In [2]: | df = pd.read csv('problem1.csv', names = ['X','Y','Z']) df.head() Out[2]: Z **0** | 13.242098 | 21.650022 | 27.425479 20.916445 7.025783 **1** 9.434795 **2** 15.485839 21.733990 14.963390 **3** 7.730181 16.332673 4.203992 18.166409 21.486609 28.191421 In [3]: def E(f,S): return float(sum(f(x) for x in S))/(len(S) or 1) def mean(X): return E(lambda x:x, X) **def** variance(X): return E(lambda x:x**2, X) - E(lambda x:x, X)**2def sd(X): return math.sqrt(variance(X)) def covariance(X,Y): return sum(X[i]*Y[i] for i in range(len(X)))/len(X) - mean(X)*mean(Y) def correlation(X,Y): return covariance(X,Y)/sd(X)/sd(Y) def prob1(df): Mean = []Var = [] Std = []for i in list(df.columns): Mean.append(mean(df[i])) Var.append(variance(df[i])) Std.append(sd(df[i])) ans1 = colored(("1-1. Mean, Var, Std for column X, Y, and Z are as follow:"), "blue", attrs = ["bold"]) print(ans1) ans1 1 = colored(("Means"), "blue") ans1 2 = colored(("Variances"), "blue") ans1_3 = colored(("Standard deviations"), "blue") print(ans1_1, Mean) print(ans1_2, Var) print(ans1 3, Std) ans1 4 = colored((" \n 1-2. The distribution of each columns are as follow:"), "blue", attrs = ["bold"]) print(ans1 4) df.hist(figsize=[8,6]) plt.show() ans1 5 = colored(("By looking at the shape of the histogram, it looks like a normal distribution, but no t perfectly"), "red") print(ans1_5) comb lst = []for i in combinations(list(df.columns),2): comb_lst.append(i) ans1 6 = colored(("\n\n1-3. The correlation and covariance for"), "blue", attrs = ["bold"]) print(ans1 6, comb lst) for i, j in comb_lst: ans1_7 = colored(("($\{\},\{\}\}$) ="), "blue").format(i,j) print("Corr", ans1_7, correlation(df[i], df[j])) print("Cov", ans1_7, covariance(df[i], df[j])) print() ans1 8 = colored((" \n 1-4. The computed expectation values are as follow:"), "blue", attrs = ["bold"]) print(ans1_8) one = [1]X = list(df.X)Y = list(df.Y)Z = list(df.Z)XXX = (df.X) **3a = df.Z-(df.X+df.Y)/2print("E[1] =", E(lambda x:x, one)) print("E[X] =", E(lambda x:x, X)) print("E[X^3] =", E(lambda x:x, XXX)) print("E[Z-(X+Y)/2] =",(E(lambda x:x, a))) ans $1_9 = \text{colored}(("\n1-5)$. The formulars for the mean, variation, covariance, and correlation \nin terms of the expectation values are as follow:"), "blue",attrs = ["bold"]) print(ans1_9) In [4]: prob1(df) 1-1. Mean, Var, Std for column X, Y, and Z are as follow: Means [9.93706828076954, 20.217017423770194, 14.862678823134607] Variances [27.009135786469884, 24.22520896822499, 37.84152597711147] Standard deviations [5.197031439819263, 4.9219111093380175, 6.151546632929923] 1-2. The distribution of each columns are as follow: 250 250 200 200 150 150 100 100 50 50 10 Ζ 200 150 100 50 By looking at the shape of the histogram, it looks like a normal distribution, but not perfe 1-3. The correlation and covariance for [('X', 'Y'), ('X', 'Z'), ('Y', 'Z')] Corr(X,Y) = -0.05040109245653239Cov (X,Y) = -1.2892260139755933Corr(X,Z) = 0.3999697204931487Cov (X,Z) = 12.786944472729914Corr (Y,Z) = 0.321761269269099Cov (Y, Z) = 9.7420836216913931-4. The computed expectation values are as follow: E[1] = 1.0E[X] = 9.93706828076954 $E[X^3] = 1777.8614071677903$ E[Z-(X+Y)/2] = -0.214364029135249021-5. The formulars for the mean, varaiance, covariance, and correlation in terms of the expectation values are as follow: Mean $\mu_{x} = E(X)$ Variance $\sigma_{x}^{2} = E[(X - \mu_{x})^{2}]$ Covariance cov(X, Y) = E[XY] - E[X]E[Y]Correlation $corr(X, Y) = \frac{cov(X, Y)}{(\sigma_x \sigma_v)}$ Problem 2 Consider a continuous random variable in the range [2,3] with probability mass function proportional to $\exp(x/2)$ Given the condition, $p(x) = c * e(\frac{x}{2})$, where x is in [2,3] Now, $F(X) = \int_{-\infty}^{x} p(x)dx = \int_{2}^{x} c \cdot \mathbf{e}(\frac{x}{2})dx = 2c[e^{\frac{x}{2}} - e]$ F(3) = 1implies $c = \frac{1}{2(e^{\frac{3}{2}} - e)} \approx 0.2835$ Therefore, To compute with E[1], E[x], $E[x^2]$, $E[x^3]$ $E(1) = \int_{2}^{3} c * e(\frac{x}{2}) dx = 2c(e^{\frac{3}{2}} - e) = 0.99999 \approx 1$ $E[X] = \int_{2}^{3} cxe^{\frac{x}{2}} dx = ce^{\frac{x}{2}} (2x - 4) \Big|_{2}^{3} = 2.54148$ $E[X^{2}] = \int_{2}^{3} cx^{2} e^{\frac{x}{2}} dx = ce^{\frac{x}{2}} (2x^{2} - 8x + 16) \Big|_{2}^{3} = 6.54145$ $E[X^{3}] = \int_{2}^{3} cx^{3} e^{\frac{x}{2}} dx = ce^{\frac{x}{2}} (2x^{3} - 12x^{2} + 48x - 96) \Big|_{2}^{3} = 17.0393$ Problem 3 Consider a continuous various random variable in the range [1,5] with probability mass function proportional to 1/x. What is the cumulative distribution function? $p(x) = c\frac{1}{x}, x \in [1, 5]$ $F(x) = \int_{-\infty}^{x} p(x)dx = c \int_{1}^{x} \frac{1}{x} dx = c(\ln(x) - \ln(1))$ F(5) = 1 = c(ln(5) - ln(1))C = 0.62133 $p(x) = \frac{dF(x)}{dx} = 0.62133\frac{1}{x}, x \in [1, 5]$ F(x) = 0.62133ln(x)In [5]: C = 1/(math.log(5) - math.log(1))def prob3(): rand 10 = []rand 100 = []rand 1000 = []for i in range(10): y = -(float(C*math.log(random.random())))**(-1)**if** y >= 1 **and** y <= 5: rand_10.append(y) **for** i **in** range (100): y = -(float(C*math.log(random.random())))**(-1)**if** y >= 1 **and** y <= 5: rand_100.append(y) for i in range(1000): y = -(float(C*math.log(random.random())))**(-1)**if** y >= 1 **and** y <= 5: rand 1000.append(y) one = [1] $X = rand_10$ XX = []for i in X: XX.append(i**2)XXX = []for i in X: XXX.append(i**3)title = colored("When range(10)", "blue") print(title) print("E[1] =", E(lambda x:x, one)) print("E[X] =", E(lambda x:x, X)) print("E[X^2] =", E(lambda x:x, XX)) $print("E[X^3] = ", E(lambda x:x, XXX))$ one = [1] $X = rand_100$ XX = []for i in X: XX.append(i**2)XXX = []for i in X: XXX.append(i**3)title = colored("\nWhen range(100)", "blue") print(title) print("E[1] =", E(lambda x:x, one)) print("E[X] =", E(lambda x:x, X)) print("E[X^2] =", E(lambda x:x, XX)) print("E[X^3] =", E(lambda x:x, XXX)) one = [1]X = rand 1000XX = []for i in X: XX.append(i**2)XXX = []for i in X: XXX.append(i**3)title = colored("\nWhen range(1000)", "blue") print(title) print("E[1] =", E(lambda x:x, one)) print("E[X] =", E(lambda x:x, X)) print(" $E[X^2]$ =", E(lambda x:x, XX)) print("E[X^3] =", E(lambda x:x, XXX)) In [6]: prob3() When range (10) E[1] = 1.0E[X] = 2.204706561233817 $E[X^2] = 5.2880367564056305$ $E[X^3] = 13.800958387233635$ When range (100) E[1] = 1.0E[X] = 2.403370147475276 $E[X^2] = 6.922273762689239$ $E[X^3] = 23.05510268166969$ When range (1000) E[1] = 1.0E[X] = 2.3864317535427695 $E[X^2] = 6.85573276662736$ $E[X^3] = 22.636828658646117$ Problem 4 Assume you receve phone calls with an average of 10 per hour. What is the probablity of receving two phone calls less than 1 minute apart wthin any one day? Write a program that computes the answer. Explain your reasoning and your program. , per minute In [7]: **def** prob4(): count = 0lst = []**for** i **in** range (500): lst.append(random.expovariate(1/6)) ans4 = colored("TIME GAP", "red") ans4 1 = colored("random.expovariate(1/6)", "blue") print("Let", ans4_1, "be the", ans4, "between two phone calls $\n"$) print("Generate the ENOUGH samples of",ans4_1) print("(I made 500 samples of", ans4_1,")") ans4_2 = colored("1440", "blue") print("\nNow, add each", ans4_1, "repeatly until the total sum equals to", ans4_2, "(= minutes in a day)") lst1=[] summ = 0sum cnt = 0for i in lst: summ += isum cnt += 1lst1.append(i) **if** summ >= 1440: break ans4 3 = colored(summ, "blue") print("By adding them up, the total sum is calculated as", ans4 3,) print("and this satisfies to be the minutes in a day\n") ans4_4 = colored("each number of the addition", "red") ans4 5 = colored("TOTAL PHONE CALLS IN A DAY", "red") ans4_6 = colored(sum_cnt, "blue") print("While adding them up,",ans4_4,"is counted.") print("The total count of", ans4_4, "is considered as", ans4_5) print("and", ans4 5, "is calculated as", ans4 6) cnt = 0for i in lst1: **if** i < 1: cnt += 1 ans4 7 = colored("Number of phone calls less than 1 minute apart from each other", "blue") ans4 8 = colored(cnt, "blue") print("\nAmong", ans4 5, ans4 6) print(ans4 7,"is seperately counted and the count of this is", ans4 8) ans4_9 = colored(cnt/sum_cnt, "blue") print("\nThe probability of recieving two phone calls less than 1 minute apart could be", cnt,"/",sum cnt,"=",ans4 9) print("\nTherefore,",ans4 9) In [8]: prob4() Let random.expovariate(1/6) be the TIME GAP between two phone calls Generate the ENOUGH samples of random.expovariate(1/6) (I made 500 samples of random.expovariate(1/6)) Now, add each random.expovariate (1/6) repeatly until the total sum equals to 1440 (= minutes in a day) By adding them up, the total sum is calculated as 1442.0099034458904and this satisfies to be the minutes in a day While adding them up, each number of the addition is counted. The total count of each number of the addition is considered as TOTAL PHONE CALLS IN A DAY and TOTAL PHONE CALLS IN A DAY is calculated as 239 Among TOTAL PHONE CALLS IN A DAY 239 Number of phone calls less than 1 minute apart from each other is seperately counted and the count of this is 43 The probability of recieving two phone calls less than 1 minute apart could be 43 / 239 = 0. Therefore, 0.1799163179916318 Problem 5 In [9]: class RandomSource(object): def __init__(self,generator=None): if not generator: import random as generator self.generator = generator def random(self): return self.generator.random() def lookup(self,table, epsilon=1e-6): if isinstance(table, dict): table = table.items() u = self.random()for key,p in table: if u<p+epsilon:</pre> return key u = u - praise ArithmeticError('invalid probability') def table(n): table = []room = 1a = 1for i in range(n): table.append((room, a * 0.5)) room +=1a = a*0.5return table table = table(1000)def test lookup(nevents, table): g = RandomSource() f=[] for i in range(nevents): f.append(0) p over4 = 0p less4 = 0for k in range(nevents): p = g.lookup(table) ############################### # This code counts total number of people who entereed the office # when 40 people entered the room #4 or greater. **if** p < 4: p_less4 += 1 else: p_over4 += 1 **if** p over4 == 40: less4 = colored(p_less4, "blue") over4 = colored(p_over4, "blue") total = colored(p_less4+p_over4, "blue") $explain_1 = "While generating enough samples (1000 samples) by "$ eq1 = colored("p=g.lookup(1000)", "blue") explain_2 = ",\ntwo counting variables, " eq2 = colored("p >= 4 ","blue") eq3 = colored("p < 3", "blue") $explain_3 = "and"$ explain $_4$ = " are counted.\n\nAt the moment the counting variable " explain_5 = "equals to 40, the program displays" explain_6 = " the total counts of the variable " explain 7 = "So when the program shows " explain_8 = "equals to 40, the total counts of the variable " explain_9 = " equals to " explain_10 = "\nTherefore, " ans5 3 1 = explain 1 + eq1 + explain 2 + eq2 + explain 3 + eq3 ans5 3 2 = explain 4 + eq2 + explain 5 + explain 6 + eq3 $ans5_3_3 = explain_7 + eq2 + explain_8 + eq3 + explain_9 + less4$ ans5 3 4 = explain 10 +less4 + "+" + over4 +"="+ total #print(p), f[p-1]=f[p-1]+1print(f,"\n") printt = colored("I just removed the rooms with zero probability", "red") print(printt) avg room = [] for i in range(len(table)): f[i]=float(f[i])/nevents **if** f[i] != 0.000000: pass ############## print('\tfrequency[%i]=%f' % (i+1,f[i])) avg_room.append((i+1) * f[i]*nevents) ans5_2 = colored(sum(avg_room)/len(avg_room), "blue") print("\nThe average room number is", ans5 2, "calculated by \n summing up all frequency by room numbers" ", which is", sum(avg room), "\nThen, dividing by the total number of frequency, which is" ,len(avg room)) print(sum(avg_room),"/",len(avg_room),"=", sum(avg_room)/len(avg_room)) title3 = colored(("\nProblem 5-3"), "blue", attrs = ["bold"]) q3 = colored("how many people enter rooms with the number greater or equal to 4", "blue") print(title3,q3) print(ans5_3_1, ans5_3_2,ans5_3_3,ans5_3_4) In [10]: **def** prob5(): title1 = colored(("Problem 5-1"), "blue", attrs = ["bold"]) q1 = colored("the average number of people who entere the first room if 100 people enter", "blue") print(title1, q1) print("E[x] = np = 100 * 0.5 =", 100*0.5,"\n") title2 = colored(("Problem 5-2"), "blue",attrs = ["bold"]) q2 = colored("the average room number", "blue") print(title2,q2) test lookup(1000, table) title4 = colored(("\nProblem 5-4"), "blue", attrs = ["bold"]) print(title4) prob5() Problem 5-1 the average number of people who entere the first room if 100 people enter E[x] = np = 100 * 0.5 = 50.0Problem 5-2 the average room number 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] I just removed the rooms with zero probability frequency[1]=0.496000 frequency[2]=0.236000 frequency[3]=0.139000 frequency[4]=0.053000 frequency[5]=0.044000 frequency[6]=0.017000 frequency[7]=0.006000 frequency[8]=0.004000 frequency[9]=0.003000 frequency[10]=0.001000 frequency[12]=0.001000 The average room number is 2.042 calculated by summing up all frequency by room numbers, which is 2042.0 Then, dividing by the total number of frequency, which is 1000 2042.0 / 1000 = 2.042Problem 5-3 how many people enter rooms with the number greater or equal to 4 While generating enough samples (1000 samples) by p=g.lookup(1000), two counting variables, $p \ge 4$ and p < 3 are counted. At the moment the counting variable p >= 4 equals to 40, the program displays the total coun ts of the variable p < 3 So when the program shows p >= 4 equals to 40, the total counts of the variable p < 3 equals to 287 Therefore, 287+40=327Problem 5-4 Since the probability that entering room 1,2,and3 is

0.5 + 0.25 + 0.125 = 0.875

40:0.125 = x:0.875

 $\therefore x * (0.125) = (40)(0.875)$

 $\therefore x = (40)(0.875)(\frac{1}{0.125})$

 $\therefore x = 280$

The probability that the entering the room with the number greater or equal to 4 is 0.125.

Therefore, 280 + 40 = 320 people entered the room

In [14]: print("done")

done

CSC 521

In [1]: import pandas as pd

import math
import random

Problem 1

Take Home Midterm

import matplotlib.pyplot as plt
from termcolor import colored
from itertools import combinations

Jonggoo Kang

import numpy as np