# Fields

A field is a region in space in which an object experiences a force.

### Gravitation fields

A gravitational field is one in which objects experience forces because of their masses.

### Electric fields

An electric field is one in which objects experience forces due to their charges.

### Comparing Fields

| Similarities | Differences |
|---|---|
| - Follows the inverse square law | - The forces in a gravitational field are always attractive |
| - The field lines around point objects are radial | - The forces in an electric field can be attractive or repulsive |
| - Equipotential lines (lines along which the potential is the same) are concentric circles or spheres (circles or spheres surrounding the points with mass/charge) | |
| - You can add field strengths from two or more particles as vectors | |

# Gravitational Fields

Core Equations:

Force:

$$F = \frac{GMm}{r^2}$$

Work done:

$$W = \frac{GMm}{r}$$

Field strength:

$$g = \frac{GM}{r^2}$$

Potential:

$$V = \frac{GM}{r}$$

### Newton's Third Law

Forces act along the line joining the two centres of the objects. The magnitude of the force acting on both objects is the same.

### Gravitational Field Lines

Field lines are always pointing towards masses because gravitational forces are attractive.

Field lines indicate the direction of the force a point mass would experience when within the field.

All the field lines should point towards the centre of mass.

Close to the surface of the earth and over small distances, the field lines seem to be equipotential, which is why we think the Earth's gravitational field is uniform.

### Gravitational Potential

Gravitational potential is the work done per unit mass ($V = \frac{W}{m}$). The zero value of gravitational potential is defined as being at an infinite distance away from the mass generating the field. This is done because work is done to move a test mass away from the object that the field lines point towards.

**Gravitational Potential Graphs** The gravitational potential surrounding a planet or point mass is given by the formula:

$$V = -\frac{GM}{r}$$

It is important that there is a minus, as we have declared the direction away from the point mass (towards infinity) to be positive, which means that V is in the opposite direction to that and therefore is attractive.

Finding the area under and gravitational field strength-distance graph will give the change in potential

### Orbits

When an object orbits a more massive body it has a set period and radius which depend on each other.

An orbital period is the time taken for an object to do one full orbit. Even if the orbit is elliptical, the period remains constant.

The orbital radius is the average distance between the centre of the body and the centre of the object.

Orbital period and radius have the relationship that:

$$T^2 \propto r^3$$

Where $T$ is the orbital period and $r$ is the orbital radius. The constant of proportionality can be found by finding the gradient of a graph of $T^2$ plotted against $r^3$. This is Kepler's third law

In a circular orbit around a planet, the satellite is always on the same equipotential line, so the total energy of an orbiting satellite is constant. The planet does no work on the satellites so there is not change in $E_k$ or $E_g$ and the radius does not change.

In non-circular orbits (ellipses and parabolas), we can show that the total energy of an orbiting satellite is always equal to half of the $E_g$ of the satellite because gravitational field strength ($g$) follows the inverse-square law.

Proof (I doubt we need to do/know this):

The two forces acting on a satellite are a gravitational force $F_g$ and a centripetal force due to its velocity $F_c$.

$$F_g = \frac{GMm}{r^2}$$

and

$$F_c = \frac{mv^2}{r}$$

(from circular motion)

However these two forces are equal in magnitude so:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Solving for $v^2$ gives:

$$v^2 = \frac{GM}{r}$$

The total energy of the satellite ($E_T$) is:

$$E_T = E_k + E_g$$

Kinetic energy is given by the equation:

$$E_k = \frac{1}{2}mv^2$$

Substituting the equation for $v^2$ derived earlier we get:

$$E_k = \frac{GMm}{2r}$$

Gravitational Potential energy is given by:

$$potential \times mass = potential\ energy$$

so:

$$E_g = Vm = -\frac{GMm}{r}$$

Combining the equations for $E_k$ and $E_g$ we get:

$$E_T = \frac{GMm}{2r} - \frac{GMm}{r}$$

Which simplifies to:

$$E_T = -\frac{GMm}{2r}$$

By taking the $\frac{1}{2}$ out we get:

$$E_T = \frac{1}{2}(-\frac{GMm}{r})$$

and since:

$$E_g = -\frac{GMm}{r}$$

We can see that the total energy is half the gravitational potential energy.

## Electric Fields

Core Equations:

These equations are for radial fields

Force:

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

Work done:

$$W = \frac{Q_1 Q_2}{4\pi\epsilon_0 r}$$

Field strength:

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Potential:

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

**Coulomb's Law**

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

A negative force means that it is attractive and a positive force means that it is repulsive.

The gravitational force between 2 protons in a nucleus is much smaller than the electrostatic force between the same protons, suggesting that there is another force (the strong force) keeping them together.

### Electric Fields

Electric field lines show the direction a positive test charge would experience a force in (away from positively charged particles and towards negatively charged particles.)

### Uniform Electric Fields

A uniform field is one in which the field strength is the same at any point in the field. One can be created by placing two charged plated at a distance apart.

To calculate the field strength of a uniform field, the equation is:

$$E = \frac{V}{d}$$

Where $E$ is the electric field strength, $V$ is the potential difference, and $d$ is the distance between the plates.

The work done in moving a positively charged particle from the negative to the positive plate is:

$$W = Fd$$

where $F$ is a constant force acting on the particle.

The work done in moving a positively charged particle is also:

$$W = Q\Delta V$$

Combining the equations gives:

$$Fd = Q\Delta V$$

Rearranging the equation gives:

$$\frac{F}{Q} = \frac{\Delta V}{d}$$

and by definition, $\frac{F}{Q}$ is the electric field strength so:

$$E = \frac{\Delta V}{d}$$

**Electric Potential**

Electric potential is the work done per unit charge. Like for gravitational potential, the electric potential is 0 at infinity, but is the largest next to the charge.