1.

1. The first cubic is $\theta_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$ $(0 \le t \le 3)$

and the second is $\theta_2(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3$ $(3 \le t \le 5)$

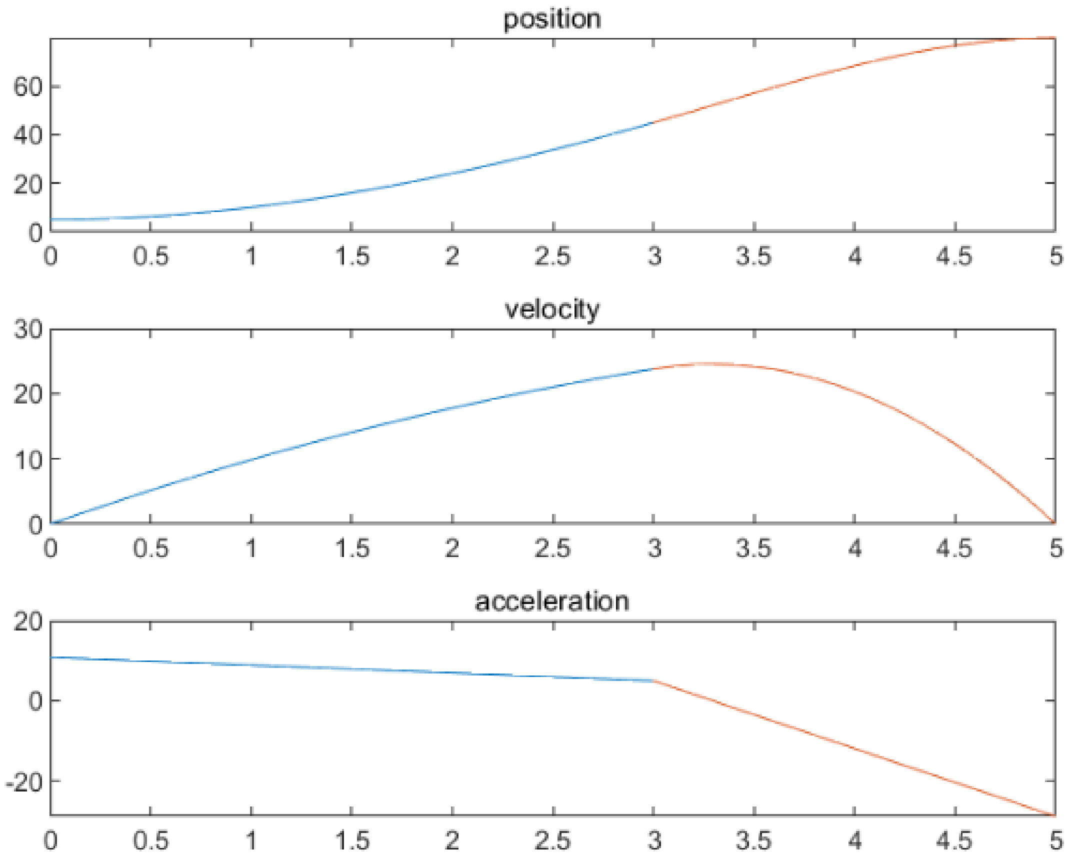
The constraints are:

$\theta_1(0) = 5^\circ$. $\dot\theta_1(0) = 0$. $\theta_1(3) = 45^\circ$

$\theta_2(3) = 45^\circ$. $\theta_2(5) = 80^\circ$. $\dot\theta_2(5) = 0$.

$\dot\theta_1(3) = \dot\theta_2(3)$. $\ddot\theta_1(3) = \ddot\theta_2(3)$

$$\Rightarrow \theta(t) = \begin{cases} 5 + 5.4167t^2 - 0.3241t^3 & (0 \le t \le 3) \\ \cancel{45 + 23.75t + 2.5t^2 - 2.8125t^3} & \\ 72.1875 - 67.1875t + \cancel{27.1875}\ 27.8125t^2 - 2.8125t^3 & (3 \le t \le 5) \\ \text{or } 45 + 23.75(t-3) + 2.5(t-3)^2 - 2.8125(t-3)^3 & \end{cases}$$



2.

2. $\theta_0 = 0^\circ \quad \theta_1 = 35^\circ - 0.5\times 5 = 32.5^\circ \quad \theta_2 = 35^\circ + 0.5\times 5 = 37.5^\circ \quad \theta_3 = 85^\circ$

$$\ddot{\theta}_0 = \mathrm{SGN}(\theta_1 - \theta_0)\,|\ddot{\theta}_0| = 80^\circ/s^2$$

$$\frac{\theta_1 - \theta_0}{t_{d01} - \frac{1}{2}t_0} = \ddot{\theta}_0 t_0 \Rightarrow \frac{32.5}{2.5 - \frac{1}{2}t_0} = 80 t_0 \Rightarrow t_0 = 0.168\ s$$

$$\dot{\theta}_{01} = \frac{\theta_1 - \theta_0}{t_{d01} - \frac{1}{2}t_0} = 13.452^\circ/s$$

$$\ddot{\theta}_1 = \mathrm{SGN}(\dot{\theta}_{12} - \dot{\theta}_{01})\,|\ddot{\theta}_1| = -80^\circ/s^2 \quad t_1 = \frac{\dot{\theta}_{12} - \dot{\theta}_{01}}{\ddot{\theta}_1} = \frac{5 - 13.452}{-80} = 0.10565\ s$$

$$t_{01} = t_{d01} - t_0 - \frac{1}{2}t_1 = 2.5 - 0.163 - \frac{1}{2}\times 0.10565 = 2.279\ s$$

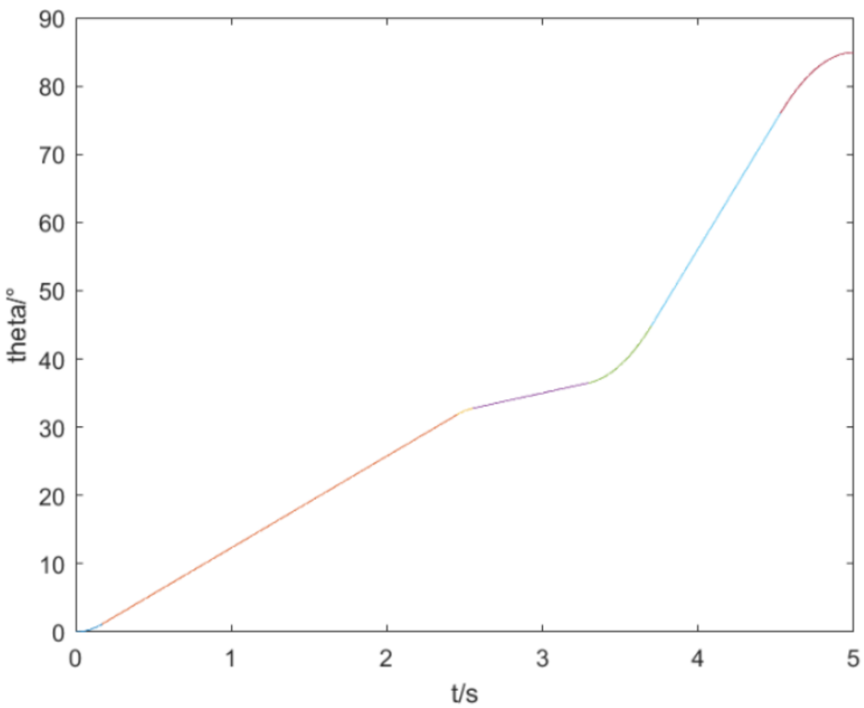
$$\ddot{\theta}_3 = \mathrm{SGN}(\theta_2 - \theta_3)\,|\ddot{\theta}_3| = -80^\circ/s^2$$

$$\frac{\theta_2 - \theta_3}{t_{d23} - \frac{1}{2}t_3} = \ddot{\theta}_3 t_3 \Rightarrow \frac{37.5 - 85^\circ}{1.5 - \frac{1}{2}t_3} = -80 t_3 \Rightarrow t_3 = 0.469\ s$$

$$\dot{\theta}_{23} = \frac{\theta_3 - \theta_2}{t_{d23} - \frac{1}{2}t_3} = 37.535^\circ/s$$

$$\ddot{\theta}_2 = \mathrm{SGN}(\dot{\theta}_{23} - \dot{\theta}_{12})\,|\ddot{\theta}_2| = 80^\circ/s^2 \quad t_2 = \frac{\dot{\theta}_{23} - \dot{\theta}_{12}}{\ddot{\theta}_2} = \frac{37.535 - 5}{80} = 0.4067\ s$$

$$t_{23} = t_{d23} - t_3 - \frac{1}{2}t_2 = 0.8277\ s \quad t_{12} = t_{d12} - \frac{1}{2}t_1 - \frac{1}{2}t_2 = 0.7438\ s$$



3.

3. SLerp: $q = \frac{\sin((1-t)\theta)}{\sin\theta} q_0 + \frac{\sin(t\theta)}{\sin\theta} q_1$, where $\theta = \cos^{-1}(q_0 \cdot q_1)$. $t \in [0,1]$

We can replace t with $f(t)$. To generate continuous angular velocity, we adopt the cubic curve here. Other curves which yield continuous angular velocity can also be used.

Assume that $f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$. The constraints are:

$$\begin{cases} f(0) = 0 \\ f(1) = 1 \\ f'(0) = f'(1) = 0 \end{cases} \Rightarrow a_0 = a_1 = 0 ,\ a_2 = 3 ,\ a_3 = -2$$

$\therefore f(t) = 3t^2 - 2t^3$

$$q = \frac{\sin((1-f(t))\theta)}{\sin\theta} q_0 + \frac{\sin(f(t)\theta)}{\sin\theta} q_1$$

$$= \frac{\sin(\theta - 3\theta t^2 + 2\theta t^3)}{\sin\theta} q_0 + \frac{\sin(3\theta t^2 - 2\theta t^3)}{\sin\theta} q_1$$