

Surface plasmons

➤ Basics :

➤ Dispersion relation of surface plasmons

➤ References

“Metal Optics”

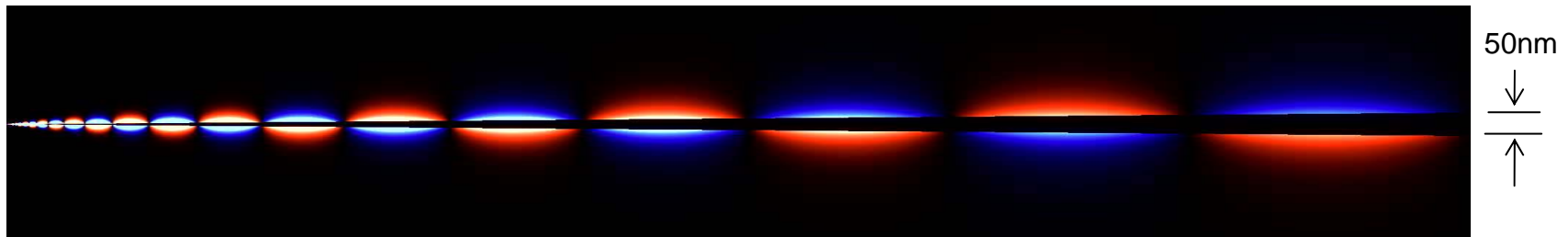
Prof. Vlad Shalaev, Purdue Univ., ECE Department,
<http://shay.ecn.purdue.edu/~ece695s/>

“Surface plasmons on smooth and rough surfaces and on gratings”

Heinz Raether(Univ Hamburg), 1988, Springer-Verlag

“Surface-plasmon-polariton waveguides”

Hyongsik Won, Ph.D Thesis, Hanyang Univ, 2005.

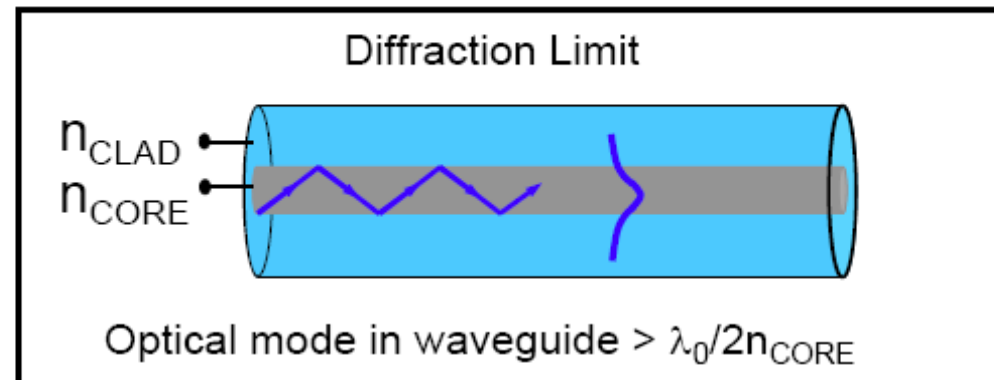
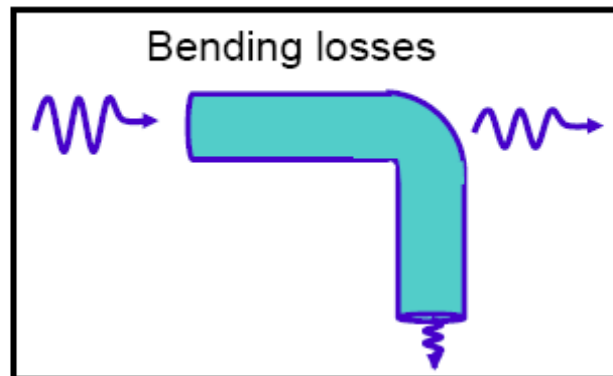


Metal Optics: An introduction

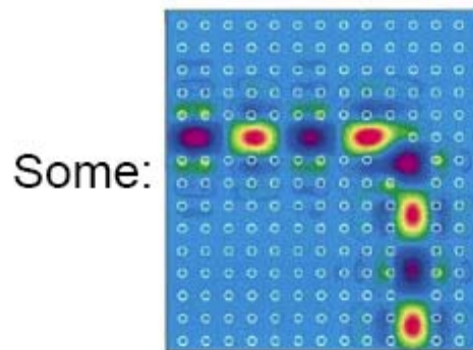
Majority of optical components based on dielectrics

- High speed, high bandwidth (ω), but...
- Does not scale well \Rightarrow Needed for large scale integration

Problems



Solutions ?



Some fundamental problems!



Photonic functionality based on metals?!

J. D. Joannopoulos, et al, Nature, vol.386, p.143-9 (1997)

What is a plasmon?

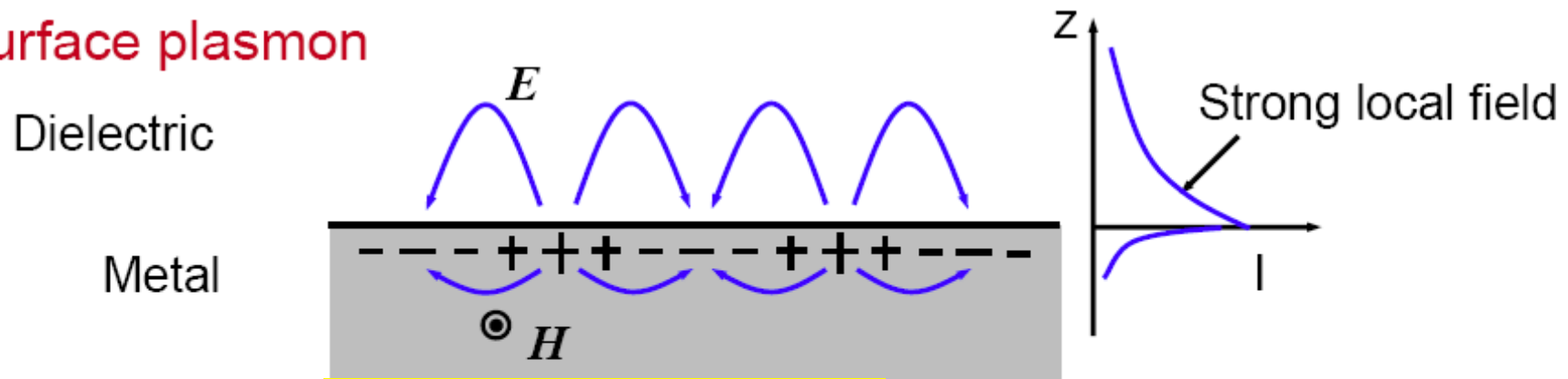
What is a plasmon ?

- Compare electron gas in a metal and real gas of molecules
- Metals are expected to allow for electron density waves: plasmons

Bulk plasmon

- Metals allow for EM wave propagation above the plasma frequency
 They become transparent!

Surface plasmon

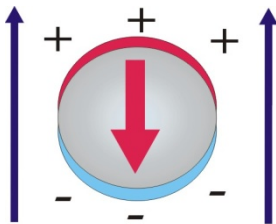
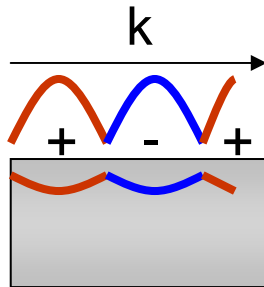
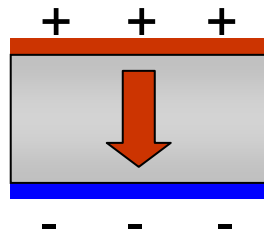


Note: SP is a TM wave!

- Sometimes called a surface plasmon-polariton (strong coupling to EM field)

Plasmon-Polaritons

Plasma oscillation = density fluctuation of free electrons



Plasmons **in the bulk** oscillate at ω_p determined by the free electron density and effective mass

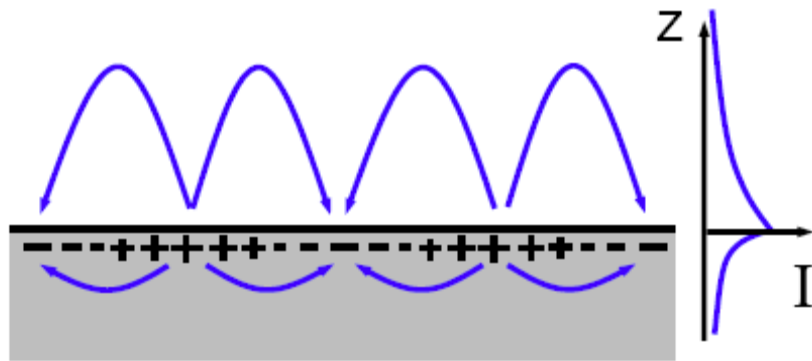
$$\omega_p^{drude} = \sqrt{\frac{Ne^2}{m\epsilon_0}}$$

Plasmons **confined to surfaces** that can interact with light to form propagating “surface plasmon polaritons (SPP)”

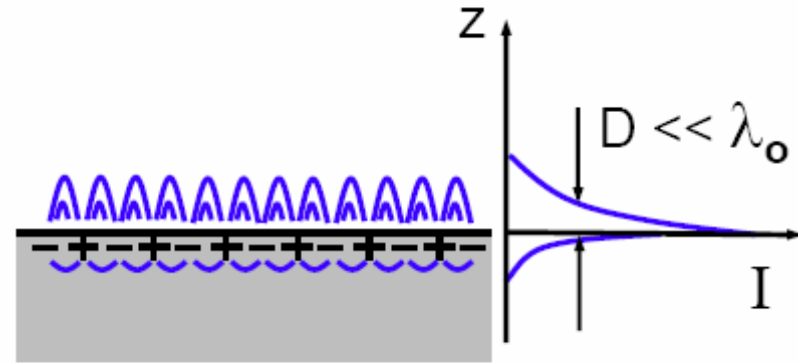
Confinement effects result in resonant SPP modes **in nanoparticles**

$$\omega_{particle}^{drude} = \sqrt{\frac{1}{3} \frac{Ne^2}{m\epsilon_0}}$$

Local field intensity depends on wavelength



Long wavelength
(small propagation constant, k)



Short wavelength
(large propagation constant, k)

- Characteristics plasmon-polariton**
- Strong localization of the EM field
 - High local field intensities easy to obtain

- Applications:**
- Guiding of light below the diffraction limit (near-field optics)
 - Non-linear optics
 - Sensitive optical studies of surfaces and interfaces
 - Bio-sensors
 - Study film growth
 -

Dielectric constant of metal : Drude model


$$\begin{aligned}\epsilon_{EFF}(\omega) &= \epsilon_B + i \frac{\sigma(\omega)}{\epsilon_o \omega} \\ &= \epsilon_B + i \frac{1}{\epsilon_o \omega} \left(\frac{\sigma_o}{1 - i\omega\tau} \right) = \epsilon_B + i \frac{\sigma_o (1 + i\omega\tau)}{\epsilon_o \omega (1 + \omega^2 \tau^2)} \\ &= \left(\epsilon_B - \frac{\omega_p^2 \tau^2}{1 + \omega^2 \tau^2} \right) + i \left(\frac{\omega_p^2 \tau^2}{\omega\tau + \omega^3 \tau^3} \right)\end{aligned}$$

where, $\omega_p^2 = \frac{\sigma_o}{\epsilon_o \tau} = \frac{ne^2}{\epsilon_o m_e}$: bulk plasma frequency ($\sim 10eV$ for metal)

Plasma frequency



Dielectric constant at $\omega \approx \omega_{\text{visible}}$

Since $\omega_{\text{vis}} \tau \gg 1$ 

$$\epsilon_{EFF} = \epsilon_B - \frac{\omega_p^2}{\omega^2} + i \frac{\omega_p^2}{\omega^3 \tau}$$

Bound electrons

Free electrons

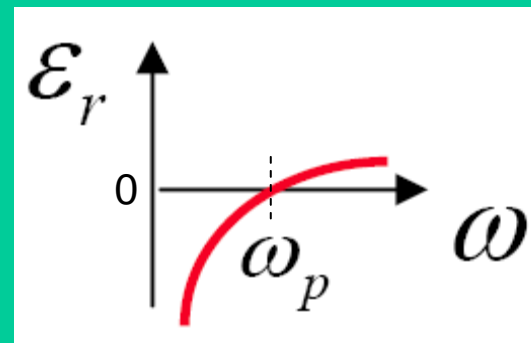
Ideal case : metal as a free-electron gas

Dielectric constant of a free electron gas

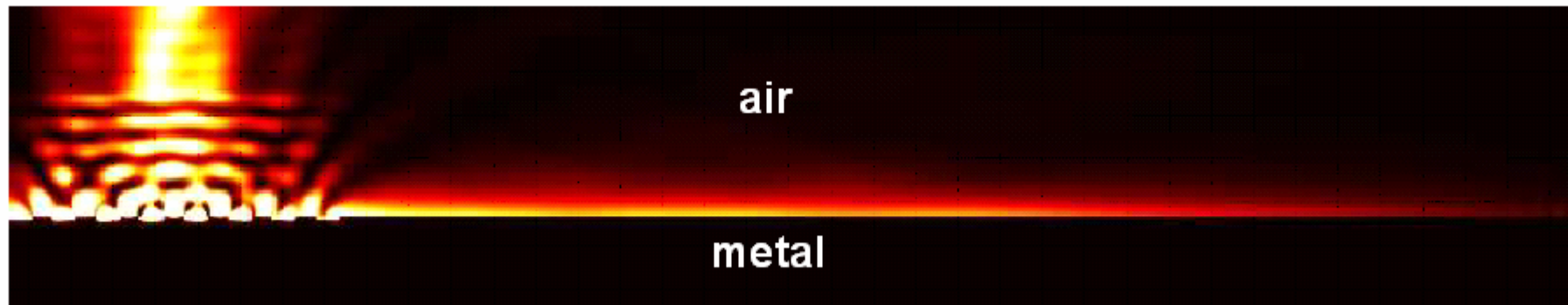
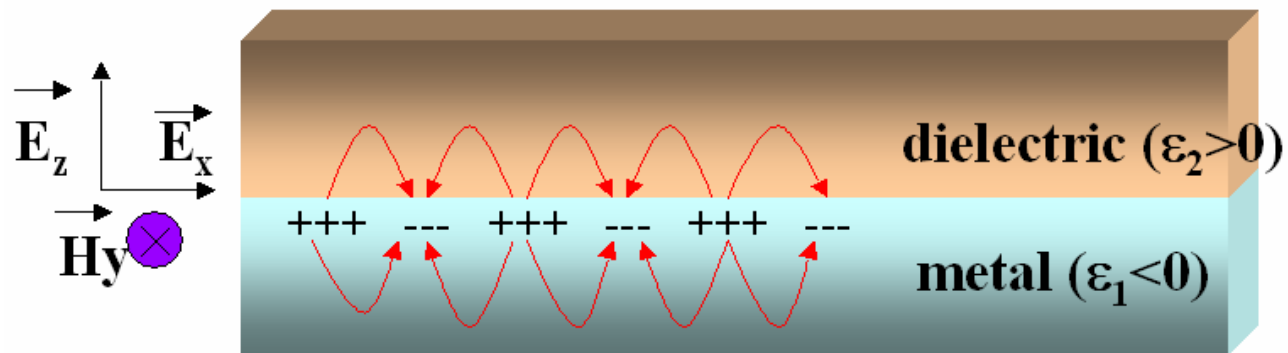
- no decay ($\gamma \rightarrow 0$: infinite relaxation time)
- no interband transitions ($\epsilon_B = 1$)

$$\begin{aligned}\epsilon_{EFF}(\omega) &= \epsilon_B + i \frac{\sigma(\omega)}{\epsilon_o \omega} \\ &= \left(\epsilon_B - \frac{\omega_p^2 \tau^2}{1 + \omega^2 \tau^2} \right) + i \left(\frac{\omega_p^2 \tau^2}{\omega \tau + \omega^3 \tau^3} \right) \xrightarrow[\epsilon_B=1]{\tau \rightarrow \infty} \epsilon_{EFF} = \left(1 - \frac{\omega_p^2}{\omega^2} \right)\end{aligned}$$

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2}$$



Surface-plasmons on dielectric-metal boundaries



Dispersion relation for EM waves in electron gas (**bulk plasmons**)

Determination of dispersion relation for bulk plasmons

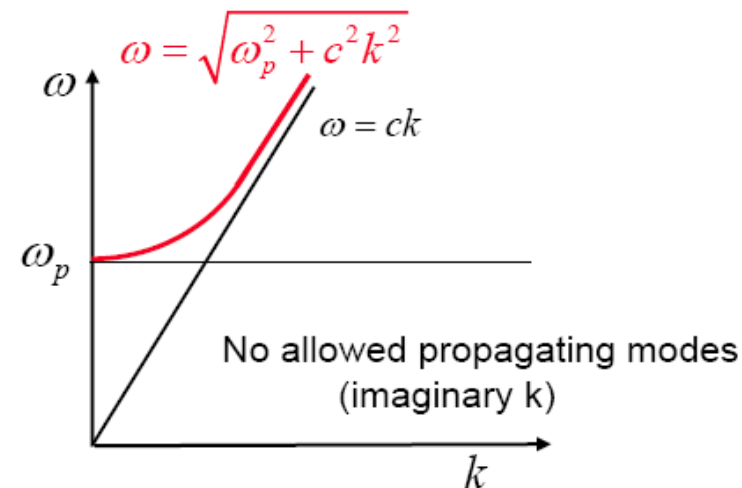
- The wave equation is given by:
- $$\frac{\epsilon_r}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = \nabla^2 \mathbf{E}(\mathbf{r}, t)$$

- Investigate solutions of the form:
- $$\mathbf{E}(\mathbf{r}, t) = \text{Re} \{ \mathbf{E}(\mathbf{r}, \omega) \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t) \}$$

- Dielectric constant: $\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2}$
- $$\left. \begin{array}{l} \omega^2 \epsilon_r = c^2 k^2 \\ \epsilon_r = 1 - \frac{\omega_p^2}{\omega^2} \end{array} \right\} \Rightarrow \omega^2 \left(1 - \frac{\omega_p^2}{\omega^2} \right) = \boxed{\omega^2 - \omega_p^2 = c^2 k^2}$$

- Dispersion relation:

$$\boxed{\omega = \omega(k)}$$



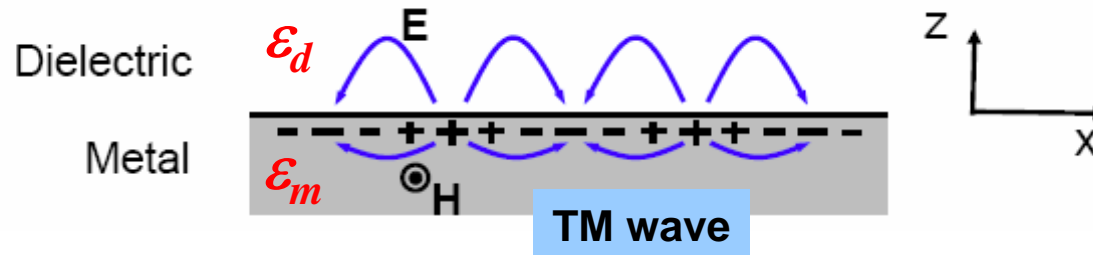
Note1: Solutions lie above light line

Note2: Metals: $\hbar\omega_p \approx 10$ eV; Semiconductors $\hbar\omega_p < 0.5$ eV (depending on dopant conc.)

Dispersion relation for **surface plasmons**

Solve Maxwell's equations with boundary conditions

- We are looking for solutions that look like:



- Mathematically:

$$\begin{aligned}
 z > 0 \quad & \begin{cases} H_d = (0, H_{yd}, 0) \exp i(k_{xd}x + k_{zd}z - \omega t) \\ E_d = (E_{xd}, 0, E_{zd}) \exp i(k_{xd}x + k_{zd}z - \omega t) \end{cases} \\
 z < 0 \quad & \begin{cases} H_m = (0, H_{ym}, 0) \exp i(k_{xm}x + k_{zm}z - \omega t) \\ E_m = (E_{xm}, 0, E_{zm}) \exp i(k_{xm}x + k_{zm}z - \omega t) \end{cases}
 \end{aligned}$$

- Maxwell's Equations in medium i (i = metal or dielectric):

$$\nabla \cdot \epsilon_i \mathbf{E} = 0 \quad \nabla \cdot \mathbf{H} = 0 \quad \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \quad \nabla \times \mathbf{H} = \epsilon_i \frac{\partial \mathbf{E}}{\partial t}$$

- At the boundary (continuity of the tangential E_x , H_y , and the normal D_z):

$$E_{xm} = E_{xd} \quad H_{ym} = H_{yd} \quad \epsilon_m E_{zm} = \epsilon_d E_{zd}$$

Dispersion relation for surface plasmon polaritons

- Start with curl equation for \mathbf{H} in medium i

$$\left. \begin{aligned} \nabla \times \mathbf{H}_i &= \epsilon_i \frac{\partial \mathbf{E}_i}{\partial t} \\ \text{where } \mathbf{H}_i &= (0, H_{yi}, 0) \exp i(k_{xi}x + k_{zi}z - \omega t) \\ \mathbf{E}_i &= (E_{xi}, 0, E_{zi}) \exp i(k_{xi}x + k_{zi}z - \omega t) \end{aligned} \right\} \Rightarrow$$

$$\left(\frac{\partial H_{zi}}{\partial y} - \frac{\partial H_{yi}}{\partial z}, \frac{\partial H_{xi}}{\partial z} - \frac{\partial H_{zi}}{\partial x}, \frac{\partial H_{yi}}{\partial x} - \frac{\partial H_{xi}}{\partial y} \right) = (\underline{-ik_{zi}H_{yi}}, 0, \underline{ik_{xi}H_{yi}}) = (\underline{-i\omega\epsilon_i E_{xi}}, 0, \underline{-i\omega\epsilon_i E_{zi}})$$

$$k_{zi}H_{yi} = \omega\epsilon_i E_{xi} \Rightarrow \left\{ \begin{aligned} k_{zm}H_{ym} &= \omega\epsilon_m E_{xm} \\ k_{zd}H_{yd} &= \omega\epsilon_d E_{xd} \end{aligned} \right\} \Rightarrow \frac{k_{zm}}{\epsilon_m} H_{ym} = \frac{k_{zd}}{\epsilon_d} H_{yd}$$

- E_{\parallel} across boundary is continuous: $E_{xm} = E_{xd}$

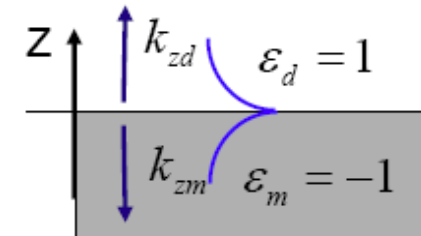
- H_{\parallel} across boundary is continuous: $H_{ym} = H_{yd}$

$$\left. \begin{aligned} \text{Combine with: } \frac{k_{zm}}{\epsilon_m} H_{ym} &= \frac{k_{zd}}{\epsilon_d} H_{yd} \end{aligned} \right\} \Rightarrow \frac{k_{zm}}{\epsilon_m} = \frac{k_{zd}}{\epsilon_d}$$

Dispersion relation for surface plasmon polaritons

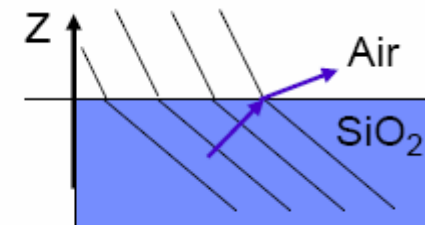
Relations between k vectors

- Condition for SP's to exist: $\frac{k_{zm}}{\epsilon_m} = \frac{k_{zd}}{\epsilon_d}$ Example



- Relation for k_x (Continuity $E_{||}$, $H_{||}$): $k_{xm} = k_{xd}$
true at any boundary

Example

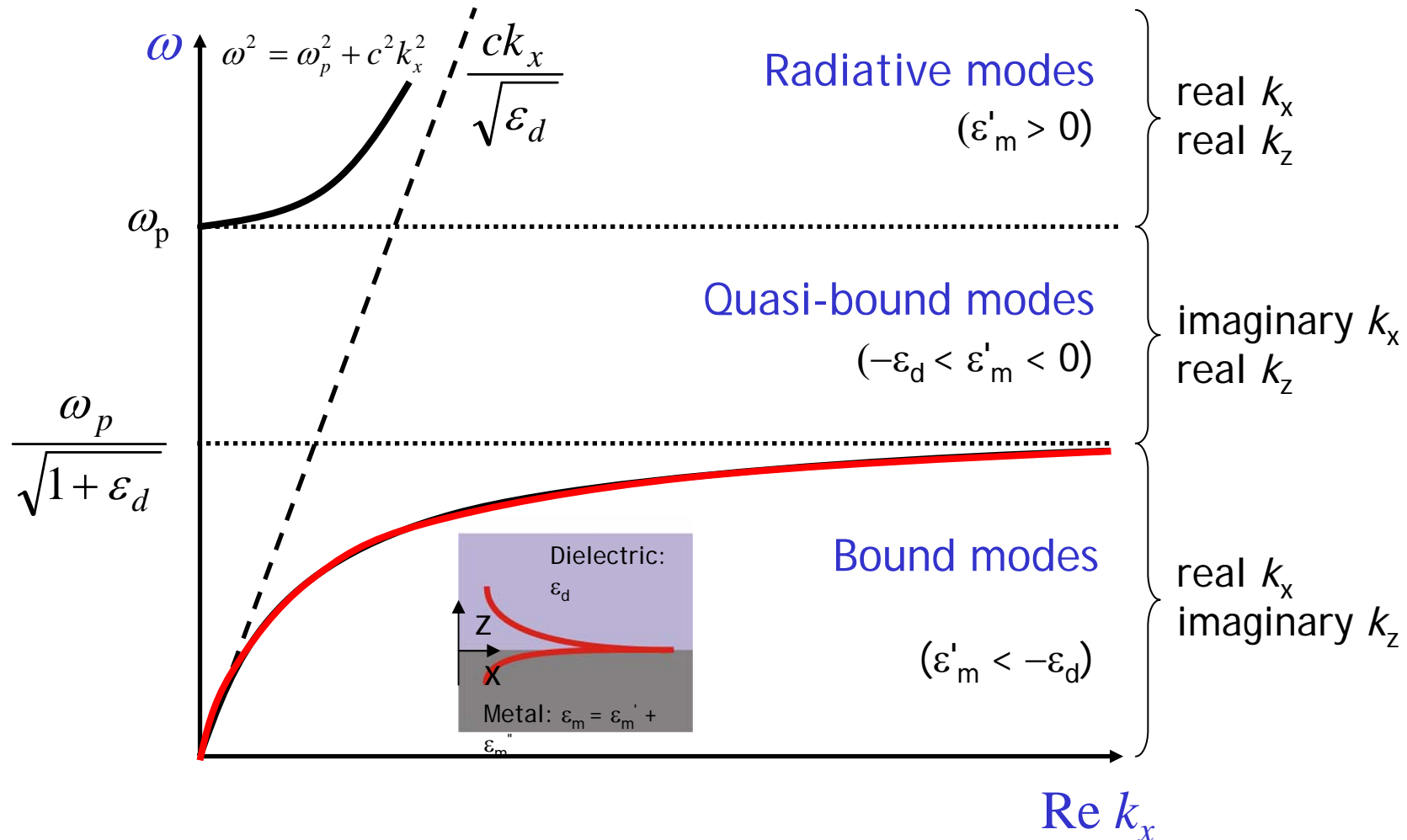


- For any EM wave: $k^2 = \epsilon_i \left(\frac{\omega}{c} \right)^2 = k_x^2 + k_{zi}^2$, where $k_x \equiv k_{xm} = k_{xd}$

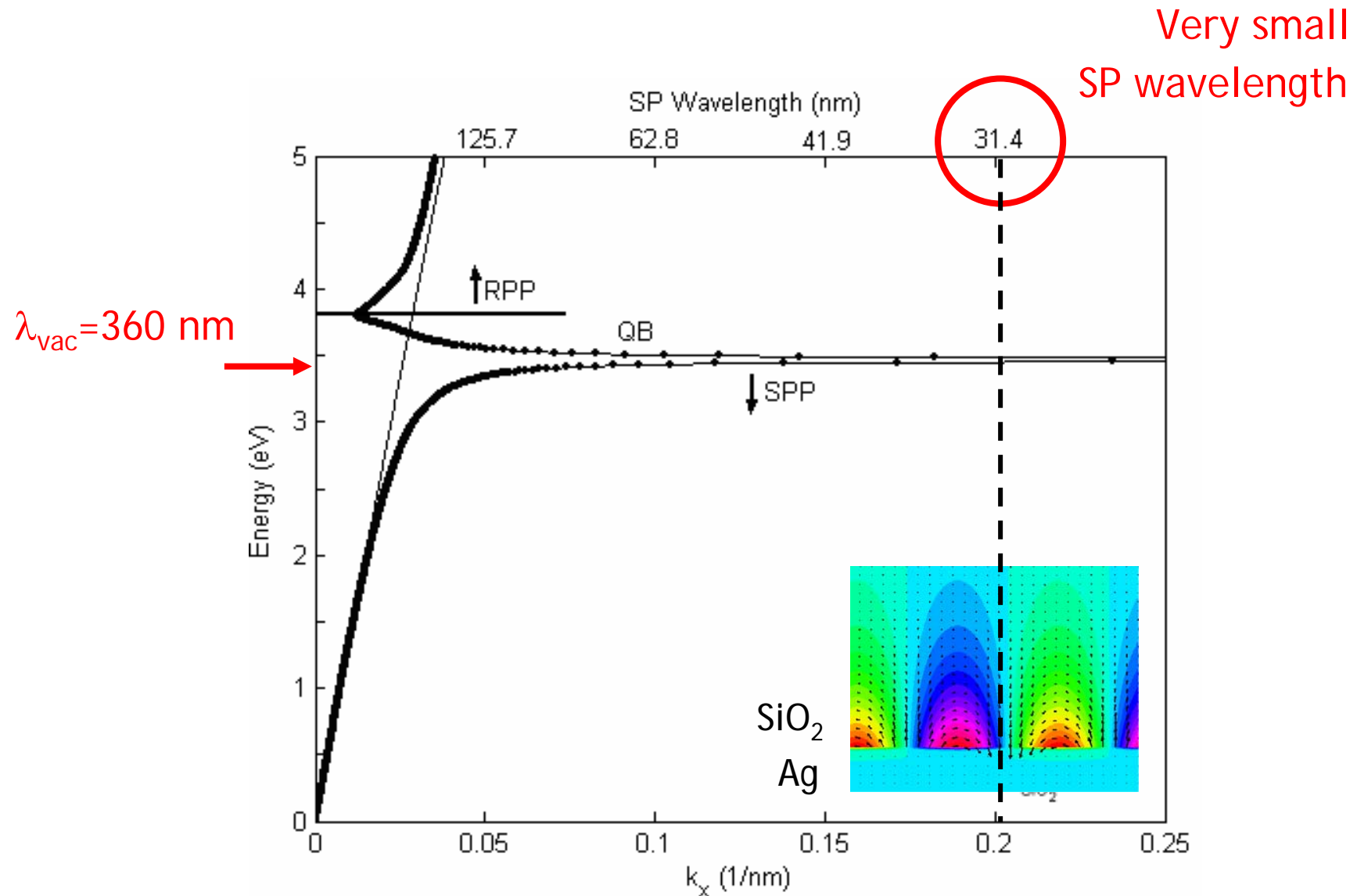
- Both in the metal and dielectric: $k_{sp} = k_x = \sqrt{\epsilon_i \left(\frac{\omega}{c} \right)^2 - k_{zi}^2}$
 $\frac{k_{zm}}{\epsilon_m} = \frac{k_{zd}}{\epsilon_d}$ } **SP Dispersion Relation**
 $k_x = \frac{\omega}{c} \sqrt{\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d}}$

Surface plasmon dispersion relation

$$k_x = \frac{\omega}{c} \left(\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d} \right)^{1/2} \quad k_{zi} = \frac{\omega}{c} \left(\frac{\epsilon_i^2}{\epsilon_m + \epsilon_d} \right)^{1/2}$$



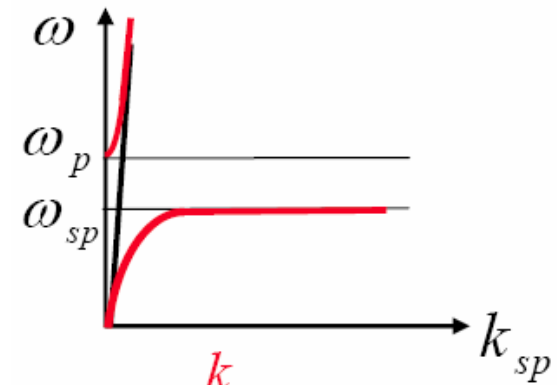
X-ray wavelengths at optical frequencies !!



Excitation of surface plasmons

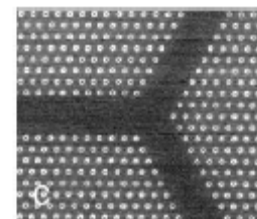
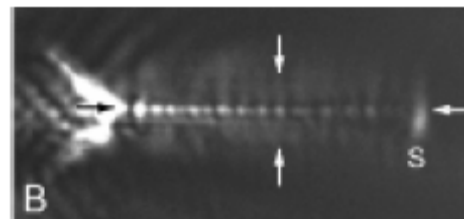
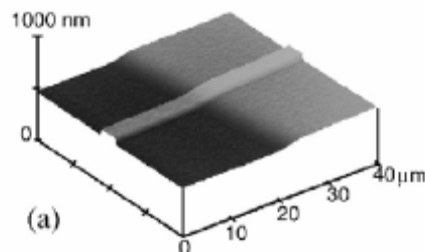
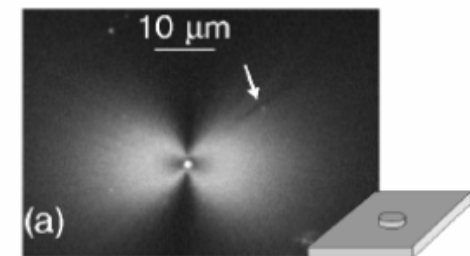
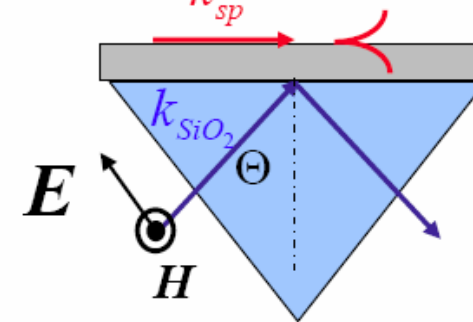
The dispersion relation for surface plasmons

- Useful for describing plasmon excitation & propagation



Coupling light to surface plasmon-polaritons

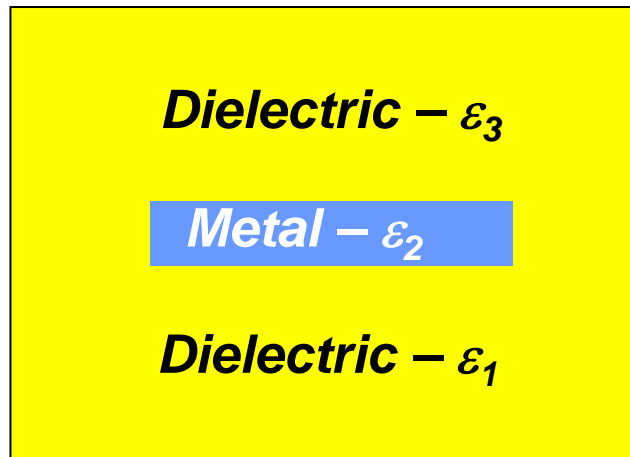
- Using high energy electrons (EELS)
- Kretschman geometry $k_{//,SiO_2} = \sqrt{\epsilon_d} \frac{\omega}{c} \sin \theta = k_{sp}$
- Grating coupling
- Coupling using subwavelength features
- A diversity of guiding geometries



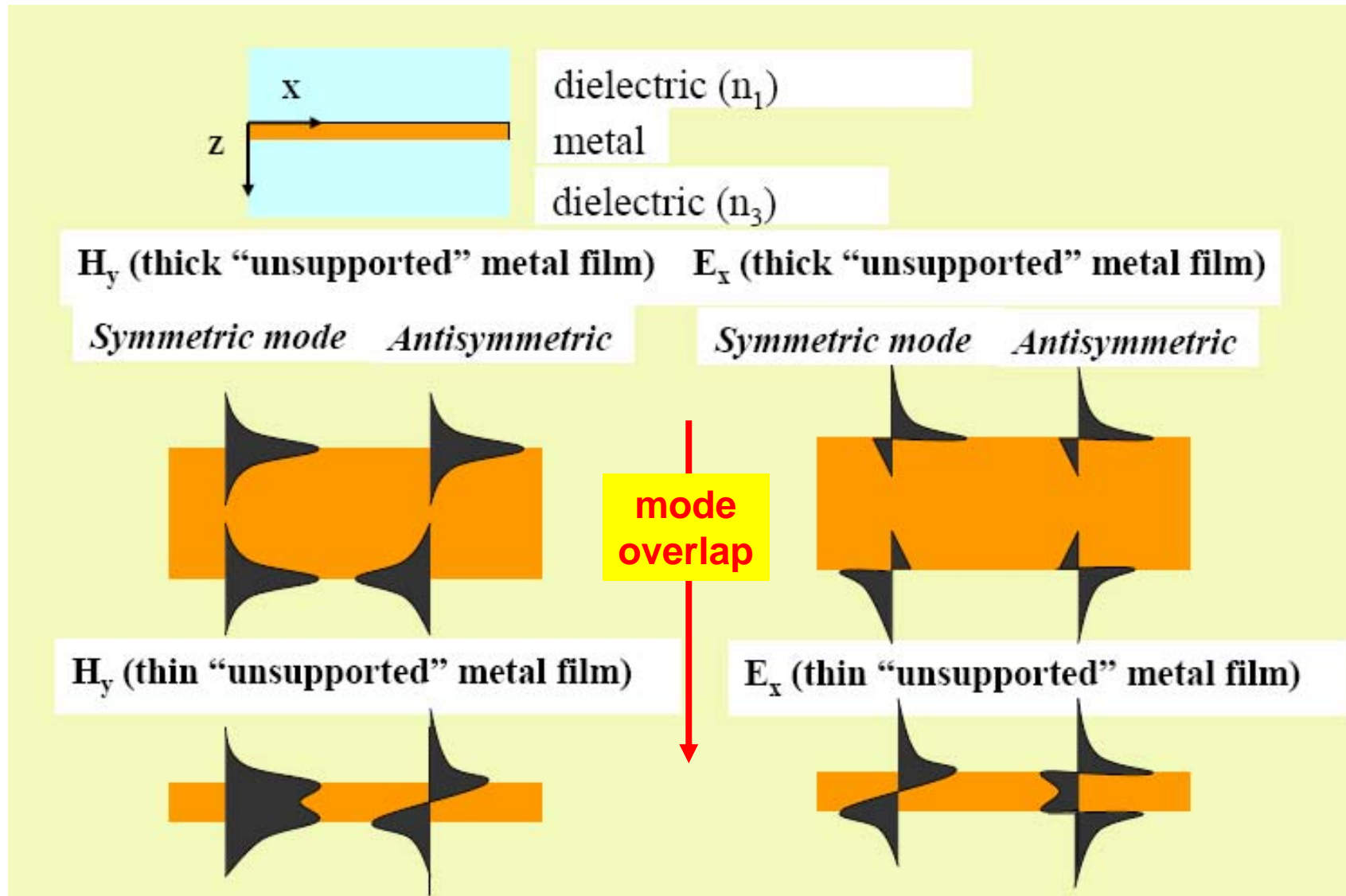
Surface-plasmon-polariton waveguides

Surface plasmon polaritons

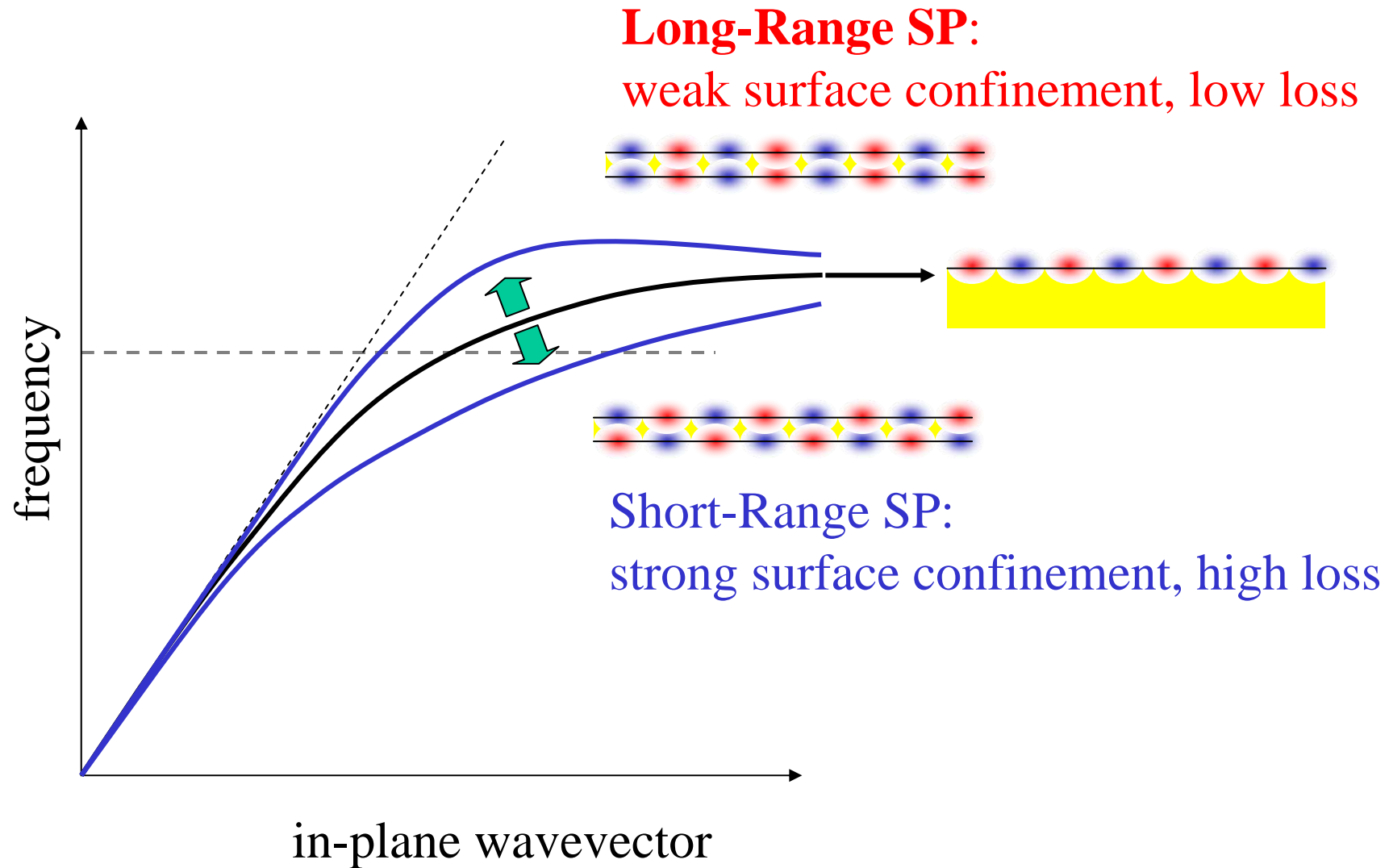
propagating along very-thin(~ 10 nm) metal strips



When the film thickness becomes finite.



Possibility of Propagation Range Extension



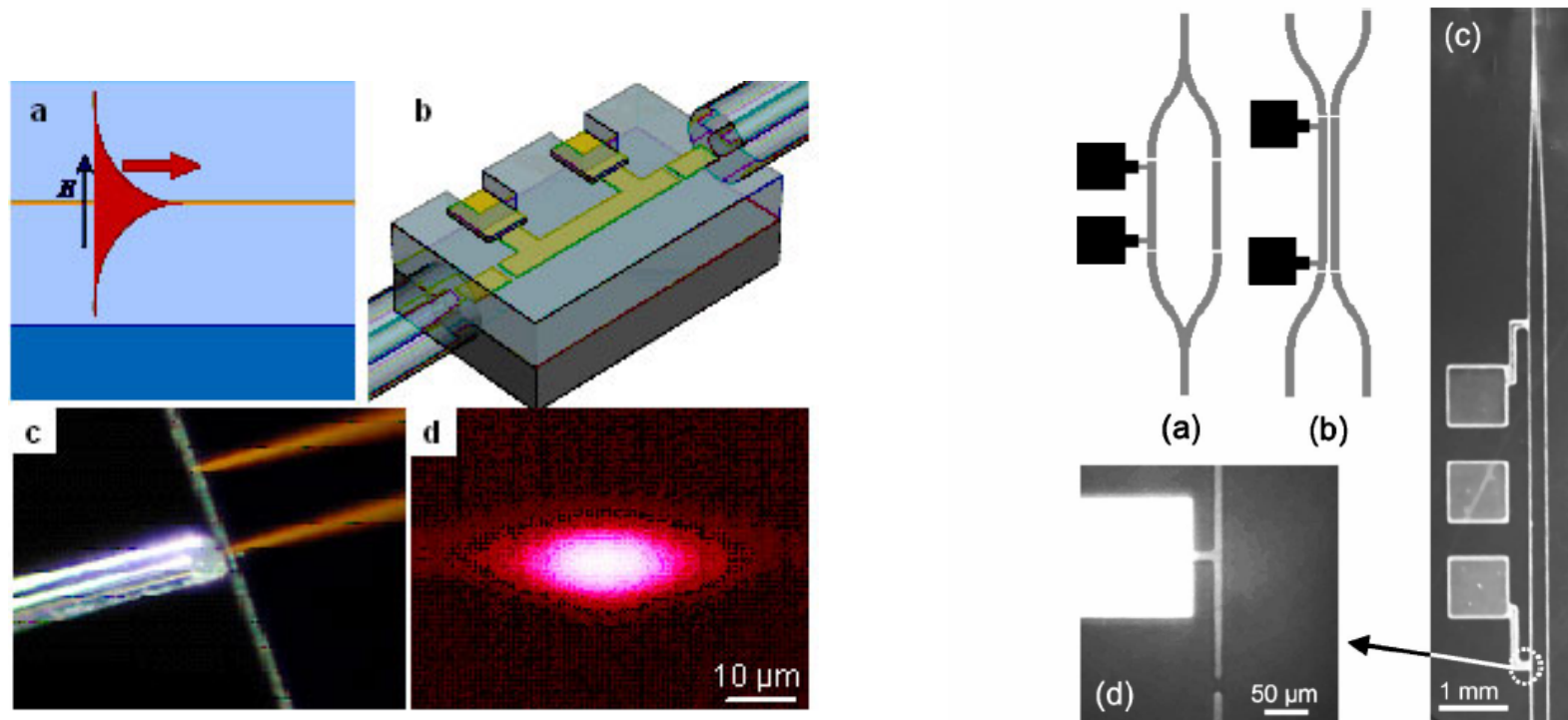
Thomas Nikolajsen ^a, Kristjan Leosson ^a, Sergey I. Bozhevolnyi ^{a,b,*}

^a *Micro Managed Photons A/S, Ryttermarken 15, DK-3520 Farum, Denmark*

Optics Communications 244 (2005) 455–459

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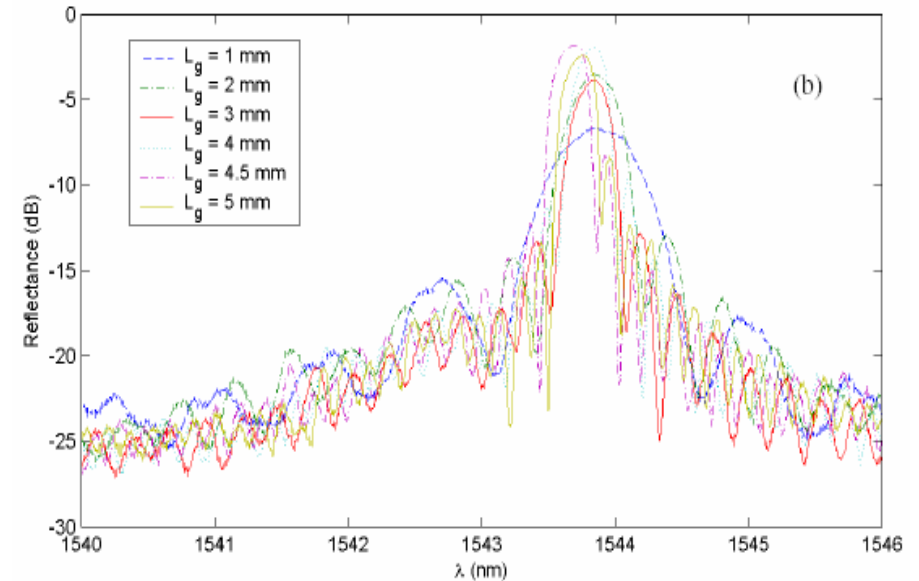
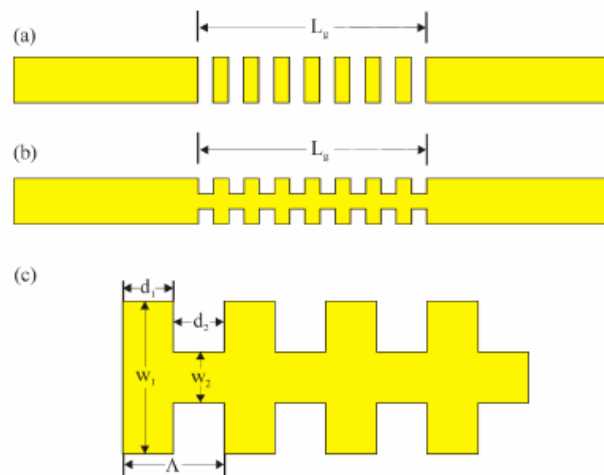
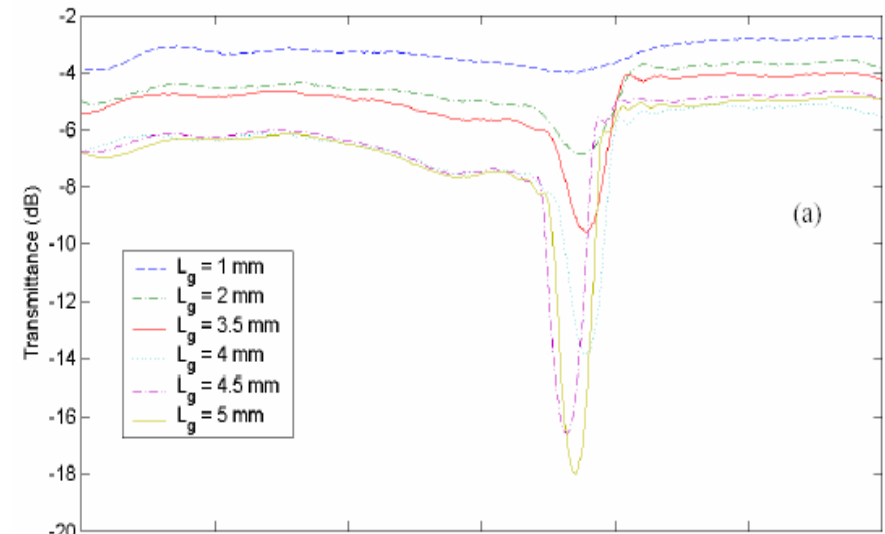
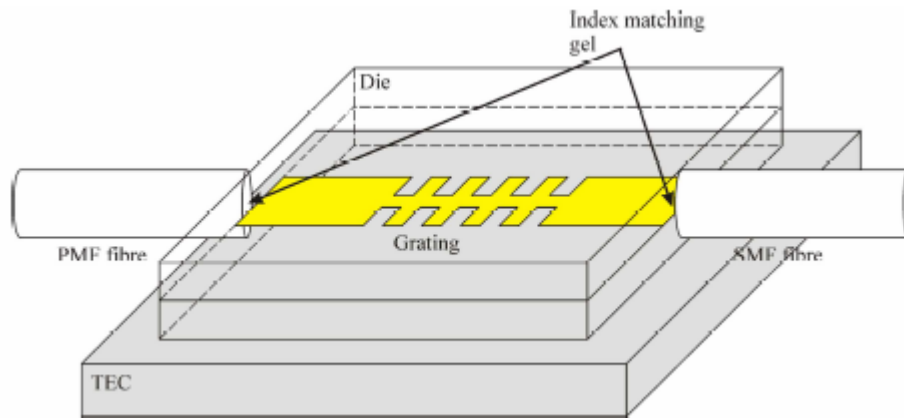
Appl. Phys. Lett., Vol. 85, No. 24, 13 December 2004



Several 1 cm long, 15 nm thin and 8 micron wide gold stripes guiding LRSPPs
3-6 mm long control electrodes
low driving powers (approx. 100 mW) and high extinction ratios (approx. 30 dB)
response times (approx. 0.5 ms)
total (fiber-to-fiber) insertion loss of approx. 8 dB when using single-mode fibers

Demonstration of Bragg gratings based on long-ranging surface plasmon polariton waveguides

Pierre Berini 13 June 2005 / Vol. 13, No. 12 / OPTICS EXPRESS 4674

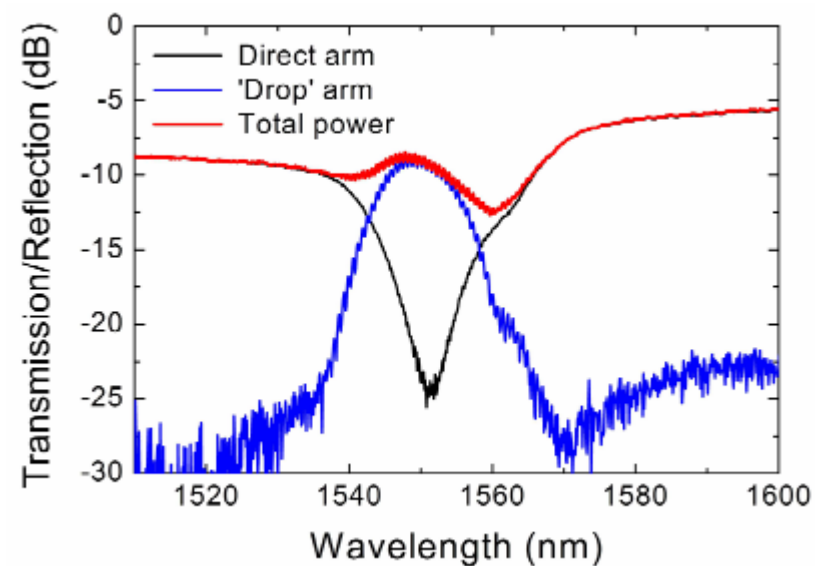
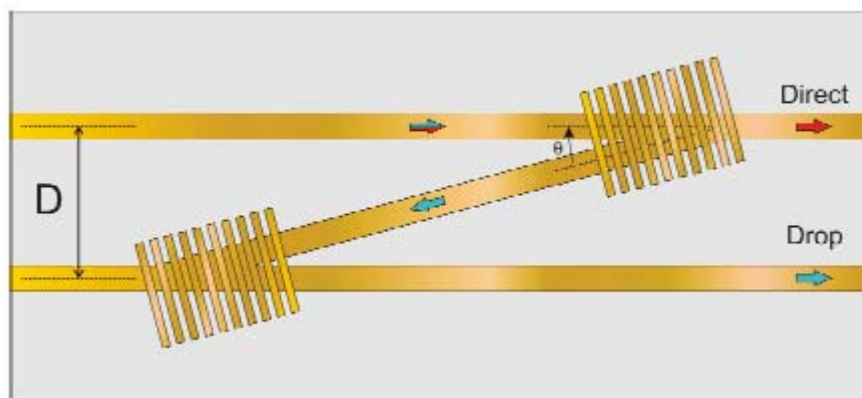
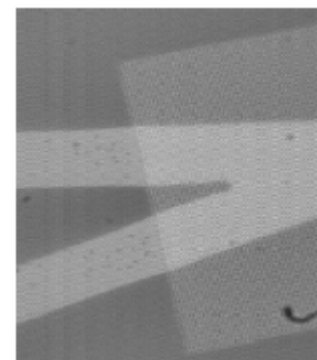
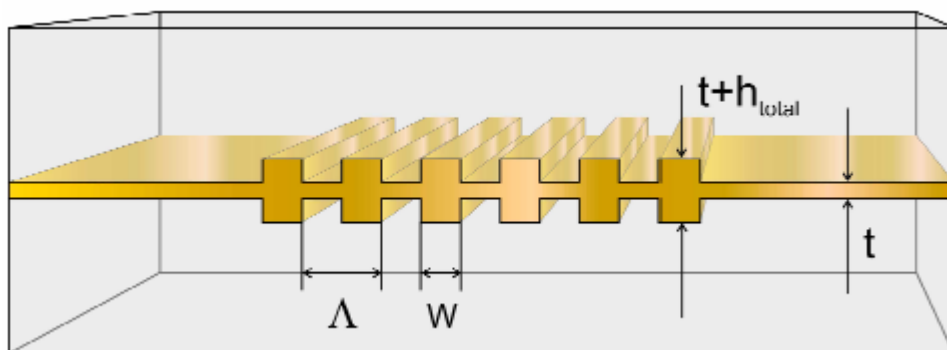


Compact Z-add-drop wavelength filters for long-range surface plasmon polaritons

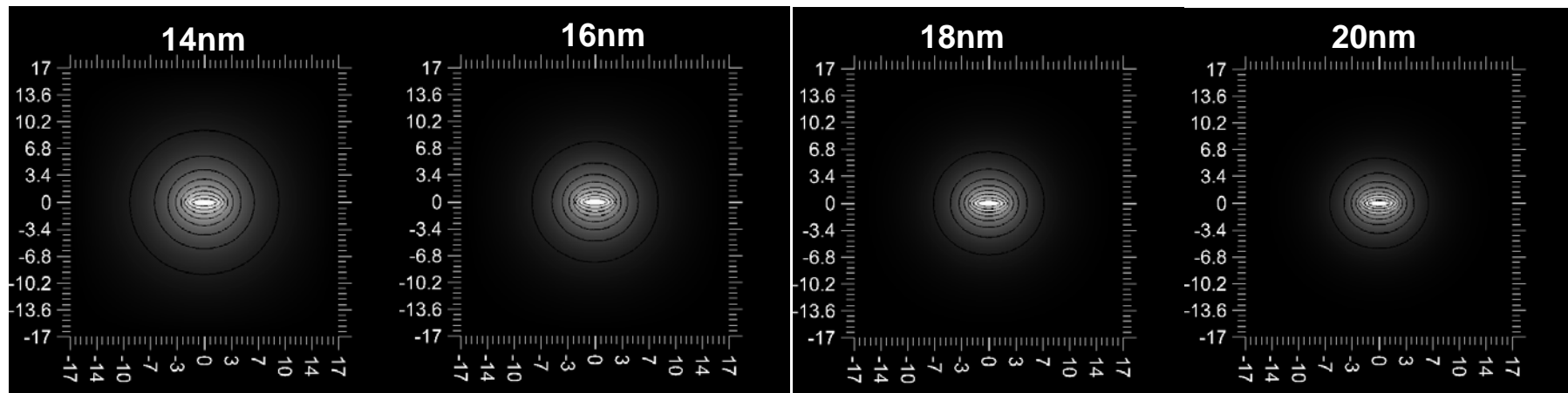
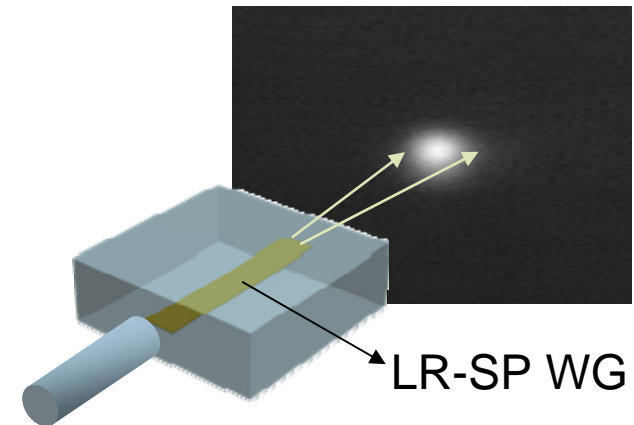
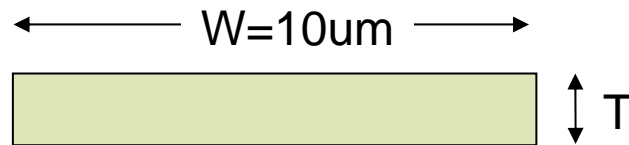
Alexandra Boltasseva

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30 May 2005 / Vol. 13, No. 11 / OPTICS EXPRESS 4237



Fundamental symmetric mode of a metal stripe : thickness (T)



Metal	Resistivity ($\mu\Omega\text{-cm}$)	Real (ϵ)	Imaginary (ϵ)	Attenuation (dB/mm)
Al	2.83	-253.93	-46.08	0.89
Au	2.35	-131.95	-12.65	0.41
Ag	1.62	-86.64	-3.1	0.11

Table 1 : Resistivity and dielectric constant values for Al, Au, and Ag, along with theoretical attenuation at 1550 nm for a 4 μm wide waveguide of thickness of 20 nm embedded in SiO_2 .

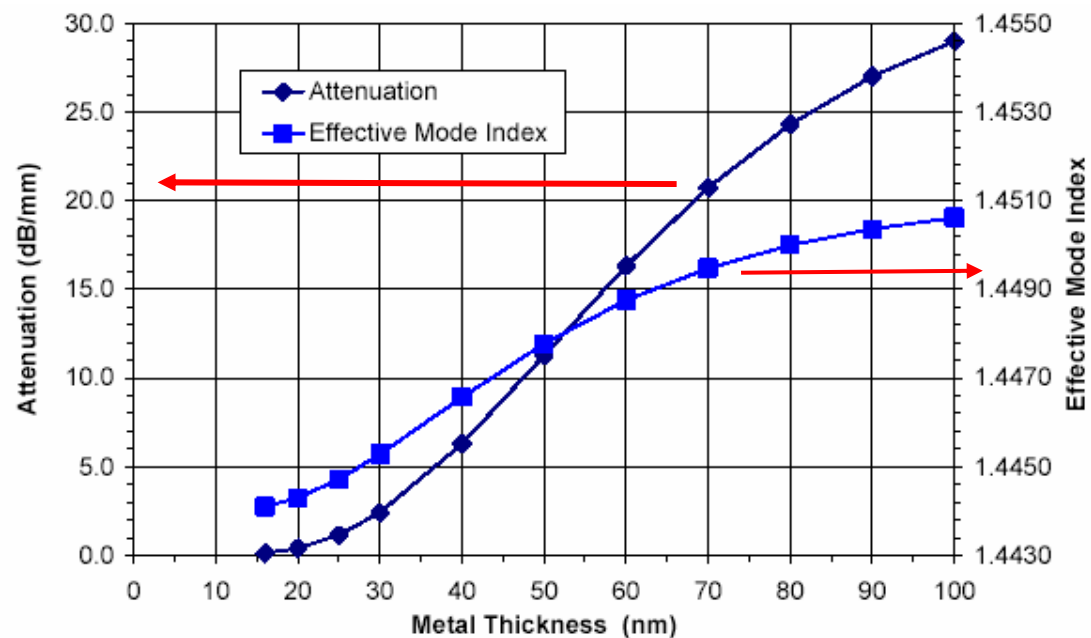
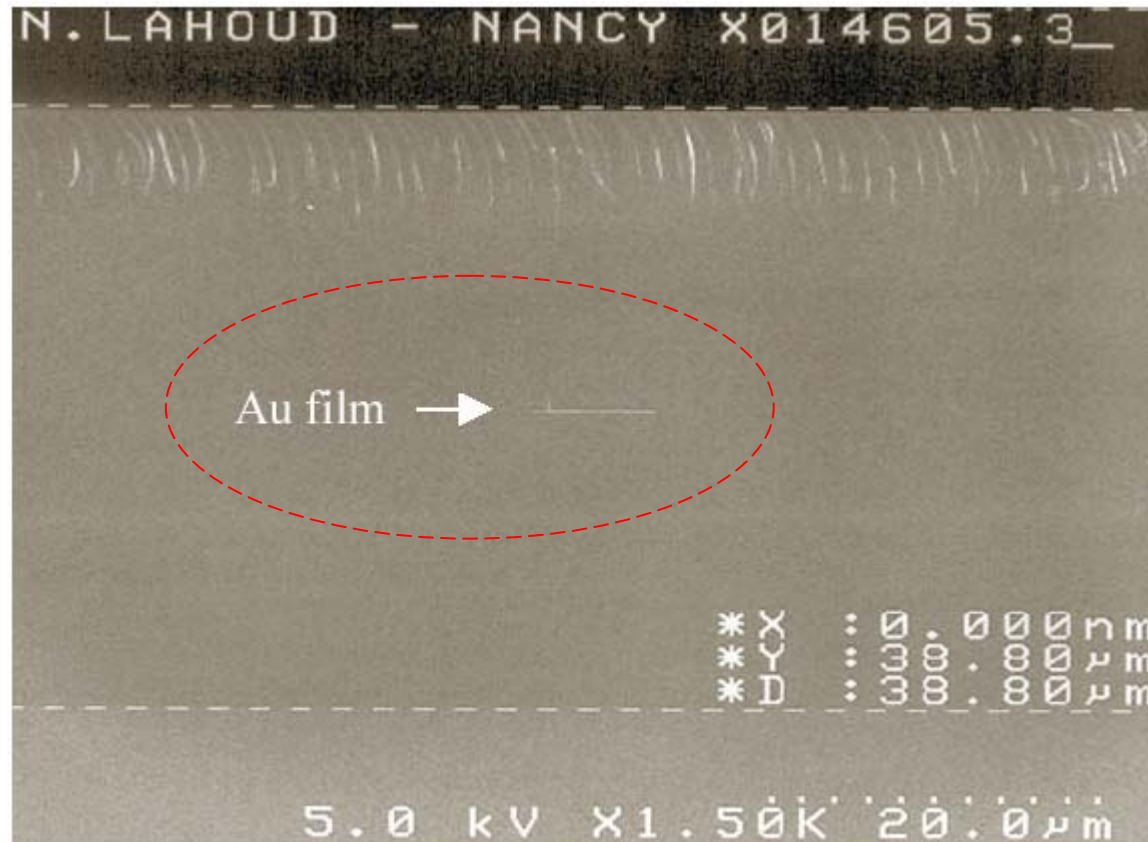


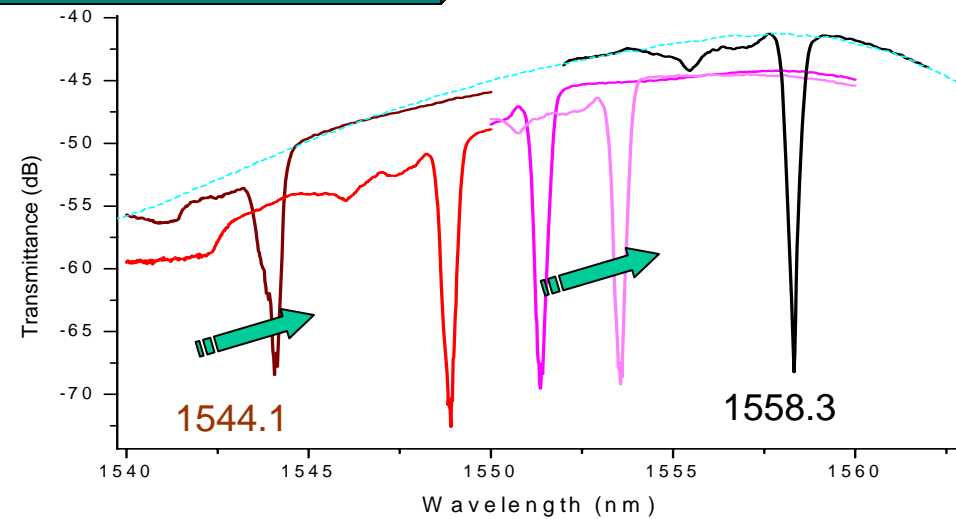
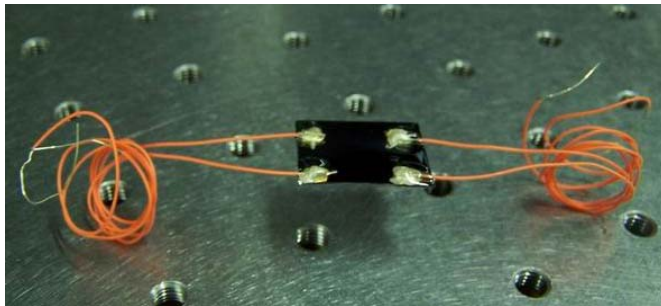
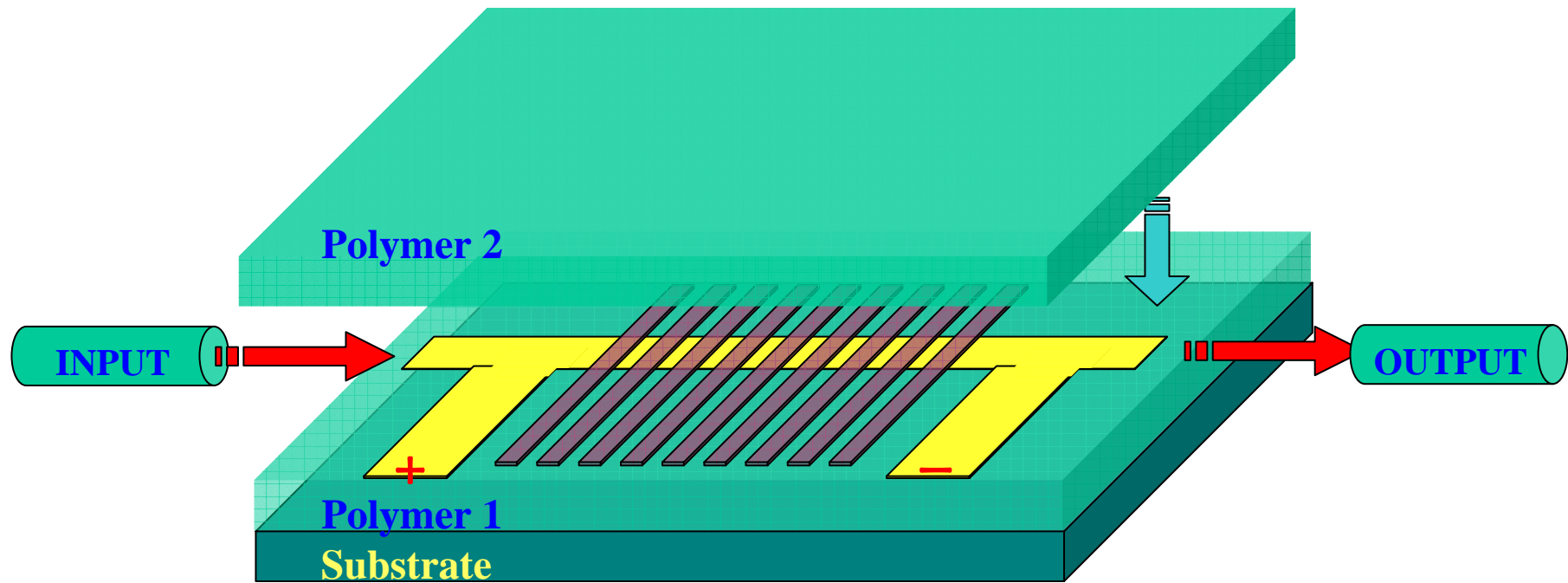
Figure 2 - Mode attenuation and effective index as a function of metal thickness for a 4 μm wide gold MWG cladded with oxide. The data presented in Figure 1 is a subset of the data presented here.



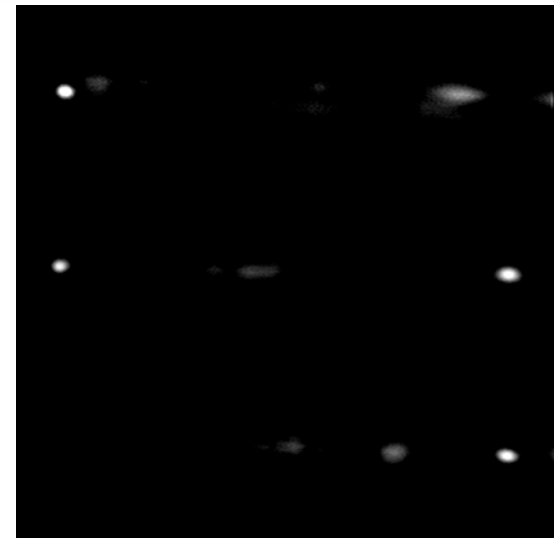
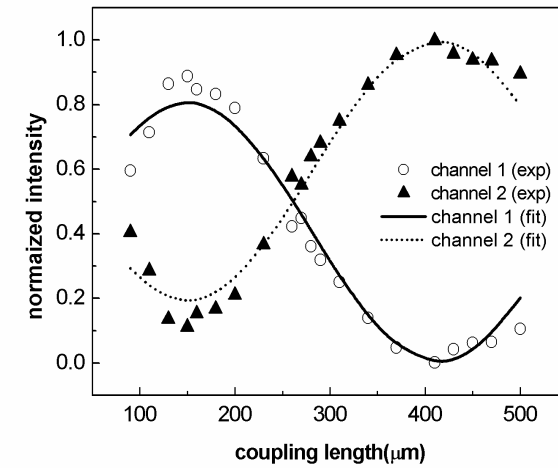
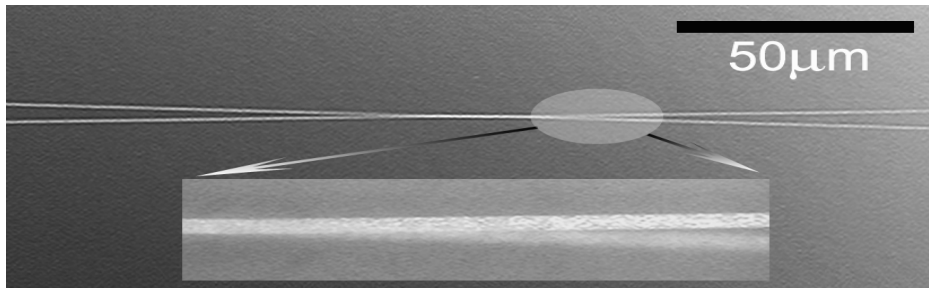
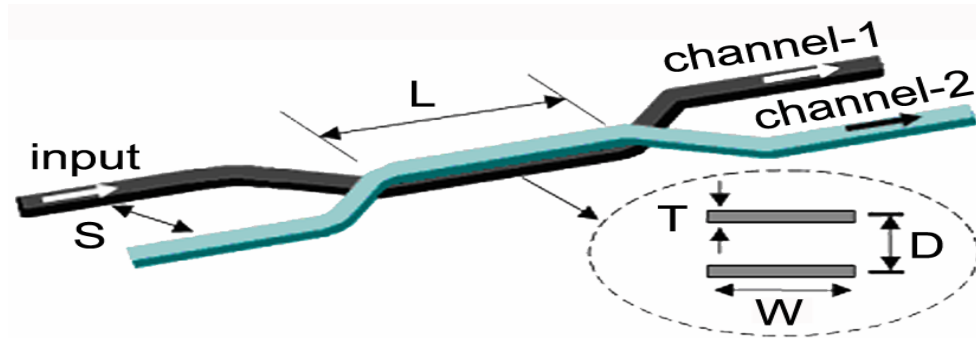
Cross-sectional FESEM image of a fabricated metal waveguide showing a thin Au film (bright horizontal line) embedded in about 40 μm of SiO₂.

(Spectalis Co.)

Tunable Wavelength filter by direct heating a metal wire



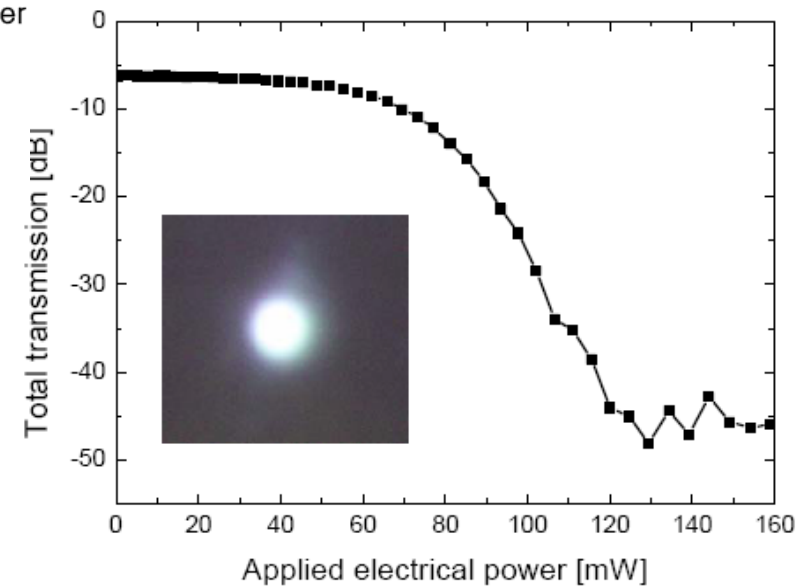
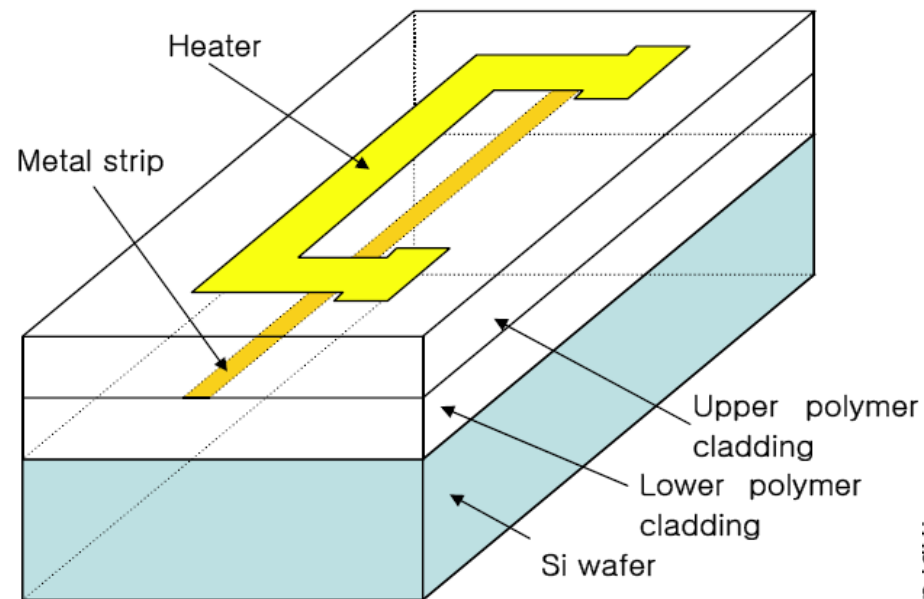
Vertical directional couplers



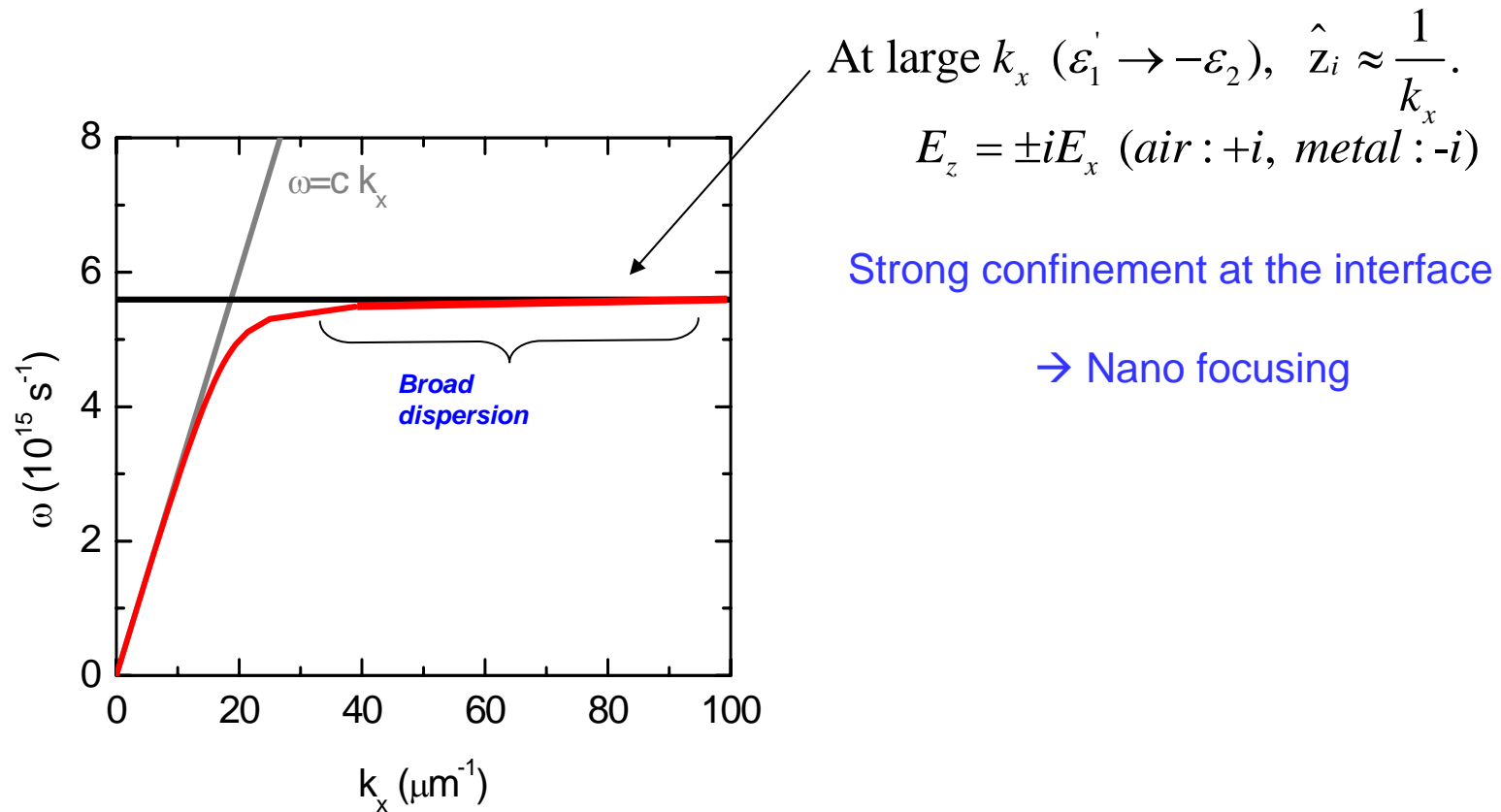
H. Won, APL vol.88, 011110 (2006)

Variable optical attenuator based on LR-SPP

Submitted to EL, S. Park & S. Song

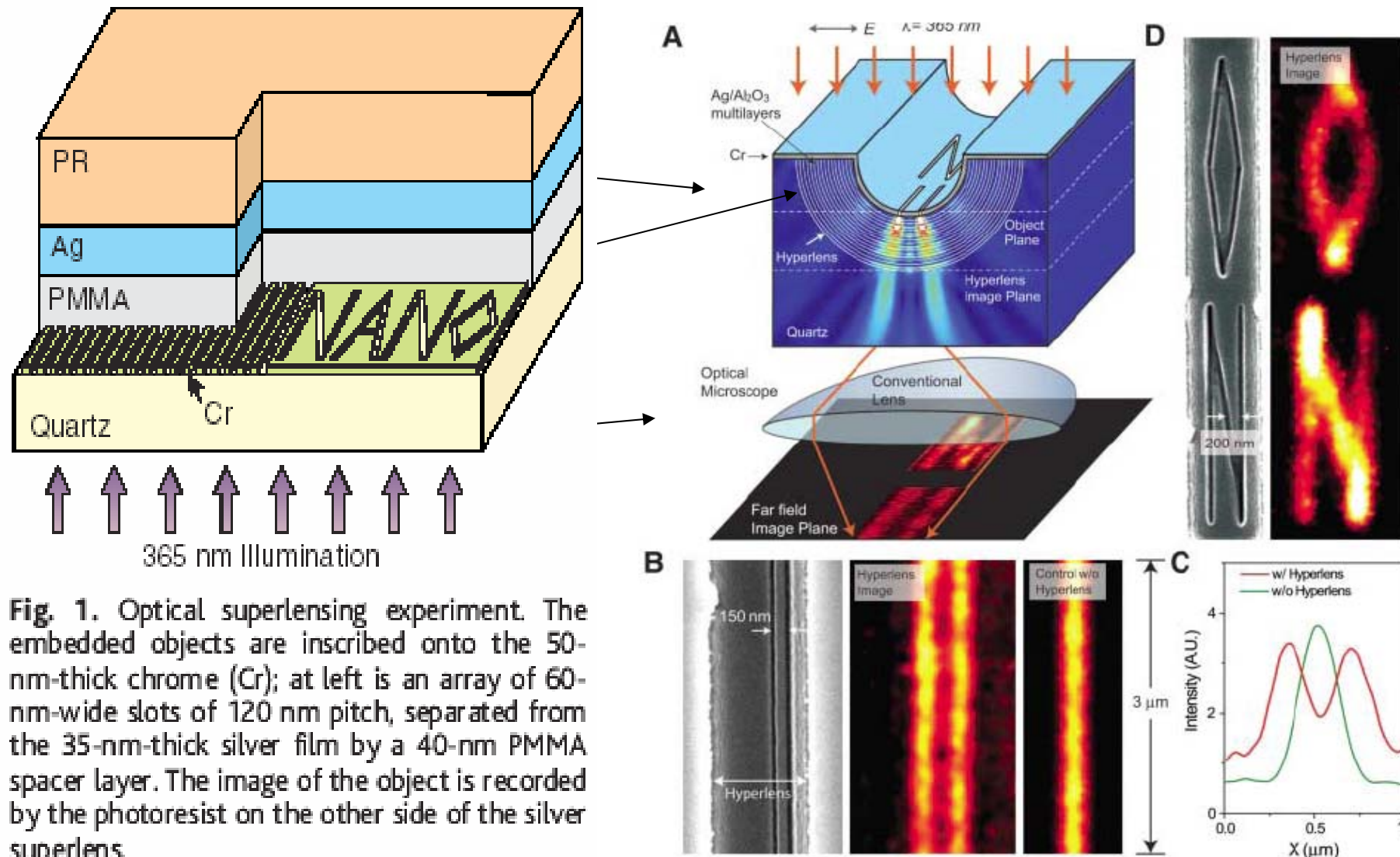


Localized Surface Plasmons : Nanofocusing and Nanolithography



Far-Field Optical Hyperlens Magnifying Sub-Diffraction-Limited Objects

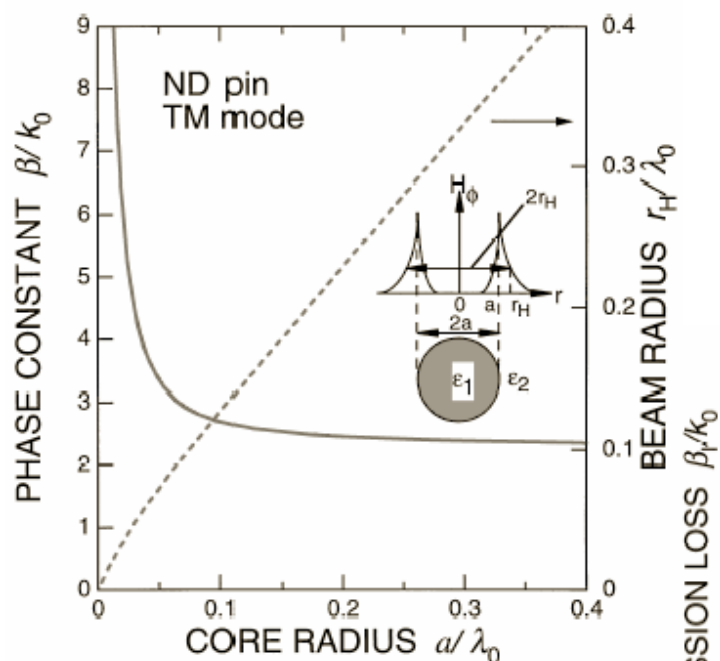
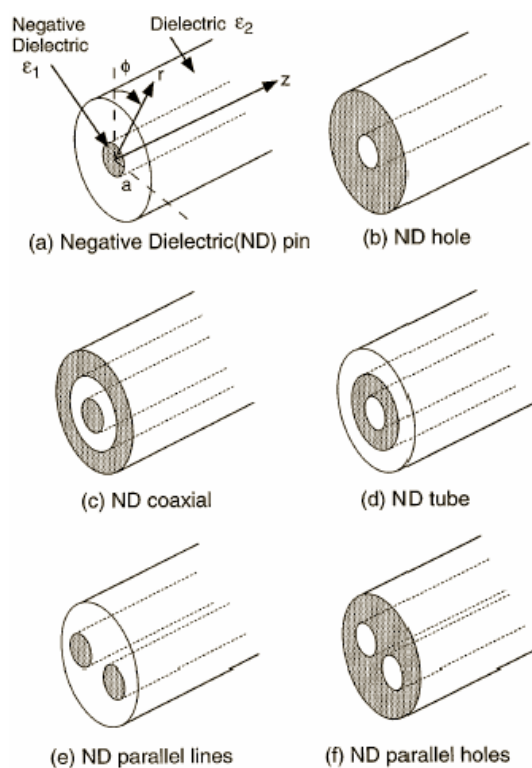
Zhaowei Liu,* Hyesog Lee,* Yi Xiong, Cheng Sun, Xiang Zhang†



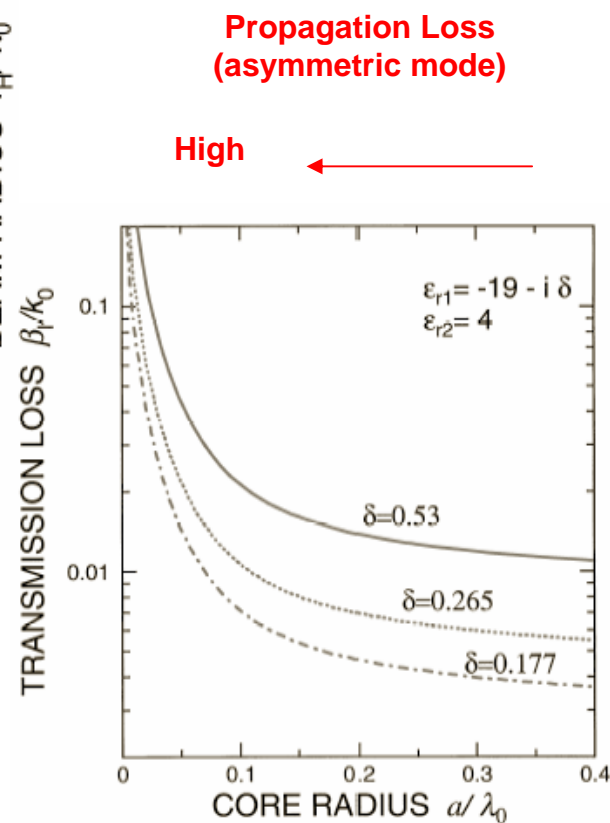
Guiding of a one-dimensional optical beam with nanometer diameter

Junichi Takahara, Suguru Yamagishi, Hiroaki Taki, Akihiro Morimoto, and Tetsuro Kobayashi

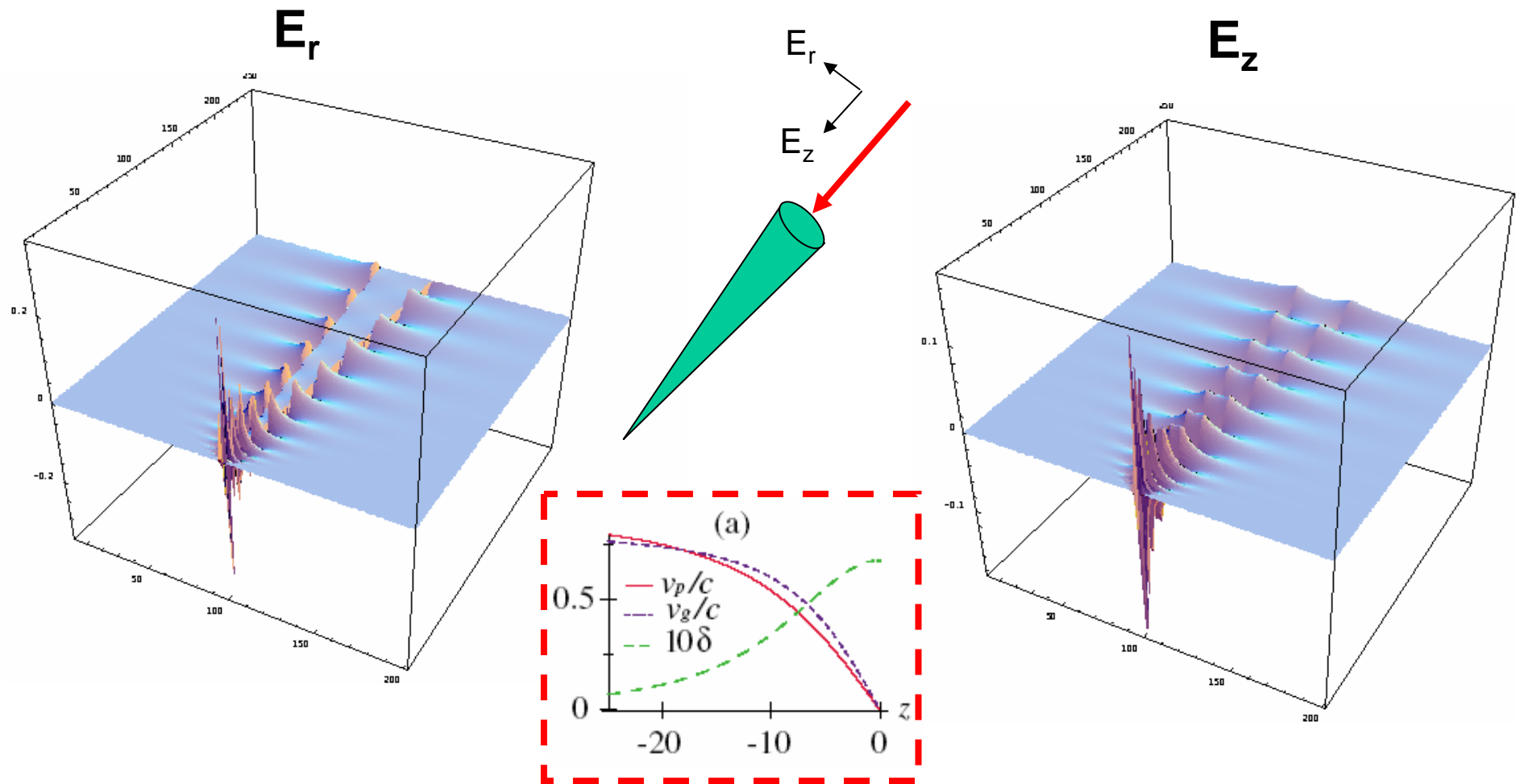
Department of Electrical Engineering, Faculty of Engineering Science, Osaka University, Toyonaka, Osaka 560, Japan



Beam radius \rightarrow zero!



Asymmetric mode : field enhancement at a metallic tip

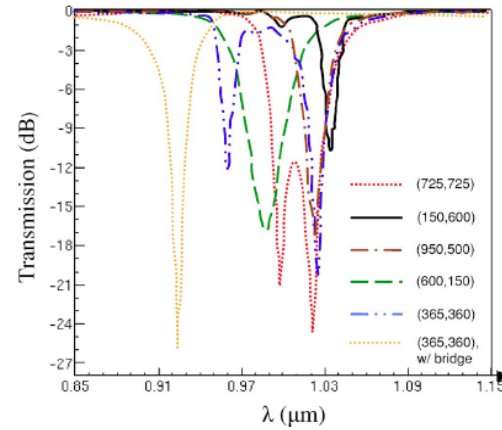
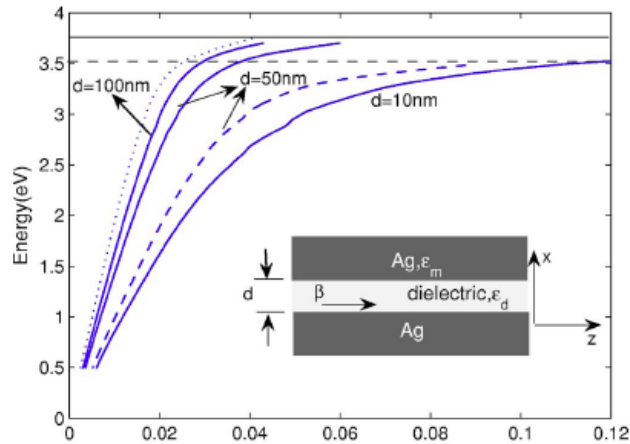


Nanoscale surface plasmon based resonator using rectangular geometry

Amir Hosseini and Yehia Massoud^{a)} *Rice University, Houston*

an optical range resonator based on single mode metal-insulator-metal plasmonic gap waveguides.
A small bridge between the resonator and the input waveguide can be used to tune the resonance frequency.

FDTD with the perfectly matched layer boundary conditions



The transmission ($T=|b_2/a_1|^2$) through the input waveguide is given by

$$T = \frac{\alpha^2 + |t|^2 - 2\alpha|t|\cos(\theta + \phi)}{1 + \alpha^2|t| - \alpha|t|\cos(\theta + \phi)}, \quad (1)$$

where α and $|t|$ represent the ring internal attenuation and the coupling coefficient, respectively, i.e., $\alpha=1$ means no internal loss, and $\theta=(2\pi/\lambda_g)L$.¹⁰ At resonance ($\phi+\theta=2N\pi$),

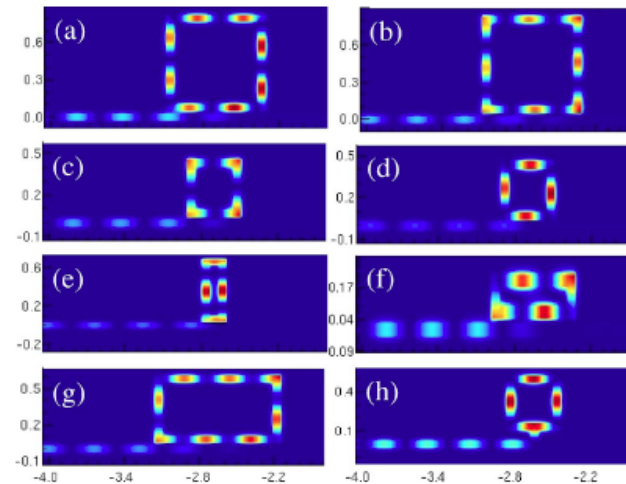
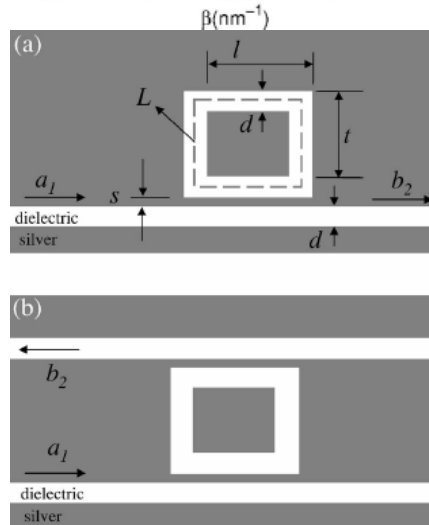
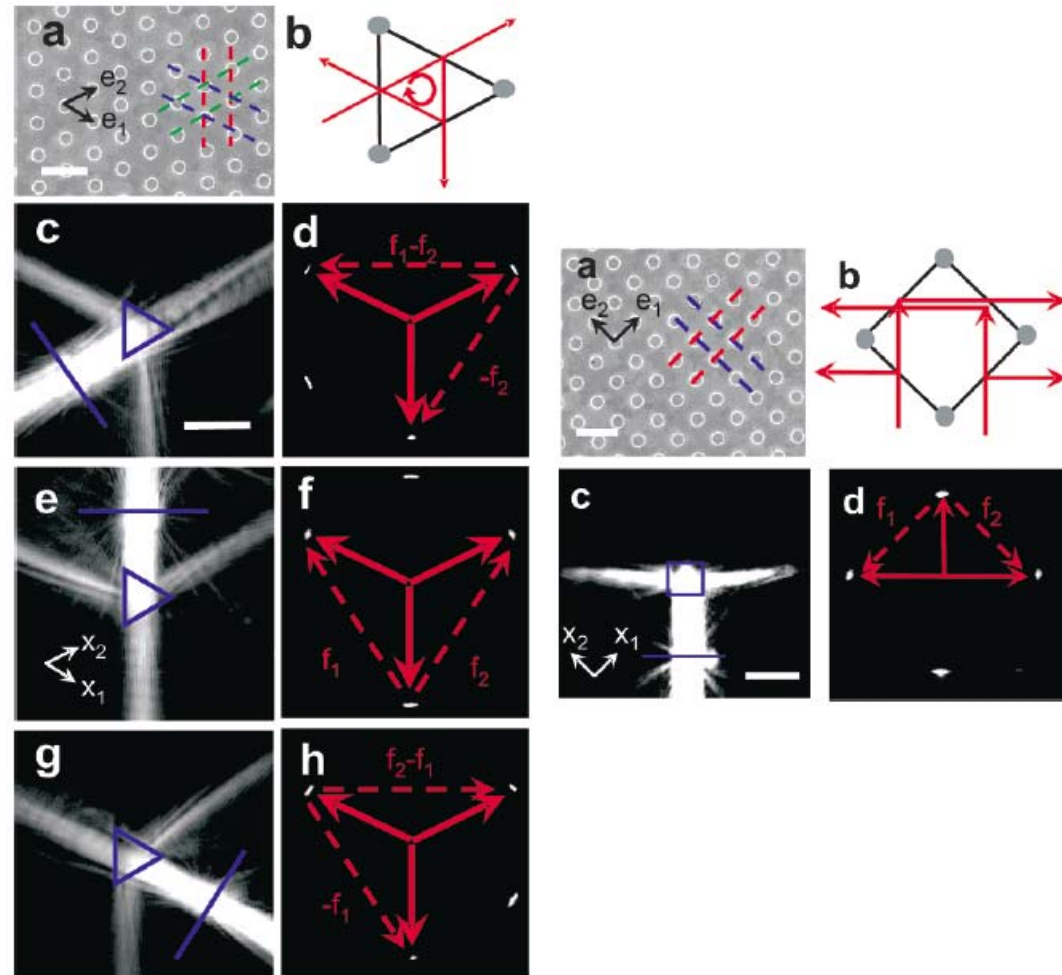
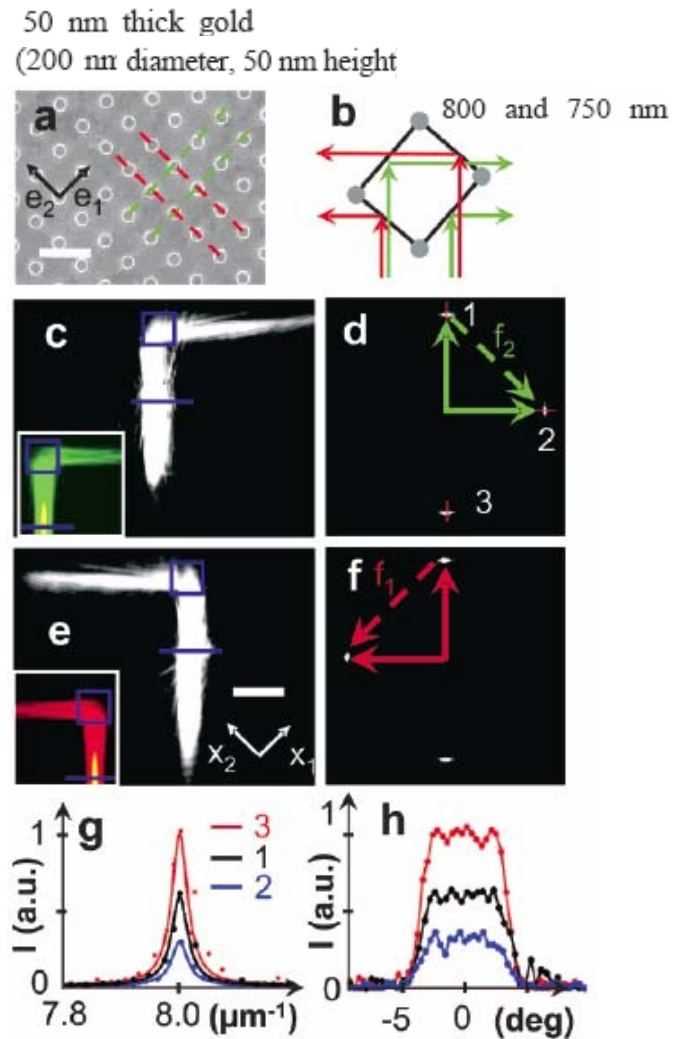


FIG. 4. (Color online) Field profile ($|H_z|^2$) for different (l, t) values in nm:

Plasmonic Crystal Demultiplexer and Multiports

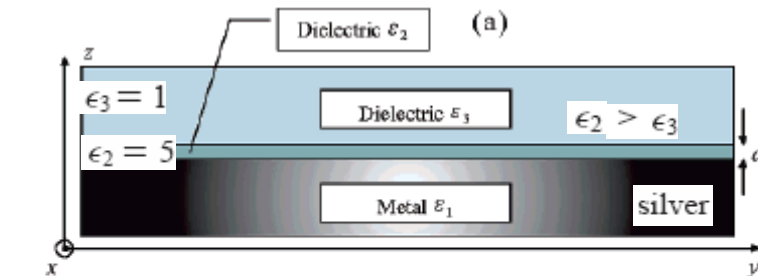
A. Drezet,* Daniel Koller, Andreas Hohenau, Alfred Leitner, Franz R. Aussenegg, and Joachim R. Krenn *Graz, Austria*

the realization of two-dimensional optical wavelength demultiplexers and multiports for surface plasmon polaritons (SPPs) based on plasmonic crystals, i.e., photonic crystals for SPPs.



Slow Propagation, Anomalous Absorption, and Total External Reflection of Surface Plasmon Polaritons in Nanolayer Systems

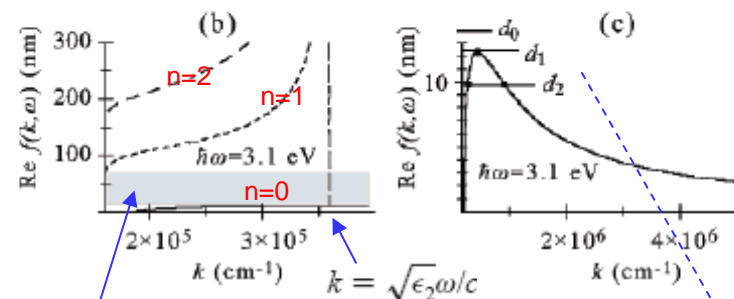
Mark I. Stockman* Georgia State University



$$\epsilon_1(\omega_2) + \epsilon_2 = 0, \quad \hbar\omega_2 = 3.02 \text{ eV}$$

$$\epsilon_1(\omega_3) + \epsilon_3 = 0, \quad \hbar\omega_3 = 3.68 \text{ eV}$$

when the frequency $\omega_3 > \omega > \omega_2$.



the gap, d for which there is no surface plasmon polariton propagation.

$$d_1 = 12 \text{ nm} \quad d_u = 82 \text{ nm} \quad \text{for } \omega = 3.1 \text{ eV}$$

$$d = f(k, \omega)$$

$$f(k, \omega) = \frac{1}{\epsilon_2 v_2} \left\{ -\text{Arctan} \left[\frac{v_2(u_1 + u_3)}{u_1 u_3 - v_2^2} \right] + n\pi \right\},$$

$n = 0, 1, 2, \dots$, where

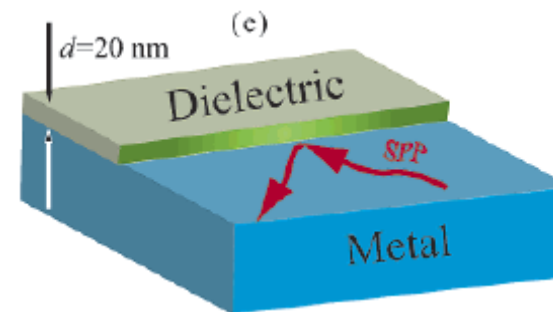
$$u_i = \frac{1}{\epsilon_i} \sqrt{q^2 - \epsilon_i}, \quad v_i = \sqrt{\epsilon_i - q^2}, \quad k = \frac{\omega}{c} q$$

SPPs can propagate for a very thin layer ($d \rightarrow 0$)

when d increases,

the situation eventually changes to resemble an infinite layer of dielectric 2 over the metal where the SPP propagation is impossible.

→ total external reflection



$d_0 > d_1 = 12 \text{ nm}$, there are no propagating SPPs.

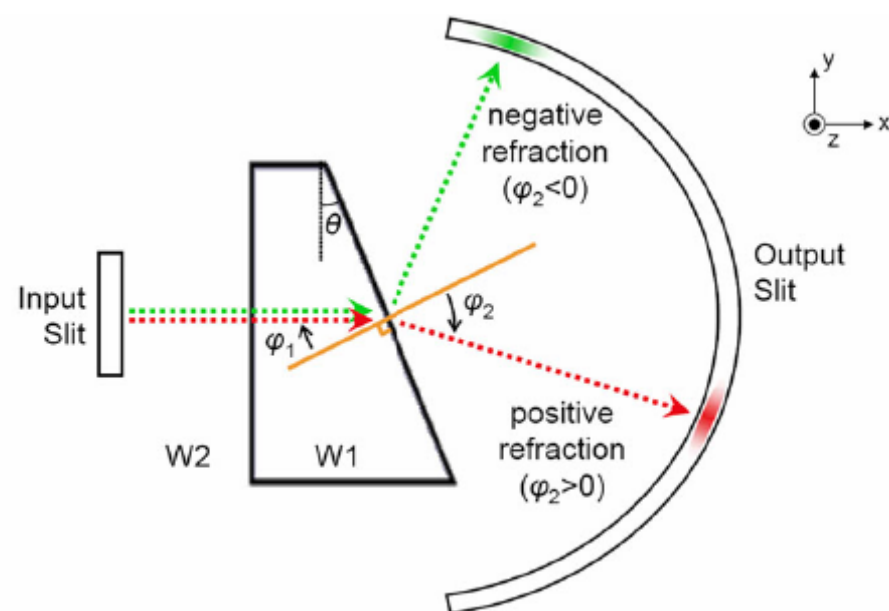
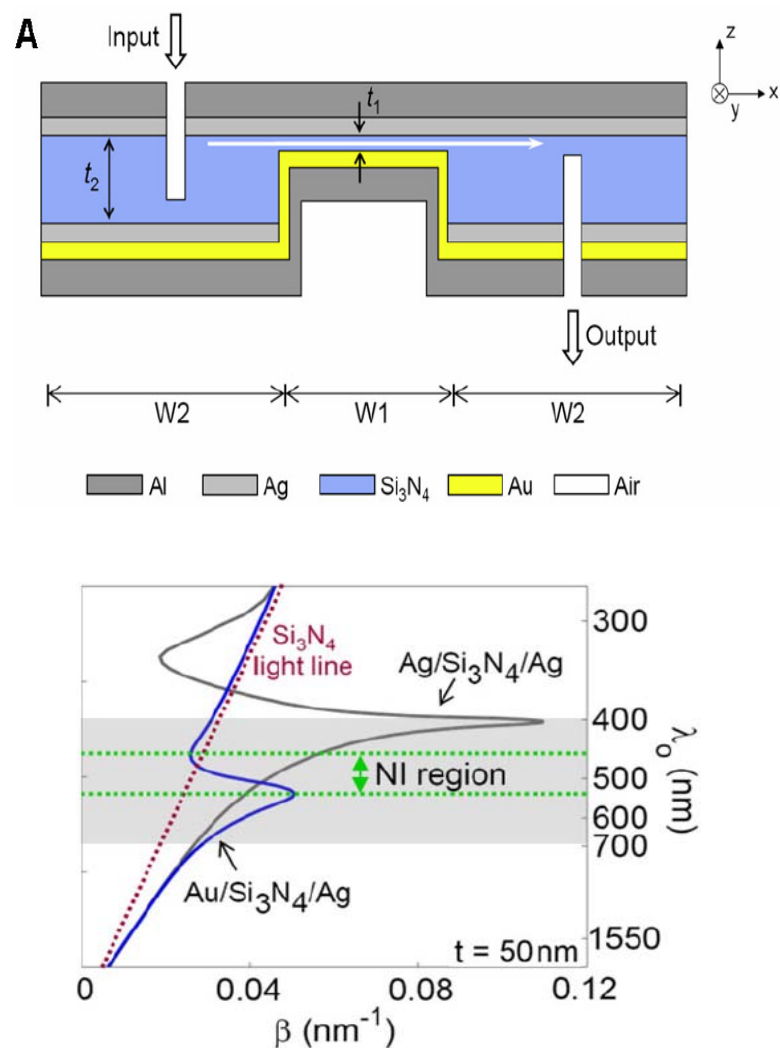
$d = d_1 = 12 \text{ nm}$ a bifurcation point

$d_2 < d_1 = 12 \text{ nm}$, two solutions. a long wavelength (small k),
a short wavelength (large k)

Negative Refraction at Visible Frequencies

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Terahertz Surface Plasmon-Polariton Propagation and Focusing on Periodically Corrugated Metal Wires

Stefan A. Maier,^{1,*} Steve R. Andrews,¹ L. Martín-Moreno,² and F. J. García-Vidal^{3,†} *University of Bath, United Kingdom*

we show how the dispersion relation of surface plasmon polaritons (SPPs) propagating along a perfectly conducting wire can be tailored by corrugating its surface with a periodic array of radial grooves.

Importantly, the propagation characteristics of these spoof SPPs can be controlled by the surface geometry, opening the way to important applications such as energy concentration on cylindrical wires and superfocusing using conical structures.

$$\sum_{n=-\infty}^{+\infty} S_n^2 \frac{g}{q_n} \frac{K_1(q_n R)}{K_0(q_n R)} \frac{N_0(gR)J_0(gr) - N_0(gr)J_0(gR)}{N_0(gr)J_1(gR) - N_1(gR)J_0(gR)} = 1,$$

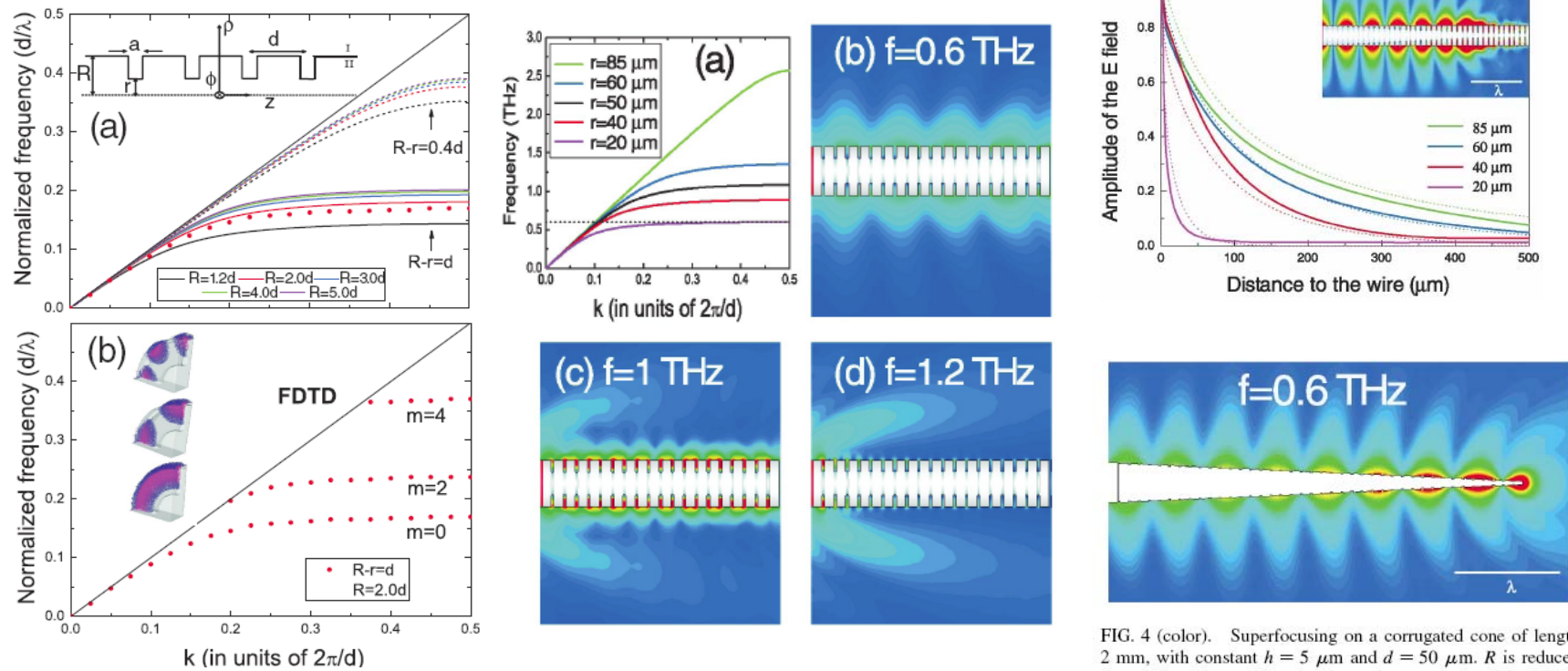


FIG. 4 (color). Superfocusing on a corrugated cone of length 2 mm, with constant $h = 5 \mu\text{m}$ and $d = 50 \mu\text{m}$. R is reduced from 100 to 10 μm . The plot shows the magnitude of the \mathbf{E} field on a logarithmic scale spanning 2 orders of magnitude.

Summary : Plasmonic Photonics

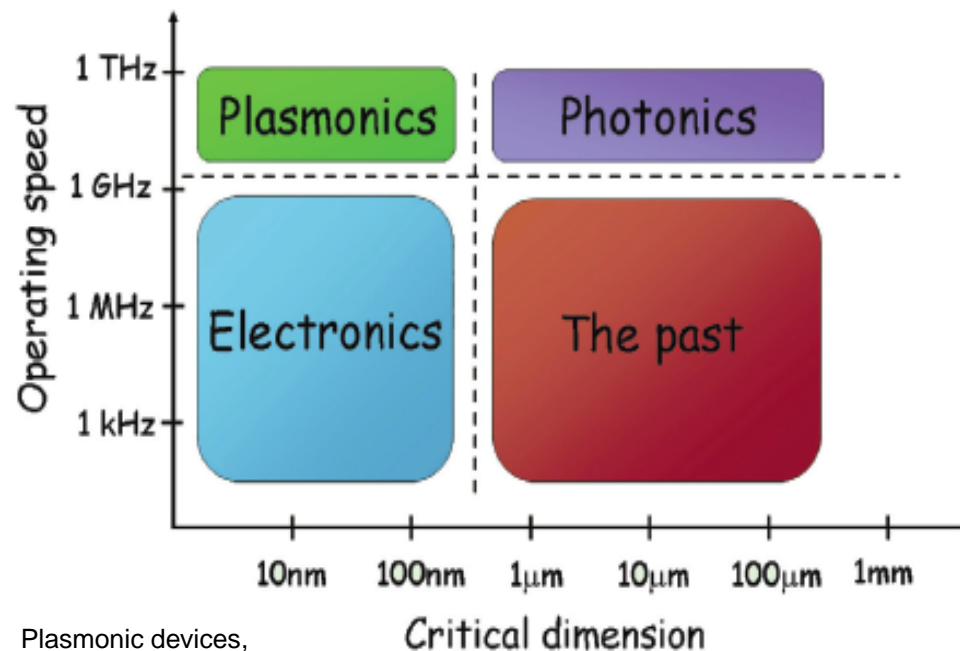
20 materials today JULY-AUGUST 2006 | VOLUME 9 | NUMBER 7-8

Plasmonics: the next chip-scale technology

Rashid Zia, Jon A. Schuller, Anu Chandran, and Mark L. Brongersma* Stanford University

Plasmonics is an exciting new device technology that has recently emerged.

A tremendous synergy can be attained by integrating plasmonic, electronic, and conventional dielectric photonic devices on the same chip and taking advantage of the strengths of each technology.



Plasmonic devices, therefore, might interface naturally with similar speed photonic devices and similar size electronic components. For these reasons, plasmonics may well serve as the missing link between the two device technologies that currently have a difficult time communicating. By increasing the synergy between these technologies, plasmonics may be able to unleash the full potential of nanoscale functionality and become the next wave of chip-scale technology.

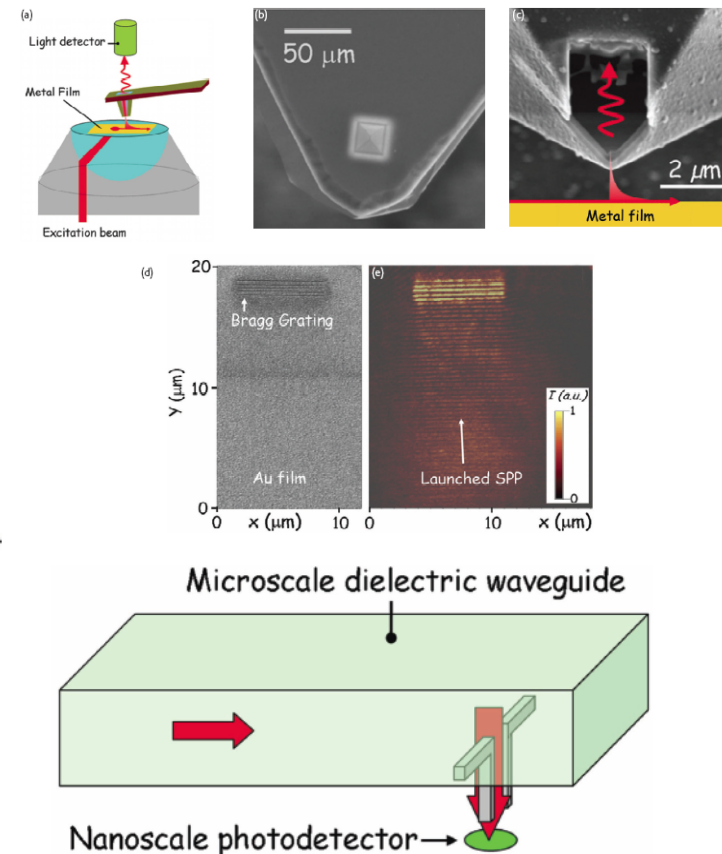


Fig. 7 Schematic of how a nanoscale antenna structure can serve as a bridge between microscale dielectric components and nanoscale electronic devices.