

Load libraries and data

```
library(tseries)
library(fBasics)

## Loading required package: timeDate
## Loading required package: timeSeries
##
## Rmetrics Package fBasics
## Analysing Markets and calculating Basic Statistics
## Copyright (C) 2005-2014 Rmetrics Association Zurich
## Educational Software for Financial Engineering and Computational Science
## Rmetrics is free software and comes with ABSOLUTELY NO WARRANTY.
## https://www.rmetrics.org --- Mail to: info@rmetrics.org
library(forecast)
library(lmtest)

## Loading required package: zoo
##
## Attaching package: 'zoo'
## The following object is masked from 'package:timeSeries':
##
##   time<-
## The following objects are masked from 'package:base':
##
##   as.Date, as.Date.numeric
setwd("~/Desktop/CSC425/hwork4/")
myd = read.table("onlinesales.csv", header = T, sep = ',')
head(myd)

##           date sales
## 1 1999-10-01  5263
## 2 2000-01-01  5556
## 3 2000-04-01  6062
## 4 2000-07-01  6891
## 5 2000-10-01  9070
## 6 2001-01-01  7874
tail(myd)

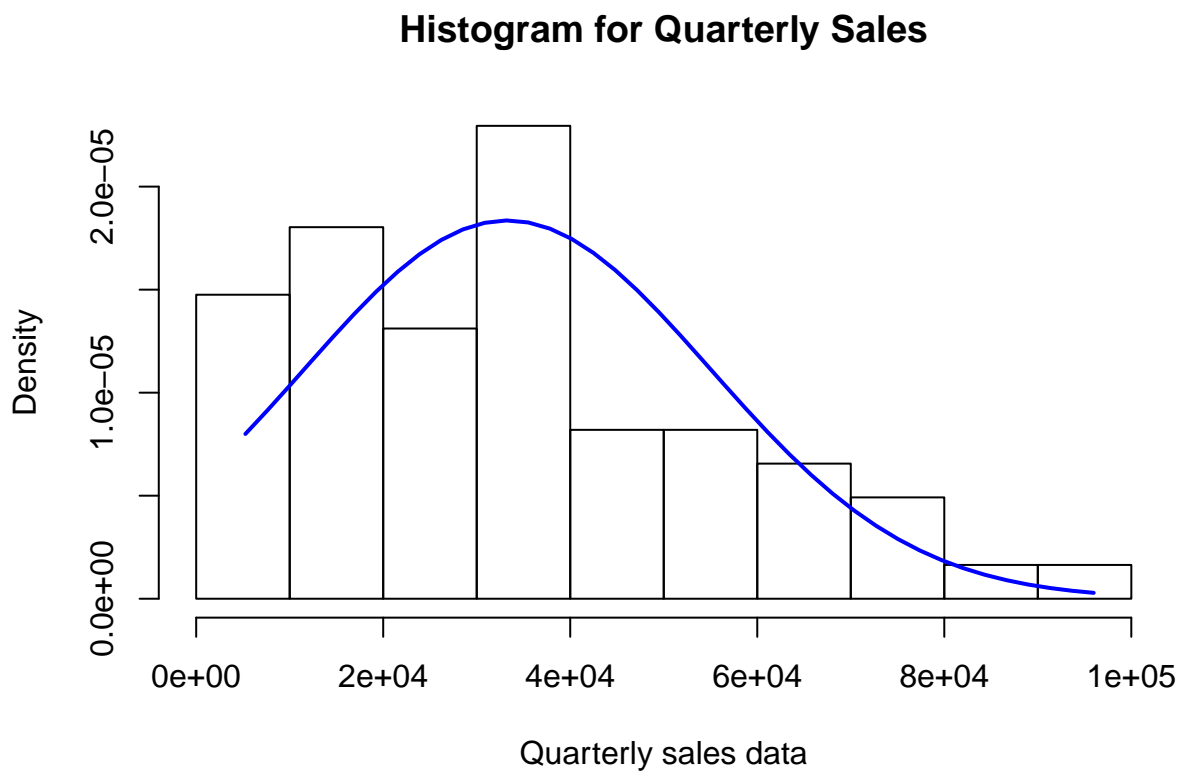
##           date sales
## 56 2013-07-01 61857
## 57 2013-10-01 83709
## 58 2014-01-01 66938
## 59 2014-04-01 70134
## 60 2014-07-01 71862
## 61 2014-10-01 95979
```

a) Analyze the distribution of the quarterly sales data. Are the data normally distributed? Provide appropriate statistics, tests and graphs to support your conclusions.

```
x = myd$sales
xts = ts(x, start = c(2000,1), frequency = 4)
```

histogram

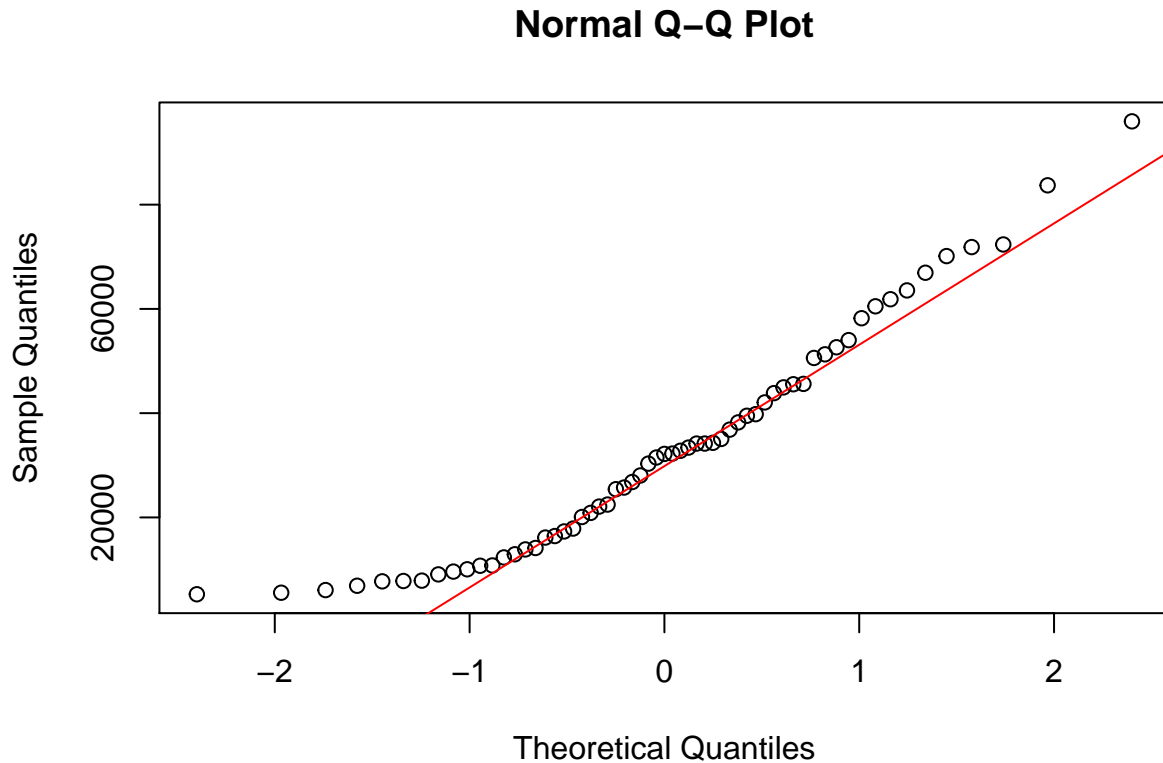
```
hist(x, xlab = 'Quarterly sales data', main = "Histogram for Quarterly Sales", prob = T)
xfit<-seq(min(x), max(x), length = 40)
yfit<-dnorm(xfit, mean = mean(x), sd = sd(x))
lines(xfit, yfit, col = "blue", lwd = 2)
```



It has a fat tail to the right and hard to say it has a perfect-normal distribution. However, it looks like it is very close to the normal distribution.

qq-line

```
qqnorm(x)
qqline(x, col = 2)
```



Many outliers are detected on the graph. However, it looks like it is very close to the normal distribution.

Perform Jarque-Bera normality test

```
normalTest(xts, method = c("jb"))
```

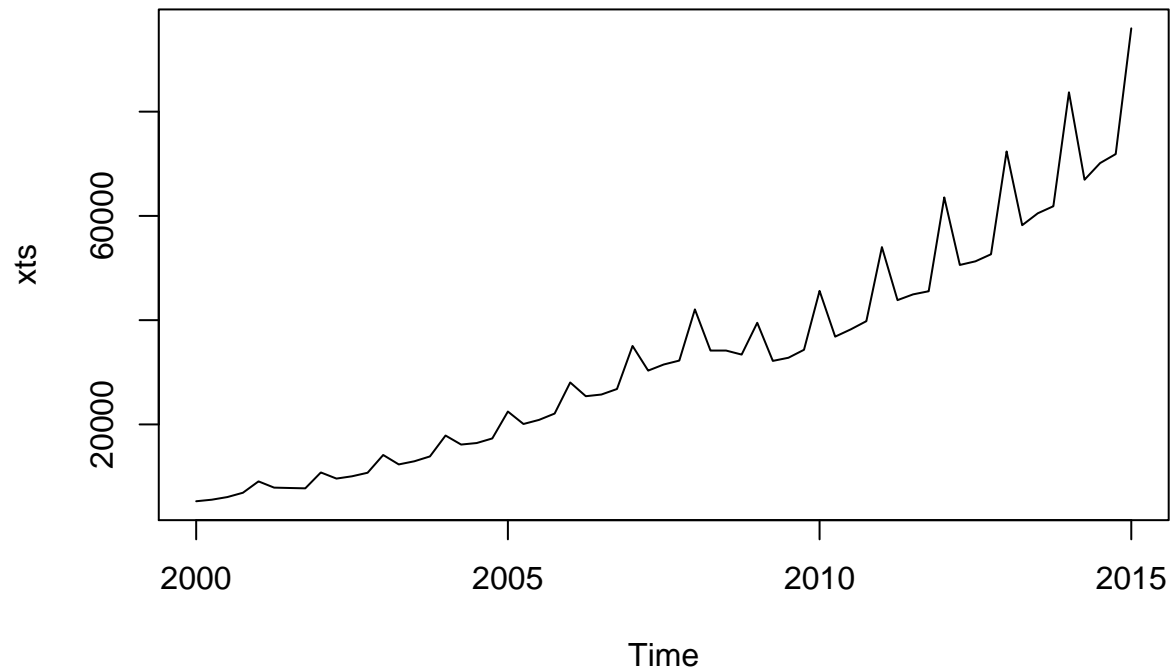
```
##
## Title:
##  Jarque - Bera Normalality Test
##
## Test Results:
##  STATISTIC:
##    X-squared: 5.525
##    P VALUE:
##    Asymptotic p Value: 0.06313
##
## Description:
##  Wed Nov  1 00:53:34 2017 by user:
```

The result shows that the p-value is not very significant as of 0.063. Thus, We cannot reject the null hypothesis. Therefore, it is normal.

b) Create a time plot for online sales. Does the plot show variations in variance? Discuss the trends displayed in the plot.

```
plot(xts, main = 'Time plot for Sales', type = 'l')
```

Time plot for Sales



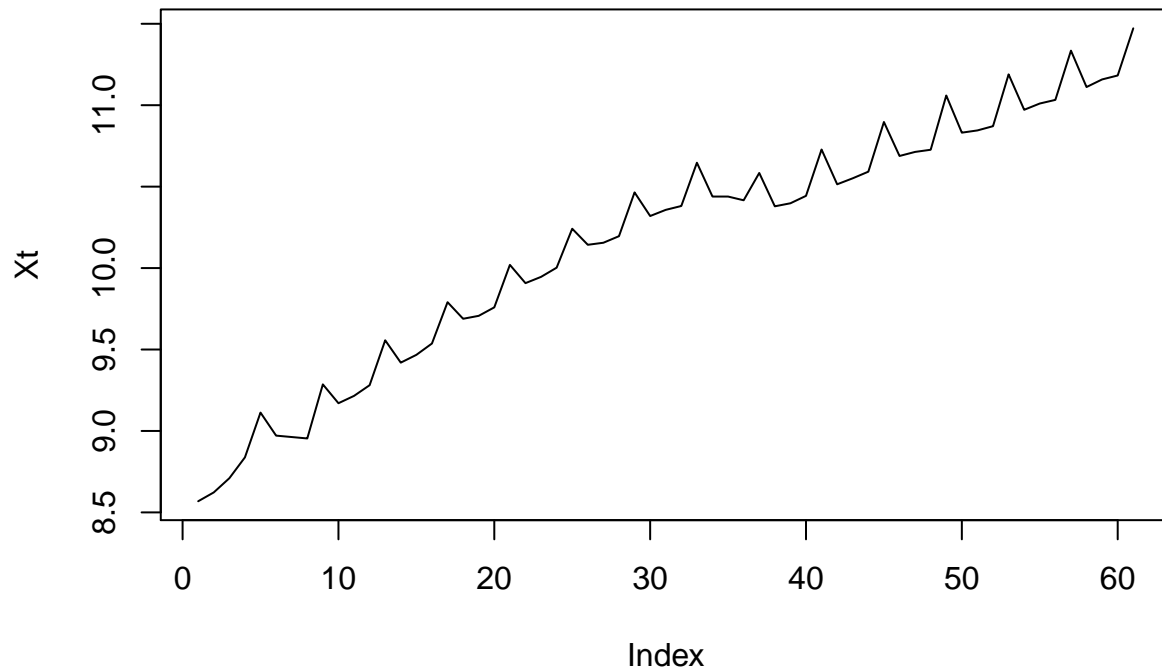
the graph shows that the it has a upward trend as well as variation is not constant over time.

c) Analyze the autocorrleation function of X_t and of its first difference. is the differenced time series stationary? Is there any evidence of a seasonal effect in the data?

log transformation

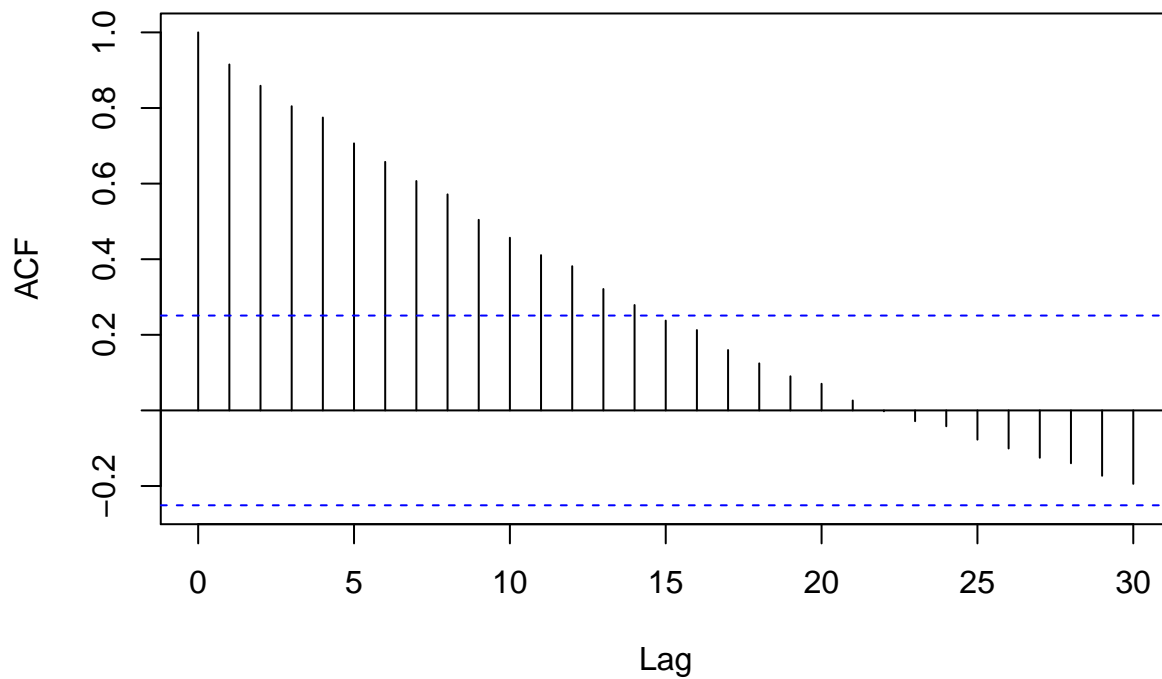
```
Xt = log(x)
plot(Xt, main = 'time plot for log sales', type = 'l')
```

time plot for log sales



```
lnts = ts(Xt, start=c(2000,1), frequency = 4)
acf(as.vector(lnts), lag.max= 30, main = "ACF of log sales")
```

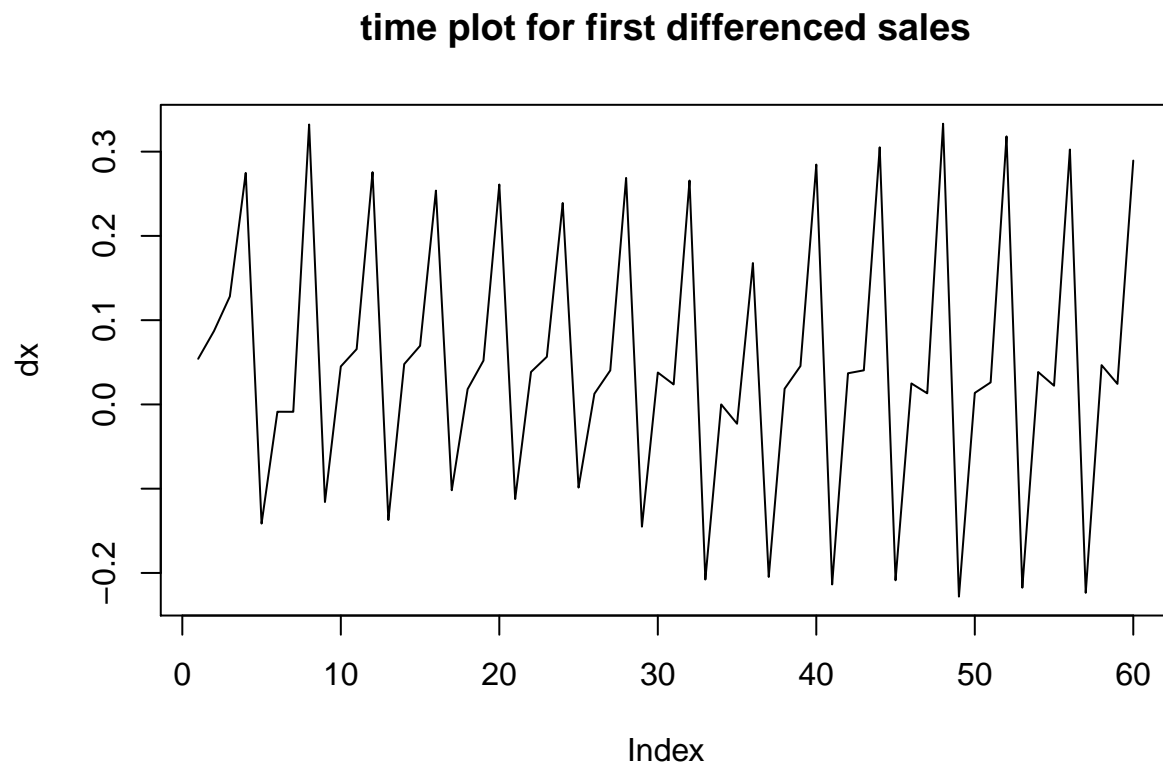
ACF of log sales



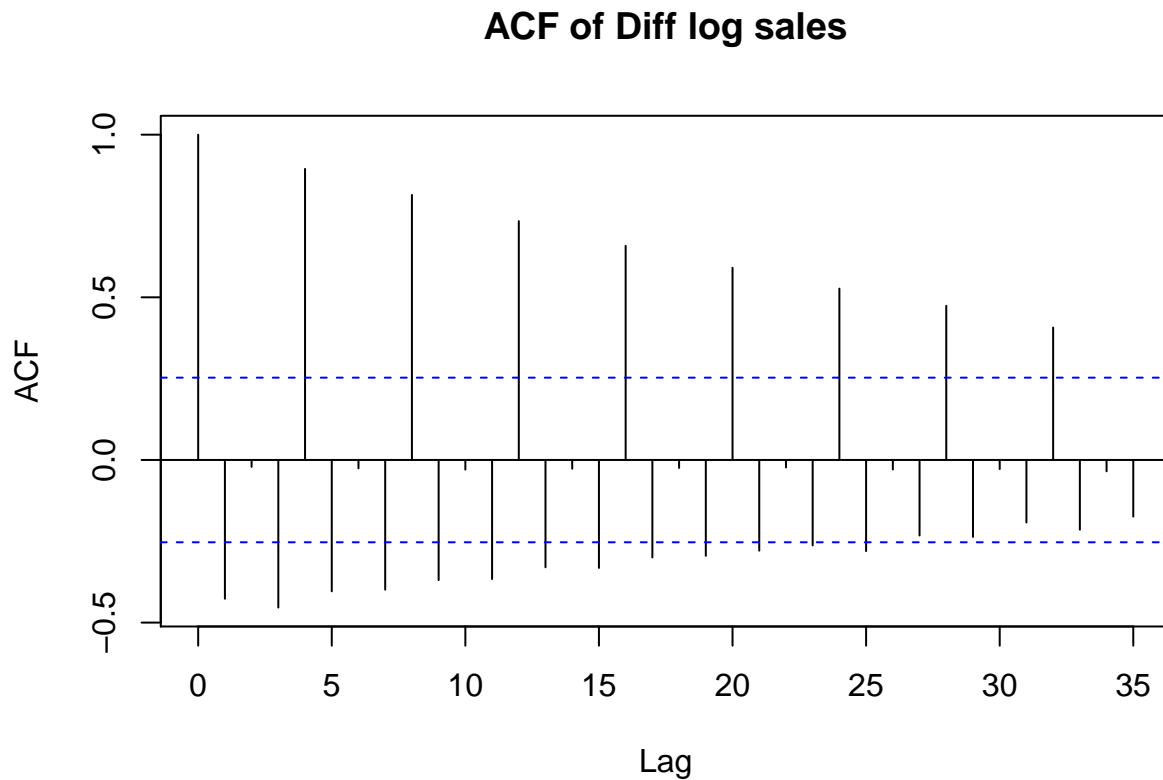
ACF for log sales is decaying slowly. This indicates a non-stationary series. Thus, we need to see the first difference

first difference

```
dx = diff(Xt)
plot(dx, main = 'time plot for first differenced sales', type = 'l')
```



```
dxts= ts(dx, start=c(2000,1), frequency = 4)
acf(as.vector(dxts), lag.max = 35, main = "ACF of Diff log sales")
```

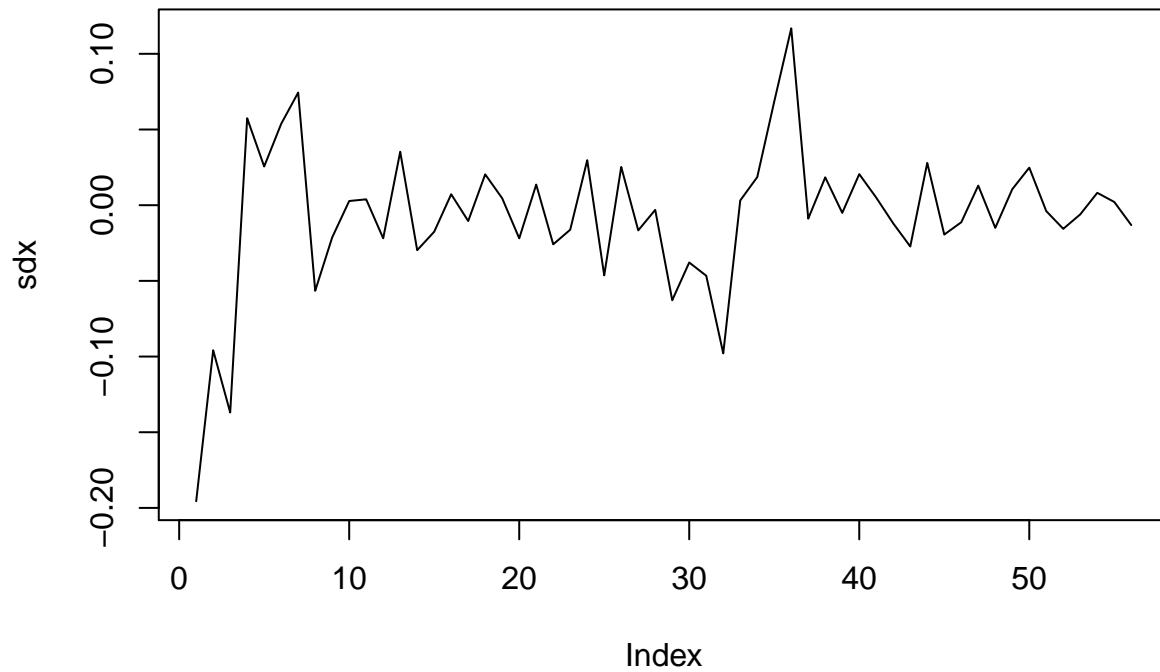


ACF is alternating while it is slowly decaying around near zero. Thus, first difference results of a non-stationary as well.

d) After de-trending and de-seasonalizing the time series, do you obtain a stationary time series?

```
sdx = diff(dx,4)
plot(sdx, main = 'time plot for seasonal difference', type = 'l')
```

time plot for seasonal difference



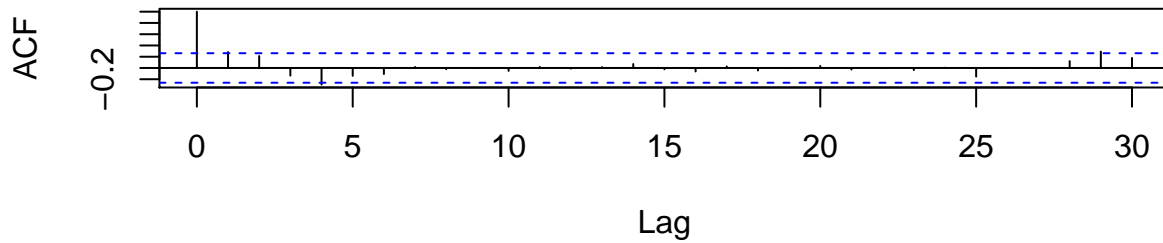
```
sdxts = ts(sdx, start = c(2000,1), frequency = 4)
```

After the seasonal difference, ACF is now stationary.

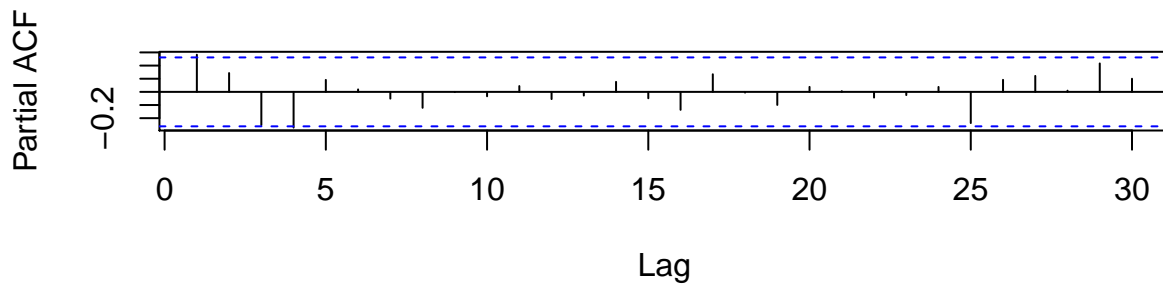
check ACF and PACF

```
par(mfcol = c(2,1))  
acf(as.vector(sdxts), lag.max = 30, main= "ACF of seasonal dx")  
pacf(as.vector(sdxts), lag.max = 30, main= "PACF of seasonal dx")
```


ACF of seasonal dx



PACF of seasonal dx



e) Identify a seasonal multiplicative model that explains the dynamic behavior of the process X_t . Evaluate the goodness of fit of your model using residual analysis. Explain the steps of your analysis and how you identified the final model

For non-seasonal behavior, the spikes at lag 1 in both ACF and PACF indicates MA(1) and AR(1). For seasonal behavior, the spike at lag 4 in both ACF and PACF indicates seasonal MA(1) and seasonal AR(1). so my seasonal arima model would be $(1,1,1) \times (1,1,1)$ period = 4

fit the model

```
m1 = Arima(xts, order = c(1,1,1), seasonal = list(order = c(1,1,1), period = 4), method = 'ML')
m1

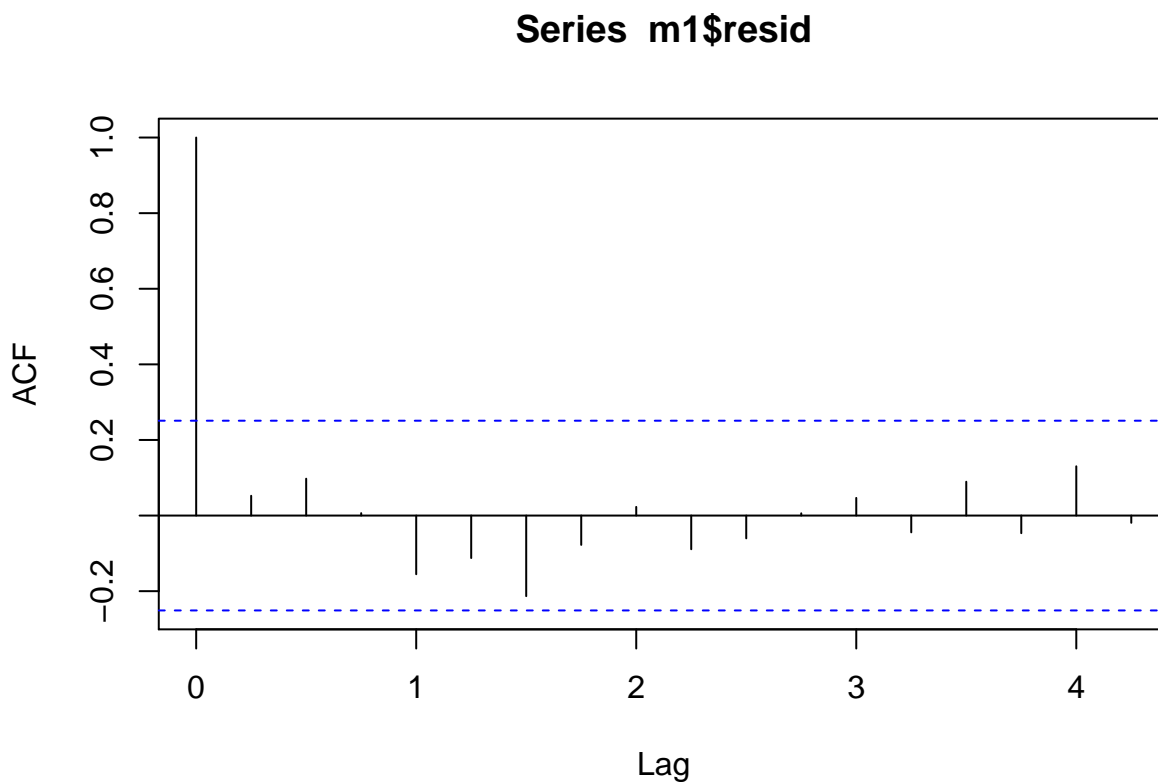
## Series: xts
## ARIMA(1,1,1)(1,1,1)[4]
##
## Coefficients:
##          ar1      ma1      sar1      sma1
##      -0.9654  0.9229  0.9920 -0.9319
## s.e.   0.0704  0.1179  0.0545  0.2383
##
## sigma^2 estimated as 1914582:  log likelihood=-484.52
## AIC=979.03  AICc=980.23  BIC=989.16
coeftest(m1)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error  z value  Pr(>|z|)
## ar1  -0.965368   0.070377 -13.7171 < 2.2e-16 ***
## ma1   0.922927   0.117914   7.8271 4.991e-15 ***
## sar1   0.992008   0.054537  18.1895 < 2.2e-16 ***
## sma1  -0.931935   0.238290  -3.9109 9.194e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

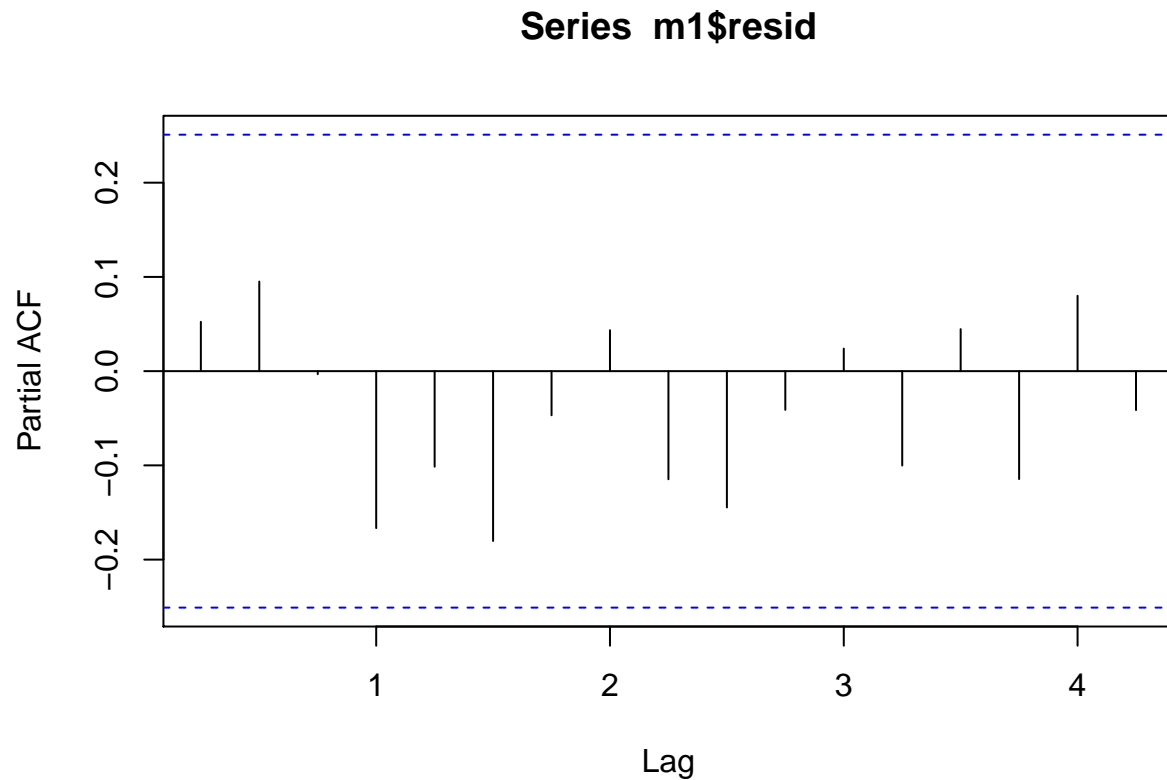
ma1 is not significant at all so it can be removed. Therefore, I will revise model 1 by get rid of MA(1)

residuals analysis

```
acf(m1$resid)
```



```
pacf(m1$resid)
```



ACF and PACF show a White Noise

Ljung box test on residuals

```
Box.test(m1$residuals, 6, "Ljung-Box", fitdf = 2)
```

```
##
## Box-Ljung test
##
## data: m1$residuals
## X-squared = 6.4622, df = 4, p-value = 0.1672
```

```
Box.test(m1$residuals, 12, "Ljung-Box", fitdf = 2)
```

```
##
## Box-Ljung test
##
## data: m1$residuals
## X-squared = 7.9634, df = 10, p-value = 0.6324
```

by Ljung-box test, residuals cannot reject hypothesis of white noise.

f)

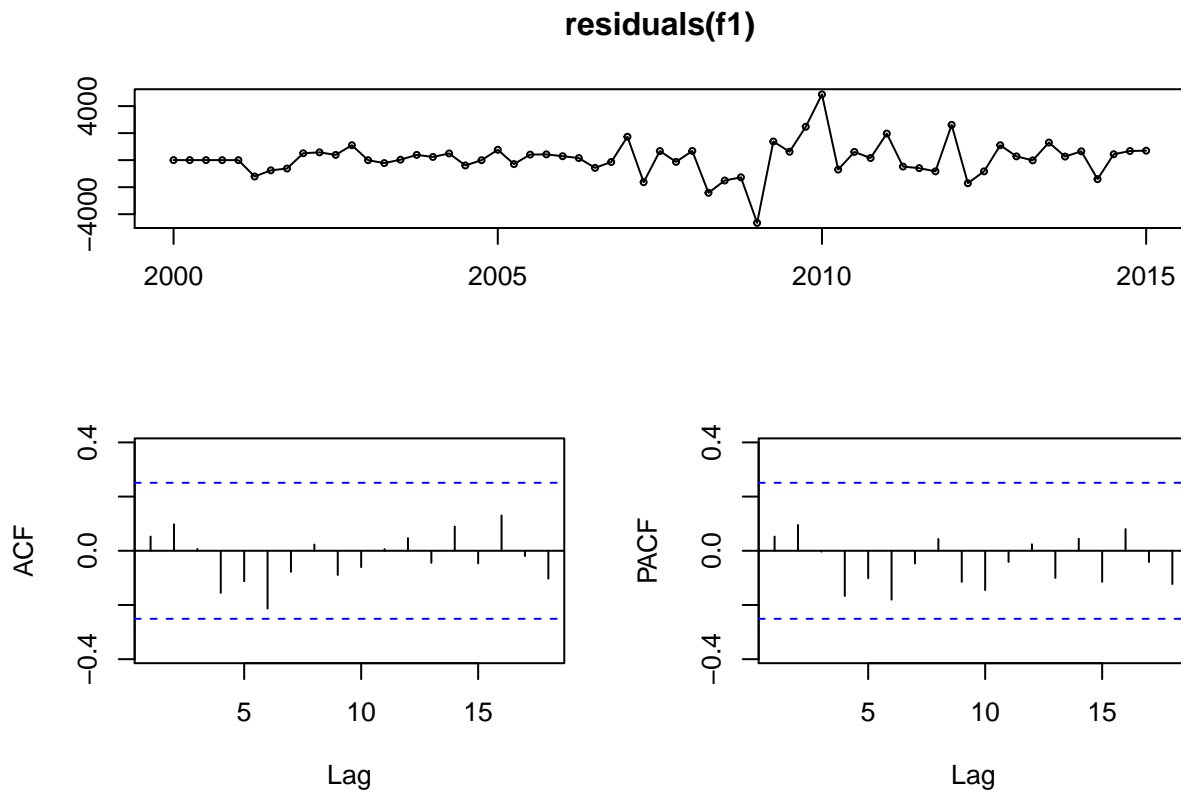
$(1+0.9654B) * (1-0.9920B^4) * (1-B) * (1-B^4) X_t = (1+0.9229B) * (1-0.9319B^4) a_t$

g) Compute forecasts for sales for the next 4 quarters.

```
f1 = forecast(m1, h=4)
f1
```

##	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
## 2015 Q2	77855.11	76072.60	79637.63	75128.99	80581.23
## 2015 Q3	81549.86	79079.22	84020.51	77771.33	85328.40
## 2015 Q4	83040.16	79995.26	86085.05	78383.39	87696.92
## 2016 Q1	108762.32	105268.39	112256.25	103418.82	114105.83

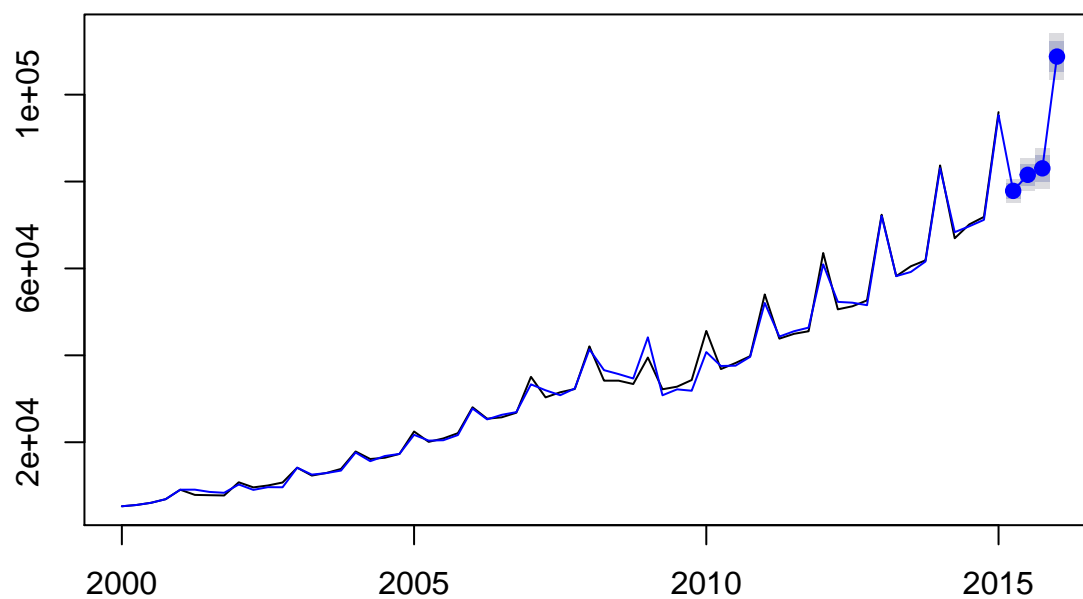
```
tsdisplay(residuals(f1))
```



h)

```
plot(f1, include = 100)
lines(ts(c(f1$fitted, f1$mean), frequency = 4, start = c(2000,1)), col = "blue")
```

Forecasts from ARIMA(1,1,1)(1,1,1)[4]



forecasts are consistent since it fluctuates around zero, and it slightly has an upward trend.