Capacitor Discharge

Jay Grinols

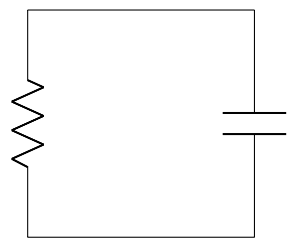

Figure 1: Setup of our circuit.

The capacitance is defined as $C = \frac{Q}{\Delta V_C}$. We know that $I \cdot R = \Delta V_R$ from Ohm's Law. By Kirchhoff's Loop Rule, we know that the sum of differences in potential in a closed loop must be equal to 0.

$$\Delta V_C - \Delta V_R = 0$$

Substituting for ΔV, we obtain that

$$\frac{Q}{C} - I \cdot R = 0$$

Recall that $I = -\dfrac{\mathrm{d}Q}{\mathrm{d}t}$, since the capacitor is losing charge. Solving for the first order separable differential equation...

$$\begin{aligned}
\frac{Q}{C} - I \cdot R = 0 \\
\frac{Q}{C} + \frac{\mathrm{d}Q}{\mathrm{d}t} \cdot R = 0 \\
\frac{\mathrm{d}Q}{\mathrm{d}t} \cdot R = -\frac{Q}{C} \\
\int \frac{dQ}{Q} = -\int \frac{dt}{R \cdot C} \\
\ln Q = -\frac{1}{R \cdot C} \cdot t + C_1 \\
Q(t) = C_2 \cdot e^{-\frac{t}{\tau}}
\end{aligned}$$

We know that $Q(t) = C \cdot \Delta V_C(t)$, and that $Q(0) = C \cdot \Delta V_0$.

$$\begin{aligned}
C \cdot \Delta V_0 = C_2 \\
Q(t) = C \cdot \Delta V_0 \cdot e^{-\frac{t}{\tau}} \\
C \cdot \Delta V_C(t) = C \cdot \Delta V_0 \cdot e^{-\frac{t}{\tau}} \\
\Delta V_C(t) = \Delta V_0 \cdot e^{-\frac{t}{\tau}} \\
\ln \Delta V_C(t) = -\frac{1}{\tau} \cdot t + \ln \Delta V_0
\end{aligned}$$