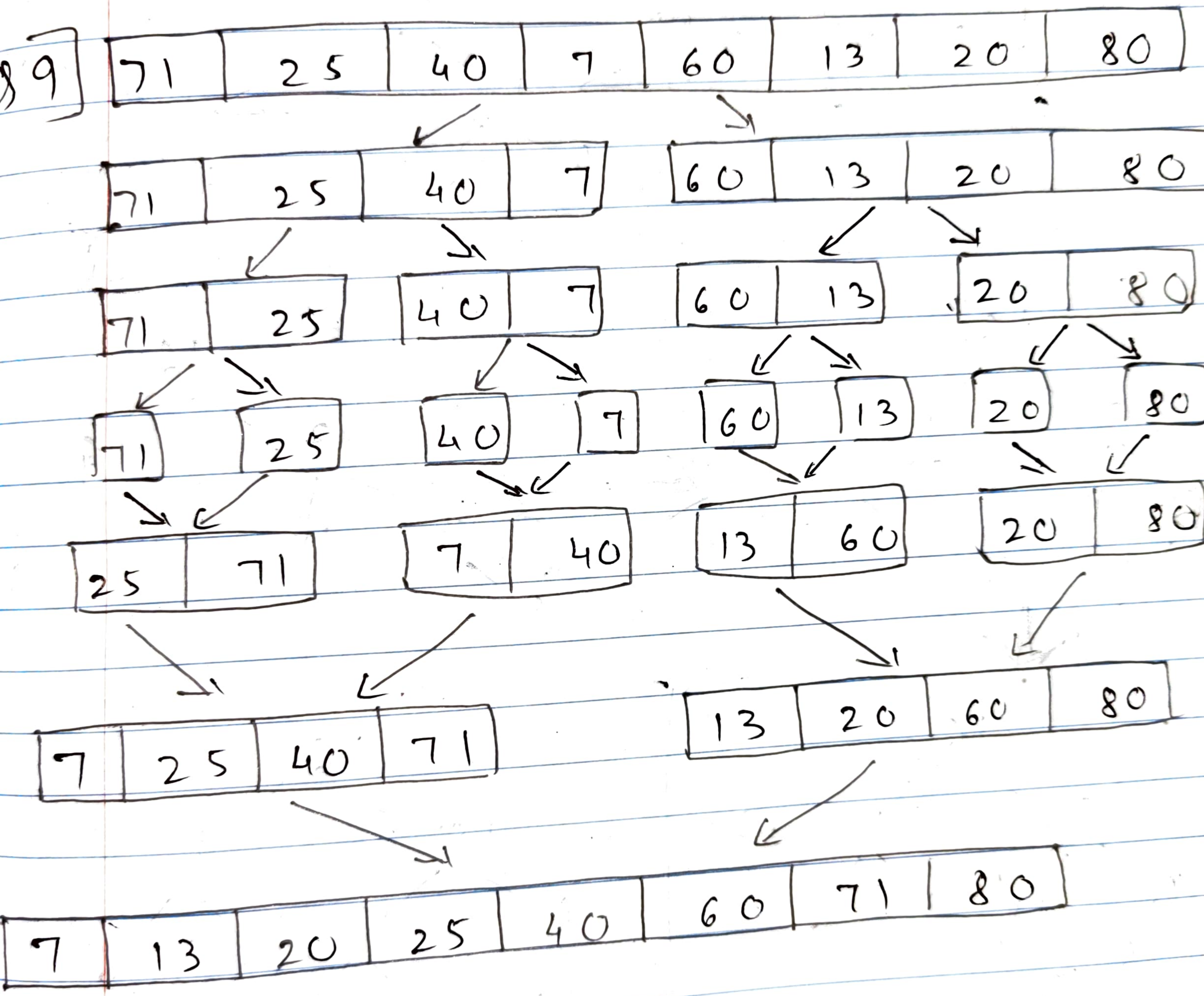
We see that the tree is imbalanced as follows



$\therefore$ Considering only the extreme left and right subtrees,

Upper bound is $O(n)$

Q9



71 25 40 7 60 13 20 80
71 25 40 7
60 13 20 80
71 25
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25
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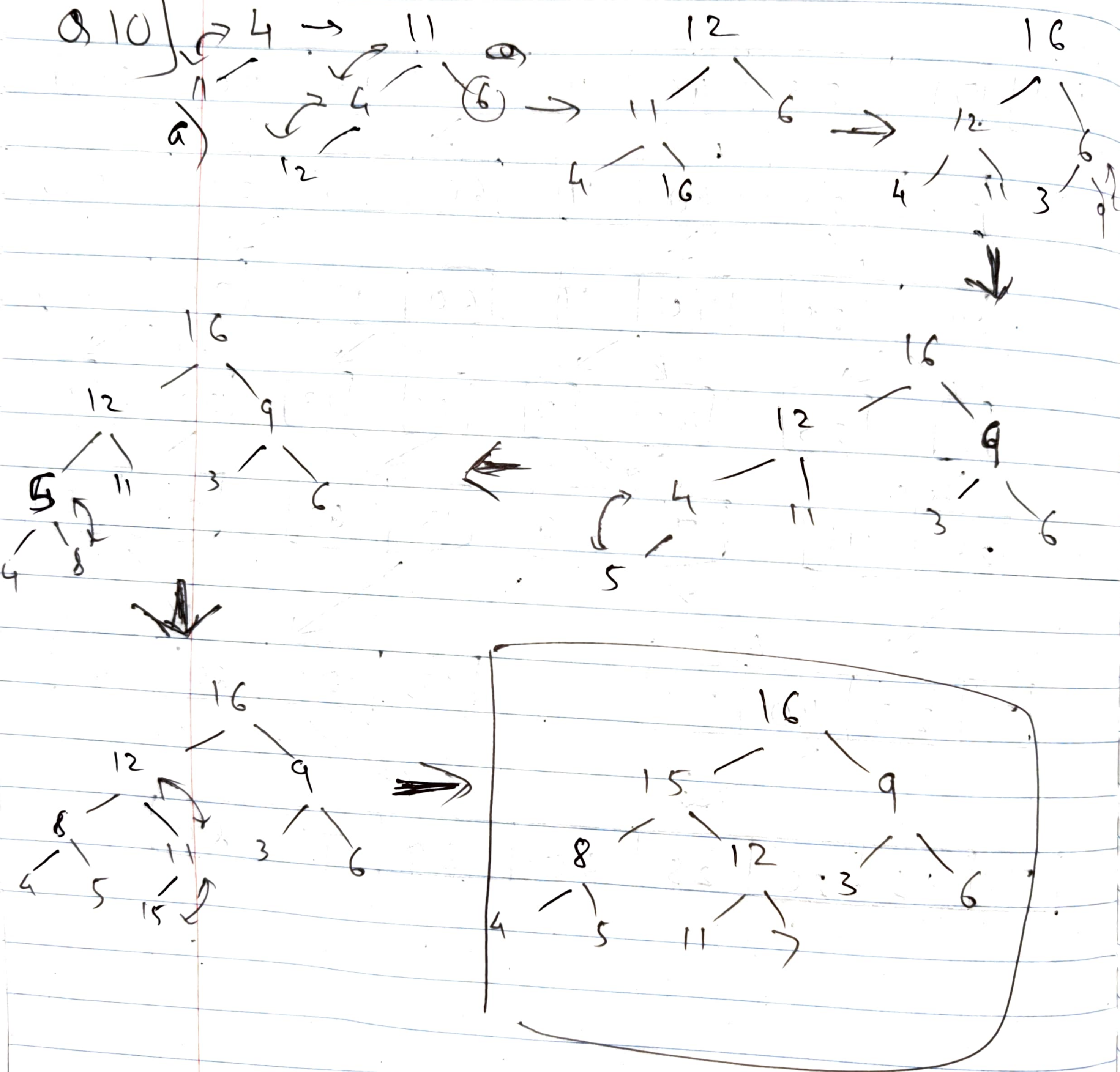
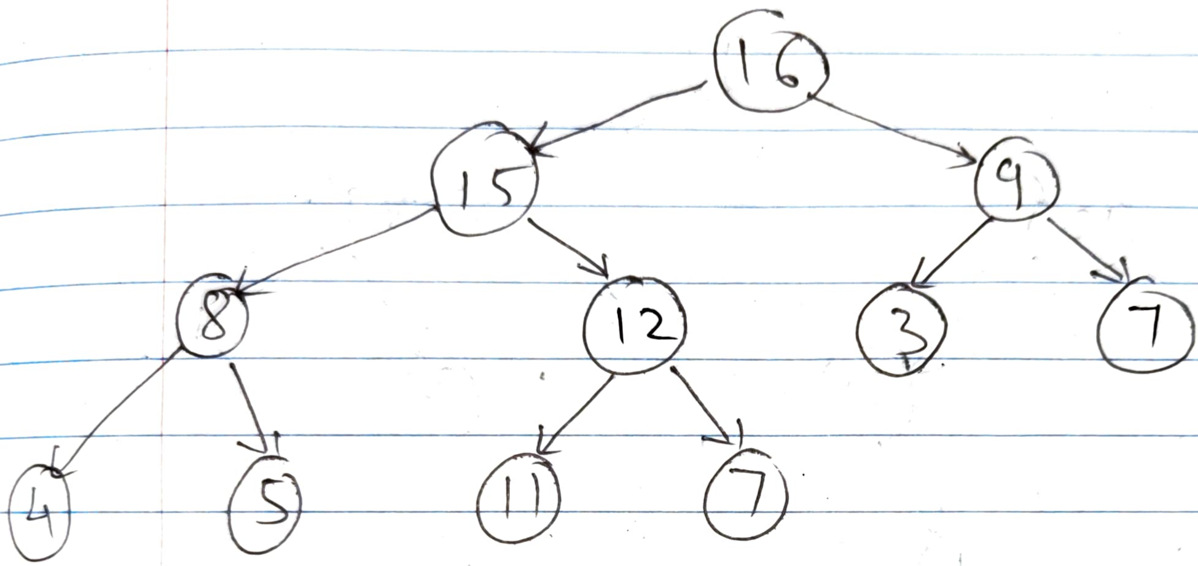
7
60
13
20
80
25 71
7 40
13 60
20 80
7 25 40 71
13 20 60 80
7 13 20 25 40 60 71 80

Q10

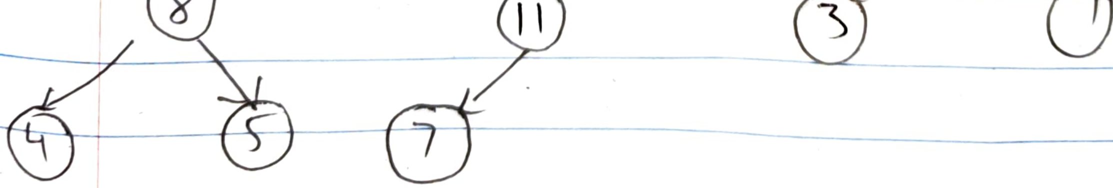
a)



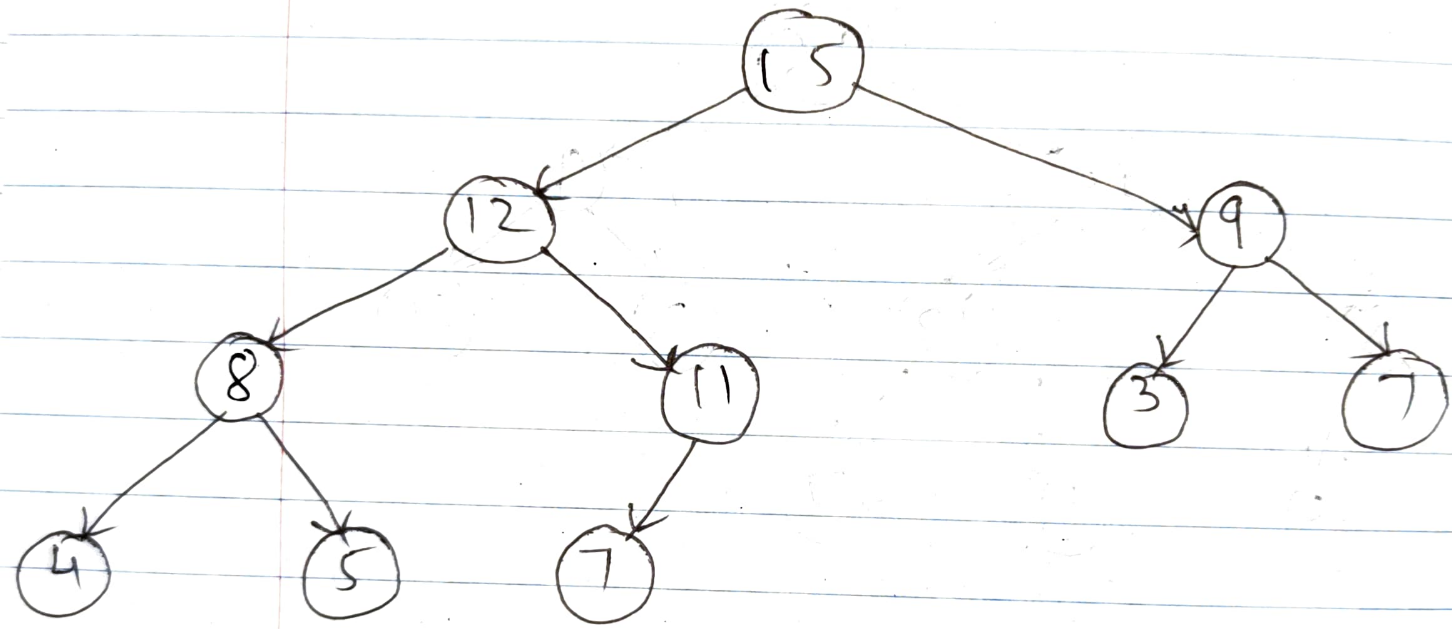
a) ∴ Final max-heap built from the array:



b) First we extract max & replace it with last element, graph looks like:



Now, we add (10) at last position, we see that the tree is still max-heap by default ∴ Final tree is:



Q11 Min (A , n)

```
{
  ~~if A[1]~~
  if n > 1
      if A[1] > A[2]
          min = A[2]
      else
          min = A[1]
  ~~for i from n-2 n-(n-2)+1 to~~ n
  ~~while~~ i
  i = n - (n-2) + 1
  while i ≤ n
      if min < A[i]
          do nothing
      else
          min = A[i]
      i++
  else if n == 1
      return A[1]
  else return " n is 0 "
```

Loop Invariant

Here, min is always smaller than or equal to all the elements in the array on the left.

Initialization → at the beginning, loop starts at index 3 and min holds the smaller of the first two elements.

Maintenance → On every iteration, the min ~~elemen~~ is compared to the i^{th} element and thus is maintained to be minimum (by swapping if necessary)

Termination → ~~A~~ When the loop terminates,

the algorithm has traversed through the array, thus we are at the last element. Thus, from the loop invariant condition, min will hold the smallest element among all the elements to the left, i.e., the entire array.

Q 13)

b) The probability is 1

Since the hash functions output hash table slot from 1 to m,

The minimum hash slot generated will be $1 + 1 = 2$

and the maximum will be $m + m = 2m$.

This range lies between $1 \leq j \leq 2m$

$\therefore$ Probability is 1

c) Unsuccessful search