

Natural Logic

Foundation for language and reasoning

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20 August 2015

Smart Data Conference, San Jose

What is Natural Logic?

Symbolic logic isn't as “natural” as Aristotle's or Euclid's logic.

Cognitive scientists have been searching for a natural logic:

- **Linguists:** Natural logic is the “language of thought.”
- **Logicians:** Any logic, natural or otherwise, must be algebraic.
- **Computational linguists:** It's a kind of knowledge representation (KR).
- **Psychologists and neuroscientists:** Mental maps, models, or imagery provide the basis for language, thought, and reasoning.
- **Neural network researchers:** The layers of neurons in the cerebral cortex are sufficient for a “unified” model of intelligence.

Questions:

- **Is there any kind of logic underlying natural languages?**
- **If not, how can people do science, engineering, law, and business?**
- **If so, can we get any clues about its structure and operations?**
- **Could it support more intelligent computer systems?**
- **How would it relate to statistical methods for processing Big Data?**

Outline

1. Semantics of natural languages

Natural languages are highly flexible, expressive, and context dependent. These properties make them easy to use by people, but difficult to process by computer.

2. Visualization in language, logic, and mathematics

Language, gestures, thought, and mental models are intimately integrated. Creative discoveries in any field begin with visualization, and formal proofs are bookkeeping aids for checking the details.

3. Existential graphs and discourse representation structures

Peirce's EGs have the simplest and most general rules of inference ever discovered. Kamp's DRS notation, which is widely used for mapping language to and from logic, is isomorphic to a subset of EGs. Therefore, any computational method developed for either notation can be adapted to the other in exactly equivalent form.

4. Reasoning with and about images

Generalized EGs can accommodate arbitrary images or icons as an integral part of the notation. Peirce's rules can be applied to them, and methods of observation can be used to derive symbolic information from an icon and insert it into a formal proof.

1. Semantics of Natural Languages

Human language is based on the way people think about everything they see, hear, feel, and do.

And thinking is intimately integrated with perception and action.

The semantics and pragmatics of a language are

- **Situated in time and space,**
- **Distributed in the brains of every speaker of the language,**
- **Dynamically generated and interpreted in terms of a constantly developing and changing context,**
- **Embodied and supported by the sensory and motor organs.**

These points summarize current views by psycholinguists.

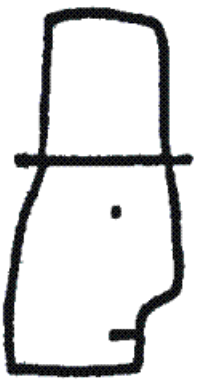
Philosophers and logicians have debated other issues:

- **NL as a formal logic; a sharp dichotomy between NL and logic; a continuum between NL and logic.**

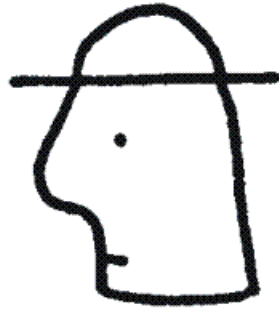
18 Theories of Language and Thought

- **Noam Chomsky:** Generative syntax is the essence of language.
- **Roman Jakobson:** Syntax without semantics is meaningless.
- **Michael Halliday:** Language is social semiotic.
- **Jerry Fodor:** Speech is generated from a language of thought.
- **Sydney Lamb:** Knowledge consists of connections in a network.
- **Richard Montague:** Any natural logic must be a formal logic.
- **Ludwig Wittgenstein:** Games (*Sprachspiele*) are the foundation.
- **Yorick Wilks:** Wittgenstein was right, but more detail is needed.
- **Roger Schank:** Background knowledge is essential.
- **Fred Jelinek:** Statistics is key to all language processing.
- **Lotfi Zadeh:** The fuzziness of language is not statistical.
- **Philip Johnson-Laird:** Language is mapped to mental imagery.
- **George Lakoff:** The mapping is done by image-based metaphors.
- **Len Talmy:** Language has the same semantics as cognition.
- **David McNeill:** Speech, gestures, and thought are intimately related.
- **Geoffrey Hinton:** Neural networks are the key to intelligence.
- **Marvin Minsky:** The Society of Mind has many keys.
- **Charles Sanders Peirce:** The mind is the totality of a lifetime of signs.

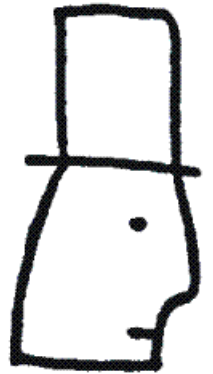
Relating Language to Perception



le curé est devant le ministre
the priest is in front of the minister

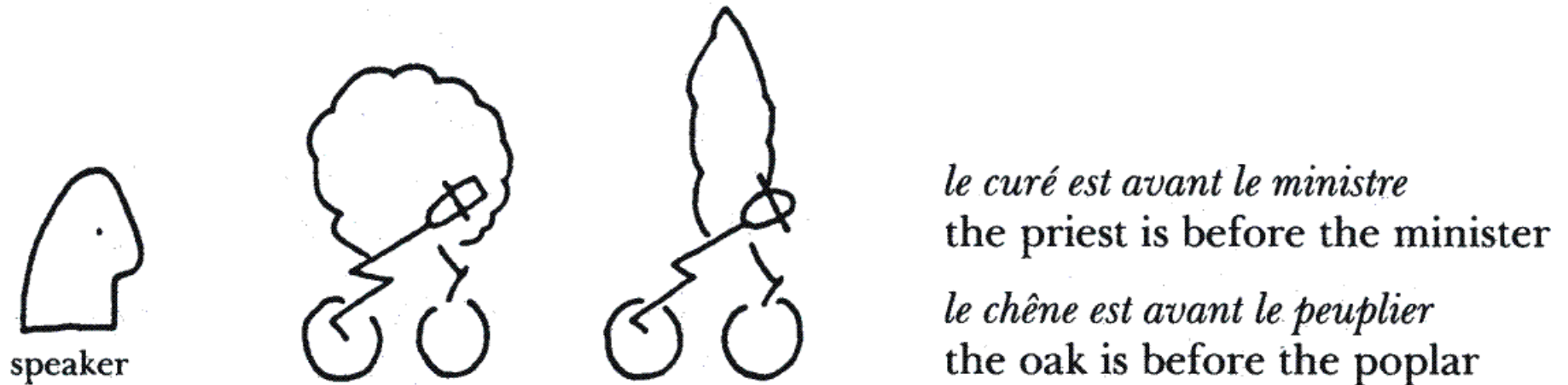


le curé est derrière le ministre
the priest is behind the minister



Vandeloise drew diagrams to explain spatial terms in French.
For most words, dictionaries list many meanings.
But the number of possible meanings is open ended.
Even the claim of a core literal meaning is doubtful.

Effect of Motion

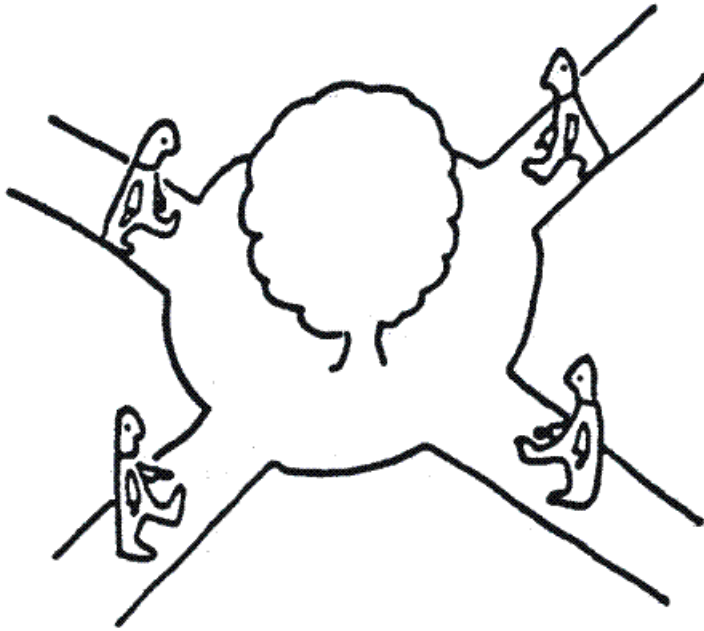


For stationary objects, such as trees, the speaker's viewpoint determines the choice of preposition.

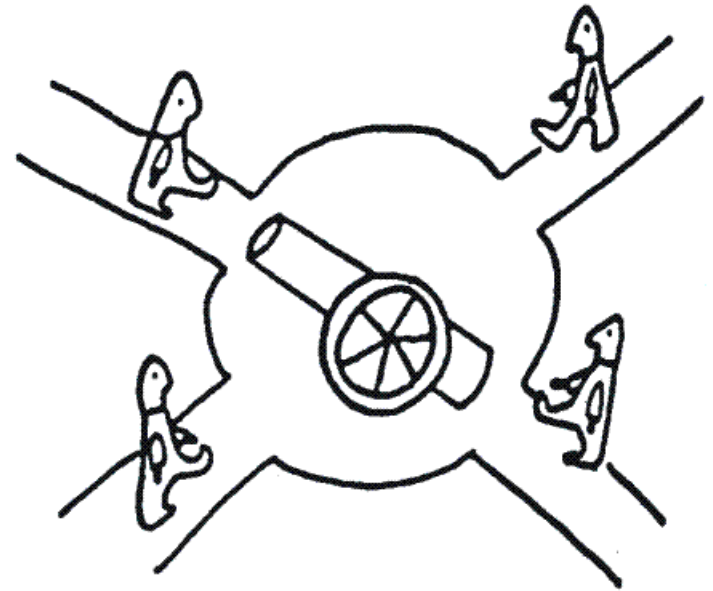
For moving objects, their relative position is more significant.

But objects like snails and turtles, which move very slowly, are treated like stationary objects (unless their motion is relevant).

Effect of Orientation



les présidents sont devant l'arbre
the presidents are in front of the tree



? *les présidents sont devant le canon*
the presidents are in front of the cannon

For a tree, any side could be considered the front.

But a cannon has distinct front, back, and sides.

Effect of Function



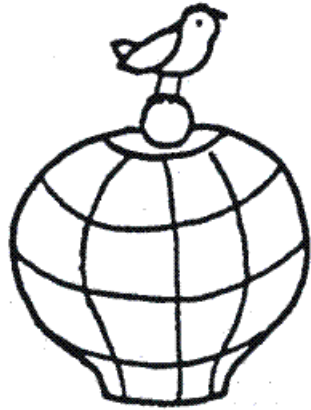
la poire est dans la coupe
the pear is in the bowl

The French preposition *dans* or the English *in* normally relates something to a container.

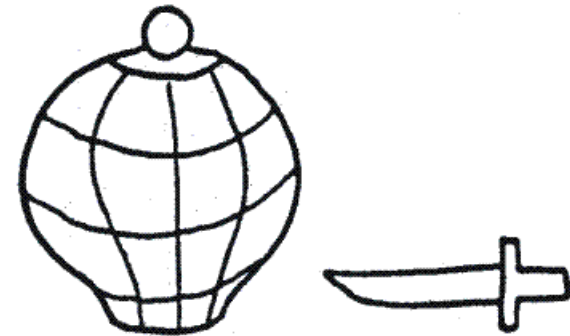
The primary function of a bowl is to serve as a container.

That function is more relevant than the question whether the bowl actually encloses the pear.

Effect of Background Knowledge



l'oiseau est à l'extérieur de la cage
the bird is outside the cage



? le couteau est à l'extérieur de la cage
the knife is outside the cage

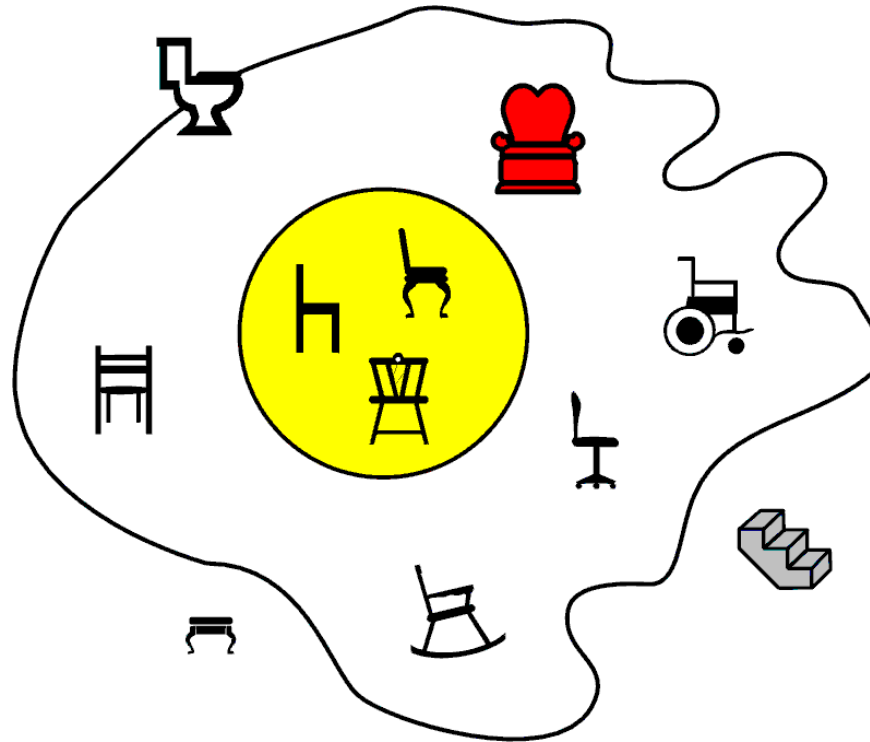
A cage is sometimes used to enclose a bird.

But a cage is an unlikely container for a knife.

Normal comment: “The knife is to the right of the cage.”

To say “The knife is outside the cage” implies that there is some reason why it might have been in the cage.

What is a Chair?



An egg-yolk diagram puts typical examples in the yolk and less common variants in the egg white. (Lehmann & Cohn 1994)

Boundaries resemble the *level cuts* of fuzzy set theory: the fuzzy value 0.9 could be the boundary for the yolk, and 0.7 for the egg white.

But the reasons for the variations are more significant than the numbers.

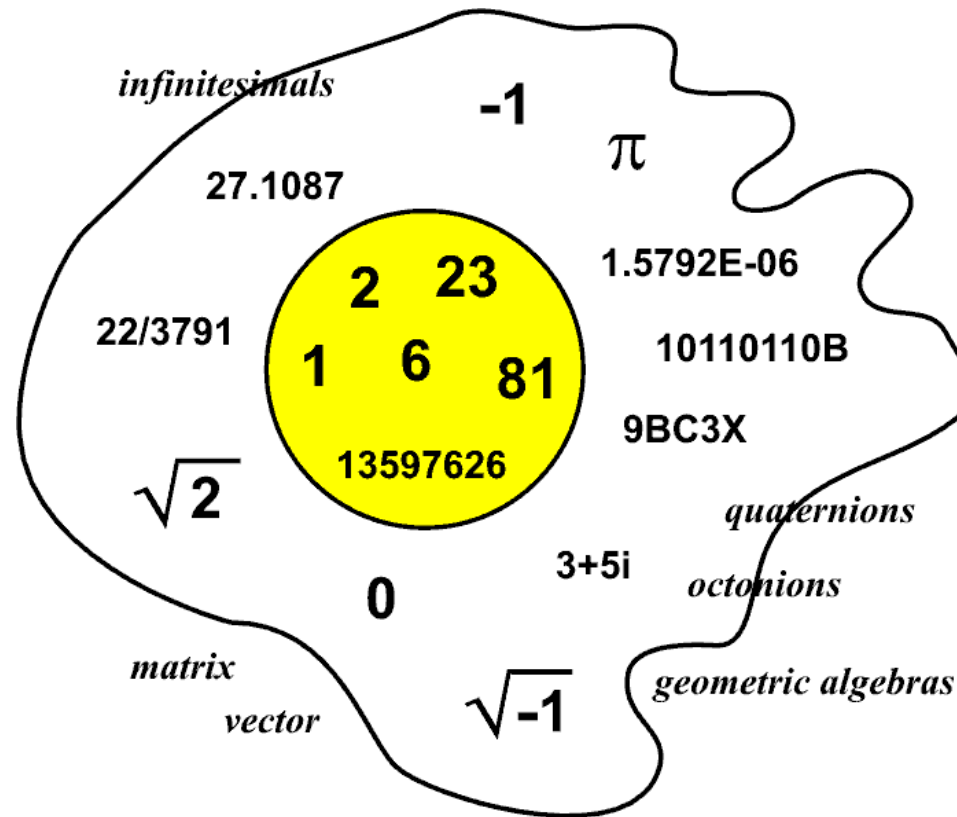
Is it a Chair? Art? Humor? Fantasy?



In a museum, it's funny. But suppose you saw it at night in an old castle. Meaning is always context dependent.

Claw and Ball Chair by Jake Cress. At the Smithsonian Renwick Gallery.

What is a Number?



Concepts in science and mathematics grow and change.

Consider the evolution in the basic terms of physics during the past century: *mass, energy, force, momentum, space, time, gravity, light, heat.*

Engineers often use different definitions of those terms for different components of the same system.

Relating Actions and Descriptions

Vagueness, fuzziness, and ambiguities are inevitable:

- No discrete set of words can precisely describe a continuous world.
- For any task, actions flow from one step to another with no breaks between the steps described by separate verbs.
- Observers emphasize different aspects of a task, describe them at varying levels of detail, and omit anything they consider “obvious”.

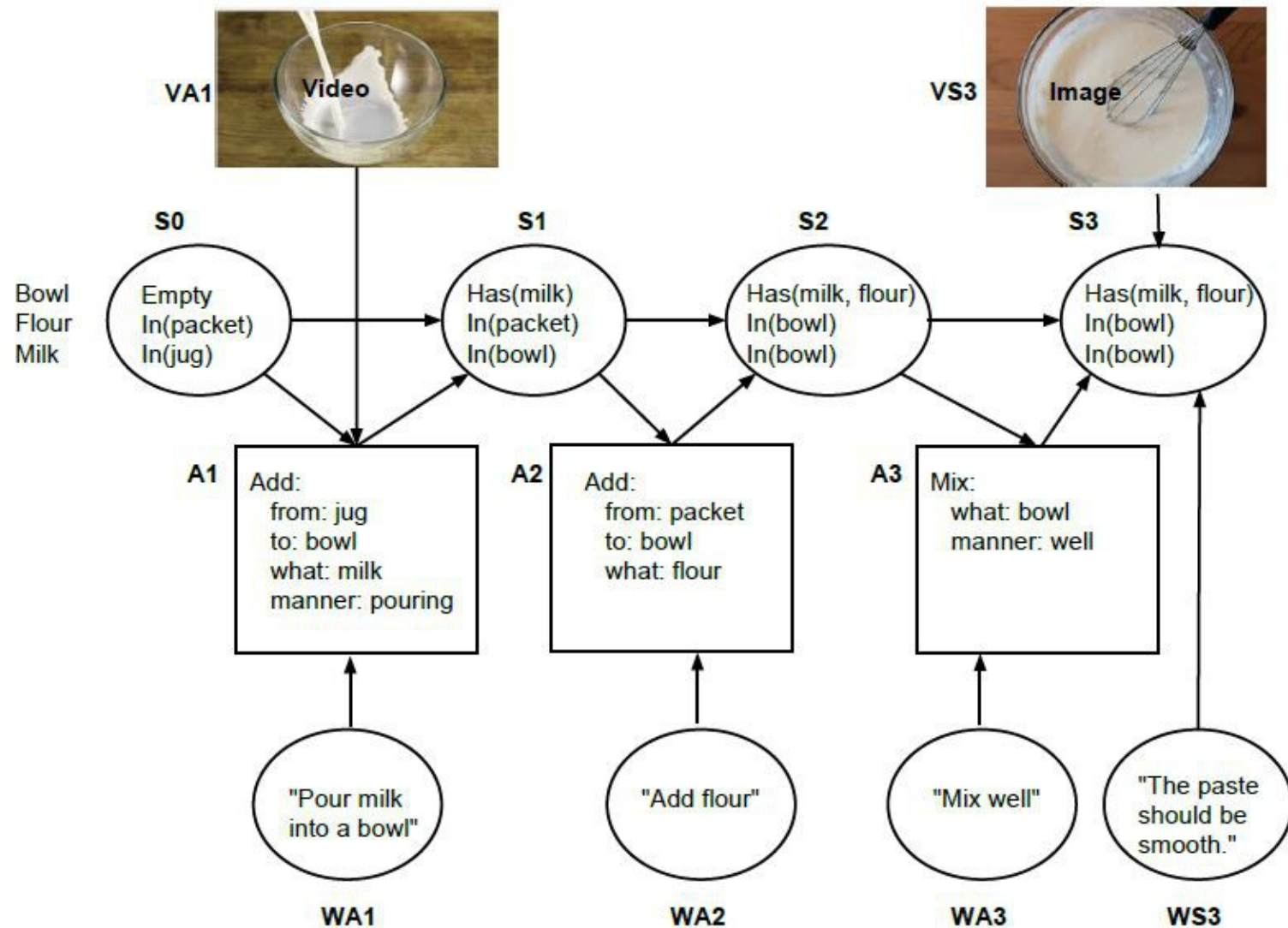
Two simple tasks in the domain of cooking:

- Making pancakes: Specify each step in a knowledge representation.
- Preparing a carrot: Relate a video to a sequence of KR statements.

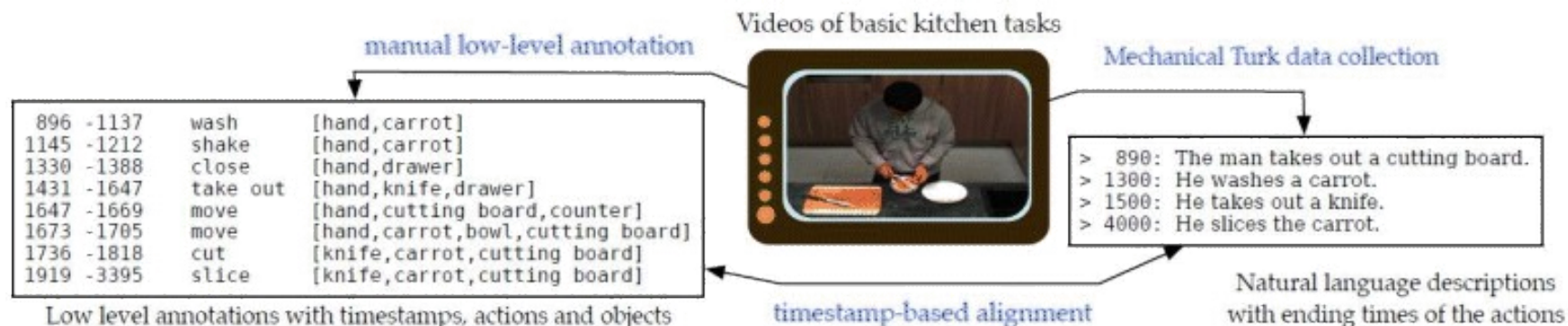
In each case, the KR is a pale reflection of the physical process:

- A computer system might answer some questions from the KR.
- A human cook would use non-linguistic habits to fill in the details.
- But no human or robot who was unfamiliar with cooking methods could successfully perform the task from the information in the KR.

Making Pancakes



Preparing a Carrot



Sample frame	Start	End	Action	Participants
	743	911	wash	hand, carrot
	982	1090	cut	knife, carrot, cutting board
	1164	1257	open	hand, drawer
	1679	1718	close	hand, drawer
	1746	1799	trash	hand, carrot

Sample frame	Start	End	Action	Participants
	1854	2011	wash	hand, carrot
	2011	2045	shake	hand, carrot
	2083	2924	slice	knife, carrot, cutting board
	2924	2959	scratch off	hand, carrot, knife, cutting board
	3000	3696	slice	knife, carrot, cutting board

Microsenses

The linguist Allen Cruse coined the term *microsense* for a specialized sense of a word in a particular application.

Examples of microsenses:

- Spatial terms in different situations and points of view.
- The many kinds of chairs or numbers in the egg whites.
- Methods of frying, baking, or slicing vary with the ingredients, the ethnic cuisine, the cook's training, and the available utensils.
- Computer science requires precise definitions, but the meanings change whenever programs are revised or extended.
- Consider the term *file system* in Unix, Apple OS X, Microsoft Windows, and IBM mainframes.

No finite set of words can have a fixed, precise set of mappings to a dynamically changing world.

“I don’t believe in word senses.”

The title is a quotation by the lexicographer Sue Atkins, who devoted her career to writing and analyzing word definitions.

In an article with that title,* Adam Kilgarriff observed that

- “A task-independent set of word senses for a language is not a coherent concept.”**
- The basic units of meaning are not the word senses, but the actual “occurrences of a word in context.”**
- “There is no reason to expect the same set of word senses to be relevant for different tasks.”**
- “The set of senses defined by a dictionary may or may not match the set that is relevant for an NLP application.”**
- Professional lexicographers are well aware of these issues.**
- The senses they select for a dictionary entry are based on editorial policy and assumptions about the readers’ expectations.**

*** See <http://www.kilgarriff.co.uk/Publications/1997-K-CHum-believe.pdf>**

Using Background Knowledge

People resolve ambiguities and choose the correct microsenses by retrieving background knowledge about the options.

Choosing the microsense: *My dog bit the visitor's ear.*

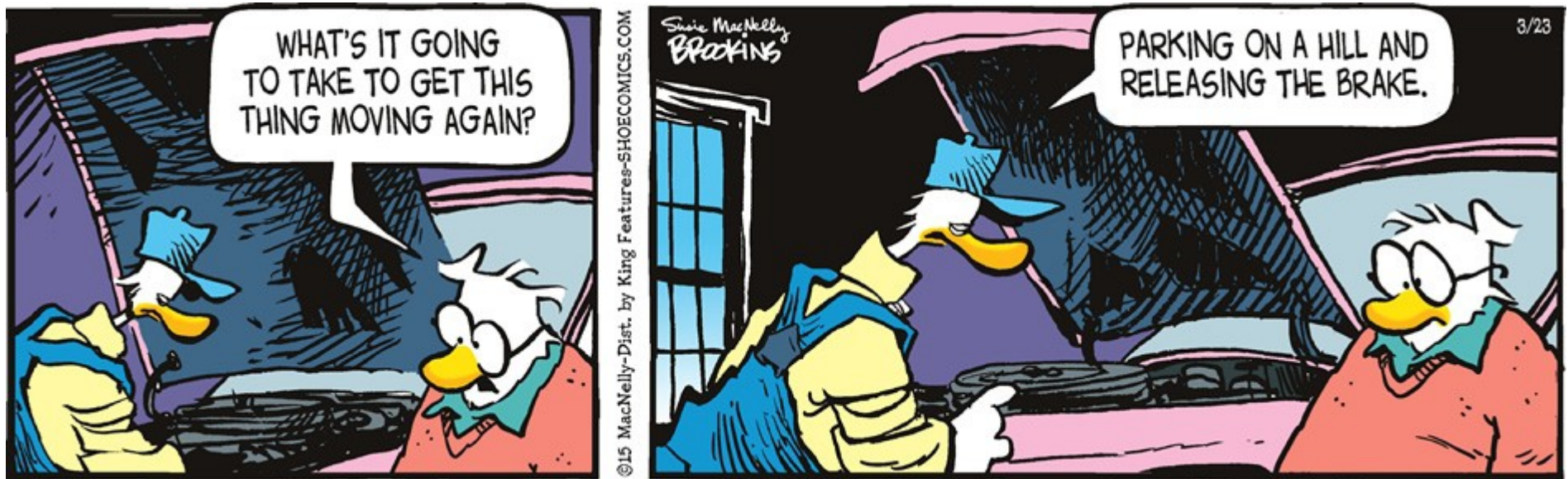
- From knowledge about the size of dogs, one would assume it was more likely to be a doberman than a dachshund.
- But if one knew the visitor was in the habit of bending over to pet a dog, it might even be a chihuahua.

Resolving an ambiguous parse: *The chicken is ready to eat.*

- From knowledge about typical food, one would assume the chicken had been cooked and prepared as a meal.
- If the word *chicken* were replaced with *dog*, one might assume the dog was begging for food.
- But people in different cultures may make different assumptions.

The many microsenses and the dependence on background knowledge require highly flexible methods of reasoning.

Understanding Cartoons and Comics



Relatively easy: Parse the question and the answer.

Much harder: Find and use background knowledge in order to

- Recognize the situation type and the roles of the two agents,
- Relate the word 'thing' to the picture and to the concept Car,
- Determine what would be taken and how the car would move.
- Use elementary physics to understand the answer.

Major challenge: Explain the irony and the humor.

* Search for 'moving' at <http://www.shoecomics.com/>

2. Visualization in Language and Logic

Mental models are more fundamental than language or logic.

- **Meanings expressed in language are based on perception.**
- **Thinking and reasoning are based on mental models that use the same mechanisms as perception and action.**
- **The notations of mathematics and logic are abstractions from the symbols and patterns in natural languages.**

Computers can manipulate symbols faster than any human.

- **But they are much less efficient in perception and action.**
- **That limitation makes them unable to process language in the same way that people do.**

How could computers support human-like methods?

Bird Nest Problem

Robots can perform many tasks with great precision.

But they don't have the flexibility to handle unexpected shapes.

They can't wash dishes the way people do — with an open-ended variety of shapes and sizes.

And they can't build a nest in an irregular tree with irregular twigs, straw, and moss.

If a human guides a robot through a complex task with complex material, the robot can repeat the same task in the same way.

But it doesn't have the flexibility of a bird, a beaver, or a human.



Intelligence and Tools

The ability to make tools is a sign of intelligence:

- All animals, including humans, are born with some built-in tools.
- Birds can't wash dishes for the same reason that humans can't wash dishes with a sewing machine: they have the wrong tools.
- Humans make and use the most elaborate tools, but biologists keep discovering many species that make and use tools.

The role of instinct:

- Birds have an instinct to build nests, beavers have an instinct to build dams, and humans have an instinct to speak a language.
- But the details of the nests, dams, and languages depend on the animals' built-in tools, the environment, learning from parents, and creativity.

Language is a kind of mental tool: *

- The power and flexibility of language results from its integration with every aspect of perception, action, reasoning, and learning.

* Stout, Toth, Schick, & Chaminade (2008) [Neural correlates of early stone age toolmaking: technology, language and cognition in human evolution.](#)

American Sign Language



MANY

DEAF

LEARN

AGENT



ENTER



SPEAK



COLLEGE

Many deaf students enter hearing colleges.

**The order of signs in ASL is similar to English word order.
But many syntactic features are absent; others are different.**

Diagram adapted from Lou Fant (1983) *The American Sign Language Phrase Book*.

Spoken and Signed Language

The same neural mechanisms are used to produce and interpret spoken and signed languages. (Petitto 2005)

Studies of bilingual infants of parents with different languages:

- **All pairs of four languages: English, French, American Sign Language (ASL), and Langue des Signes Québécoise (LSQ).**
- **Monolingual and bilingual babies go through the same stages and at the same ages for both spoken and signed languages.**
- **Hearing babies born to profoundly deaf parents babble with their hands, but not vocally.**
- **Babies bilingual in a spoken and a signed language babble in both modalities – vocally and with their hands.**
- **And they express themselves with equal fluency in their spoken and signed language at every stage of development.**

Petitto's conclusion: Any hypothesis about a Language Acquisition Device (LAD) must be independent of modality.

Spatio-Temporal Syntax

Signed and spoken languages have a time-ordered sequence.

But signed languages take advantage of 3-D space:

- **For anything visible, pointing serves the role of pronouns.**
- **But references to people and things that left the scene are also possible by pointing to where they had been.**
- **The signer can also introduce new characters and things, place them in fixed locations in the air, and refer to them by pointing.**
- **For spatial relations, signing is more “natural” than spoken language.**

Observation: The index finger is the most natural indexical.

Some theoretical questions:

- **Must a language of thought include geometry of the environment?**
- **If so, should it still be called a “language” of thought?**
- **A better term might be “cognitive map” or “mental model.”**

Visualization in Mathematics

Paul Halmos, mathematician:

“Mathematics — this may surprise or shock some — is never deductive in its creation. The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas, and becomes convinced of their truth long before he can write down a logical proof... the deductive stage, writing the results down, and writing its rigorous proof are relatively trivial once the real insight arrives; it is more the draftsman’s work not the architect’s.” *

Albert Einstein, physicist:

“The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be *voluntarily* reproduced and combined... The above-mentioned elements are, in my case, of visual and some of muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage, when the mentioned associative play is sufficiently established and can be reproduced at will.” **

*** Halmos (1968). ** Quoted by Hadamard (1945).**

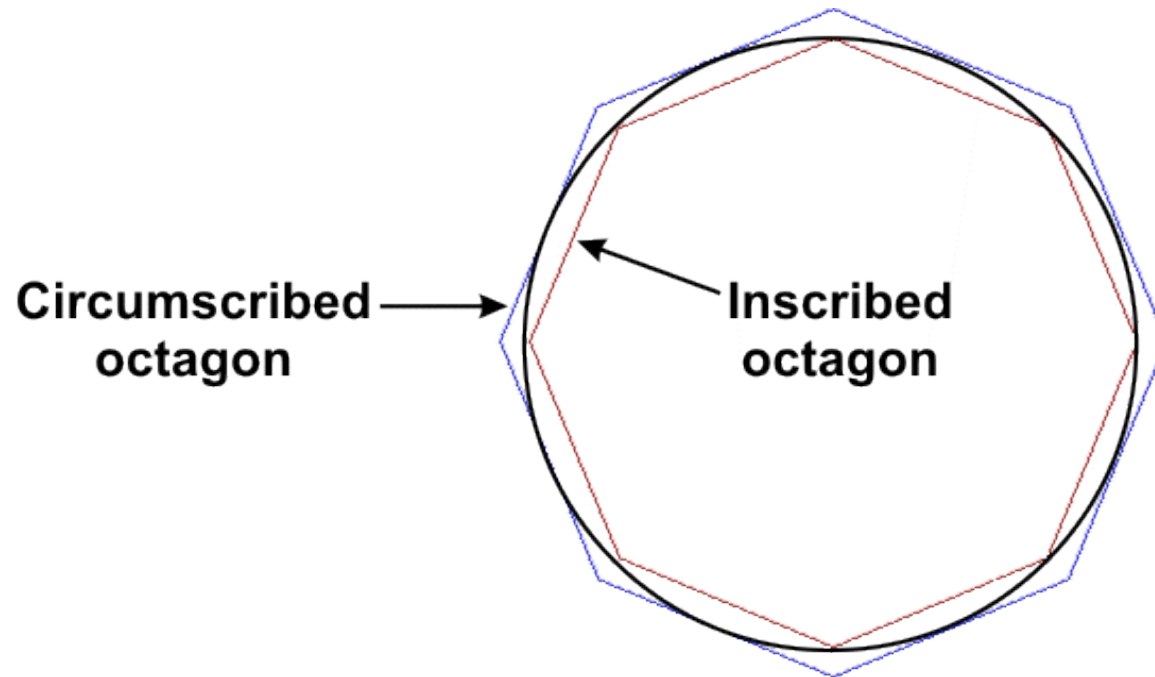
Archimedes' Eureka Moment



Insight: A submerged body displaces an equal volume of water.

- It's a mathematical principle, a property of Euclidean space.
- Scientists and engineers have used it ever since.
- They don't prove it. They use it to define *incompressible fluid*.

Determining the Value of π

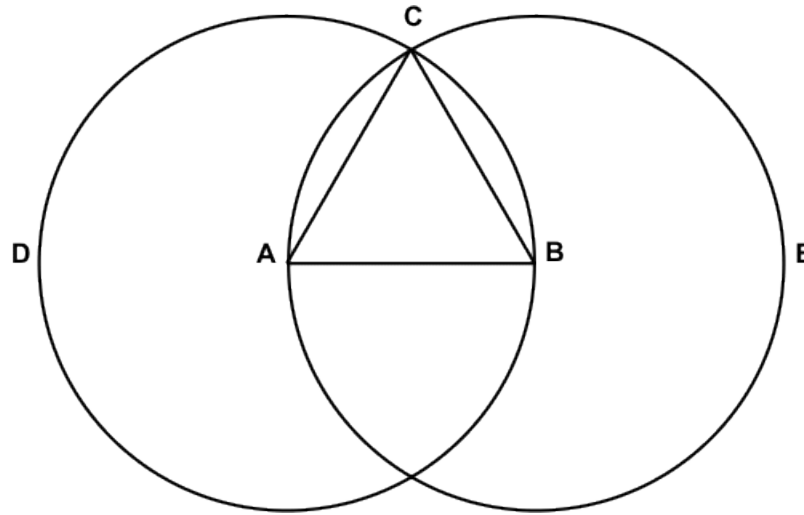


Archimedes had two creative insights:

- The circumference of the circle is greater than the perimeter of the inner polygon and less than that of the outer polygon.
- As the number of sides increases, the inner polygon expands, and the outer polygon shrinks. They converge to the circle.

Given these insights, a good mathematician could compute π to any desired precision. Archimedes used 96-gons.

Euclid's Proposition 1



Euclid's statement, as translated by Thomas Heath:

- **On a given finite straight line, to draw an equilateral triangle.**

The creative insight is to draw two circles:

- **The circle with center at A has radii AB and AC.**
- **The circle with center at B has radii BA and BC.**
- **Since all radii of a circle have the same length, the three lines AB, AC, and BC form an equilateral triangle.**

Icons and Diagrams

Three kinds of signs: Icons, indexes, and symbols.

- **An icon has a structural resemblance to its referent.**
- **An index points to its referent by some connection.**
- **A symbol indicates its referent by a habit or convention.**

Algebraic notations combine symbols and indexes with linear icons for the operators and transformation rules.

Diagrams use more general icons to represent relations:

- **A road map is an icon of a road system with symbolic labels.**
- **A topographic map is a labeled icon of some land surface.**
- **A map of roads plus topography can combine both.**

Implicit Information in an Icon



A map (icon) of the Atlantic Ocean and surrounding countries.

- Which country is closer to Africa: Canada or the USA?
- Observe the two congruent lines, which are determined by 4 data points.
- The possible observations grow as D^n — where D is the number of data points in the icon, and n is the number used in the observation.

The information implicit in an icon can be far greater than the information explicitly encoded in symbols and indexes.

The Role of Icons in Logic

Icons can make logic more readable.

But they can also play a more fundamental role:

- **Visual images used for discovery or teaching can be used in deduction.**
- **Implicit information in icons can be much larger than the explicit information in symbols.**
- **That information can support heuristics for selecting axioms or background knowledge.**
- **A logic with icons is more natural than a logic without icons.**

Questions:

- **How can icons relate logic to natural languages?**
- **How would they relate to the other branches of cognitive science?**
- **How would icons affect computational complexity?**
- **Can they do anything that could not be computed by a Turing machine?**

Semantics is Based on Icons

Richard Montague (1970):

“I reject the contention that an important theoretical difference exists between formal and natural languages.”

Hans Kamp (2001):

“The basic concepts of linguistics — and especially those of semantics — have to be thought through anew... Many more distinctions have to be drawn than are dreamt of in current semantic theory.”

Barbara Partee (2005):

“The present formalizations of model-theoretic semantics are undoubtedly still rather primitive compared to what is needed to capture many important semantic properties of natural languages...”

Those “many more distinctions” are derived from icons.

Icons map the world to perception and action. They relate the world to the symbols and indexes of a language or logic.

Situated Simulation

Neural and psychological research by Lawrence Barsalou: *

- Mental simulation is the re-enactment of perceptual, motor and introspective states acquired during experience.
- Unconscious re-enactments occur during memory and reasoning.
- Conscious re-enactments are usually called mental imagery.

Cognition is grounded in perception, action, and internal states.

- Simulations can re-enact social interactions in situations.
- Simulated imagery can stimulate the same emotions as perception.

Mirror neurons promote learning and social understanding:

- The neurons used in performing an action are also activated in seeing another person perform the same action.
- Simulations in motor and emotional systems are critical to empathy, social understanding, and successful cooperation.

* See <http://www.psychology.emory.edu/cognition/barsalou/onlinepapers.html>

Mental Maps, Images, and Models

Quotation by the neuroscientist Antonio Damasio (2010):

“The distinctive feature of brains such as the one we own is their uncanny ability to create maps... But when brains make maps, they are also creating images, the main currency of our minds. Ultimately consciousness allows us to experience maps as images, to manipulate those images, and to apply reasoning to them.”

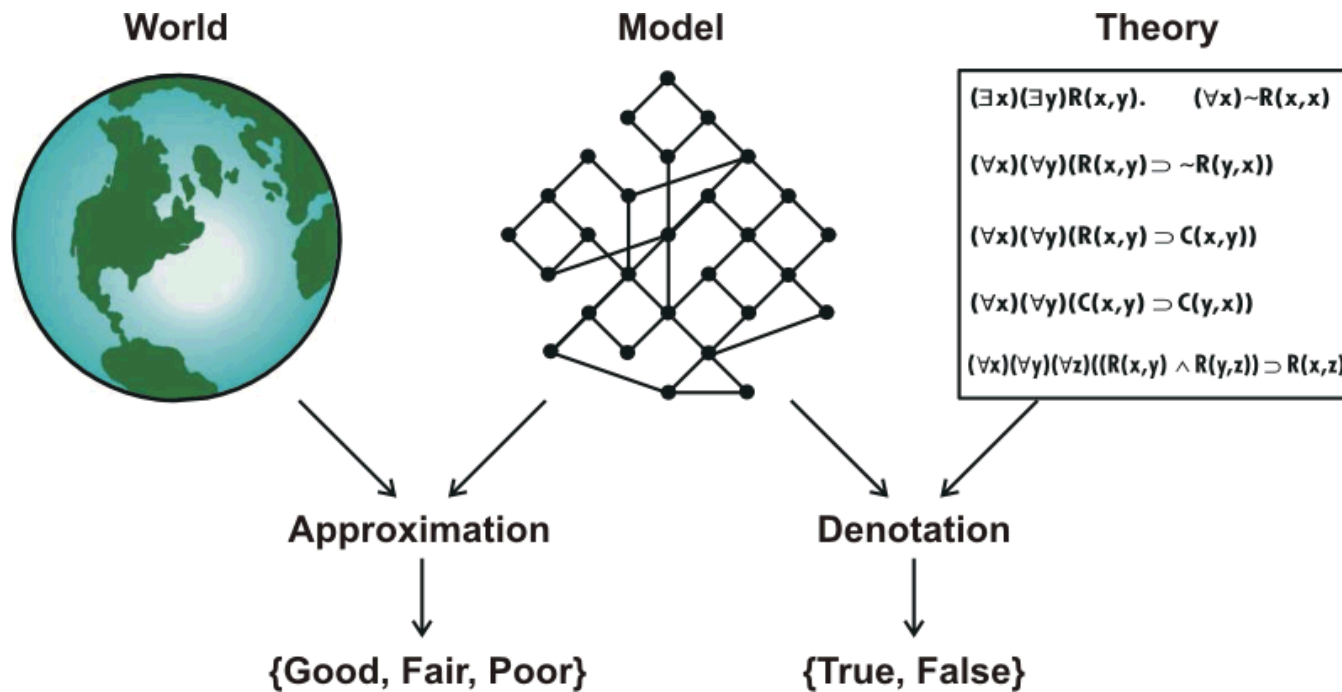
The maps and images form mental models of the real world or of the imaginary worlds in our hopes, fears, plans, and desires.

Words and phrases of language can be generated from them.

They provide a “model theoretic” semantics for language that uses perception and action for testing models against reality.

Like Tarski’s models, they define the criteria for truth, but they are flexible, dynamic, and situated in the daily drama of life.

Model-Theoretic Semantics



In the middle is a Tarski-style model that relates a theory to the world.

Logicians: The model is a subset of the world.

Engineers: *“All models are wrong, but some are useful.”*

The engineering view is more realistic.

For a natural logic, the models used by logicians and engineers are just a small subset of the full range of mental models.

Feelings and Emotions

Damasio and Carvalho (2013),

- **“Feelings are mental experiences of body states.”**
- **“They signify physiological need, tissue injury, optimal function, threats to the organism, or specific social interactions.”**
- **“Feelings constitute a crucial component of the mechanisms of life regulation, from simple to complex.”**
- **“Their neural substrates can be found at all levels of the nervous system, from individual neurons to subcortical nuclei and cortical regions.”**

Damasio (2014), *

- **“I’m ready to give the very teeny brain of an insect – provided it has the possibility of representing its body states – the possibility of having feelings.”**
- **“Of course, what flies don’t have is all the intellect around those feelings that could make use of them: to found a religious order, or develop an art form, or write a poem.”**

* Interview, <http://www.technologyreview.com/qa/528151/the-importance-of-feelings/>

The Ultimate Understanding Engine

Sentences uttered by a child named Laura before the age of 3. *

Here's a seat. It must be mine if it's a little one.

I went to the aquarium and saw the fish.

I want this doll because she's big.

*When I was a little girl, I could go “geek geek” like that,
but now I can go “This is a chair.”*

Laura used a larger subset of logic than Montague formalized.

**No computer system today has Laura's ability to learn, speak,
and understand language.**

* John Limber, The genesis of complex sentences. In T. Moore (Ed.), *Cognitive development and the acquisition of language*. New York: Academic Press, 1973.
http://pubpages.unh.edu/~jel/JLimber/Genesis_complex_sentences.pdf

3. Existential Graphs

Peirce claimed that existential graphs (EGs) represent “a moving picture of the action of the mind in thought.”

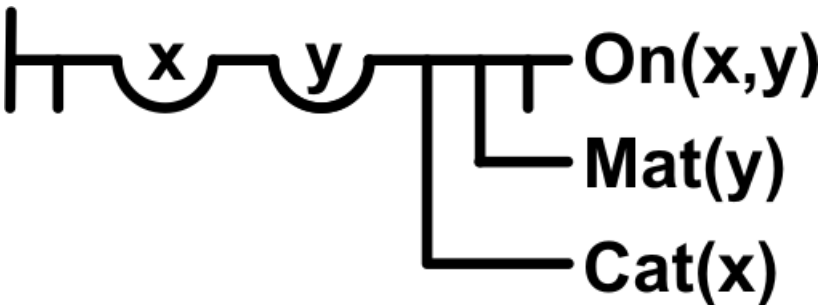
The psycholinguist Philip Johnson-Laird agreed: Peirce’s EGs and rules of inference are a good candidate for a natural logic.

Discourse representation structures, which Hans Kamp designed to represent NL semantics, are isomorphic to a subset of EGs.

Three pairs of simple, very general rules:

- **Simplification and generalization of Gentzen’s natural deduction.**
- **The same rules can be used for propositional logic, first-order logic, higher-order logic, and many variations and extensions.**
- **They can be used with graphic or linear notations, including Common Logic, the Semantic Web, and controlled natural languages (CNLs).**
- **Many advantages: Philosophical, psychological, linguistic, formal, computational, and pedagogical.**

How to say “A cat is on a mat.”

Gottlob Frege (1879): 

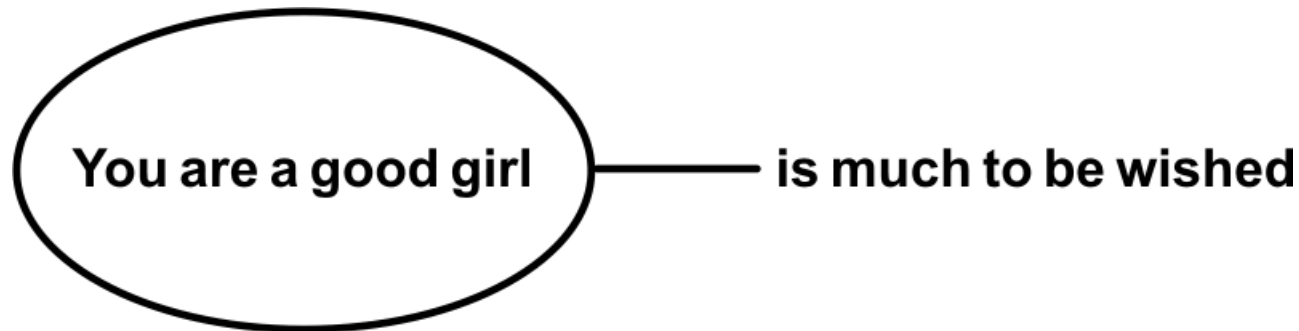
Charles Sanders Peirce (1885): $\Sigma_x \Sigma_y \text{Cat}_x \bullet \text{Mat}_y \bullet \text{On}_{x,y}$

Giuseppe Peano (1895): $\exists x \exists y \text{Cat}(x) \wedge \text{Mat}(y) \wedge \text{On}(x,y)$

Charles Sanders Peirce (1897): $\text{Cat} \text{ — } \text{On} \text{ — } \text{Mat}$

Metalanguage

A relation attached to an oval makes a metalevel comment about the proposition expressed by the nested graph. *



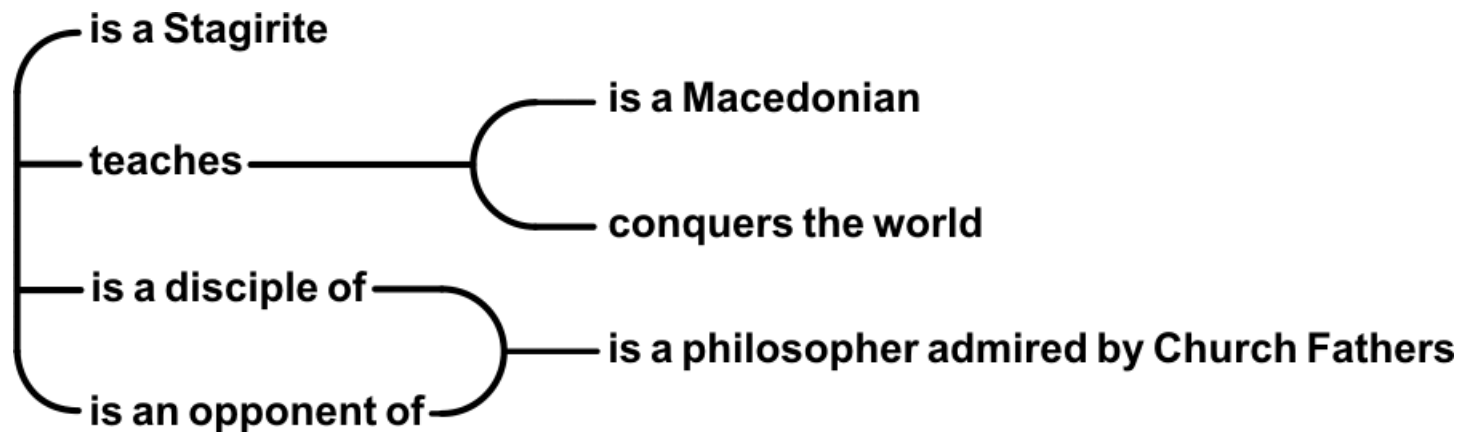
Peirce allowed the names of relations to contain blanks.

**The relation named 'You are a good girl' has zero pegs.
It is an EG that expresses a proposition p .**

**The relation named 'is much to be wished' has one peg, which
is attached to a line that states the existence of proposition p .**

*** From Charles Sanders Peirce, *Reasoning and the Logic of Things*, The Cambridge Conferences Lectures of 1898, Harvard University Press, p. 151.**

Expressing Simple Relations



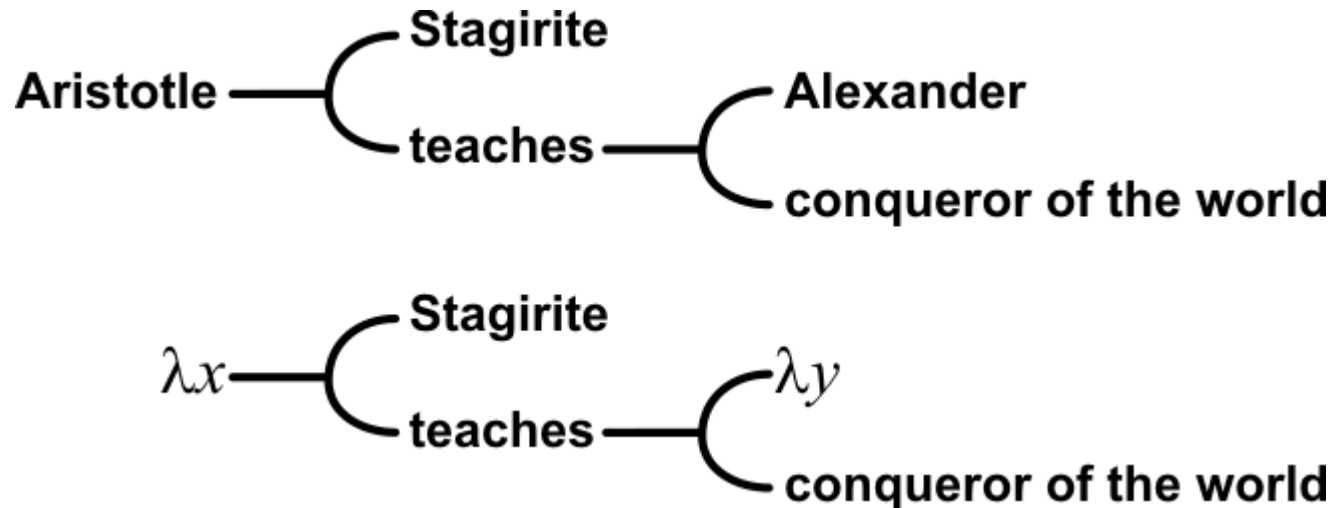
English: *“There exists an x , which is a Stagirite, x teaches some y , which is a Macedonian that conquers the world, x is a disciple of some z , which is a philosopher admired by Church Fathers, and x is an opponent of z .”*

The only logical operators are existence and conjunction:

$$\begin{aligned} \exists x \exists y \exists z & (\text{isaStagirite}(x) \wedge \text{teaches}(x,y) \wedge \text{isaMacedonian}(y) \wedge \\ & \text{conquersTheWorld}(y) \wedge \text{isaDiscipleOf}(x,z) \wedge \text{isanOpponentOf}(x,z) \\ & \wedge \text{isaPhilosopherAdmiredByChurchFathers}(z)). \end{aligned}$$

This subset of logic can represent the content of any relational DB or any RDF DB (including blank nodes in RDF).

Lambda Abstraction



The top EG says *Aristotle is a Stagirite who teaches Alexander who conquers the world.*

In the EG below it, the names Aristotle and Alexander are erased, and their places are marked with the Greek letter λ .

That EG represents a dyadic relation: ____ *is a Stagirite who teaches* ____ *who conquers the world.*

Peirce used an underscore to mark those empty places, but Alonzo Church marked them with λ .

Syntax of Existential Graphs

Example: Cat — On — Mat

- Two lines mean there exists something x and something y .
- Cat and Mat are *monadic* relations; On is a *dyadic* relation.

Four primitives:

- Relation: A character string with zero or more *pegs*.
- Existence: A *line* that means “There exists something.”
- Conjunction: Two or more graphs in the same area.
- Metalanguage: An *oval* that encloses some graph or subgraph.

Four combinations:

- Argument: A line attached to a peg of some relation.
- Equality: Two or more connected lines (called a *ligature*).
- Metacomment: A line that connects an oval to a relation.
- Negation: A *shaded oval* that represents the comment *not*.

First-Order Logic

Shaded ovals are sufficient to express full FOL:

Existence: —

Negation: 

Relations: Cat- -On- -Under- -With- -Mat

A cat is on a mat: Cat—On—Mat

Something is under a mat: —Under—Mat

Some cat is not on a mat: Cat——Mat

Some cat is on something that is not a mat: Cat—On—Mat

Boolean Combinations

Areas nested inside an odd number of negations are shaded.

p q

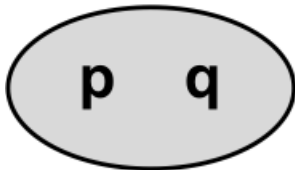
p and q



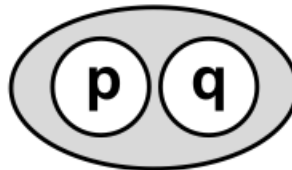
not p and not q



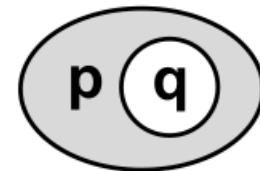
p and not q



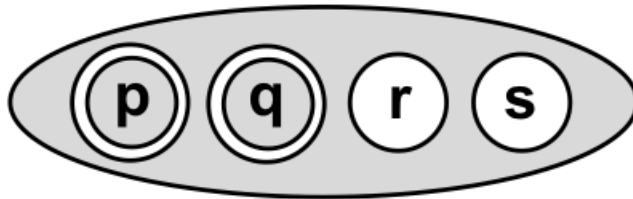
not (p and q)



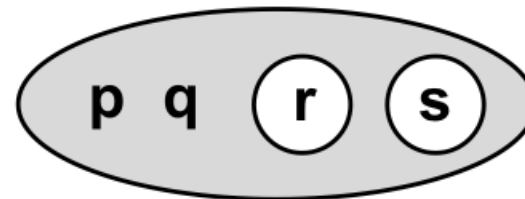
p or q



if p, then q



not p or not q or r or s



if p and q, then r or s

The Scope of Quantifiers

The scope is determined by the outermost point of any line.

Cat—Black

Some cat is black.

Cat—Black

Some cat is not black.

Cat—Black

No cat is black.

Cat—Black

*It is false that
some cat is not black.*

Cat—Black

*If there is a cat,
then it is black.*

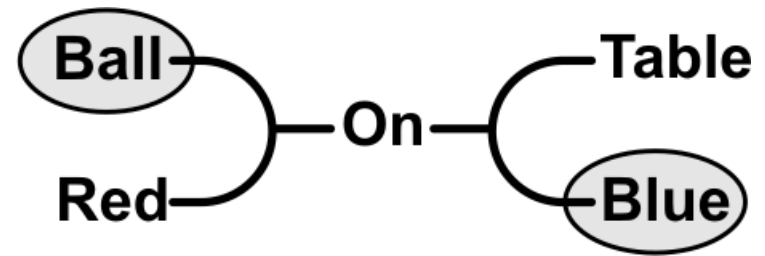
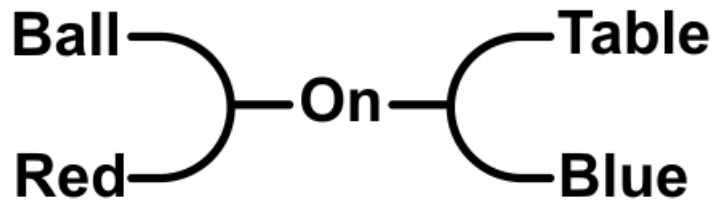
Cat—Black

Every cat is black.

The bottom graph can be read in three equivalent ways.

Translating EGs to and from English

Most existential graphs can be read in several equivalent ways.



Left graph:

A red ball is on a blue table.

Some ball that is red is on some table that is blue.

Right graph:

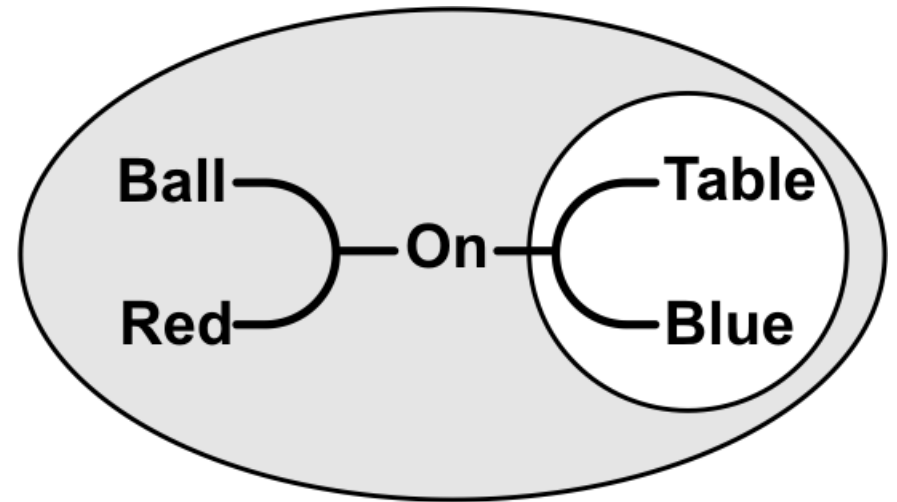
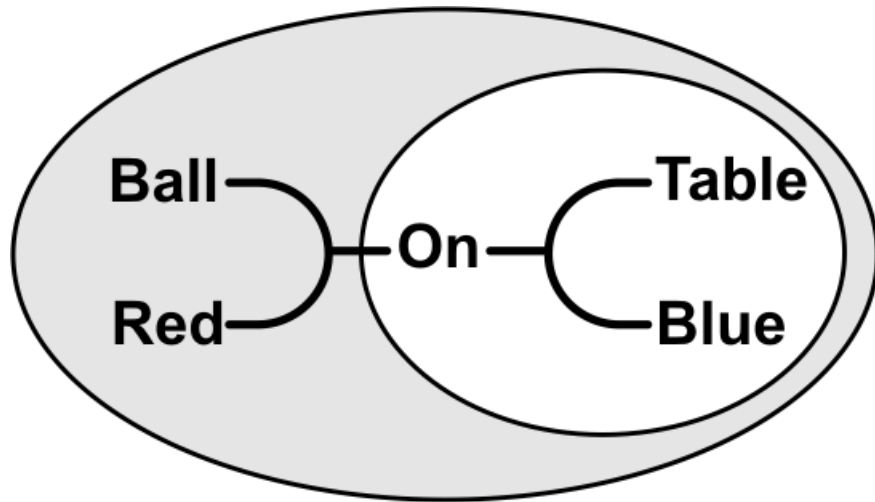
Something red that is not a ball is on a table that is not blue.

A red non-ball is on a non-blue table.

On some non-blue table, there is something red that is not a ball.

Scope of Quantifiers and Negations

Ovals define the scope for both quantifiers and negations.



Left graph:

If there is a red ball, then it is on a blue table.

Every red ball is on some blue table.

Right graph:

If a red ball is on something x , then x is a blue table.

Existential Graph Interchange Format

A subset of the Conceptual Graph Interchange Format (CGIF):

Existence: — [*x]

Negation:  ~[]

Relations: (Cat ?x) (On ?x ?y) (Under ?x ?y) (Mat ?y)

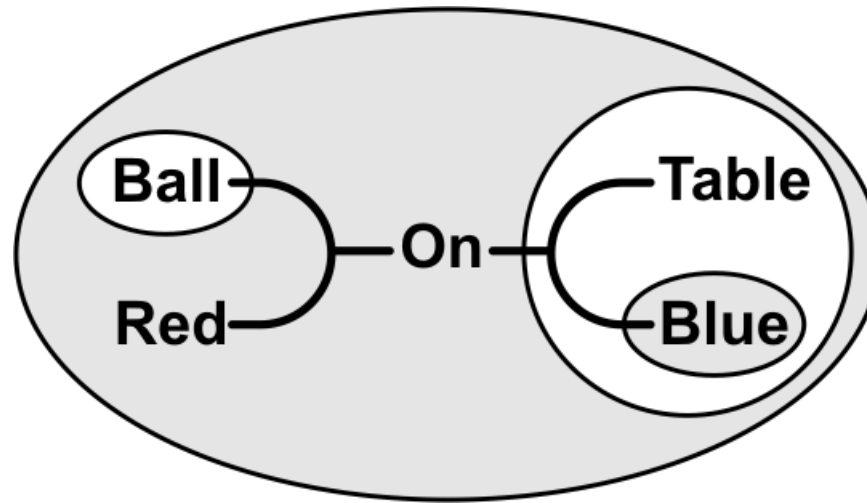
A cat is on a mat: [*x] [*y] (Cat ?x) (On ?x ?y) (Mat ?y)

Something is under a mat: [*x] [*y] (Under ?x ?y) (Mat ?y)

Some cat is not on a mat: [*x] (Cat ?x) ~[*y] (On ?x ?y) (Mat ?y)]

Some cat is on something that is not a mat:
[*x] [*y] (Cat ?x) (On ?x ?y) ~[(Mat ?y)]

Mapping EG to EGIF



If something red that is not a ball is on something y, then y is a table that is not blue.

$\sim[[[*x] [*y] (\text{Red } ?x) \sim[(\text{Ball } ?x)] (\text{On } ?x ?y) \sim[(\text{Table } ?y) \sim[(\text{Blue } ?y)]]]$

A one-to-one mapping of EG features to EGIF features:

- Two ligatures of connected lines map to $[*x]$ and $[*y]$.
- Four ovals map to four negations, represented as $\sim[]$.
- Five EG relation names map to five EGIF relation names.
- Six pegs of the relations map to six occurrences of $?x$ or $?y$.

Issues in Mapping Language to Logic

Hans Kamp observed that the features of predicate calculus do not have a direct mapping to and from natural languages. *

Pronouns can cross sentence boundaries, but variables cannot.

- Example: *Pedro is a farmer. He owns a donkey.*
- $(\exists x)(\text{Pedro}(x) \wedge \text{farmer}(x)). (\exists y)(\exists z)(\text{owns}(y,z) \wedge \text{donkey}(z)).$
- There is no operator that can relate x and y in different formulas.

The rules for scope of quantifiers are different.

- Example: *If a farmer owns a donkey, then he beats it.*
- In English, quantifiers in the if-clause govern the then-clause.
- But in predicate calculus, the quantifiers must be moved to the front.
- Formula: $(\forall x)(\forall y)((\text{farmer}(x) \wedge \text{donkey}(y) \wedge \text{owns}(x,y)) \supset \text{beats}(x,y)).$

In narratives, the default operator between NL sentences is usually equivalent to *and then*.

* Hans Kamp & Uwe Reyle (1993) *From Discourse to Logic*, Dordrecht: Kluwer.

Translating the Word *is* to Logic

Three different translations in the algebraic notation:

- Existence: *There is x.* $\leftrightarrow \exists x$
- Predication: *x is a cat.* $\leftrightarrow \text{Cat}(x)$
- Identity: *x is y.* $\leftrightarrow x=y$

Do these three translations imply that English is ambiguous?

Or is the algebraic notation unnecessarily complex?

In EGs, all three uses of the word *is* map to a line of identity:

- Existence: *There is x.* $\leftrightarrow \text{—}$
- Predication: *x is a cat.* $\leftrightarrow \text{—Cat}$
- Identity: *x is y.* $\leftrightarrow \text{——}$ (a ligature of two lines)

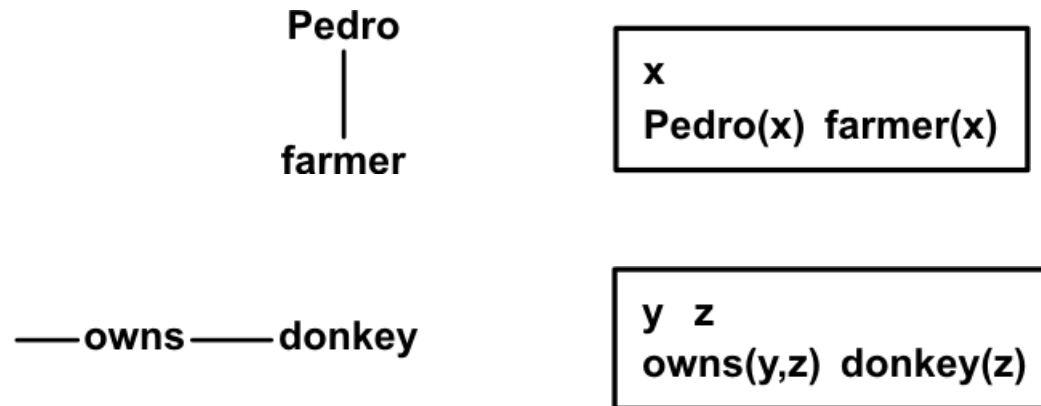
As Peirce said, EGs are more iconic than the algebraic notation: they relate language to logic more clearly and directly.

Frege and Russell were misled by their notations.

Linking Existential Quantifiers

Kamp invented Discourse Representation Structure (DRS) as a logic with a simpler mapping to and from NLS.

EGs (left) and DRSES (right) support equivalent operations.



By connecting EG lines or merging DRS boxes,

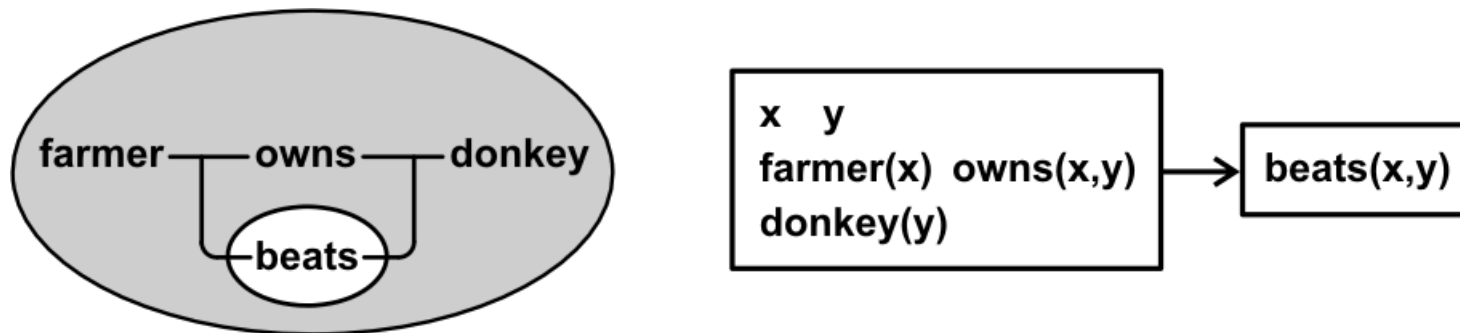


Quantifiers in EG and DRS

Peirce and Kamp independently chose isomorphic structures.

- Peirce chose nested ovals for EG with lines to show coreference.
- Kamp chose boxes for DRS with variables to show coreference.
- But the boxes and ovals are isomorphic: they have the same constraints on the scope of quantifiers, and they have the same mapping to NLS.

Example: *If a farmer owns a donkey, then he beats it.*

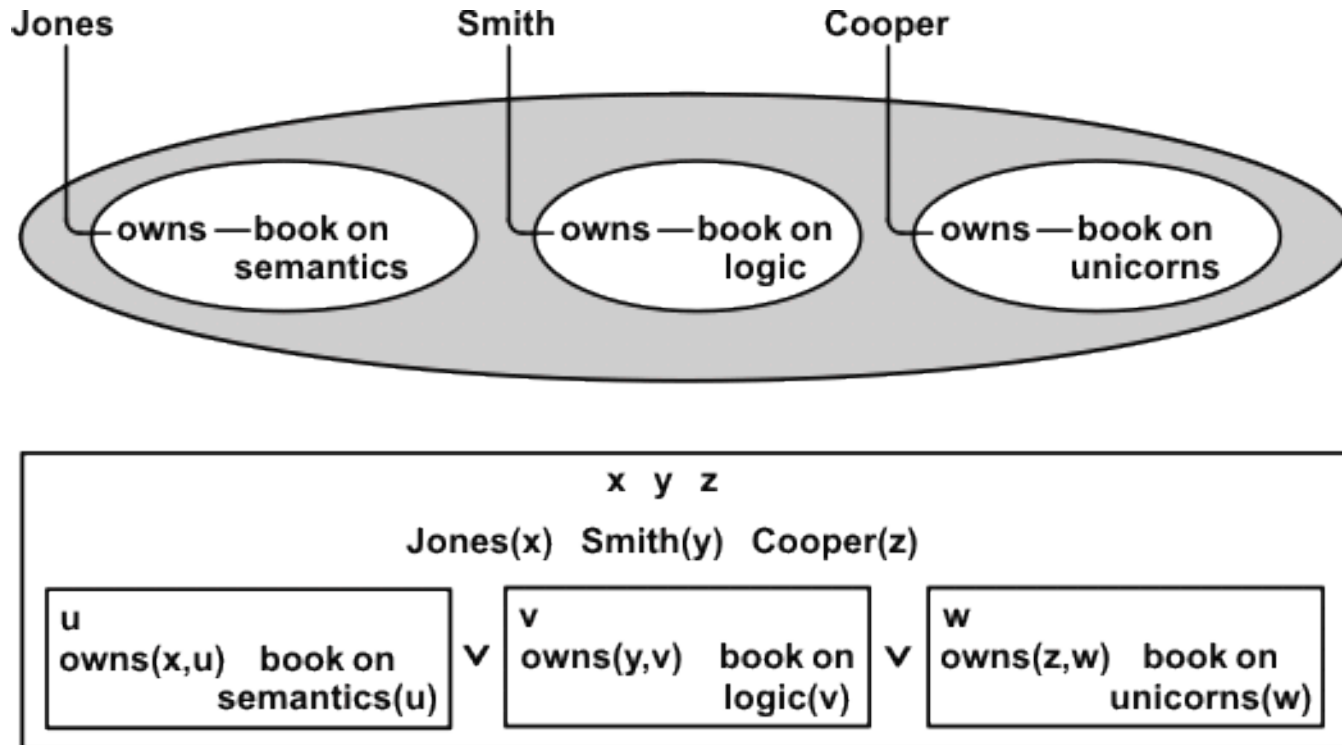


In both EG and DRS, quantifiers in the *if*-area are existential, and they include the *then*-area within their scope.

Disjunctions in EG and DRS

Example by Kamp and Reyle (1993):

Either Jones owns a book on semantics, or Smith owns a book on logic, or Cooper owns a book on unicorns.



Peirce's Rules of Inference

Peirce's rules support the simplest, most general reasoning method ever invented for any logic.

Three pairs of rules, which insert or erase a graph or subgraph:

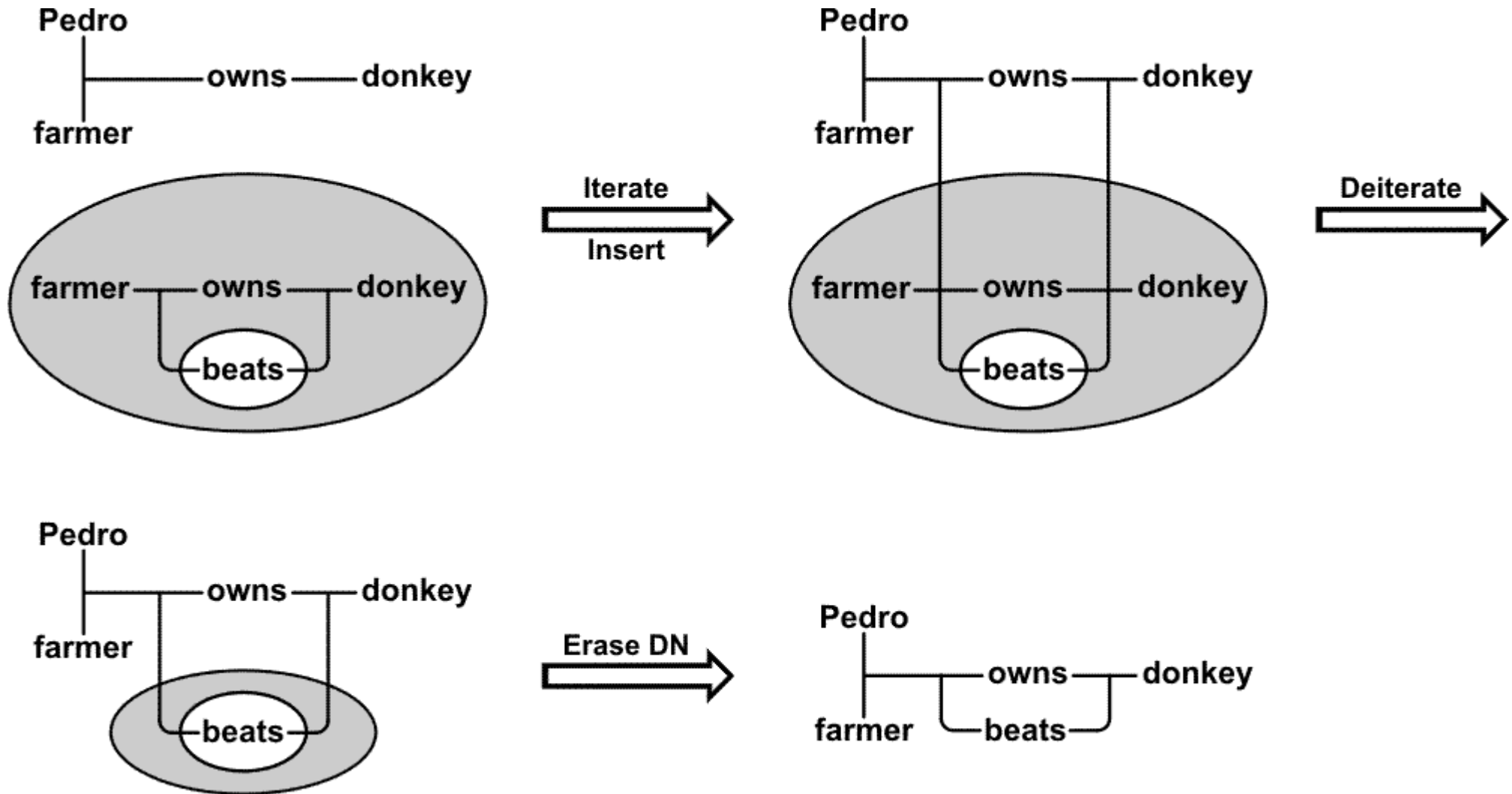
- 1. Insert/Erase: Insert anything in a negative area; erase anything in a positive area.**
- 2. Iterate/Deiterate: Iterate (copy) anything in the same area or any nested area; deiterate (erase) any iterated copy.**
- 3. Double negation: Insert or erase a double negation (pair of ovals with nothing between them) around anything in any area.**

These rules are stated in terms of EGs.

But they can be adapted to many syntaxes, including DRS, predicate calculus, frame notations, and even natural language.

For details, see <http://www.jfsowa.com/pubs/egtut.pdf> .

A Proof by Peirce's Rules



Conclusion: *Pedro is a farmer who owns and beats a donkey.*

Peirce and Kamp

Peirce and Kamp independently developed isomorphic logics, while they were working on different, but related problems:

- **Peirce considered logic to be an integral part of semiotics, and he extended logic to include all methods of reasoning with signs.**
- **Peirce was also employed as an associate editor of the *Century Dictionary*, for which he wrote or edited over 16,000 definitions.**
- **Kamp developed a version of logic that had a simple mapping to and from natural languages.**
- **Peirce and Kamp had common interests, which led to some convergence in their choice of representations.**
- **Kamp and his colleagues showed how the DRS (or EG) notations could facilitate research in linguistics, especially anaphora.**
- **Peirce discovered very general rules of inference that could be applied to a wide range of notations, including EG and DRS.**
- **Their results are complementary.**

Applying Peirce's Rules to English



If a cat is on a mat, then it is happy.

Start with any English sentence that can be mapped to or from a DRS:

- Draw ovals around negative areas.
- Draw the ovals through words like *not*, *if*, *then*, *every*, *either*, *or*.
- Shade negative areas, and leave positive areas unshaded.

A generalization of Peirce's first pair of rules:

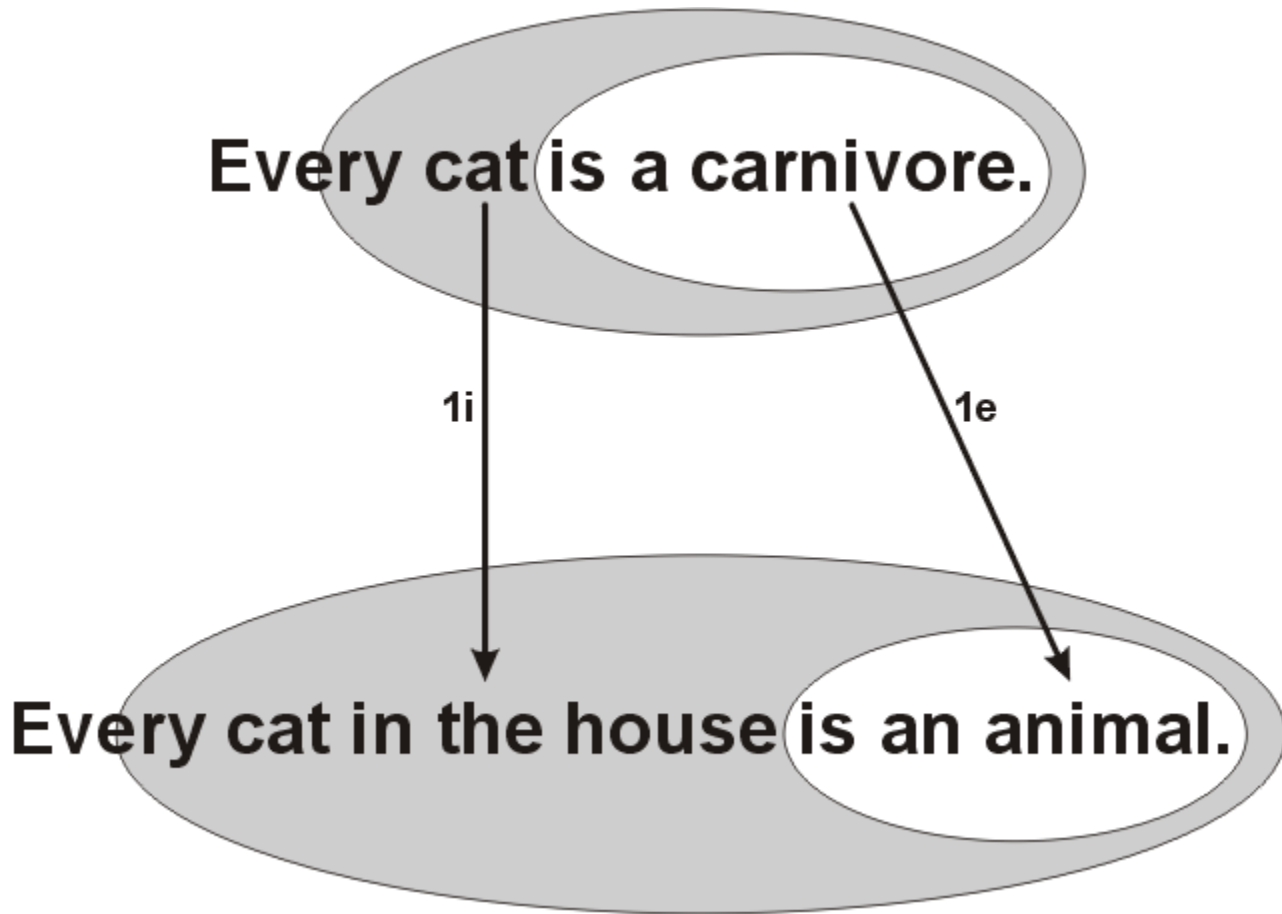
- **Insert:** In a negative context (shaded), any propositional expression may be replaced by a more specialized expression.
- **Erase:** In a positive context (unshaded), any propositional expression may be replaced by a more general expression.

An Inference in English

Use shading to mark the positive and negative parts of each sentence.

Rule 1i specializes 'cat' to 'cat in the house'.

Rule 1e generalizes 'carnivore' to 'animal'.



This method of reasoning is sound for sentences that can be mapped to a formal logic. It can also be used on propositional parts of sentences that contain some nonlogical features.

A Proof in English

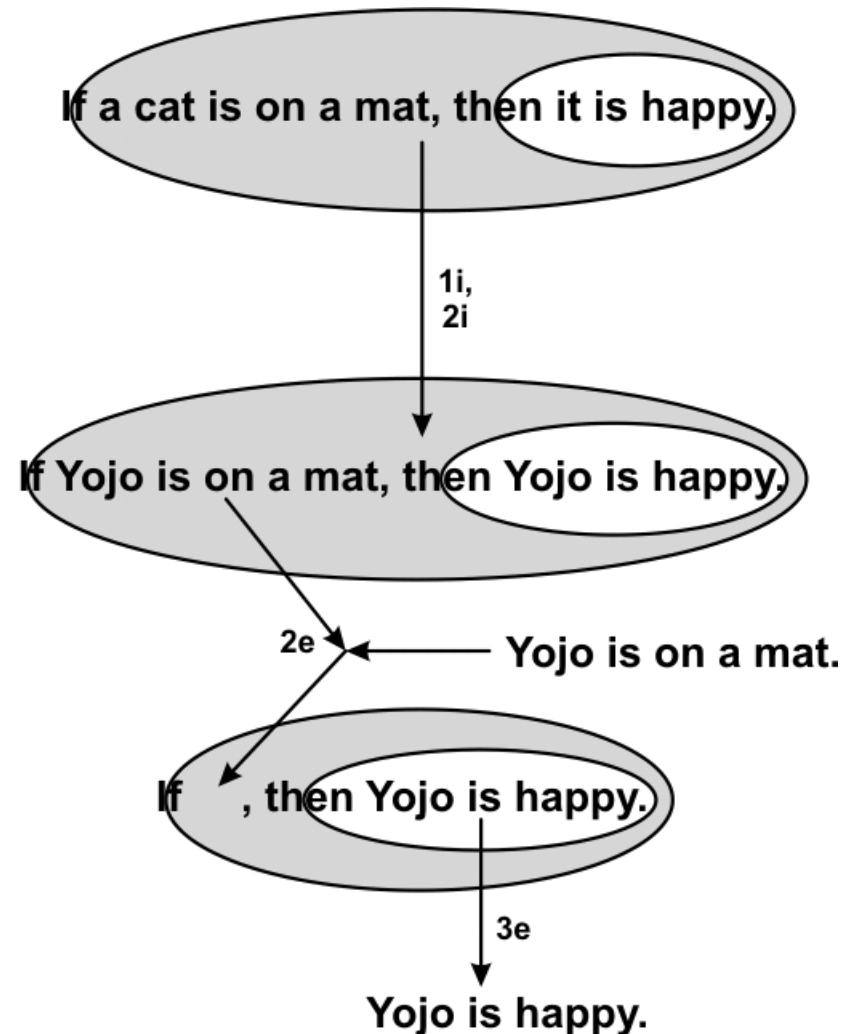
Use shading to mark positive and negative parts of each sentence.

Rule 1i specializes 'a cat' to 'Yojo', and Rule 2i iterates 'Yojo' to replace the pronoun 'it'.

Rule 2e deiterates the nested copy of the sentence 'Yojo is on a mat'.

As a result, there is nothing left between the inner and outer negation of the if-then nest.

Finally, Rule 3e erases the double negation to derive the conclusion.



Natural Deduction

Gerhard Gentzen developed a proof procedure that he called *natural deduction*.

But Peirce's method is a version of natural deduction that is simpler and more general than Gentzen's:

Peirce's Method	Gentzen's Method
6 rules	16 rules
3 symmetric pairs	Many irregularities
Simple operations	Requires provability
Straight-line proofs	Complex bookkeeping
Date: 1897-1909	Date: 1935

For a proof, see <http://www.jfsowa.com/pubs/egtut.pdf>

Psychological Issues

Endorsement by the psychologist Philip Johnson-Laird (2002):

Peirce's existential graphs... anticipate the theory of mental models in many respects, including their iconic and symbolic components, their eschewal of variables, and their fundamental operations of insertion and deletion.

*Much is known about the psychology of reasoning... But we still lack a comprehensive account of how individuals represent multiply-quantified assertions, and so the graphs may provide a guide to the future development of psychological theory. **

Johnson-Laird published a book on mental models.

His comments on that topic are significant, and the option of using icons in generalized EGs strengthens the claim.

* See <http://mentalmodels.princeton.edu/papers/2002peirce.pdf>

Mental Maps, Images, and Models

The neuroscientist Antonio Damasio (2010):

The distinctive feature of brains such as the one we own is their uncanny ability to create maps... But when brains make maps, they are also creating images, the main currency of our minds. Ultimately consciousness allows us to experience maps as images, to manipulate those images, and to apply reasoning to them.

The maps and images form mental models of the real world or of the imaginary worlds in our hopes, fears, plans, and desires.

Words and phrases of language can be generated from them.

They provide a “model theoretic” semantics for language that uses perception and action for testing models against reality.

With Generalized EGs, the model theory can be based on direct mappings between icons in the logic and icons in the model.

Reasoning with Mental Models

From Damasio and other neuroscientists:

- Mental models are patterns in the sensory projection areas that resemble patterns generated during perception.
- But the stimuli that generate mental models come from the frontal lobes, not from sensory input.
- The content of the mental models is generated by assembling fragments of earlier perceptions in novel combinations.

From suggestions by Johnson-Laird:

- The nodes of an existential graph could represent images or fragments of images from long-term memory.
- The connecting lines of an EG would show how those fragments are assembled to form a mental model.
- The logical features of EGs could be used to represent rules and constraints for reasoning about those models.

Teaching Logic

EGs are an excellent pedagogical tool for teaching logic at every level from beginners to the most advanced.

For people who were exposed to predicate calculus and hate it:

- **First hour: EG syntax.**
- **Second hour: Theorem proving (with many more examples).**
- **Third hour: Draw EGs and ask the class how to prove them.**
- **After 3 hours, they say it's the first time they understood logic.**

For advanced students:

- **Cover everything in a one-hour seminar.**

Observation by Don Roberts at the University of Waterloo:

- **Students who start with EGs and move to predicate calculus score higher on exams than students who study only predicate calculus.**
- **The biggest improvement is in their ability to prove theorems.**

4. Reasoning With and About Images

Human language is based on the way people think.

And thinking is intimately integrated with perception and action.

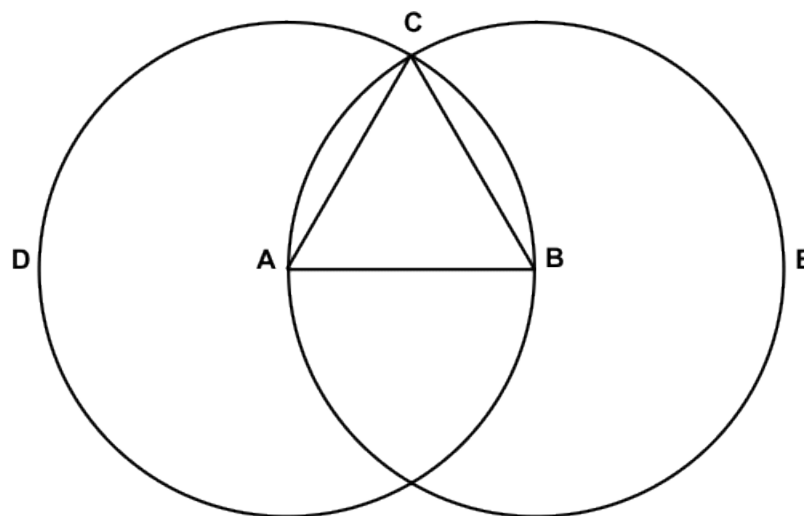
Icons are more fundamental than indexes and symbols.

- **As Damasio said, “Images are the currency of our thought.”**
- **Icons are derived from perception and serve as goals for action.**

How could computer systems reason with icons?

- **Start with a graphic notation for logic.**
- **Incorporate icons as an integral part of the logic.**
- **Generalize the rules of inference to use icons.**
- **Generalize the model-theoretic semantics.**
- **Result: A multimedia logic.**

Euclid's Proposition 1



On a given finite straight line, to draw an equilateral triangle.

Let AB be the given finite straight line. Thus it is required to construct an equilateral triangle on the straight line AB. With center A and distance AB, let the circle BCD be described [Postulate 3]. Again with center B and distance BA, let the circle ACE be described [Post. 3]. And from the point C, in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined [Post. 1]. Now, since the point A is the center of the circle CDB, AC is equal to AB [Definition 15]. Again, since the point B is the center of the circle CAE, BC is equal to BA [Def. 15]. But CA was also proved equal to AB. Therefore, each of the straight lines CA, CB is equal to AB. And things which are equal to the same thing are also equal to one another [Common Notion 1]. Therefore, CA is also equal to CB. Therefore, the three straight lines CA, AB, BC are equal to one another. Therefore, the triangle ABC is equilateral, and it has been constructed on the given finite straight line. QED

Theorems and Corollaries

Peirce's observation about Euclid's *Elements*:

- Every theorem has a new diagram.
- Every corollary uses the same diagram as the main theorem.

The creative insight is the visualization of a new diagram:

- The statement of Proposition 1 mentioned only a line and a triangle.
- It did not mention circles, their relationships to each other, or their relationships to the line and the triangle.

New insights, good or bad, can come from anywhere:

- Taking a bath, walking in the park, or wild images in a dream.
- Drawing a diagram is a guess — an abduction that introduces novel ideas. They may be brilliant, irrelevant, or wrong.
- Deduction is systematic perception — a careful examination of the diagram to make implicit relationships explicit.

Deduction cannot generate anything new. Its primary use is to develop and test the results of abduction and induction.

Ontology of Space



If the line segment AB is bisected at the midpoint C, what happens to the point C? Which “half” should contain C? Either, both, or neither?

With Cantor’s ontology,

- A line consists of a set of points.
- Therefore, C must go to one side or the other.
- The two sides cannot be identical (congruent).

With Aristotle’s ontology, which Euclid and Peirce adopted, *

- The proper parts of a line are smaller lines.
- A point may designate a locus on a line, but it is not part of the line.
- Similarly, a line may bisect an area, and a plane may bisect a solid.
But they are not parts of the area or the solid.

In translating Euclid to any notation for logic, use Aristotle’s ontology.

* See Ketner & Putnam (1992) Introduction to Peirce’s *Reasoning and the logic of things*.

Euclid's Drawings in a Proof

Euclid stated his proofs in procedural form:

- His proofs mix imperative statements about what to draw with declarative statements about the result.

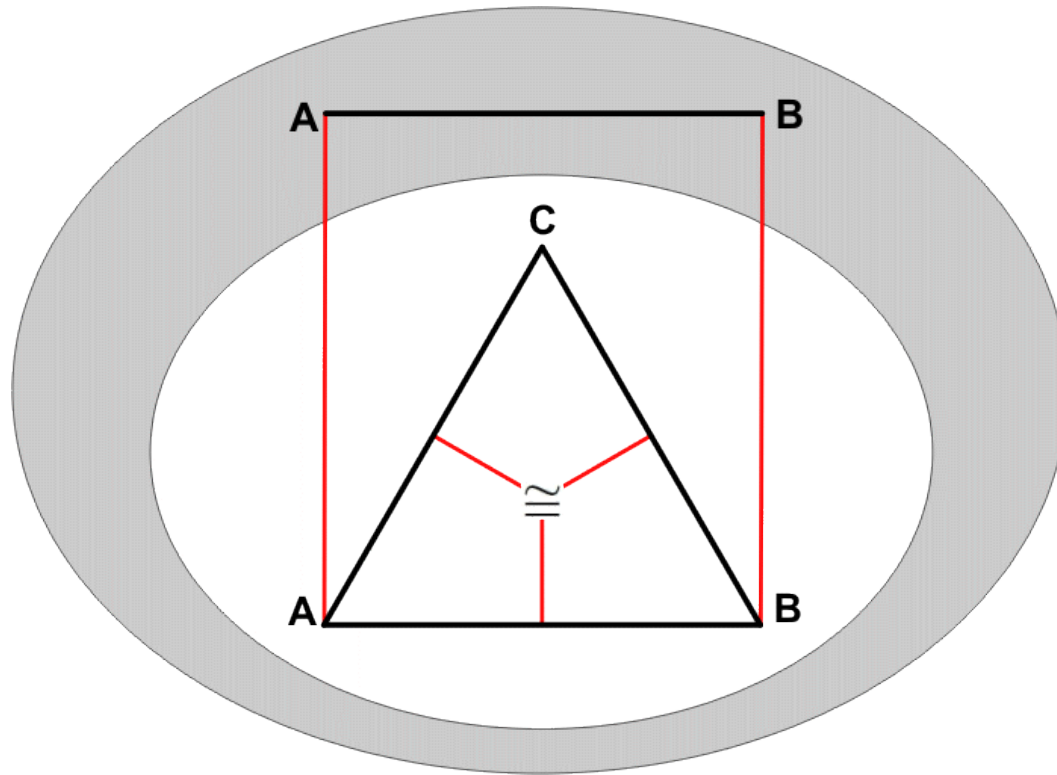
To simplify the logic, use only declarative sentences:

- Euclid's Proposition 1, as translated by Thomas Heath:
On a given finite straight line, to draw an equilateral triangle.
- Restate that proposition as a conditional:
If there is a finite straight line AB , then there is an equilateral triangle with AB as one of its sides.
- Treat every imperative sentence beginning with *let* as a directive for the next step in carrying out the proof.

When a drawing appears in an EG, represent lines of identity in a color that is not used in the diagrams.

Euclid's naming conventions may be used in EG or EGIF.

Proposition 1 as a Generalized EG

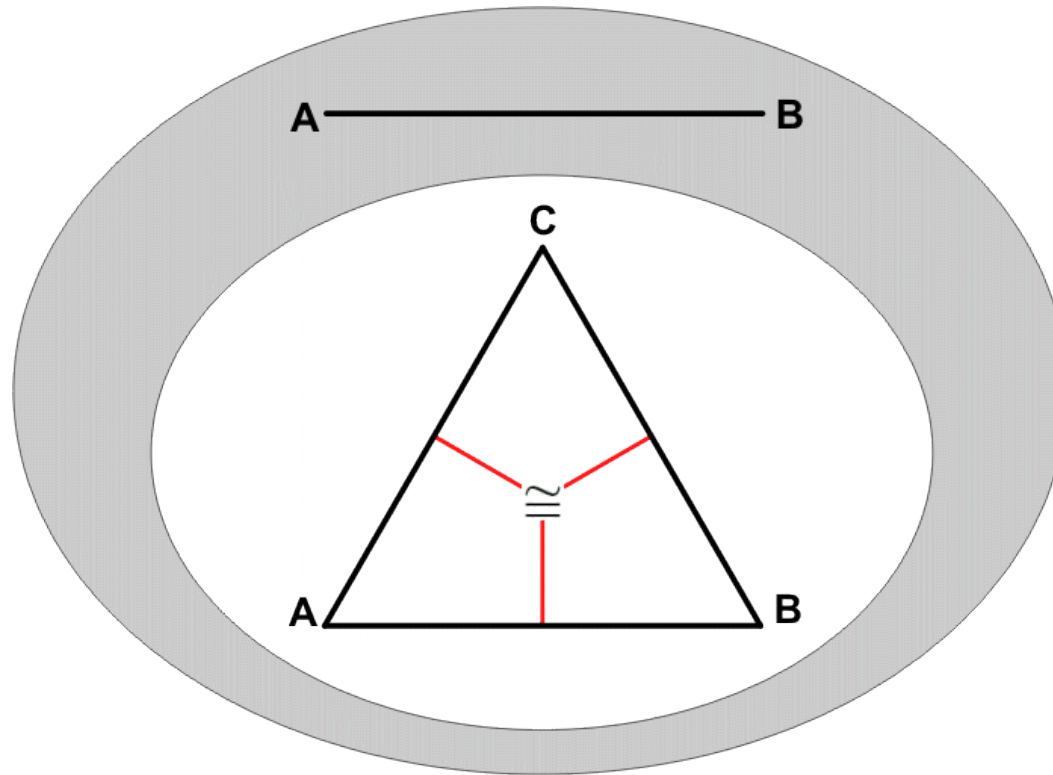


English: *If there is a finite straight line AB , then there is an equilateral triangle with AB as one of its sides.*

The symbol ' \cong ' names a relation that says its arguments (the three sides of the triangle) are congruent.

The lines of identity are in red. The lines from A to A and B to B are redundant and may be omitted.

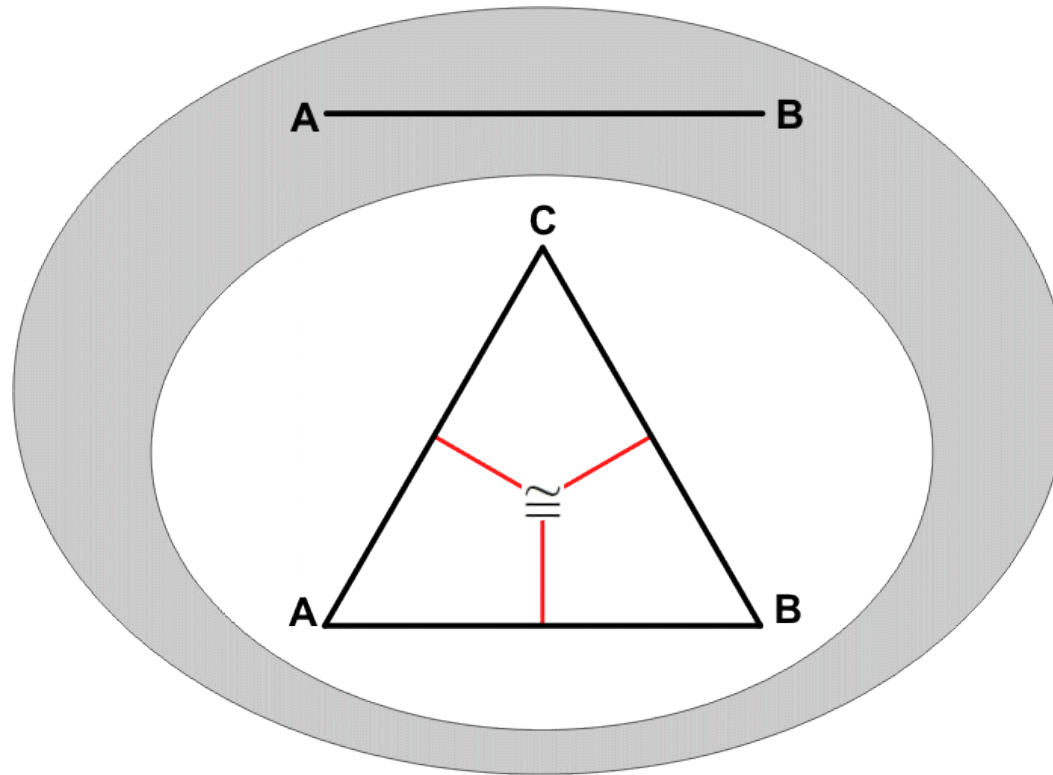
Proposition 1 in EGIF



**EGIF: ~[[*A] [*B] [*AB] (Point ?A) (Point ?B) (Line ?AB)
~[[*ABC] [*AC] [*BC] (Triangle ?ABC) (Line ?AC)
(Line ?BC) (HasSides ?ABC ?AB ?AC ?BC)
("≅" ?AB ?AC ?BC)]]**

The EGIF is not as readable as the EG because it requires more symbols and indexes to represent the icons in the diagram.

Proposition 1 in CGIF



**CGIF: [If [Point *A] [Point *B] [Line *AB]
[Then [Triangle *ABC] [Line *AC] [Line *BC]
(HasSides ?ABC ?AB ?AC ?BC)
("≅" ?AB ?AC ?BC)]]**

Conceptual Graph Interchange Format (CGIF) is a superset of EGIF that is somewhat more compact and readable.

Peirce's Rules of Inference

Peirce's rules support the simplest, most general reasoning method ever invented for any logic.

Three pairs of rules, which insert or erase a graph or subgraph:

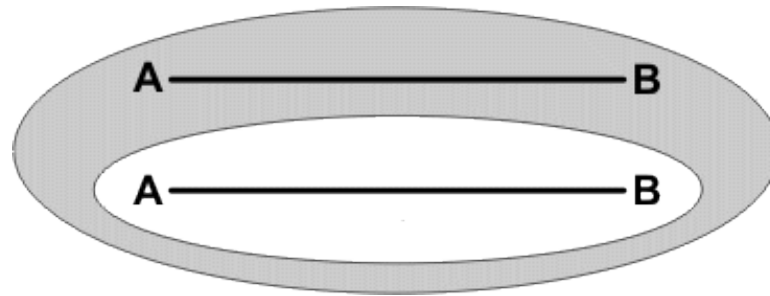
- 1. Insert/Erase: (i) insert anything in a negative area (shaded); (e) erase anything in a positive area (unshaded).**
- 2. Iterate/Deiterate: (i) iterate (copy) anything into the same area or any nested area; (e) deiterate (erase) any such copy.**
- 3. Double negation: (i) insert or (e) erase a double negation (a shaded area with nothing in it) around any graph in any area.**

There is only one axiom: the empty graph, which is always true.

Peirce stated these rules in terms of EGs.

But they can be adapted to many notations, including predicate calculus, natural languages, and Euclid's diagrams.

Proving Proposition 1



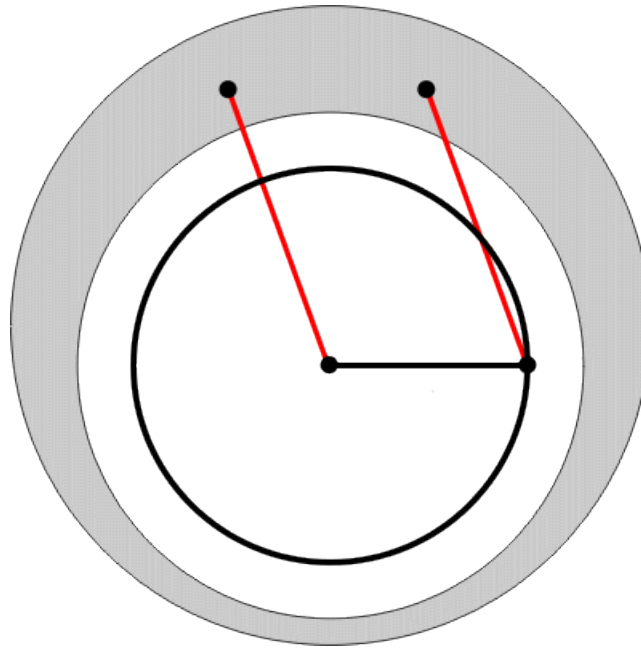
The first steps in proving most theorems in EG form:

- The only axiom is the empty graph — a blank sheet of paper.
- By rule 3i, insert a double negation around the blank.
- By rule 1i, insert the original hypothesis into the shaded area.
- By rule 2i, iterate (copy) the hypothesis into the unshaded area.
- The result is obvious true: If there is a line AB, then there is a line AB.

Continuation of the proof:

- By repeated use of rule 2i, copy axioms and definitions into the unshaded area and use other rules to apply them in that area.
- Finally, the condition of Proposition 1 remains in the shaded area and the complete diagram of Proposition 1 appears in the unshaded area.

Euclid's Postulate 3



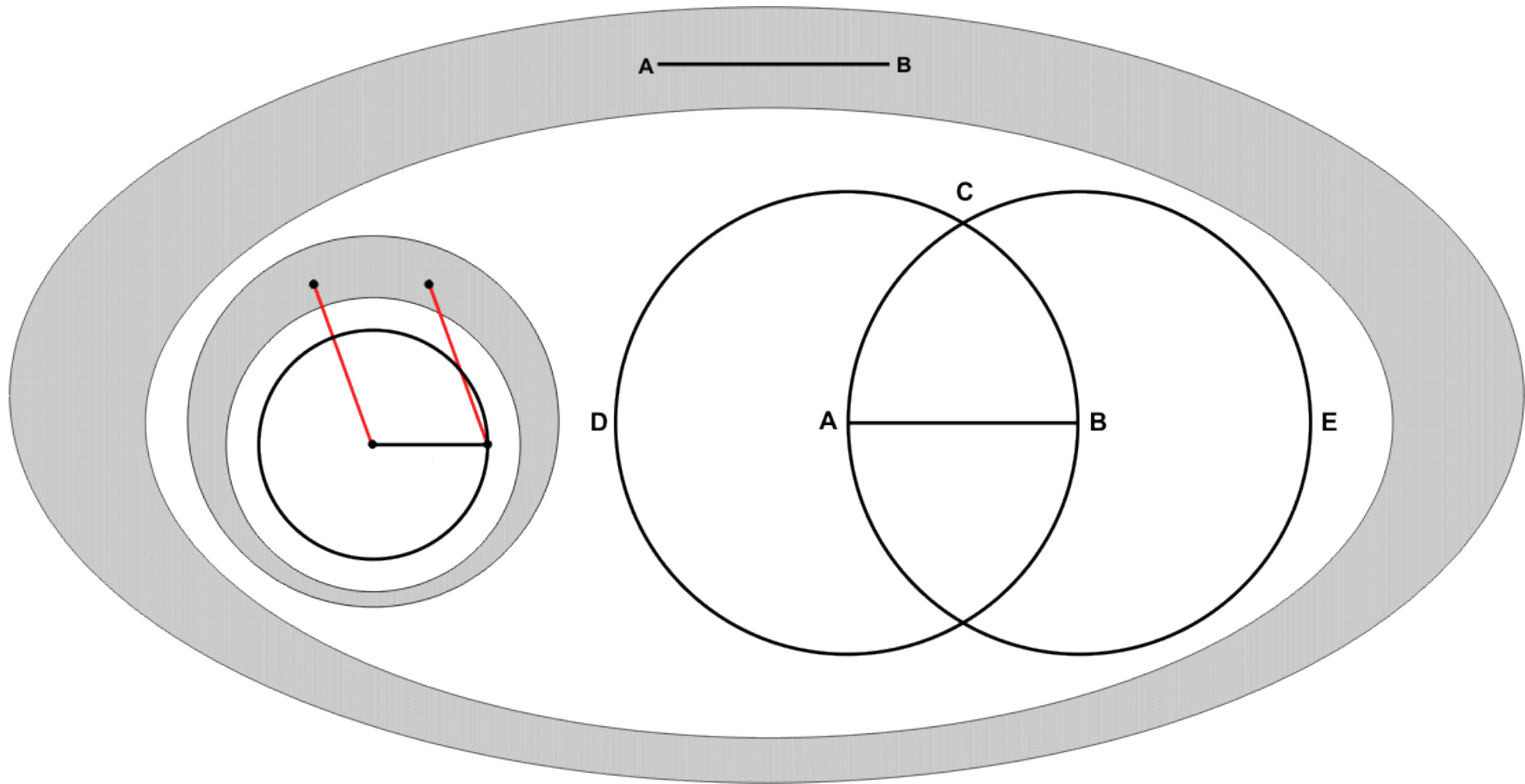
Euclid: *To describe a circle with any center and distance.*

English for the EG: *If there are two points, then there exists a circle with one point at its center and a radius from the center to the other point.*

Next steps in proving Proposition 1:

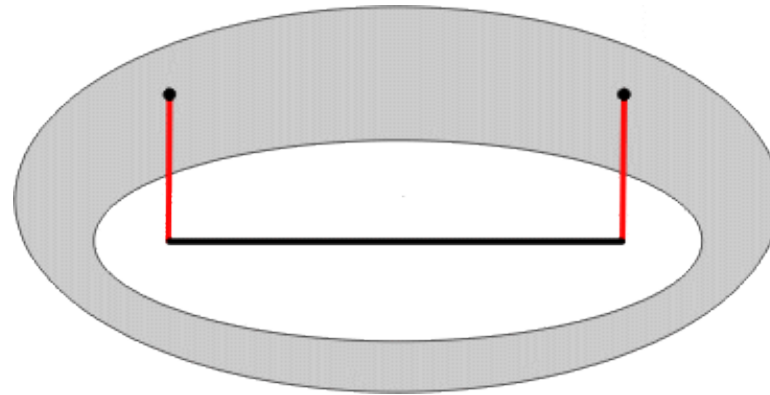
- By rule 2i, insert Postulate 3 into the unshaded area of the EG in slide 77.
- Then use modus ponens, a derived rule of inference for EGs.

Continuation



**By modus ponens, insert a circle with center A and radius AB.
By modus ponens, insert a circle with center B and radius BA.
Then by rule 1e, erase Postulate 3 from the unshaded area.**

Euclid's Postulate 1



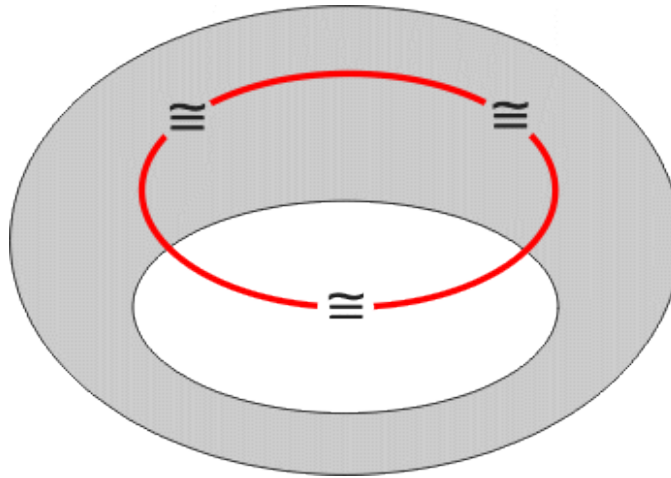
Euclid: *To draw a straight line from any point to any point.*

English for the EG: *If there are two points, then there exists a straight line from one to the other.*

Continuation:

- By rule 2i, iterate this EG into the unshaded area of the current EG.
- By modus ponens, use it to insert a line from C to A.
- By modus ponens, insert another line from C to B.
- By rule 1e, erase Postulate 1 in the unshaded area.
- By the definition of triangle, lines AB, CA, and CB form a triangle ABC.

Euclid's Common Notion 1



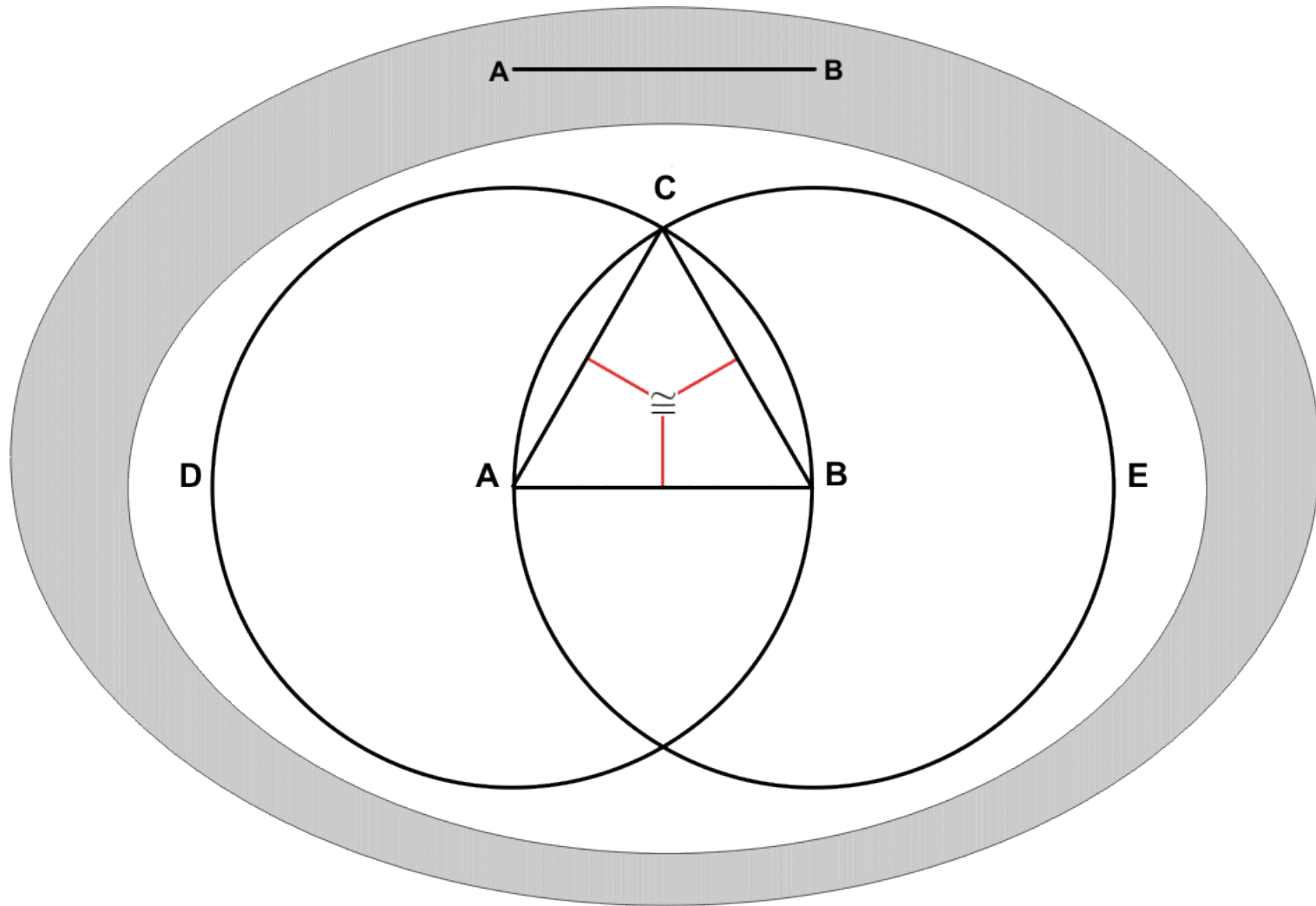
Euclid: *Things which are equal to the same thing are also equal to one another.*

English for the EG: *If two things are congruent to a third, then they are congruent to each other.*

Concluding steps:

- By rule 2i, iterate this EG into the unshaded area of the current EG.
- By definition 15, the radii AC and AB of the circle CDB are congruent.
- By definition 15, the radii BC and BA of the circle CAE are congruent.
- By common notion 1, the lines AC, AB, and BC are congruent.
- By rule 1, erase Common Notion 1 from the unshaded area.

Quod Erat Demonstrandum



This EG is the result of carrying out every step in Euclid's proof. By rule 1e, erase the circles to derive the EG for Proposition 1.

The Complete Proof in CGIF

Every EG can be translated to EGIF, but CGIF is more compact.

In either notation, the icons of figures must be replaced with symbols that name the figures: Point, Line, Circle, Triangle.

Use Euclid's names, but with the letters in alphabetical order.

1. Start with a blank sheet of paper.
2. By rule 3i, [If [Then]].
3. By 1i, [If [Point *A] [Point *B] [Line *AB] [Then]].
4. By 2i, [If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]]].
5. Postulate 3: [If [Point *X] [Point *Y]
[Then [Circle *XYZ] (Center ?XYZ ?X) (Radius ?XYZ ?X ?Y)]].
6. By 2i, insert Postulate 3,
[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[If [Point *X] [Point *Y]
[Then [Circle *XYZ] (Center ?XYZ ?X) (Radius ?XYZ ?X ?Y)]]].

Comments

Slide 83 shows the details summarized in Slide 77.

Differences between EGs, EGIF, and CGIF:

- Peirce's EGs have a direct mapping to EGIF.
- But the icons in Generalized EGs require a more complex mapping.
- The CGIF notation `[If [Then]]` is “syntactic sugar” for the EGIF `~[~[]]`.
- CGIF `[Point *A]` is a “typed version” of the EGIF `[*A] (Point ?A)`.
- EGIF and CGIF do not use shading to mark negative areas.

Comments about the mapping from EGs to EGIF:

- In line 4, the notation `[?A] [?B] [?AB]` indicates lines of identity.
- The '*' prefix of `*A`, `*B`, and `*AB` shows the start of a line or ligature.
- The '?' prefix shows a continuation of a line or ligature.
- It's permissible, but not necessary to iterate (copy) everything from the If-area to the nested Then-area.
- In line 5, `(Center ?XYZ ?X)` may be read “the center of XYZ is X”.
- `(Radius ?XYZ ?X ?Y)` may be read “a radius of XYZ extends from X to Y”.

Continuation

Observation derives information by looking at an icon. Peirce called it diagrammatic reasoning. Polya called it heuristics. Kant and Frege used the word *Anschauung*.

7. By two applications of modus ponens,

[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[If [Point *X] [Point *Y]
[Then [Circle *XYZ] (Center ?XYZ ?X) (Radius ?XYZ ?X ?Y)]]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?A ?B)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?B ?A)]].

8. Erase Postulate 3 by 1e,

[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?A ?B)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?B ?A)]].

9. Observation: the two circles intersect at point C.

[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?A ?B)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?B ?A) [Point *C]]].

Continuation

CGIF supports polyadic relations such as (Line AB), (Line AB A B).

10. Postulate 1: [If [Point *X] [Point *Y] [Then [Line *XY] (Line ?XY ?X ?Y)]].

11. Insert Postulate 1 by 2i,

**[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?A ?B)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?B ?A) [Point *C]
[If [Point *X] [Point *Y] [Then [Line *XY] (Line ?XY ?X ?Y)]]].**

12. By two applications of modus ponens,

**[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?A ?B)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?B ?A) [Point *C]
[If [Point *X] [Point *Y] [Then [Line *XY] (Line ?XY ?X ?Y)]]
[Line *AC] [Line *BC]]].**

13. Erase Postulate 1 by 1e,

**[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?A ?B)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?B ?A) [Point *C]
[Line *AC] [Line *BC]]].**

Continuation

**Triad (Radius XYZ X Y) and dyad (Radius XYZ XY) are equivalent.
By Definition 15, all radii of the same circle are congruent.**

- 14. Observation: the lines AB, AC, and BC form a triangle ABC.**
[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?A ?B)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?B ?A) [Point *C]
[Line *AC] [Line *BC] [Triangle *ABC] (HasSides ?ABC ?AB ?AC ?BC)]].
- 15. By replacing the triadic Radius with the dyadic version,**
[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?AB)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?AB) [Point *C]
[Line *AC] [Line *BC] [Triangle *ABC] (HasSides ?ABC ?AB ?AC ?BC)]].
- 16. Observation: AC is a radius of BCD, and BC is a radius of ACE,**
[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?AB)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?AB) [Point *C]
[Line *AC] [Line *BC] [Triangle *ABC] (HasSides ?ABC ?AB ?AC ?BC)
(Radius ?BCD ?AC) (Radius ?ACE ?BC)]].

Conclusion

Triad (\cong X Y Z) is equivalent to 3 dyads (\cong X Y) (\cong X Z) (\cong Y Z).

17. By Definition 15, all radii of the same circle are congruent,

**[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?AB)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?AB) [Point *C]
[Line *AC] [Line *BC] [Triangle *ABC] (HasSides ?ABC ?AB ?AC ?BC)
(Radius ?BCD ?AC) (Radius ?ACE ?BC) (\cong ?AC ?AB) (\cong ?BC ?AB)]].**

18. Common Notion 1:

[If [*X] [*Y] [*Z] (\cong ?X ?Z) (\cong ?Y ?Z) [Then (\cong ?X ?Y)]].

19. Insert Common Notion 1, apply it by modus ponens, and erase it,

**[If [Point *A] [Point *B] [Line *AB] [Then [?A] [?B] [?AB]
[Circle *BCD] (Center ?BCD ?A) (Radius ?BCD ?AB)
[Circle *ACE] (Center ?ACE ?B) (Radius ?ACE ?AB) [Point *C]
[Line *AC] [Line *BC] [Triangle *ABC] (HasSides ?ABC ?AB ?AC ?BC)
(Radius ?BCD ?AC) (Radius ?ACE ?BC) (\cong ?AC ?AB) (\cong ?BC ?AB)
(\cong ?AC ?BC)]].**

20. Use the triad for \cong . Erase everything not mentioned in Proposition 1.

**[If [Point *A] [Point *B] [Line *AB] [Then [Point *C] [Line *AC] [Line *BC]
[Triangle *ABC] (HasSides ?ABC ?AB ?AC ?BC) (\cong ?AC ?AB ?BC)]].**

Comments About the Proof

The proof clarifies some obscure points by Peirce and Euclid.

Issues in mapping Euclid to Generalized Existential Graphs:

- **The relationship between rules for drawing and rules of inference.**
- **Creativity in discovering a diagram that makes the proof “obvious”.**
- **Deduction for generating an identical diagram by rules of inference.**
- **A new rule of inference: observation of an icon to add new information.**

Issues in mapping GEGs to EGIF,

- **Every icon is replaced by a symbol or a graph of symbols.**
- **Different kinds of observations may lead to different kinds of symbols.**
- **Distinctions such as the dyadic and triadic variants of the Radius relation may be represented in the icons and in EGIF.**
- **Some observations, such as recognizing a triangle, may be simple.**
- **Other observations, such as recognizing intersecting circles, may be easy for people to detect, but difficult for computer programs.**

Every EGIF can be translated to predicate calculus. With some reordering, Gentzen’s natural deduction can be used in the proof.

The Role of Icons in Logic

As the examples show, icons can make logic more readable.

But icons can play a more fundamental role:

- **The visual images used for induction and abduction can also be integral components of the notation used in deduction.**
- **The implicit information in icons is potentially much larger than the explicit information in the symbols of a linear notation.**
- **That information can support heuristics for selecting appropriate axioms or background knowledge.**
- **The open-ended amount of information can make a logic with icons more expressive than a logic without icons.**

Questions:

- **How can icons be used to relate logic to natural languages?**
- **How would they relate to the other branches of cognitive science?**
- **How would icons affect the computational complexity?**
- **Can they prove anything that could not be proved by a Turing machine?**

Proof Procedure With Icons

Generalized EGs can relate language, logic, and mental models.

Two-dimensional GEGs use observation as a rule of inference:

- **In any context, observation may be used to determine whether one icon is more generalized or specialized than another.**
- **Information about generalizations may be used in Peirce's rules.**
- **A GEG that contains icons may be translated to EGIF by replacing each icon with a character string that names the icon.**
- **If icon x is more general than icon y, then the relation named by the string for x is more general than the relation named by the string for y.**
- **Therefore, the proofs with GEGs or EGIF are step by step equivalent.**

The 2-D rules may be generalized to any dimension N.

- **The ovals may be replaced by closed surfaces of N-1 dimensions.**
- **Generalizations of 3-D and 4-D icons may be determined by observation, and the icons may be replaced by their names in EGIF.**
- **Icons for other sensory modalities may also be represented by names.**

Peirce's rules are formal, but observations depend on context.

Theoretical Issues

Peirce's rules have some remarkable properties:

- **Simplicity:** Each rule inserts or erases a graph or subgraph.
- **Symmetry:** Each rule has an exact inverse.
- **Depth independence:** Rules depend on the positive or negative areas, not on the depth of nesting.

They allow short proofs of remarkable theorems:

- **Reversibility Theorem.** Any proof from p to q can be converted to a proof of $\sim p$ from $\sim q$ by negating each step and reversing the order.
- **Cut-and-Paste Theorem.** If q can be proved from p on a blank sheet, then in any positive area where p occurs, q may be substituted for p .
- **Resolution and natural deduction:** Any proof by resolution can be converted to a proof by Peirce's version of natural deduction by negating each step and reversing the order.

For proofs of these theorems and further discussion of the issues, see Section 6 of <http://www.jfsowa.com/pubs/egtut.pdf>

Natural Deduction

Gerhard Gentzen developed a proof procedure that he called *natural deduction*.

But Peirce's method is a version of natural deduction that is simpler and more general than Gentzen's:

Peirce's Method	Gentzen's Method
6 rules	16 rules
3 symmetric pairs	Many irregularities
Simple operations	Requires provability
Straight-line proofs	Complex bookkeeping
Date: 1897-1909	Date: 1935

For details, see pp. 24 to 26 of <http://www.jfsowa.com/pubs/egtut.pdf>

Gentzen's Natural Deduction

	Introduction Rules	Elimination Rules
\wedge	$\frac{A, B}{A \wedge B}$	$\frac{A \wedge B}{A} \quad \frac{A \wedge B}{B}$
\vee	$\frac{A}{A \vee B} \quad \frac{B}{A \vee B}$	$\frac{A \vee B, A \vdash C, B \vdash C}{C}$
\supset	$\frac{A \vdash B}{A \supset B}$	$\frac{A, A \supset B}{B}$
\sim	$\frac{A \vdash \perp}{\sim A} \quad \frac{\perp}{A}$	$\frac{A, \sim A}{\perp} \quad \frac{\sim \sim A}{A}$
\forall	$\frac{A(a)}{(\forall x)A(x)}$	$\frac{(\forall x)A(x)}{A(t)}$
\exists	$\frac{A(t)}{(\exists x)A(x)}$	$\frac{(\exists x)A(x), A(a) \vdash B}{B}$

Like Peirce, Gentzen assumed only one axiom: a blank sheet of paper.

But Gentzen had more operators and more complex, nonsymmetric pairs of rules for inserting or erasing operators.

Role of the Empty Sheet

Both Peirce and Gentzen start a proof from an empty sheet.

In Gentzen's syntax, a blank sheet is not a well-formed formula.

- Therefore, no rule of inference can be applied to a blank.
- A method of making and discharging an assumption is the only way to begin a proof.

But in EG syntax, an empty graph is a well-formed formula.

- Therefore, a double negation may be drawn around a blank.
- Then any assumption may be inserted in the negative area.

Applying Peirce's rules to predicate calculus:

- Define a blank as a well-formed formula that is true by definition.
- Define the positive and negative areas for each Boolean operator.
- Show that each of Gentzen's rules is a derived rule of inference in terms of Peirce's rules.

Then any proof by Gentzen's rules is a proof by Peirce's rules.

A Problem in Automated Reasoning

Larry Wos (1988), a pioneer in automated reasoning methods, stated 33 unsolved problems. His problem 24:

Is there a mapping between clause representation and natural-deduction representation (and corresponding inference rules and strategies) that causes reasoning programs based respectively on the two approaches or paradigms to attack a given assignment in an essentially identical fashion?

The answer in terms of Peirce's rules is yes:

- **The inference rules for Gentzen's clause form and natural deduction are derived rules of inference in terms of the EG rules.**
- **Any proof in clause form (by resolution) can be converted, step by step, to a proof by EG rules.**
- **Any such proof can be converted to a proof by Peirce's version of natural deduction by negating each step and reversing the order.**
- **Convert the proof by Peirce's rules to a proof by Gentzen's rules.**

Computability

Can icons enable proofs with Generalized EGs to go beyond what can be computed with a Turing machine?

With the kinds of icons Euclid used, no:

- Every diagram has a finite number of figures, each determined by a finite number of points in a finite number of possible patterns.
- As slides 32 to 37 showed, those proofs can be mapped to EGIF or CGIF and then to predicate calculus.
- They may be more readable, but in theory, no more powerful.

But with continuous icons, maybe:

- A continuous image may have an infinity of points, shapes, relationships, and possible transformations.
- Repeated observations may extract an open-ended amount of information from a single image.
- But the observations may require different views or perspectives at different distances, angles, and magnifications.

Turing Oracle Machines

How could Generalized EGs go beyond a Turing machine?

Turing a-machines can represent any algorithm:

- They can perform any computation by any digital computer.
- But they do not allow external input during a computation.

But Turing (1939) also discussed oracles for o-machines:

- An o-machine is an a-machine that can interrogate an oracle.
- Emil Post expanded Turing's one-page summary to show how an o-machine can extend the computational power of an a-machine.
- Robert Soare (2009) presented a detailed history and analysis of the Post-Turing hypothesis. *
- Soare claims that a digital computer connected to I/O devices can have the power of an o-machine.
- Generalized EGs with the option of extracting information from continuous images could also have the power of an o-machine.

* See <http://www.people.cs.uchicago.edu/~soare/History/turing.pdf>

A Multimedia Logic

Peirce called existential graphs “the logic of the future.”

Computer graphics and virtual reality can implement them:

- **Icons in two-dimensional graphs may be generalized to three dimensions or to 3+1 dimensions for motion and change.**
- **Conjunctions and lines of identity may be represented in any dimension.**
- **For negation, the ovals may be generalized to closed shapes in any number of dimensions.**
- **Viewers with VR goggles could wander through 4-dimensional EGs, watch the movies, and manipulate icons according to Peirce’s rules.**

Peirce’s claim is consistent with neuroscience:

- **As Damasio said, images are “the main currency of our minds.”**
- **As Johnson-Laird observed, Peirce’s rules of inference insert and erase graphs and subgraphs — operations that neural processes can perform.**
- **Generalized EGs can include arbitrary images in the graphs.**
- **When Peirce claimed that EGs represent “a moving picture of the action of the mind in thought,” he may have envisioned something similar.**

Related Readings

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- Johnson-Laird, Philip N. (2002) Peirce, logic diagrams, and the elementary operations of reasoning, *Thinking and Reasoning* 8:2, 69-95. <http://mentalmodels.princeton.edu/papers/2002peirce.pdf>
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<http://www.helsinki.fi/science/commens/papers/magiclantern.pdf>
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<http://www.helsinki.fi/~pietarin/publications/Semiotica-Diagrams-Pietarinen.pdf>
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<http://www.jfsowa.com/pubs/rolelog.pdf>
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<http://www.jfsowa.com/pubs/csp21st.pdf>
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[http://standards.iso.org/ittf/PubliclyAvailableStandards/c039175_ISO_IEC_24707_2007\(E\).zip](http://standards.iso.org/ittf/PubliclyAvailableStandards/c039175_ISO_IEC_24707_2007(E).zip)
- For other references, see the general bibliography,
<http://www.jfsowa.com/bib.htm>