部分思考题、习题参考答案与提示

思考题一

1. 全对.

2. $f_n(A)$ 是一个变化的量 (其实就是第 2 章中的随机变量), $P(A)$ 是个实数, 当 n 充分大时, $f_n(A) \approx P(A)$ (严格表述见第 5 章). 不一定成立.

3. 略.

4. $P(A|A \cup B)$ 一般不等于 1.

5. 不能同时成立.

6. 不一定.

习题一

1. (1) 9; (2) $A = \{(0,a),(1,a),(2,a)\}$; (3) $B = \{(0,a),(0,b),(0,c)\}$.

2. (1) $AB \cup AC \cup BC$ 或 $AB\overline{C} \cup A\overline{B}C \cup \overline{A}BC \cup ABC$; (2) $\overline{A}\,\overline{B} \cup \overline{B}\,\overline{C} \cup \overline{A}\,\overline{C}$; (3) $\overline{A}BC \cup A\overline{B}C \cup AB\overline{C}$; (4) $\overline{A} \cup \overline{B} \cup \overline{C}$ 或 $\overline{ABC}$.

3. (1) 0.5; (2) 0.3.

4. (1) 0.8; (2) 0.2; (3) 0.5.

5. (1) 0.1; (2) 0.6; (3) 0.3.

6. 当不放回抽样时: (1) $\dfrac{28}{45}$; (2) $\dfrac{16}{45}$; (3) $\dfrac{4}{5}$; 当放回抽样时:(1) 0.64; (2) 0.32; (3) 0.8.

7. (1) $\dfrac{28}{435}$; (2) $\dfrac{1}{435}$.

8. (1) $\dfrac{12}{35}$; (2) $\dfrac{1}{35}$; (3) $\dfrac{2}{35}$.

9. (1) $\dfrac{1}{9}$; (2) $\dfrac{7}{72}$.

10. $\dfrac{80}{243}$.

11. $\dfrac{1}{1\ 225}$.

12. (1) $\dfrac{7}{9}$; (2) $\dfrac{2}{5}$; (3) $\dfrac{2}{9}$.

13. 0.86.

14. (1) $\dfrac{1}{9}$; (2) $\dfrac{1}{2}$.

15. (1) $\dfrac{37}{80}$; (2) $\dfrac{1}{37}$.

16. (1) 0.22; (2) 0.54.

17. (1) 0.014 54; (2) 0.606 60.

18. 0.834.

19. (1) 0.541 7; (2) 0.394 4.

20. $\dfrac{6}{25}$.

21. 略.

22. (1) 对; (2) 错; (3) 错; (4) 对.

23. (1) $\alpha = p_1p_2p_3 + p_1p_2p_4 + p_2p_3p_4 + p_1p_3p_4 - 3p_1p_2p_3p_4$; (2) $\beta = \dfrac{p_1p_2p_3p_4}{\alpha}$; (3) $\gamma = C_3^2\alpha^2(1-\alpha)$.

24. (1) $P(A_i) = p(1-p)^{i-1}, i = 1, 2, \cdots, P(B_4) = p^2(1-p)$;(2) $p^2(1-p)$; (3) p.

25. (1) $\dfrac{1}{19}$; (2) 0.998 4.

26. 14.

27. (1) 0.086; (2) 0.213.

思考题二

1. 略.
2. 不一定.
3. 不对.
4. (D).
5. (B).
6. 仅与 σ 有关.
7. 不可以.

习题二

1. $P\{X=k\} = \dfrac{C_{k-1}^1 \cdot C_{7-k}^1}{C_7^3} = \dfrac{(k-1)(7-k)}{35}, k = 2, 3, \cdots, 6$.

2. (1)

X	0	1	2	4
p	0.80	0.16	0.032	0.008

(2) 0.008; (3) 0.2.

3.

X	0	1	2
p	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$

4. (1) $(1-10^{-7})^n$; (2) $(1-10^{-6})^n$.

5. (1) 0.882; (2) 0.367; (3) 0.127; (4) 0.260.

6. (1) 0.204 8; (2) 0.737 3; (3) 0.057 9.

7. (1) $P\{X=0\} = \prod_{i=1}^{3}(1-p_i)$,

$P\{X=1\} = p_1(1-p_2)(1-p_3) + (1-p_1)p_2(1-p_3) + (1-p_1)(1-p_2)p_3$,

$P\{X=2\} = p_1p_2(1-p_3) + p_1(1-p_2)p_3 + (1-p_1)p_2p_3, P\{X=3\} = p_1p_2p_3$;

(2)

Y	0	1	2	3
p	p_1	$(1-p_1)p_2$	$(1-p_1)(1-p_2)p_3$	$(1-p_1)(1-p_2)(1-p_3)$

8. (1) $P\{X=k\}=p(1-p)^{k-1}, k=1,2,3,4, P\{X=5\}=(1-p)^4$; (2) $p(2-p)$.

9. (1) $1-2\mathrm{e}^{-1}$; (2) $\dfrac{2}{3(\mathrm{e}-2)}$.

10. (1) $1-11\mathrm{e}^{-10}$; (2) $\mathrm{e}^{-0.5}$.

11. (1) $1-\dfrac{11}{2}\mathrm{e}^{-4.5}$; (2) $\dfrac{16}{5(\mathrm{e}^{3.2}-1)}$.

12. (1) $\dfrac{324}{5}\mathrm{e}^{-6}$; (2) $\dfrac{324}{5(\mathrm{e}^{6}-115)}$.

13. 0.801.

14. (1) $P\{X=k\}=\dfrac{\mathrm{C}_3^k\mathrm{C}_7^{3-k}}{\mathrm{C}_{10}^3}, k=0,1,2,3$;

(2) $P\{Y=k\}=\dfrac{\mathrm{C}_3^k}{8}, k=0,1,2,3$;

(3) $P\{Z=k\}=\dfrac{9^{k-1}}{10^k}, k=1,2,3,\cdots$;

(4) $\dfrac{3}{16}$.

15. (1) $F(x)=\begin{cases}0, & x<0,\\ \dfrac{x}{2}, & 0\leqslant x<1,\\ \dfrac{1}{2}, & 1\leqslant x<2,\\ \dfrac{x-1}{2}, & 2\leqslant x<3,\\ 1, & x\geqslant 3;\end{cases}$ (2) $\dfrac{3}{4}$.

16. (1) $\dfrac{3}{16}$; (2) $F(x)=\begin{cases}0, & x<0,\\ \dfrac{12x-x^3}{16}, & 0\leqslant x<2,\\ 1, & x\geqslant 2;\end{cases}$ (3) $\dfrac{11}{16}$; (4) 0.144 2.

17. (1) $a=b=0.5$; (2) $f(x)=\begin{cases}x, & 0<x<1,\\ 0.5, & 1<x<2,\\ 0, & \text{其他};\end{cases}$ (3) 0.625.

18. (1) $\dfrac{3}{5}$; (2) $\dfrac{3}{5}$; (3) $\dfrac{1}{4}$.

19. $f_X(x)=\begin{cases}\dfrac{1}{4}, & -1<x<3,\\ 0, & \text{其他},\end{cases}$ $P\{Y=k\}=\mathrm{C}_n^k\left(\dfrac{3}{4}\right)^k\left(\dfrac{1}{4}\right)^{n-k}, k=0,1,2,\cdots,n$.

20. (1) 0.993 8; (2) 0.069 4; (3) 0.682 6; (4) 0.045 6.

21. 0.308 5.

22. (1) 0.5; (2) 0.682 6; (3) 0.655 4.

23. (1) 0.111; (2) 0.244; (3) 0.297.

24. (1) 0.064; (2) 0.662; (3) 0.972.

25. $\mu = 14\ 109, \sigma = 2\ 498$.

26. $x_1 = 15, x_2 = 17$.

27. (1) $\dfrac{1}{\sqrt{\pi}}$; (2) 0.239.

28. (1) $f(x) = \begin{cases} \dfrac{1}{8}\mathrm{e}^{-x/8}, & x > 0, \\ 0, & x \leqslant 0; \end{cases}$ (2) $\mathrm{e}^{-1.25}$; (3) $\mathrm{e}^{-1} - \mathrm{e}^{-2}$.

29. (1) 0.275; (2) $\mathrm{e}^{-2/9}$.

30. (1) $f(x) = \begin{cases} 0.2\mathrm{e}^{-0.2x}, & x > 0, \\ 0, & x \leqslant 0; \end{cases}$ (2) $\mathrm{e}^{-1} - \mathrm{e}^{-2}$; (3) $(1 - \mathrm{e}^{-1})^6(6\mathrm{e}^{-1} + 1)$.

31. (1) 0.116; (2) 0.127.

32.

Y	2	8	10
p	0.216	0.294	0.490

33. (1) $c = \dfrac{1}{9}$; (2) $f_Y(y) = \begin{cases} \dfrac{1}{27}\left[4 - \left(\dfrac{y}{3}\right)^2\right], & -3 < y < 6, \\ 0, & \text{其他}; \end{cases}$

(3) $F_Z(t) = \begin{cases} 0, & t \leqslant 0, \\ \dfrac{2}{9}\left(4t - \dfrac{t^3}{3}\right), & 0 < t \leqslant 1, \\ \dfrac{1}{9}\left(4t - \dfrac{t^3}{3} + \dfrac{11}{3}\right), & 1 < t < 2, \\ 1, & t \geqslant 2, \end{cases}$ $f_Z(t) = \begin{cases} \dfrac{2}{9}(4 - t^2), & 0 < t \leqslant 1, \\ \dfrac{1}{9}(4 - t^2), & 1 < t < 2, \\ 0, & \text{其他}. \end{cases}$

34. (1) $F_T(t) = \begin{cases} 1 - \mathrm{e}^{-\lambda t}, & t > 0, \\ 0, & t \leqslant 0; \end{cases}$ (2) $\mathrm{e}^{-\lambda t}$.

35. $f_Y(y) = \begin{cases} \dfrac{1}{n}y^{\frac{1}{n}-1}, & 0 < y < 1, \\ 0, & \text{其他}. \end{cases}$

36. $F_Y(y) = \begin{cases} 0, & y < -1, \\ \dfrac{4(\pi - \arccos y)}{3\pi}, & -1 \leqslant y < 0, \\ 1 - \dfrac{2\arccos y}{3\pi}, & 0 \leqslant y < 1, \\ 1, & y \geqslant 1. \end{cases}$

37. $f_Y(y) = \begin{cases} \dfrac{1}{2\sqrt{2\pi y}\sigma}\left[\mathrm{e}^{-(\sqrt{y}-\mu)^2/(2\sigma^2)} + \mathrm{e}^{-(\sqrt{y}+\mu)^2/(2\sigma^2)}\right], & y > 0, \\ 0, & \text{其他}. \end{cases}$

38. (1) $a = \dfrac{1}{3}, b = \dfrac{1}{6}$; (2) $f_Y(y) = \begin{cases} \dfrac{y(2y^2 + 1)}{3}, & 0 < y < \sqrt{2}, \\ 0, & \text{其他}. \end{cases}$

39. (1) $f_Y(y) = \begin{cases} \dfrac{1}{\sqrt{2\pi}y}\mathrm{e}^{-(\ln y)^2/2}, & y > 0, \\ 0, & y \leqslant 0; \end{cases}$ (2) $f_Z(y) = \sqrt{\dfrac{2}{\pi}}\mathrm{e}^{y - \frac{\mathrm{e}^{2y}}{2}}, |y| < +\infty$.

思考题三

1. 联合分布可以决定边际分布, 而边际分布不能决定联合分布.
2. (3) 正确.
3. 不对.
4. (2) 正确.
5. 第一个说法对, 第二个说法不对.

习题三

1. $P\{X=2,Y=4\}=P\{X=4,Y=2\}=\dfrac{6}{25}, P\{X=3,Y=3\}=\dfrac{13}{25};$
   $P\{X=2\}=P\{X=4\}=\dfrac{6}{25}, P\{X=3\}=\dfrac{13}{25}.$
2. $a=0.3, b=0.2.$
3. $a=c=0.2, b=0.3,$

X	1	2
p	0.6	0.4

Y	-1	0	1
p	0.3	0.2	0.5

4. (1)

		Y		
		0	1	2
X	0	0.2	0.1	0.1
	1	0	0.4	0.2

(2) $P\{Y=k|X=0\}=\begin{cases}\dfrac{1}{2}, & k=0,\\ \dfrac{1}{4}, & k=1,\\ \dfrac{1}{4}, & k=2.\end{cases}$

5. (1)

		Y						$P\{X=i\}$
		1	2	3	4	5	6	
X	1	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
	2	0	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
	3	0	0	$\frac{3}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
	4	0	0	0	$\frac{4}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
	5	0	0	0	0	$\frac{5}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
	6	0	0	0	0	0	$\frac{6}{36}$	$\frac{1}{6}$
$P\{Y=j\}$		$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$	

(2)

X	1	2	3	4	5	6
$P\{X=k\|Y=6\}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{6}{11}$

6. (1)

		Y		
		1	2	3
X	0	$\frac{1}{15}$	$\frac{11}{30}$	$\frac{1}{15}$
	1	$\frac{7}{18}$	$\frac{1}{18}$	$\frac{1}{18}$

(2)

Y	1	2	3
p	$\frac{41}{90}$	$\frac{38}{90}$	$\frac{11}{90}$

(3) $P\{X=k|Y=1\}=\begin{cases}\dfrac{6}{41}, & k=0,\\ \dfrac{35}{41}, & k=1.\end{cases}$

7. (1) $P\{X=i,Y=j\}=\dfrac{\mathrm{e}^{-\lambda}\lambda^i}{i!}\cdot \mathrm{C}_i^j(0.1)^j(0.9)^{i-j}, i=0,1,2,\cdots,j=0,1,2,\cdots,i;$

(2) $P\{Y=j\}=\dfrac{\mathrm{e}^{-0.1\lambda}(0.1\lambda)^j}{j!}, j=0,1,2,\cdots.$

8. (1)

		Y		
		0	a	$2a$
X	0	0.6	0	0
	1	$0.3(1-p)$	$0.3p$	0
	2	$0.1(1-p)^2$	$0.2p(1-p)$	$0.1p^2$

(2) $P\{Y=t|X=1\}=\begin{cases}1-p, & t=0,\\ p, & t=a.\end{cases}$

9. (1)

		Y	
		0	1
X	1	0.1	0.2
	2	0.3	0.4

(2) $F_{X|Y}(x|0)=\begin{cases}0, & x<1,\\ 0.25, & 1\leqslant x<2,\\ 1, & x\geqslant 2.\end{cases}$

10. (1)

		Y	
		0	1
X	0	0.35	0.35
	1	0.25	0.05

(2) $F_X(x)=\begin{cases}0, & x<0,\\ 0.7, & 0\leqslant x<1,\\ 1, & x\geqslant 1;\end{cases}$

(3) $F_{Y|X}(y|1)=\begin{cases}0, & y<0,\\ \dfrac{5}{6}, & 0\leqslant y<1,\\ 1 & y\geqslant 1.\end{cases}$

11. $F(x,y)=\begin{cases}0, & x<0 \text{ 或 } y<0,\\ 0.1+0.8xy, & 0\leqslant x<1, 0\leqslant y<1,\\ 0.1+0.8x, & 0\leqslant x<1, y\geqslant 1,\\ 0.1+0.8y, & x\geqslant 1, 0\leqslant y<1,\\ 1, & x\geqslant 1, y\geqslant 1.\end{cases}$

12. (1) $c=6$; (2) 0.5; (3) $\dfrac{7}{8}$.

13. (1) $f_X(x)=\begin{cases}2x, & 0<x<1,\\ 0, & \text{其他},\end{cases}$ $f_Y(y)=\begin{cases}\dfrac{1}{2}, & 0<y<2,\\ 0, & \text{其他};\end{cases}$

(2) $\dfrac{2}{3}$.

14. (1) $c=3$;

(2) $f_X(x)=\begin{cases}6(x-1)(2-x), & 1<x<2,\\ 0, & \text{其他},\end{cases}$ $f_Y(y)=\begin{cases}\dfrac{3(y-1)^2}{2}, & 1<y\leqslant 2,\\ \dfrac{3(3-y)^2}{2}, & 2<y<3,\\ 0, & \text{其他}.\end{cases}$

15. (1) $f_X(x)=\begin{cases}x\mathrm{e}^{-x}, & x>0,\\ 0, & x\leqslant 0,\end{cases}$ $f_Y(y)=\begin{cases}\mathrm{e}^{-y}, & y>0,\\ 0, & y\leqslant 0;\end{cases}$

(2) 当 $x>0$ 时, $f_{Y|X}(y|x)=\begin{cases}\dfrac{1}{x}, & 0<y<x,\\ 0, & \text{其他};\end{cases}$

(3) 当 $\{X=x\}$ 时, Y 的条件分布为区间 $(0,x)$ 上均匀分布.

16. (1) $f(x,y)=\begin{cases}\lambda^2\mathrm{e}^{-\lambda x}\mathrm{e}^{-y/x}, & x>0, y>0,\\ 0, & \text{其他};\end{cases}$

(2) 当 $x>0$ 时, $F_{Y|X}(y|x)=\begin{cases}1-\mathrm{e}^{-y/x}, & y>0,\\ 0, & y\leqslant 0;\end{cases}$

(3) e^{-1}.

17. (1) $f_Y(y)=\begin{cases}\dfrac{5(1-y^4)}{8}, & |y|<1,\\ 0, & \text{其他};\end{cases}$

(2) 当 $|y|<1$ 时, $f_{X|Y}(x|y)=\begin{cases}\dfrac{2x}{1-y^4}, & y^2<x<1,\\ 0, & \text{其他};\end{cases}$

(3) 0.8.

18. (1) $f(x,y)=\begin{cases}\dfrac{1}{1-x}, & 0<x<y<1,\\ 0, & \text{其他};\end{cases}$

(2) 当 $0<y<1$ 时, $f_{X|Y}(x|y)=\begin{cases}\dfrac{-1}{(1-x)\ln(1-y)}, & 0<x<y,\\ 0, & \text{其他}.\end{cases}$

19. (1) $f(x,y)=\begin{cases}\dfrac{2(4-y)}{(3-x)^2}, & 1<x<2, x+1<y<4,\\ 0 & \text{其他},\end{cases}$ $P\{Y<3\}=\dfrac{1}{2}$;

(2) $f_Y(y)=\begin{cases}y-2, & 2<y<3,\\ 4-y, & 3<y<4,\\ 0 & \text{其他};\end{cases}$

(3) $\dfrac{1}{3}$.

20. $f_Z(t)=\begin{cases}\dfrac{2(m-t)}{m^2}, & 0<t<m,\\ 0, & \text{其他}.\end{cases}$

21. (1) $f_X(x)=\begin{cases}\dfrac{4\sqrt{1-x^2}}{\pi}, & 0<x<1,\\ 0, & \text{其他};\end{cases}$

(2) $\dfrac{1}{3}+\dfrac{\sqrt{3}}{2\pi}$;

(3) X 与 Y 不独立, 因为 $f(x,y)\neq f_X(x)\cdot f_Y(y), (x,y)\in D$.

22. (1) $f_X(x)=\dfrac{1}{\sqrt{2\pi}}\mathrm{e}^{-x^2/2}, |x|<+\infty, f_Y(y)=\dfrac{1}{2\sqrt{\pi}}\mathrm{e}^{-(y-1)^2/4}, |y|<+\infty$;

(2) $f_{Y|X}(y|0)=\dfrac{1}{\sqrt{3\pi}}\mathrm{e}^{-(y-1)^2/3}, |y|<+\infty$;

(3) 0.5.

23. (1) $f_X(x)=\dfrac{1}{\sqrt{2\pi}}\mathrm{e}^{-x^2/2}, |x|<+\infty, f_Y(y)=\dfrac{1}{\sqrt{2\pi}}\mathrm{e}^{-y^2/2}, |y|<+\infty$;

(2) X, Y 相互独立.

24. $F_T(t)=\begin{cases}1-2\mathrm{e}^{-2\lambda t}+\mathrm{e}^{-3\lambda t}, & t>0,\\ 0, & t\leqslant 0,\end{cases}$

$f_T(t)=\begin{cases}4\lambda\mathrm{e}^{-2\lambda t}-3\lambda\mathrm{e}^{-3\lambda t}, & t>0,\\ 0, & t\leqslant 0.\end{cases}$

25. (1) $Z \sim B(n,p), P\{Z=k\} = \mathrm{C}_n^k p^k(1-p)^{n-k}, k=0,1,2,\cdots,n;$

(2) $W \sim B(m+n,p), P\{W=k\} = \mathrm{C}_{m+n}^k p^k(1-p)^{m+n-k}, k=0,1,2,\cdots,m+n.$

26. $f_Z(t) = \dfrac{1}{2a}\left[\Phi\left(\dfrac{t+a-\mu}{\sigma}\right) - \Phi\left(\dfrac{t-a-\mu}{\sigma}\right)\right], |t| < +\infty.$

27. $$f_Z(t) = \begin{cases} \dfrac{t(3-t)}{3}, & 0 < t \leqslant 1, \\ \dfrac{3-t}{3}, & 1 < t \leqslant 2, \\ \dfrac{(3-t)^2}{3}, & 2 < t \leqslant 3, \\ 0 & \text{其他}. \end{cases}$$

28. $f_Z(t) = 0.5f(t) + 0.3f(t-1\ 000) + 0.2f(t-5\ 000).$

29. (1) $\dfrac{3}{8}$;

(2) $$F_Z(z) = \begin{cases} 0, & z < 48, \\ \dfrac{z}{32} - \dfrac{3}{2}, & 48 \leqslant z < 60, \\ \dfrac{9z}{160} - 3, & 60 \leqslant z < 64, \\ \dfrac{z}{40} - 1, & 64 \leqslant z < 80, \\ 1, & z \geqslant 80. \end{cases}$$

30. (1) $1 - \mathrm{e}^{-10\lambda} - 10\lambda\mathrm{e}^{-10\lambda}$;

(2) $1 - \mathrm{e}^{-10\lambda}(1+\lambda)^{10}$;

(3) $1 - \dfrac{\mathrm{e}^{-10\lambda}[(1+\lambda)^{10} - \lambda^{10}]}{1-(1-\mathrm{e}^{-\lambda})^{10}}$.

31.

Z	1	2	3	4	5
p	0.04	0.14	0.30	0.32	0.20

M	1	2	3
p	0.10	0.50	0.40

N	0	1	2
p	0.20	0.40	0.40

32. (1) $f_T(t) = \begin{cases} (\lambda_1+\lambda_2)\mathrm{e}^{-(\lambda_1+\lambda_2)t}, & t > 0, \\ 0, & t \leqslant 0; \end{cases}$

(2) $f_T(t) = \begin{cases} \lambda_1\mathrm{e}^{-\lambda_1 t} + \lambda_2\mathrm{e}^{-\lambda_2 t} - (\lambda_1+\lambda_2)\mathrm{e}^{-(\lambda_1+\lambda_2)t}, & t > 0, \\ 0, & t \leqslant 0; \end{cases}$

(3) 当 $\lambda_1 \neq \lambda_2$ 时, $f_T(t) = \begin{cases} \dfrac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}[\mathrm{e}^{-\lambda_1 t} - \mathrm{e}^{-\lambda_2 t}], & t > 0, \\ 0, & t \leqslant 0, \end{cases}$

当 $\lambda_1 = \lambda_2 = \lambda$ 时, $f_T(t) = \begin{cases} \lambda^2 t\mathrm{e}^{-\lambda t}, & t > 0, \\ 0, & t \leqslant 0. \end{cases}$

33. $f_Z(t) = \begin{cases} \dfrac{1}{2} - \dfrac{t}{8}, & 0 < t \leqslant 4, \\ 0, & \text{其他}. \end{cases}$

34. (1) $P\{W=k\}=\mathrm{C}_n^k p^k(1-p)^{n-k}, k=0,1,2,\cdots,n;$

(2)

		Z	
		0	1
X	0	$(1-p)^2$	$p(1-p)$
	1	p^2	$p(1-p)$

35. $f_M(s)=\begin{cases}3s^2, & 0<s<1,\\ 0, & \text{其他},\end{cases}\qquad f_N(t)=\begin{cases}1+2t-3t^2, & 0<t<1,\\ 0, & \text{其他}.\end{cases}$

思考题四

1. 不对. 随机变量 X 的数学期望按定义应该是

$$\begin{aligned}E(X)&=\int_{-\infty}^{+\infty}x\cdot f(x)\mathrm{d}x\\&=\int_{-\infty}^{-1}x\cdot 0\mathrm{d}x+\int_{-1}^{0}x\cdot(1+x)\mathrm{d}x+\int_{0}^{1}x\cdot(1-x)\mathrm{d}x+\int_{1}^{+\infty}x\cdot 0\mathrm{d}x.\end{aligned}$$

2. 随机变量 X 与 Y 同分布, 那么它们的任意阶矩 (如果存在) 全部相等. 反之, 若有 $E(X)=E(Y)$ 且 $\mathrm{Var}(X)=\mathrm{Var}(Y)$, 不能推出随机变量 X 与 Y 分布一定相同. 反例, 当 $X\sim P(1), Y\sim N(1,1)$ 时, $E(X)=E(Y)=1$ 且 $\mathrm{Var}(X)=\mathrm{Var}(Y)=1$, 但显然两者的分布不一样.

3. 方差是 2×2.5^2.

4. 两个随机变量如果相互独立则它们一定不相关, 反之则不然.

5. (1) 对于 $n\geqslant 1$, 有 $E\left(\sum\limits_{i=1}^n X_i\right)=\sum\limits_{i=1}^n E(X_i)$ 成立, 但 $\mathrm{Var}\left(\sum\limits_{i=1}^n X_i\right)=\sum\limits_{i=1}^n \mathrm{Var}(X_i)$ 不一定成立, 因为

$$\mathrm{Var}\left(\sum_{i=1}^n X_i\right)=\sum_{i=1}^n \mathrm{Var}(X_i)+\sum_{i\neq j}\mathrm{Cov}(X_i,X_j).$$

且只有当 $\{X_i, i\geqslant 1\}$ 两两不相关时, $\mathrm{Var}\left(\sum\limits_{i=1}^n X_i\right)=\sum\limits_{i=1}^n \mathrm{Var}(X_i)$ 才成立;

(2) 若 $\{X_i, i\geqslant 1\}$ 相互独立, 那么对于 $n\geqslant 1$, 有 $E\left(\prod\limits_{i=1}^n X_i\right)=\prod\limits_{i=1}^n E(X_i)$ 成立, 但 $\mathrm{Var}\left(\prod\limits_{i=1}^n X_i\right)=\prod\limits_{i=1}^n \mathrm{Var}(X_i)$ 不一定成立, 仅知

$$\mathrm{Var}\left(\prod_{i=1}^n X_i\right)=E\left(\prod_{i=1}^n X_i^2\right)-\left(E\left(\prod_{i=1}^n X_i\right)\right)^2=\prod_{i=1}^n E(X_i^2)-\prod_{i=1}^n (E(X_i))^2.$$

6. 错. 应为 $\mathrm{Var}(X-2Y)=\mathrm{Var}(X)+(-2)^2\mathrm{Var}(Y)+2\mathrm{Cov}(X,-2Y)=5-4\mathrm{Cov}(X,Y)$.

7. 错. 应根据定理 4.1.1 来计算,

$$E\left(\frac{1}{X}\right)=\int_1^3\frac{1}{x}\cdot\frac{1}{2}\mathrm{d}x=\frac{1}{2}\cdot(\ln 3-\ln 1)=\ln\sqrt{3}.$$