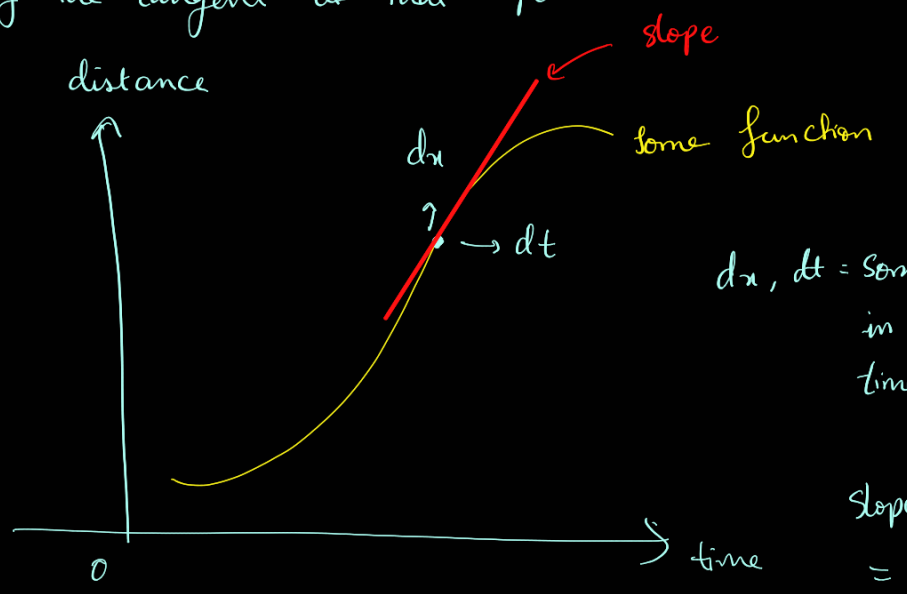
Intro to derivatives:

A Derivative is the instantaneous rate of change of any function.

more formally: the derivative of a function at a point is the slope of the tangent at that point.

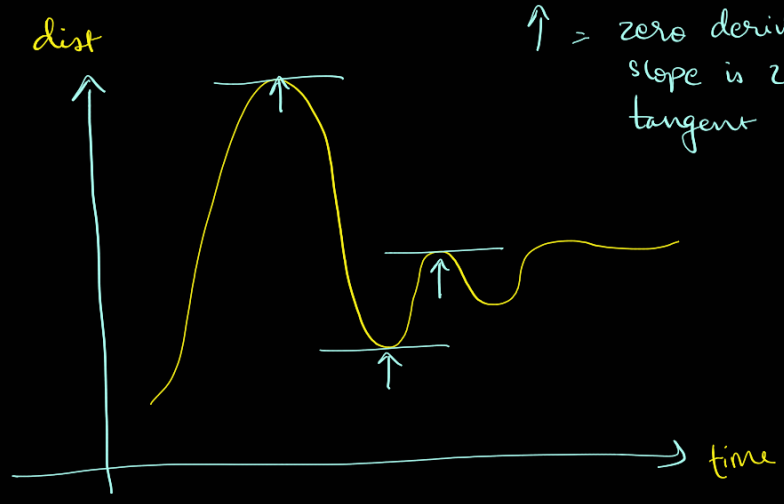
Eg:



dx, dt = Some small change in x over a small time period t.

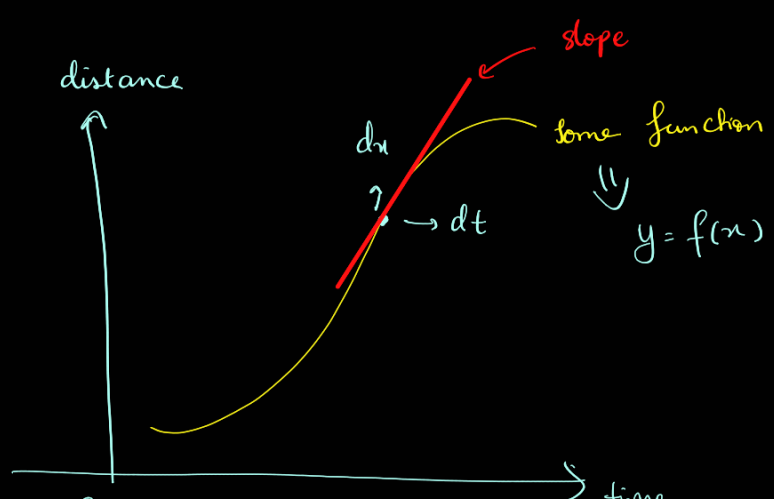
Slope is given by $= \frac{dx}{dt}$

Zero derivative:



↑ = zero derivative as slope is zero and tangent is horizontal.

Notation:



Here the slope $= \frac{dx}{dt}$

∴ Derivative of f is given by

$f'(x)$ ← Lagrange's notation

OR

$dy = d\,f(x)$ ← leibniz's

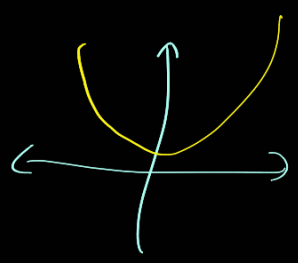
$\frac{dy}{dx}$ $\frac{}{dx}$ notation.

Common derivatives:

$y = f(x) = c \longrightarrow f'(x) = 0$ ($\because$ no change in slope)

$f(x) = ax + b \longrightarrow f'(x) = a$ ($\because$ entire line has same slope ie a)

$f(x) = x^2 \longrightarrow f'(x) = 2x$



Similarly: $f(x) = x^3 \longrightarrow f'(x) = 3x^2$

$f(x) = \frac{1}{x} = x^{-1} \longrightarrow f'(x) = -1x^{-2} = \frac{-1}{x^2}$

$$\boxed{\therefore\ f(x) = x^n \longrightarrow f'(x) = nx^{n-1}}$$

if $f(x)$ and $g(x)$ are inverse functions of each other, then

$$g'(y) = \frac{1}{f'(x)}$$

$f(x) = \sin x \longrightarrow f'(x) = \cos(x)$

$f(x) = \cos x \longrightarrow f'(x) = -\sin(x)$

$f(x) = e^x \longrightarrow f'(x) = e^x$

log function :

$e^{x} = 3$ then x may be $\log(3)$

Similarly $e^{\log(x)} = x$

$\therefore$ if $f(x) = e^{x}$ then $f^{-1}(x) = \log(x)$

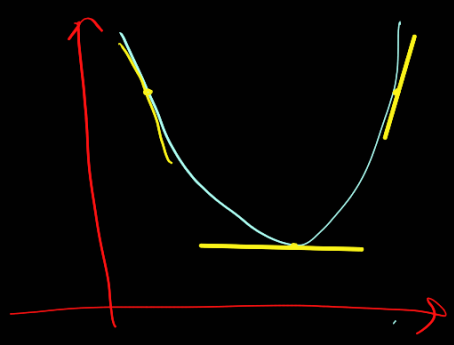
$$\therefore \boxed{\frac{d}{dy} \log(y) = \frac{1}{y}}$$

Non differentiable functions :

Some functions may not be differentiable at all points.

now we calculate derivative at a point by drawing a tangent in the graph.

Eg :-

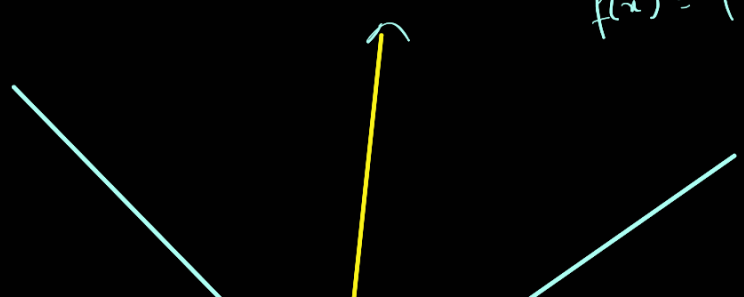


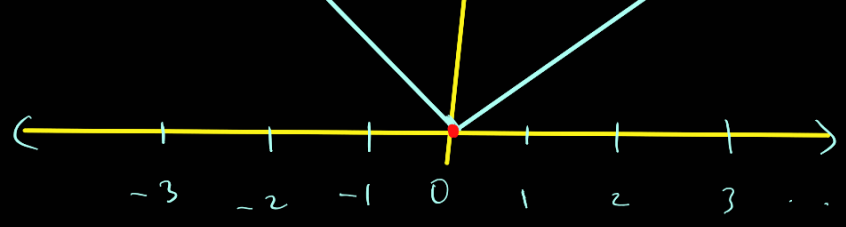
But in some functions, it is not possible to draw tangents at every point.

Eg :

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

$f(x) = |x|$





At the corner point "Origin" a tangent can't be drawn.

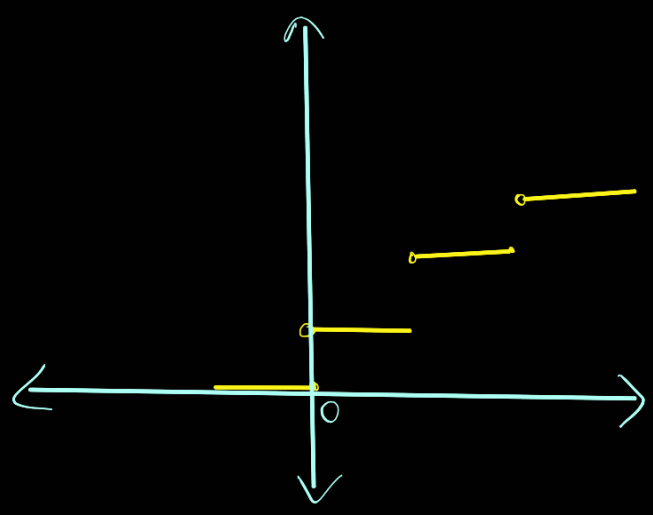
$\therefore$ At $x=0$, $f(x)=|x|$ is not differentiable.

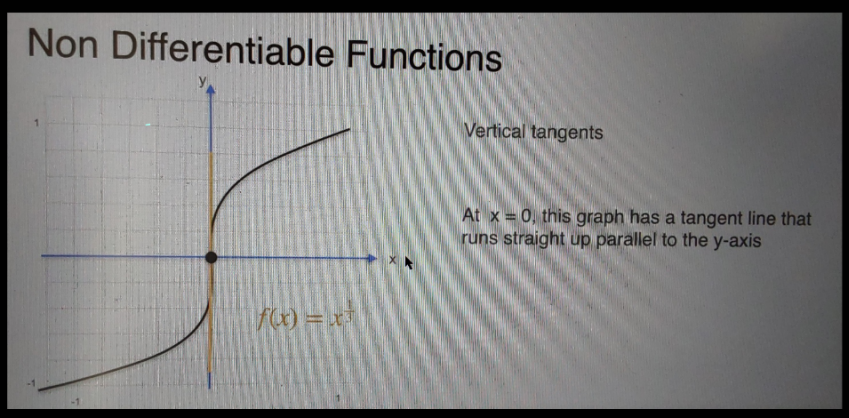
<u>Note</u>: A fn is only called differentiable if it has a derivative at "<u>all</u>" points.

<u>jump discontinuity</u>:

Any fn that can't be drawn without lifting the pen is not continuous and therefore non-differentiable.

Eg:





<u>Vertical tangents are not defined.</u>

Thus this is <u>non-differentiable</u>.

Properties of derivatives:

① Multiplication by scalars:

Eg: $f = 4g$

then $f' = 4g'$

$\therefore$ c = constant : $f' = c \times g'$

② Sum rule

If $f = g + h$ (where f, g, h are some functions)

then $\boxed{f' = g' + h'}$

③ Product rule:

If $f = g \times h$

then $\boxed{f' = g'h + gh'}$

④ The Chain Rule:

Say you have a composite function $f = g(h(t))$

you want to find $\frac{df}{dt} = \frac{d}{dt}\left(g(h(t))\right)$

$$\therefore \quad \boxed{\frac{df}{dt} = \frac{dg}{dh} \cdot \frac{dh}{dt}}$$

Rough (Quiz)

$x_1 = 0 \quad y_1 = 0.8$

$x_2 = 0.4 \quad y_2 = 1.6$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.6 - 0.8}{0.4 - 0} = \frac{0.8}{0.4} = 2$$

$$e^{-x} = \frac{1}{e^{x}}$$

$$\boxed{1 \times \frac{1}{e^{x}}}$$